Light-cone distribution amplitudes of pion and kaon from the Heavy-quark OPE method

THE HOPE COLLABORATION

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References

W. Detmold and CJDL, Phys. Rev. **D 73** (2006) 014501

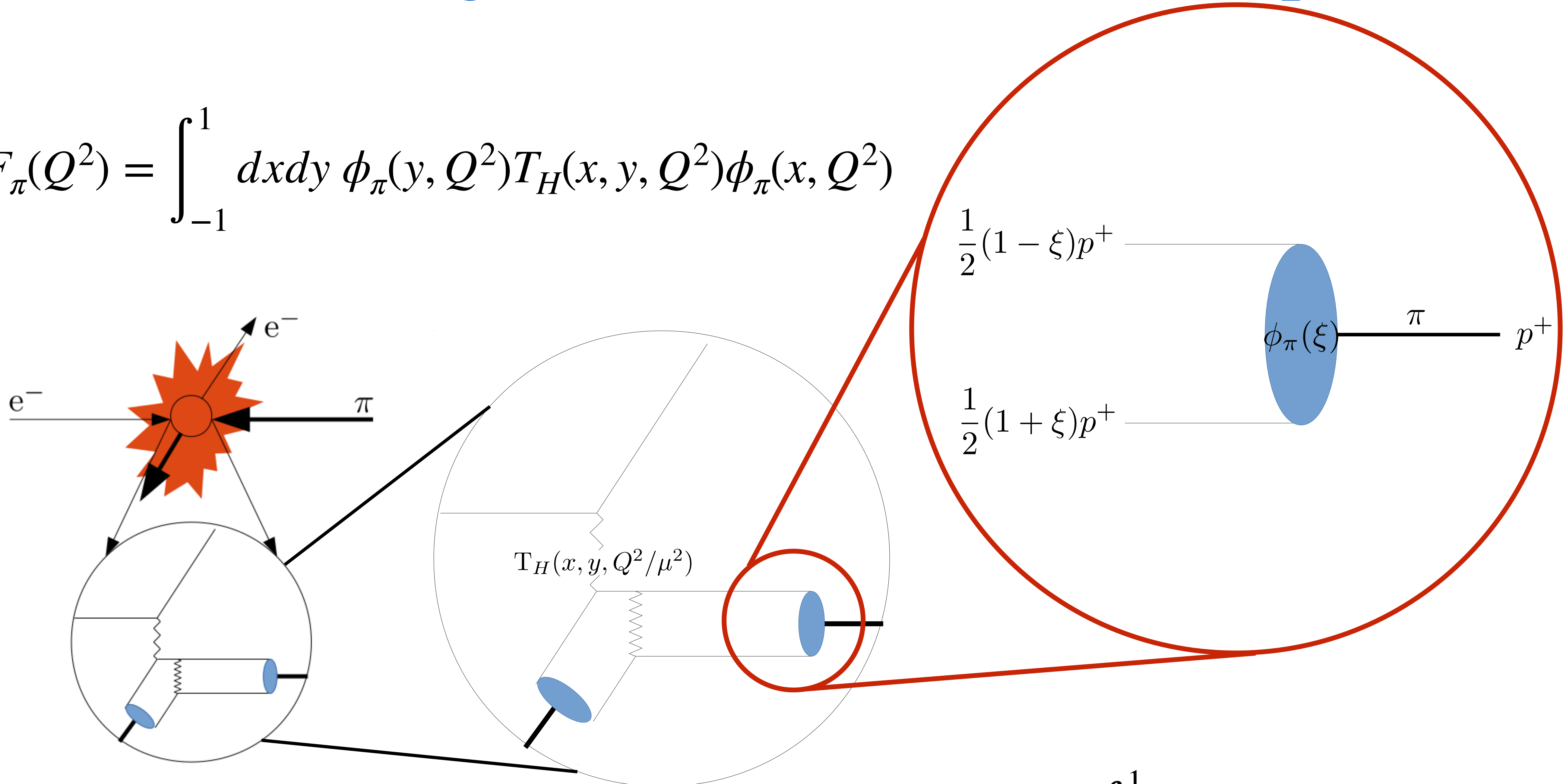
HOPE Collaboration, W. Detmold *et al*., Phys. Rev. **D 104** (2021) 7, 074511

HOPE Collaboration, W. Detmold *et al*., Phys. Rev. **D 105** (2022) 3, 034506

HOPE Collaboration, poster by R. Perry at Lattice 2024

Phenomenological relevance: Exclusive process

$$F_\pi(Q^2) = \int_{-1}^{1} dxdy\ \phi_\pi(y, Q^2) T_H(x, y, Q^2) \phi_\pi(x, Q^2)$$



$$\langle \Omega \,|\, \bar\psi(z_-)\gamma_\mu\gamma_5 W[z_-, -z_-]\psi(-z_-) \,|\, \pi(\mathbf{p})\rangle = ip_\mu f_\pi \int_{-1}^{1} d\xi\ \mathrm{e}^{i\xi p^+ z_-}\phi_\pi(\xi, \mu)$$

Light-cone, challenging for Euclidean lattice

Extremely high $Q^2 \Rightarrow F_\pi(Q^2) = \dfrac{16\pi\alpha_s(Q^2)}{Q^2} f_\pi^2 \int dxdy \phi_\pi(x)\phi_\pi(y)\ ,\ \ \phi_\pi(\xi) = \dfrac{3}{4}(1-\xi^2)$

Phenomenological relevance: Exclusive process

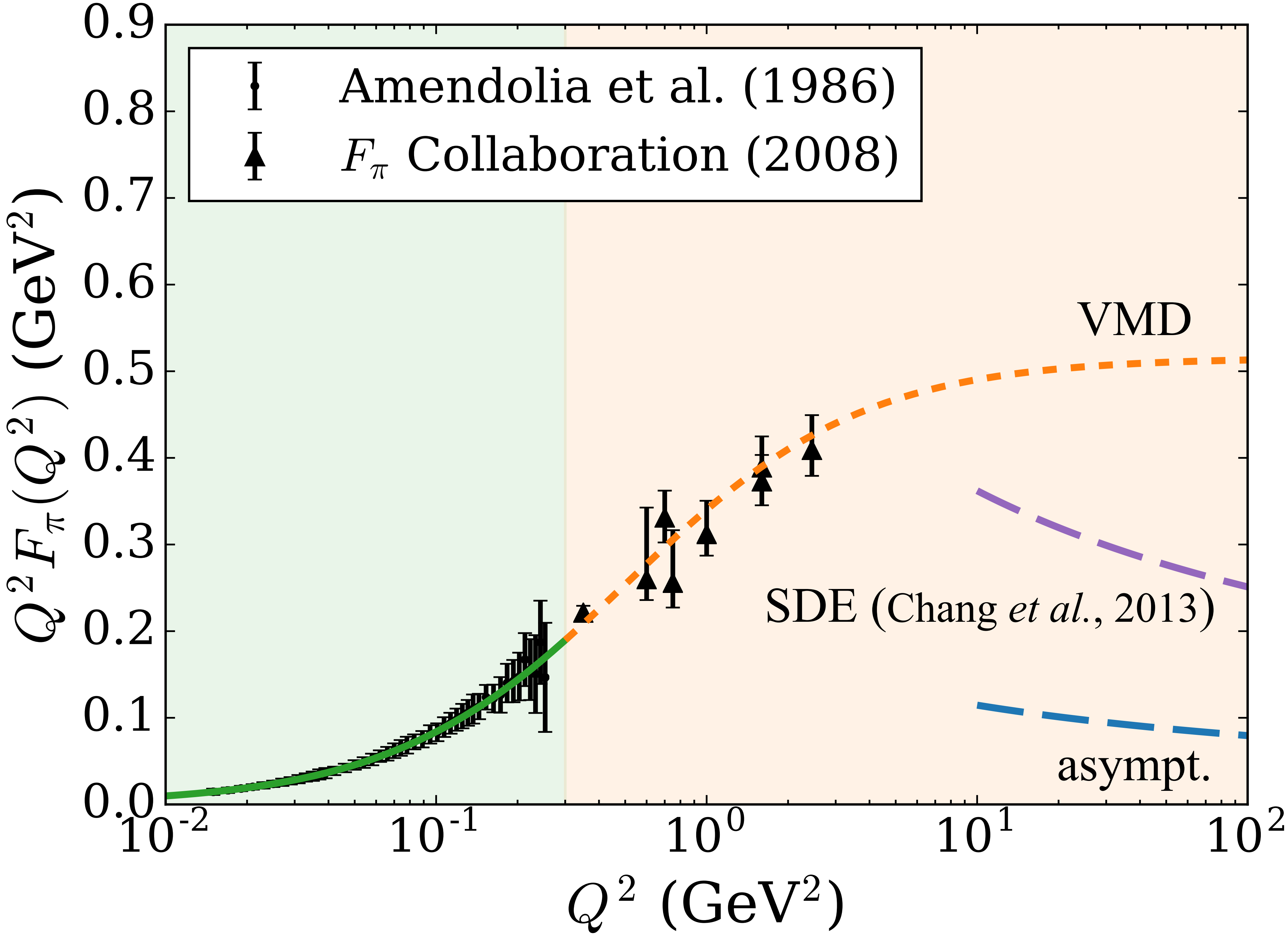
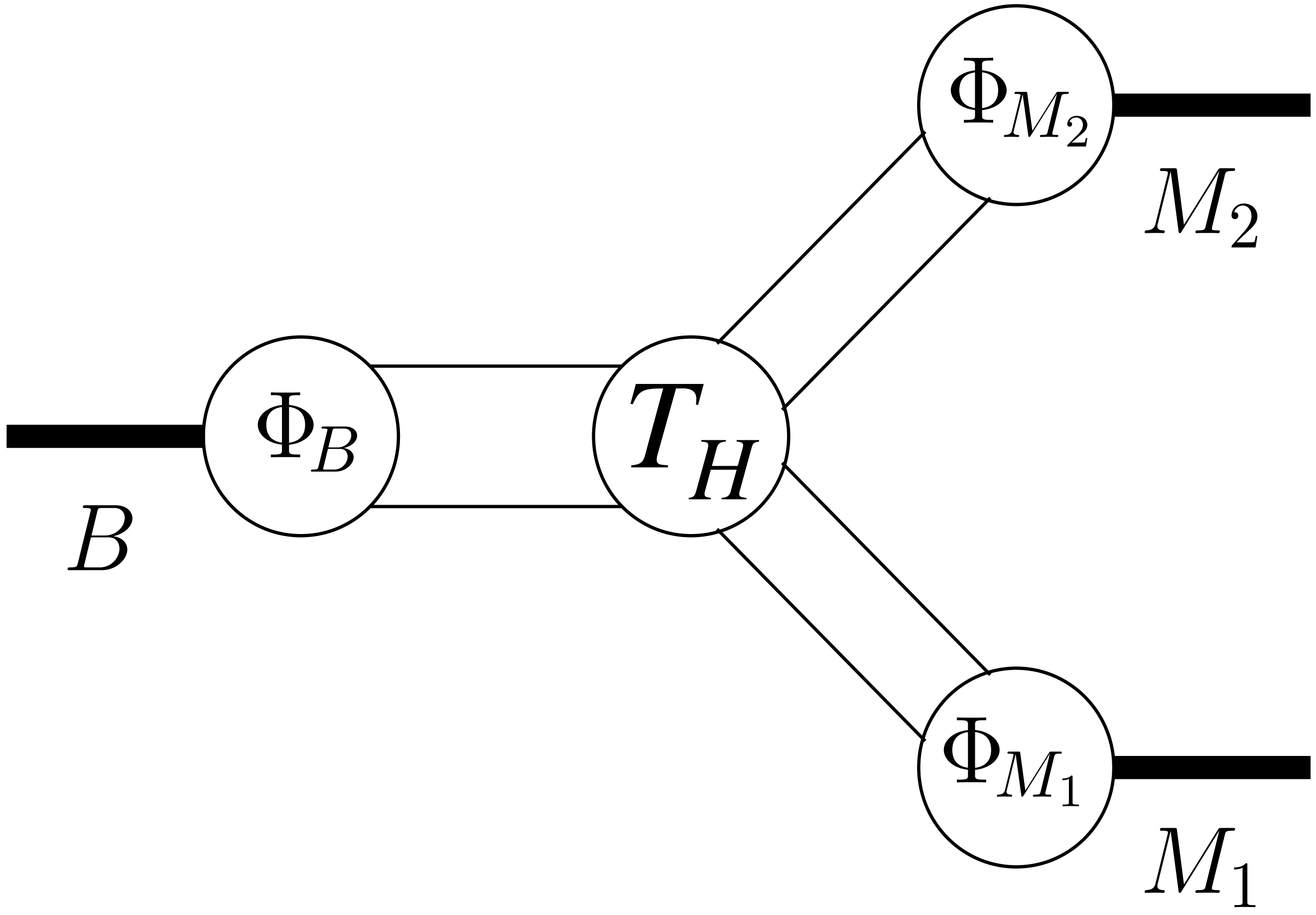


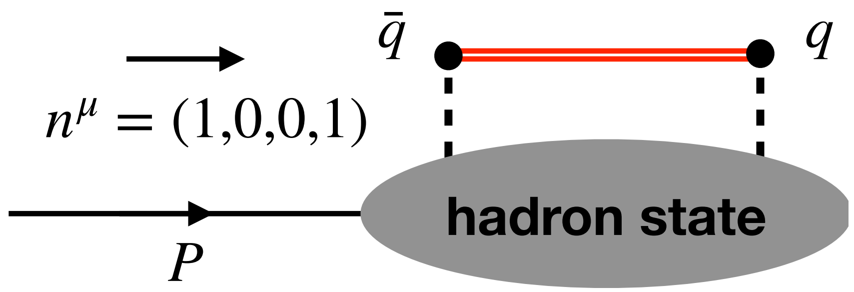
Figure from R.J. Perry *et al.*, PLB 807 (2020) 135581

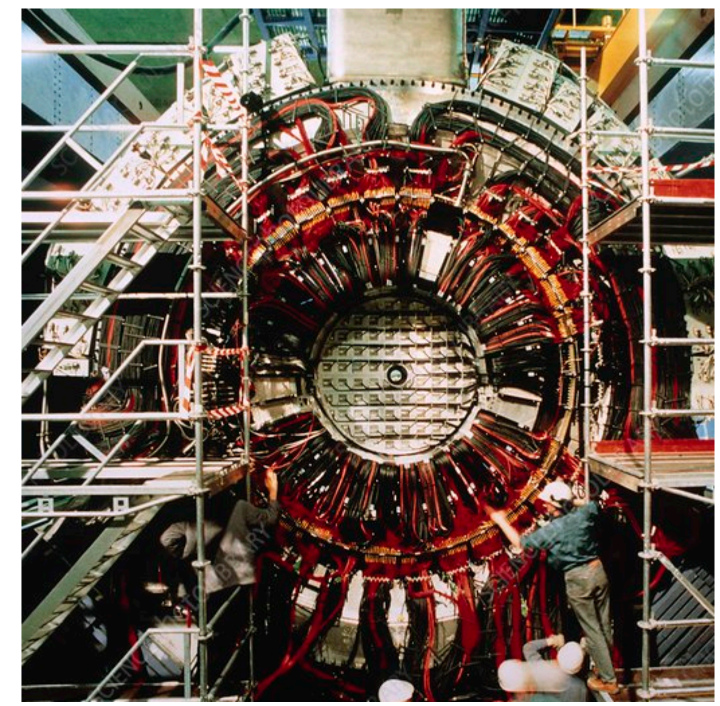
Phenomenological relevance

Input for flavour physics

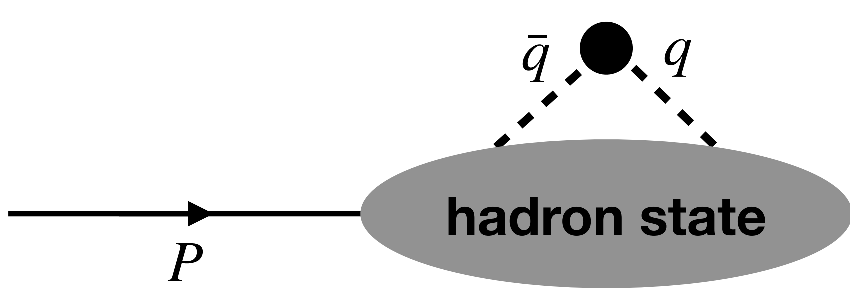


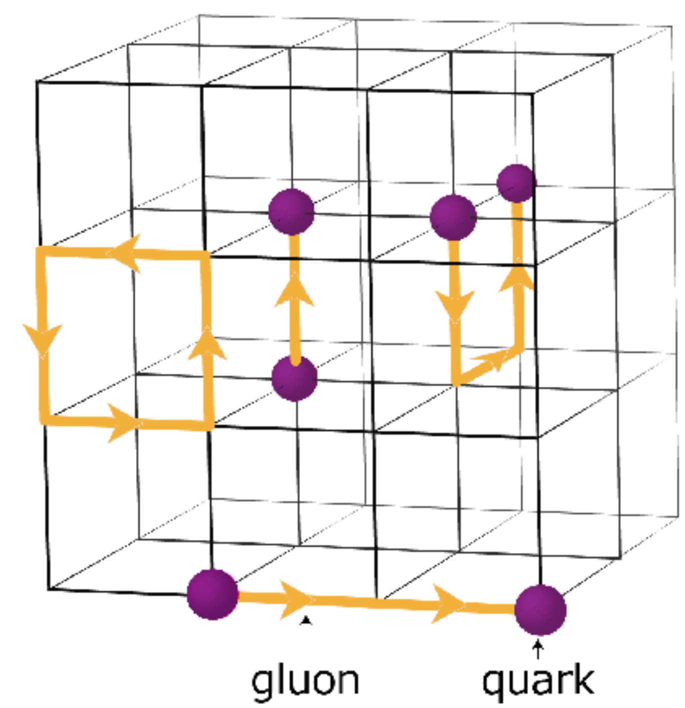
Conventional LQCD approach





light-cone operator product expansion





twist-2 Mellin moments

twist = dim - spin

Conventional LQCD approach

★ Light-cone OPE

$$T[J^\mu(x)J^\nu(0)] = \sum_{i,n} C_i(x^2,\mu^2)\, x_{\mu_1}\ldots x_{\mu_n}\, \mathcal{O}_i^{\mu\nu\mu_1\ldots\mu_n}(\mu) + \text{higher twists}$$

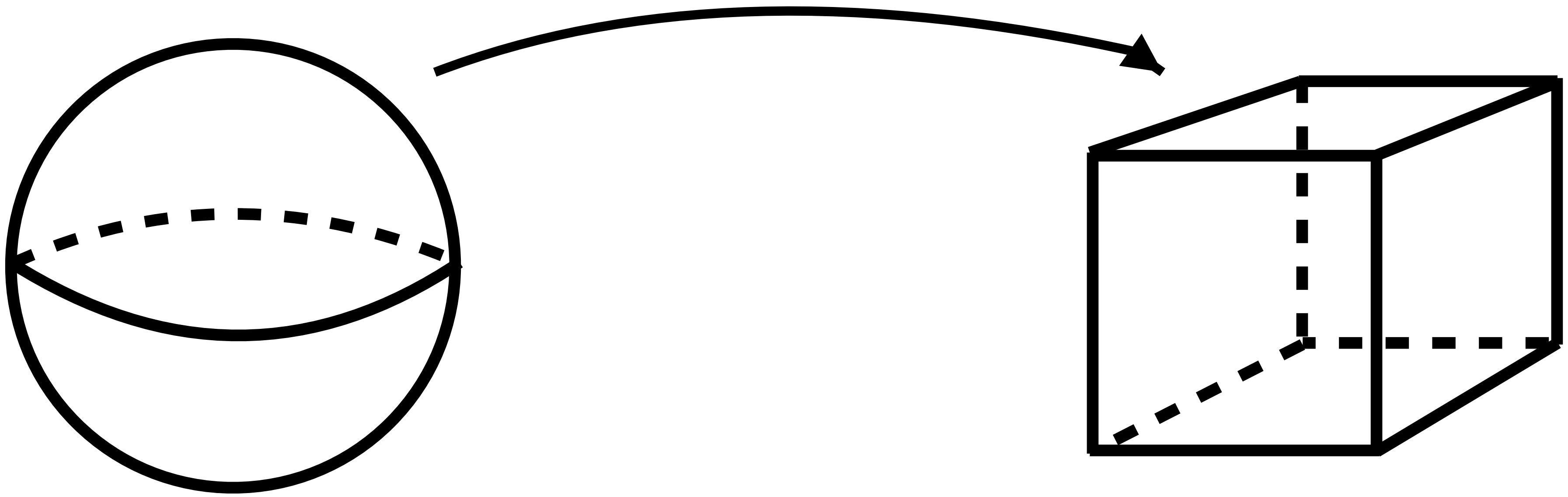
Twist-2 Mellin moments ⇒ parton distribution functions

★ The twist-2 operators

$$\mathcal{O}_i^{\nu\mu\mu_1\ldots\mu_n} = \bar{\psi}\Gamma_{i,\nu} D^\mu D^{\mu_1}\ldots D^{\mu_n}\psi - \text{ traces}$$

Issue with computing the Mellin moments

Continuum	Lattice
SU(2) symmetry	Octahedral symmetry
Twist = Dim - Spin	~~Spin~~

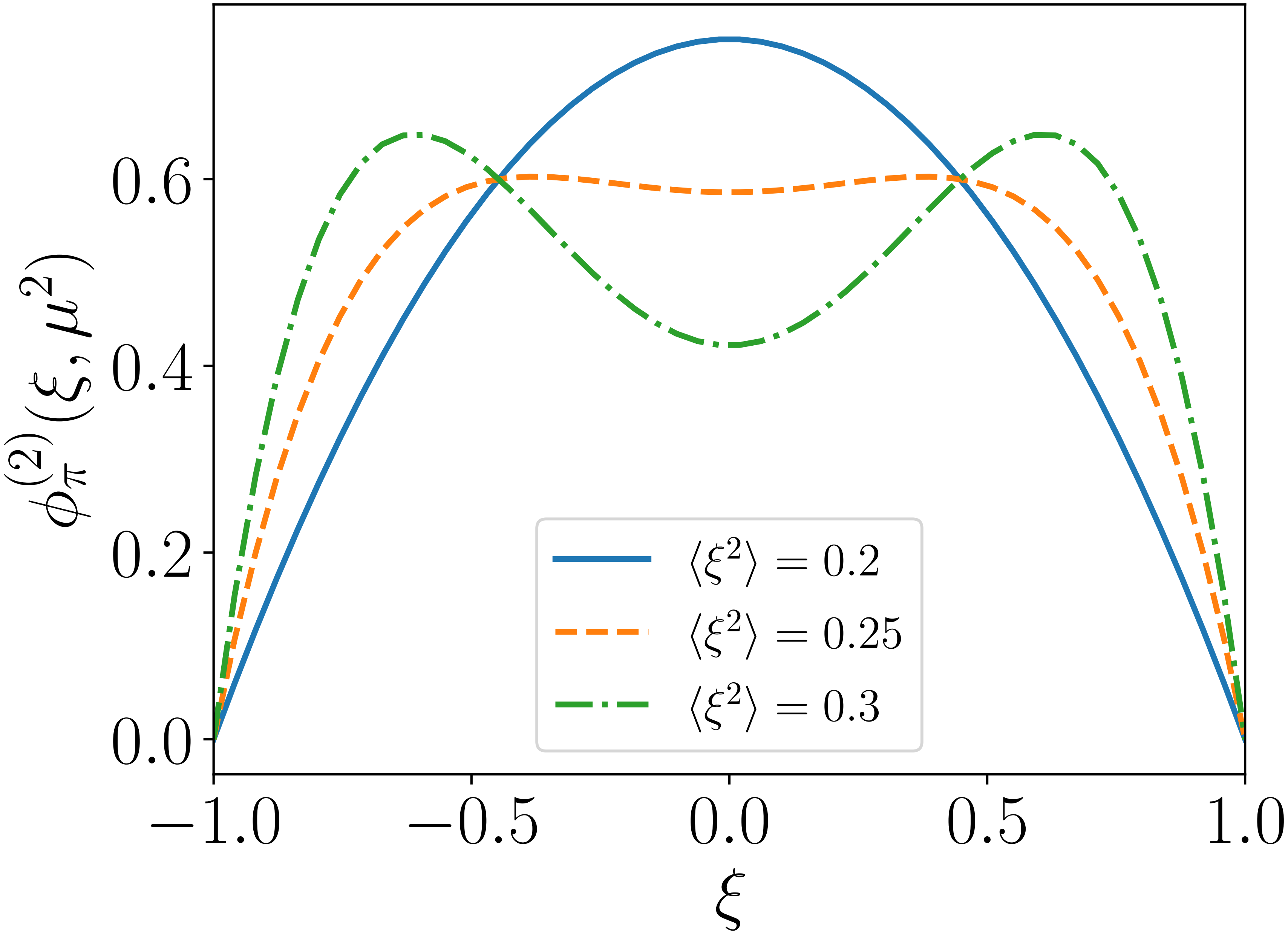


Operator mixing under renormalisation, power ($1/a^n$) divergence

→ Only the first few moments can be extracted in practice

OPE and ξ-dependence

ξ: the fraction of p_π carried by one of the valence quarks (parton limit)



- Power divergence already shows up in LQCD calculation for $\langle \xi^2 \rangle$

Parton distribution from lattice QCD

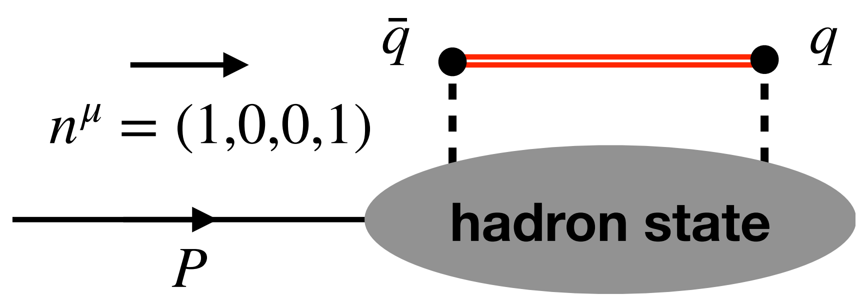
through *unphysical* non-local operators

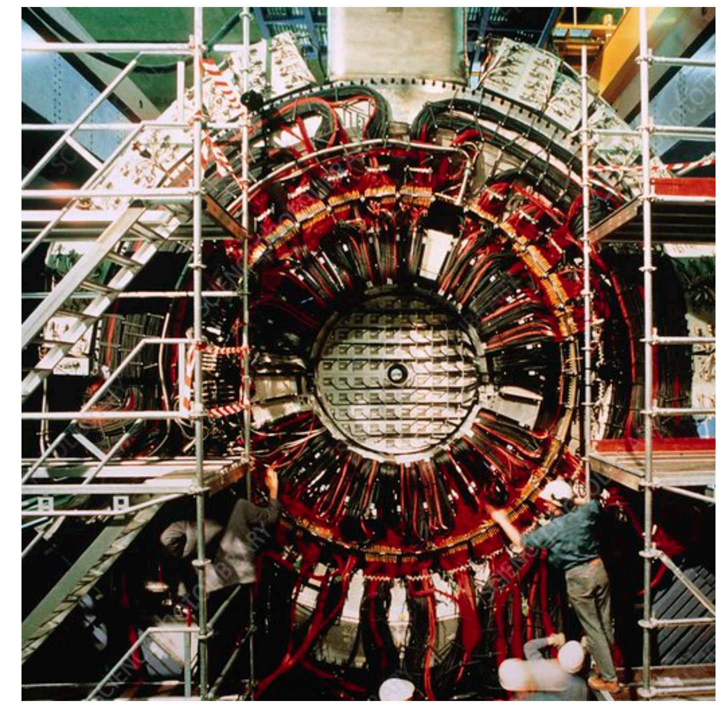
See C. Dawson *et al.*, Nucl. Phys. B 514 (1998) for same idea in kaon physics

- A space-like Wilson line (quasi-PDF and pseudo-PDF)
  X. Ji, PRL 110 (2013); A. Radyushkin, PRD 96 (2017)
- Two currents separated by space-like distance
  V. Braun and D. Mueller, EPJC 55 (2008)
- Two flavour-changing currents with valence heavy quark (HOPE method)
  W. Detmold and CJDL, PRD 73 (2006)
- More A. Chambers *et al.*, PRL 118 (2017); Y. Ma & J.-W. Qiu, PRL 120 (2018)…

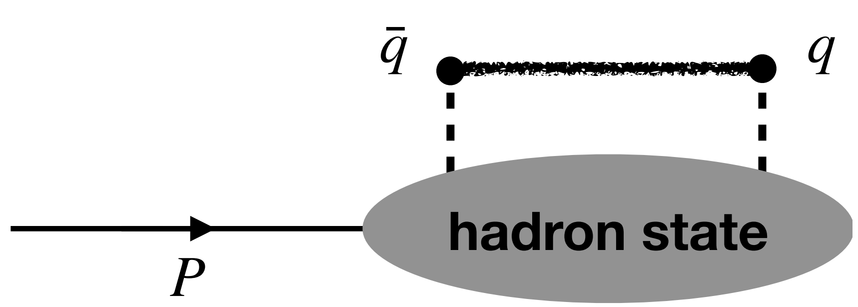
Complementarity discussed in X. Ji, arXiv:**2209.09332**

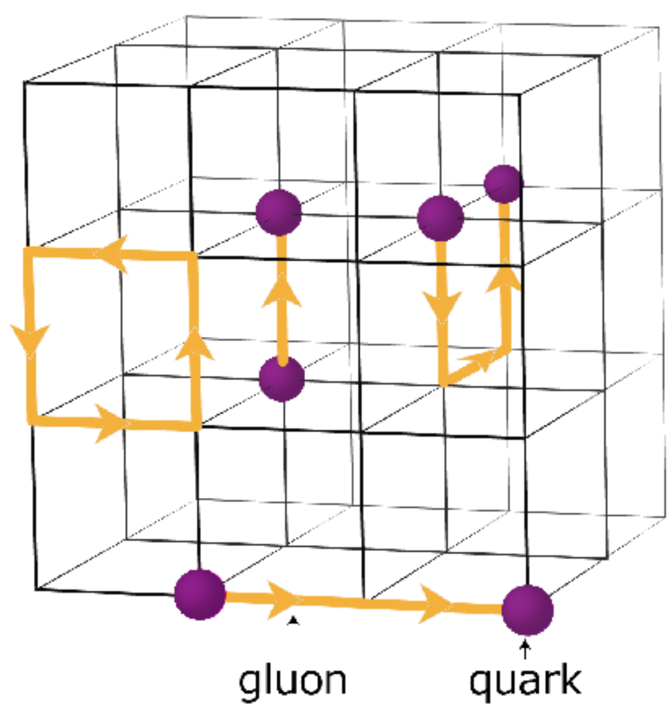
"Novel" LQCD approach



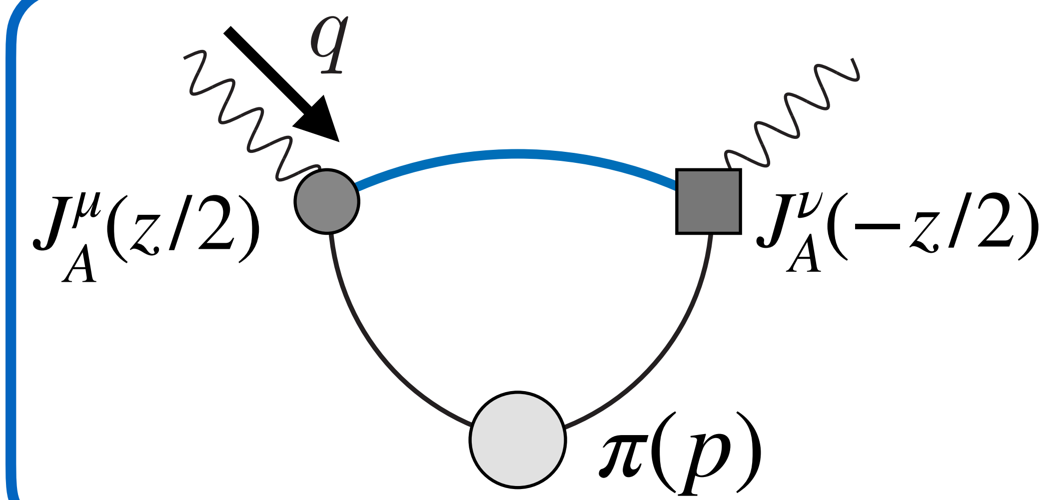


Perturbation theory





HOPE amplitude for computing pion LCDA



+ the cross term

See also S. Brodsky *et al.*, Phys. Lett. B 91 (1980)

$$V^{\mu\nu}(p,q) = \int d^4z\, e^{iq\cdot z} \langle 0 | T[J_A^\mu(z/2) J_A^\nu(-z/2)] | \pi(\mathbf{p}) \rangle$$

$$J_A^\mu = \bar{\Psi}\gamma^\mu\gamma^5\psi + \bar{\psi}\gamma^\mu\gamma^5\Psi$$

Ψ is the valence, relativistic heavy quark

$$V^{[\mu\nu]}(p,q) = \frac{1}{2}\left[V^{\mu\nu}(p,q) - V^{\nu\mu}(p,q)\right]$$

OPE for HOPE amplitude

$$V^{[\mu\nu]}(p,q) = \frac{2\epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_{n\ \mathrm{even}}^{\infty} \frac{\zeta^n \mathcal{C}_n^2(\eta)}{2^n(n+1)} C_W^{(n)}(\tilde{Q}^2) f_\pi \langle \xi^n \rangle + \mathcal{O}(1/\tilde{Q}^3)$$

higher-twist

$$\tilde{Q}^2 = q^2 + m_\Psi^2$$

$$\eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}, \quad \zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}$$

$\mathcal{C}_n^2(\eta)$: target-mass effect

- tree-level OPE
- one-loop
- fit lattice data

$\sim$ Expansion in powers of $\tilde{\omega} = \dfrac{2p \cdot q}{\tilde{Q}^2}$

Lattice technical issues

- Need large pion momentum for high moments

- Strategies for enhancing sensitivity to high moments

$$\tilde{\omega} = \frac{2p \cdot q}{\tilde{Q}^2} = \frac{2(iE_\pi q_4 - \mathbf{p} \cdot \mathbf{q})}{\tilde{Q}^2}$$

$\Rightarrow$ Choose p and q, such that

$\mathrm{Im}(V^{[\mu\nu]}) \sim$ leading moment

$\mathrm{Re}(V^{[\mu\nu]}) \sim$ subleading moment

- Excited-state contamination

Quenched calculation @ $M_\pi \approx 560$ MeV

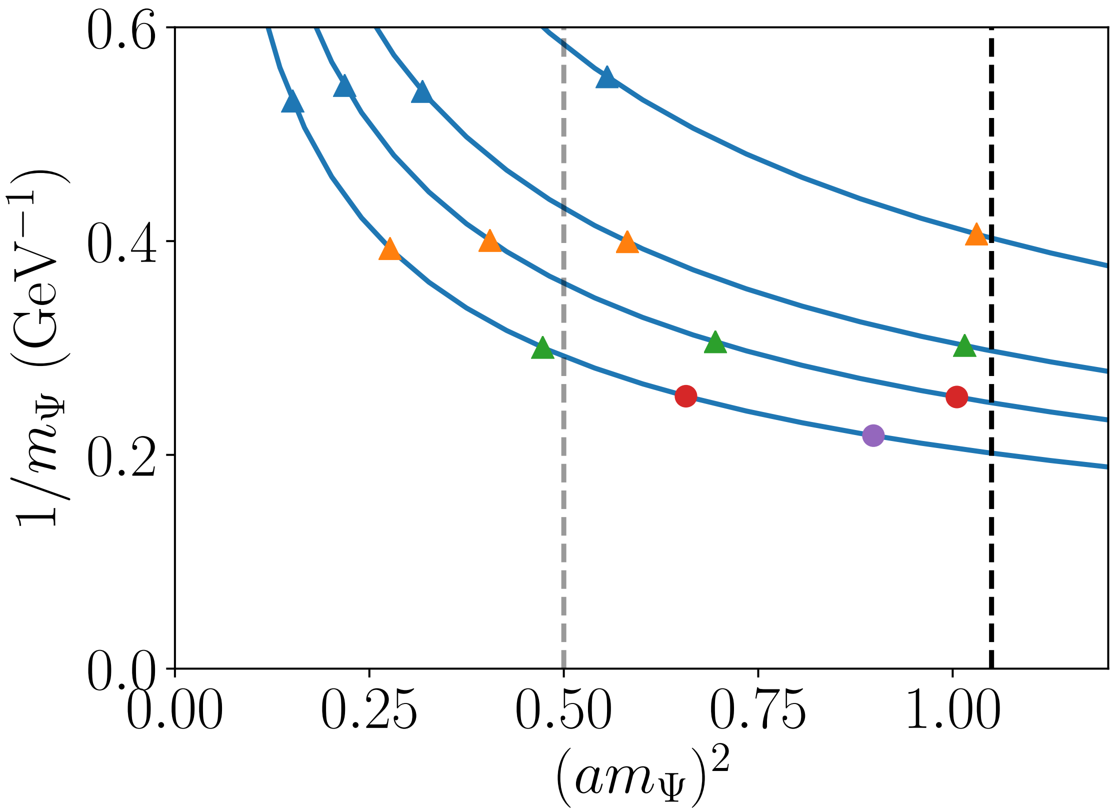
- Proof-of-principle nature
- 4 lattice spacings: 0.04 to 0.08 fm
- Learn how to control errors
- Good result for $\langle \xi^2 \rangle$
- Reasonable preliminary result for $\langle \xi^4 \rangle$
- 88 Intel KNL nodes @ NYCU & Mare Nostrum in Barcelona

Lattice setting for determining $\langle \xi^2 \rangle$

Wilson plaquette and non-perturbatively improved clover actions

a (fm)	$\hat{L}^3 \times \hat{T}$	N_{config}	N_{src}
0.081	$24^3 \times 48$	650	12
0.060	$32^3 \times 64$	450	10
0.048	$40^3 \times 80$	250	6
0.041	$48^3 \times 96$	341	10

$L \sim 2$ fm



- $\mathbf{p}$ = (1,0,0) $\mathbf{q}$ = (1/2,0,1) in units of $2\pi/L \sim 0.64$GeV
- $V^{\mu\nu}$ is $O(a)$ improved without improving the axial current

Analysis strategy

★ Momentum space

$$V^{[\mu\nu]}(p,q) \equiv \int d^4z\, \mathrm{e}^{iq\cdot z}\, \langle 0\,|\, T[J^{[\mu}(z/2)J^{\nu]}(-z/2)]\,|\,\pi(\mathbf{p})\rangle$$

★ Time-momentum representation (TMR)

$$R^{[\mu\nu]}(\tau;\mathrm{p},\mathrm{q}) = \int dz_4\, \mathrm{e}^{-iq_4 z_4}\, V^{[\mu\nu]}(p,q)$$

$$= \int d^3\mathbf{z}\, \mathrm{e}^{\mathbf{q}\cdot\mathbf{z}}\, \langle 0\,|\, T[J^{[\mu}(z/2)J^{\nu]}(-z/2)]\,|\,\pi(\mathbf{p})\rangle$$

→ Fourier transform of Wilson coeff numerically

Analysis strategy

★ Momentum space

$$V^{[\mu\nu]}(p,q) \equiv \int d^4z\, e^{iq\cdot z} \langle 0 | T[J^{[\mu}(z/2)J^{\nu]}(-z/2)] | \pi(\mathbf{p})\rangle$$

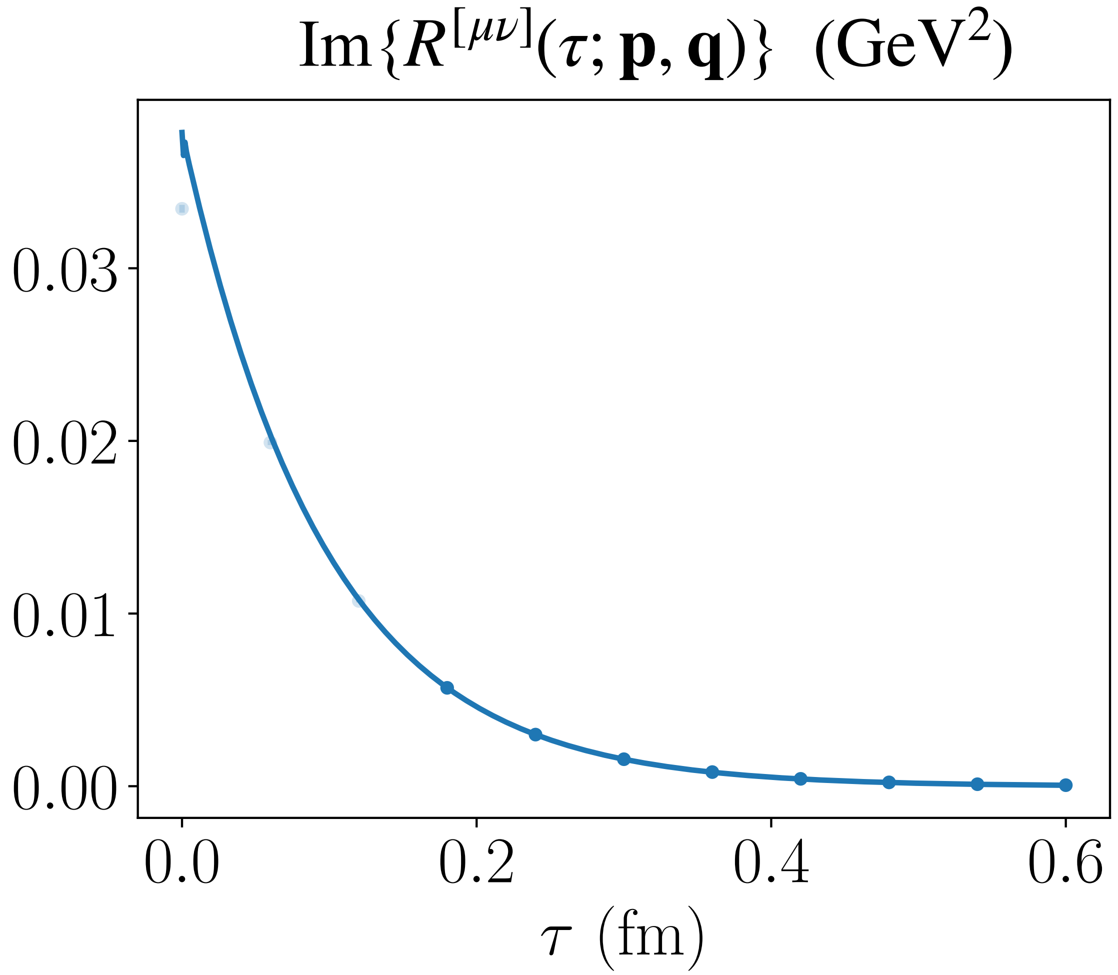
★ Time-momentum representation (TMR) ✔

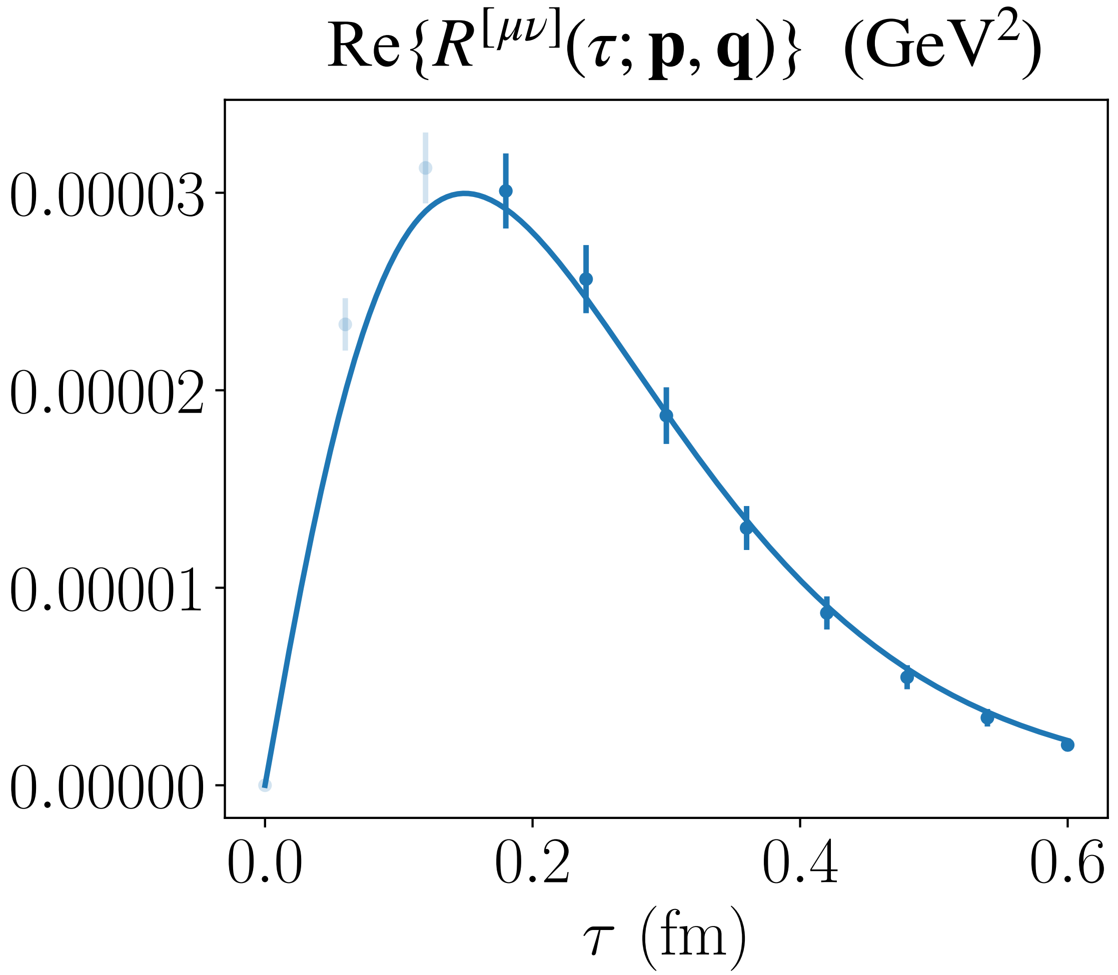
$$R^{[\mu\nu]}(\tau;\mathrm{p},\mathrm{q}) = \int dz_4\, e^{-iq_4 z_4}\, V^{[\mu\nu]}(p,q)$$

$$= \int d^3\mathbf{z}\, e^{\mathbf{q}\cdot\mathbf{z}} \langle 0 | T[J^{[\mu}(z/2)J^{\nu]}(-z/2)] | \pi(\mathbf{p})\rangle$$

→ Fourier transform of Wilson coeff numerically

Extracting $\langle \xi^2 \rangle$ from HOPE formula

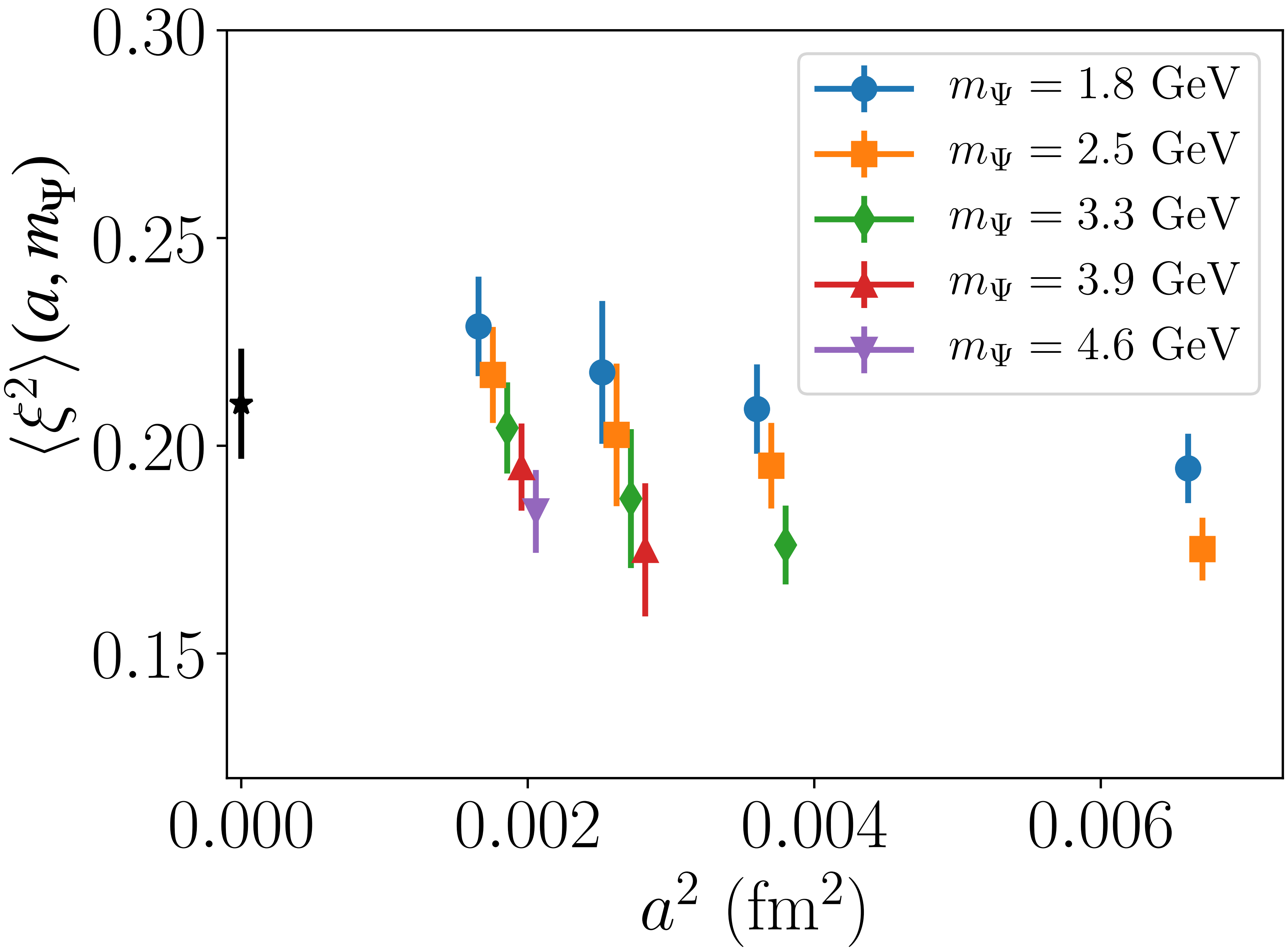




fit f_π and m_Ψ

fit $\langle \xi^2 \rangle$

Lattice artefacts and higher-twist effects in $\langle \xi^2 \rangle (a, m_\Psi)$

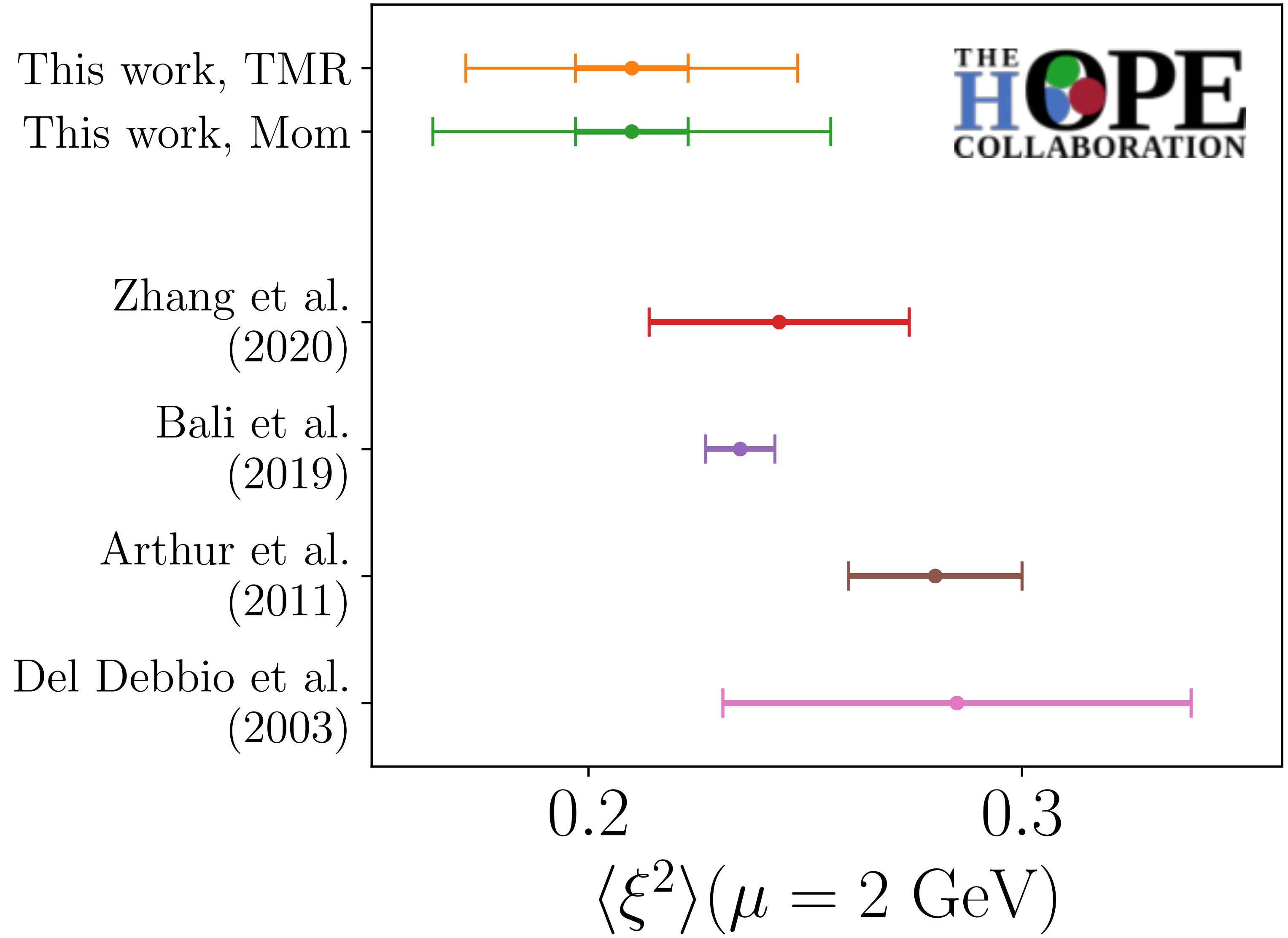


$$\langle \xi^2 \rangle (a, m_\Psi) = \langle \xi^2 \rangle + \frac{A}{m_\Psi} + Ba^2 + Ca^2 m_\Psi + Da^2 m_\Psi^2$$

Our 2021 result for $\langle \xi^2 \rangle$

TMR analysis errors

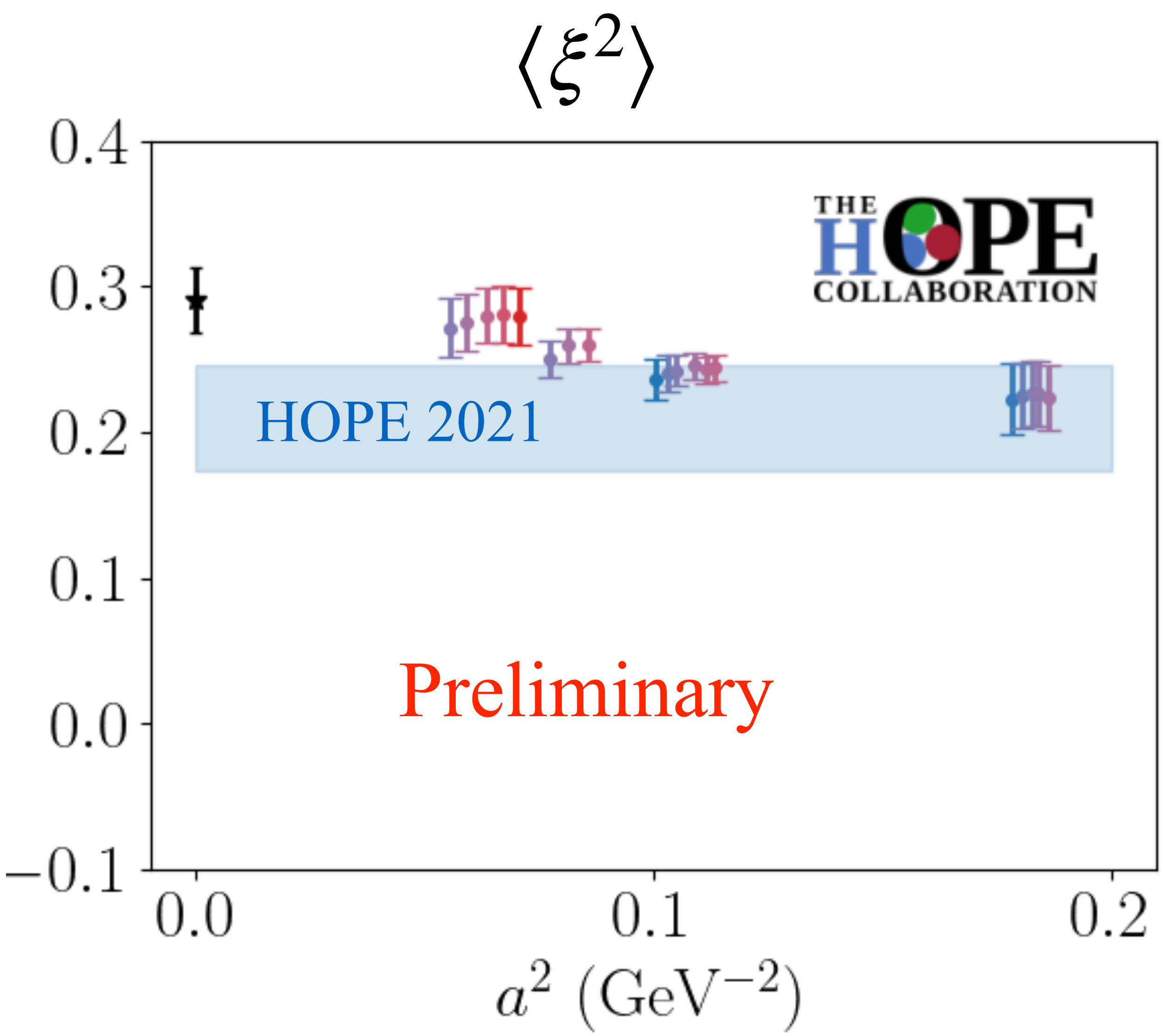
Source of error	Size
Statistical	0.013
Continuum extrapolation	0.016
Higher-twist	0.025
Excited-state contamination	0.002
Unphysical m_π	0.014
Fit range	0.002
Running coupling	0.008
Total (exc. quenching)	**0.036**

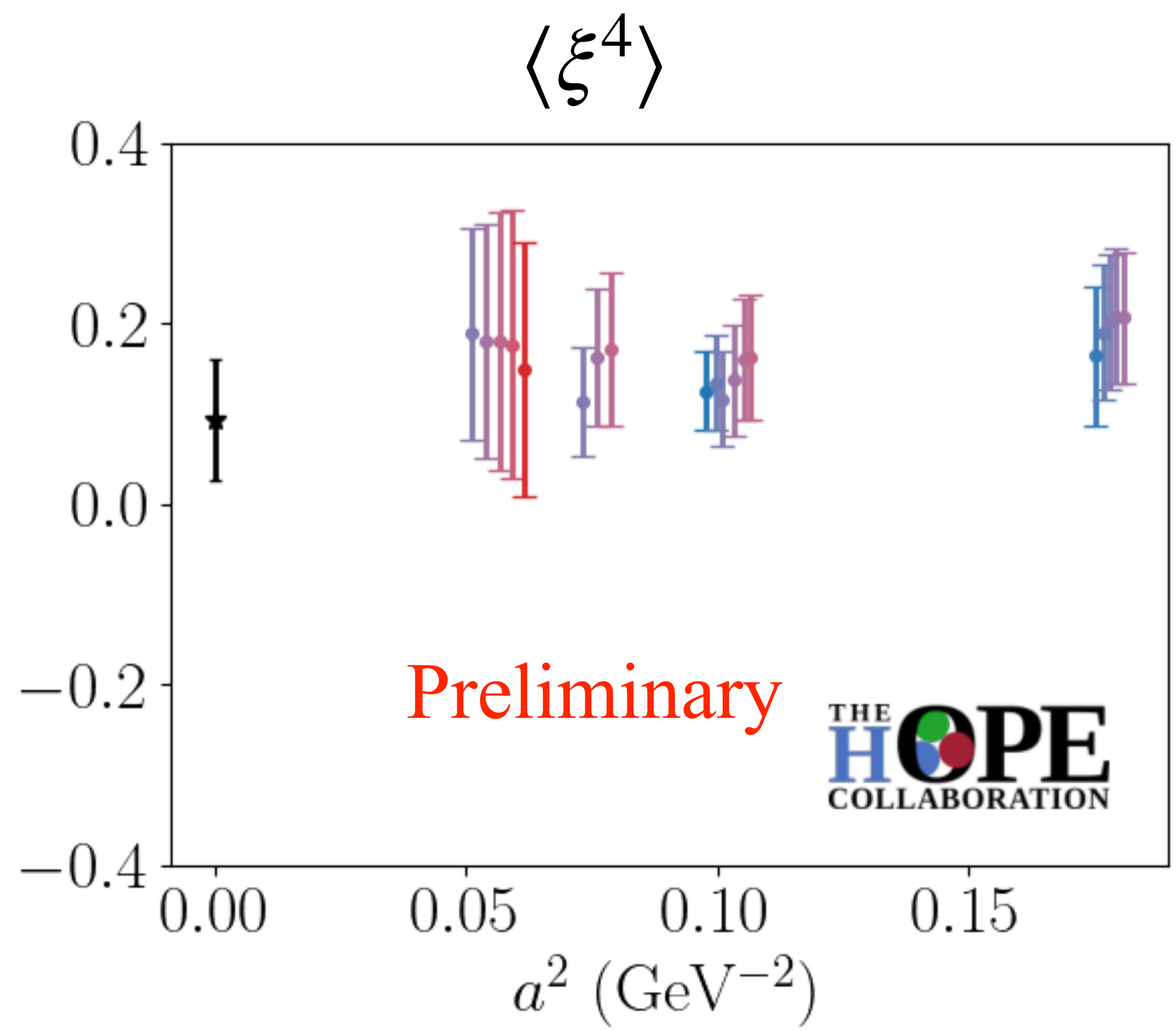


$$\langle \xi^2 \rangle_{\text{TMR}} (\mu = 2\ \text{GeV}) = 0.210 \pm 0.013\ (\text{stat.}) \pm 0.034\ (\text{sys.}) = 0.210 \pm 0.036$$

$$\langle \xi^2 \rangle_{\text{Mom}} (\mu = 2\ \text{GeV}) = 0.210 \pm 0.013\ (\text{stat.}) \pm 0.044\ (\text{sys.}) = 0.210 \pm 0.046$$

Latest status of $\langle \xi^4 \rangle$ calculation



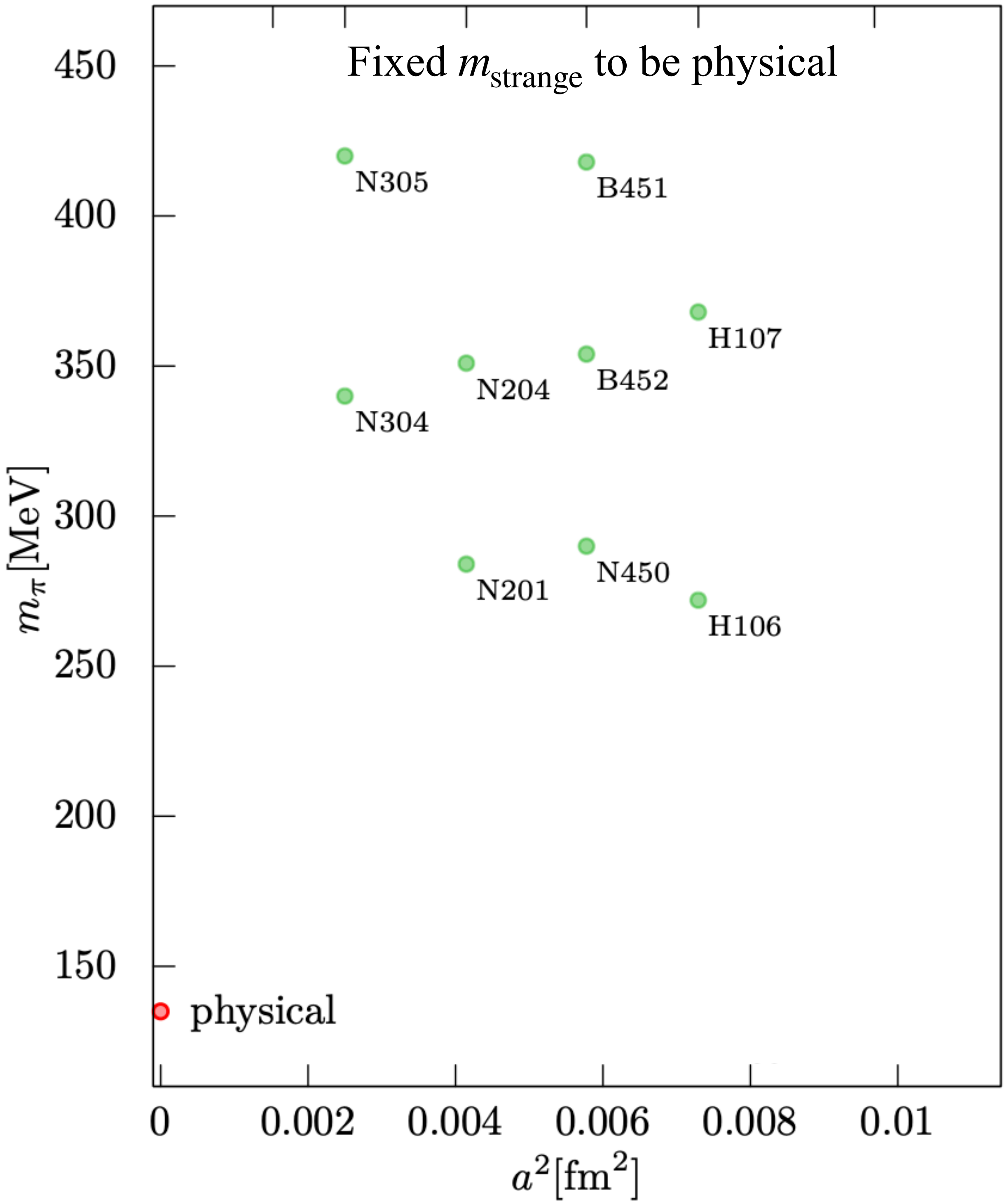


- **p** = (2,0,0) and **q** = (1/2,0,1) in units of $2\pi/L \sim 0.64$GeV

~10000 measurements each point

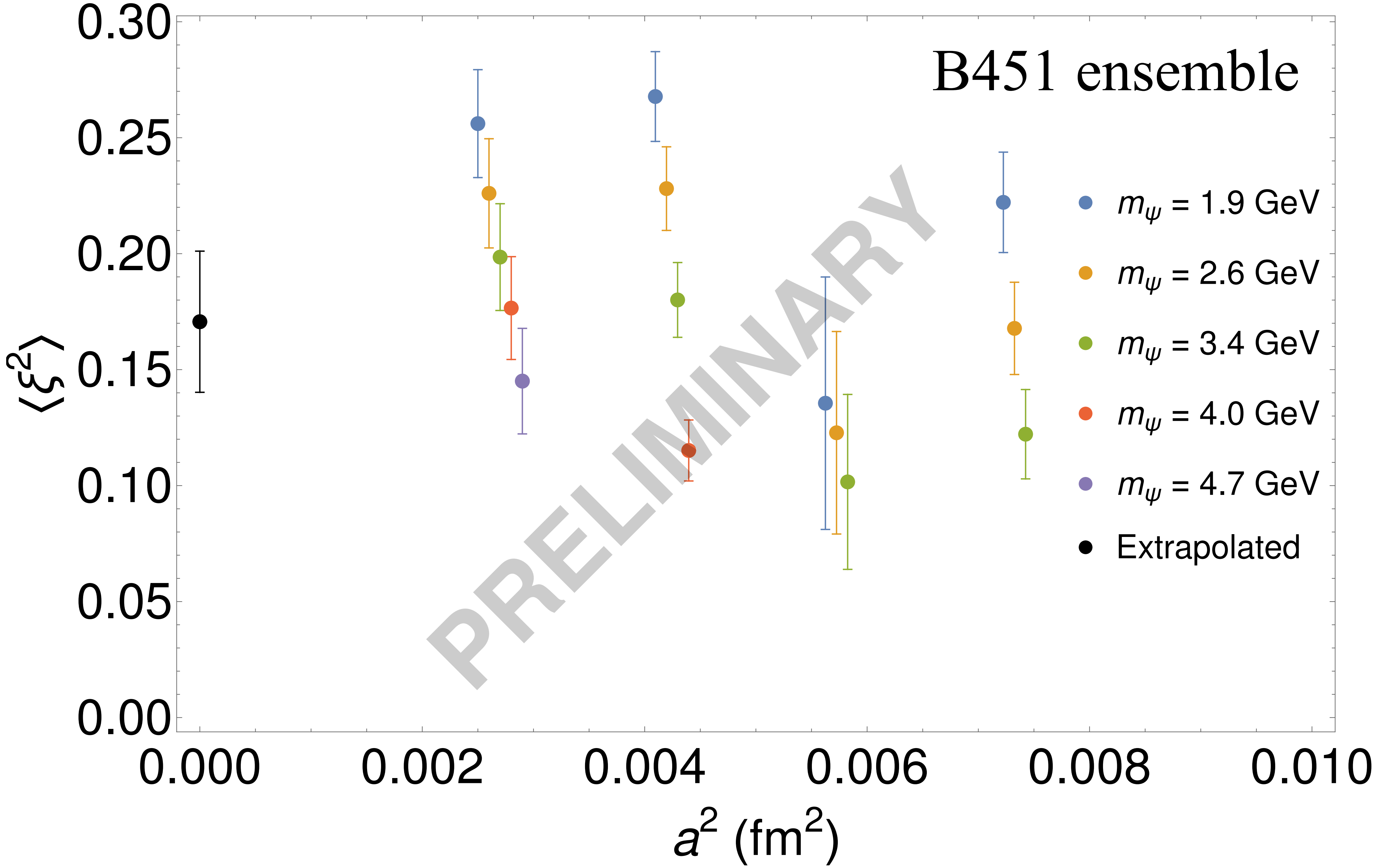
Dynamical calculations

Full-QCD dynamical calculation for $\phi_{\pi,K}(\xi,\mu)$ commenced



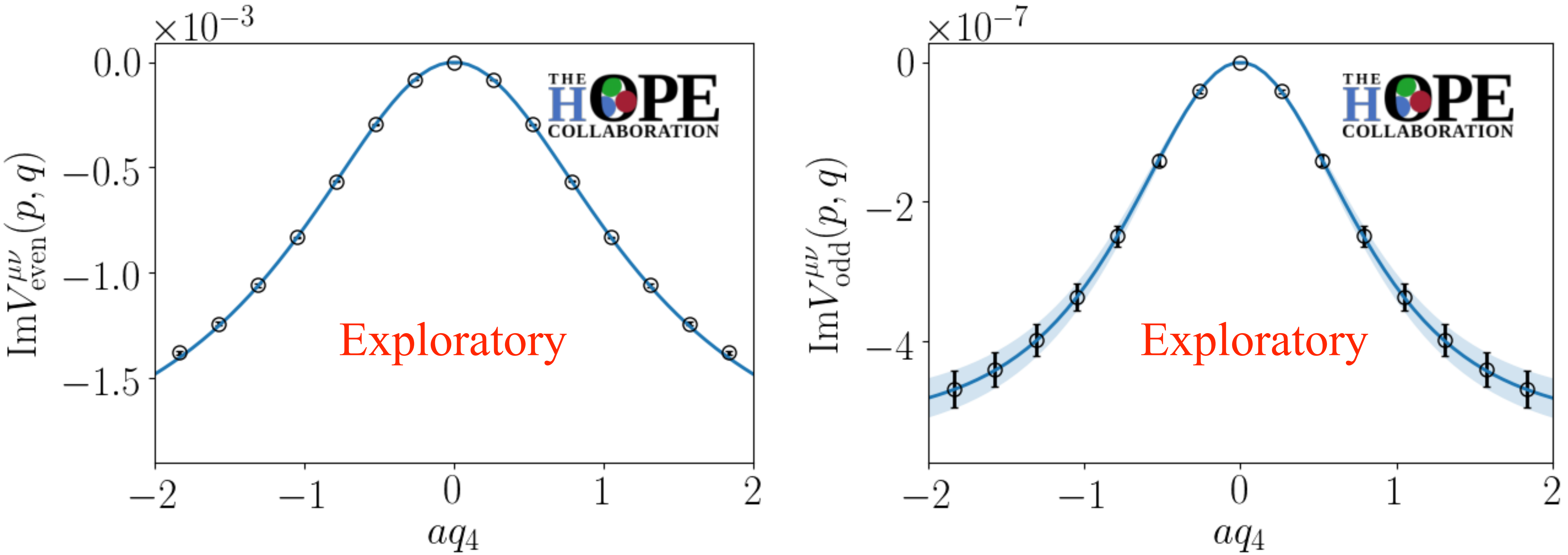
Planned our calculations on these CLS ensembles

Latest status of $\langle \xi^2 \rangle$ calculation



Kaon LCDA

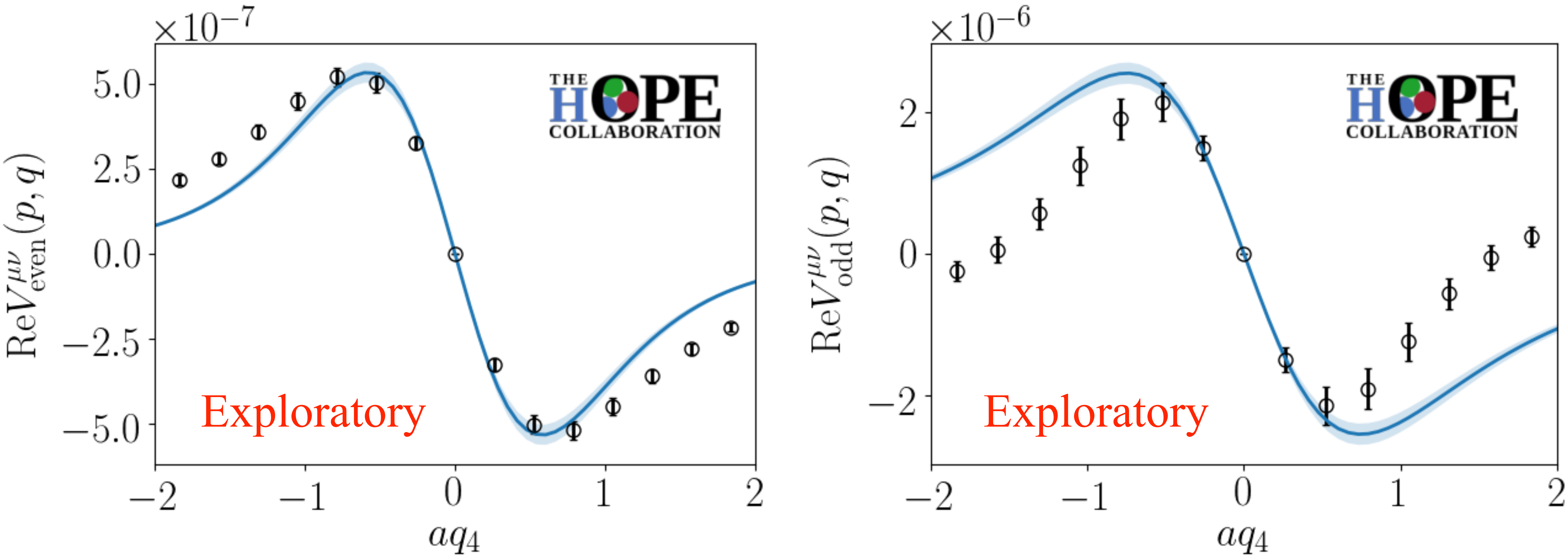
$$V_{\text{even}} = \frac{1}{2}(V_{sl} - V_{ls}) \Rightarrow \text{even moments} \; ; \; V_{\text{odd}} = \frac{1}{2}(V_{sl} + V_{ls}) \Rightarrow \text{odd moments}$$



Zeroth and first moments

Kaon LCDA

$$V_{\text{even}} = \frac{1}{2}(V_{sl} - V_{ls}) \Rightarrow \text{even moments} \; ; \; V_{\text{odd}} = \frac{1}{2}(V_{sl} + V_{ls}) \Rightarrow \text{odd moments}$$



Second and third moments

Tree level Wilson coefficients ⇒ One-loop needed

Conclusion and outlook

- HOPE method facilitates high-moments calculations
- Numerically well tested *via* $\langle \xi^2 \rangle$ of $\phi_\pi(\xi, \mu)$
- Reasonable preliminary result of $\langle \xi^4 \rangle$ of $\phi_\pi(\xi, \mu)$
- Dynamical calculations for pion and kaon LCDAs
- Pion PDF in the near future
- Direct calculation for ξ-dependence from HOPE

HOPE Collaboration, W. Detmold *et al*., Phys. Rev. **D 104** (2021) 7, 074511

Backup slides

Pion LCDA: definition and OPEs

$$\langle 0|\bar{d}(z)\gamma_\mu\gamma_5 W[z,-z]u(-z)|\pi^+(\mathbf{p})\rangle = ip_\mu f_\pi \int_{-1}^{1} d\xi\ e^{-i\xi p\cdot z}\phi_\pi(\xi,\mu)$$

Gegenbauer (conformal) OPE in the isospin limit

$$\phi_\pi(\xi,\mu) = \frac{3}{4}(1-\xi^2)\sum_{n=0,\text{even}}^{\infty}\phi_n(\mu)\mathcal{C}_n^{3/2}(\xi) \xrightarrow[\text{RG}]{\mu\to\infty} \frac{3}{4}(1-\xi^2)$$

Gegenbauer moments

Light-cone OPE

$$\langle 0|\left[\bar{d}\gamma^{\{\mu_0}\gamma_5(i\overleftrightarrow{D}^{\mu_1})\dots(i\overleftrightarrow{D}^{\mu_n\}})u - \text{traces}\right]|\pi^+(\mathbf{p})\rangle$$
$$= f_\pi\,\langle\xi^n\rangle\,(\mu^2)\,[p^{\mu_0}p^{\mu_1}\dots p^{\mu_n} - \text{traces}]$$

Mellin moments $\langle\xi^n\rangle(\mu) = \int_{-1}^{1} d\xi\ \xi^n\phi_\pi(\xi,\mu)$

$$\phi_0 = \langle\xi^0\rangle = 1\,,\ \phi_2 = \frac{7}{12}\left(5\langle\xi^2\rangle - \langle\xi^0\rangle\right)\,,\ \phi_4 = \frac{11}{24}\left(21\langle\xi^4\rangle - 14\langle\xi^2\rangle + \langle\xi^0\rangle\right)\,,\dots.$$

Generic issue in HOPE for higher moments

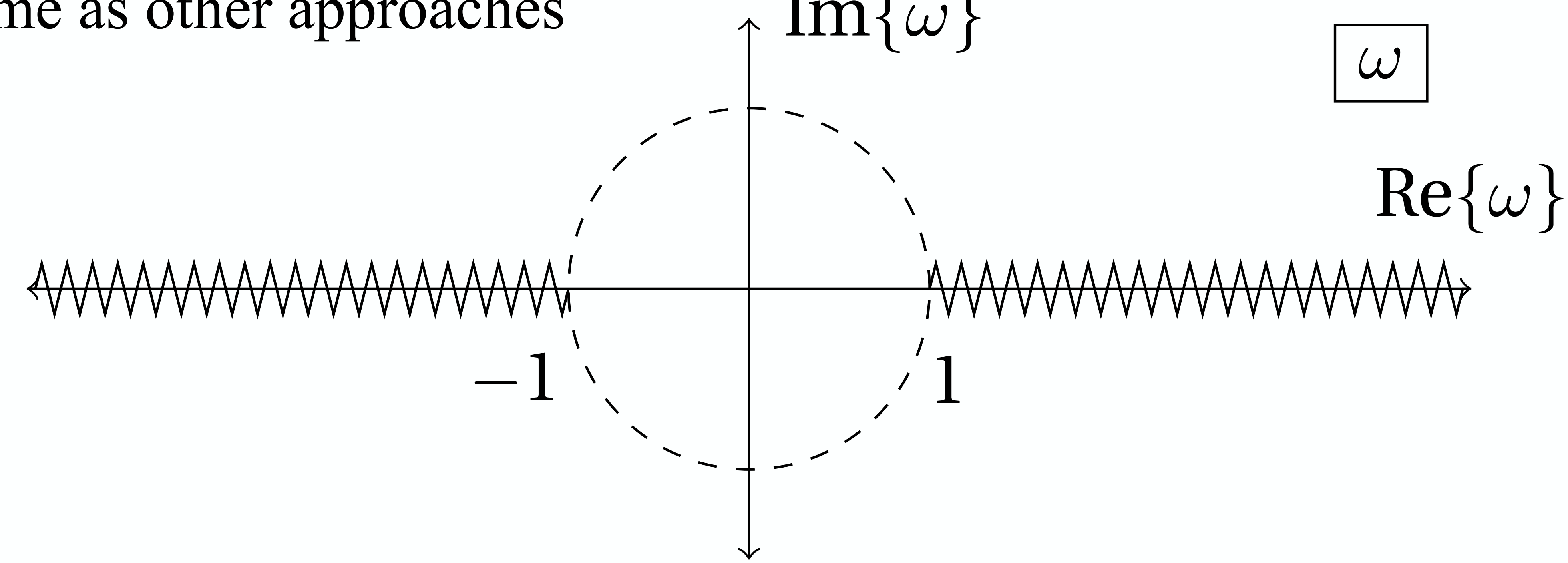
$$T^{\mu\nu}_{\Psi,\psi}(p,q) \sim \sum_{n=0}^{\infty} \langle \xi^n \rangle \omega^n + \text{higher twist}, \; \omega = \frac{2p\cdot q}{\tilde{Q}^2} = \frac{2\mathbf{p}\cdot\mathbf{q} + 2iE_\pi q_4}{q_4^2 + \mathbf{q}^2 + m_\Psi^2}$$

simulate fit

★ Need large $\tilde{Q}^2$ to suppress higher-twist effects [$\sim (\Lambda_{QCD}/\tilde{Q})^m$]

★ Need large $\mathbf{p}$ to make $|\omega| \to 1$ (sensitivity to higher moments)

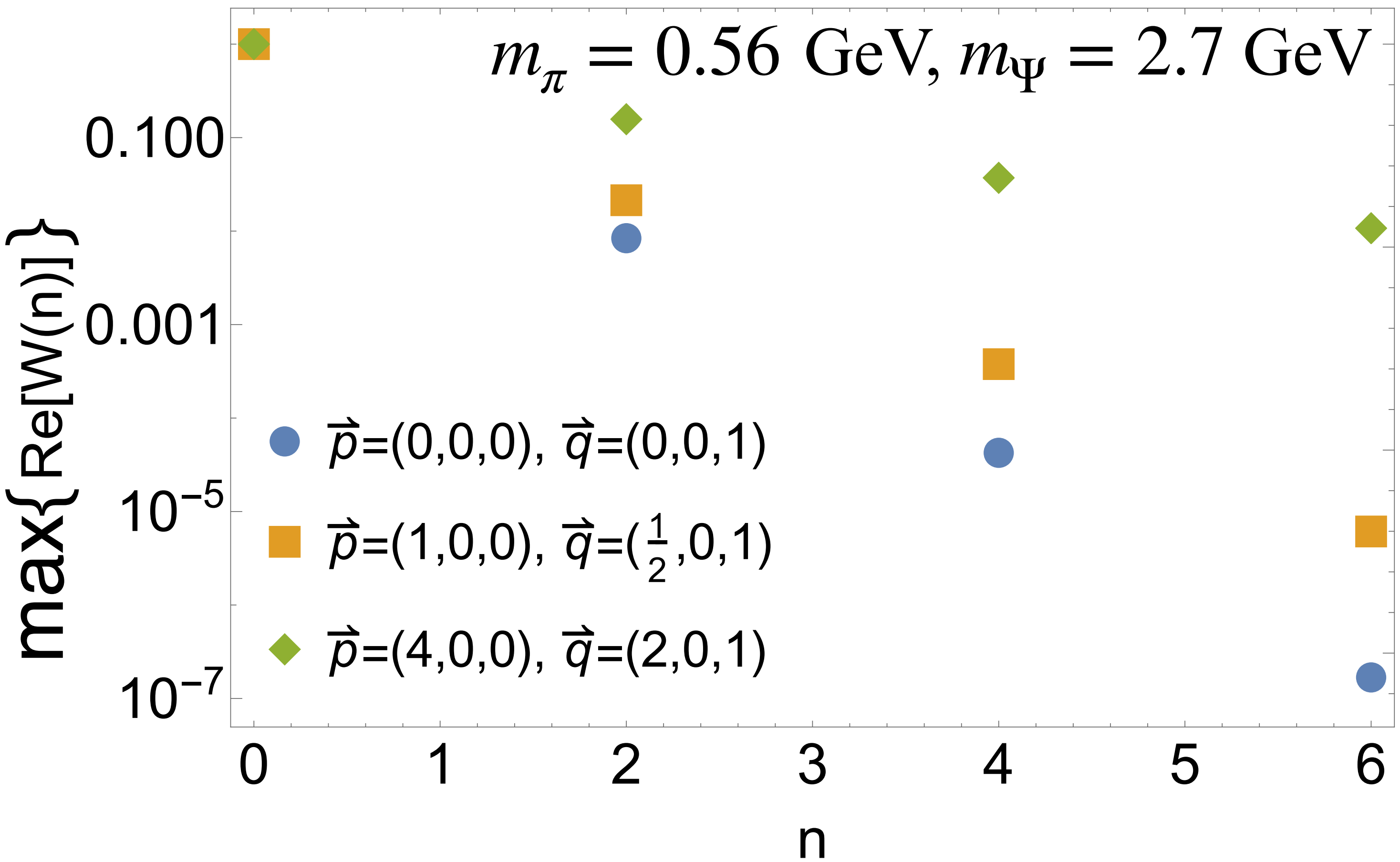
Same as other approaches



HOPE for $V^{[\mu\nu]}$: issue in fitting higher moments

$$V^{[\mu\nu]}(p,q) = \frac{2\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta}{\tilde{Q}^2}\sum_{n \text{ even}}^{\infty}\frac{\zeta^n \mathcal{C}_n^2(\eta)}{2^n(n+1)}C_W^{(n)}(\tilde{Q}^2)f_\pi\langle\xi^n\rangle + \mathcal{O}(1/\tilde{Q}^3)$$

$$= \frac{2\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta}{\tilde{Q}^2}\sum_{n \text{ even}}^{\infty}W(n)C_W^{(n)}(\tilde{Q}^2)f_\pi\langle\xi^n\rangle + \mathcal{O}(1/\tilde{Q}^3)$$



In general, need large **p** to access non-leading moments

Strategy for enhancing sensitivity to $\langle \xi^n \rangle$

$$V^{[12]}(p,q) = \frac{2\epsilon^{12\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_{n \text{ even}}^{\infty} \frac{\zeta^n \mathcal{C}_n^2(\eta)}{2^n(n+1)} C_W^{(n)}(\tilde{Q}^2) f_\pi \langle \xi^n \rangle + \mathcal{O}(1/\tilde{Q}^3)$$

$$= \frac{2(q_3 p_4 - q_4 p_3)}{\tilde{Q}^2} \left[C_W^{(0)}(\tilde{Q}^2) f_\pi + \frac{6(p\cdot q)^2 - p^2 q^2}{6(\tilde{Q}^2)^2} C_W^{(2)}(\tilde{Q}^2) f_\pi \langle \xi^2 \rangle + \dots \right] + \mathcal{O}(1/\tilde{Q}^3)$$

$p_4 = iE_\pi$

choose $\mathbf{p}\cdot\mathbf{q} \neq 0$ while $p_3=0$, $q_3 \neq 0$ and q_4 being real

$$= \frac{2iq_3 E_\pi}{\tilde{Q}^2} \left[C_W^{(0)}(\tilde{Q}^2) f_\pi + \frac{6(p\cdot q)^2 - p^2 q^2}{6(\tilde{Q}^2)^2} C_W^{(2)}(\tilde{Q}^2) f_\pi \langle \xi^2 \rangle + \dots \right] + \mathcal{O}(1/\tilde{Q}^3)$$

imaginary — real — complex

→ The largest contribution to $\mathrm{Re}[V^{[12]}]$ is from $\langle \xi^2 \rangle$

Enhancing the signal: the idea

We work with $|\omega| < 1$ where Minkowskian $V^{\mu\nu}$ is imaginary.

From $V^{\mu\nu}_{\text{Minkowski}}(p,q) = \int_{-\infty}^{\infty} d\tau\, e^{-q_0\tau} R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q})$.

→ $R^{\mu\nu}$ is imaginary.

Back to Euclidean space:

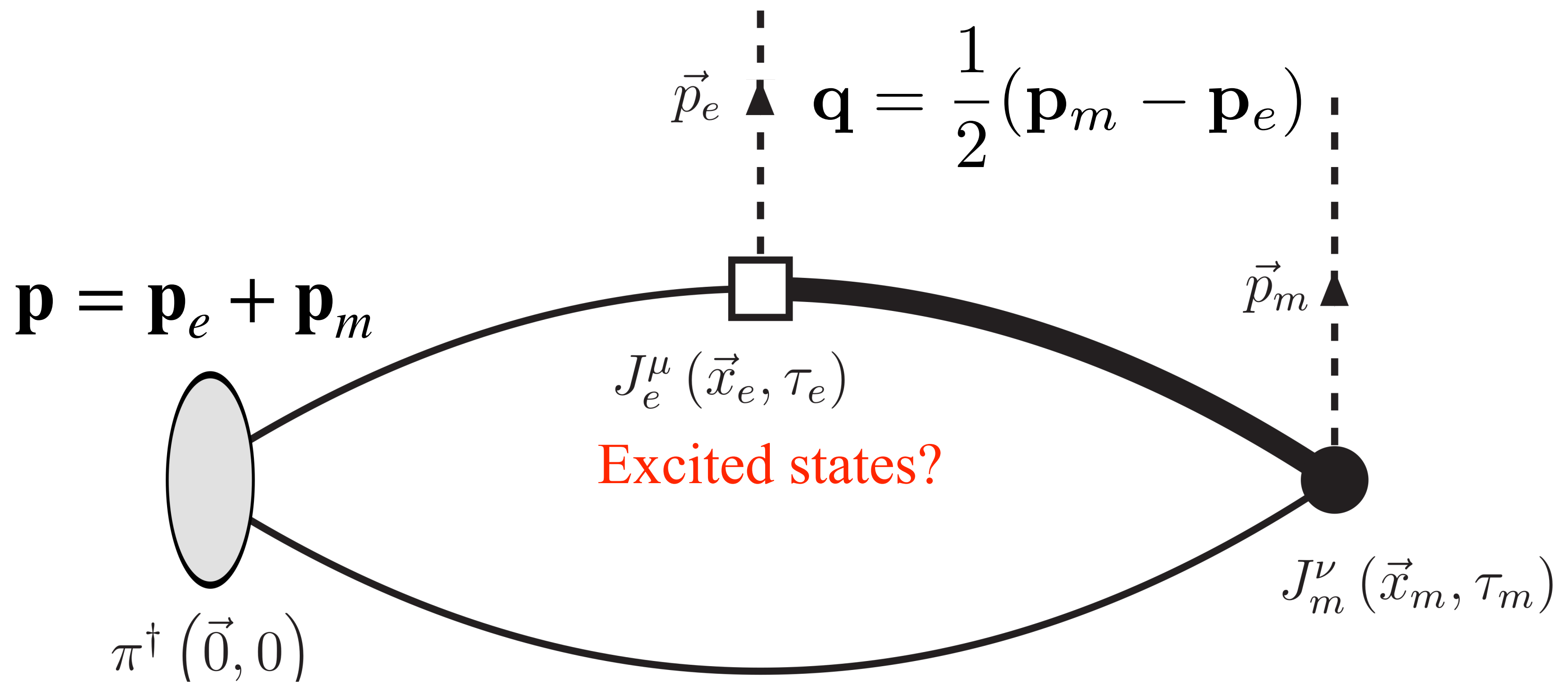
$$\mathrm{Re}[U^{\mu\nu}(\mathbf{p},q)] = \mathrm{Re}\left[\int_{-\infty}^{\infty} d\tau\, R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q})e^{-iq_4\tau}\right]$$

$$\propto \int_0^{\infty} d\tau\, [R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q}) - R^{\mu\nu}(-\tau;\mathbf{p},\mathbf{q})]\sin(q_4\tau)$$

γ_5 hermiticity → $= R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q}) + R^{\mu\nu}(\tau;-\mathbf{p},\mathbf{q})$

More correlated → reduced error

Correlators for lattice calculation



$$C_3^{\mu\nu}(\tau_e, \tau_m; \mathbf{p}_e, \mathbf{p}_m) = \int d^3x_e\, d^3x_m\, e^{i\mathbf{p}_e\cdot\mathbf{x}_e} e^{i\mathbf{p}_m\cdot\mathbf{x}_m} \langle 0|\mathcal{T}\left[J_A^\mu(\tau_e, \mathbf{x}_e) J_A^\nu(\tau_m, \mathbf{x}_m)\mathcal{O}_\pi^\dagger(\mathbf{0})\right]|0\rangle$$

$$= R^{\mu\nu}(\tau_e - \tau_m; \mathbf{p}, \mathbf{q}) \frac{Z_\pi(\mathbf{p})}{2E_\pi(\mathbf{p})} e^{-E_\pi(\mathbf{p})(\tau_e+\tau_m)/2}$$

$z = x_e - x_m$

$$\int d^3z\, e^{iq\cdot z}\langle 0|\,T[J_A^\mu(z/2)J_A^\nu(-z/2)]\,|\pi(p)\rangle$$

HOPE hadronic amplitude in TMR

Fit from

$$C_2(\tau_\pi, \mathbf{p}) = \int d^3\mathbf{x}\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\mathcal{O}_\pi(\mathbf{x}, \tau_\pi)\mathcal{O}_\pi^\dagger(\mathbf{0}, 0)|0\rangle$$

Excited state contamination in $R^{[\mu\nu]}(\tau; \mathbf{p}, \mathbf{q})$