

Correlation-driven electronic states in low-dimensional nanoscale systems (research overview)

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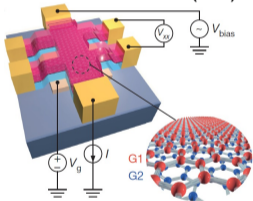
National Science and Technology Council (NSTC), Institute of Physics & Academia Sinica, Taiwan

Nanoscale systems in low dimensions

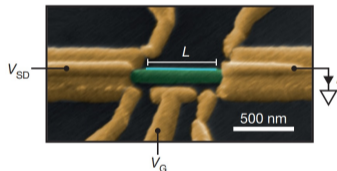
- Target systems:
quantum wires, nanotubes, quantum point contacts, quantum dots, topological superconductors, topological insulators, two-dimensional materials ...

- Low-dimensional nanoscale systems:

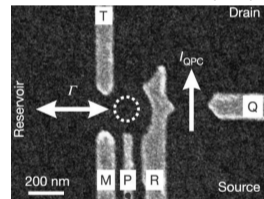
- two dimension (2D)



- one dimension (1D)



- zero dimension (0D)

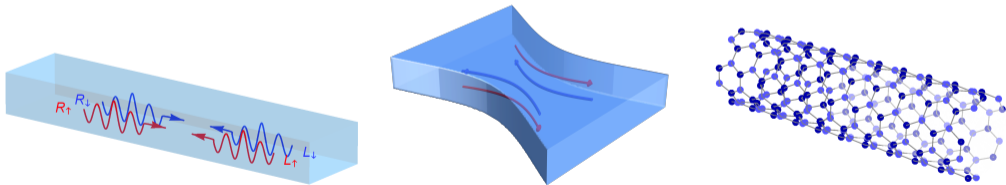


Cao et al., Nature 556, 43 (2018); Albrecht et al., Nature 531, 206 (2016); Elzerman et al., Nature 430, 431 (2004)

- Controllable ingredients in nanoscale systems:
confinement potential, carrier density, spin-orbit coupling, micromagnets, superconductivity ...
⇒ platform for quantum matter and quantum phenomena

Correlated 1D channels in nanoscale systems

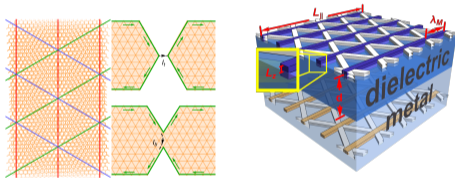
- 1D channels formed due to electric potential or band structures
- Interacting systems in 1D: no well-defined quasiparticle excitations
- Power-law suppression of density of states at the Fermi level: $\rho(\epsilon_F) \rightarrow 0$
 \Rightarrow **power-law** behavior of correlation functions (observable features)
- Correlation effects in any interacting 1D systems
- Tomonaga-Luttinger liquid (TLL): excitations with bosonic nature
ex: nanowires, quantum point contact, nanotubes ...



- Research at IoP: generalizations to other platforms

Correlated electron systems in low-dimensional nanoscale systems

- Generalization of the usual Tomonaga-Luttinger liquid (TLL)
- *Coupled TLL* beyond 1D:
a network of domain wall modes in
twisted bilayer graphene (TBG)
- *TLL* beyond spin-degenerate systems:
spin-momentum-locked boundary states
⇒ **helical liquids**



Quantum anomalous Hall states in TBG network

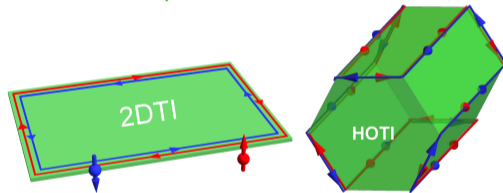
[CHH et al., PRB 108, L121409 \(2023\)](#)

Superconductivity in TBG network

[Wang & CHH, 2D Mater. 11, 035007 \(2024\)](#)

2D helix in TBG network

[Chang & CHH, arXiv:2412.14065 \(2024\)](#)



Topological superconductivity in double helical liquids

[CHH, Nanoscale Horiz. 9, 1725 \(2024\)](#)

Time-reversal soliton corner modes in HOTI

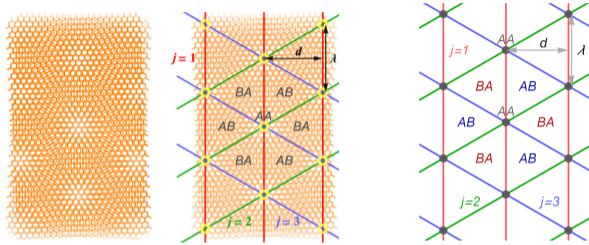
[Hung, Wang, CHH et al., PRB 110, 035125 \(2024\)](#)

Majorana zero modes in high spin Chern insulators

[Hung, CHH et al., arXiv:2412.08632 \(2024\)](#)

Domain wall network in twisted bilayer graphene

- Moiré pattern from twisted structure + interlayer bias
 \Rightarrow local spectral gap with alternating signs between AB- and BA-stacking areas



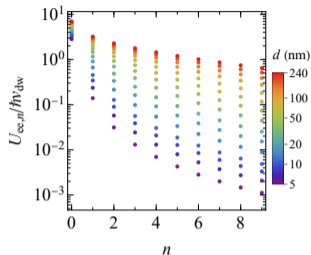
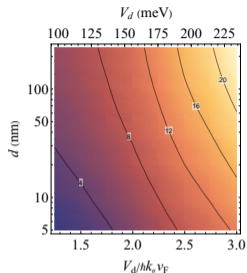
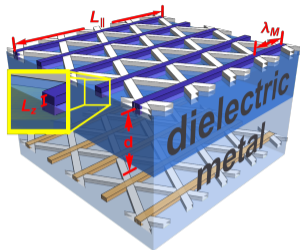
- Domain wall network with general quadratic interaction terms:

$$H_{0,c}^{(j)} = \sum_{mm'} \int \frac{dx}{2\pi} \left[V_{\phi,mm'}^j \partial_x \phi_{c,m}^j \partial_x \phi_{c,m'}^j + V_{\theta,mm'}^j \partial_x \theta_{c,m}^j \partial_x \theta_{c,m'}^j \right]$$

$$H_{0,s}^{(j)} = \sum_m \int \frac{\hbar dx}{2\pi} \left[\frac{u_s}{K_s} (\partial_x \phi_{s,m}^j)^2 + u_s K_s (\partial_x \theta_{s,m}^j)^2 \right]$$

- $V_{\phi,mm'}^j, V_{\theta,mm'}^j, K_s$: forward-scattering terms ($R \leftrightarrow R$ & $L \leftrightarrow L$)
- $\phi_{c,m}^j, \theta_{c,m}^j, \phi_{s,m}^j, \theta_{s,m}^j$: boson fields in the charge/spin sector

Electrically tunable correlated domain wall network

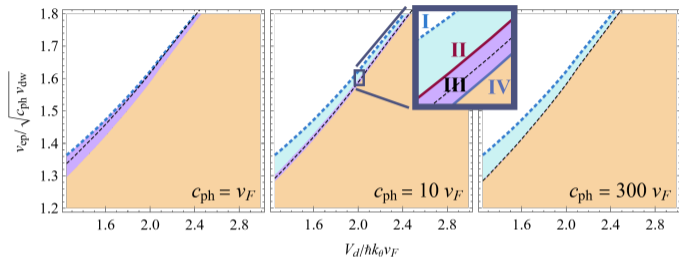
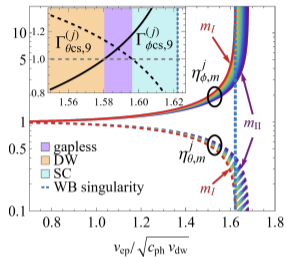


- Spatial profile of the charge density $\rho(\vec{r})$ of the domain wall modes
- Electrically tunable interaction strength in the domain wall network

$$U_{ee,n} = \frac{e^2 L_y}{4\pi\epsilon_0} \int d^3\mathbf{x} \int d^3\mathbf{x}' \left[\frac{\rho_m^{\text{dw}}(\mathbf{x})\rho_{m+n}^{\text{dw}}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} + \frac{\rho_m^{\text{dw}}(\mathbf{x})\rho_{m+n}^{\text{image}}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \right]$$

- Interaction strength tunable through device design and interlayer bias
 \Rightarrow input for bosonized model of the correlated domain wall network

Electrically tunable correlated domain wall network

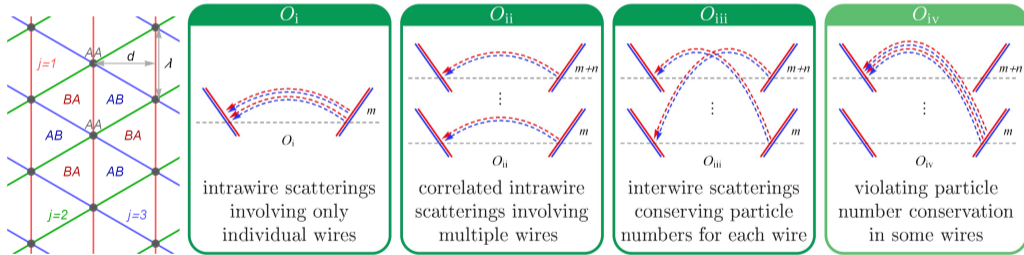


Wang & [CHH](#), 2D Mater. 11, 035007 (2024)

- Various phases: correlated network, density wave, superconductivity, e-ph-coupled liquid
- Distinct behavior upon for different phonon velocity:
 - low-velocity regime: no pairing instability
 - higher-velocity regime: **pairing instability** for sufficiently large electron-phonon coupling
- Even larger e-ph coupling destabilizes the network: Wentzel-Bardeen (WB) singularity

General scatterings in the domain wall network

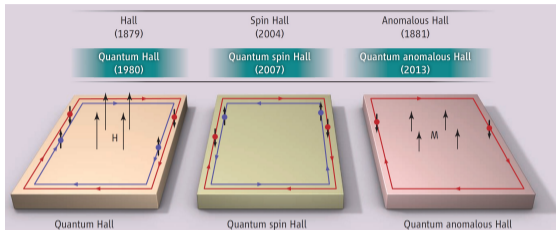
- We consider general scatterings in the domain wall network of twisted bilayer graphene
- We systematically construct scattering operators allowed by the conservation laws
- Moiré pattern allows for novel correlated states and fractional excitations
⇒ **moiré umklapp scatterings**



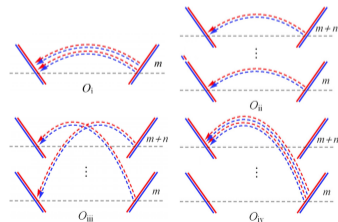
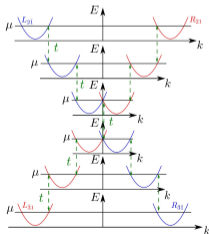
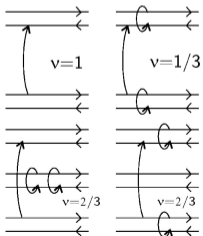
- O_{iv} : correlated states hosting a gapped bulk and gapless edge modes at fractional fillings
⇒ integer and fractional quantum anomalous Hall effects

Trio of quantum Hall-related phenomena

- coupled-wire or network construction



Oh, Science 2013



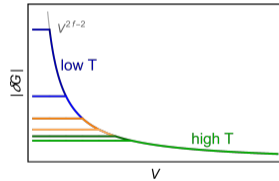
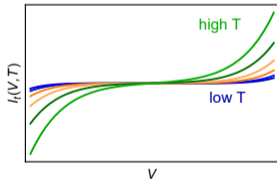
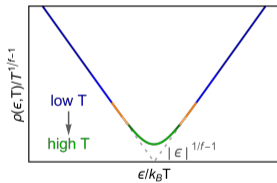
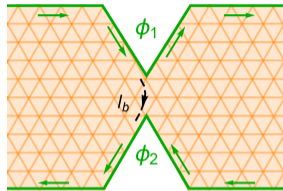
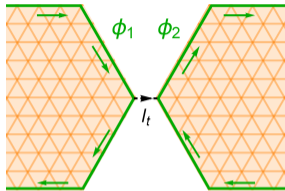
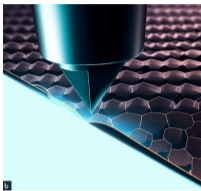
Kane et al., PRL 2002;

Klinovaja and Tserkovnyak, PRB 2014;

CHH et al., PRB 108, L121409 (2023)

Moiré correlated states at fractional fillings

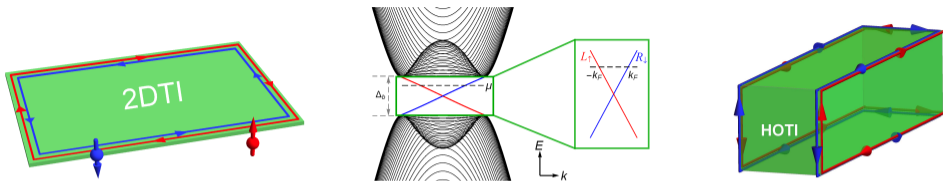
- We proposed spectroscopic and transport setups for experimental verification



- Universal scaling curve for current-bias (I_t - V) curve at temperature T :

$$I_t \propto T^{\frac{2}{f}-1} \sinh\left(\frac{eV}{2k_B T}\right) \left| \Gamma\left(\frac{1}{f} + i\frac{eV}{2\pi k_B T}\right) \right|^2$$

Helical liquids formed by interacting electrons in helical channels



- Electrons in 2DTI edges or HOTI hinges: $H_{hl} = H_{kin} + H_{ee}$

- Kinetic energy:

$$H_{kin} = -i\hbar v_F \int dr \left(R_{\downarrow}^{\dagger} \partial_r R_{\downarrow} - L_{\uparrow}^{\dagger} \partial_r L_{\uparrow} \right)$$

- e - e interaction (g_2, g_4 : interaction strength):

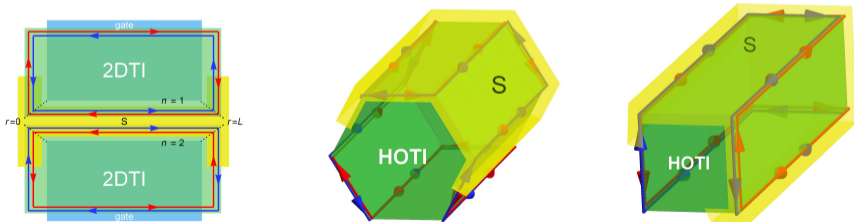
$$H_{ee} = g_2 \int dr R_{\downarrow}^{\dagger} R_{\downarrow} L_{\uparrow}^{\dagger} L_{\uparrow} + \frac{g_4}{2} \int dr \left[\left(R_{\downarrow}^{\dagger} R_{\downarrow} \right)^2 + \left(L_{\uparrow}^{\dagger} L_{\uparrow} \right)^2 \right]$$

- Spin-momentum locking nature + correlation effects in 1D confinement
 \Rightarrow **helical liquids**

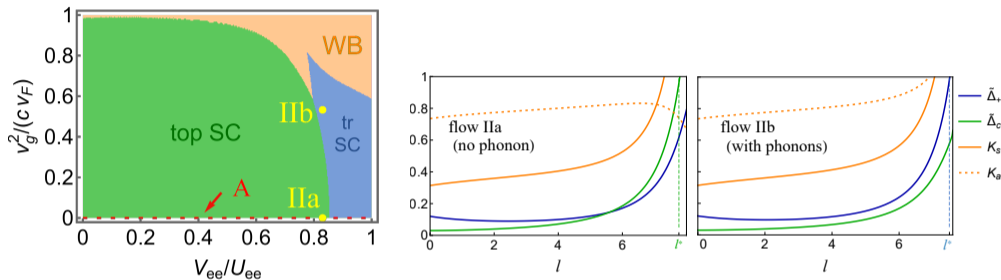
Review article: [CHH et al., SST 36, 123003 \(2021\)](#)

Nanoscale platforms for topological superconductivity and zero modes

- Synthesizing nanoscale systems with nontrivial topology + superconductivity
- Intensively investigated setup in proximitized 1D nanowires with strong spin-orbit coupling
⇒ fine-tuning μ and external B field are needed
Sato PLB 2003; Sato et al., PRL 2009; Sato & Fujimoto, PRB 2009; Lutchyn et al., PRL 2010; Oreg et al., PRL 2010 ...
- Alternative setups proposed to avoid external B fields or fine-tuning μ
- Proposals based on **double helical liquids** with proximity-induced pairing
⇒ dominant nonlocal pairing over local pairing

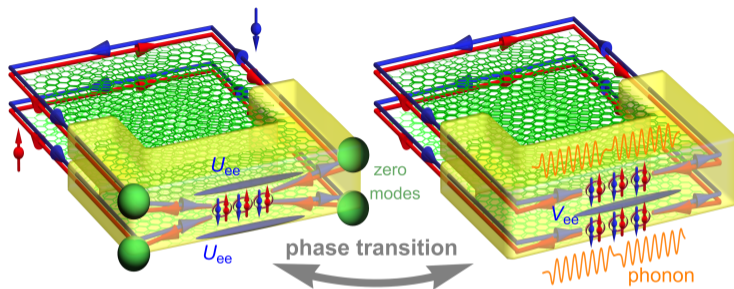


Interaction and phonon effects on pairing in double helical liquids



- e-ph coupling $g \propto v_g^2/(cV_F)$ vs inter-channel to intra-channel interaction ratio V_{ee}/U_{ee}
- Phonons: effectively mediate attractive interactions within each channel
 \Rightarrow enhancing local pairing Δ_n more significantly (compared to Δ_c)
- Electron-phonon coupling can push the system from topological SC to trivial SC phase
 \Rightarrow **phonon-induced topological phase transitions**
- Reaching the Wentzel-Bardeen (WB) singularity in a non-monotonic way

Interaction- and phonon-induced topological phase transitions



- Electrically tunable topological phase transitions through the ratio V_{ee}/U_{ee} of the inter-channel to intra-channel interaction strength
- Omnipresence of $e-e$ interactions and phonons
⇒ practical constraints in utilizing helical channels to realize topological zero modes

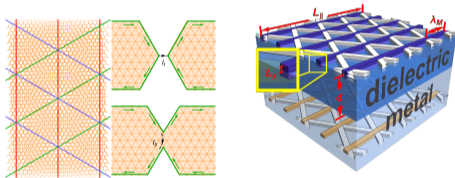
[CHH](#), *Nanoscale Horiz.* 9, 1725 (2024); [CHH et al.](#), *Phys. Rev. Lett.* 121, 196801 (2018);

topical review: [CHH et al.](#), *SST* 36, 123003 (2021)

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- Correlation effects in any interacting 1D systems

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Quantum anomalous Hall states in TBG network

[CHH et al., PRB 108, L121409 \(2023\)](#)

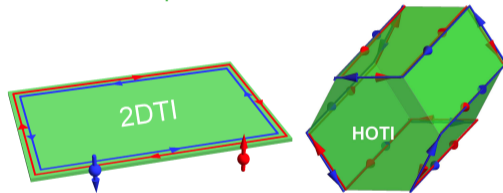
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- *TLL* beyond spin-degenerate systems:
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Majorana zero modes in high spin Chern insulators

[Hung, CHH et al., arXiv:2412.08632 \(2024\)](#)

Topical review on helical liquids

[CHH et al., SST 36, 123003 \(2021\)](#)

Collaboration and recruitment

- Research interests:
Quantum matter and quantum phenomena in nanoscale systems
- Research positions at IoP, AS
 - Postdoctoral Researcher
 - Graduate Student
 - Research Assistant
- Webpage: <https://sites.google.com/view/qmtheory>
- Contact: chenhsuan@gate.sinica.edu.tw
- We welcome highly motivated people to join us
- Welcome to share the information!
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