Implementing parametric amplifiers in Advanced Gravitational Wave Detectors

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OutLine

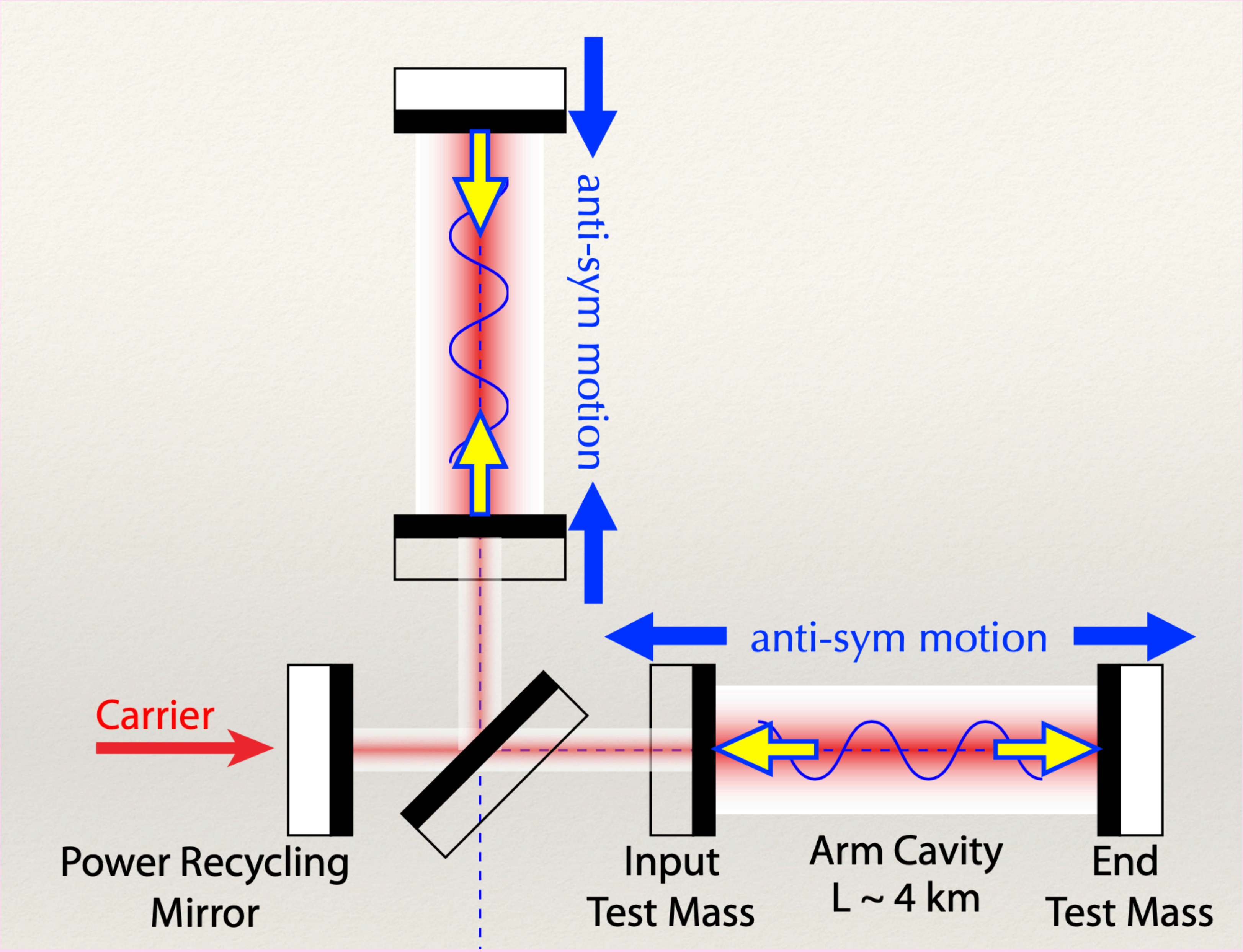
Quantum Limit of Gravitational Wave Detectors

Parametric Amplifiers

Parametric Amplifiers as the tool to manipulation quantum optical field

Summary

Quantum Limit of Gravitational Wave Detectors



Michelson Interferometer, 4km arms, and kg-scale mirrors

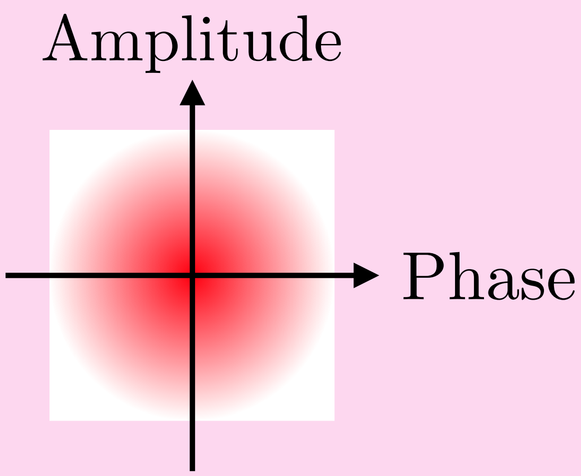
Audio band window

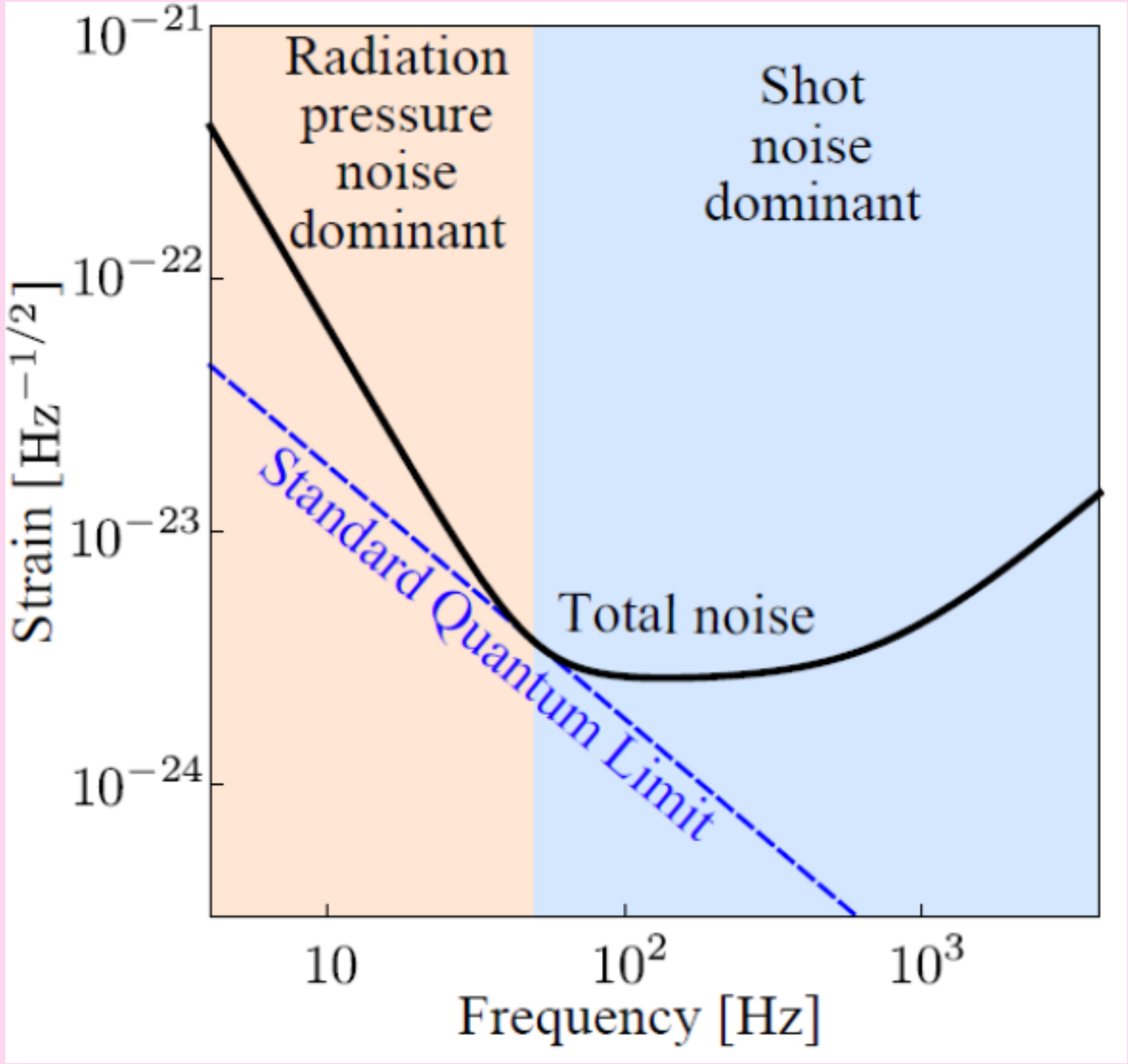
$$\Delta L \sim \lambda_{\mathrm{mirror}} \sim \frac{h}{m\Omega_{\mathrm{GW}}\Delta L}$$

To sense a motion roughly the size of matter wavelength

Quantum property of detector!

Standard Quantum Limit





Amplitude fluctuation

Radiation Pressure Noise

Phase fluctuation

Shot Noise

Trade-off Between

Shot/Radiation-Pressure Noise

Standard Quantum Limit

Energetic Quantum Limit

$$\Delta E \Delta \phi \geq \hbar \omega \quad \longrightarrow$$

$$\int_0^\infty \frac{d\Omega}{2\pi} \frac{1}{S_h(\Omega)} \leq \frac{\Delta E^2}{4\hbar^2}$$

Mizuno Ph.D Thesis, 1995
Braginsky et al. AIP Conference Proceedings **523**, 180 (2000);
Miao et.al Phys. Rev. Letts 2017

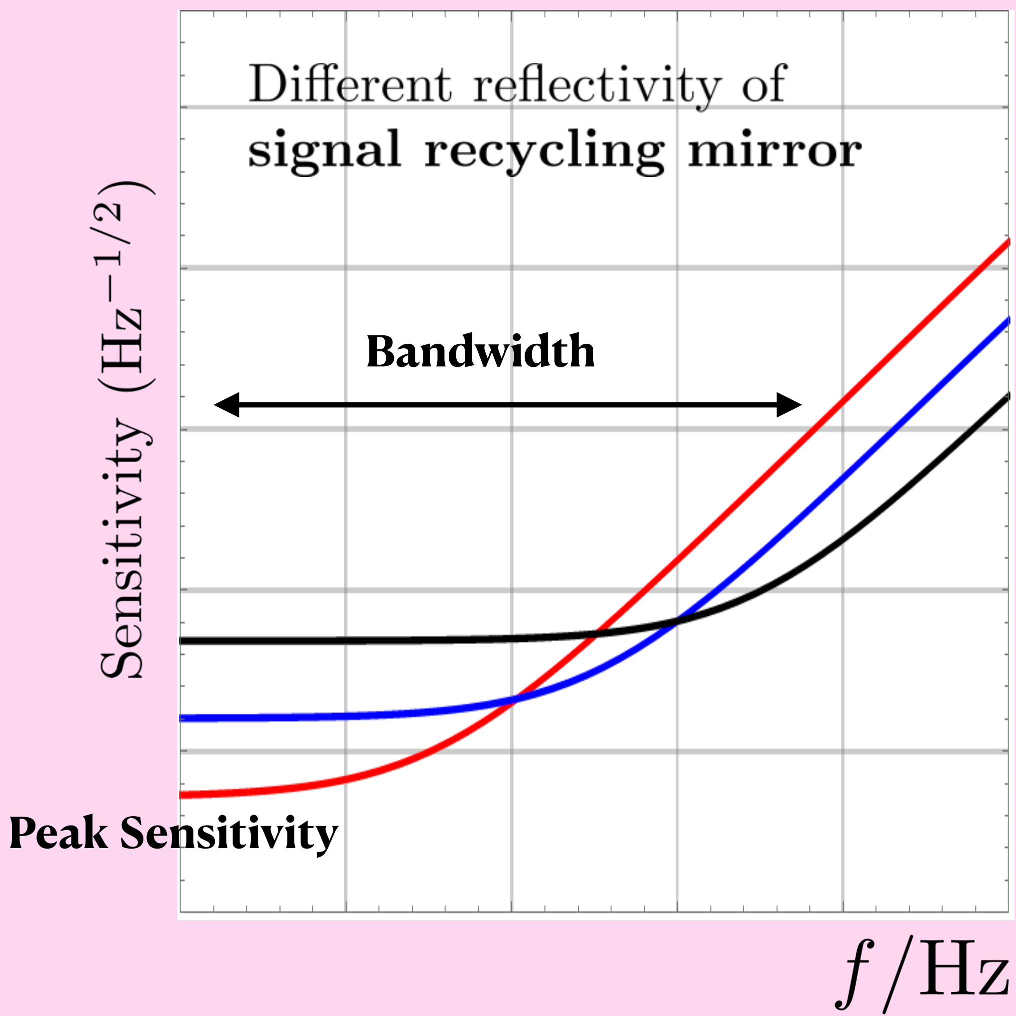
Increase the Bandwidth

↓

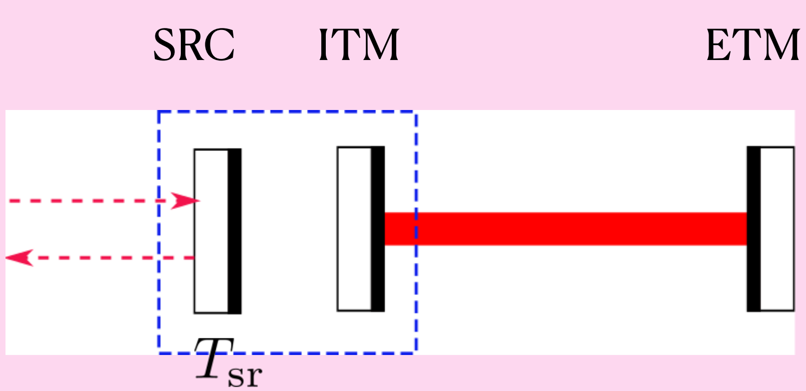
Optical Cavity case:
Increase T.

Decrease the peak sensitivity

Bandwidth-Sensitivity trade-off



Differential Mode of LIGO:



Surpassing these quantum limits

Parametric Amplifier Can help us!

- **Squeezing generation to surpass Standard Quantum Limit!**
- **Intracavity field Manipulation to Surpass bandwidth-peak Limit (EQL)**

Parametric Oscillator

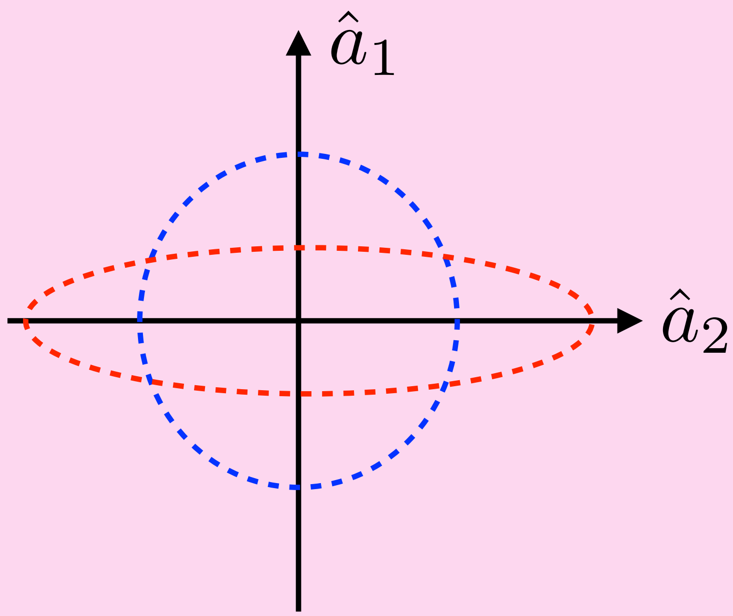
Vacuum injection → **Pumped Nonlinearity** → Entangled photons $\hat{a}$ $\hat{b}$

$$\hat{H}_{\text{int}} = g\bar{A}_p(\hat{a}\hat{b} + \hat{a}^\dagger\hat{b}^\dagger)$$

Degenerate Amplifier

$$\hat{a} = \hat{b}$$

Phase Sensitive Amplification



"Squeezing!"

Non-Degenerate Amplifier

$$\hat{a} \neq \hat{b}$$

Phase insensitive Amplification

$$S_{a_1 b_1} \neq 0!$$

Quantum Entanglement between two fields

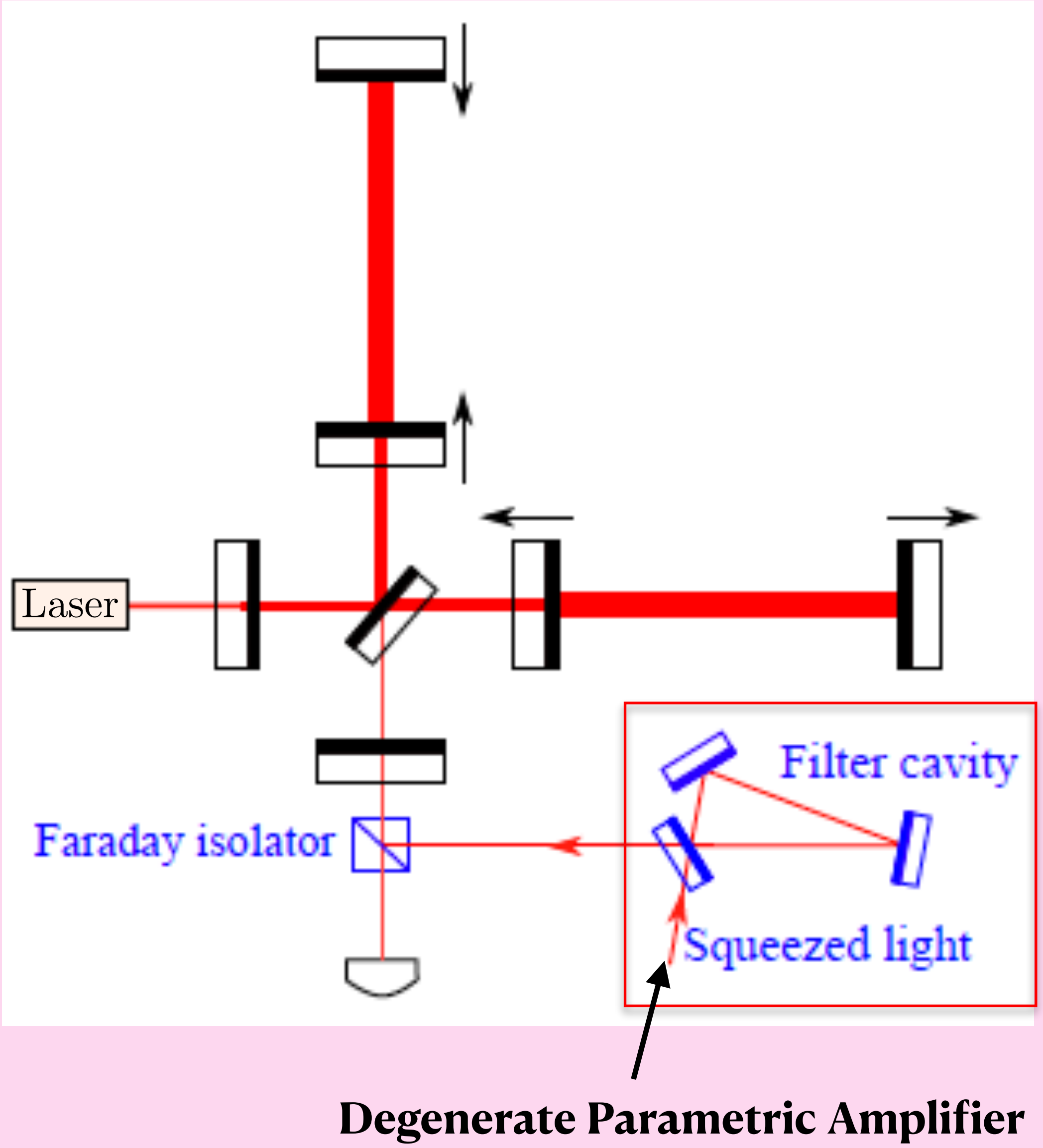
Measure a — conditionally reduce the uncertainty of b

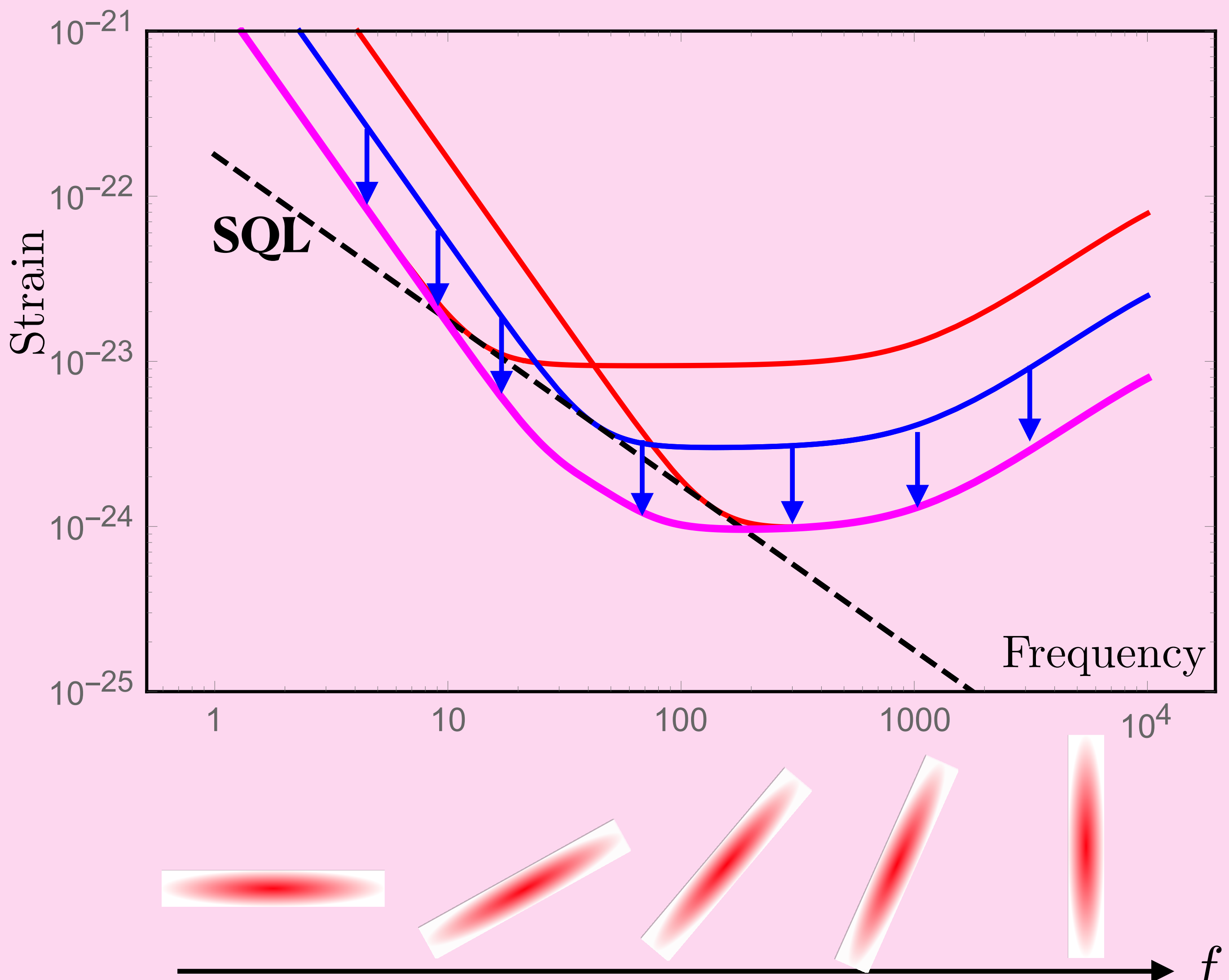
Modify dynamics of optical system

Manipulating Input Fields:

Frequency Dependent Squeezing

Degenerate Parametric Amplifier + Fiter Cavity





16 meters Filter Cavity experiment: McCuller et al. Phys. Rev. Letts, 124, 171102, 2020 (Matt Evans' group)

Manipulating Input Fields:

Frequency Dependent CONDITIONAL Squeezing

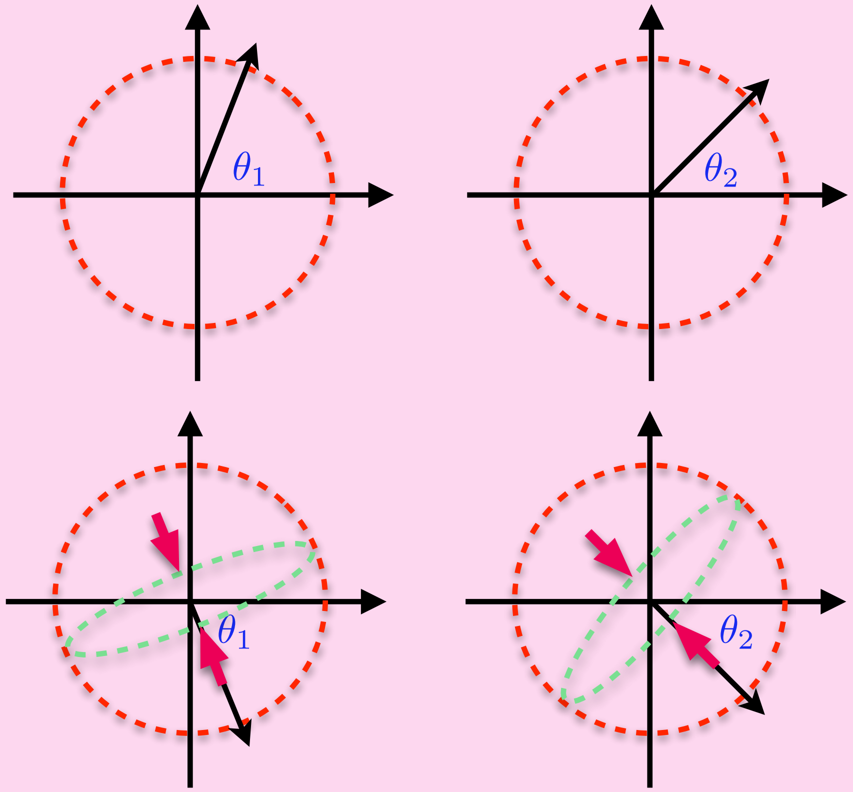
Non-Degenerate Parametric Amplifier + **Arm Cavity**

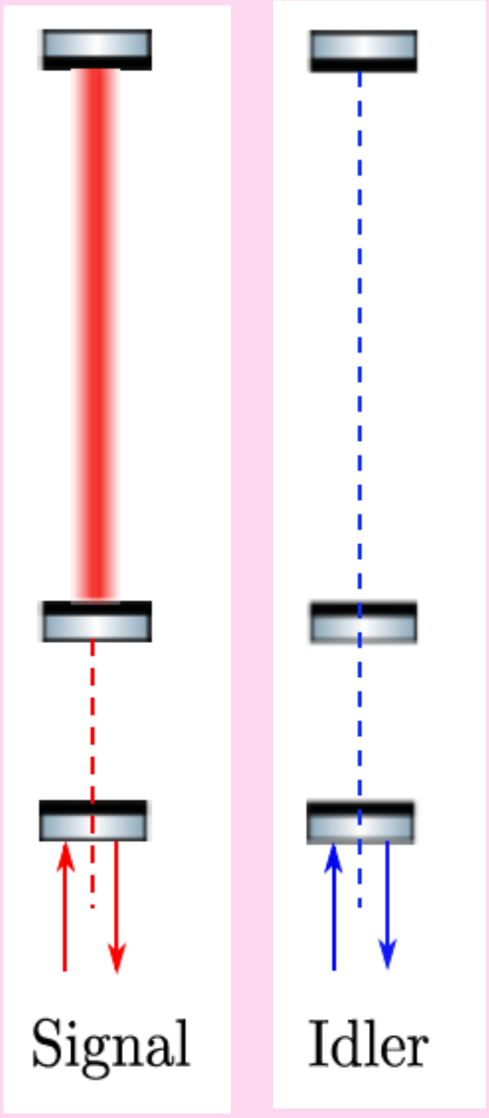
Yiqiu Ma et. al. Nat. Phys. 13,776-780 (2017)

Entangled beam from ND Parametric Amplifier
Injected into the Interferometer!

Measurement →

Squeezing! →





Experimental Follow up:

Jan Sudbeck et al. Nat.Photonics, 14, 240-244 (2020)

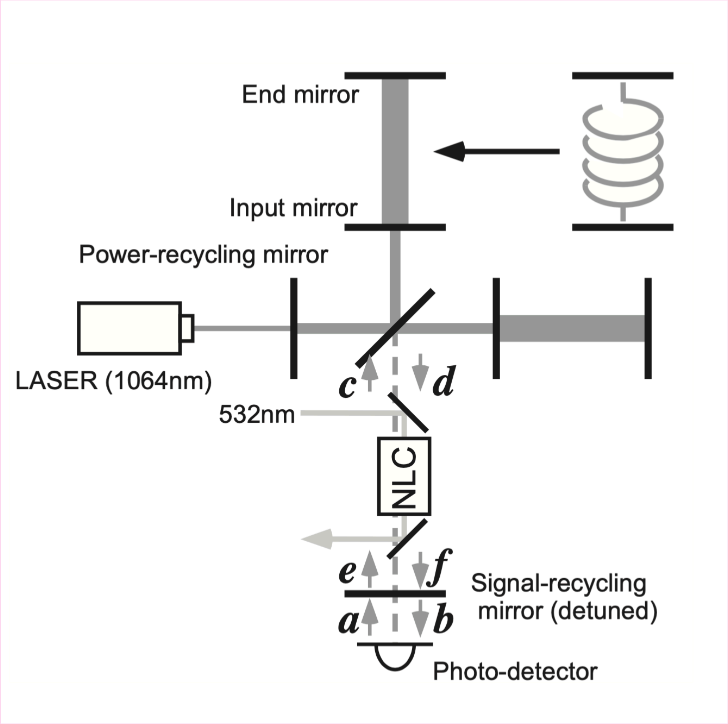
Min-Jet Yap et al., arXiv:1908.08685 (2019)

Theoretical Follow up:

Beckey, et.al, Phys. Rev. D 100,083011 (2019)

Zhang, et.al, Phys. Rev. D 101,124052 (2020)

Manipulating Intracavity Fields:



Pioneering Work: by Professor Kentaro Somiya

Somiya et. al. Parametric Signal amplification to create a stiff optical bar. Phys. Lett. A 380, 521-524, (2016)

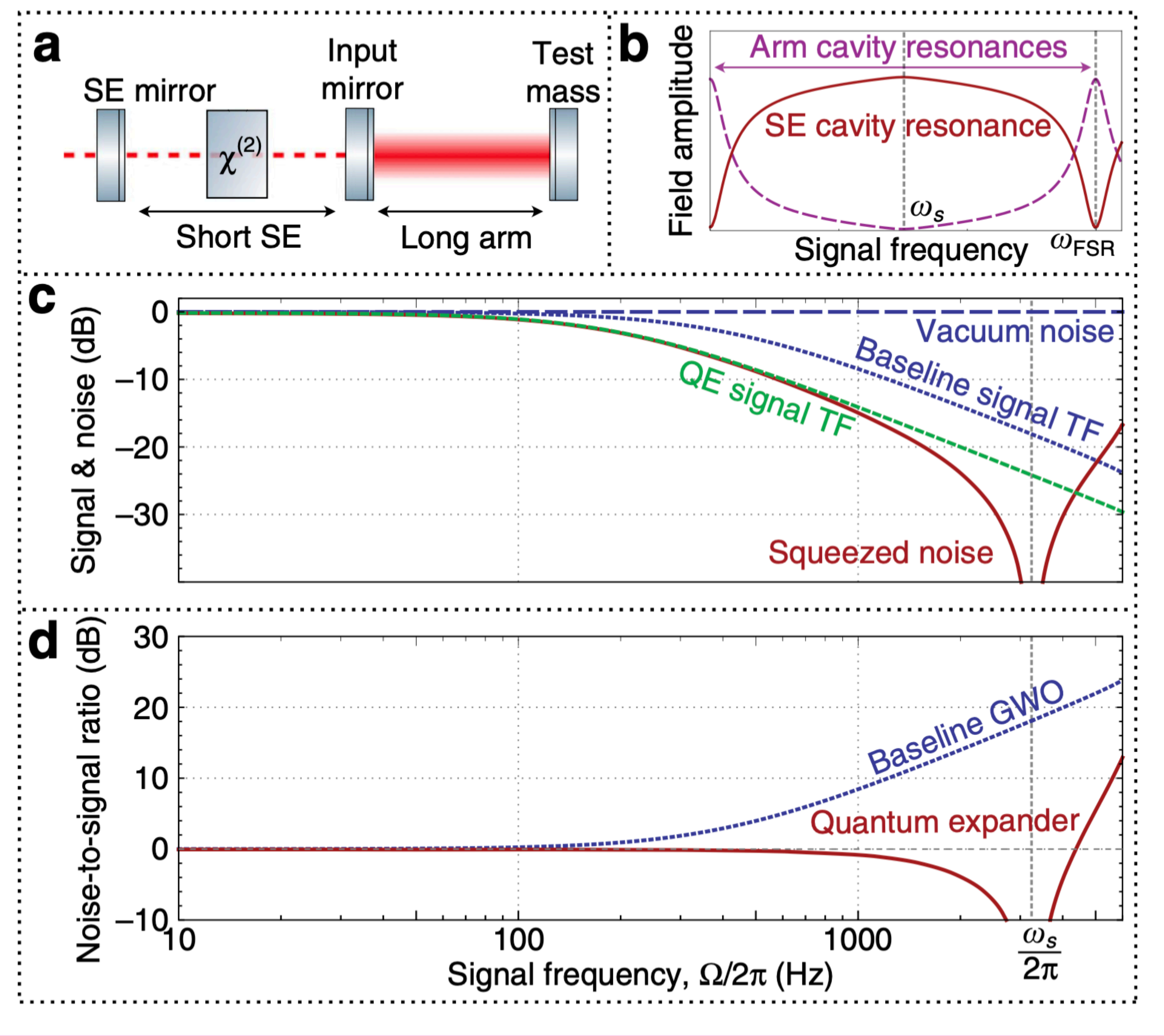
Detuned Case! Parametric amplification affects the optical rigidity!

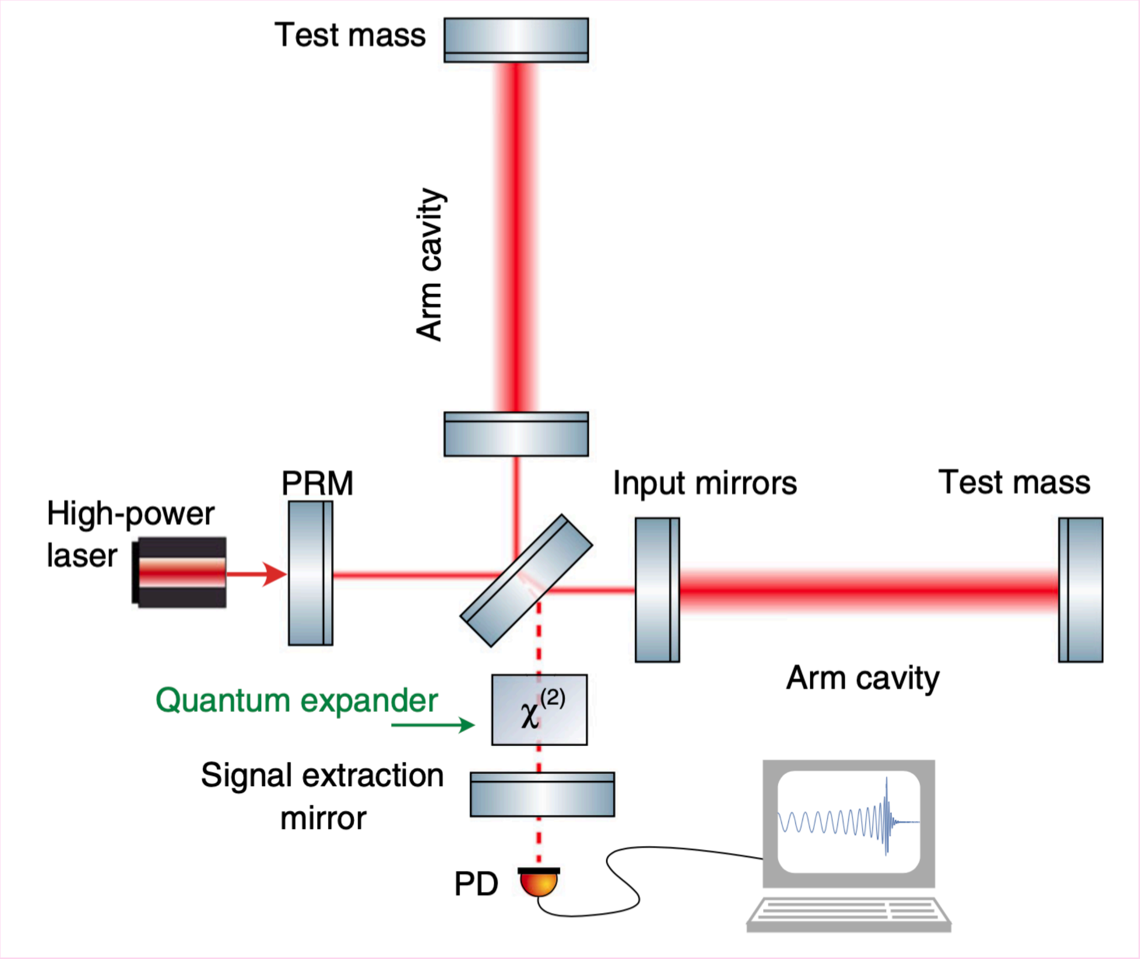
Our Target: Surpassing the Energetic Quantum Limit

- **Quantum Expander: the use of Degenerate Parametric Amplifier**
- **Coherent quantum noise cancelation using PT-symmetry: Non-degenerate amplifier**

Manipulating Intracavity Fields:

Quantum Expander: the use of Degenerate Parametric Amplifier





Simply another region (tuned case) of Kentaro's design!

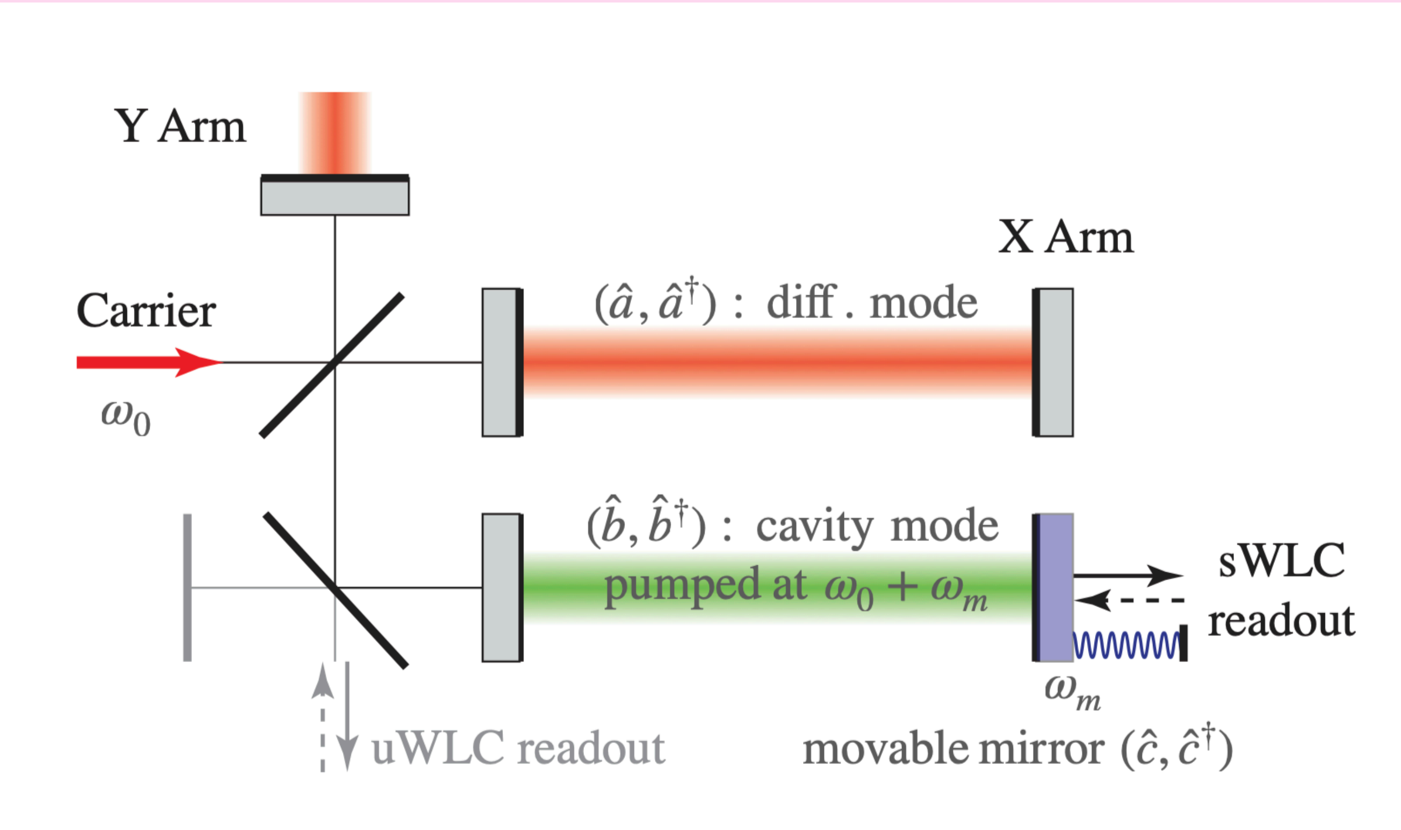
Signal is "squeezed" by the amplifier, but the noise **squeezed More at high frequencies!**

Enhancement Factor: $$\Lambda = \frac{\int^\infty d\Omega 1/S_h^{\rm exp}(\Omega)}{\int^\infty d\Omega 1/S_h^{\rm trad}(\Omega)} \propto \frac{1}{(\gamma-\chi)^2}$$

Korobko, Ma, Chen and Schnabel: Light: Science and Application (2019)8:118

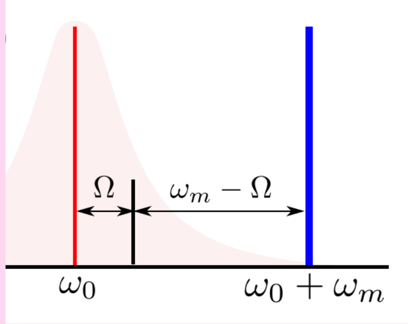
Manipulating Intracavity Fields:

Coherent Noise Cancellation Using Nondegenerate Parametric Amplifier



An optomechanical ND amplifier example

$$\hat{V} = i\kappa(\hat{a}\hat{b}^\dagger - \hat{a}^\dagger\hat{b}) + i\chi(\hat{b}^\dagger\hat{c}^\dagger - \hat{b}\hat{c})$$



Equation of motion:

$$\dot{\hat{a}} = -\kappa\hat{b} + i\alpha h$$

$$\dot{\hat{c}}^\dagger = \chi\hat{b}$$

Signal extraction mode

$$\dot{\hat{b}} = -\gamma_R\hat{b} + \kappa\hat{a} + \chi\hat{c}^\dagger + \sqrt{2\gamma_R}\hat{u}$$

$$\frac{d}{dt}(\chi\hat{a} + \kappa\hat{c}^\dagger) = i\chi\alpha h$$

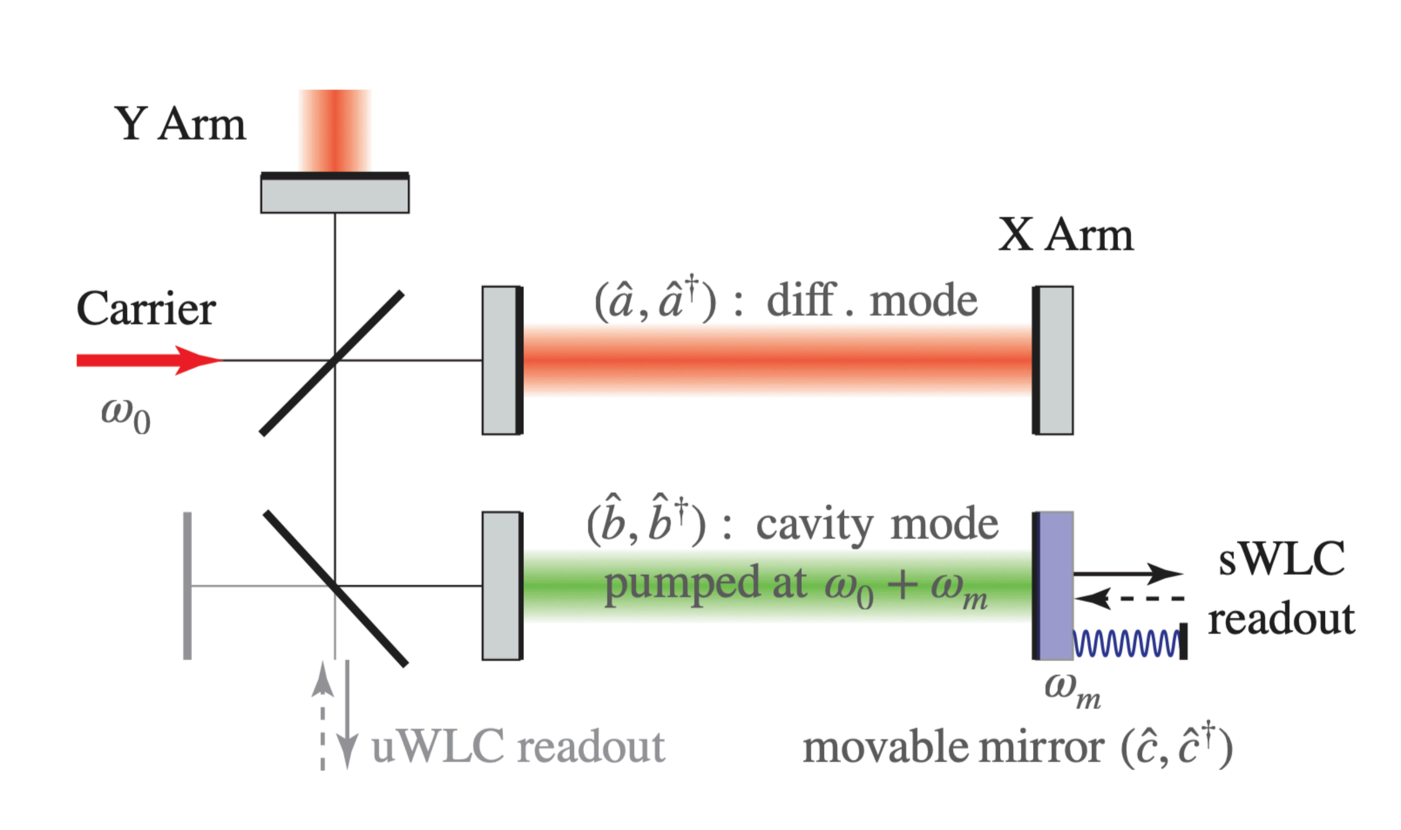
+ Zero Noise field in the ideal Case!

Infinite response to signal at low frequency!

Li, Goryachev, Ma, Tobar, Adkhari and Chen: arXiv:2012.00836

Manipulating Intracavity Fields:

Coherent Noise Cancellation Using Nondegenerate Parametric Amplifier



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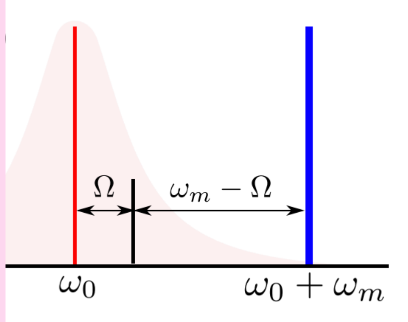

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Signal extraction mode

$$\dot{\hat{b}} = -\gamma_R\hat{b} + \kappa\hat{a} + \chi\hat{c}^\dagger + \sqrt{2\gamma_R}\hat{u}$$

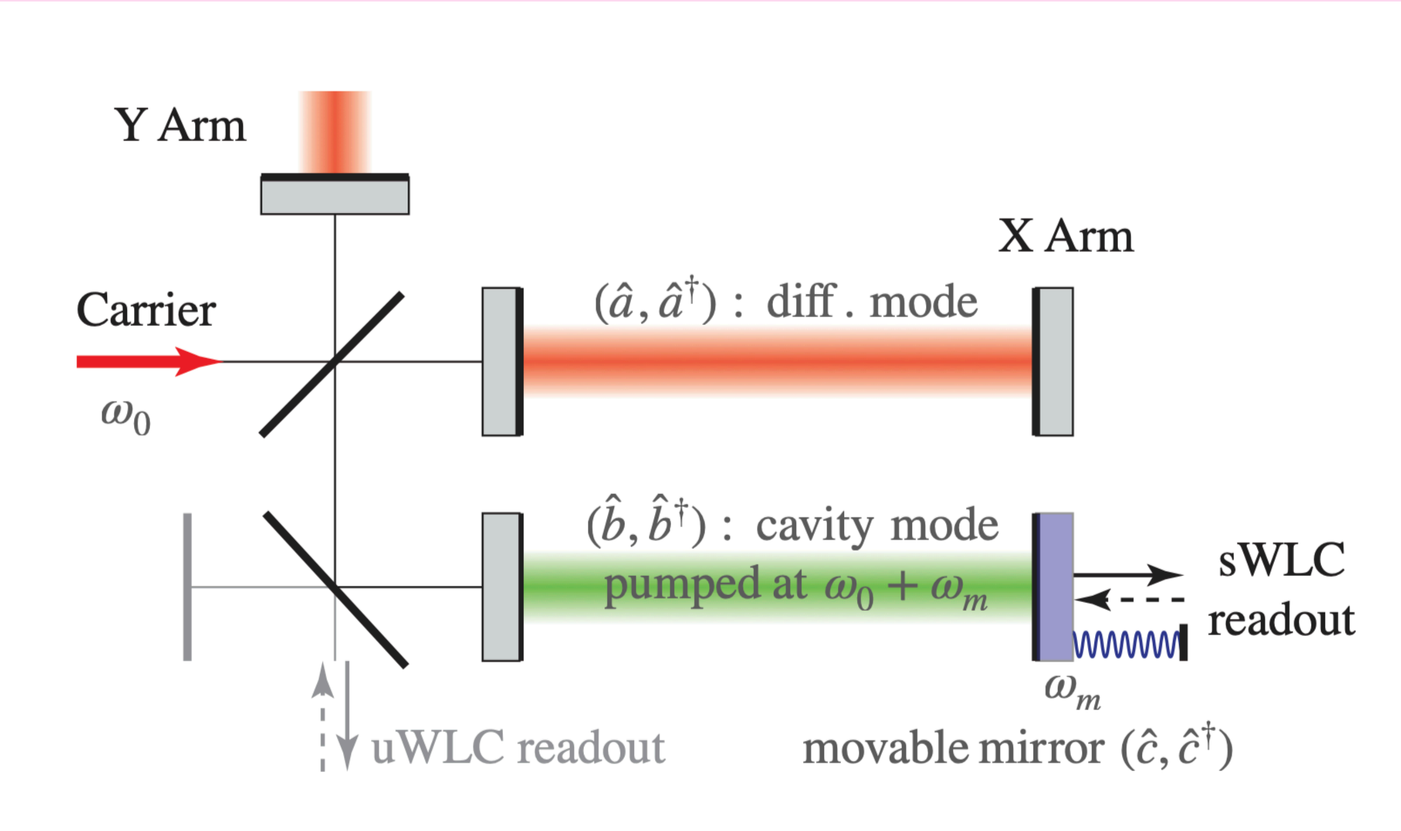
The gain in integrated sensitivity approaches *infinity* as $\chi \to \kappa$

$$\Lambda \equiv \frac{\int_0^{+\infty} d\Omega/(2\pi)[S_h^{\rm sWLC}]^{-1}}{\int_0^{+\infty} d\Omega/(2\pi)[S_h^{\rm conv}]^{-1}} = \frac{1}{1-\chi^2/\kappa^2}$$

Li, Goryachev, Ma, Tobar, Adkhari and Chen: arXiv:2012.00836

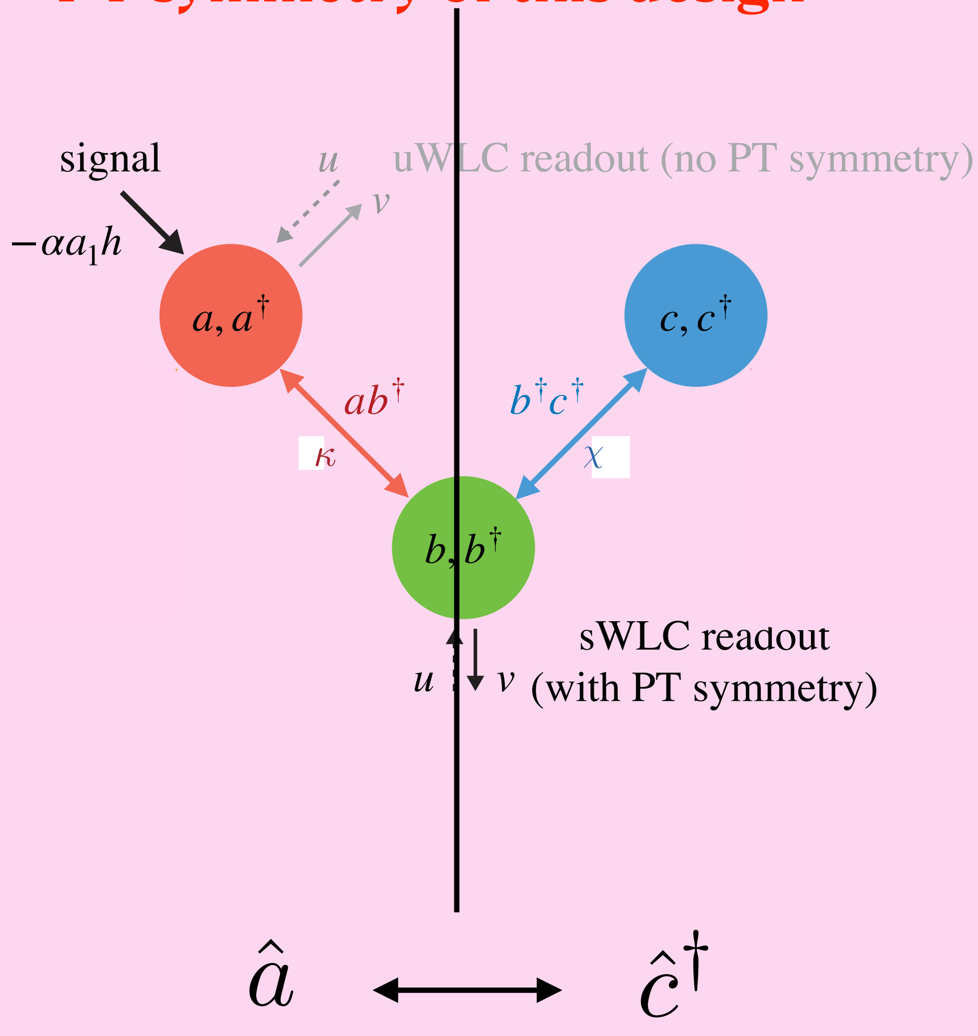
Manipulating Intracavity Fields:

Coherent Noise Cancellation Using Nondegenerate Parametric Amplifier



An optomechanical ND amplifier example

PT symmetry of this design



$$\hat{V} = i\kappa(\hat{a}\hat{b}^\dagger - \hat{a}^\dagger\hat{b}) + i\chi(\hat{b}^\dagger\hat{c}^\dagger - \hat{b}\hat{c})$$

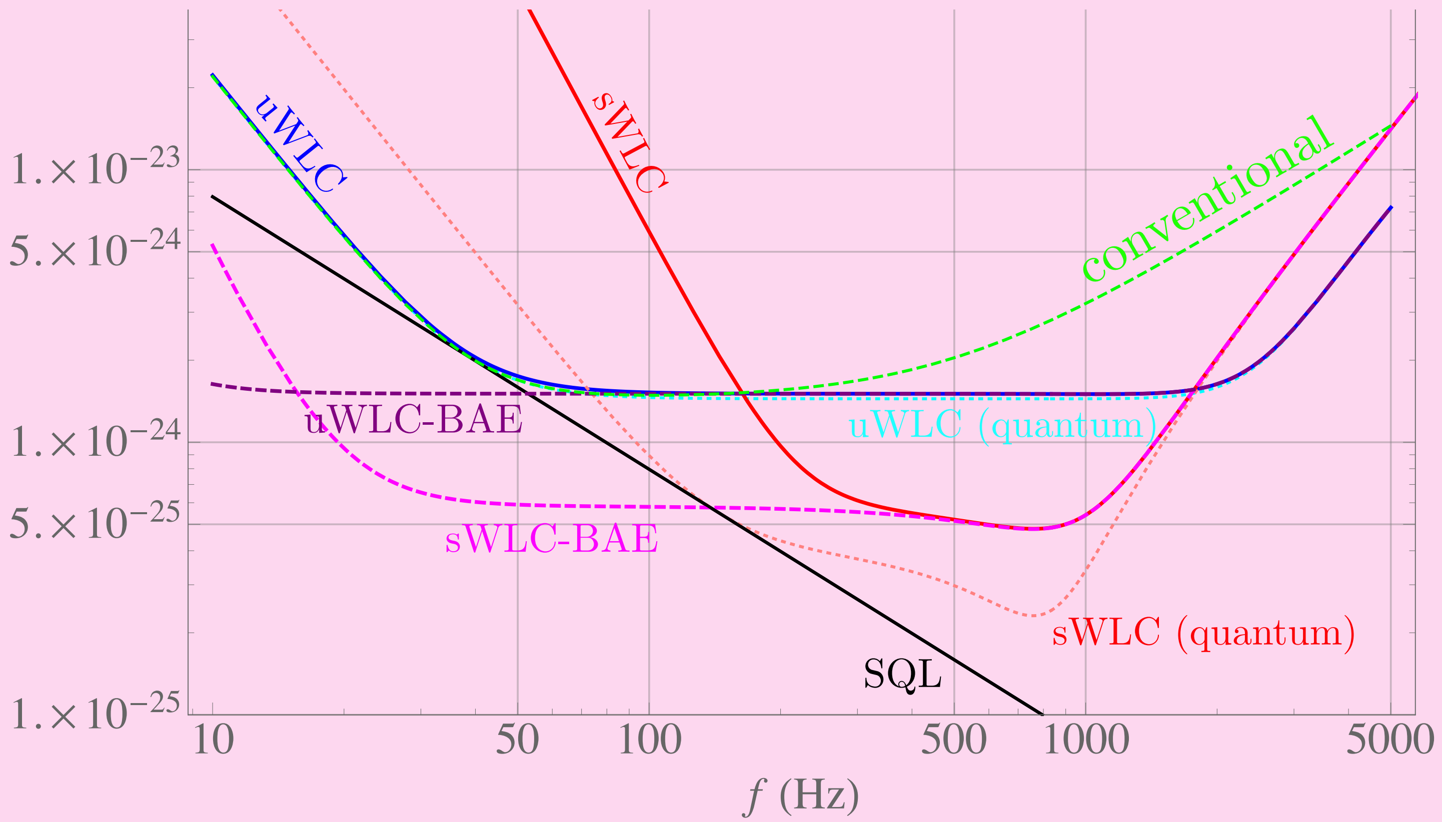
When $\chi \to \kappa$ System has a PT -symmetry

Li, Goryachev, Ma, Tobar, Adkhari and Chen: arXiv:2012.00836

Manipulating Intracavity Fields:

Coherent Noise Cancellation Using Nondegenerate Parametric Amplifier

Sensitivity Curve!



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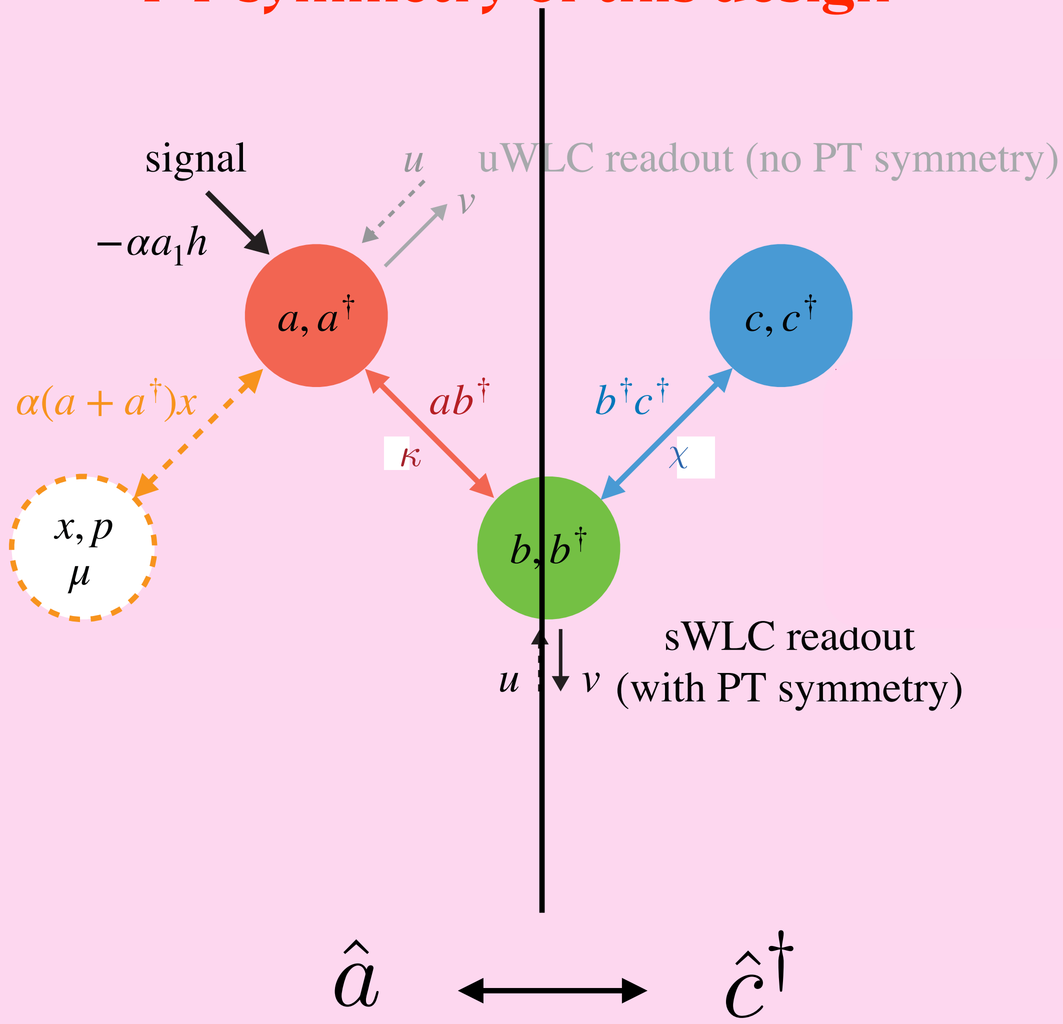

But do not forget the radiation pressure back-action Noise!

Li, Goryachev, Ma, Tobar, Adkhari and Chen: arXiv:2012.00836

Manipulating Intracavity Fields:

Coherent Noise Cancellation Using Nondegenerate Parametric Amplifier

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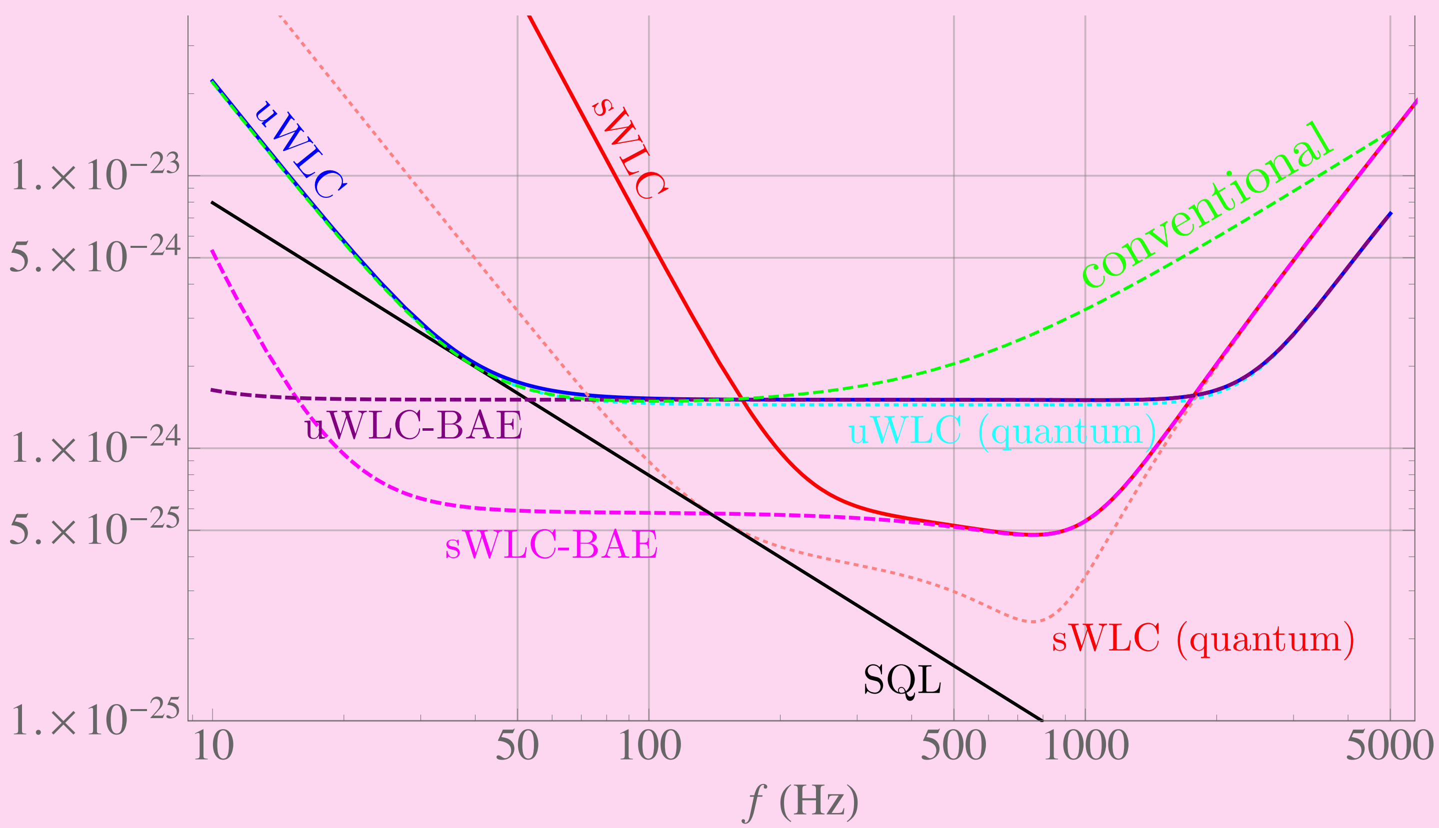

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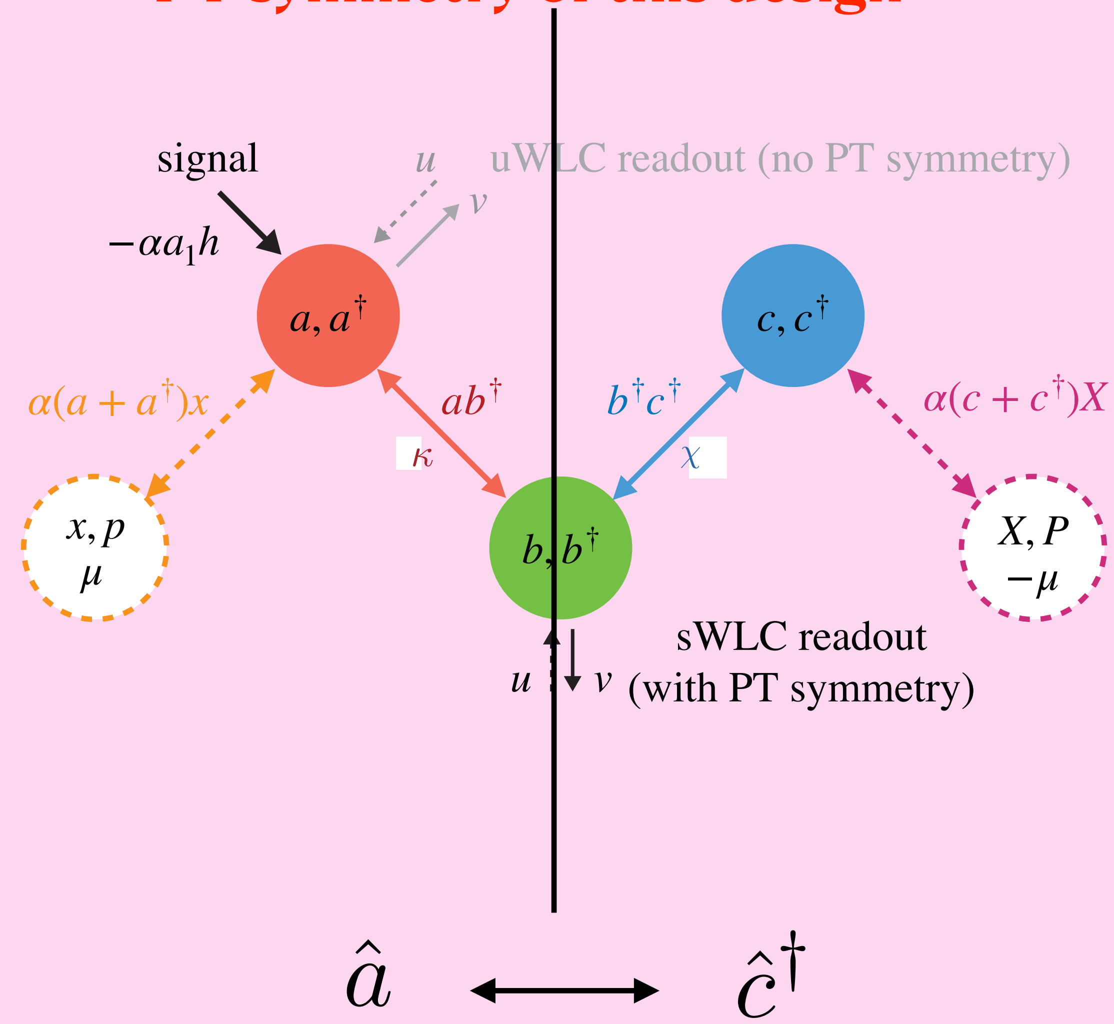


Fully PT-symmetric with negative inertia!

We need the negative inertia to do Back-action evasion (BAE!)

Li, Goryachev, Ma, Tobar, Adkhari and Chen: arXiv:2012.00836

Manipulating Intracavity Fields:

Connection to the Gain-balance symmetry interpretation:

$$\dot{\hat{a}} = -\kappa\hat{b} + i\alpha h$$

$$\dot{\hat{c}}^{\dagger} = \chi\hat{b}$$

$$\dot{\hat{b}} = -\gamma_R\hat{b} + \kappa\hat{a} + \chi\hat{c}^{\dagger} + \sqrt{2\gamma_R}\hat{u}$$

We adiabatically eliminate b-field:

Gain

$$\dot{\hat{a}} = \boxed{-\frac{\kappa^2}{\gamma}\hat{a}} - \frac{\kappa\chi}{\gamma}\hat{c}^{\dagger} + \ldots$$

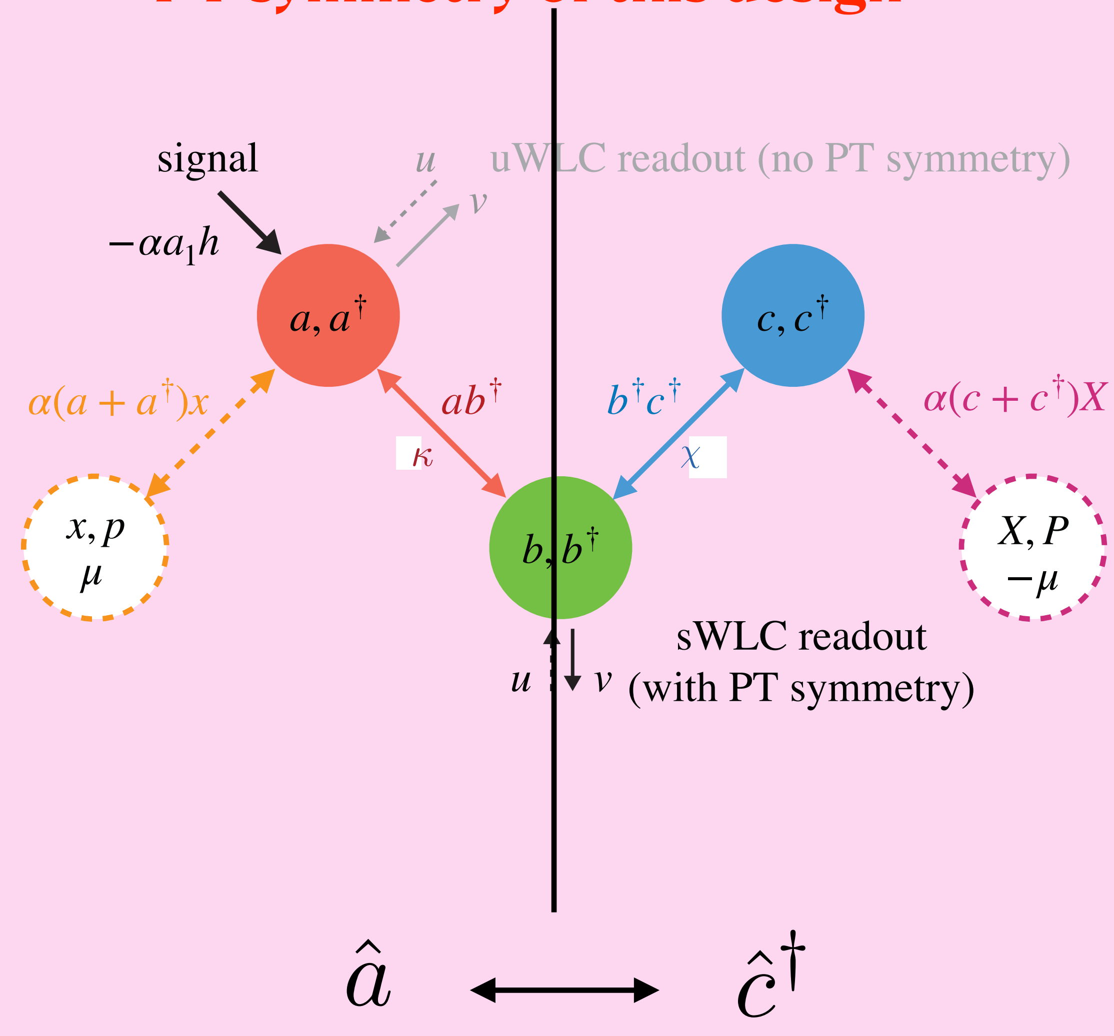
$$\dot{\hat{c}}^{\dagger} = \boxed{\frac{\kappa^2}{\gamma}\hat{c}} + \frac{\kappa\chi}{\gamma}\hat{a} + \ldots$$

Loss

$$\chi \to \kappa$$

Gain=Loss

PT symmetry of this design



Li, Goryachev, Ma, Tobar, Adkhari and Chen: arXiv:2012.00836

Summary:

Parametric Amplifier Can help us to surpass Quantum Limit!

- **Squeezing generation to surpass Standard Quantum Limit!**
  - Degenerate PA can generate squeezing to beat quantum fluctuations
  - NDegenerate PA can generate EPR entanglement light to generate conditional squeezing
- **Intracavity field Manipulation to Surpass bandwidth-peak Limit (EQL)**
  - Quantum Expander using Degenerate PA can significantly broaden the detection bandwidth
  - New Concept Based on PT symmetry system coherently cancel the quantum noise

These works are done based on the Collaboration and stimulated discussions with:

Albert Einstein Institute: Roman Schnabel, Mikhail Korobko

California Institute of Technology: Xiang Li, Yanbei Chen

University of Birmingham: Jacob Beckey and Haixing Miao

Tokyo Institute of Technology Kentaro Somiya

University of Western Australia Chunnong Zhao, David Blair and Michale Page

Thank you for your attention!