From quantum damped harmonic oscillators to quantum field theory of open system: the path integral approach

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Work in progress, with Nishant Agarwal, Brenden Bowen, and Yi-Zen Chu

Outline

1. Motivation
2. Formalism
3. Results
4. Future direction

Motivation

Motivation

- In the recent years, effective field theories of open systems draws several attentions from different areas.
- Most literature focus on the asymptotic behaviour of observables with specific initial conditions.
- We begin with establishing a theory of quantum mechanical damped harmonic oscillators (DHO) with general initial condition through path integral.

Formalism

Formalism

- The real-time dynamics of open quantum system cannot be described by the scattering amplitudes. The observables which we care is the expectation value:

$$\langle\widehat{\mathcal{O}}(t)\rangle = \mathrm{Tr}[\widehat{\mathcal{O}}\hat{\rho}]. \tag{1}$$

- We exploit the Schwinger-Keldysh formalism (in-in formalism) to construct the generating functional of correlation functions.

$$\begin{aligned} Z_{\rm SK}[J_\pm] &= \mathcal{N}\int \mathrm{d}X_f \\ &\times \int \mathrm{d}X_i^+ \int \mathrm{d}X_i^- \int_{X_i^+}^{X_f} [\mathrm{d}X_+] \int_{X_i^-}^{X_f} [\mathrm{d}X_-]\, \rho[X_i^+, X_i^-; t_i] \\ &\times \exp\left[i\mathcal{S}_{\rm eff}[X_\pm] + i\int_{t_i}^{t_f} \mathrm{d}t\, \big(X_+J_+ - X_-J_-\big)\right] \end{aligned} \tag{2}$$

Formalism

- Effective action can be written as

$$\mathcal{S}_{\rm eff}[X_+, X_-] = \underbrace{\mathcal{S}[X_+] - \mathcal{S}[X_-]}_{\text{analogue of } \widehat{U}\hat{\rho}(t)\widehat{U}^\dagger} + \underbrace{\mathcal{S}_{\rm IF}[X_+, X_-]}_{\text{effect of environment}} \tag{3}$$

where $\mathcal{S}_{\rm IF}[X_+, X_-]$ is called influence functional.

Feynman and Vernon, Annals Phys. 24 (1963)

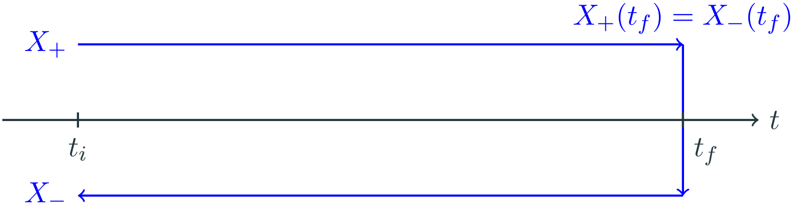


Figure 1: Illustration of the time evolution in Schwinger-Keldysh formalism

Formalism

- We consider the effective action Agarwal and Chu, Phys.Rev.Res. 6 (2024)

$$\begin{aligned}
&\mathcal{S}_{\text{eff}}[X_+, X_-] \\
&= \int_{t_i}^{t_f} \mathrm{d}t \left[\frac{1}{2}\left(\dot{X}_+^2 - \omega_0^2 X_+^2\right) - \frac{1}{2}\left(\dot{X}_-^2 - \omega_0^2 X_-^2\right) \right. \\
&\left. - \frac{i\alpha}{2}(X_+ - X_-)^2 - \gamma(X_+ - X_-)(\dot{X}_+ + \dot{X}_-) + L_{\text{int}} \right] \quad (4)
\end{aligned}$$

where $\alpha, \gamma > 0$.

- We treat L_{int} as perturbations.

Results

Results: Cubic interaction

- Consider
$$L_{\text{int}} = -\frac{\lambda}{3!}X_+^3 + \frac{\lambda}{3!}X_-^3. \tag{5}$$
- We evaluate the expectation value of $\hat{X}$ and obtain

$$\begin{aligned}\langle\hat{X}(t)\rangle = &\ \langle\hat{X}(t_i)\rangle_0\,\Delta(t-t_i) + \langle\hat{P}(t_i)\rangle_0\,\mathcal{G}^+(t-t_i)\\ &- \frac{\lambda}{3!}\int_{t_i}^{t_f}\mathrm{d}t'\,\Big(3[\mathcal{G}^+(t'-t_i)]^2\mathcal{G}^+(t-t')\,\langle\hat{P}^2(t_i)\rangle_0\\ &+ 3\mathcal{G}^+(t-t')\mathcal{G}^+(t'-t_i)\Delta(t'-t_i)\,\langle\{\hat{X}(t_i),\hat{P}(t_i)\}\rangle_0\\ &+ 3[\Delta(t'-t_i)]^2\mathcal{G}^+(t-t')\,\langle\hat{X}^2(t_i)\rangle_0\\ &+ 3\mathcal{G}^+(t-t')\mathcal{G}_\alpha(t',t';t_i)\Big) + \mathcal{O}(\lambda^2)\end{aligned} \tag{6}$$

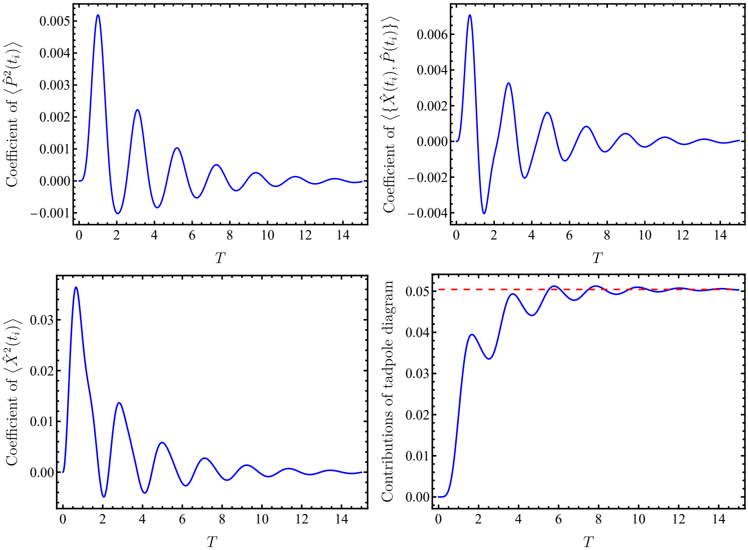
Results: Cubic interaction

- The first order correction from cubic interaction contains
  1. Terms involve initial two-point functions,
  2. Terms are independent to initial condition. (Tadpole diagram)
- As $t \to \infty$,

$$\langle \hat{X}(t) \rangle \to -\frac{\alpha\lambda}{8\gamma\omega_0^2} \tag{7}$$

- We visualise the first order correction of one-point function $\lambda^{-1}(\langle \hat{X}(t) \rangle_0 - \langle \hat{X}(t) \rangle)$ with parameters $\omega = 3$, $\gamma = 0.3$, $\alpha = 10$, and $\Omega_0 = 1$. We set $t_i = 0$ and $t = T$

Results: Cubic interaction



Future direction

Future direction

- Extension to quantum field theory of open system
- The loop calculation and renormalisation with generic initial conditions.
- Relation between renormalisation group flows and the quantities in quantum information

References

References

- R. P. Feynman and F. L. Vernon, Jr., “The Theory of a general quantum system interacting with a linear dissipative system,” Annals Phys. **24**, 118-173 (1963) doi:10.1016/0003-4916(63)90068-X
- N. Agarwal and Y. Z. Chu, “Initial value formulation of a quantum damped harmonic oscillator,” Phys. Rev. Res. **6**, no.2, 023113 (2024) doi:10.1103/PhysRevResearch.6.023113 [arXiv:2303.04829 [hep-th]].

Thank you for your attention