Cool dense QCD and inhomogeneous phases a.k.a. chiral spirals

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NYCU
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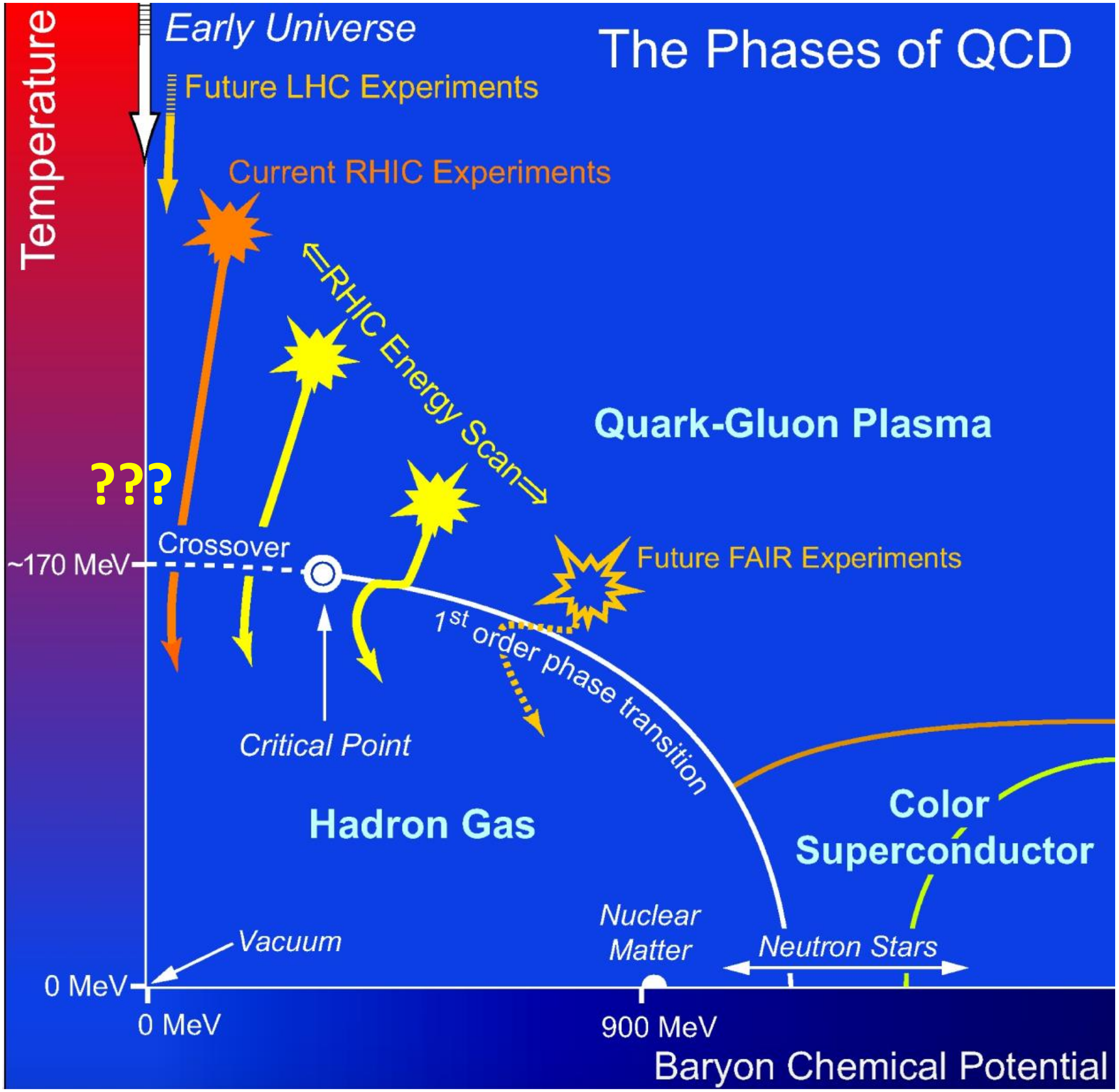
Based on Phys. Rev. D 102, 016015 (2020)
and arXiv:2403.18640

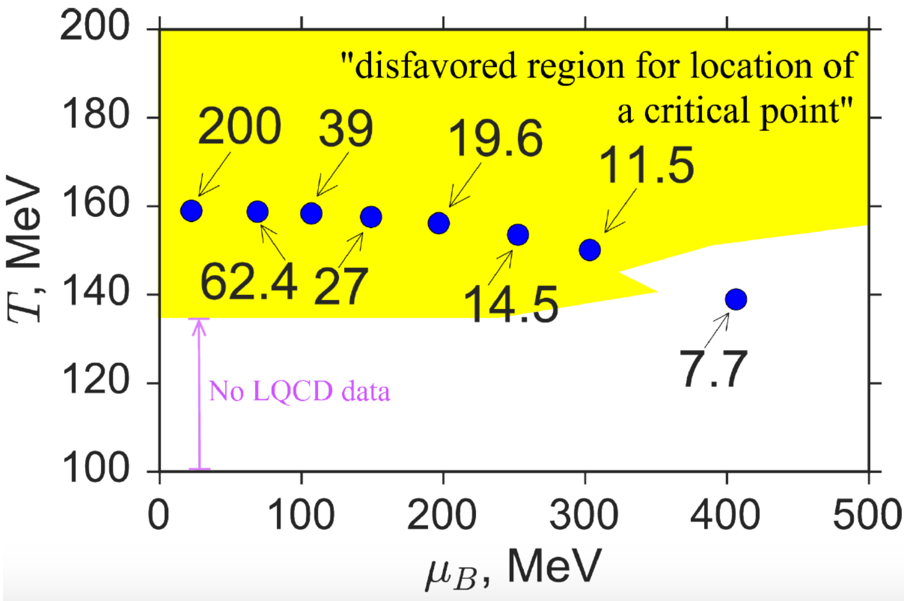
QCD at finite density

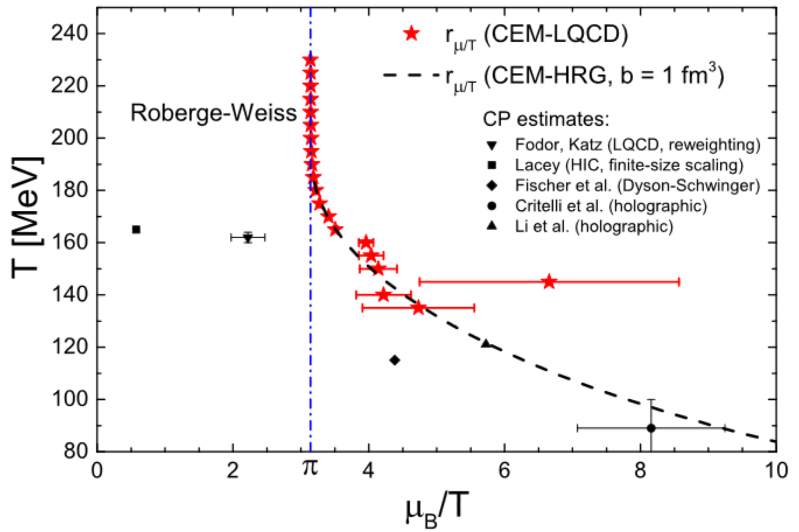


Signs of CEP?

Lattice: ***small mu,*** ***Taylor expansion & estimate radius of convergence, Cluster expansion***

No signs of CEP ***from lattice so far***

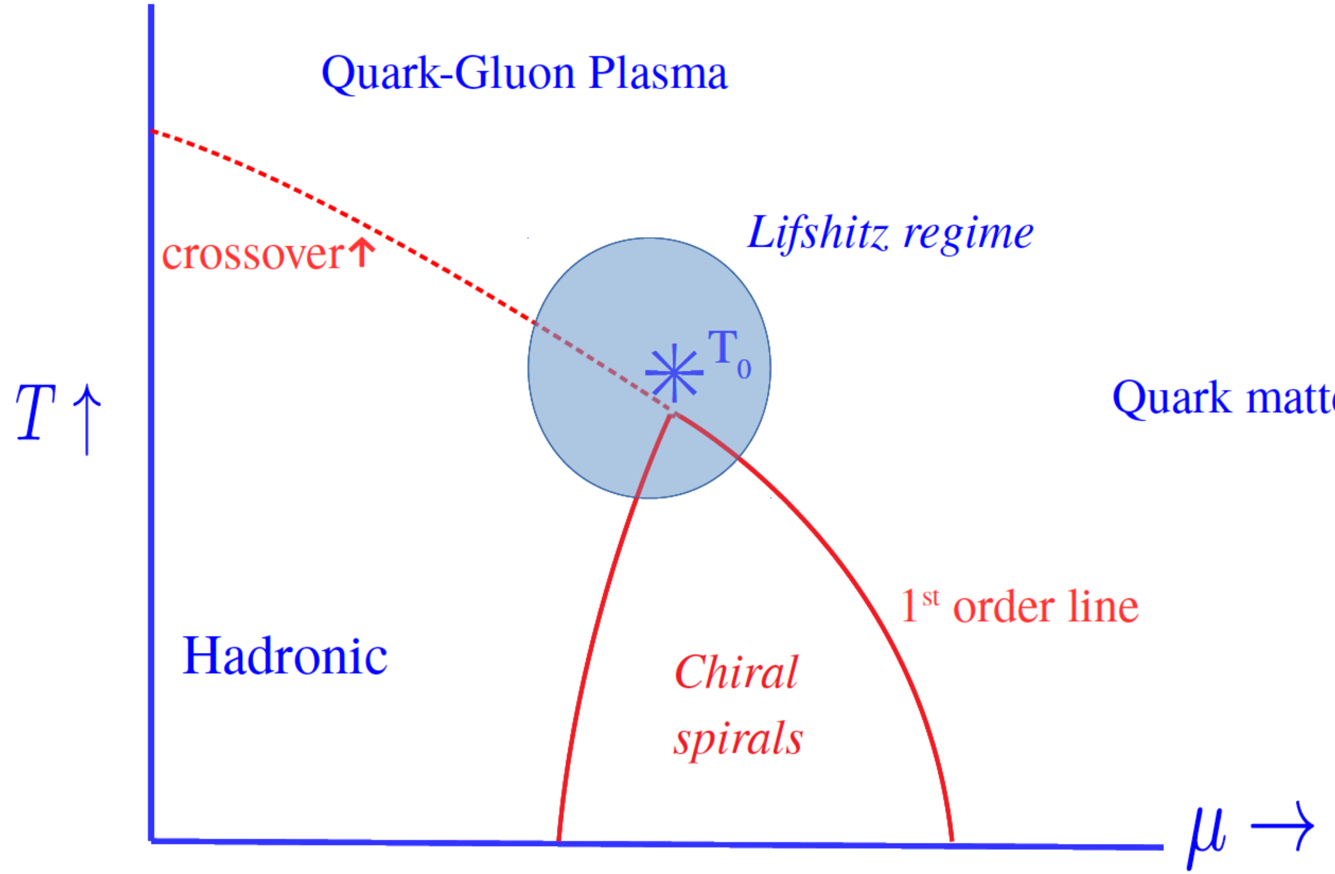




Hot QCD
Arxiv: 1701.04325

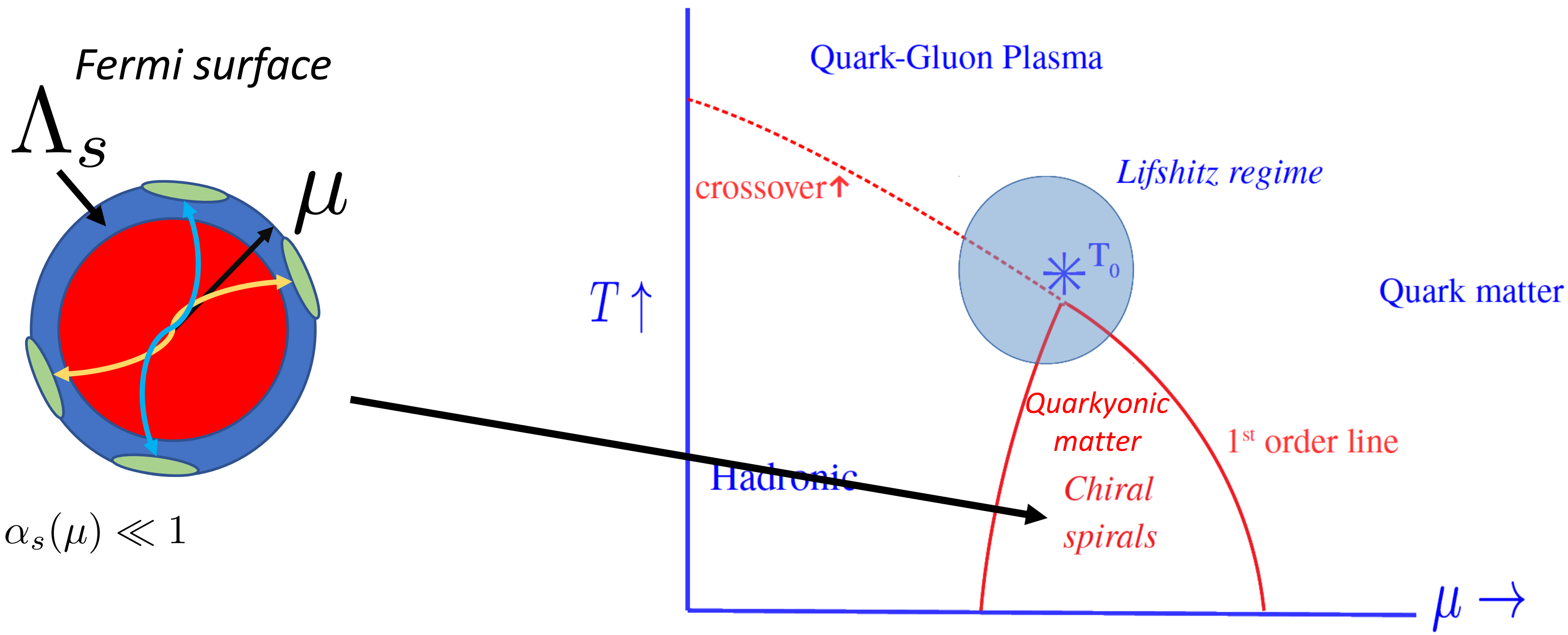
Vovchenko, Steinheimer, Philipsen, Stoecker
Arxiv:1711.01261

Alternative scenario?

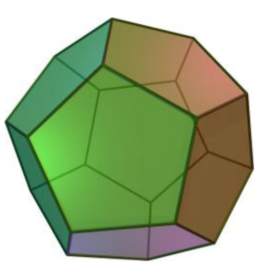


R. Pisarski, V. Skokov, A. Tsvelik,
Arxiv:1801.08156

Quarkyonic matter



Effective reduction to 1+1-dim SU(2 Nf) QCD in large Nc limit



Kojo, Hidaka, McLerran, Pisarski
Arxiv:0912.3800

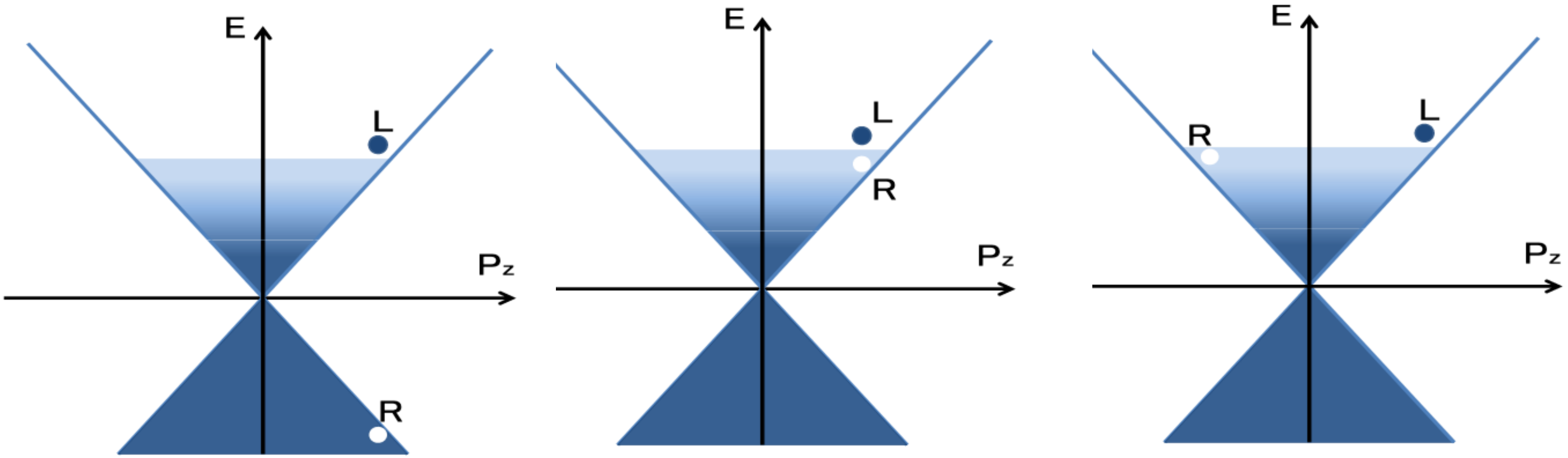
Evidence from lattice in two-color QCD (no sign problem!):

Bornyakov et al, 1711.01869

Chiral symmetry breaking

Suggestive argument in 1+1 dim: μ ***can be eliminated in the expense of Chiral Spiral:***

$$\bar{q}q \rightarrow \cos(2\mu z)\bar{q}q + i\sin(2\mu z)\bar{q}\gamma_5 q$$



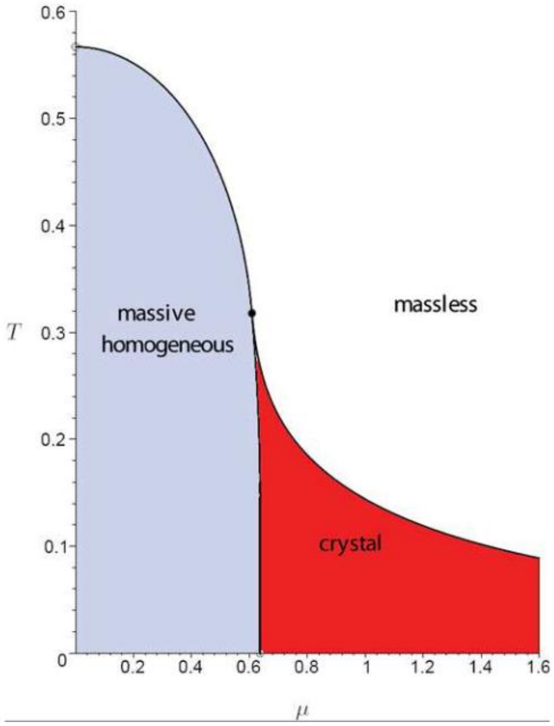
Uniform condensate

Exciton

Chiral density wave/
Chiral Spiral

Chiral symmetry breaking

Exact solutions in 1+1 (GN, NJL models at infinite Nf):

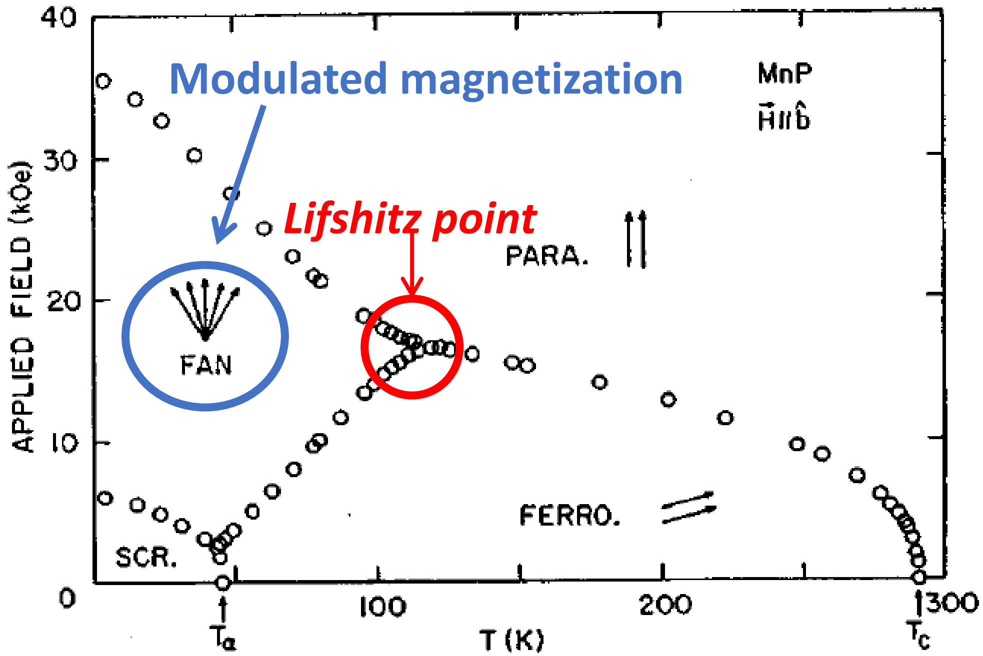


Basar, Dunne, Thies
Arxiv:0903.1868

Ginzburg-Landau analysis in 3+1 NJL models:

Nickel, Arxiv:0902.1778
Buballa, Carignano, Arxiv:1406.1367

Lifshitz point and Lifshitz regime



Observation of Lifshitz point in magnetic structures

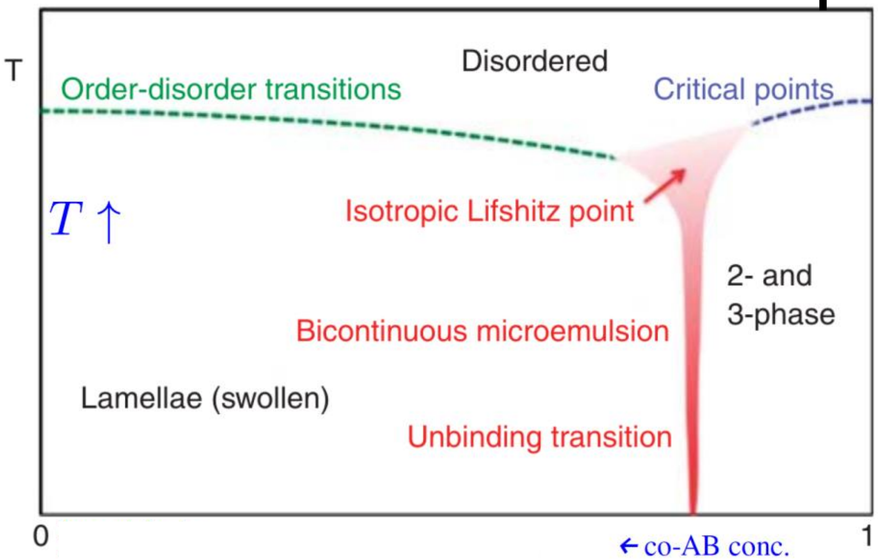
Shapira, Becerra, Oliveira Jr, Chang
Phys. Rev. B 24, 2780

Ginzburg-Landau: $$\mathcal{L} = \frac{1}{2}(\nabla_\perp \phi)^2 + \frac{Z}{2}(\nabla_\parallel \phi)^2 + \frac{1}{2M^2}(\Delta_\parallel \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \ldots$$

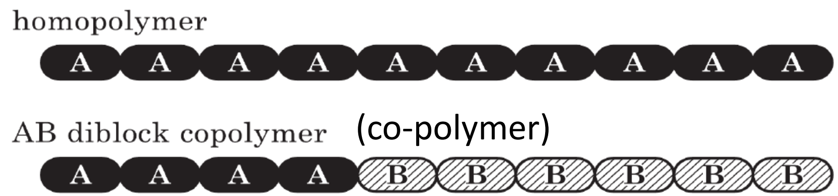
Lifshitz point: $m^2 = Z = 0$

Anisotropic fluctuations: $k_\perp^2$ **and** $k_\parallel^4$

Lifshitz point and Lifshitz regime: Inhomogeneous polymers



Mixture of polymers A & B and co-polymer:



(co-polymer)

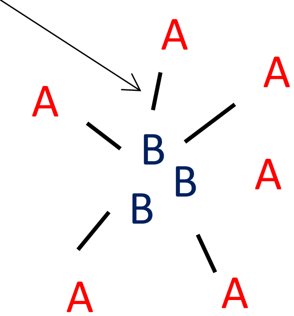
Lamellar phase:

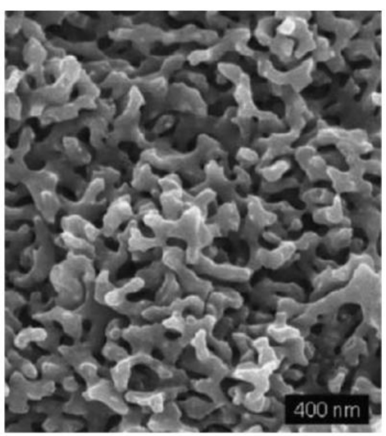
- *Anisotropic fluctuations*

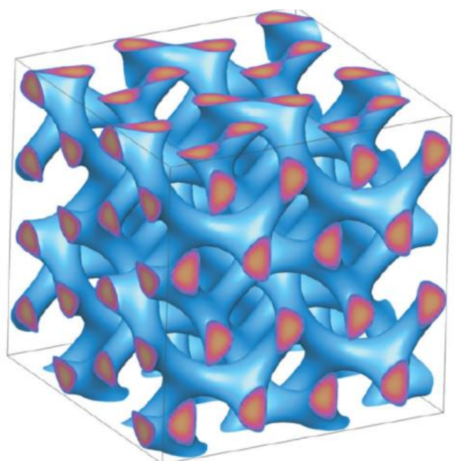
AAAAAAAAAAA
||||||||||||||
BBBBBBBBBBB
||||||||||||||
AAAAAAAAAAA

Co-polymers

Microemulsion:







Experiment

Self-consistent field theory

Lifshitz regime: bicontinuous microemulsion

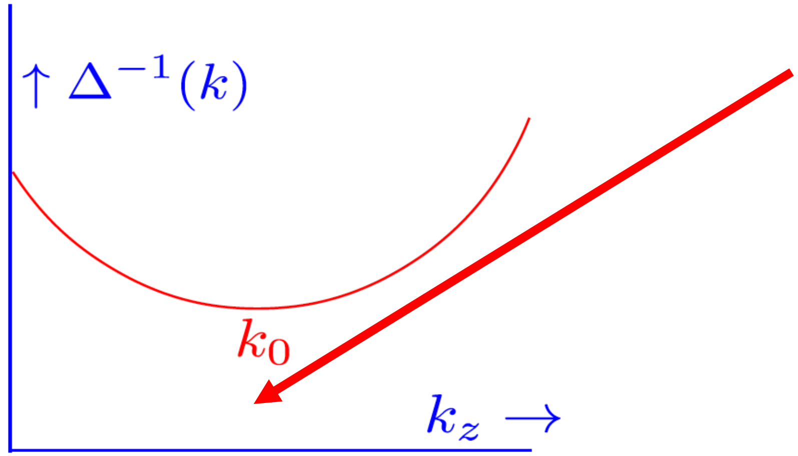
Jones, Lodge
Polymer Journal (2012) 44, 131-146

Lifshitz point: O(N) effective model

$$\mathcal{L} = \frac{1}{2}(\partial_0\phi)^2 + \frac{1}{2M^2}(\partial_i^2\phi)^2 + \frac{Z}{2}(\partial_i\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}(\phi^2)^2 + \ldots$$

3+1-dimensional model, we implicitly assume finite density and temperature

Z is allowed to be negative. In this case:



Gap may close if Z gets sufficiently negative

Condensate in this case: chiral spiral

$$\phi(x) = \phi_0(\cos(k_0 z), \sin(k_0 z))$$

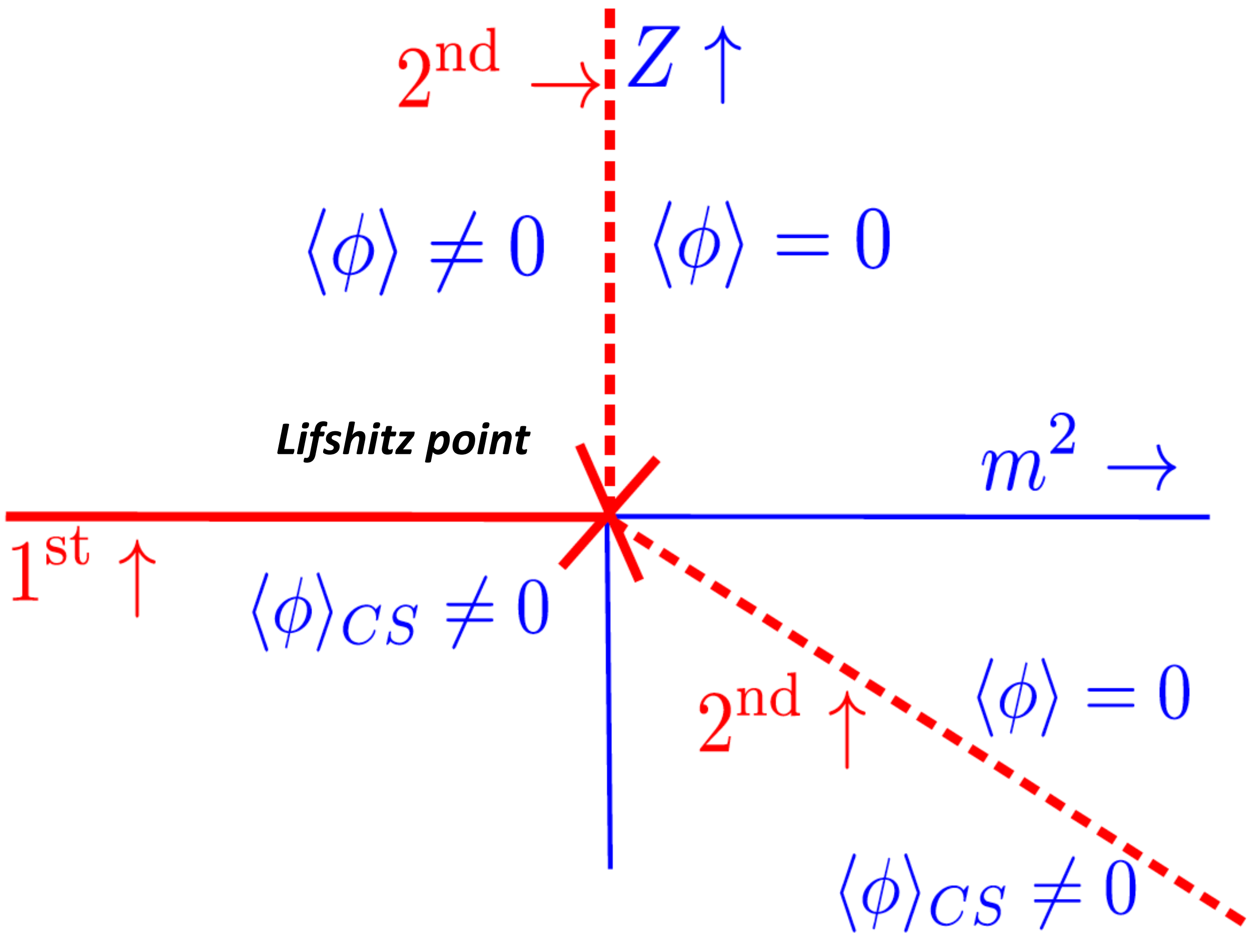
A'la roton condensation in superfluid?

$$\Delta^{-1} = m_{eff}^2 - 2Z(k_z - k_0)^2 + \frac{1}{M^2}(4k_0 k_z \vec{k}^2 + (\vec{k}^2)^2)$$

Anisotropic fluctuations

O(N) effective model

Mean-field phase diagram



$$\phi(x) = \phi_0(\cos(k_0 z), \sin(k_0 z))$$

What to expect from fluctuations?

No long-range order is possible due to anisotropy:

$$\int d^2k_\perp dk_\parallel \frac{1}{(k_\parallel - k_0)^2 + (k_\perp^2)^2} \sim \int d^2k_\perp \frac{1}{k_\perp^2} \sim \log \Lambda_{IR}$$

Effective 1-dimensional reduction =>
phonon fluctuations of chiral spiral destroy long range order

No Lifshitz point is possible, only Lifshitz regime in d<= 4: $\int d^4k_\perp \frac{1}{k_\perp^4} \sim \log \Lambda_{IR}$

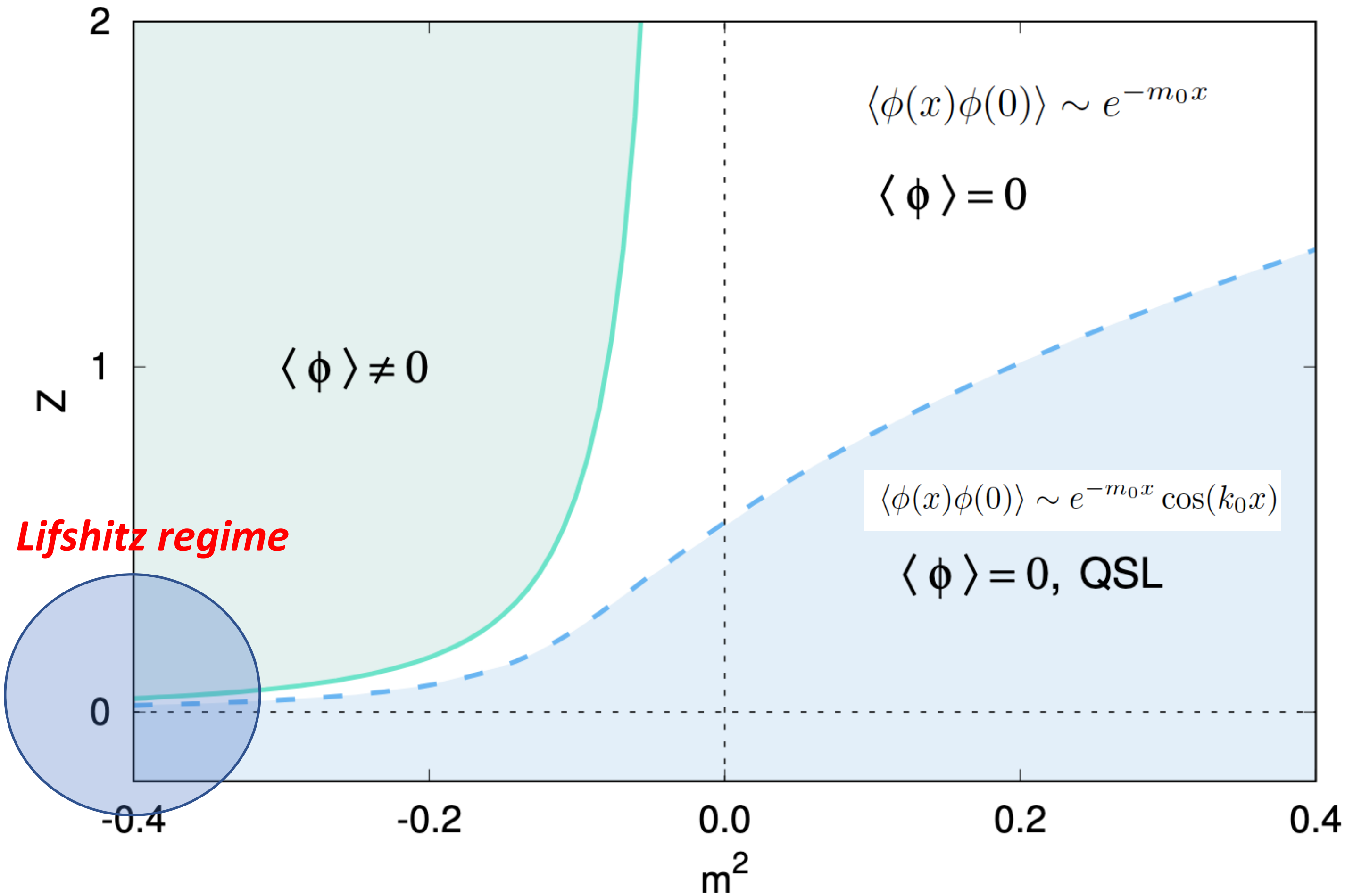
Either mass squared of Z must be generated non-perturbatively

In order to address these issue we use large N analysis

Infinite N phase diagram

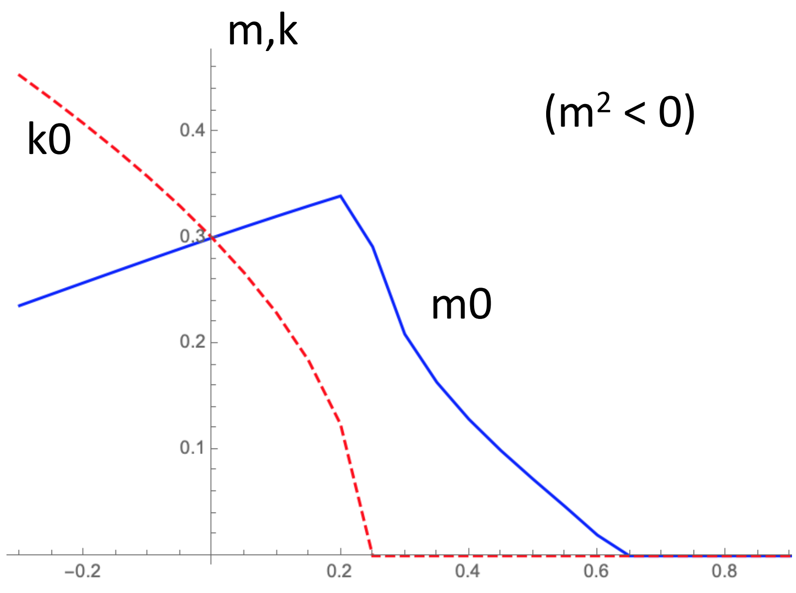
Effects of fluctuations: dramatic change

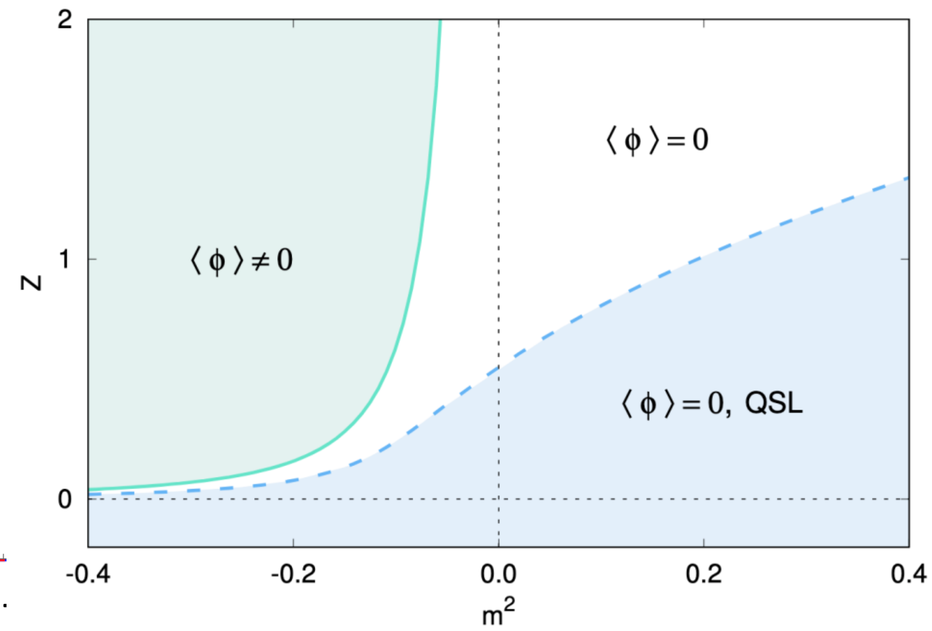
No Lifshitz point, but Lifshitz regime



No evidence for 2nd order phase transition to CS

Quantum Spin Liquid regime





Infinite N and Z large and negative: $m_0 \sim \frac{1}{2\pi}\frac{\lambda N}{Z^2}, \qquad k_0 \sim \frac{1}{\sqrt{2}} M\sqrt{|Z|};$

Symmetric solution exists for any negative Z

Possible phase transition to chiral spirals must be of 1st order if it exist

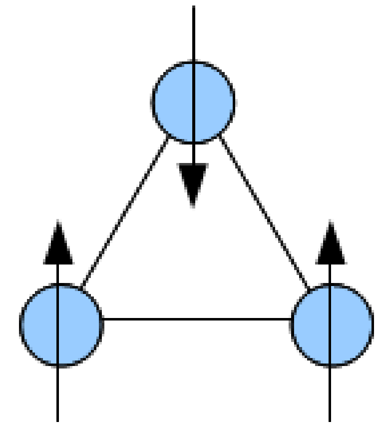
Quantum Spin Liquid regime

Infinite N and Z large and negative:

$$m_0 \sim \frac{1}{2\pi}\frac{\lambda N}{Z^2}, \qquad k_0 \sim \frac{1}{\sqrt{2}} M\sqrt{|Z|}.$$

Quantum spin liquid: quantum fluctuations prevent condensate formation

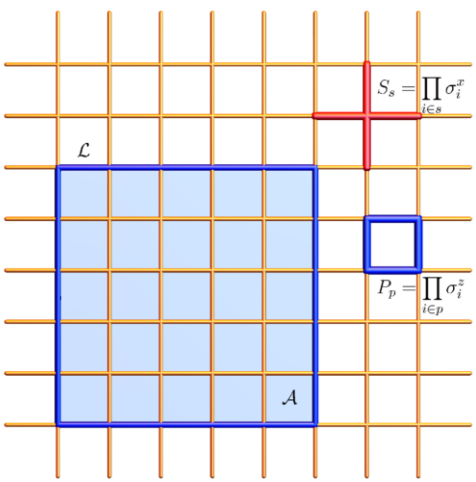
Entanglement over long distances instead of long-range order

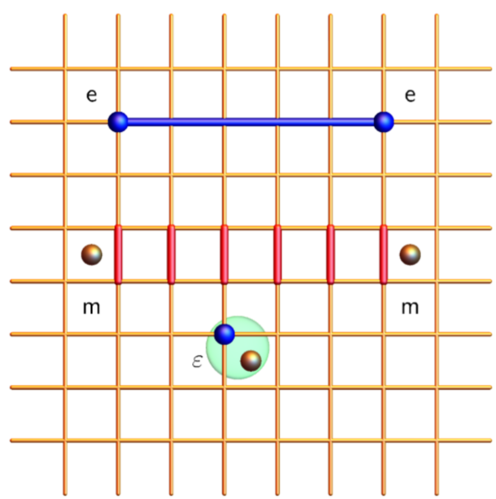


Example:

1) Anti-ferromagnetically coupled spins on triangular lattice
2) Kitaev's Toric Code

$$H_{\rm tc} = -K\sum_p P_p - K'\sum_s S_s$$





Figures from an excellent review:
arXiv:1601.03742

Finite N: perturbation theory

$$\mathcal{L} = \frac{1}{2}(\partial_0\phi)^2 + \frac{1}{2M^2}(\partial_i^2\phi)^2 + \frac{Z}{2}(\partial_i\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}(\phi^2)^2 + \dots$$

We assume Z < 0 and the following ansatz: $\phi_0 = \phi_0(\cos(k_0 z), \sin(k_0 z), 0)$

$$\phi_0^2 = \text{const}$$

Minimize w.r.t. k_0 **and** ϕ_0**.**

Find a double pole in the propagator of transverse fluctuations:

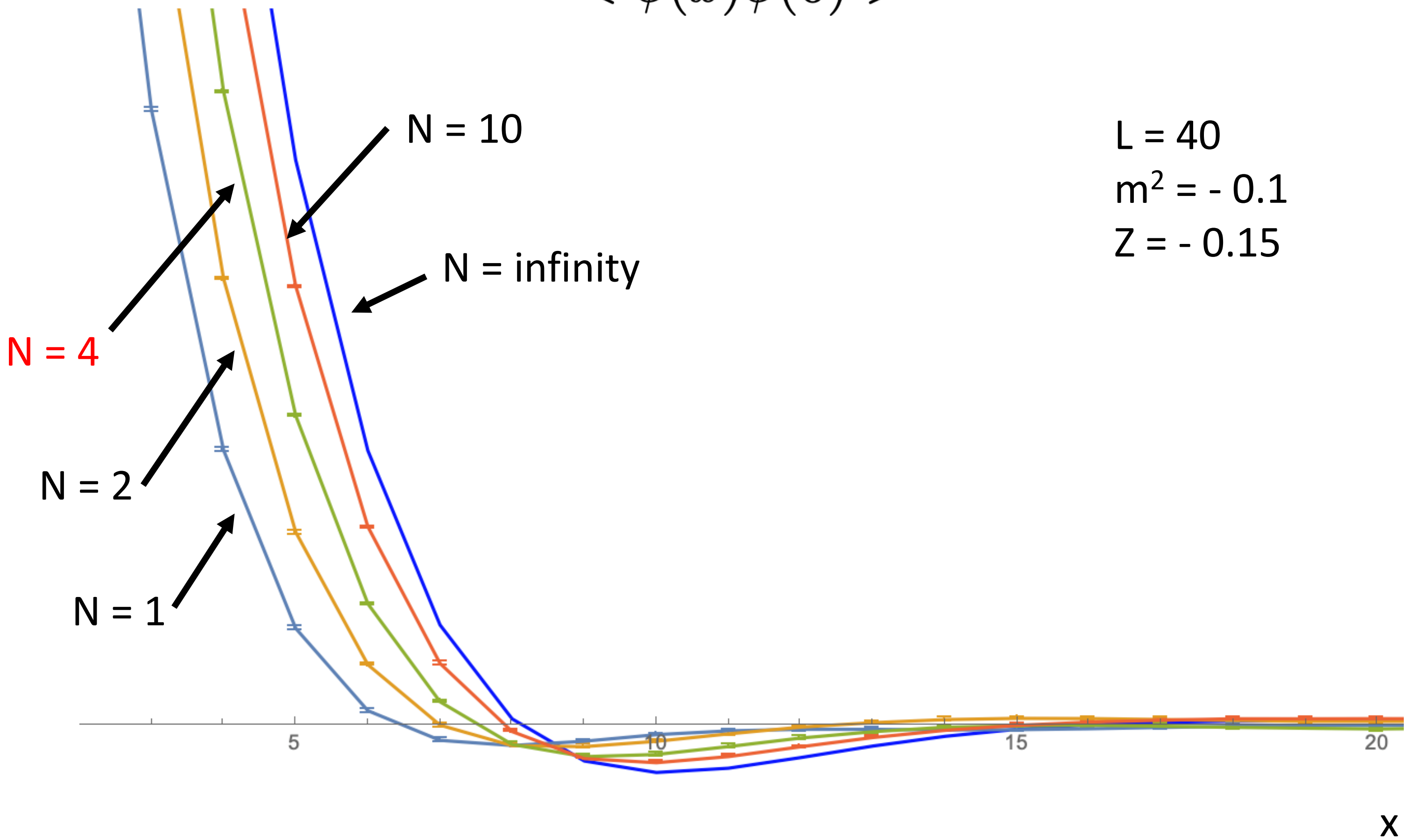
$$\Delta_\chi^{-1}(\boldsymbol{k}) = \frac{1}{2M^2}\left(\boldsymbol{k}^2 - k_0^2\right)^2$$

Simple ansatz is not consistent, system is disordered at very short distances

We cannot exclude more complex solution, but these are very hard to find

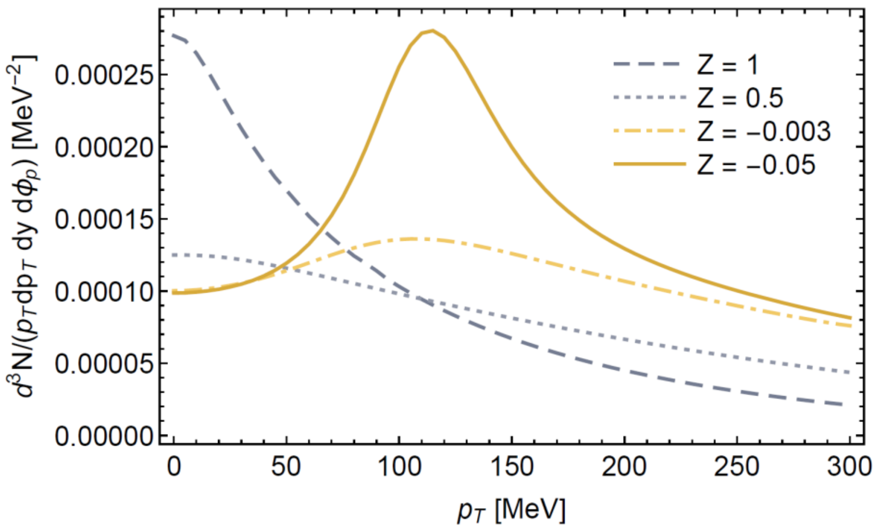
Finite N: lattice computation

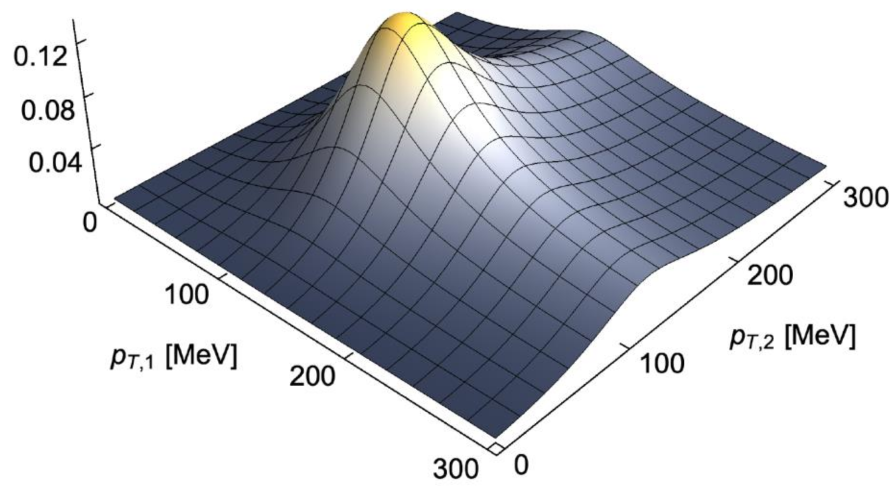
$$< \phi(x)\phi(0) >$$



No evidence for 1st / 2nd order phase transition

Experimental signatures





R. Pisarski and Fabian Rennecke arXiv:2103.06890

Conclusions

More open questions than answers:

1) Interplay of Critical End Point and Lifshitz regime?

Fluctuations in Lifshitz regime are only logarithmic, weaker than critical fluctuation.

Possible experimental signatures? Modulation momentum k0 should appear in fluctuations

2) Is there a room for chiral spirals anyway?

Lattice computations with external field?