Extending the Range and Precision of Lattice Flavourdynamics

Chris Sachrajda
University of Southampton

TQCD 2nd Meeting
NYCU Taipei
September 24th 2024

University of Southampton

Precision Flavour Physics

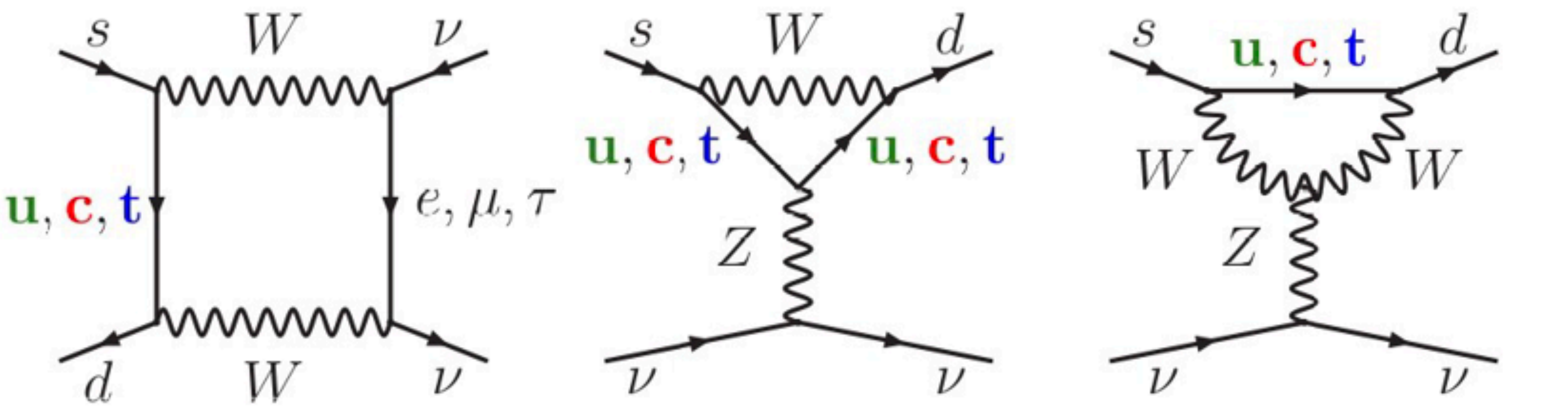
- Precision Flavour Physics, is a key approach, complementary to the large E_T searches at the LHC, in exploring the limits of the standard model and in searches for New Physics.
    - If the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to unravel the underlying framework.
    - The discovery potential of precision flavour physics would also not be underestimated. (In principle, the reach may be about two orders of magnitude deeper than the LHC!

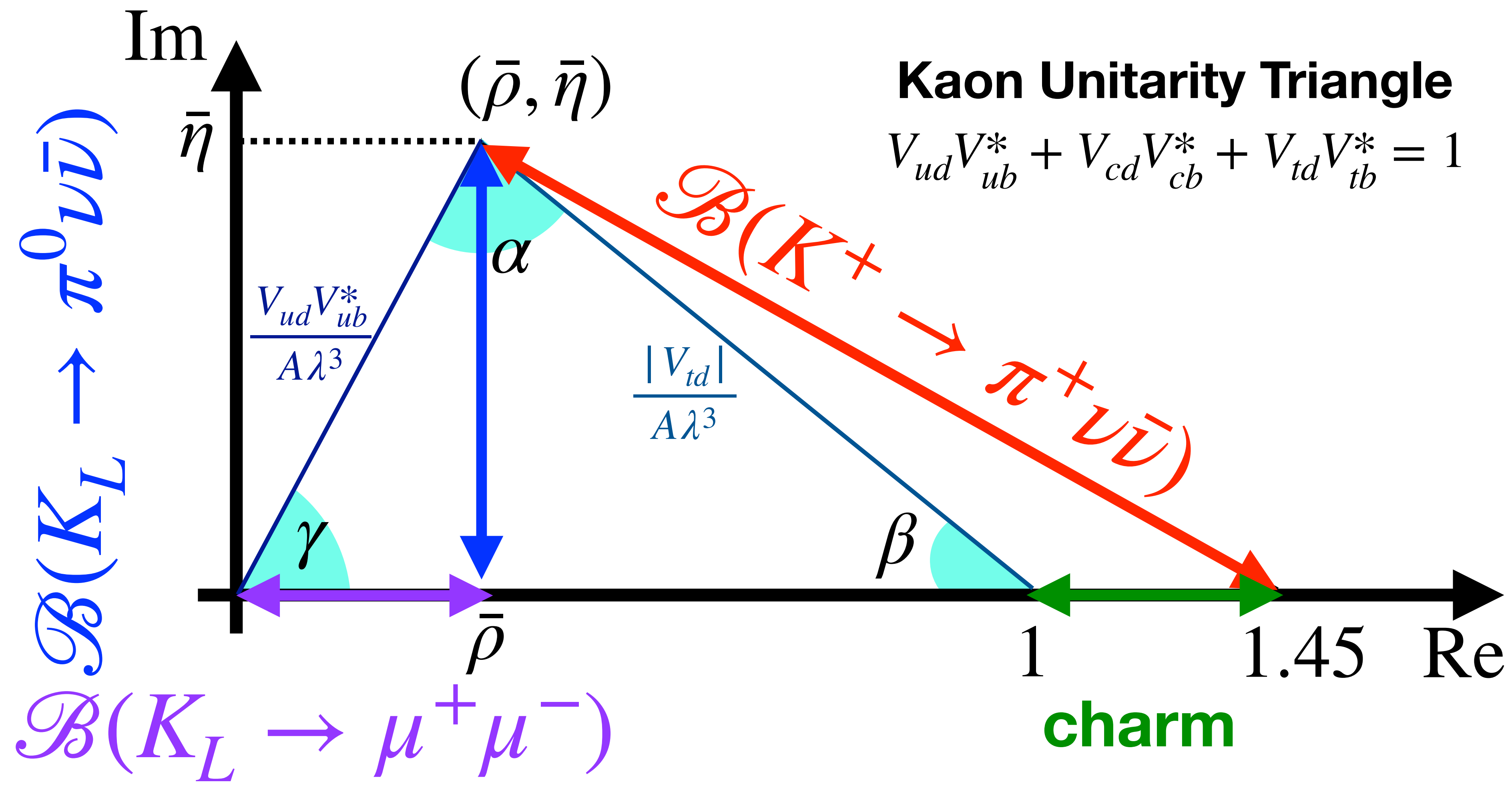
- To illustrate very recent experimental results, I show two slides from the NA62 experiment measuring the branching ratio for the very rare decay $K^+ \to \pi^+\nu\bar{\nu}$, presented at CERN this Tuesday 24/09/2024.

$K \to \pi\nu\bar{\nu}$: Precision test of the Standard Model

NA62

SM: Z-penguin & box diagrams





- $\mathcal{B}(K \to \pi\nu\bar{\nu})$ **highly suppressed** in SM
  - GIM mechanism & maximum CKM suppression $s \to d$ transition: $\sim \frac{m_t}{m_W}\left|V_{ts}^* V_{td}\right|$
- Theoretically clean ⇒ high precision SM predictions
  - Dominated by short distance contributions.
  - Hadronic matrix element extracted from $\mathcal{B}(K \to \pi^0\ell^+\nu_\ell)$ decays via isospin rotation.

Mode	SM Branching Ratio [1]	SM Branching Ratio [2]	Experimental Status
$K^+ \to \pi^+\nu\bar{\nu}$	$(8.60 \pm 0.42) \times 10^{-11}$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6 \pm 4.0) \times 10^{-11}$ NA62 16—18
$K_L \to \pi^0\nu\bar{\nu}$	$(2.94 \pm 0.15) \times 10^{-11}$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 2 \times 10^{-9}$ KOTO (2021 data)

^Recent SM calculations [1:Buras et al. EPJC 82 (2022) 7, 615][2:D'Ambrosio et al. JHEP 09 (2022) 148]
(Differences in SM calculations from choice of CKM parameters: see [Eur.Phys.J.C 84 (2024) 4, 377])

Results in context

BNL E787/E949 experiment
[Phys.Rev.D 79 (2009) 092004]

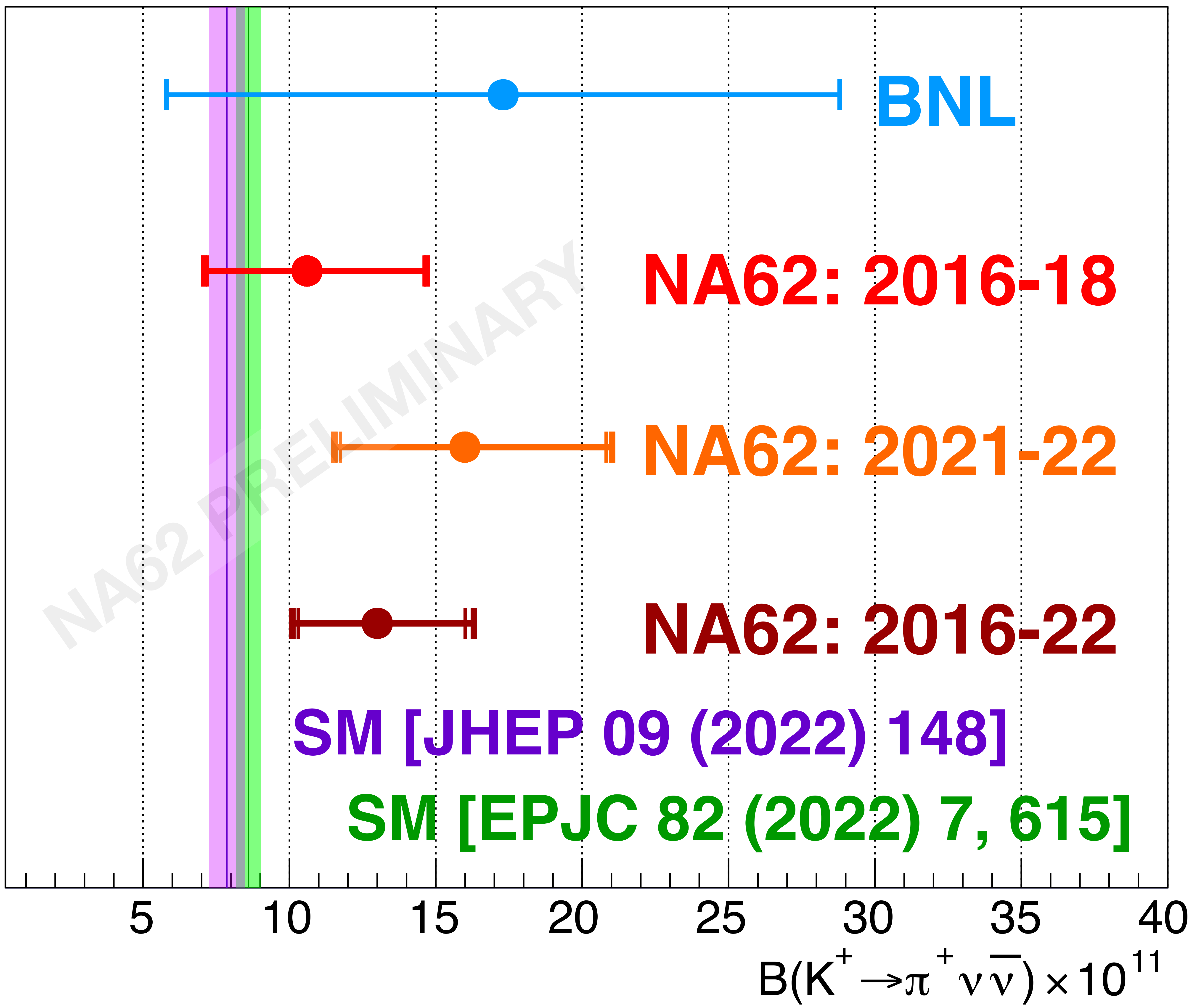
$$\mathcal{B}^{16-18}_{\pi\nu\bar{\nu}} = \left(10.6^{+4.1}_{-3.5}\right) \times 10^{-11}$$

[JHEP 06 (2021) 093]

$$\mathcal{B}^{21-22}_{\pi\nu\bar{\nu}} = \left(16.0^{+5.0}_{-4.5}\right) \times 10^{-11}$$

$$\mathcal{B}^{16-22}_{\pi\nu\bar{\nu}} = \left(13.0^{+3.3}_{-2.9}\right) \times 10^{-11}$$

- NA62 results are consistent
- Central value moved up (now 1.5–1.7σ above SM)
- Fractional uncertainty decreased: 40% to 25%
- Bkg-only hypothesis rejected with significance Z>5

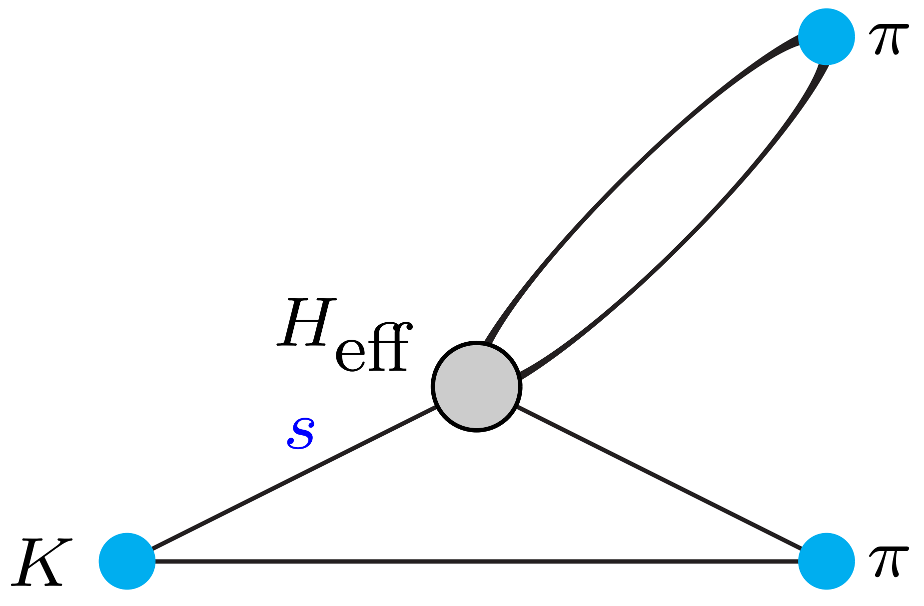


Precision Flavour Physics

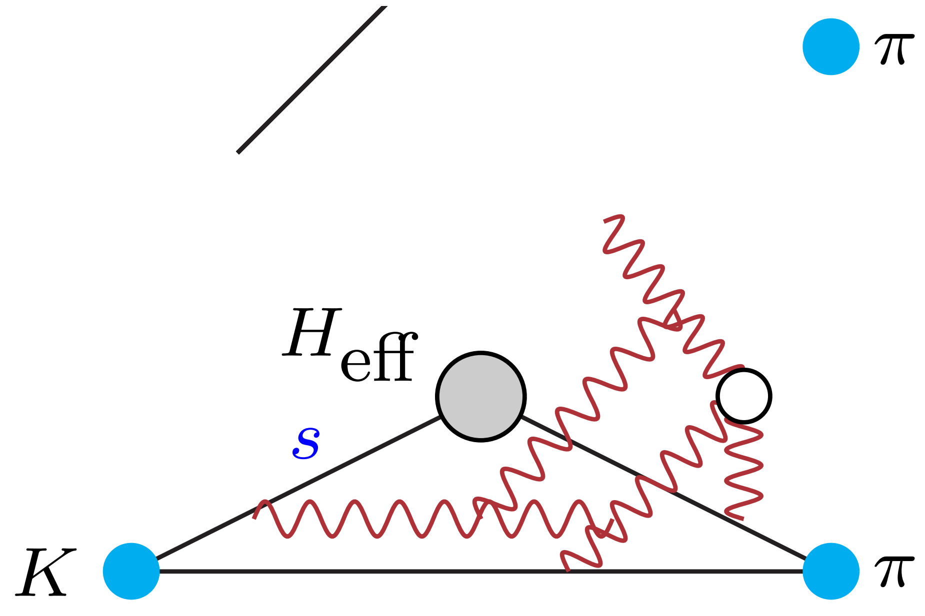
- Precision Flavour Physics, is a key approach, complementary to the large E_T searches at the LHC, in exploring the limits of the standard model and in searches for New Physics.
    - If the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to unravel the underlying framework.
    - The discovery potential of precision flavour physics would also not be underestimated. (In principle, the reach may be about two orders of magnitude deeper than the LHC!

- Precision Flavour Physics requires control of hadronic effects for which Lattice QCD computations are essential.

- For illustration - a schematic diagram of $K \to \pi\pi$ decays:



means



Lattice Flavour Physics

- It is a pleasure to acknowledge the continuing collaboration with Guido Martinelli and colleagues in Rome123, which started in 1986 and which currently counts 86 joint publications.

- It is also a pleasure to acknowledge my continuing collaboration with the RBC/UKQCD collaborations on a number of the topics discussed in this talk.

THE KAON *B*-PARAMETER AND K–π AND K–ππ TRANSITION AMPLITUDES ON THE LATTICE

M.B. GAVELA[1]

Dept. de Fisica Teorica, Universität Autonoma de Madrid, Madrid, Spain

L. MAIANI, S. PETRARCA and F. RAPUANO

Istituto di Fisica "G. Marconi", Università degli Studi di Roma "La Sapienza", Rome, Italy, and INFN, Sezione di Roma, Rome, Italy

G. MARTINELLI

CERN, Geneva, Switzerland

O. PENE

LPTHE, Orsay, France

C.T. SACHRAJDA

University of Southampton, Southampton S09 5NH, UK

Received 22 January 1988

Parton Distribution Amplitudes

- Given the interest in PDAs at this meeting I also mention this early paper.

A LATTICE CALCULATION OF THE SECOND MOMENT OF THE PION'S DISTRIBUTION AMPLITUDE

G. MARTINELLI and C.T. SACHRAJDA [1]

CERN, CH-1211 Geneva 23, Switzerland

Received 13 February 1987

We calculate $\langle \xi^2 \rangle$, the second moment of the pion's distribution amplitude on a $10^3 \times 20$ lattice, with Wilson fermions in the quenched approximation and at $\beta = 6.0$. We find $\langle \xi^2 \rangle = 0.26 \pm 0.13$, in the lattice renormalisation scheme at $a \simeq (1.8\ \mathrm{GeV})^{-1}$. This is in disagreement with the previous lattice determination of this quantity. The reasons for this discrepancy are discussed.

Outline of Talk(s)

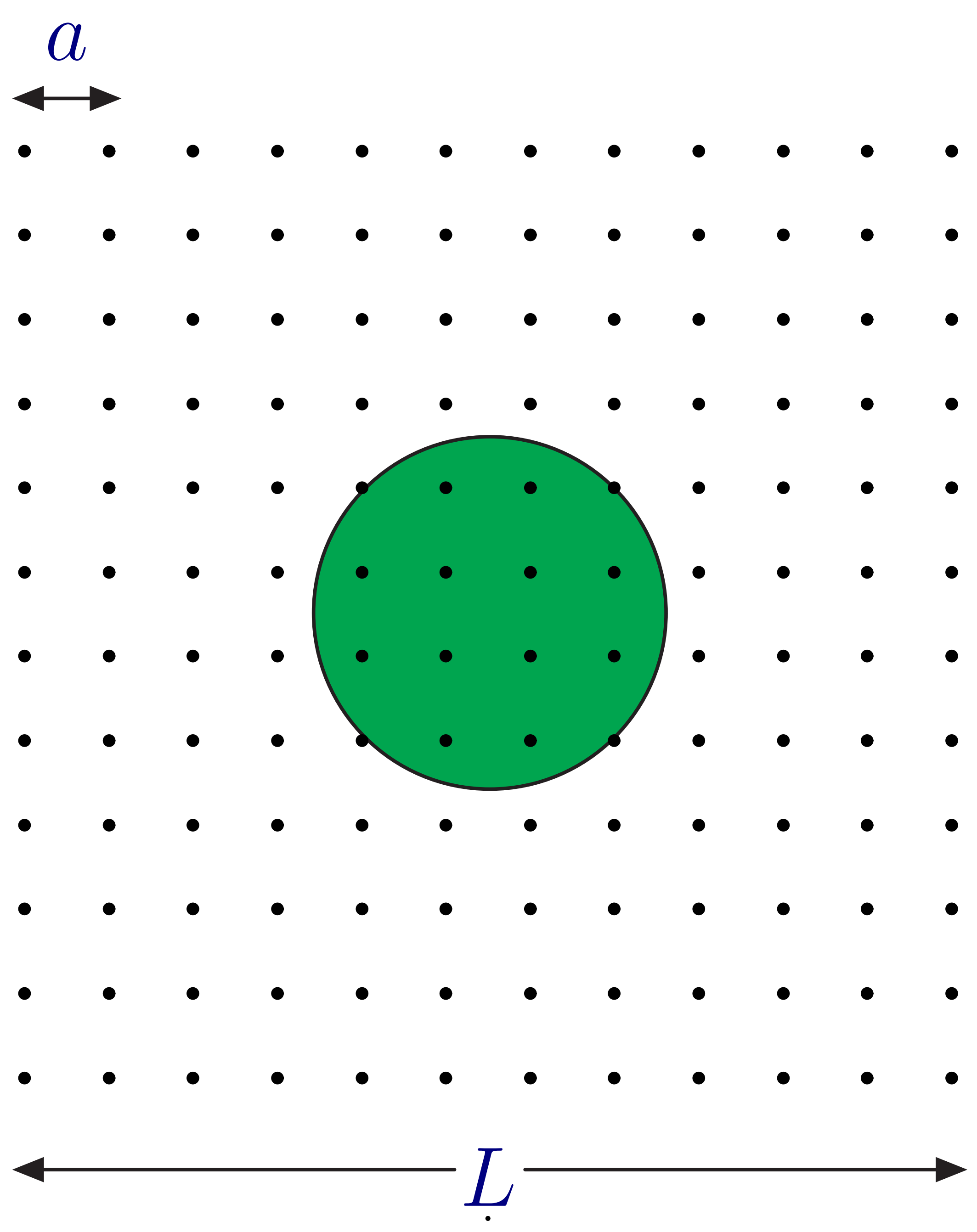
1. Introductory Remarks and Examples

2. Illustrative example: $\bar{B}_s \to \mu^+\mu^-\gamma$ at large q^2 .

R.Frezzotti, **G.Gagliardi,** V.Lubicz, G.Martinelli, CTS, F.Sanfilippo, S.Simula, N.Tantalo, arXiv:2402.03262

3. QED Corrections to Decay Amplitudes
(To be presented at Academia Sinica next Monday)

1. Introductory Remarks and Examples



- Lattice QCD is a general *first-principles* technique used to compute non-perturbative QCD effects in a huge variety of applications.

- In principle the systematic errors are controllable, and can be progressively reduced.

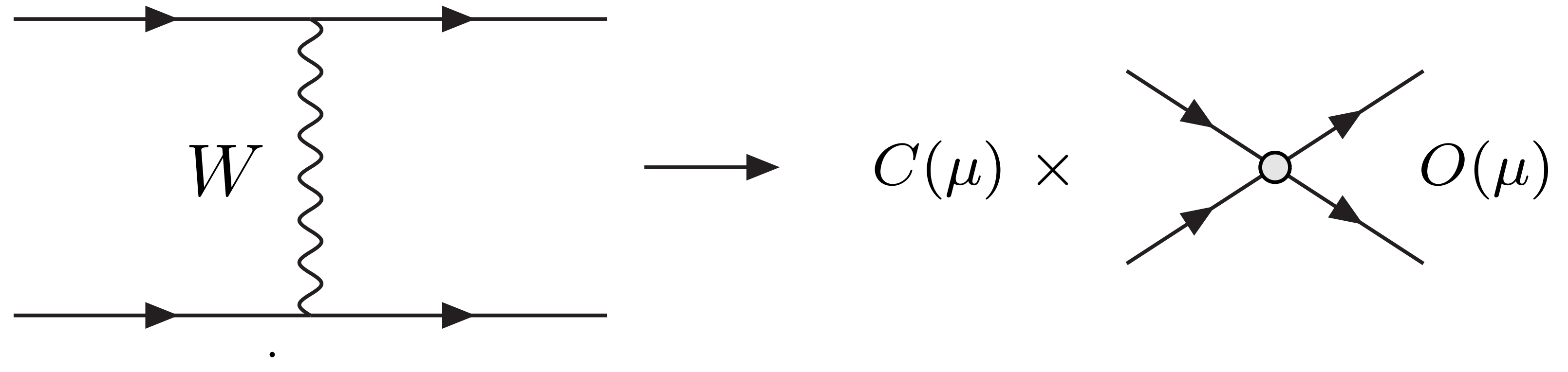
i) Continuum extrapolation $a \to 0$.

ii) Extrapolation to infinite-volume $L \to \infty$.

iii) Minkowski → Euclidean continuation.

- For some simple quantities in spectroscopy and flavour physics, the M→E continuation is not an issue, the discretisation and finite-volume effects are under control and results can be obtained with a precision at the sub-percent level.

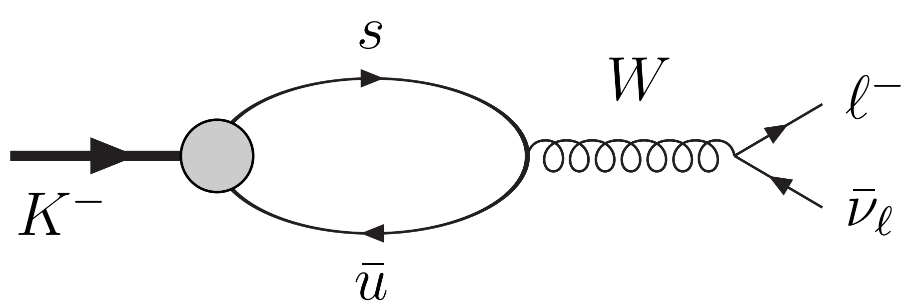
- The lattice spacing a (typically $0.05 - 0.1$ fm) is far too large to allow for propagating W,Z - bosons ⇒ use the Operator Product Expansion.



$C(\mu)$ - perturbative
Matrix element of $O(\mu)$ non-perturbative

Well-studied quantities in lattice kaon physics

1. Leptonic decay constant f_K



$\Rightarrow V_{us}$

$$\langle 0|A_\mu|K(p)\rangle = f_K p_\mu \,,$$

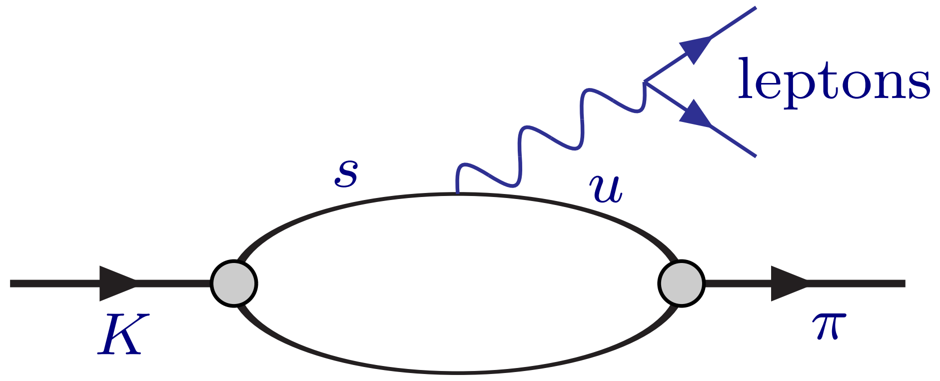
$$\Gamma^{(0)} = \frac{G_F^2|V_{us}|^2 f_K^2}{8\pi} m_K^3 r_\ell^2 \left(1 - r_\ell^2\right)^2$$

$$r_\ell = \frac{m_\ell}{m_K}$$

$f_K = 155.7(3)$ MeV

FLAG Review 2021, Y.Aoki et al., arXiv:2111.09849

2. $K_{\ell 3}$ decays



$\Rightarrow V_{us}$

$$\langle \pi(p_\pi)|\bar{s}\gamma_\mu u|K(p_K)\rangle = f_0(q^2)\frac{m_K^2 - m_\pi^2}{q^2} q_\mu + f_+(q^2)\left[(p_\pi + p_K)_\mu - \frac{m_K^2 - m_\pi^2}{q^2} q_\mu\right]$$

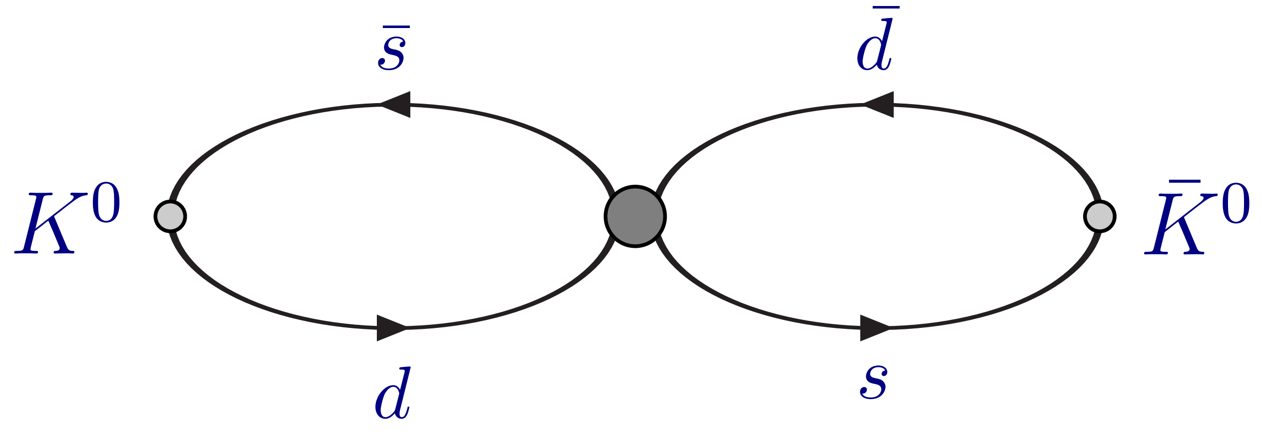
where $q = p_K - p_\pi$.

$f_0(0) = 0.9698(17)$

- Shape of form factor also computed.

FLAG Review 2021, Y.Aoki et al., arXiv:2111.09849 from ETM (arXiv:1602.04113) and FNAL/MILC (arXiv:1809.02827) collaborations.

3. K^0-$\bar{K}^0$ mixing



$$\langle \bar{K}^0|\bar{s}\gamma^\mu(1-\gamma^5)d\;\bar{s}\gamma_\mu(1-\gamma^5)d|K^0\rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

$$\hat{B}_K \equiv \alpha_s(\mu)^{-\gamma_0/2\beta_0}\left(1 + O(\alpha_s(\mu)\right) B_K(\mu)$$

$\hat{B}_K = 0.717(18)(16)$

FLAG Review 2021, Y.Aoki et al., arXiv:2111.09849 from ETM (arXiv:1505.06639) collaboration.

Lattice QCD and Flavour Physics

- In the past, most lattice computations in flavour physics have been of matrix elements of the form

$$\langle\, f \,|\, O(0) \,|\, i \,\rangle$$

where $|\, i \,\rangle$ is a single-hadron state, $|\, f \,\rangle$ is the vacuum or single-hadron state and $O(0)$ is a local composite operator.

- In recent years, together with my collaborators in Rome and in the RBC-UKQCD collaboration, we have been working to extend the range of physical processes for which the hadronic effects can be computed:

- Matrix elements of bifocal operators: $\displaystyle\int d^4y\, \langle\, f \,|\, O_1(0)\, O_2(y) \,|\, i \,\rangle\,.$ For example:

  (i) Δm_K and long distance contributions to ϵ_K. Here O_1 and O_2 are both 4-quark weak operators.

  N.H.Christ, T.Izubuchi, CTS, A.Soni and J.Yu , arXiv:1212.5931; Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni and J.Yu, arXiv:1406.0916
  Z.Bai, N.H.Christ and CTS, EPJ WebConf. 175 (2018) 13017; Z.Bai, N.H.Chris, J.Karpie, CTS, A.Soni and B.Wang, arXiv:2309.01193

  (ii) The rare kaon decays $K \to \pi\ell^+\ell^-$ and $K \to \pi\nu\bar{\nu}$. Here O_1 and O_2 can both be weak operators ($K \to \pi\nu\bar{\nu}$) or a weak operator and an electromagnetic current ($K \to \pi\ell^+\ell^-$).

  N.H.Christ, X.Feng, A.Portelli and CTS, arXiv:1507.03094, arXiv:1605.04442 + a series of numerical studies

- For these processes, the theoretical frameworks have been developed, exploratory numerical computations have been performed, but computations on the next generation of machines will have to be performed to achieve, precise, robust results.

K → $\pi\pi$ Decays

- For these decays $|\,f\,\rangle$ consists of two hadrons which interact in the finite volume.
- $K \to \pi\pi$ decays are a very important class of processes with a long and noble history.
  - It is in these decays that both indirect and direct CP-violation was discovered.
- Bose symmetry ⇒ the two-pion state has isospin 0 or 2 ,

$$_{I=2}\langle \pi\pi\,|\,H_W\,|\,K^0\rangle = A_2\,e^{i\delta_2}\,, \qquad _{I=0}\langle \pi\pi\,|\,H_W\,|\,K^0\rangle = A_0\,e^{i\delta_0}\,.$$

- Among the very interesting issues are the origin of the $\Delta I = 1/2$ rule ($\mathrm{Re}A_0/\mathrm{Re}A_2 \simeq 22.5$) and an understanding of the experimental value of ϵ'/ϵ, the parameter which was the first experimental evidence for direct CP-violation.
- See the following two RBC-UKQCD papers, which however represent the culmination of many years of preparatory work:

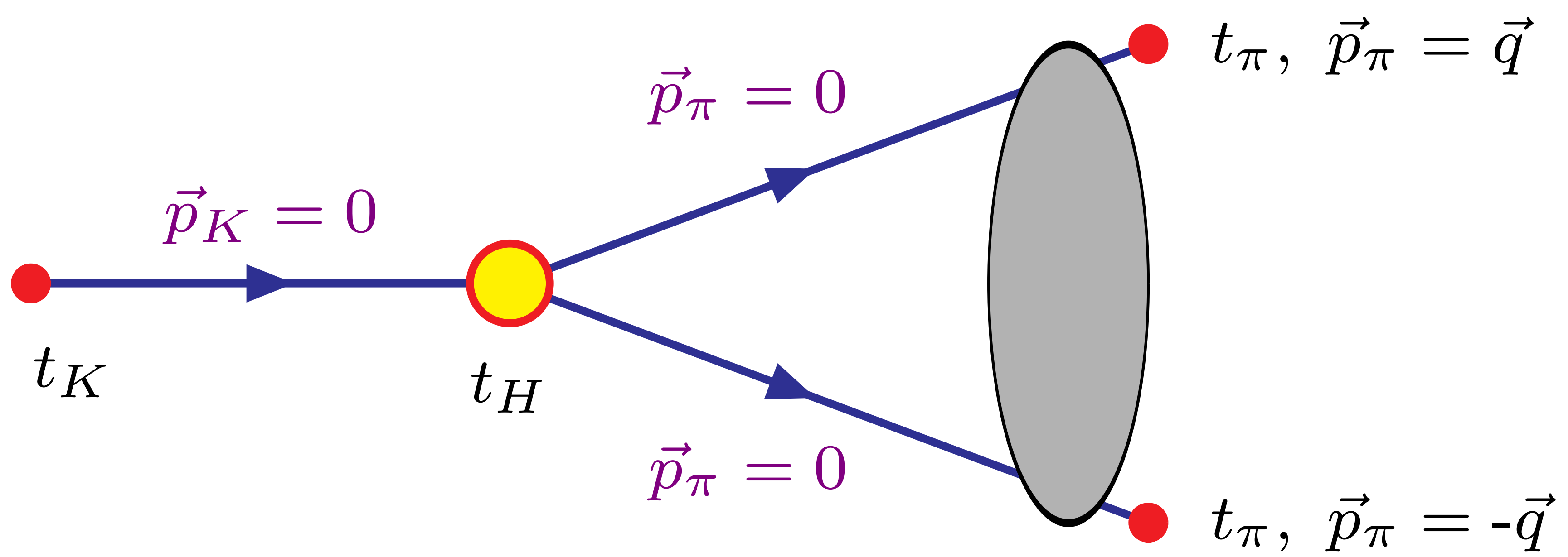
1. *"$K \to \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit"*
T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Janowski, C.Jung, C.Kelly, C.Lehner, A.Lytle, R.D.Mawhinney, CTS., A.Soni, H.Yin, and D.Zhang arXiv:1502.00263

2. *"Direct CP violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay in the Standard Model"*
R.Abbott, T.Blum, P.A.Boyle, M.Bruno, N.H.Christ, D.Hoying, C.Jung, C.Kelly, C.Lehner, R.D.Mawhinney, D.J.Murphy, CTS, A. Soni, M.Tomii and T.Wang, arXiv:2004.09440

(Building on RBC-UKQCD, Z.Bai et al. arXiv:1505.07863)

  - Detailed references to earlier work can be found in these papers.

Why are the amplitudes difficult to compute?



- $K \to \pi\pi$ correlation function is dominated by the lightest intermediate state. L.Maiani and M.Testa, Phys.Lett. B245 (1990) 585
  - With periodic boundary conditions this is the $\pi\pi$ state with both pions at rest for A_2 and the vacuum state for A_0 .
  - We have chosen to use anti periodic boundary conditions for the d-quark for A_2 and G-parity boundary conditions for A_0 .
  - Work is in progress to compute the amplitudes with periodic boundary conditions with excited $\pi\pi$ states. M.Tomii, Lattice 2023
- Volume must be tuned to ensure $E_{\pi\pi} = m_K$.
  - Moreover, the s-wave $I = 0$ and $I = 2$ channels are attractive and repulsive respectively and so the two cases must be treated separately.
- Finite-volume effects are not exponentially small and must be corrected. L.Lellouch and M.Lüscher, hep-lat/00030023, C.J.D.Lin, G.Martinelli, CTS and M.Testa, hep-lat/0104006 C-h.Kim, CTS and S.Sharpe, hep-lat/0507006

Summary of our Results

- $\mathrm{Re}\,A_0 = 2.99\,(0.32)\,(0.59)\times 10^{-7}\,\mathrm{GeV}$ (Experiment $3.3201(18)\times 10^{-7}\,\mathrm{GeV}$);

  $\mathrm{Im}\,A_0 = -\,6.98\,(0.62)\,(1.44)\times 10^{-11}\,\mathrm{GeV}$.

- $\mathrm{Re}\,A_2 = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}}\times 10^{-8}\,\mathrm{GeV}$, (Experiment $1.4787(31)\times 10^{-8}\,\mathrm{GeV}$) ;

  $\mathrm{Im}\,A_2 = -\,6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}}\times 10^{-13}\,\mathrm{GeV}$.

- We find $\dfrac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} = 19.9 \pm 2.3 \pm 4.4$ in good agreement with the experimental result of 22.45(6) .

- Combining the result for $\mathrm{Im}\,A_0$ and $\mathrm{Im}\,A_2$ and using the experimental results for the real parts we obtain

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = 0.00217\,(26)_{\mathrm{stat}}\,(62)_{\mathrm{syst}}\,(50)_{\mathrm{IB}}\,.$$

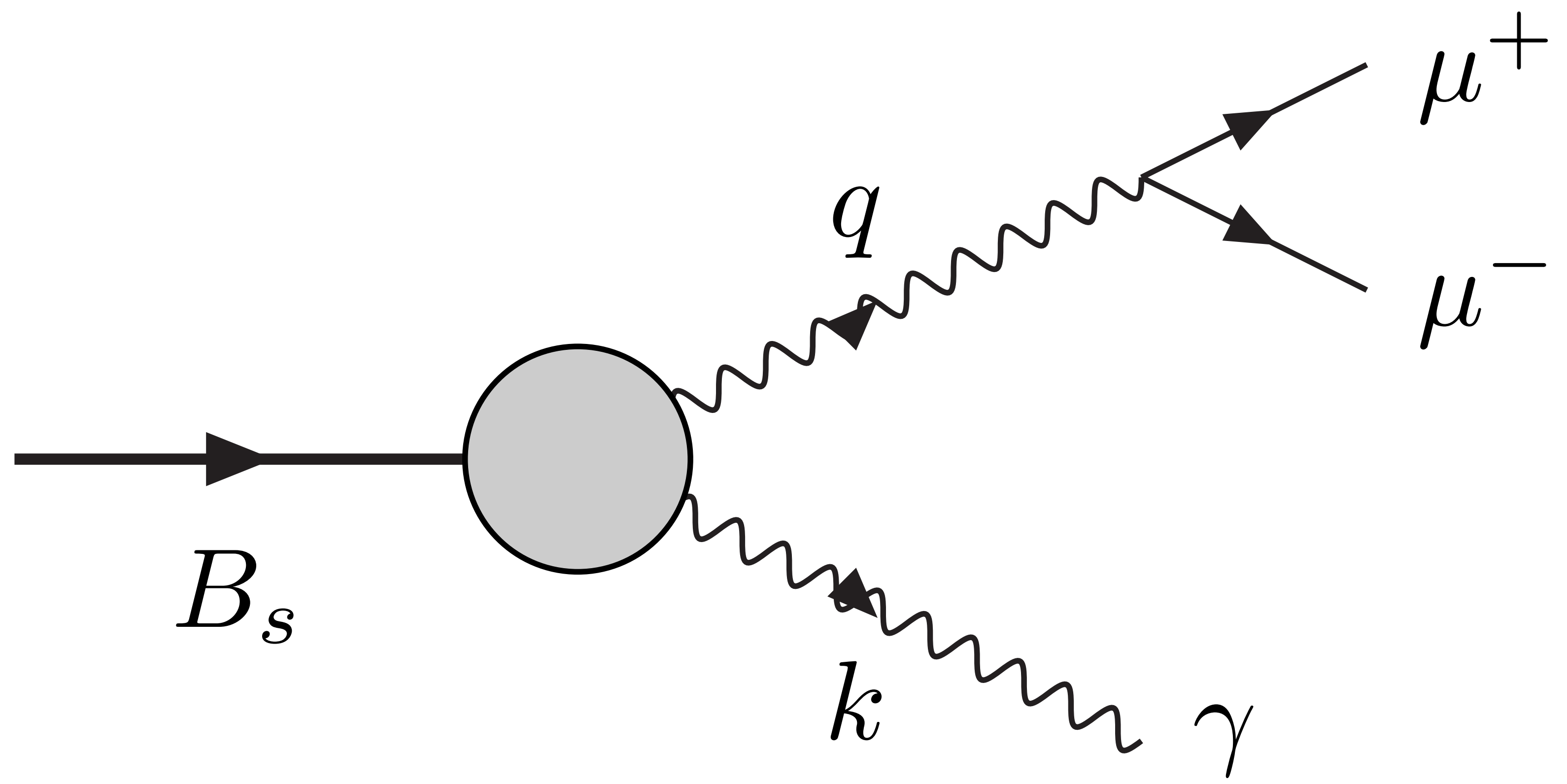
  The result is consistent with the experimental value of 0.00166 (23) .

- The RBC/UKQCD Collaboration continues work to reduce the uncertainties. Important priority is to control the IB effects.

2. The $B_s \to \mu^+\mu^-\gamma$ Decay Rate at Large q^2

R.Frezzotti, **G.Gagliardi,** V.Lubicz, G.Martinelli, CTS, F.Sanfilippo, S.Simula, N.Tantalo, arXiv:2402.03262

- I use this interesting FCNC process to illustrate the elements which we are able to compute and to highlight the important theoretical issues which we are still working to resolve.
    - Preview: We can compute the dominant contribution, but are working to solve the problems which will enable an improved precision.



$x_\gamma = \dfrac{2E_\gamma}{m_{B_s}}$, E_γ is the energy of the real photon in rest frame of the B_s meson.

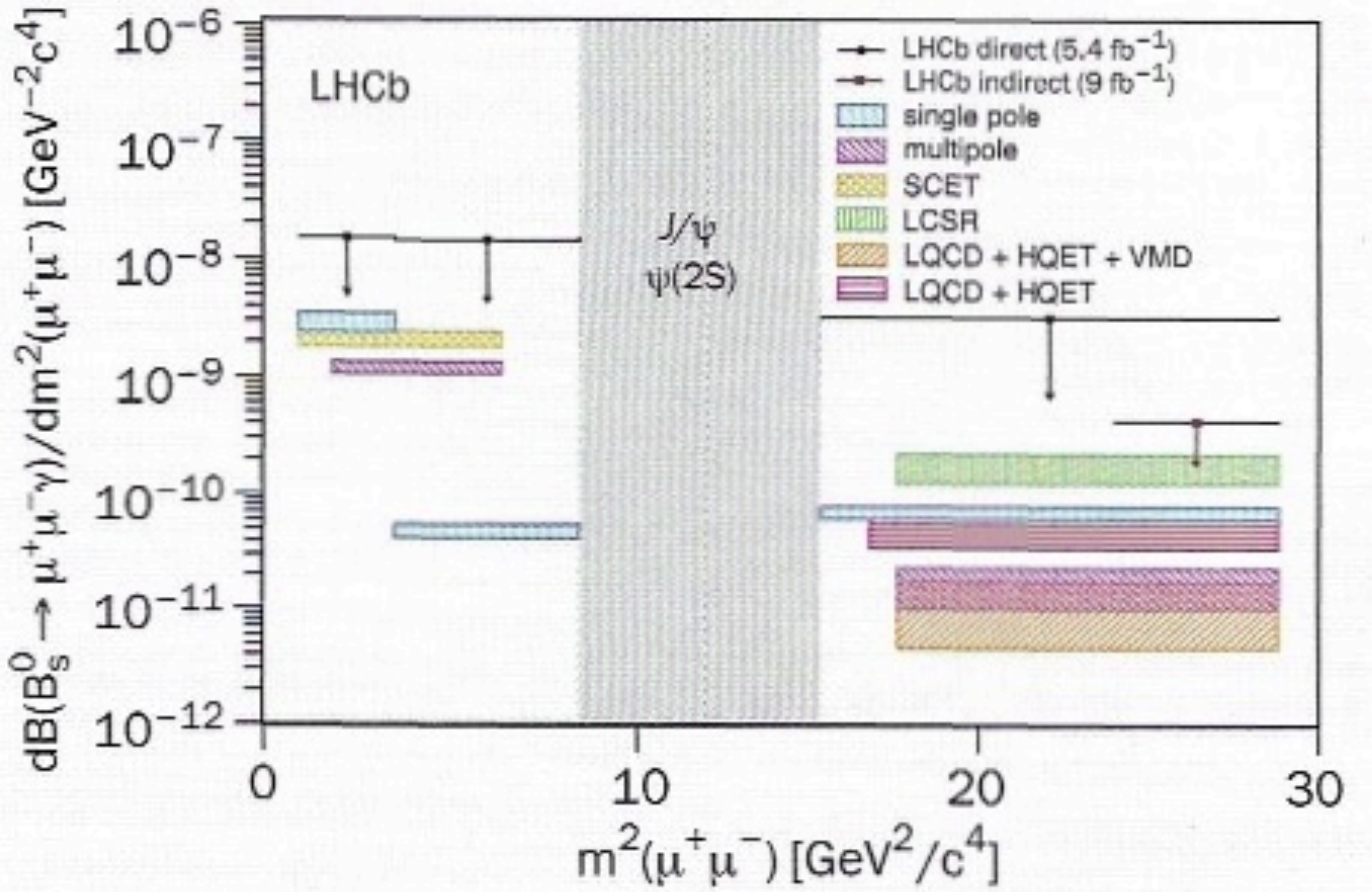
$$q^2 = m_{B_s}^2(1 - x_\gamma), \qquad 0 \le x_\gamma \le 1 - \frac{4m_\mu^2}{m_{B_s}^2}$$

- LHCb: $B(B_s \to \mu^+\mu^-\gamma)|_{\sqrt{q^2}>4.9\,\mathrm{GeV}} < 2.0 \times 10^{-9}$, arXiv:2108.09283/4

From the May/June 2024 issue of the Cern Courier

LHCb targets rare radiative decay

Rare radiative b-hadron decays are powerful probes of the Standard Model (SM) sensitive to small deviations caused by potential new physics in virtual loops. One such process is the decay of $B_s^0 \to \mu^+\mu^-\gamma$. The dimuon decay of the B_s^0 meson is known to be extremely rare and has been measured with unprecedented precision by LHCb and CMS. While performing this measurement, LHCb also studied the $B_s^0 \to \mu^+\mu^-\gamma$ decay, partially reconstructed due to the missing photon, as a background component of the $B_s^0 \to \mu^+\mu^-$ process and set the first upper limit on its branching fraction to 2.0×10^{-9} at 95% CL (red arrow in figure 1). However, this search was limited to the high-dimuon-mass region, whereas several theoretical extensions of the SM could manifest themselves in lower regions of the dimuon-mass spectrum. Reconstructing the photon is therefore essential to explore the spectrum thoroughly and probe a wide range of physics scenarios.



Source: LHCb

Fig. 1. *95% confidence limits on differential branching fractions for $B_s^0 \to \mu^+\mu^-\gamma$ in intervals of dimuon mass squared (q^2). The shaded boxes illustrate SM predictions for the process, according to different calculations.*

The LHCb collaboration now reports the first search for the $B_s^0 \to \mu^+\mu^-\gamma$ decay with a reconstructed photon, exploring the full dimuon mass spectrum. Photon reconstruction poses additional experimental challenges, such as degrading the mass resolution of the B_s^0 candidate and introducing additional background contributions. To cope with this ambitious search, machine-learning algorithms and new variables have been specifically designed with the aim of discriminating the signal among background processes with similar signatures. The analysis ▷

The Effective $b \to s$ Hamiltonian

$$\mathscr{H}^{b\to s}_{\text{eff}} = 2\sqrt{2} G_F V_{tb} V^*_{ts} \left[\sum_{i=1,2} C_i O^c_i + \sum_{i=3}^{6} C_i O_i + \frac{\alpha_{\text{em}}}{4\pi} \sum_{i=7}^{10} C_i O_i \right]$$

$$O^c_1 = (\bar{s}_i \gamma^\mu P_L c_j)\,(\bar{c}_j \gamma_\mu P_L b_i) \qquad O^c_2 = (\bar{s} \gamma^\mu P_L c)\,(\bar{c} \gamma_\mu P_L b)$$

$$\left(P_{L,R} = \frac{1}{2}(1 \mp \gamma^5) \right)$$

O_{3-6} are QCD Penguins with small Wilson Coefficients

$$O_7 = -\frac{m_b}{e}(\bar{s}\sigma^{\mu\nu} F_{\mu\nu} P_R b) \qquad O_8 = -\frac{g_s m_b}{4\pi\alpha_{\text{em}}}(\bar{s}\sigma^{\mu\nu} G_{\mu\nu} P_R b)$$

$F_{\mu\nu}$ and $G_{\mu\nu}$ are the QED and QCD Field Strength Tensors

$$O_9 = (\bar{s}\gamma^\mu P_L b)\,(\bar{\mu}\gamma_\mu \mu) \qquad O_{10} = (\bar{s}\gamma^\mu P_L b)\,(\bar{\mu}\gamma_\mu \gamma^5 \mu)$$

The amplitude is given by:

$$\mathscr{A} = \langle \gamma(k,\epsilon)\,\mu^+(p_1)\,\mu^-(p_2)\,|\, -\mathscr{H}^{b\to s}_{\text{eff}} \,|\, B_s(p)\rangle_{\text{QCD+QED}}$$

$$= -e\frac{\alpha_{\text{em}}}{\sqrt{2}\pi} V_{tb} V^*_{ts}\, \epsilon^*_\mu \left[\sum_{i=1}^{9} C_i H^{\mu\nu}_i L_{V\nu} + C_{10}\left(H^{\mu\nu}_{10} L_{A\nu} - i\frac{f_{B_s}}{2} L^{\mu\nu}_A p_\nu \right) \right]$$

The $H^{\mu\nu}$ and L are hadronic and leptonic tensors respectively

Contribution from "Semileptonic" Operators - F_V and F_A

$$H_9^{\mu\nu}(p\,.\,k) = H_{10}^{\mu\nu}(p\,.\,k) = i\int d^4y \langle\, 0\,|\, T\big[\,\bar{s}\,\gamma^\nu\, P_L\, b\,(0)\;\, J^\mu_{\rm em}(y)\big]\,|\,\bar{B}_s(p)\,\rangle$$

$$= -\, i(g^{\mu\nu}\,(k\cdot q) - q^\mu k^\nu)\,\frac{F_A(q^2)}{2m_{B_s}} + \epsilon^{\mu\nu\rho\sigma}\,k_\rho\, q_\sigma\,\frac{F_V(q^2)}{2m_{B_s}}$$

- These form factors can be computed from Euclidean correlation functions (at accessible values of m_b).
- We choose $\mathbf{p} = \mathbf{0}$ and $\mathbf{k} = (0,0,k_z)$ and use twisted boundary conditions for k_z.
- With such a choice of kinematics: $\frac{1}{2k_z}\left(H_V^{12}(t,k) - H_V^{21}(t,k)\right) \to F_V(x_\gamma)$ and $\frac{i}{2E_\gamma}\left(H_A^{11}(t,k) + H_A^{22}(t,k)\right) \to F_A(x_\gamma)\,,$

where t is the temporal position of the weak current.

The form factors F_{TV} and F_{TA}

- In a similar way the following contributions can be computed:

$$H_{7A}^{\mu\nu}(p\,.\,k) = \frac{2m_b}{q^2}\int d^4y \langle\, 0\,|\, T\big[\,\bar{s}\,\sigma^{\nu\rho}\,P_R\,b\,(0)\;J^{\mu}_{\rm em}(y)\big]\,|\,\bar{B}_s(p)\,\rangle$$

$$= -\,i(g^{\mu\nu}\,(k\cdot q) - q^{\mu}k^{\nu})\,\frac{m_bF_{TA}(q^2)}{q^2} + \epsilon^{\mu\nu\rho\sigma}\,k_\rho\,q_\sigma\,\frac{m_bF_{TV}(q^2)}{q^2}$$

- Here, for now, we are isolating the contribution in which it is the virtual photon which is emitted from O_7 .

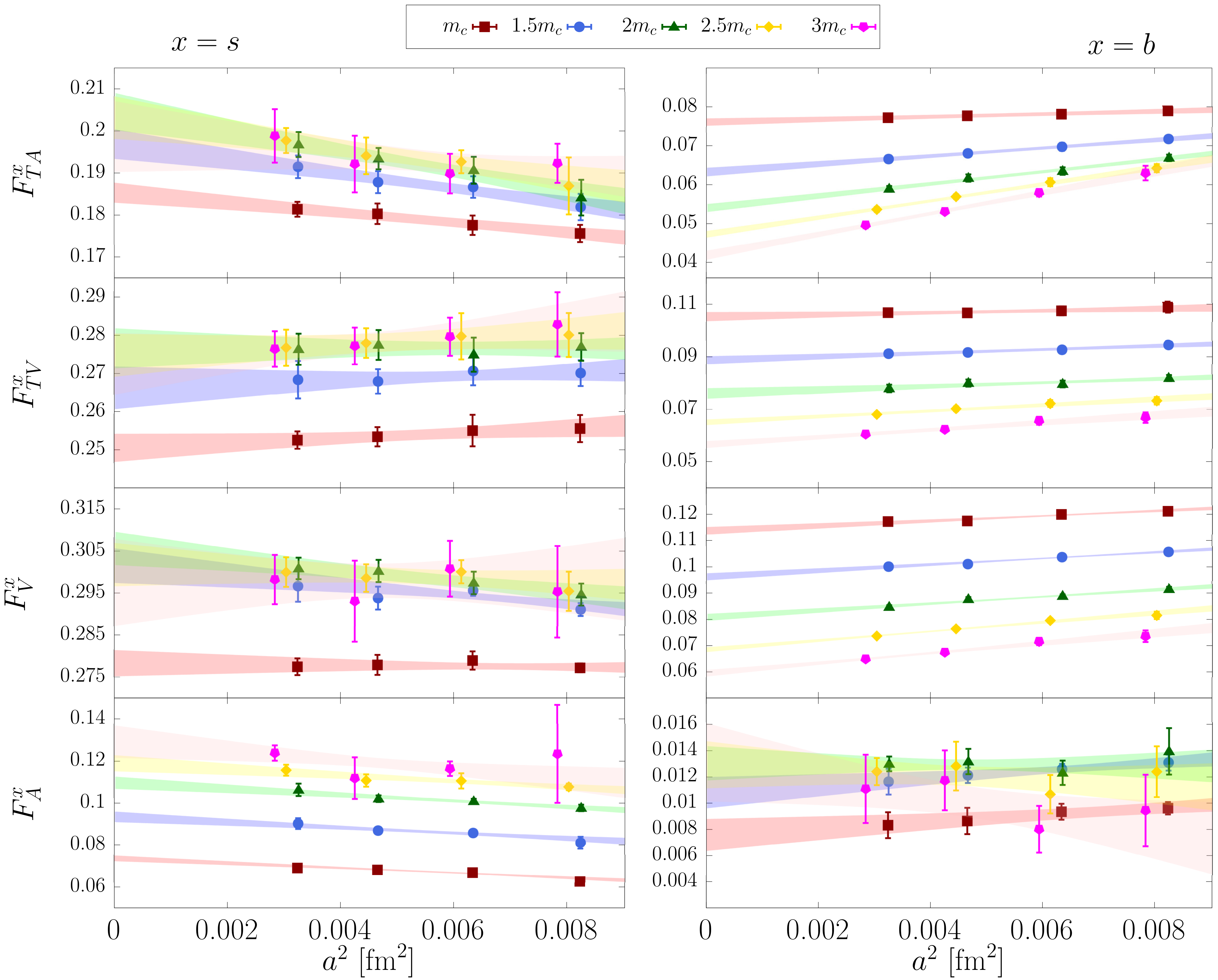
- With our choice of kinematics: $\frac{1}{2k_z}\left(H_{TV}^{12}(t,k) - H_{TV}^{21}(t,k)\right) \to F_{TV}(x_\gamma)$ and $\frac{-i}{2E_\gamma}\left(H_A^{11}(t,k) + H_A^{22}(t,k)\right) \to F_{TA}(x_\gamma)\,.$

- There is also the useful kinematical constraint that $F_{TV}(1) = F_{TA}(1)$.

Numerical Results for F_V , F_A , F_{TV} , F_{TA}

- These four form-factors can be computed using “standard” methods at the available heavy quark masses.
- We use gauge field configurations generated by the European Twisted Mass Collaboration (ETMC), with the Iwasaki gluon action and $N_f = 2 + 1 + 1$ flavours of Wilson-Clover light quarks at maximal twist (four ensemble with $0.057 \text{ fm} < a < 0.091 \text{ fm}$).
- We perform the calculations at 5 values of the heavy quark mass corresponding to and at 4 values of $x_\gamma = 0.1,\ 0.2,\ 0.3,\ 0.4$.
- Much effort is then devoted to the $m_h \to m_b$ and $a \to 0$ limit, guided by the heavy-quark scaling laws and models for possible resonant contributions.

Continuum Extrapolation



- The continuum extrapolation is performed separately at each value of m_{H_s} and x_γ .
- The illustration plots are for $x_\gamma = 0.4$.

Extrapolation of the results to $m_{B_s} = 5.367$ GeV

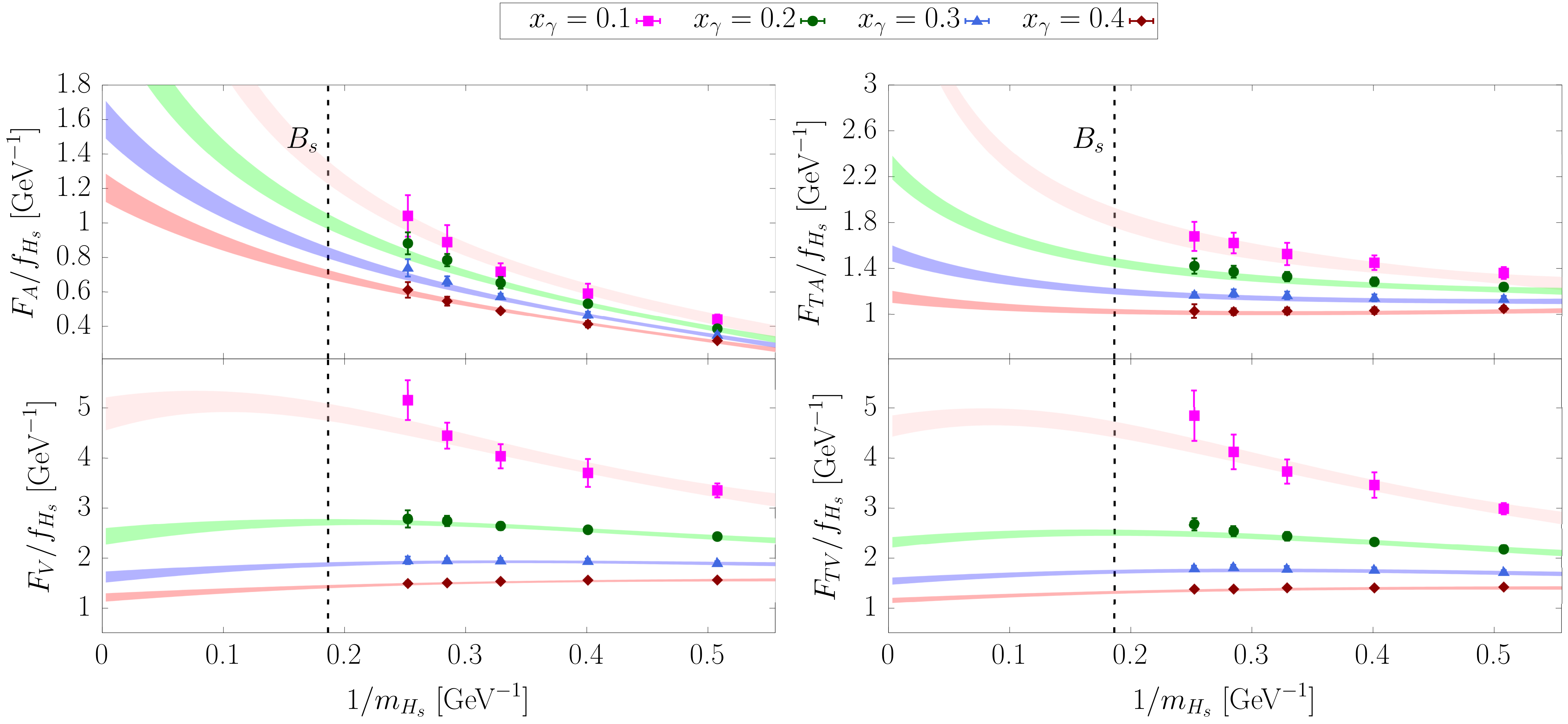
- Having performed the continuum extrapolation, we need to extrapolate the results to the physical value of m_{B_s}.
- In the heavy-quark and large E_γ limits, scaling laws were derived up to $O(1/m_{H_s}, 1/E_\gamma)$:

M.Beneke and J.Rohrwild, arXiv:1110.3228;
M. Beneke, C. Bobeth and Y.-M. Wang, arXiv:2008.12494

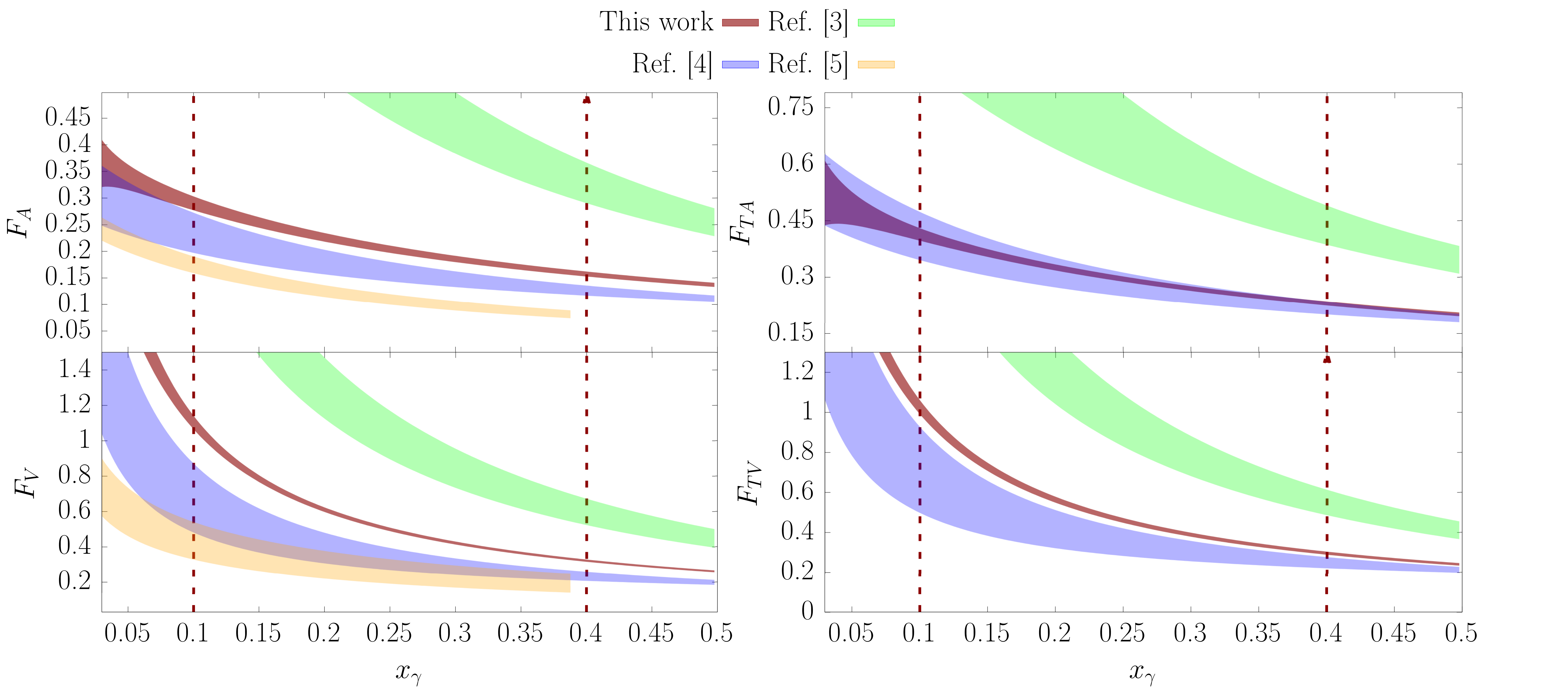
$$\frac{F_{V/A}}{f_{H_s}} = \frac{|q_s|}{x_\gamma}\left(\frac{R(E_\gamma,\mu)}{\lambda_B(\mu)} + \xi(x_\gamma, m_{H_s}) \pm \frac{1}{m_{H_s}x_\gamma} \pm \frac{|q_b|}{|q_s|}\frac{1}{m_h}\right) \quad ; \quad \frac{F_{TV/TA}}{f_{H_s}} = \frac{|q_s|}{x_\gamma}\left(\frac{R_T(E_\gamma,\mu)}{\lambda_B(\mu)} + \xi(x_\gamma, m_{H_s}) \pm \frac{1-x_\gamma}{m_{H_s}x_\gamma} + \frac{|q_b|}{|q_s|}\frac{1}{m_{H_s}}\right)$$

- $R(E_\gamma,\mu)$, $R_T(E_\gamma,\mu)$ are radiative correction factors $= 1 + O(\alpha_s)$; λ_B is the first inverse moment of the B_s-meson LCDA, $\xi(x_\gamma, m_{H_s})$ are power corrections.
- Photon emission from the b-quark suppressed relative to the emission from the s-quark.
- Tensor form-factors are presented in the $\overline{\text{MS}}$ scheme at $\mu = 5$ GeV.
- However, useful though these scaling laws are, they apply at large E_γ (as well as large m_h), are there are significant corrections at our lightest values of m_h and smaller values of E_γ. We therefore us an ansatz which includes the above scaling laws at large E_γ as well as VDM behaviour.

Extrapolation of the results to $m_{B_s} = 5.367$ GeV



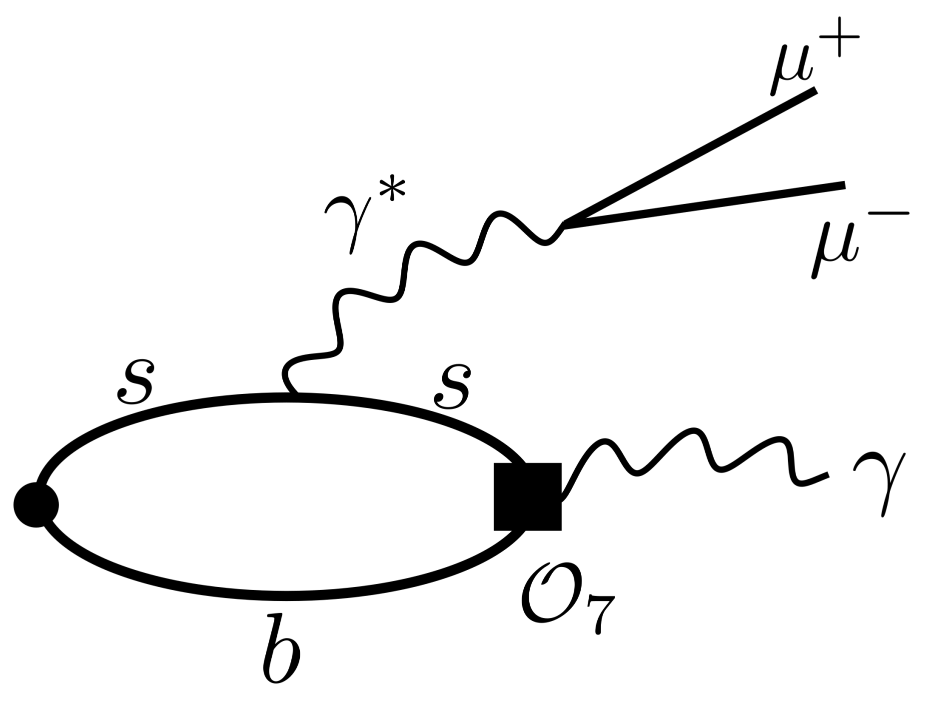
Comparison with Previous Determinations of the Form Factors

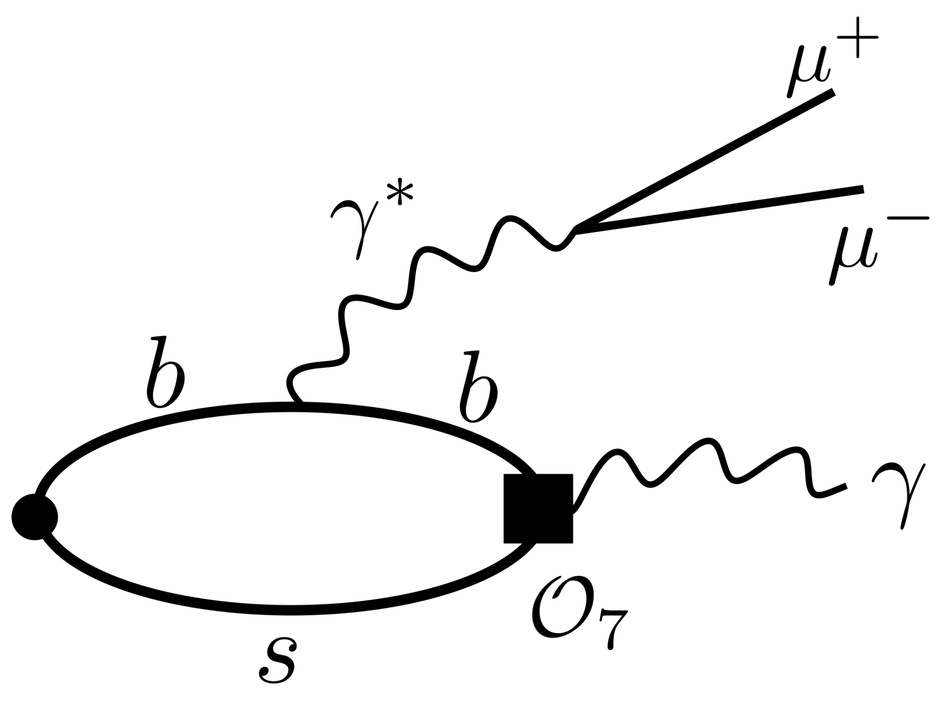


- Ref.[3] = T.Janowski, B.Pullin and R.Zwicky, arXiv:2106.13616, LCSR
- Ref.[4]= A.Kozachuk, D.Melikhov and N.Nikitin, arXiv:1712.07926, relativistic dispersion relations
- Ref.[5]= D.Guadagnoli, C.Normand, S.Simula and L.Vittorio, arXiv:2303.02174, VMD+quark model+lattice at charm

- In general our results for the form factors differ significantly from earlier estimates.

Other Contributions





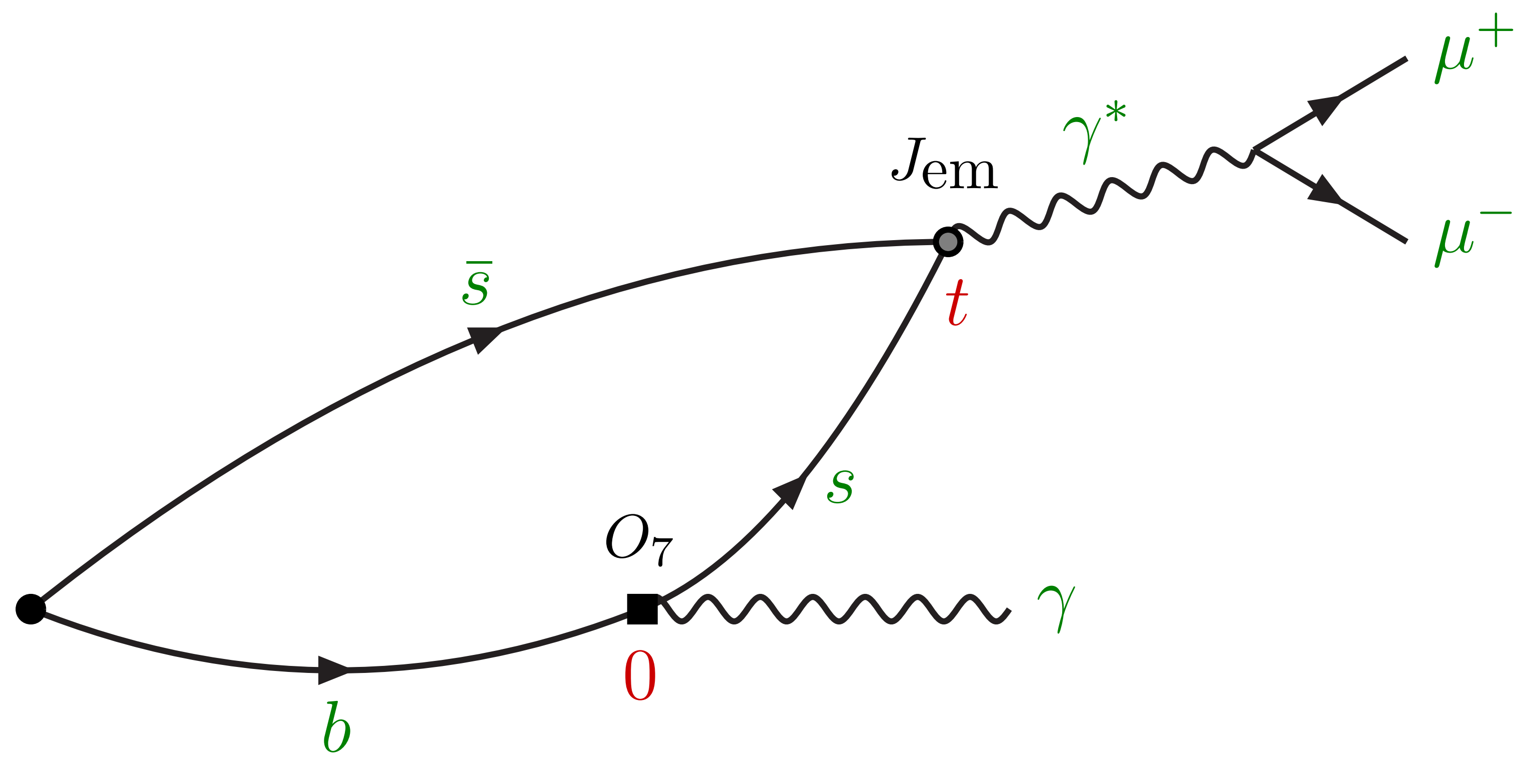
$$H^{\mu\nu}_{\bar{T}}(p,k) = i\int d^4y\; e^{i(p-k)\cdot y}\,\langle\, 0 \,|\, T\big[J^{\nu}_{\bar{T}}(0)\;J^{\mu}_{\rm em}(y)\big]\,|\,\bar{B}_s(\mathbf{0})\,\rangle \equiv -\,\epsilon^{\mu\nu\rho\sigma}\,k_\rho\,p_\sigma \frac{\bar{F}_T}{m_{b_s}} \quad \text{where}$$

$$J^{\nu}_{\bar{T}} = -\,i\,Z_T(\mu)\;\bar{s}\sigma^{\nu\rho}b\;\frac{k^\rho}{m_{B_s}}\;.$$

- The difficulty arises from the first diagrams above when $t_y > 0$.
- In that case we potentially have a hadronic intermediate state (e.g. an $s\bar{s}$ 1^- state) with smaller mass than $\sqrt{(p-k)^2}$, leading to an imaginary part and problems with the continuation to Euclidean space.

$$\sqrt{m_V^2 + E_\gamma^2} + E_\gamma < m_{B_s} \;\Rightarrow\; x_\gamma < 1 - \frac{m_V^2}{m_{B_s}^2} \simeq 1 - \frac{4m_K^2}{m_{B_s}^2} \simeq 0.96\,.$$

$\bar{\mathbf{F}}_{\mathbf{T}}$ (cont.)



- Large amount of effort is being devoted to developing techniques based on the spectral density representation,

M.Hansen, A.Lupo and N.Tantalo, arXiv:1903.06476
R.Frezzotti et al., arXiv:2306.07228

- For $t > 0$ define $C_s(t, \mathbf{k}) = \langle\, 0\,|\, J^{\mu}_{\text{em},s}(t, -\mathbf{k})\, J^{\nu}_{\bar{T}}(0)\,|\, B_s(\mathbf{0})\rangle \;=\; \int_{-\infty}^{\infty} dt'\, \delta(t'-t)\, C_s(t', -\mathbf{k})$

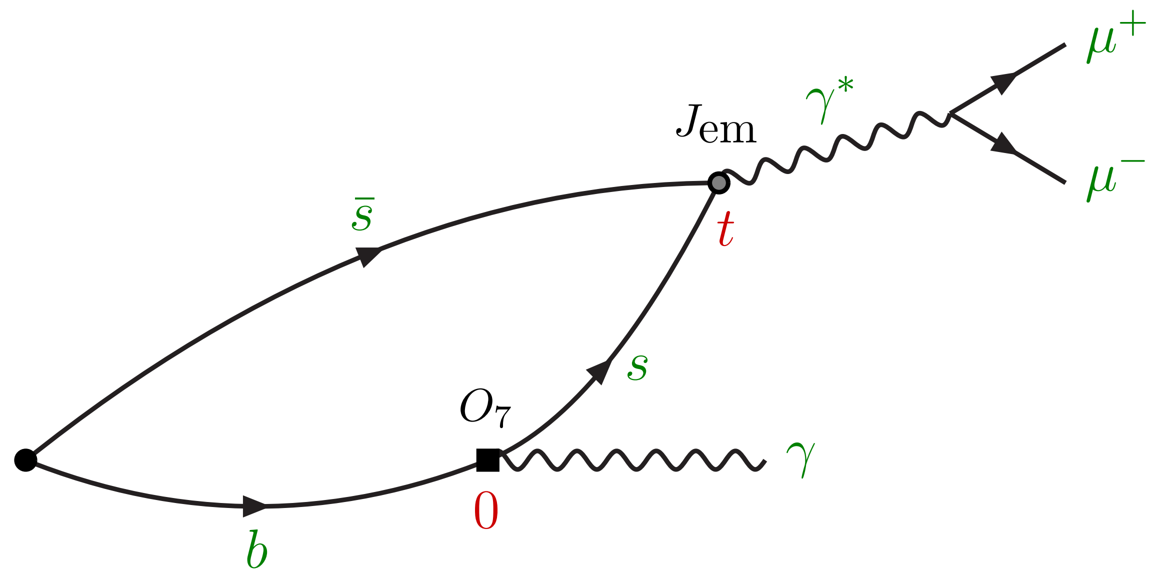
$$= \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} \frac{dE'}{2\pi}\, e^{iE'(t'-t)}\, C_s(t', -\mathbf{k}) = \int_{-\infty}^{\infty} \frac{dE'}{2\pi}\, e^{-iE't} \int d^4x'\, e^{ik'\cdot x'} \langle 0\,|\, J^{\mu}_{\text{em},s}(x')\, J^{\nu}_{\bar{T}}(0)\,|\, B(\mathbf{0})\rangle \qquad \big(k' = (E', -\mathbf{k})\big)$$

$$= \int_{-\infty}^{\infty} \frac{dE'}{2\pi}\, e^{-iE't} \int d^4x' \langle 0\,|\, J^{\mu}_{\text{em},s}(0)\, e^{-i(\hat{P}-k')\cdot x'}\, J_{\bar{T}}T^{\nu}(0)\,|\, B(\mathbf{0})\rangle = \int_{-\infty}^{\infty} \frac{dE'}{2\pi}\, e^{-iE't} \underbrace{\langle 0\,|\, J^{\mu}_{\text{em},s}(0)\, (2\pi)^4\, \delta(\hat{P}-k')\, J^{\nu}_{\bar{T}}(0)\,|\, B(\mathbf{0})\rangle}_{\rho_s(E', \mathbf{k})}$$

$$\equiv \int_{-\infty}^{\infty} \frac{dE'}{2\pi}\, e^{-iE't}\, \rho_s^{\mu\nu}(E', \mathbf{k})$$

- In Euclidean space $C_s(t, \mathbf{k}) = \int_{E^*}^{\infty} \frac{dE'}{2\pi}\, e^{-E't}\, \rho_s^{\mu\nu}(E', \mathbf{k})\,.$

$\bar{\mathrm{F}}_{\mathrm{T}}$ (cont.)



- For $t>0$ define $C_s(t,\mathbf{k}) = \langle 0\,|\,J^{\mu}_{\mathrm{em},s}(t,-\mathbf{k})\,J^{\nu}_{T}(0)\,|\,B_s(\mathbf{0})\rangle \;=\; \int_{E^*}^{\infty}\frac{dE'}{2\pi}\,e^{-iE't}\,\rho_s^{\mu\nu}(E',k)\,.$
- In Euclidean space $C_s(t,\mathbf{k}) \;=\; \int_{E^*}^{\infty}\frac{dE'}{2\pi}\,e^{-E't}\,\rho_s^{\mu\nu}(E',k)\,.$

- For the amplitude we require

$$H^{\mu\nu}_{\bar{T}_s}(m_B,\mathbf{k}) \;=\; i\int_0^{\infty} dt\; e^{i(m_B-\omega)t}\,C_s^{\mu\nu}(t,\mathbf{k}) \;=\; \lim_{\epsilon\to 0}\int_{E^*}^{\infty}\frac{dE'}{2\pi}\,\frac{\rho_s^{\mu\nu}(E',\mathbf{k})}{E'-(m_B-\omega)-i\epsilon}\,. \qquad \big(\omega=|\,\mathbf{k}\,|\big)$$

- The question is how (best) to extract the information about the spectral density, $\rho_s^{\mu\nu}(E,k)$, contained in the Euclidean correlation function in order to determine the amplitude (both the real and imaginary parts).

- We use the HLT method, in which computations are performed at several values of ϵ, and the kernel $\dfrac{1}{E'-(m_B-\omega)-i\epsilon}$ is approximated by a series of exponentials in time.

$$\frac{1}{E'-E-i\epsilon} \simeq \sum_{n=1}^{n_{\rm max}} g_n(E,\epsilon)\,e^{-anE'}$$ where the g_n are complex coefficients.

- Finally $H^{\mu\nu}_{\bar{T}_s}(m_B,\mathbf{k}) \;=\; \lim_{\epsilon\to 0}\int_{E^*}^{\infty}\frac{dE'}{2\pi}\,\frac{\rho_s^{\mu\nu}(E',\mathbf{k})}{E'-(m_B-\omega)-i\epsilon} \;=\; \lim_{\epsilon\to 0}\sum_{n=1}^{n_{\rm max}} g_n(m_B-\omega,\epsilon)\,C_s(an,\mathbf{k})$

$\bar{\mathbf{F}}_{\mathbf{T}}$ (cont.)

- Determining the g_n requires a balance between the systematic error due to the approximation of $1/(E' - E - i\epsilon)$ by a finite number of exponentials (in which the coefficients are large with alternating signs) and the statistical errors in the correlation functions $C_s(an, \mathbf{k})$.

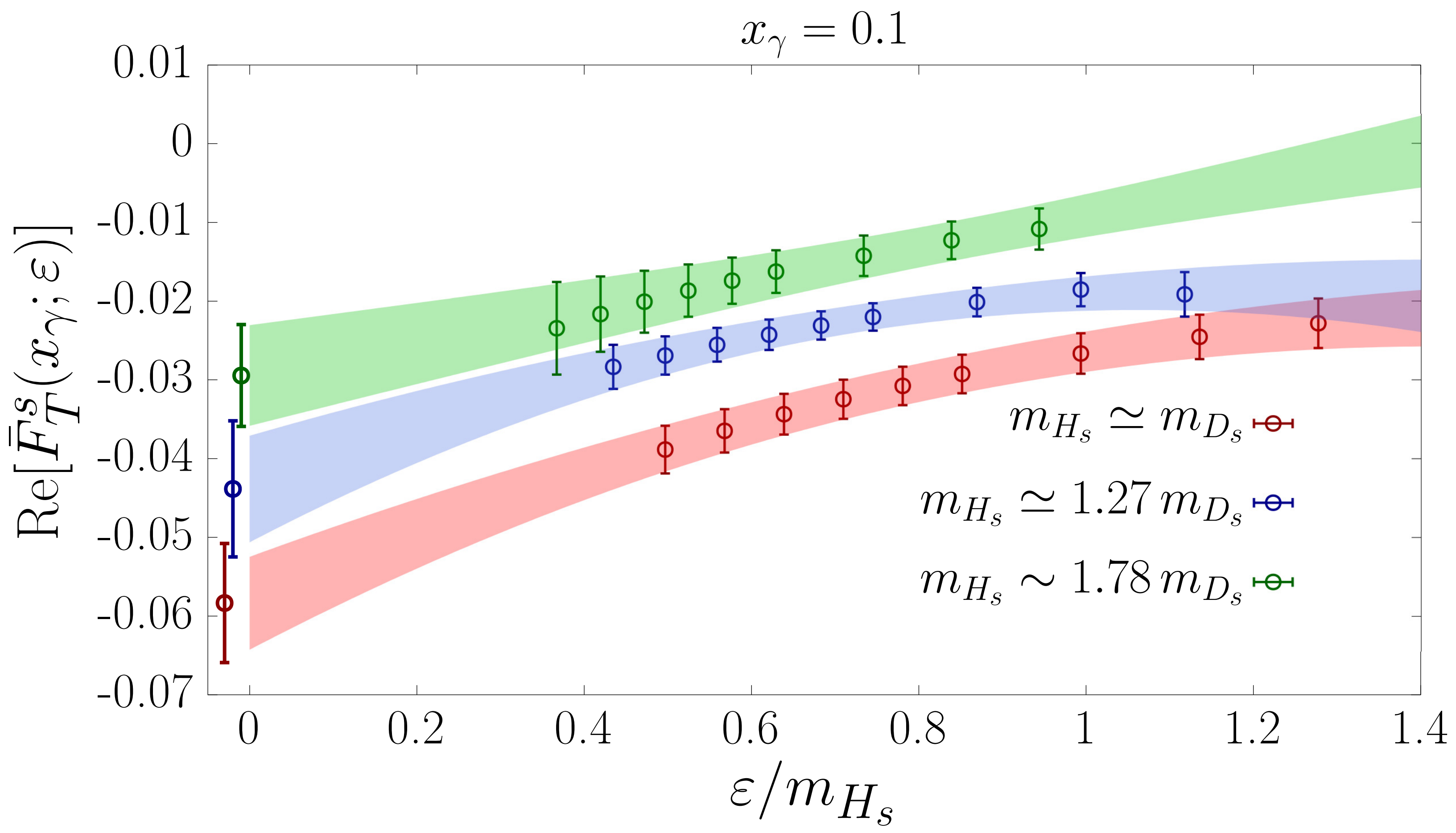
- We have computed $\bar{F}_T$ at all four values of x_γ, at three of the five values of m_h $(m_h/m_c = 1, 1.5, 2.5)$ and on two of the gauge-field ensembles ($a = 0.0796(1)$ fm and $0.0569(1)$ fm).
  - i) $\bar{F}_T$ only gives a very small contribution to the rate and is therefore not needed with great precision.
  - ii) The spectral density method is computationally expensive.

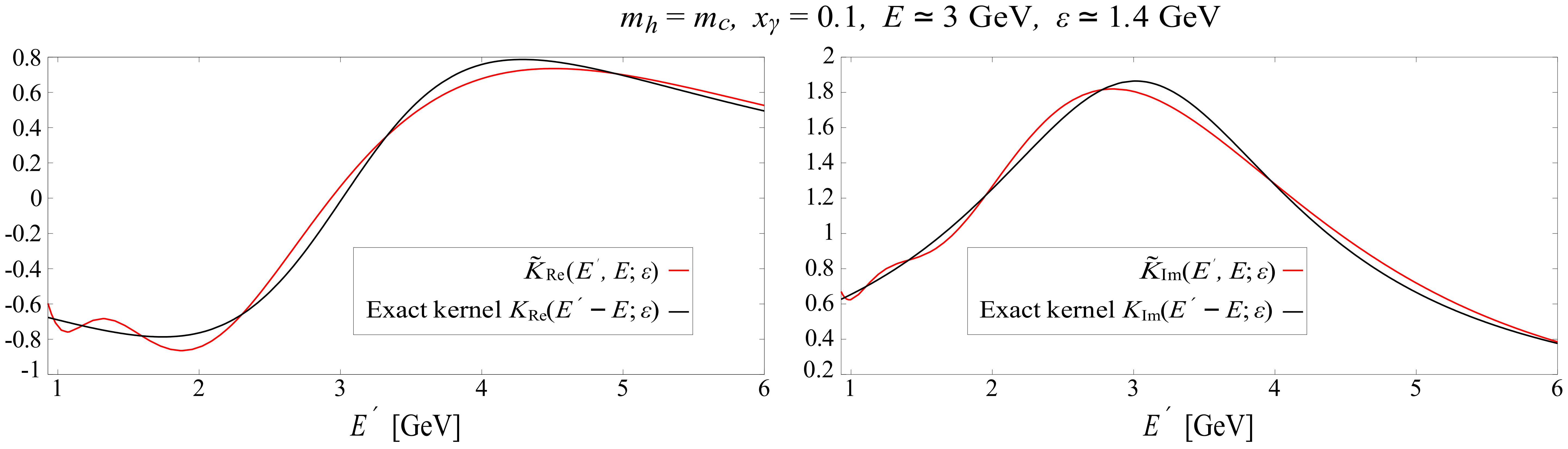
- An extrapolation in ϵ is required, as well as those in a and m_h .

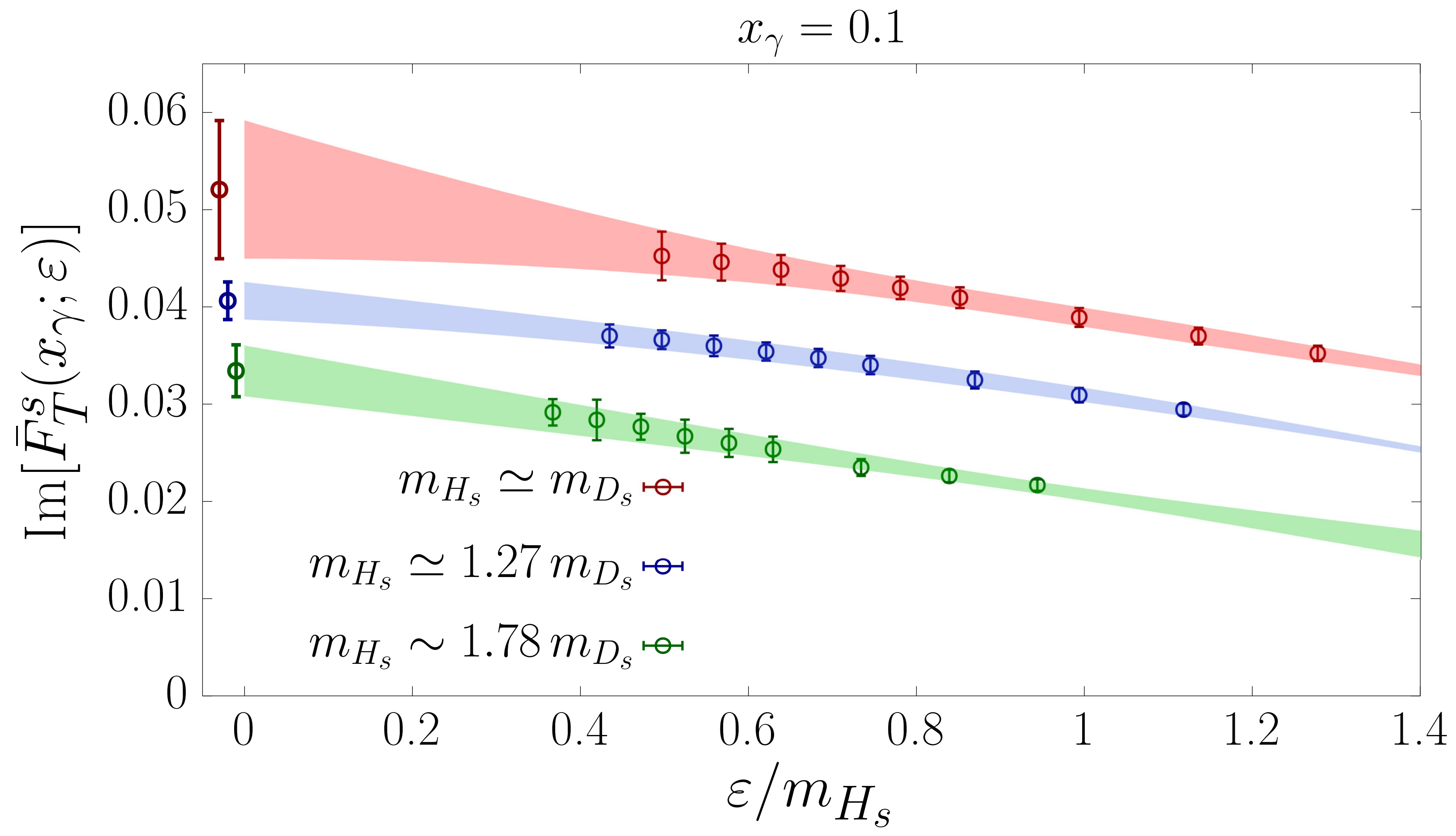
- Resulting error is $O(100\%)$ but $\bar{F}_T \ll F_{TV}, F_{TA}$. No clear x_γ dependence is observed in our data and we quote:

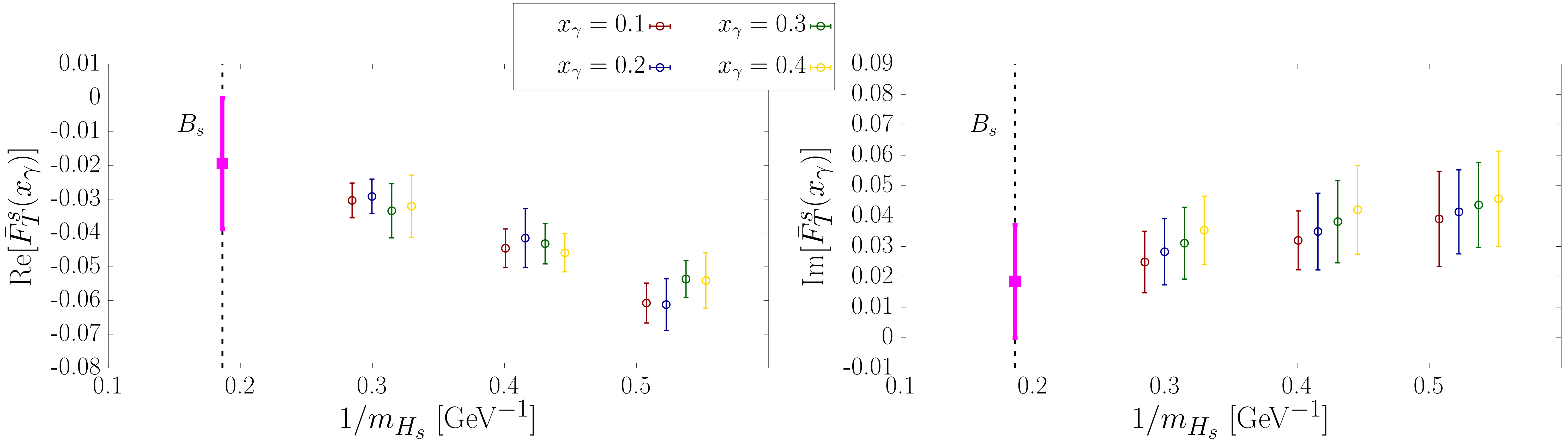
$$\mathrm{Re}\,\bar{F}^s_T(x_\gamma) = -0.019(19) \;\; \text{and} \;\; \mathrm{Im}\,\bar{F}^s_T(x_\gamma) = 0.018(18)\,.$$

$\bar{F}_T^s$ - Illustrative Plots

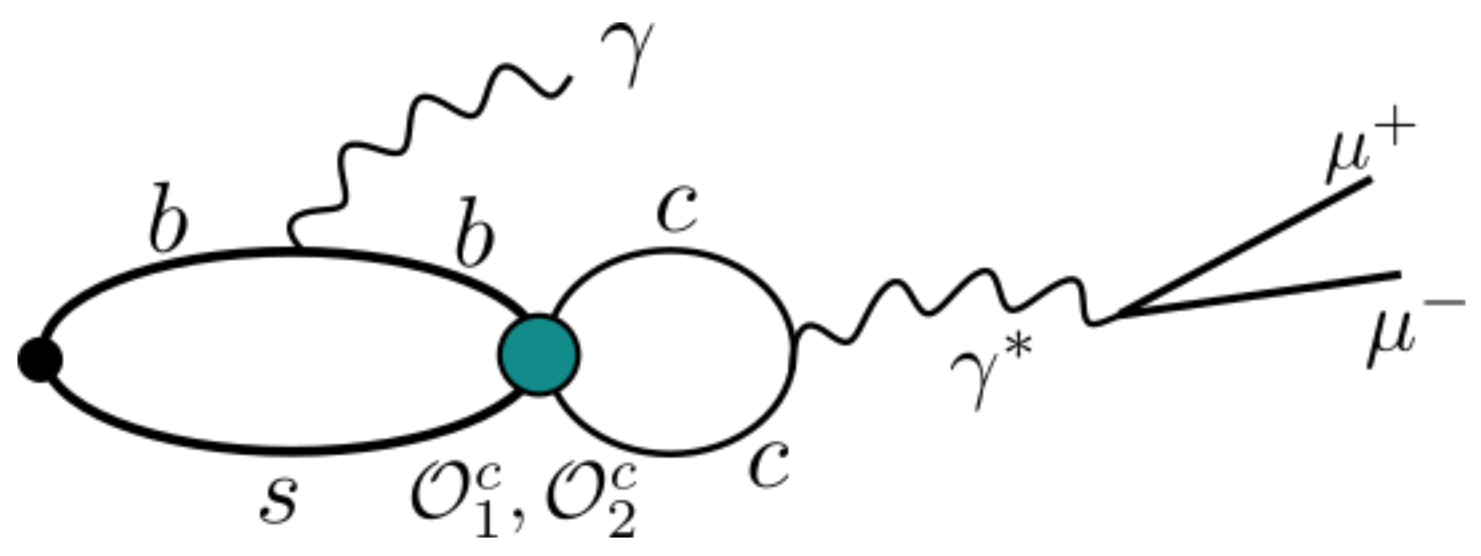








Other Contributions - Charming Penguins



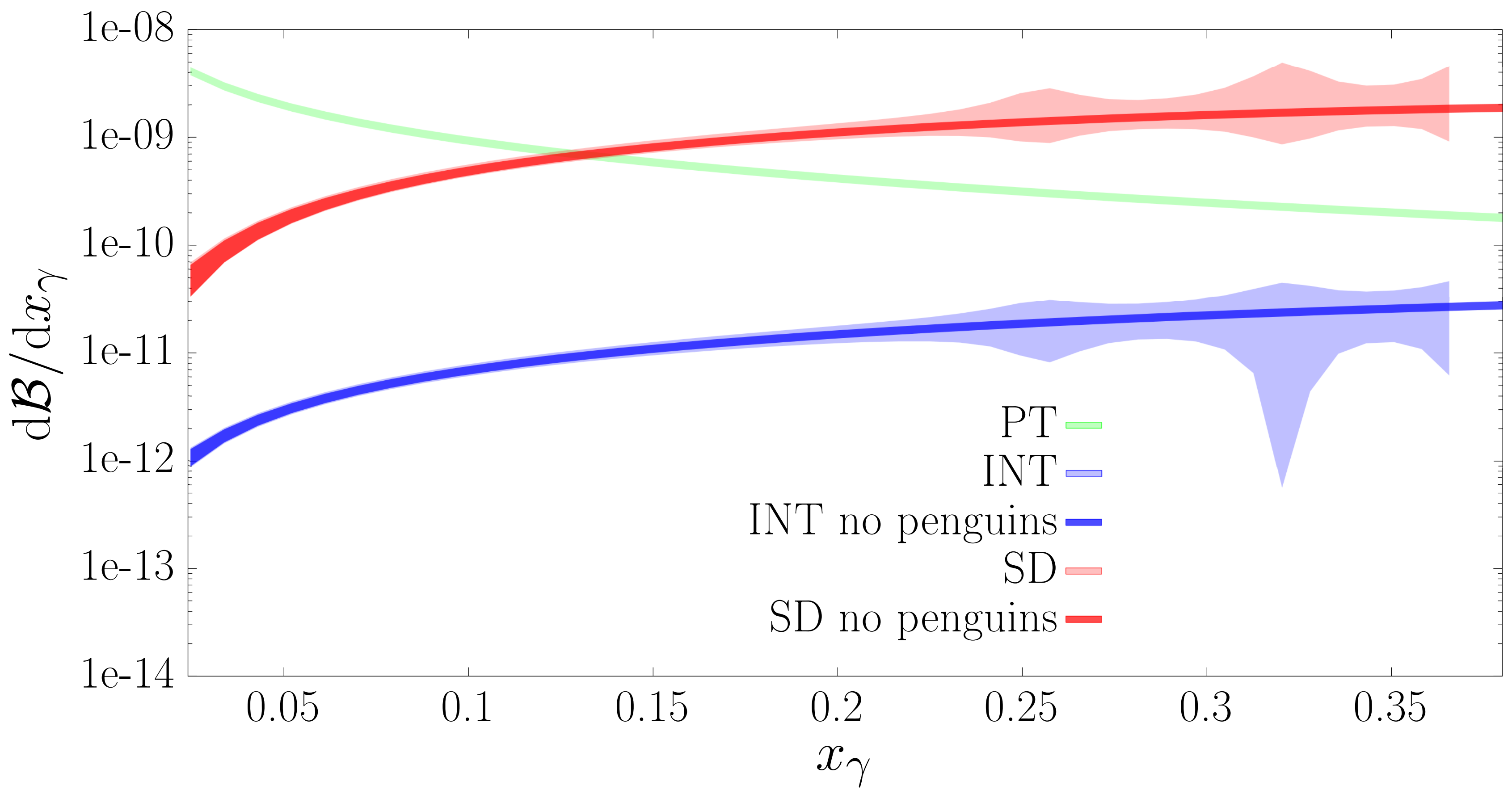
- Of the contributions we have not computed directly , the most significant one at large q^2 is expected to be that from the operators $O^c_{1,2}$ (charming penguins) and we are working on developing methods to overcome this.

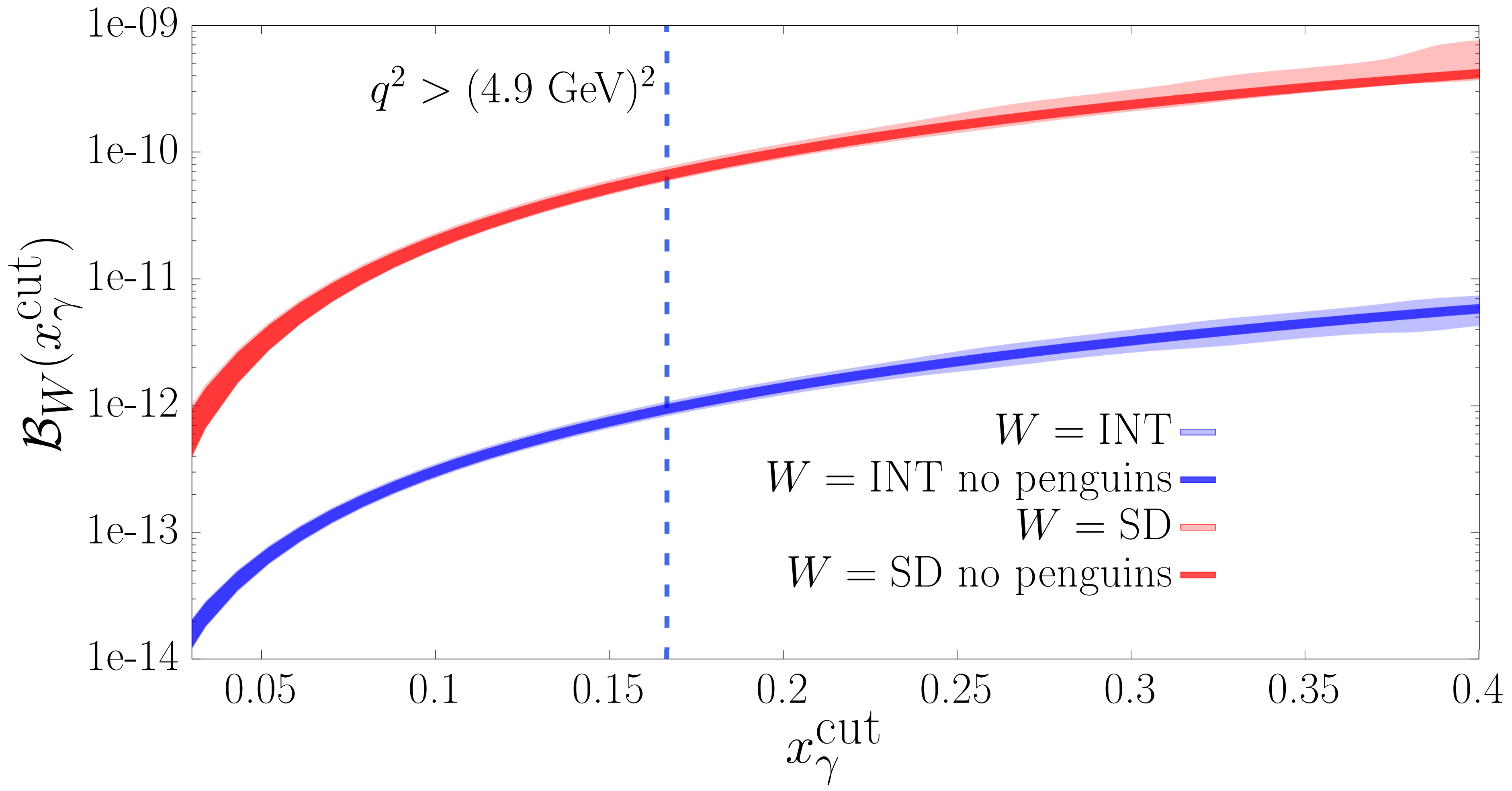
- In the meantime we follow previous ideas and estimate the contribution based on VMD inserting all $c\bar{c}$ resonances from the J/Ψ to the $\Psi(4660)$. It can be viewed as a shift in $C_9 \to C_9^{\text{eff}}(q^2) = C_9 + \Delta C_9(q^2)$:

$$\Delta C_9(q^2) = -\frac{9\pi}{\alpha_{\text{em}}^2}\left(C_1 + \frac{C_2}{3}\right) \sum_V |k_V| e^{i\delta_V} \frac{m_V \Gamma_V B(V \to \mu^+\mu^-)}{q^2 - m_V^2 + i m_V \Gamma_V} .$$

- k_V and δ_V parametrise the deviation from the factorisation approximation (in which $\delta_V = k_V - 1 = 0$). We allow δ_V to vary over $(0,2\pi)$ and $|k_V|$ to vary in the range 1.75 ± 0.75 .

Branching Fractions

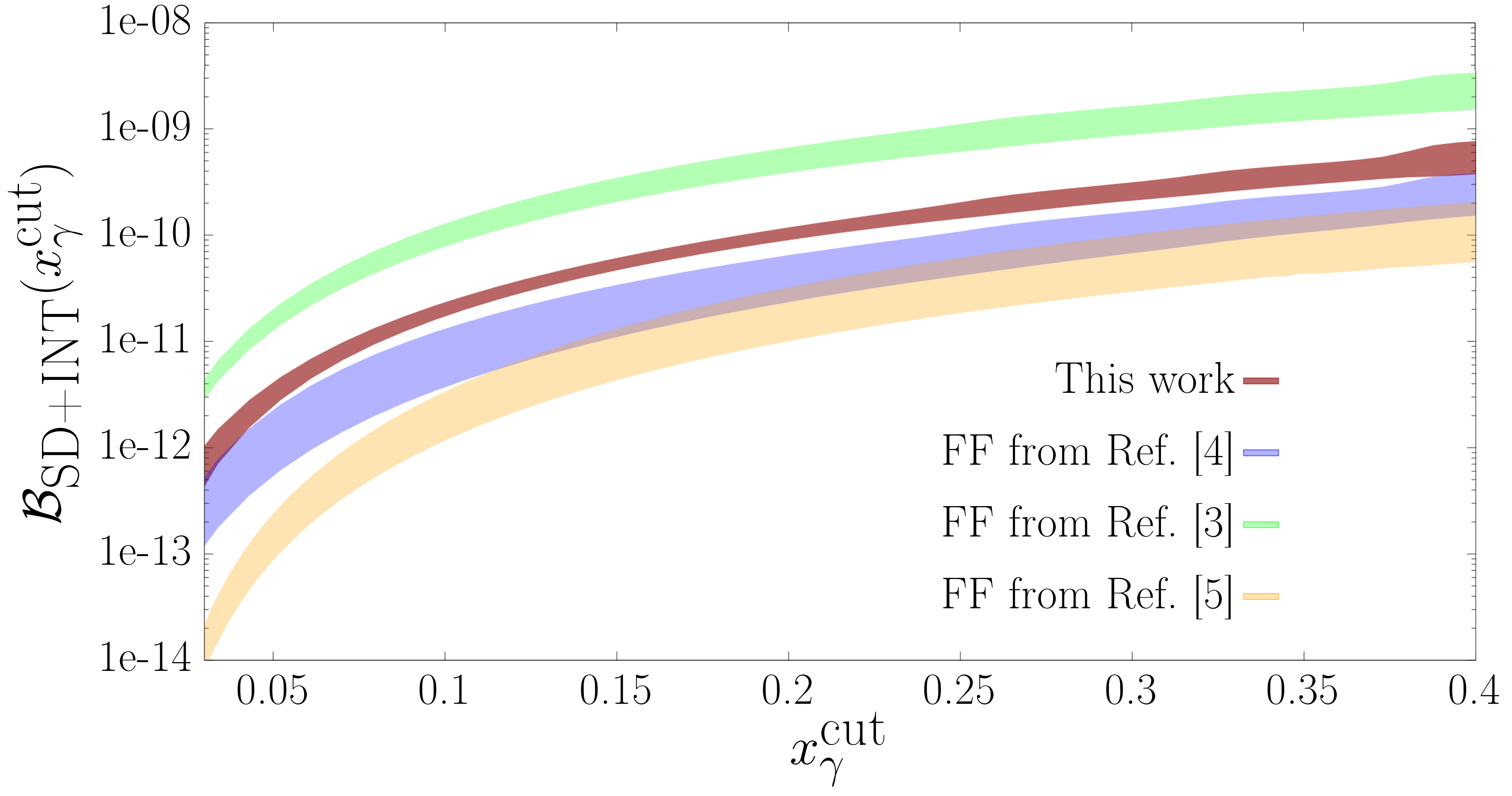


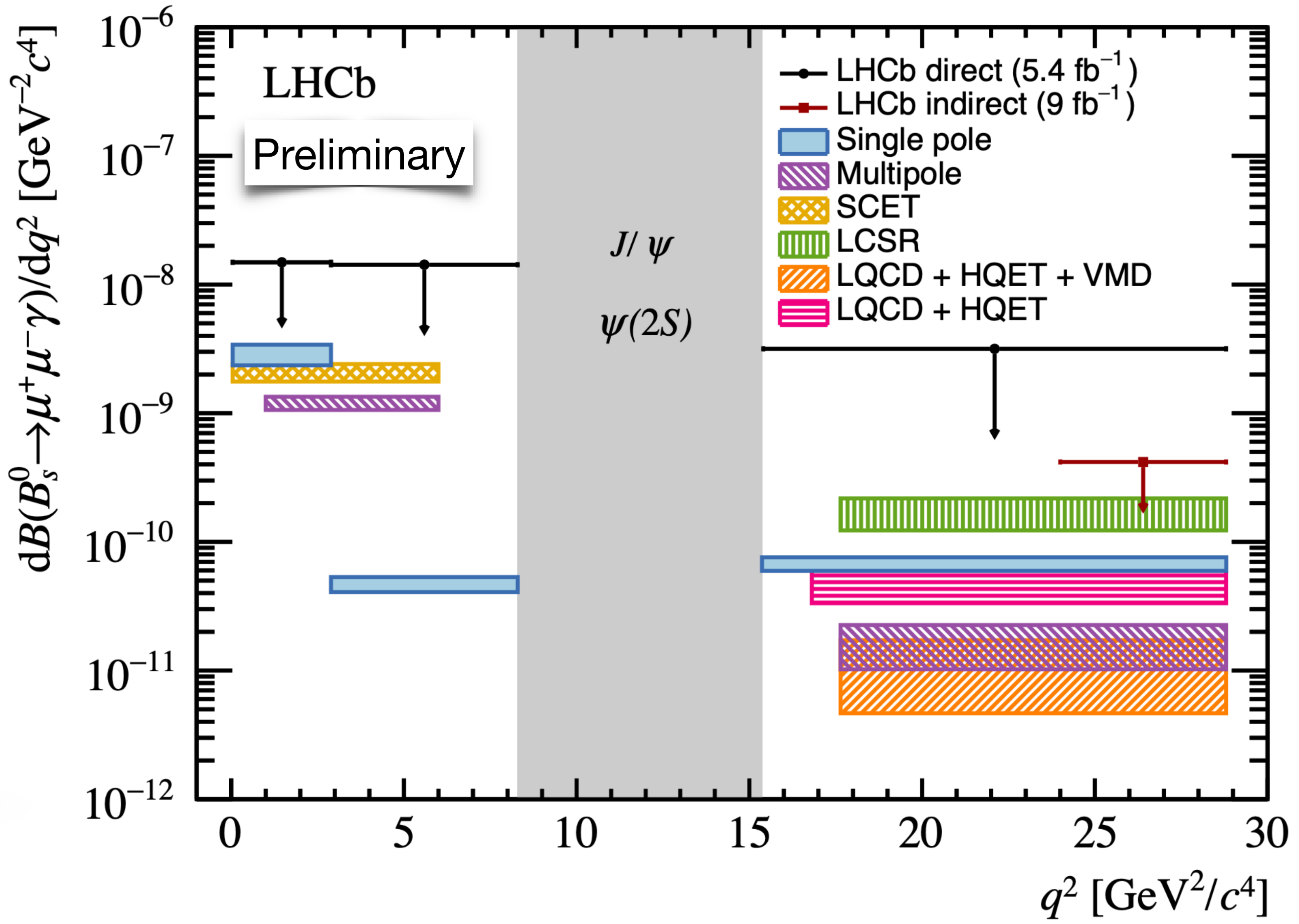


$$\mathscr{B}(x_\gamma^{\mathrm{cut}}) = \int_0^{x_\gamma^{\mathrm{cut}}} dx_\gamma \, \frac{d\mathscr{B}(x_\gamma)}{dx_\gamma}$$

- Structure Dependent (SD) contribution dominated by F_V.
- The error from the charming penguins increases with x_γ (at $x_\gamma = 0.4$ it is about 30 %).
- Our Result - $\mathscr{B}_{\mathrm{SD}}(0.166) = 6.9(9) \times 10^{-11}$; LHCb - $\mathscr{B}_{\mathrm{SD}}(0.166) < 2 \times 10^{-9}$.

Comparisons





- Ref.[3] = T.Janowski, B.Pullin and R.Zwicky, arXiv:2106.13616, LCSR
- Ref.[4]= A.Kozachuk, D.Melikhov and N.Nikitin, arXiv:1712.07926, relativistic dispersion relations
- Ref.[5]= D.Guadagnoli, C.Normand, S.Simula and L.Vittorio, arXiv:2303.02174, VMD+quark model+lattice at charm
- Discrepancy persists since rate dominated by F_V

- New LHCb update with direct detection of final state photon.

I.Bachiller, La Thuile 2024
LHCb, 2404.07648

- For $q^2 > 15\,\mathrm{GeV}^2$ the bound is about an order of magnitude higher than before.

$\bar{B}_s \to \mu^+\mu^-\gamma$ — Conclusions

- We have computed the form factors F_V, F_A, F_{TV} and F_{TA} which contribute to the amplitude. The amplitude is dominated by F_V.
  There are significant discrepancies with earlier estimates of the form factors obtained using other methods.

- As q^2 is decreased towards the region of charmonium resonances, the uncertainties grow, from 15 % with $\sqrt{q^2_{\text{cut}}} = 4.9\,\text{GeV}$ to about 30 % for $\sqrt{q^2_{\text{cut}}} = 4.2\,\text{GeV}$, largely due to the charming penguins for which we have included a phenomenological parametrisation.

Outlook

- Develop methods which would allow the evaluation of the charming penguin contributions, also for $B \to K^{(*)}\mu^+\mu^-$ decays etc.. *This is one of our top priorities!*

- Continue developing methods to evaluate the disconnected diagrams.

- Continue performing simulations on finer lattices so that the uncertainties due to the $m_h \to m_b$ extrapolation are reduced.