Dispersive analysis of excited glueball states

Hsiang-nan Li, Academia Sinica

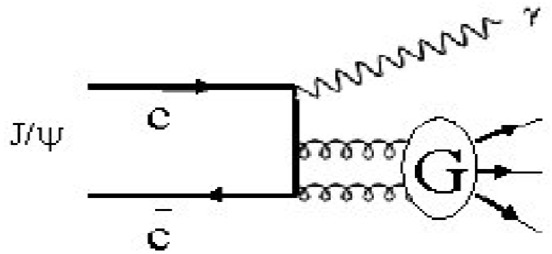
At TQCD Meeting

Sep. 27, 2024

Li, 2408.06738

Glueballs

- Quest for glueballs lasted for decades
- Quenched Lattice QCD (LQCD), sum rules (SR) gave scalar glueball mass 1.5-1.7 GeV (Chen et al. 06, Narison 98)
- Large $B(J/\psi \to \gamma f_0(1710)) \approx 10^{-3}$ supports f0(1710) as a candidate
- Quenched LQCD, SR gave pseudoscalar glueball mass > 2 GeV (Morningstar, Peardon 99; Narison 98)
- No candidate of large mass before 2023



- Quantum numbers 0-+ of X(2370) determined by BESIII (PRL 132, 181901 (2024)); BR of J/psi radiative decay ~ 10E-4
- X(2370) claimed to be lightest pseudoscalar glueball, but LQCD reliable for pseudoscalar glueball with axial anomaly?

Our postulation

- It is imperative to investigate this subject in a different approach and find out whether alternative aspects exist
- We developed dispersive approach, improved version of QCD SR, with great phenomenological success recently
- Predicted lightest scalar (pseudoscalar) glueball to be admixture of f0(1370), f0(1500) and f0(1710) (eta(1760))
- f0(500) (admixture of eta and eta') contains small glue content
- Extended to excited rho mesons, establish rho(1450), rho(1700)
- Postulate f_0(2200) (X(2370)) as first excited state of scalar (pseudiscalar) glueball

Formalism

Contour integration

- Two-current correlator

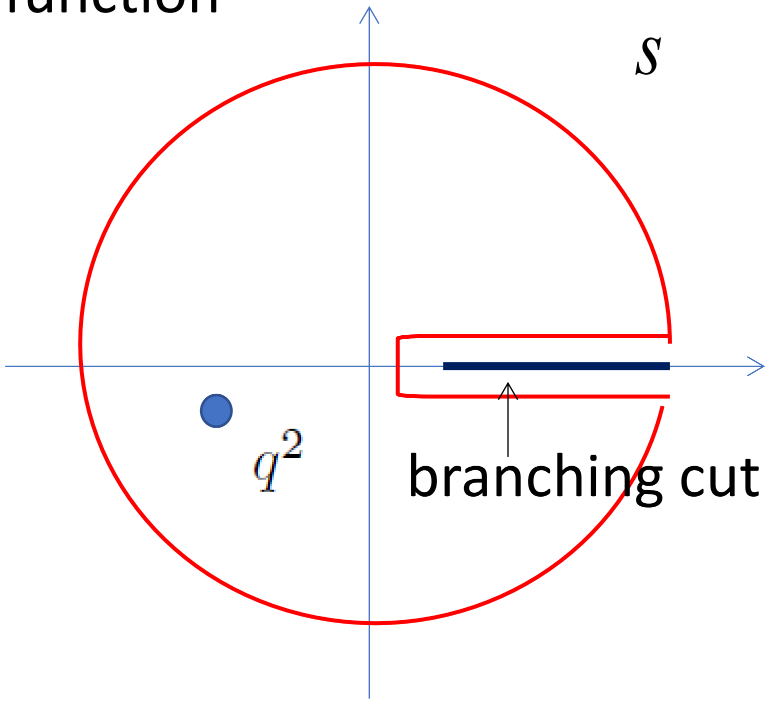
$$J_\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/\sqrt{2}.$$

$$\Pi_{\mu\nu}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T[J_\mu(x)J_\nu(0)]|0\rangle$$
$$= (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2)$$

← vacuum polarization function

- Identity from contour integration

$$\Pi(q^2) = \frac{1}{2\pi i}\oint ds \frac{\Pi(s)}{s-q^2}$$



Quark side

- Correlator at large q^2 (deep Euclidean region)
- Operator product expansion (OPE) reliable

parameter characterizing factorization breakdown

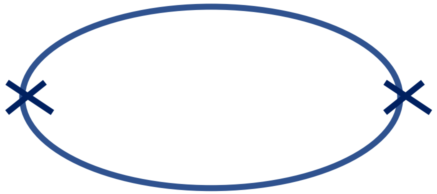
higher order

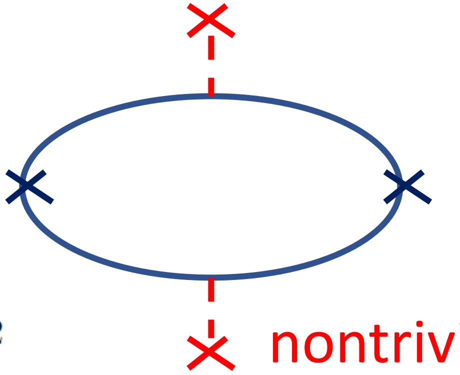
higher powers

$$\Pi^{\mathrm{OPE}}(q^2) = \frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\frac{\mu^2}{-q^2} + \frac{1}{12\pi}\frac{\langle\alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q\bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{(q^2)^3}$$

perturbative piece

4-quark condensate factorized into product of 2-quark condensates





nontrivial vacuum

$$\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\frac{\mu^2}{-q^2} \equiv c\ln\frac{\mu^2}{-q^2}$$

Hadron side

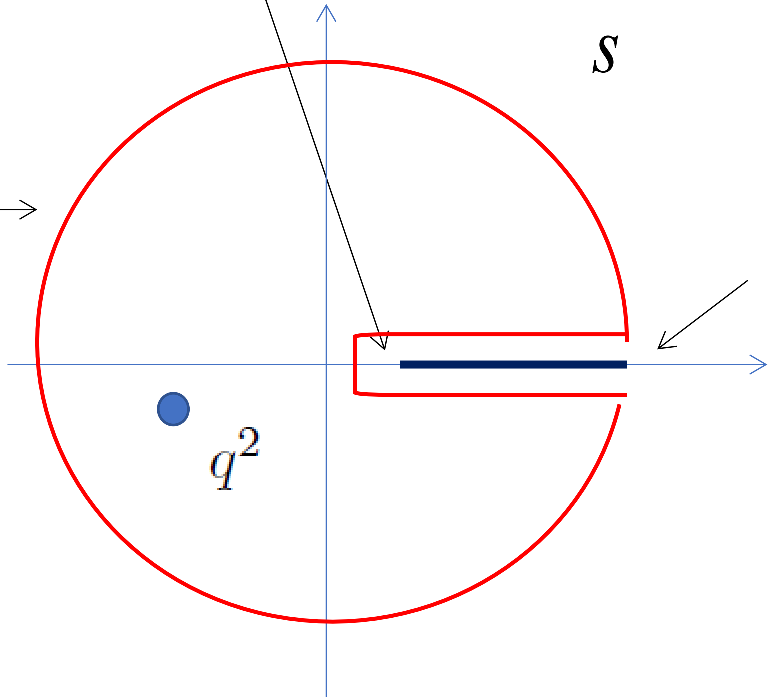
Dispersive integral

nonperturbative spectral function

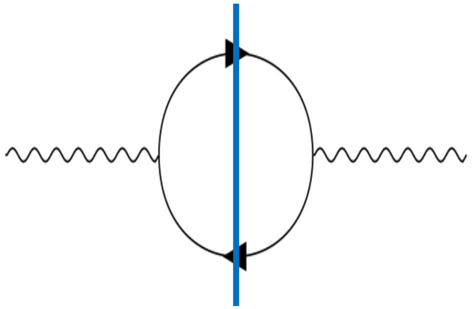
perturbative result

$$\frac{1}{2\pi i}\oint ds\frac{\Pi(s)}{s-q^2}=\frac{1}{\pi}\int_0^R ds\frac{\mathrm{Im}\Pi(s)}{s-q^2}+\frac{1}{2\pi i}\int_C ds\frac{\Pi^{\mathrm{pert}}(s)}{s-q^2}$$

contribution from large circle C of radius R will be cancelled



branch cut caused by real intermediate states due to time-like $q^2>0$ (log term)



Dispersion relation

- Rewrite pert piece as contour integral

$$\Pi^{\mathrm{OPE}}(q^2) = \frac{1}{2\pi i}\oint ds \frac{\Pi^{\mathrm{pert}}(s)}{s-q^2} + \frac{1}{12\pi}\frac{\langle \alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q \bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{(q^2)^3}$$

due to analyticity of perturbation theory

- Equality of two sides gives dispersion relation
- Contributions from big circles cancel, and unknown spectral function from branch cuts remains

arbitrary radius

$$\frac{1}{\pi}\int_0^R ds \frac{\mathrm{Im}\Pi(s)}{s-q^2} = \frac{1}{\pi}\int_0^R ds \frac{\mathrm{Im}\Pi^{\mathrm{pert}}(s)}{s-q^2} + \frac{1}{12\pi}\frac{\langle \alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q \bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{(q^2)^3}$$

UV subtraction

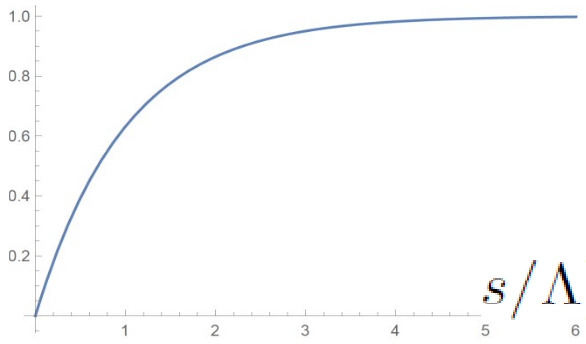
- Subtracted spectral function

arbitrary R turned into arbitrary scale

$$\Delta\rho(s,\Lambda) = \rho(s) - \frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{pert}}(s)[1-\exp(-s/\Lambda)]$$

- Maintain low-energy behavior $\rho(s) \sim s$ at $s \to 0$
- Bear resonance structure the same as $\rho(s)$
- Circle radius R can be pushed to infinity

Kwon et al 2008



$$\int_0^\infty ds \frac{\Delta\rho(s,\Lambda)}{s-q^2} = \int_0^\infty ds \frac{ce^{-s/\Lambda}}{s-q^2} + \frac{1}{12\pi}\frac{\langle\alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q\bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{(q^2)^3}$$

- No duality assumed at any finite s

Fredholm equation of the 1st kind

Weakness of sum rules

- Presume existence of ground state, parametrized as pole
- How to handle excited-state contribution?
- Rely on parametrization, quark-hadron duality

$$\mathrm{Im}\Pi(q^2) = \pi f_V^2 \delta(q^2 - m_V^2) + \mathrm{Im}\Pi^{\mathrm{pert}}(q^2)\theta(q^2 - s_0)$$

observables: decay constant, mass

continuum threshold

- Duality may fail

equivalent to q, introduced by Borel transform

- Stability in unphysical Borel mass?
- Usually not; rely on discretionary prescription; tune s0 to make 70% (30%) perturbative (nonperturbative) contribution

Sum rules as inverse problem

- Once dispersion relation is solved directly,
- Existence of a resonance not presumed
- No need to parametrize spectral function
- Free continuum threshold absent
- Quark-hadron duality not implemented
- Borel transformation not required
- Discretionary prescription not necessary
- Rely only on analyticity
- Precision of predictions enhanced systematically by adding higher-order and higher-power corrections to OPE inputs.

Ground-state solutions

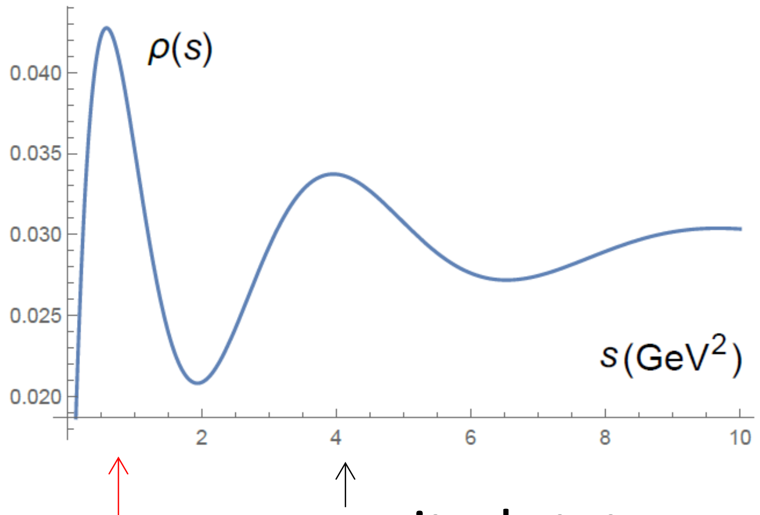
Set aside technical detail of solving the integral equation $\int_0^\infty dy \frac{\rho(y)}{x-y} = \omega(x)$

rho meson spectral function

- OPE input known in the literature

$$\langle m_q \bar{q}q \rangle = 0.007 \times (-0.246)^3 \ \text{GeV}^4, \ \ \langle \alpha_s G^2 \rangle = 0.08 \ \text{GeV}^4$$
$$\alpha_s \langle \bar{q}q \rangle^2 = 1.49 \times 10^{-4} \ \text{GeV}^6, \ \ \alpha_s = 0.5, \ \ \kappa = 2.5.$$

solution of
spectral function



local duality violation

excited states

rho meson peak emerges !

rho meson mass

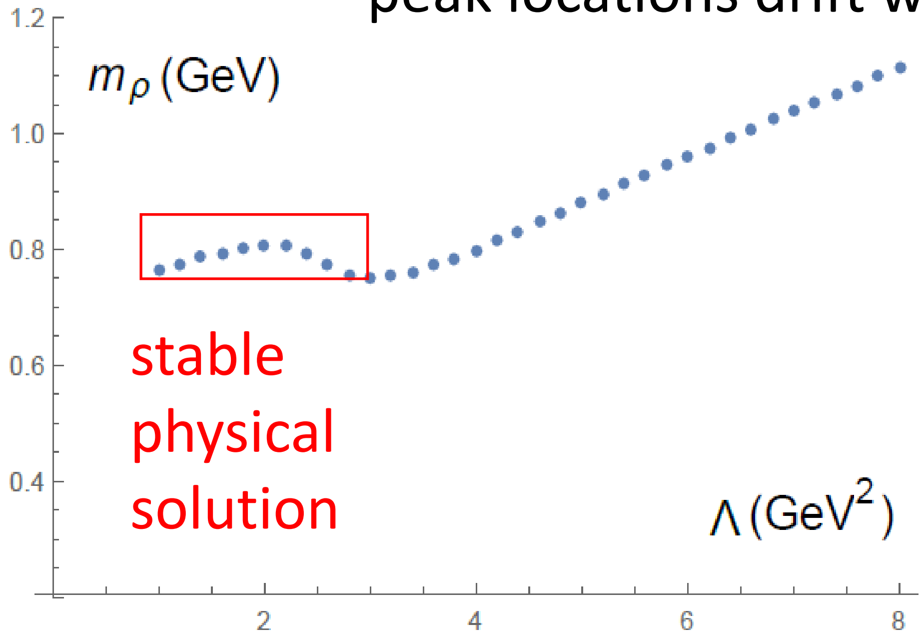
- Vary Λ , find peak location
- Physical solution insensitive to Λ
- Tiny error, stable solution

$$m_\rho = (0.77 \pm 0.02)\ \text{GeV}$$

- Including condensate variation

$$m_\rho = (0.77 \pm 0.04)\ \text{GeV}$$

scaling (unphysical) region
peak locations drift with Λ



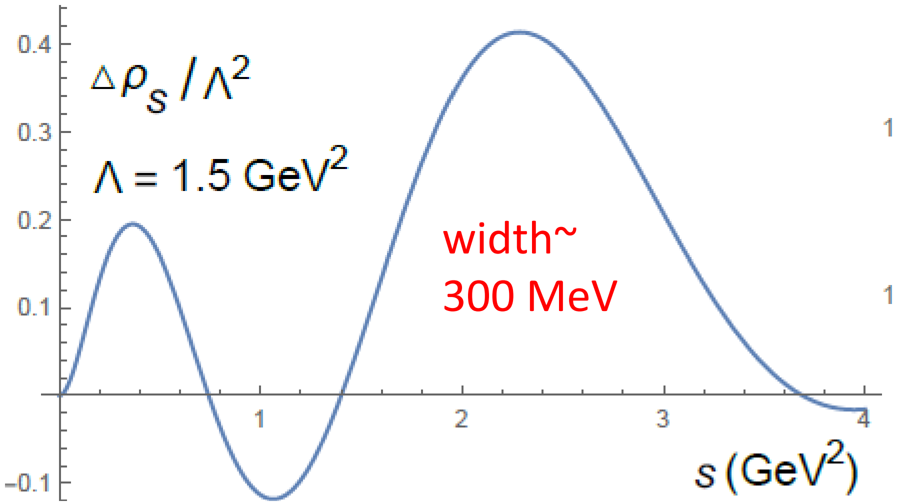
- Variable changes $x = q^2/\Lambda \quad y = s/\Lambda$

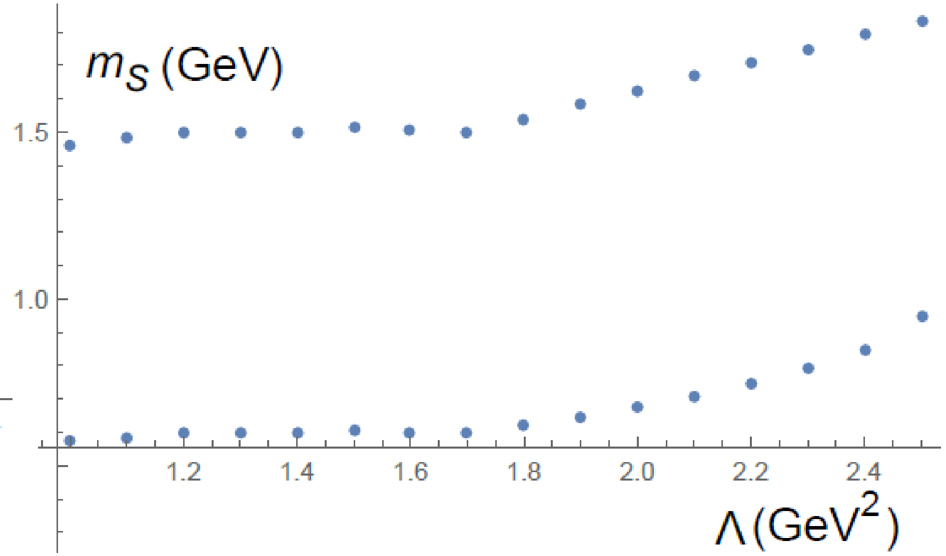
$$\int_0^\infty dy \frac{\Delta\rho(y)}{x-y} = \int_0^\infty dy \frac{ce^{-y}}{x-y} - \frac{1}{12\pi}\frac{\langle \alpha_s G^2\rangle}{x^2\Lambda^2} - 2\frac{\langle m_q \bar{q}q\rangle}{x^2\Lambda^2} - \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{x^3\Lambda^3}$$

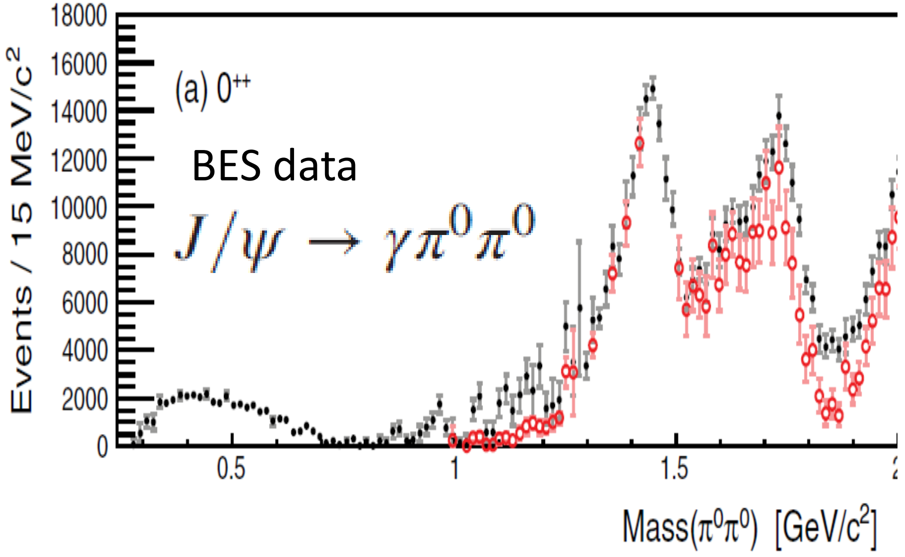
- Scaling behavior due to disappearance of power corrections at high Λ

Scalar glueball mass

subtracted spectral function, cannot resolve fine structure







$$m_{S_1} = (0.60 \pm 0.01)\ \text{GeV} \rightarrow f_0(500)$$

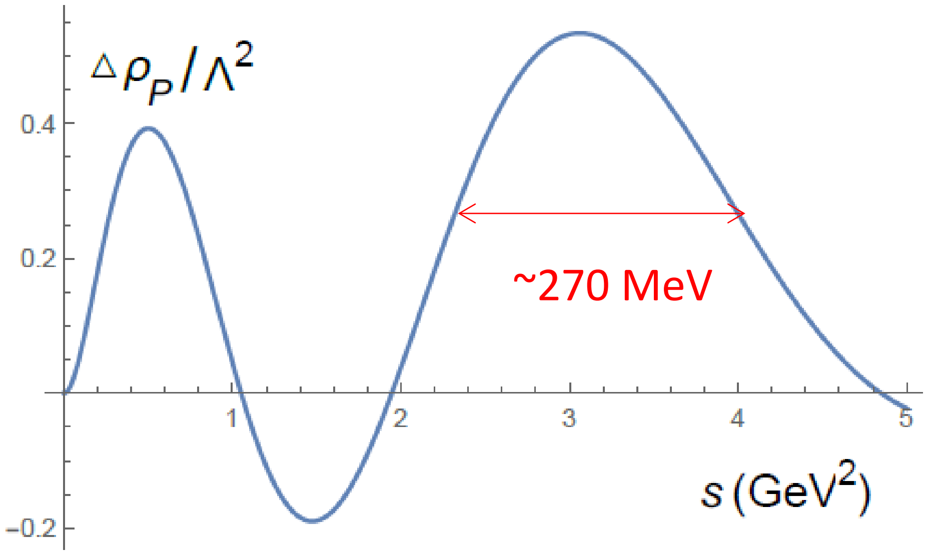
Breit-Wigner mass 400-800 MeV

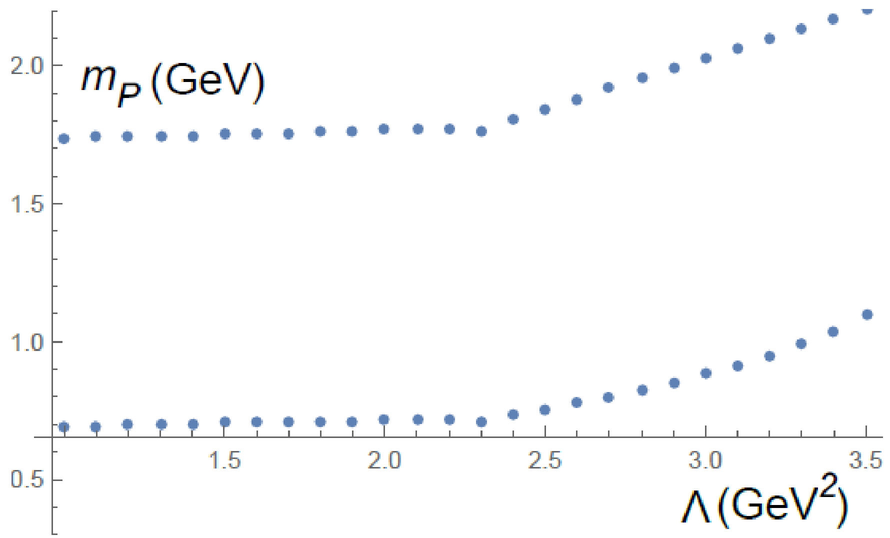
$$m_S = (1.53 \pm 0.02)\ \text{GeV}$$

$$f_0(1370),\ f_0(1500)\ \ f_0(1710)$$

width~112 MeV

Pseudoscalar glueball mass





$m_{P_1} = (0.71 \pm 0.02)\ \mathrm{GeV} \rightarrow \eta, \eta'$ $\quad$ $m_P = (1.75 \pm 0.02)\ \mathrm{GeV} \rightarrow \eta(1760)$

width ~ 240 MeV

$J/\psi \rightarrow \gamma(\eta(1760) \rightarrow)\omega\omega$ BR~10E-3

- $\eta(1760)$ proposed by Page, XQ Li in 1998
- Quenched LQCD gave 2.6 GeV in 1999
- $X(1835)$ BR ~ 10E-4, but seen in $J/\psi \rightarrow \gamma\gamma\phi$ unlikely

BESIII, 2018

Excited-state solutions

Idea

- To access excited state, ground-state contribution must be deducted from correlator, i.e., from spectral function to suppress interference
- Parametrize rho(770) contribution as delta-function $F_0\delta(s-m_\rho^2)$

$$F_0=\int_0^\infty ds\Delta\rho_0(s,\Lambda)=0.22\ \mathrm{GeV}^2$$

- Subtract it from two sides of dispersion relation

unknown

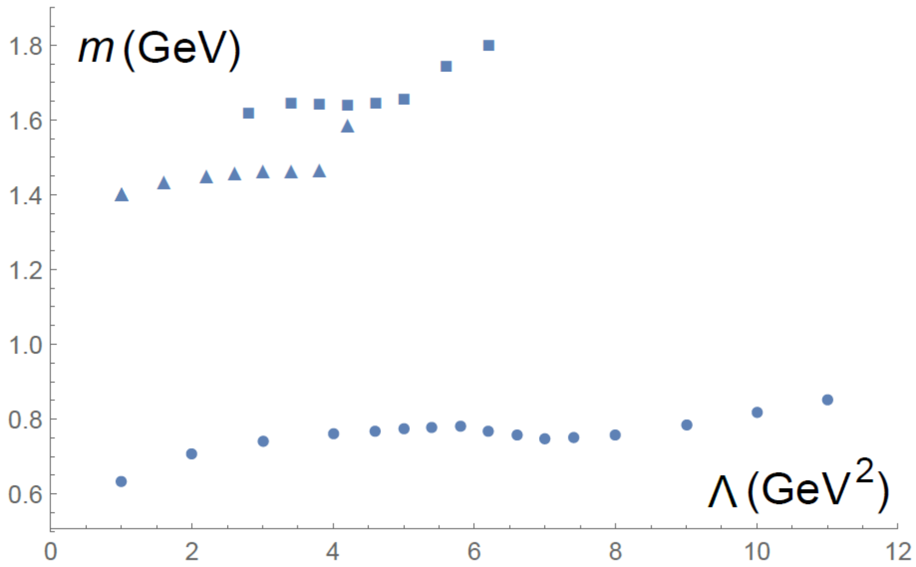
$$\int_0^\infty dy\frac{\Delta\rho(y)}{x-y}=\int_0^\infty dy\frac{ce^{-y}-f_0\delta(y-r_0)}{x-y}-\frac{1}{12\pi}\frac{\langle\alpha_s G^2\rangle}{x^2\Lambda^2}-2\frac{\langle m_q\bar{q}q\rangle}{x^2\Lambda^2}-\frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{x^3\Lambda^3}$$

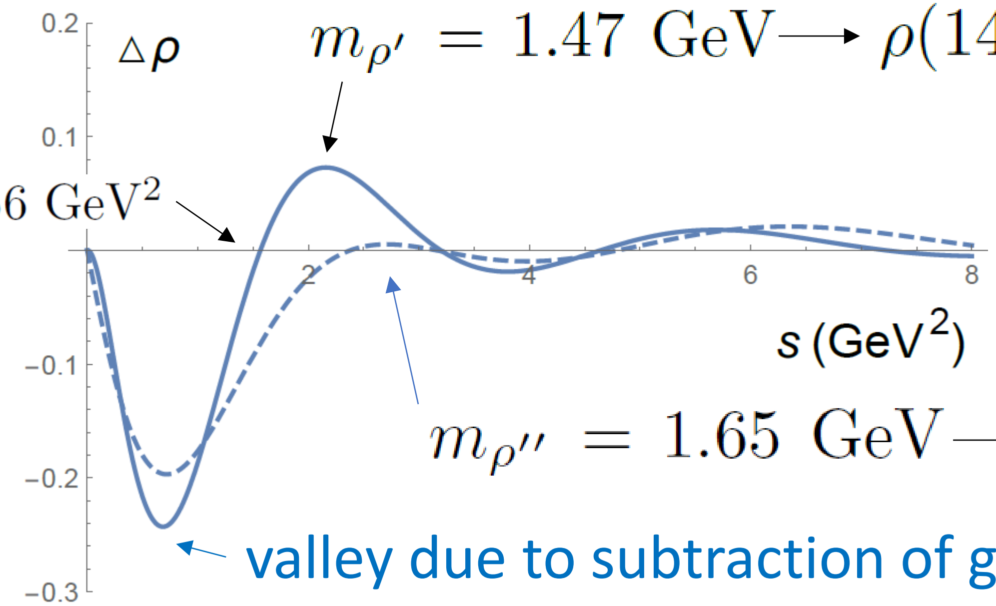
$x=q^2/\Lambda$ $\quad y=s/\Lambda$ $\quad f_0=F_0/\Lambda$ $\quad r_0=m_\rho^2/\Lambda$

Excited rho resonances

- To get 2^{nd} excited state, further subtract

$$F_1\delta(s - m_{\rho'}^2) \quad F_1 = \int_{t_1}^{\infty} ds\Delta\rho_1(s) = 0.11 \text{ GeV}^2$$





$m_{\rho'} = 1.47$ GeV $\longrightarrow \rho(1450)$

$t_1 = 1.56 \text{ GeV}^2$

$m_{\rho''} = 1.65$ GeV $\longrightarrow \rho(1700)$ 1720 ± 20 MeV

uncertainties involved in lower states propagated to higher states, enlarged through sequential subtractions

valley due to subtraction of ground state

- Adopting BW form, instead of delta-function, $m_{\rho'}$ increases by 5%

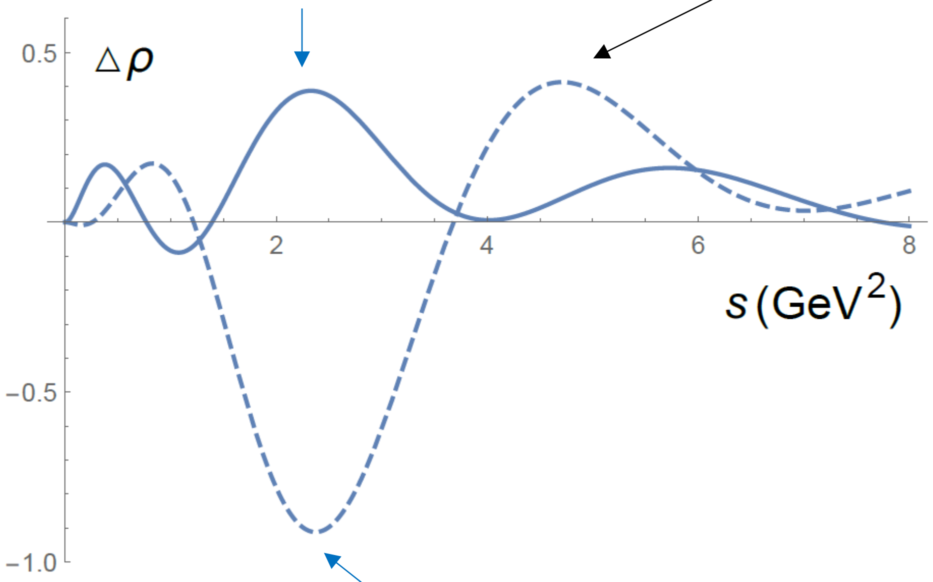
Excited scalar glueball

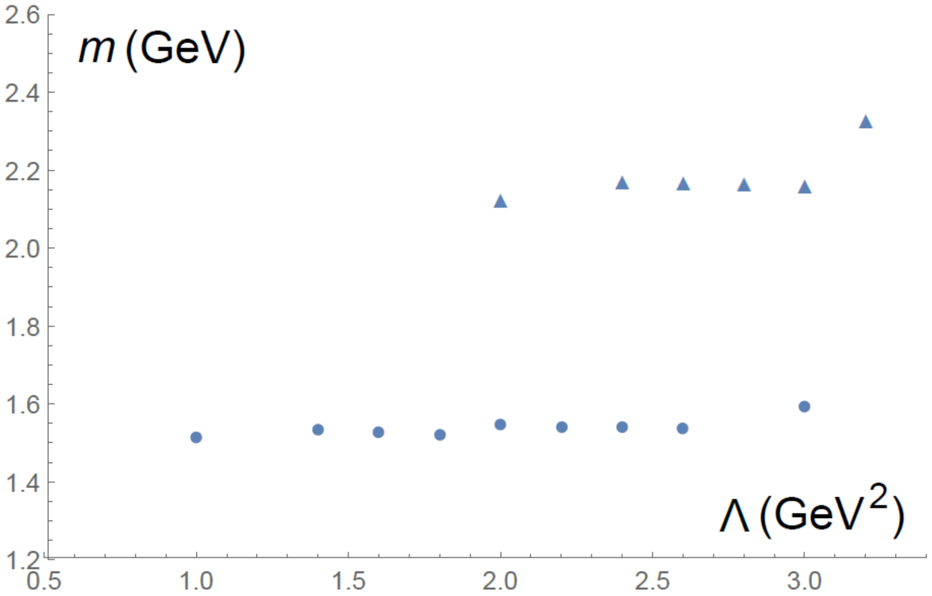
- After checking the formalism, apply it to glueballs
- For scalar glueballs

$$2187 \pm 14\ \mathrm{MeV}$$

$$m_{S'} = 2.17 \pm 0.01\ \mathrm{GeV} \longrightarrow f_0(2200)$$

ground-state solution





valley due to subtraction of ground state

Excited pseudoscalar glueball

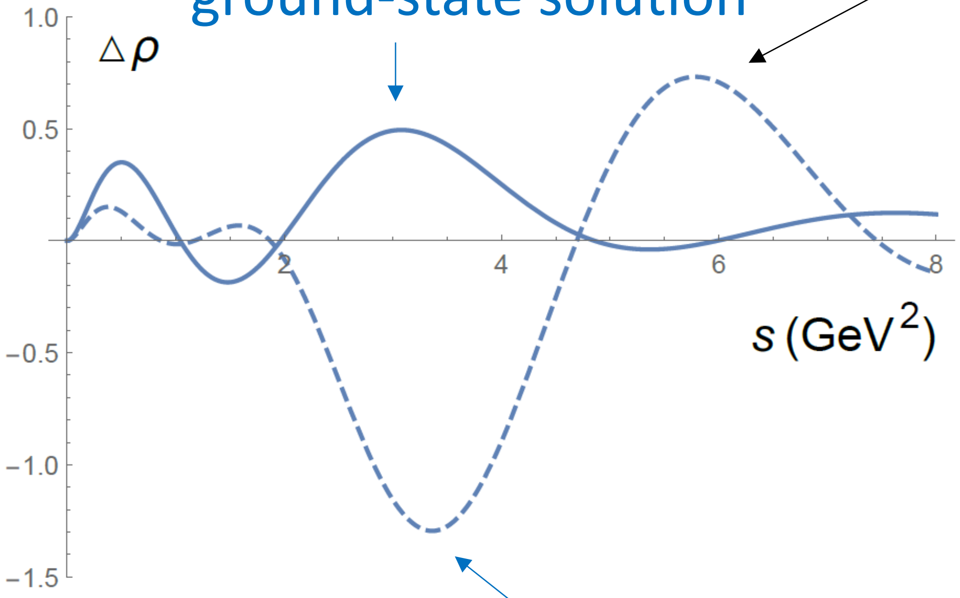
- For pseudoscalar glueballs

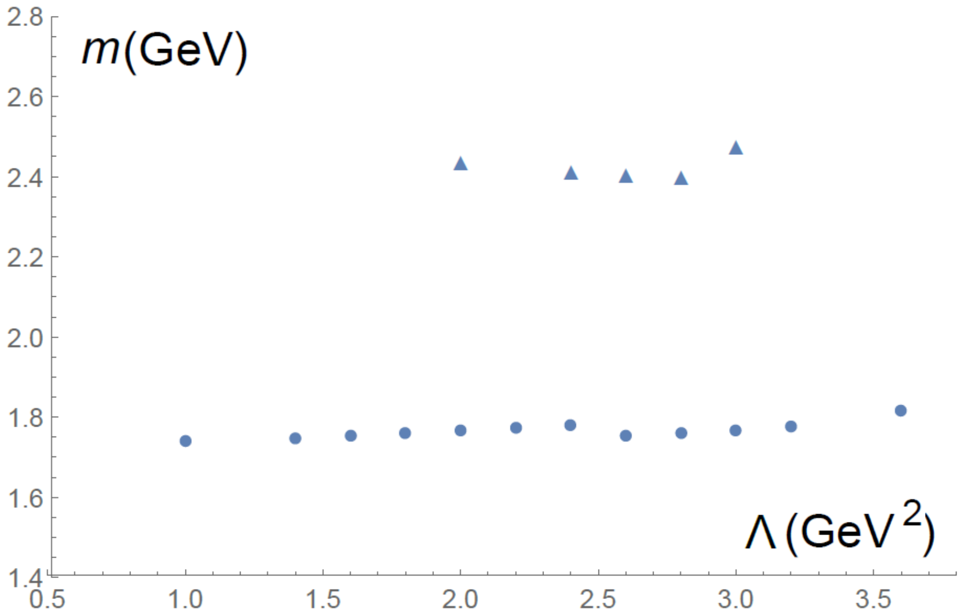
BESIII measurement

$$2395 \pm 11(\text{stat})^{+26}_{-94}(\text{syst})\ \text{MeV}$$

$$m_{P'} = 2.41 \pm 0.04\ \text{GeV} \longrightarrow X(2370)$$

ground-state solution





valley due to subtraction of ground state

Conclusion

- Our dispersive approach, compared to conventional SR, is free of arbitrary parameters, and can give definite predictions with controllable uncertainties
- Simultaneous accommodation of rho and scalar glueball spectra strongly supports our analyses
- Both f0(2200) and X(2370) are heavier than their ground states by about 700 MeV, typical energy gap induced by radial excitations
- O(10E-4) BRs of radiative decays $J/\psi \to \gamma f_0(2200)$ and $\gamma X(2370)$, lower than those of O(10E-3) associated with ground states, also make sense
- Uncertainties larger for higher states; need to be improved

Back-up slides

Fredholm integral equation

- Goal is to solve ill-posed integral equation

unknown spectral density to be solved

$$\int_0^\infty dy \frac{\rho(y)}{x-y} = \omega(x)$$

OPE input

1st kind of Fredholm integral equation

- How to solve it? Notoriously difficult
- Discretization does not work

ill-posedness

- Discretizing integral equation fails

$$\sum_i M_{ij}\rho_j = \omega_i \qquad M_{ij} = \begin{cases} 1/(i-j), & i \neq j \\ 0, & i = j \end{cases}$$

unknowns input

- Rows Mij and M(i+1)j become almost identical for fine meshes, det(M) ~ 0
- Matrix M becomes singular; M^{-1} diverges quickly
- Solution diverges and sensitive to variation of inputs

Strategy

- Suppose $\rho(y)$ decreases quickly enough
- Expansion into powers of 1/x justified

$$\frac{1}{x-y} = \sum_{m=1}^{N} \frac{y^{m-1}}{x^m} \qquad \omega(x) = \sum_{n=1}^{N} \frac{b_n}{x^n}$$

true for OPE

- Suppose $\omega(x)$ can be expanded
- Decompose

$$\rho(y) = \sum_{n=1}^{N} a_n y^\alpha e^{-y} L_{n-1}^{(\alpha)}(y)$$

generalized Laguerre polynomials

depend on $\rho(y)$ at $y \to 0$

- Orthogonality

$$\int_0^\infty y^\alpha e^{-y} L_m^{(\alpha)}(y) L_n^{(\alpha)}(y)dy = \frac{\Gamma(n+\alpha+1)}{n!}\delta_{mn}$$

Solution

- Equating coefficients of $1/x^n$

$$Ma = b \qquad M_{mn} = \int_0^\infty dy y^{m-1+\alpha} e^{-y} L_{n-1}^{(\alpha)}(y)$$

matrix (→ M)

input $b = (b_1, b_2, \cdots, b_N)$

unknown $a = (a_1, a_2, \cdots, a_N)$

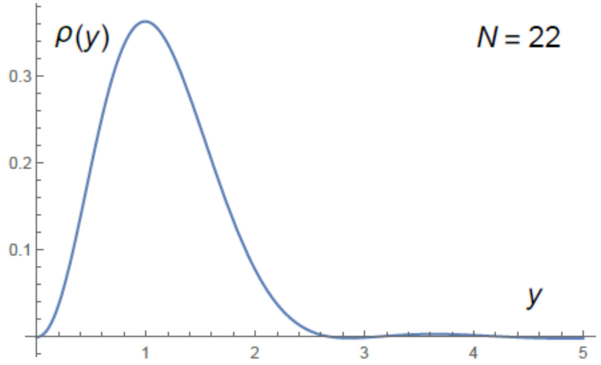
- Solution $a = M^{-1}b$
- True solution can be approached by increasing N, but M^{-1} diverges with N
- Additional polynomial gives $1/x^{N+1}$ correction due to orthogonality, beyond considered precision

Test examples

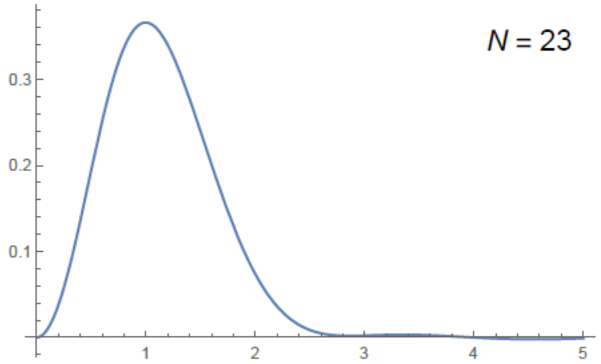
- Generate mock data from $\rho(y) = y^2 e^{-y^2}$

$$b_n = \int_0^\infty dy y^{n-1} y^2 e^{-y^2} \Longleftarrow \int_0^\infty dy \frac{\rho(y)}{x-y} = \omega(x)$$

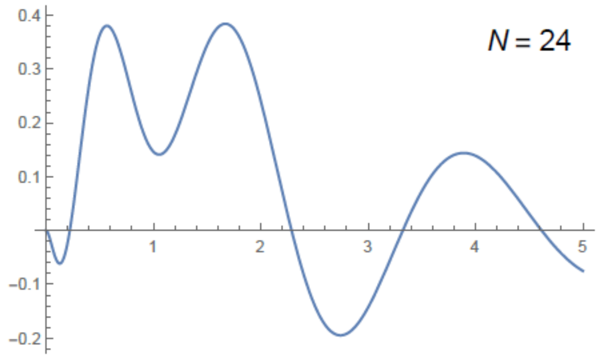
- Compute matrix M with $\alpha = 2$
- Solution stable for N > 20, becomes oscillatory as N=24 due to divergent M^{-1}



$a_{22}/a_{21} \approx 1$



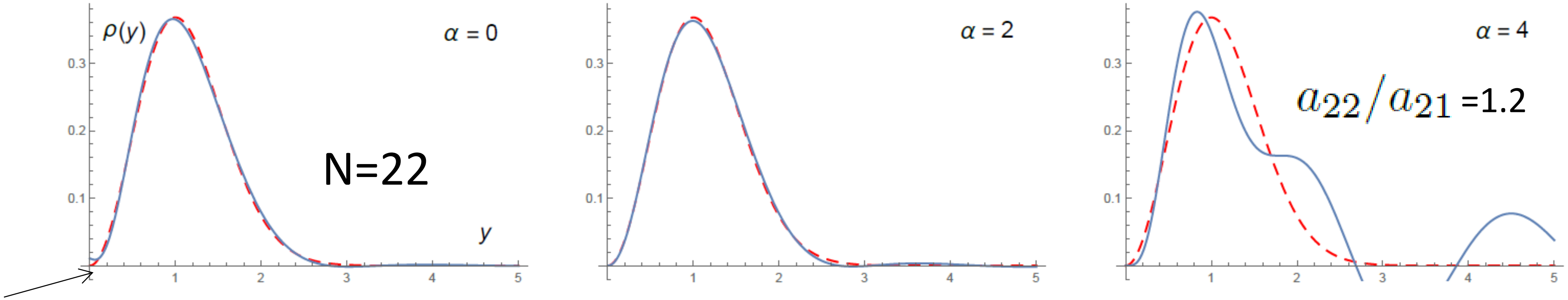
$a_{23}/a_{22} \approx 2$



$a_{24}/a_{23} \approx 58$

Boundary conditions

- Test choices of α (red: true solution)



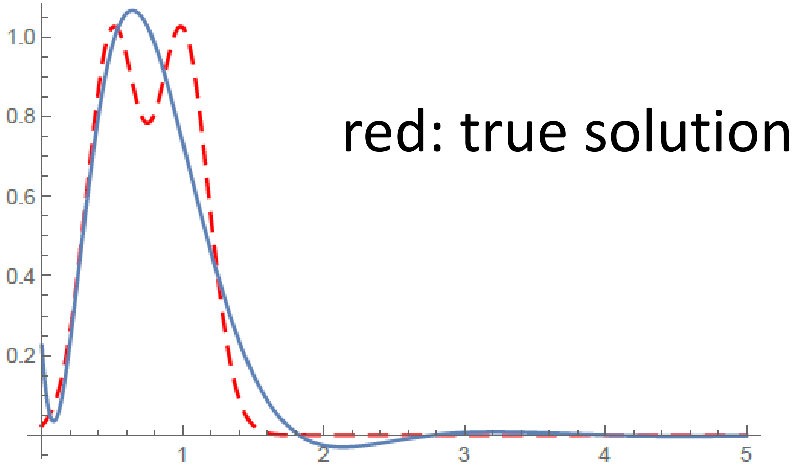
deviation almost perfect completely different

- Parameter α determined by boundary conditions of solution
- Boundary conditions help getting correct solutions

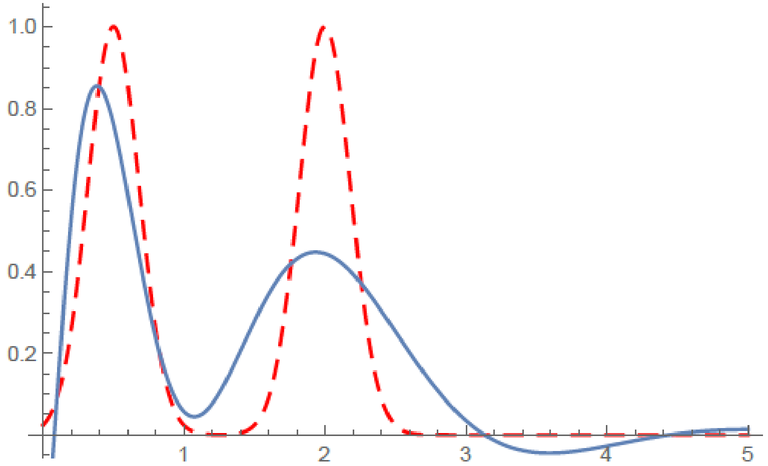
Resolution

- e^{-y} implies resolution power $\Delta y \sim 1$
- Test double peak functions

$$\rho_1(y) = e^{-20(y-0.5)^2} + e^{-20(y-1.0)^2} \qquad \Delta y \sim 0.5$$
$$\rho_2(y) = e^{-20(y-0.5)^2} + e^{-20(y-2.0)^2} \qquad \Delta y \sim 1.5$$



red: true solution



- Fine structure cannot be resolved (ill-posed)