Data analysis of GWs from core-collapse supernova with Hilbert-Huang Transform

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Outline

1. GW from core-collapse supernova
2. Hilbert-Huang Transform
3. results
4. summary

GW from core-collapse supernova (CCSN)

- A theoretical explanation of the mechanism of CCSNe, it is not yet fully understood.
- The importance of GWs in the CCSNe research is demonstrated by the results of numerical simulations
  - ⇨ different of models : EoS, mass or rotating of star etc.

high-resolution time-frequency analysis

We may be able to investigate above from GW data analysis.

A simplified picture of the flow to the explosion.

$M_{\rm star} \gtrsim 10 M_\odot$

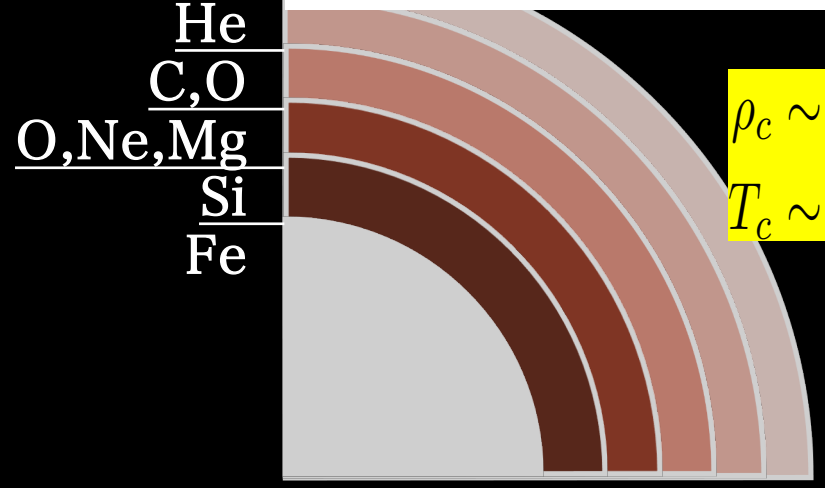
H

hydrogen
nuclear fusion reaction
in the center

Center of temprature T_c · density ρ_c
are increase, the reaction do to.

$\mathrm{H} \to \mathrm{He} \to \mathrm{C} \to \cdots \to \mathrm{Fe}$

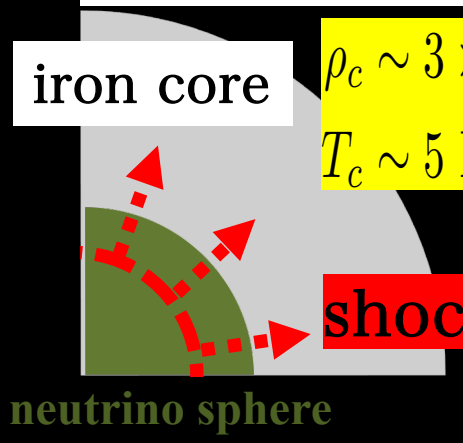
1. Gravitational collapse



$\rho_c \sim 10^{10}\ \mathrm{g/cm^3}$

$T_c \sim 1\ \mathrm{MeV}$

2. core-bounce

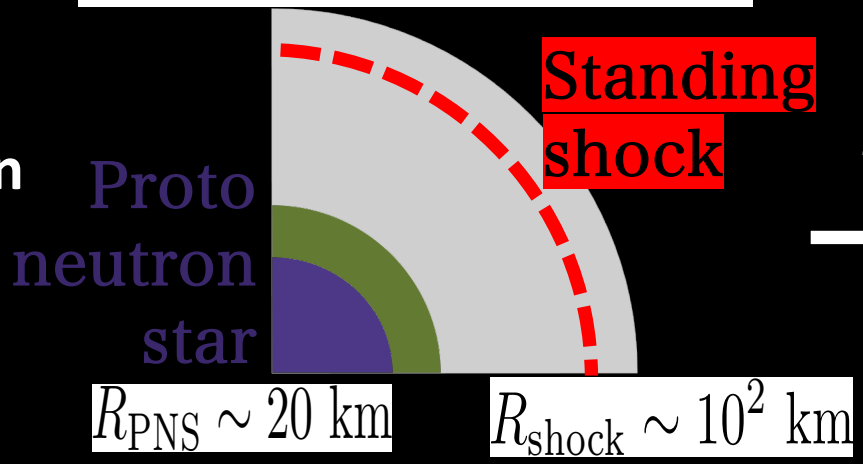


$\rho_c \sim 3 \times 10^{14}\ \mathrm{g/cm^3}$

$T_c \sim 5\ \mathrm{MeV}$

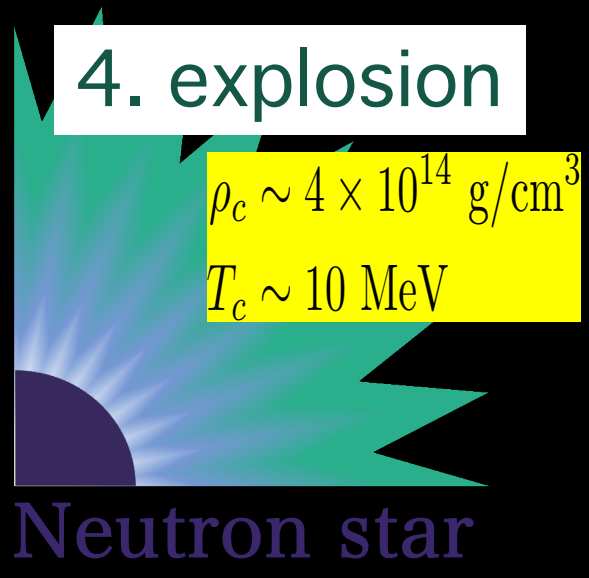
~~Prompt explosion~~

3. standing shock



?

4. explosion



$\rho_c \sim 4 \times 10^{14}\ \mathrm{g/cm^3}$

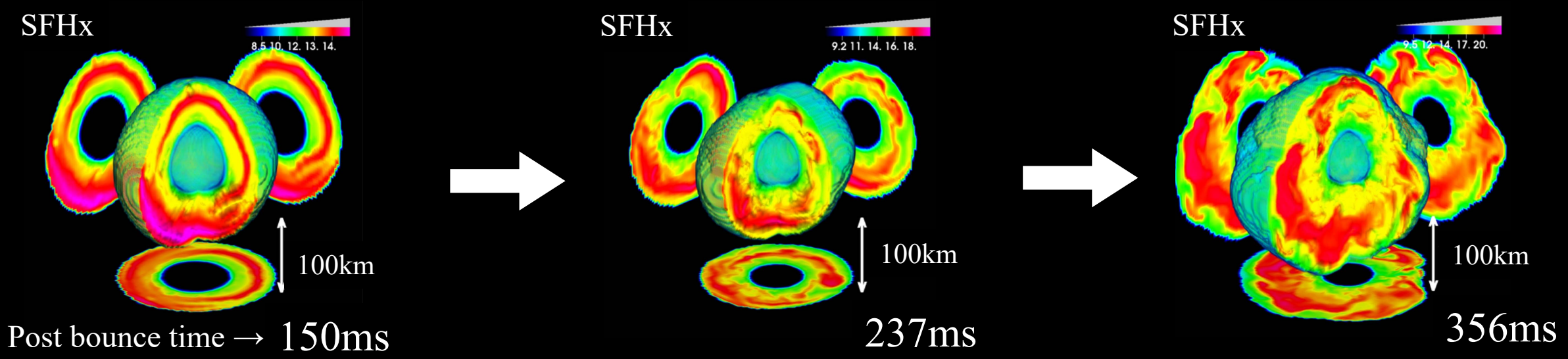
$T_c \sim 10\ \mathrm{MeV}$

GWs…smoking gun for the phenomena that take place inside a star !

GW from core-collapse supernova (CCSN)

standing accretion shock instability : SASI

- The SASI is a type of fluid instability that distorts a standing shock wave into a non-spherical wave, causing motion of the core



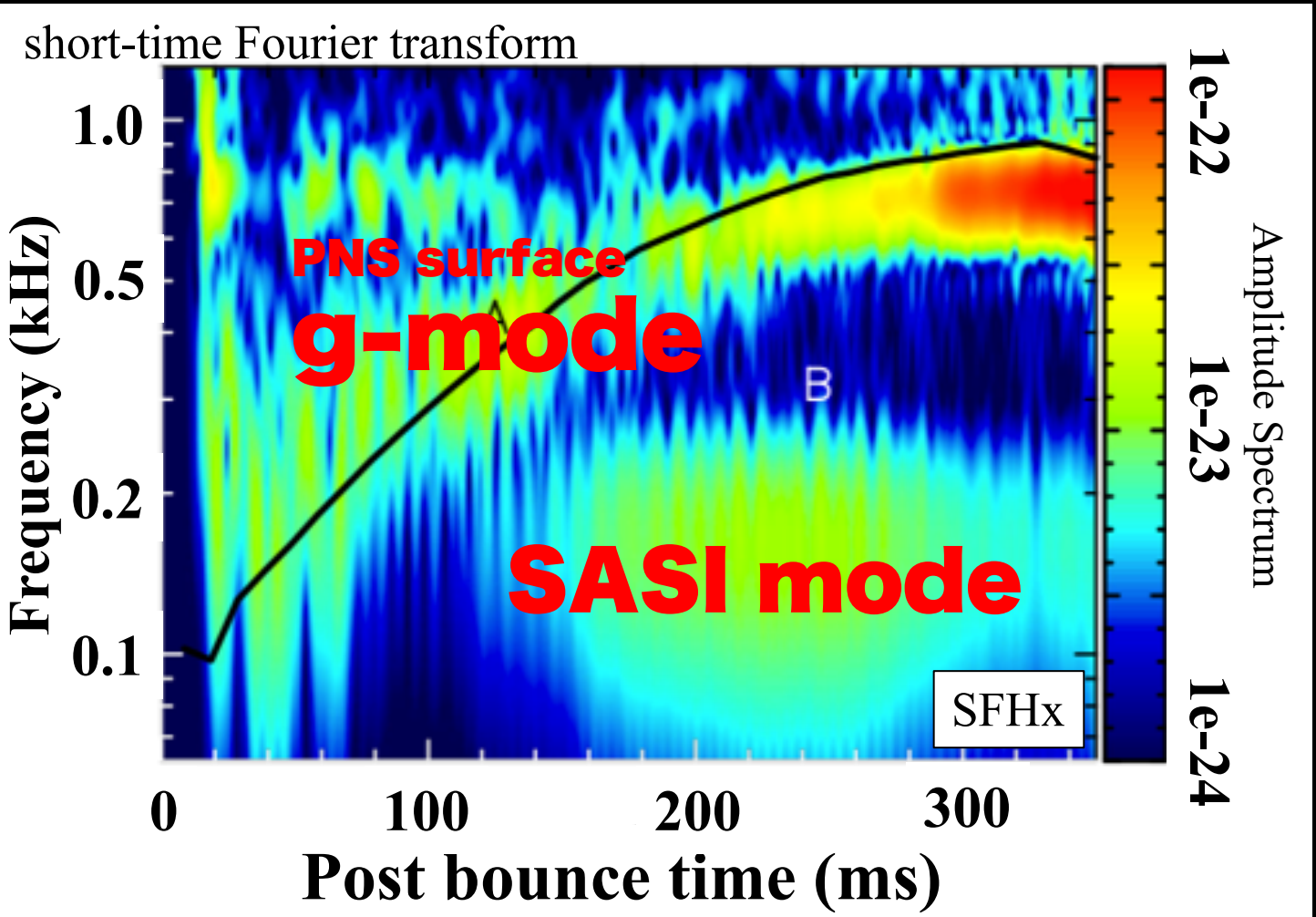
↑The time evolution of entropy space distribution of 3D-GR numerical simulation model SFHx [T. Kuroda+, ApJ(2016)] This is a snapshot of the interior from the shock wave. The entire shock wave is distorted.

✓ When nucluear equation of state is soft (SFHx), SASI is more vigorous.

GW from core-collapse supernova (CCSN)

standing accretion shock instability : SASI

◆ **Frequency of the SASI-derived GW : 0.1 - 0.2 kHz**



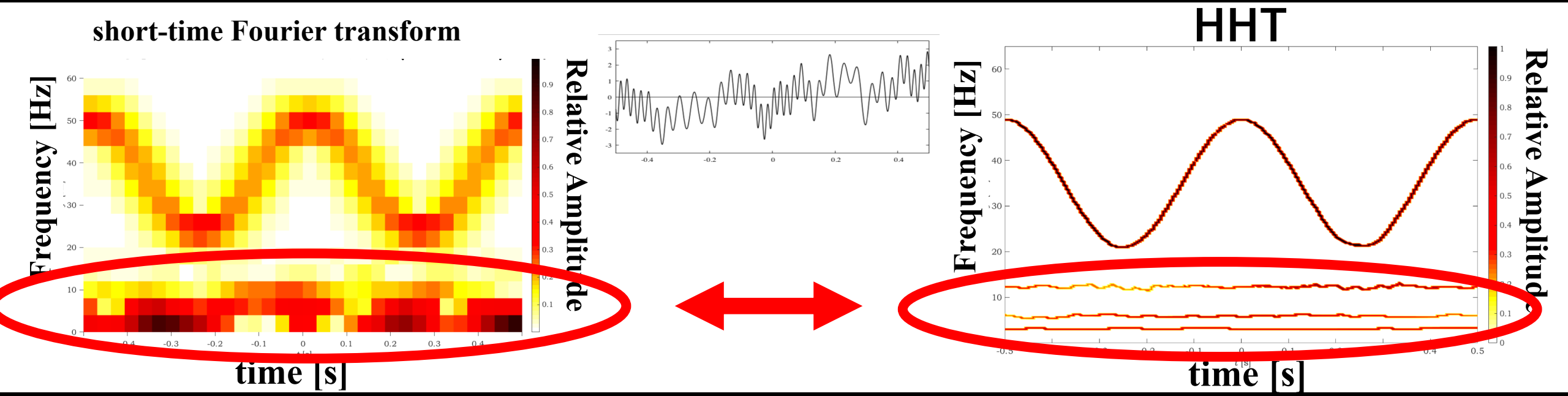
← GW (h_+@10kpc)

[T. Kuroda+, ApJ(2016)]

※this simulation calculated up to 0.35 s from the core bounce time

It is said that a low frequency component will appear in the GW spectrum if the SASI is occurring.

※ although it is difficult with electromagnetic observations



The GWs from CCSN has multiple modes, the SASI-derived GWs are low-frequency

- ✓ The HHT decomposes the signal into multiple mode functions and does not have the time and frequency resolution trade-off.

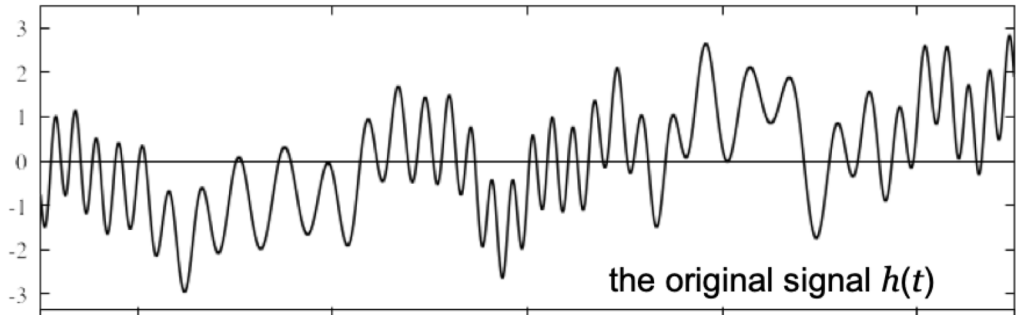
SASI frequency fluctuations may have inside information about the star before the explosion.

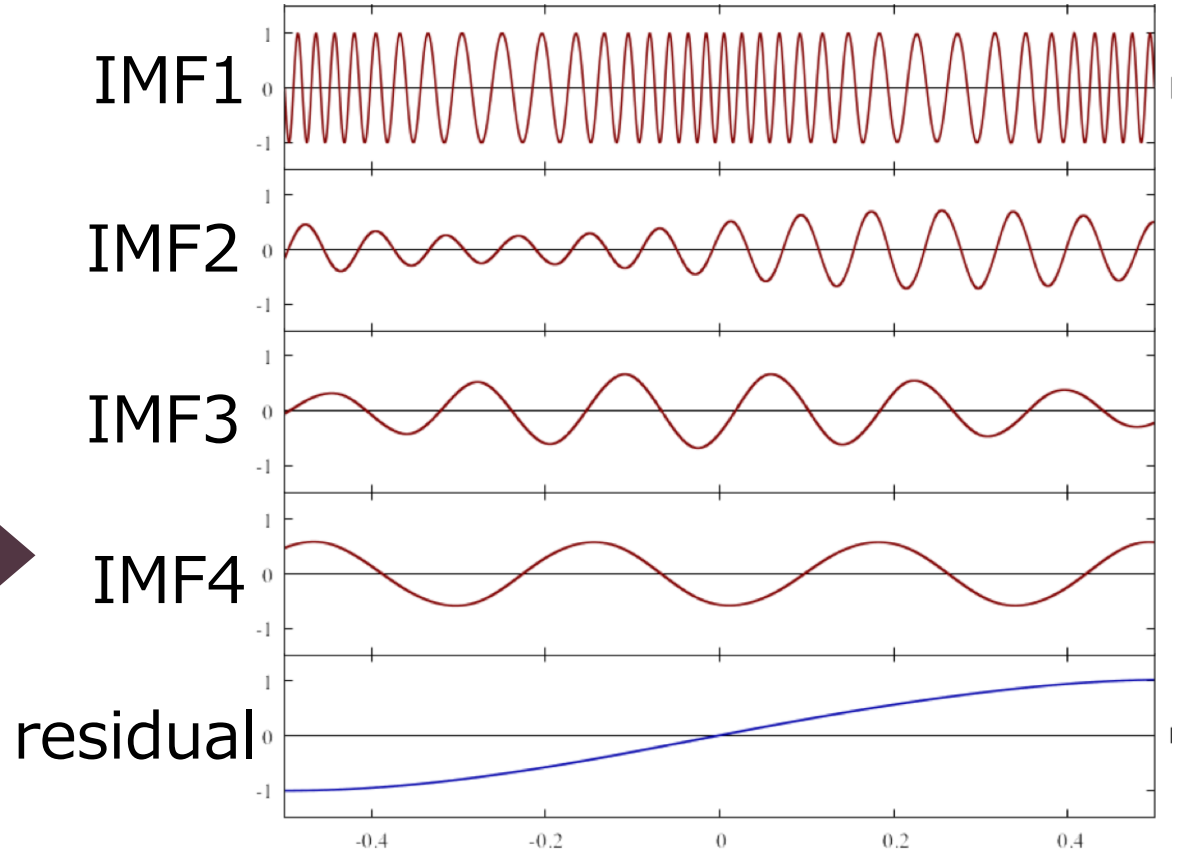
- ✓ Since the HHT defines frequency as a function of time, detailed information in the time frequency domain can be obtained.

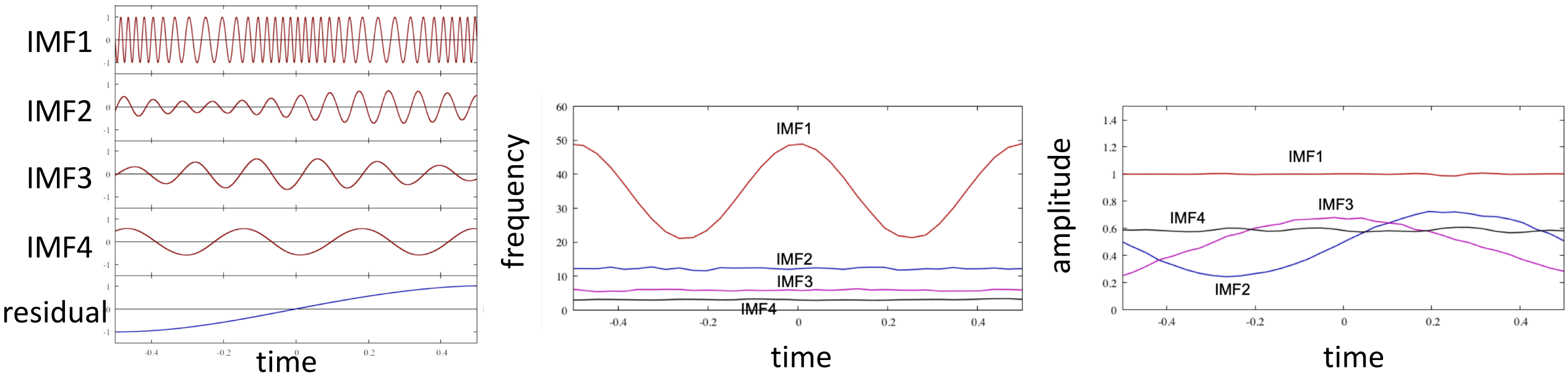
Hilbert-Huang Transform (HHT)

1. The first step is to decompose the signal into the intrinsic mode functions (IMFs).
   The IMF is a function of time that oscillates around zero in the whole data set.

$$h(t) = \sum_{k=1}^{N_{\mathrm{IMF}}} \mathrm{IMF}_k(t) + r(t)$$







In the second step, Hilbert spectral analysis (HSA) is performed on each IMF to obtain the instantaneous amplitude and frequency.

$$z_k(t) = \mathrm{IMF}_k(t) + i\mathcal{H}[\mathrm{IMF}_k(t)] = a_k(t)e^{i\phi_k(t)}$$

the polar form

↓Hilbert transform

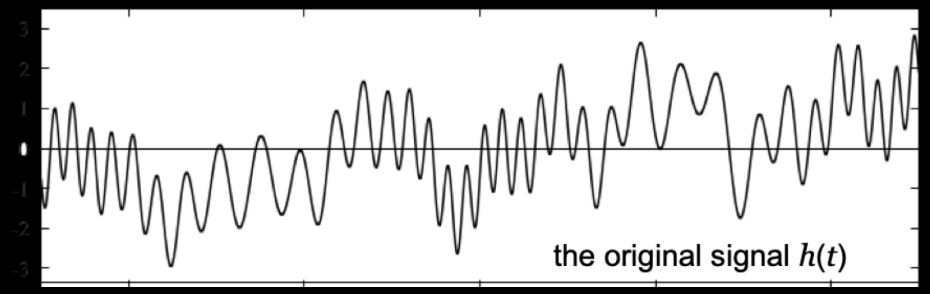
$$\mathcal{H}[x(t)] = \frac{1}{\pi}\mathrm{P}\int_{-\infty}^{\infty}\frac{x(t')}{t'-t}dt'$$

$$f_k(t) \equiv \frac{1}{2\pi}\frac{d\phi_k(t)}{dt}$$

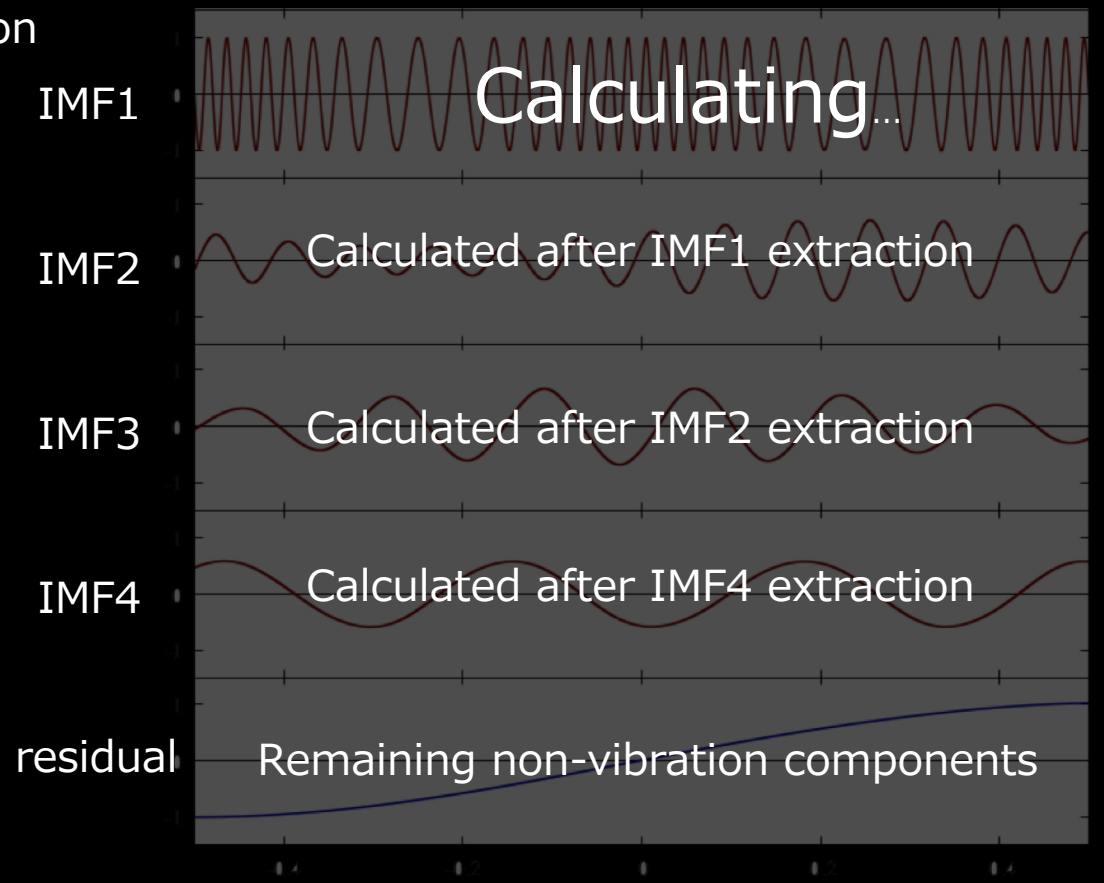
⇦ it does not have the time and frequency resolution trade-off

Hilbert-Huang Transform (HHT)

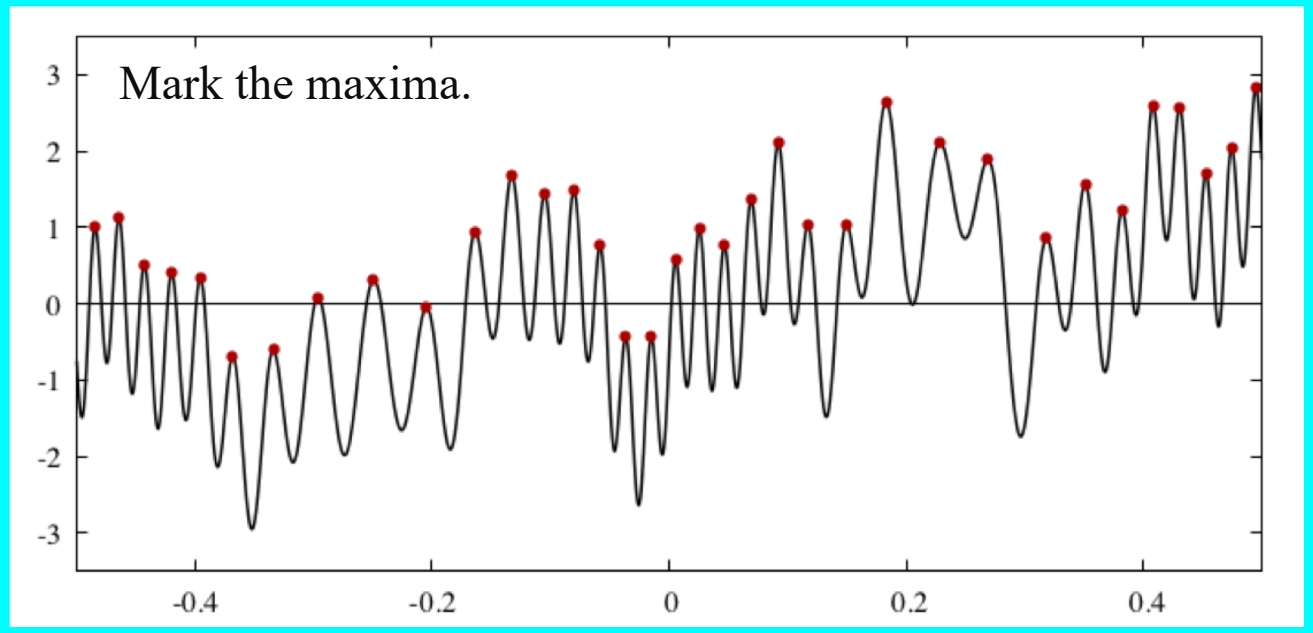
Decompose the signal into the IMFs



Mode decomposition



EMD : Empirical mode decomposition



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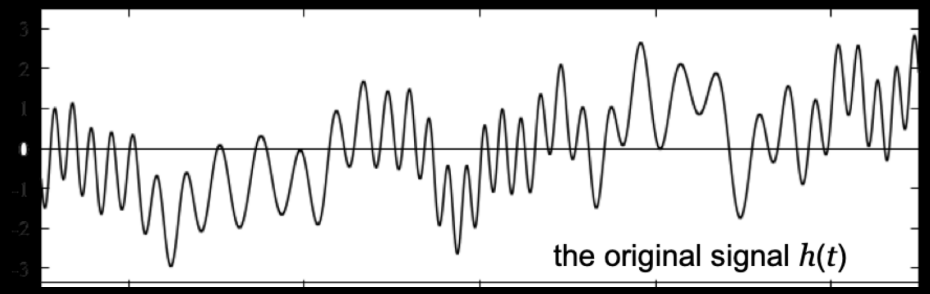

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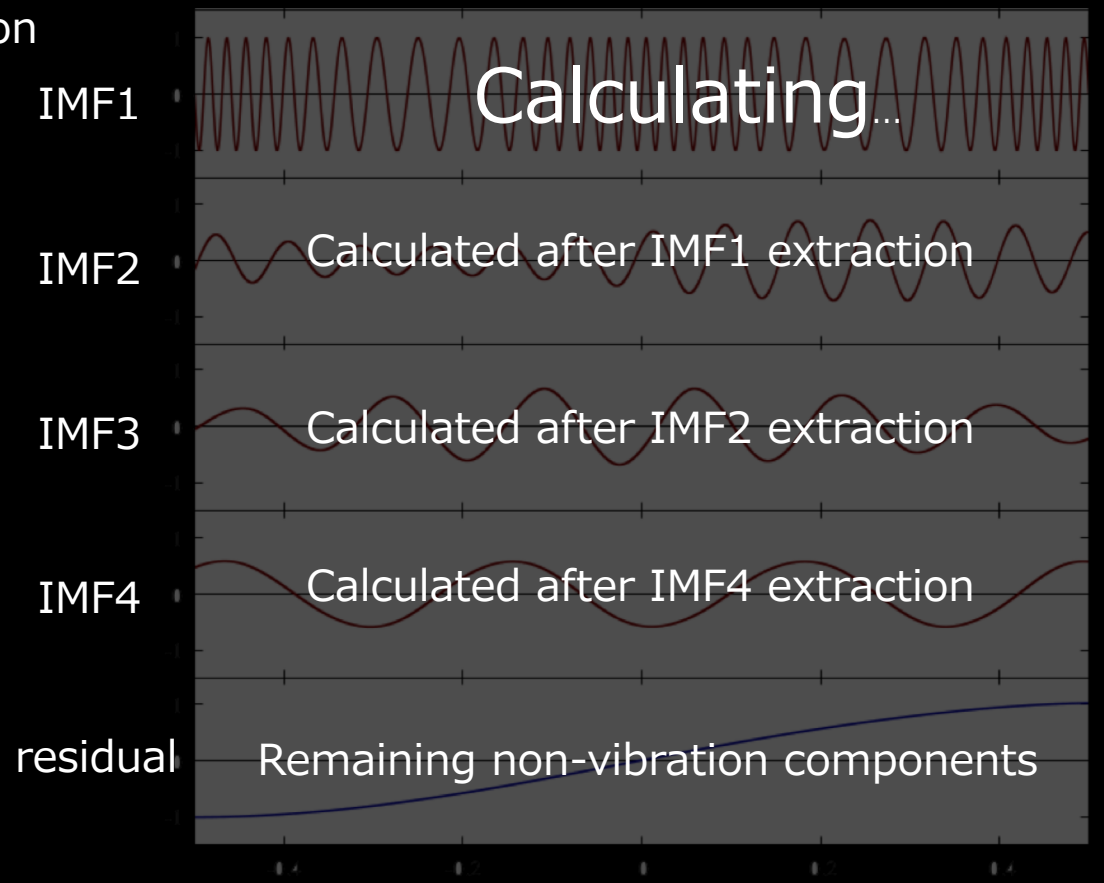

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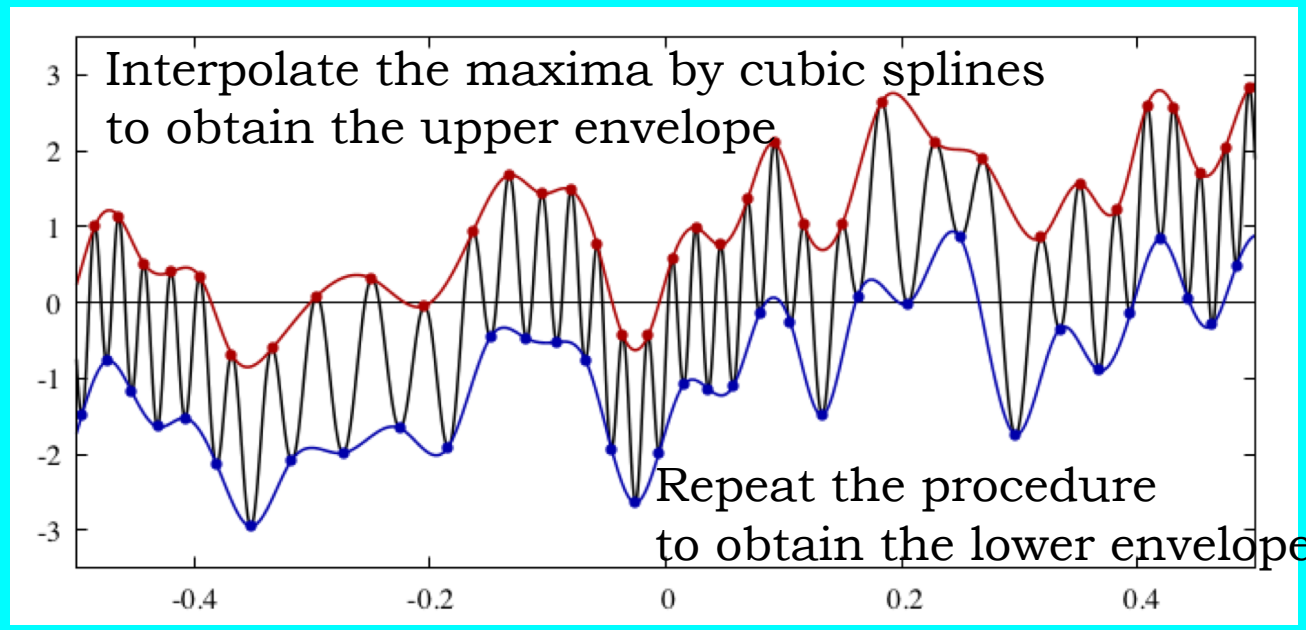

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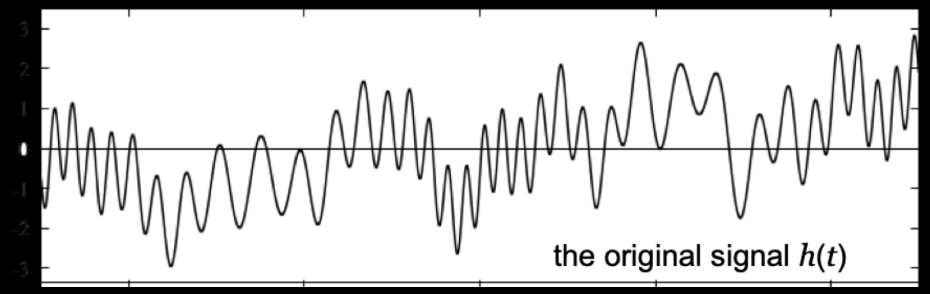

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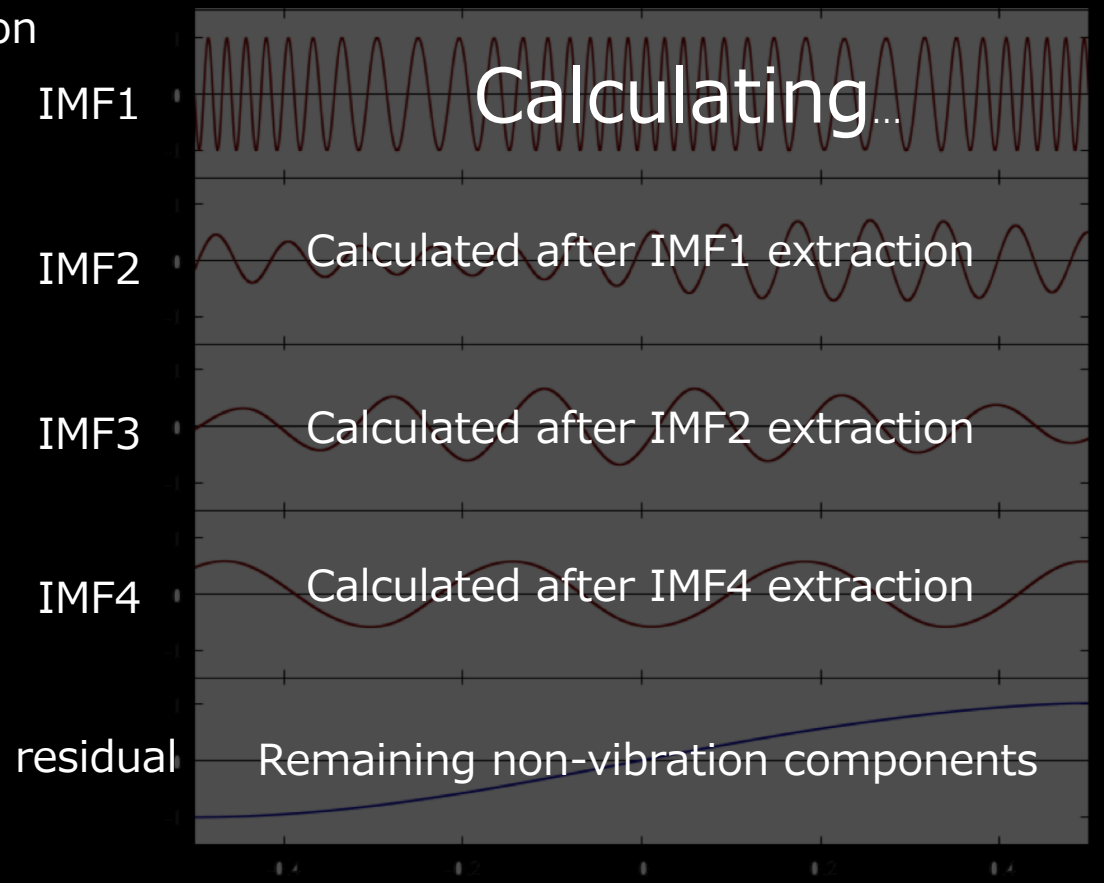

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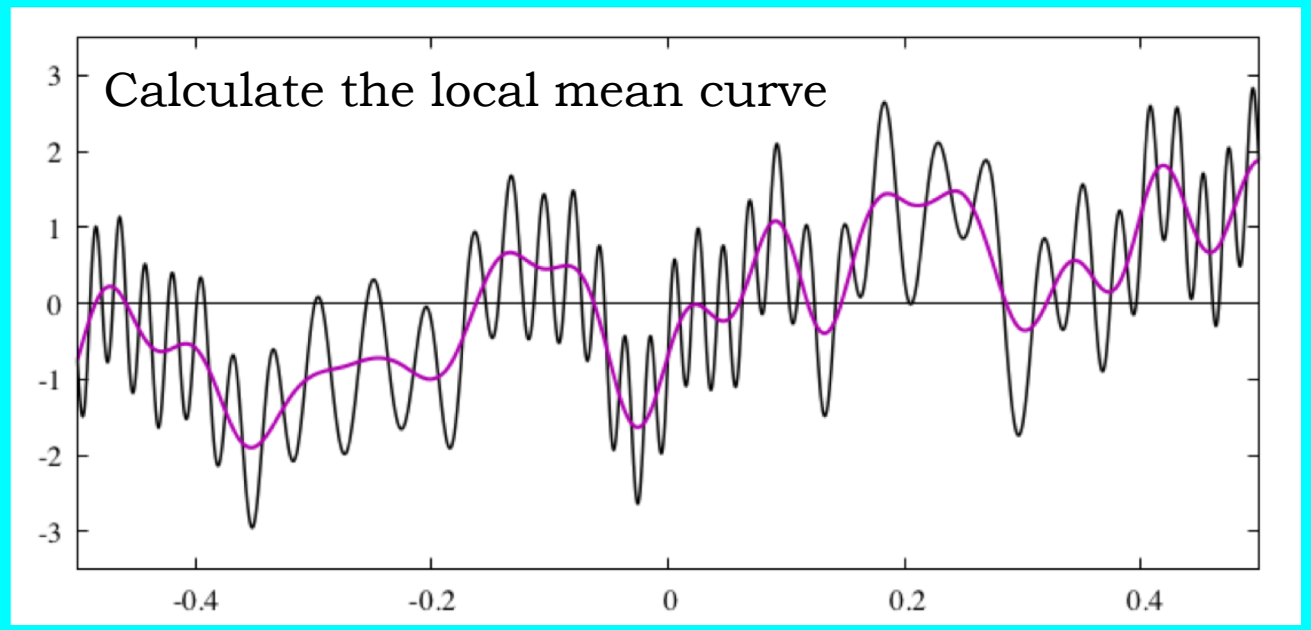

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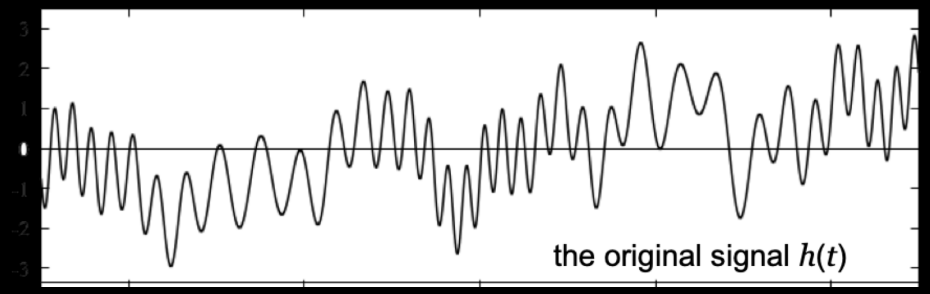

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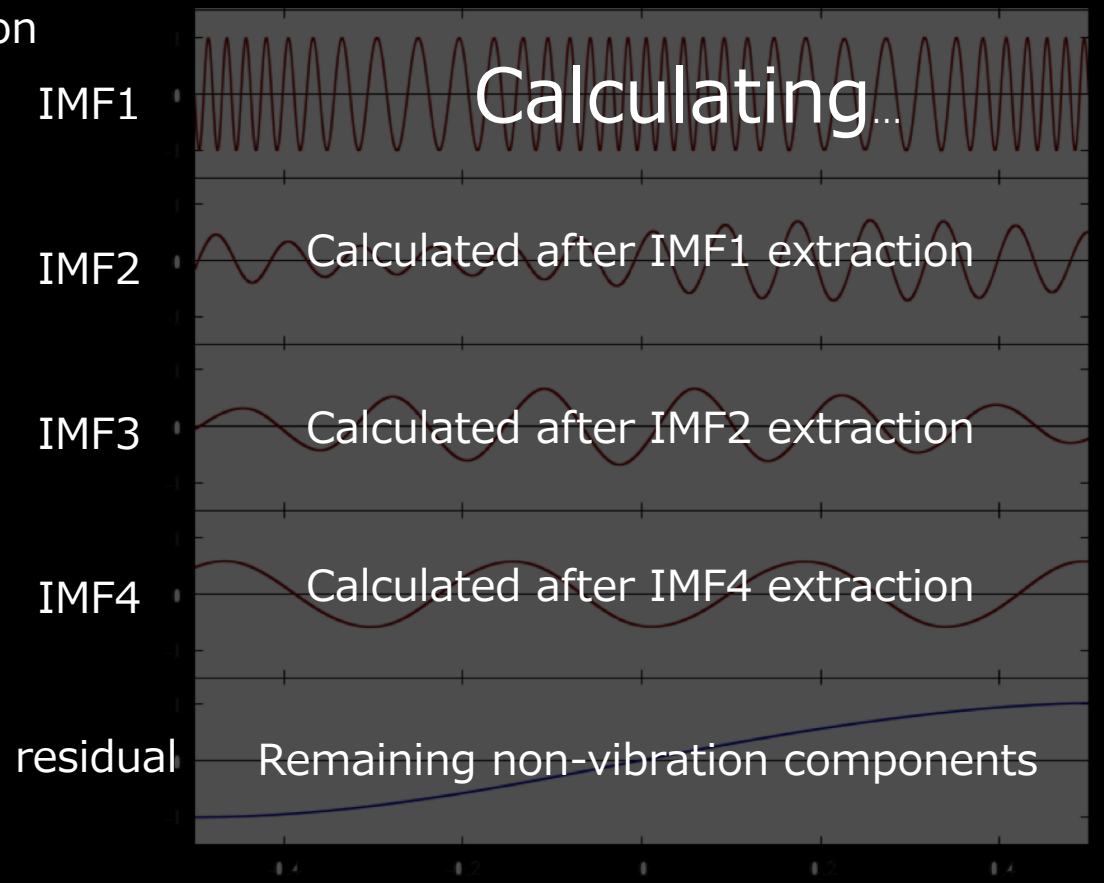

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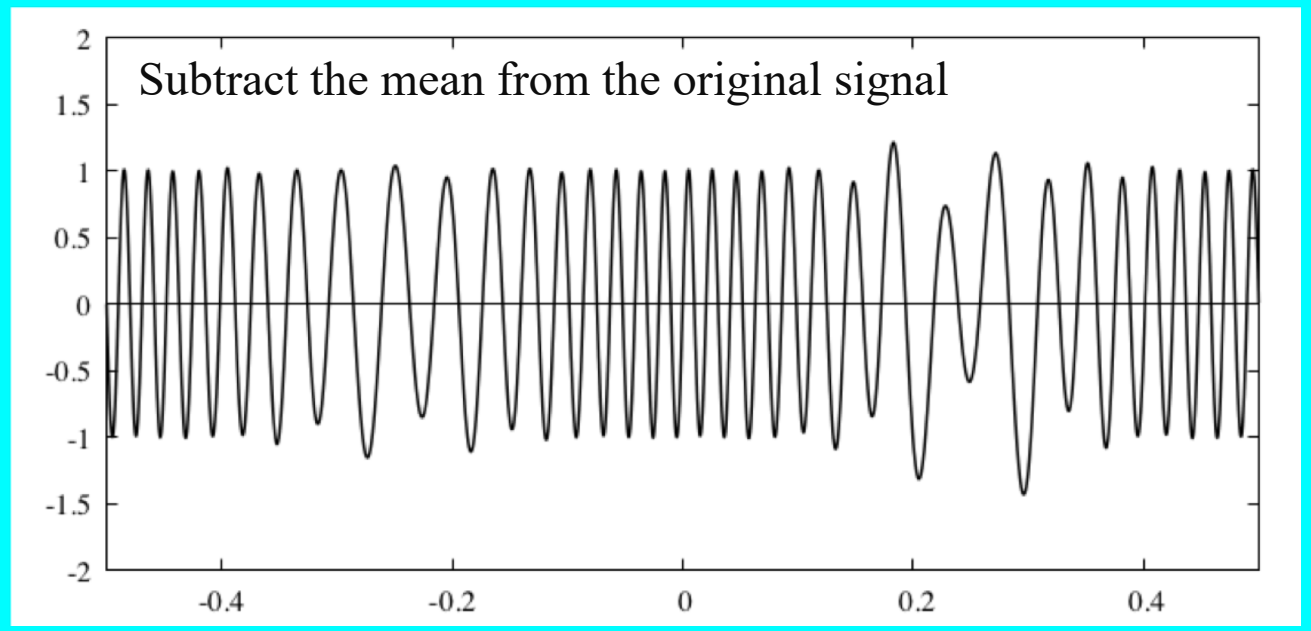

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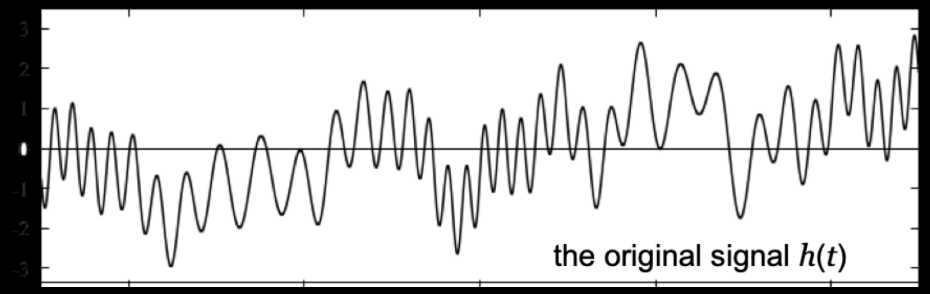

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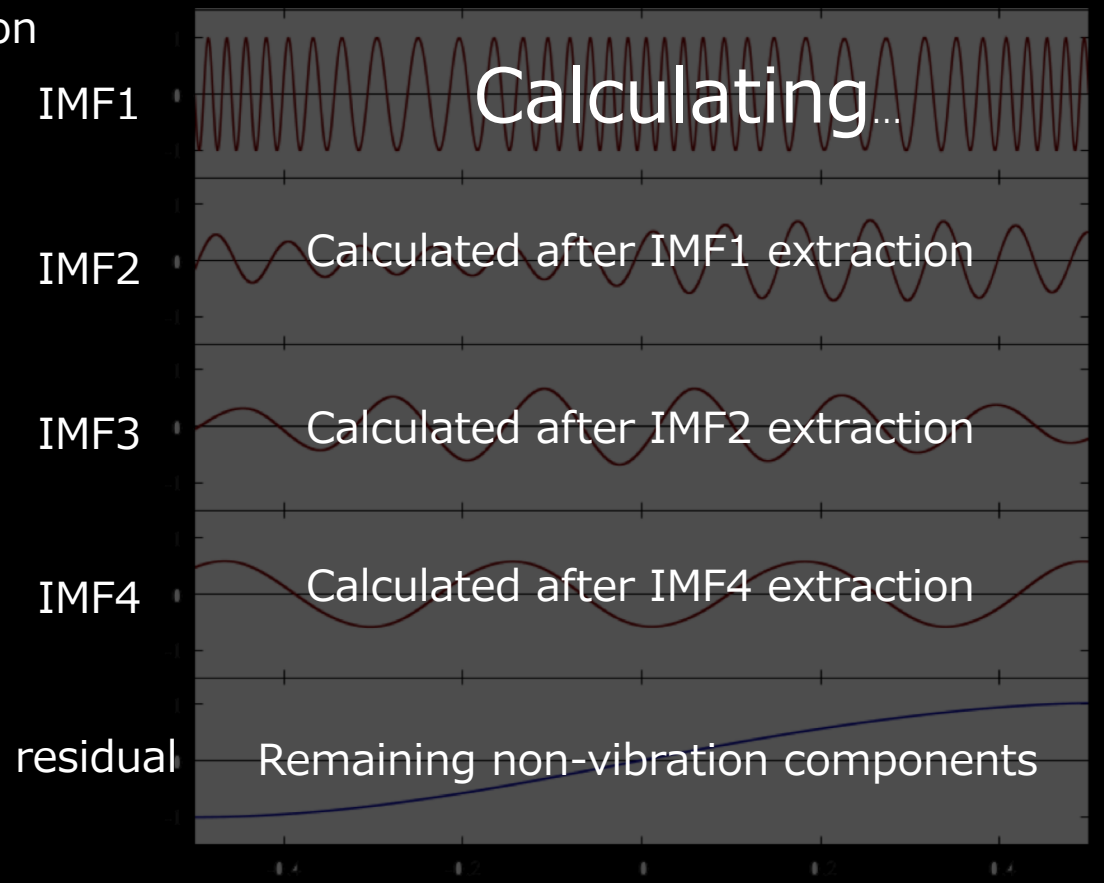

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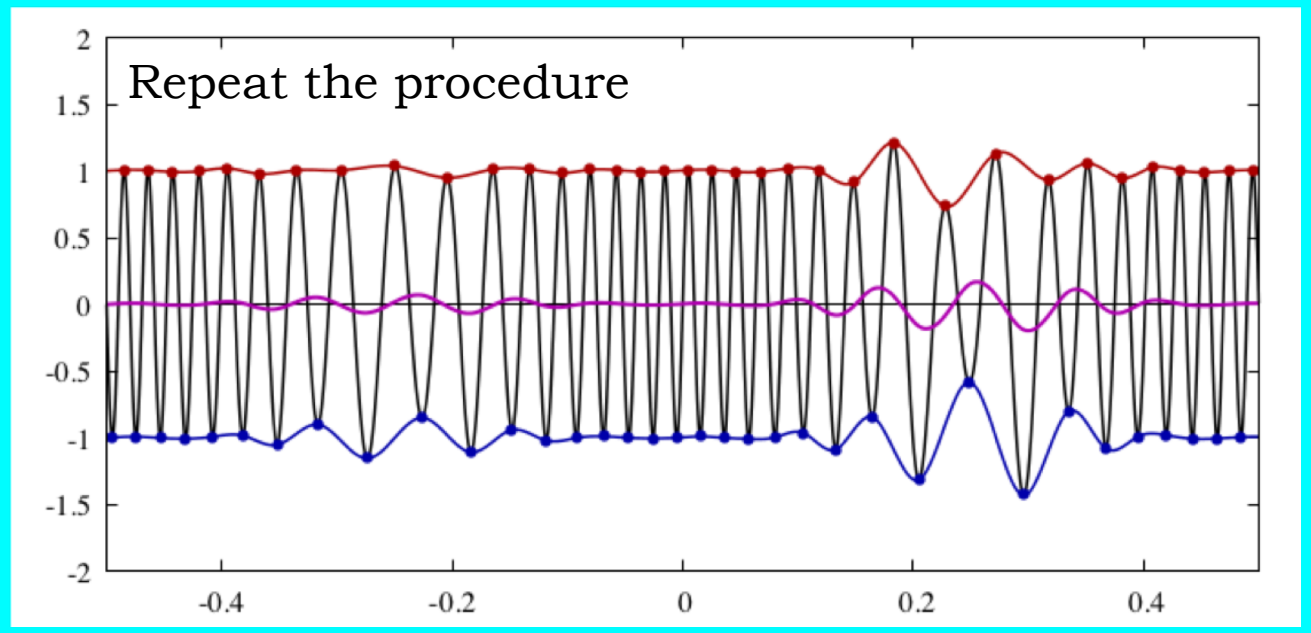

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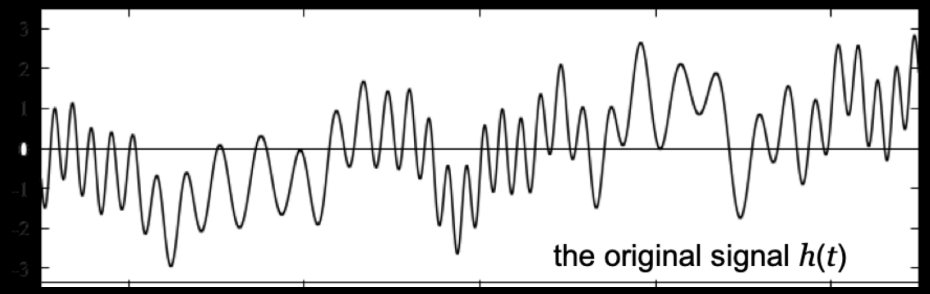

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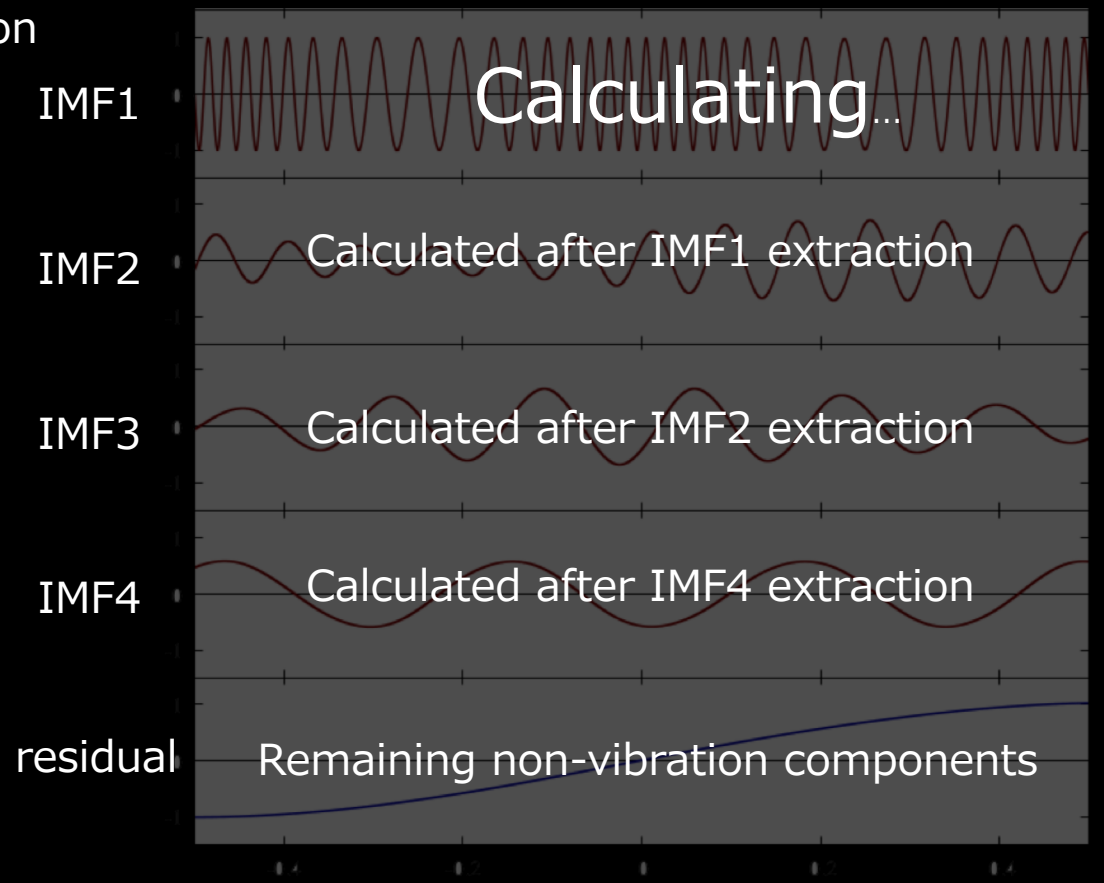
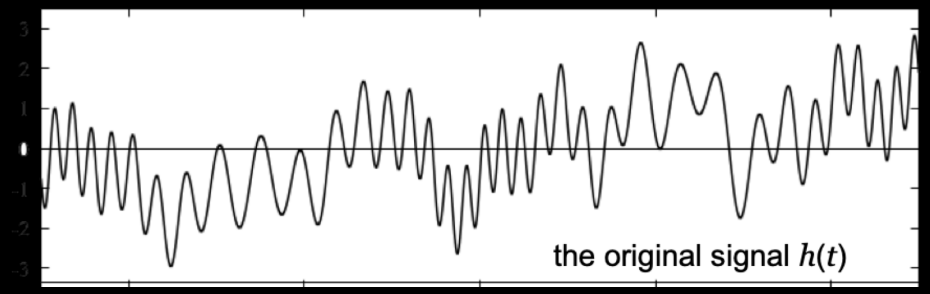


EMD：Empirical mode decomposition

Repeat the procedure

Hilbert-Huang Transform (HHT)

Decompose the signal into the IMFs



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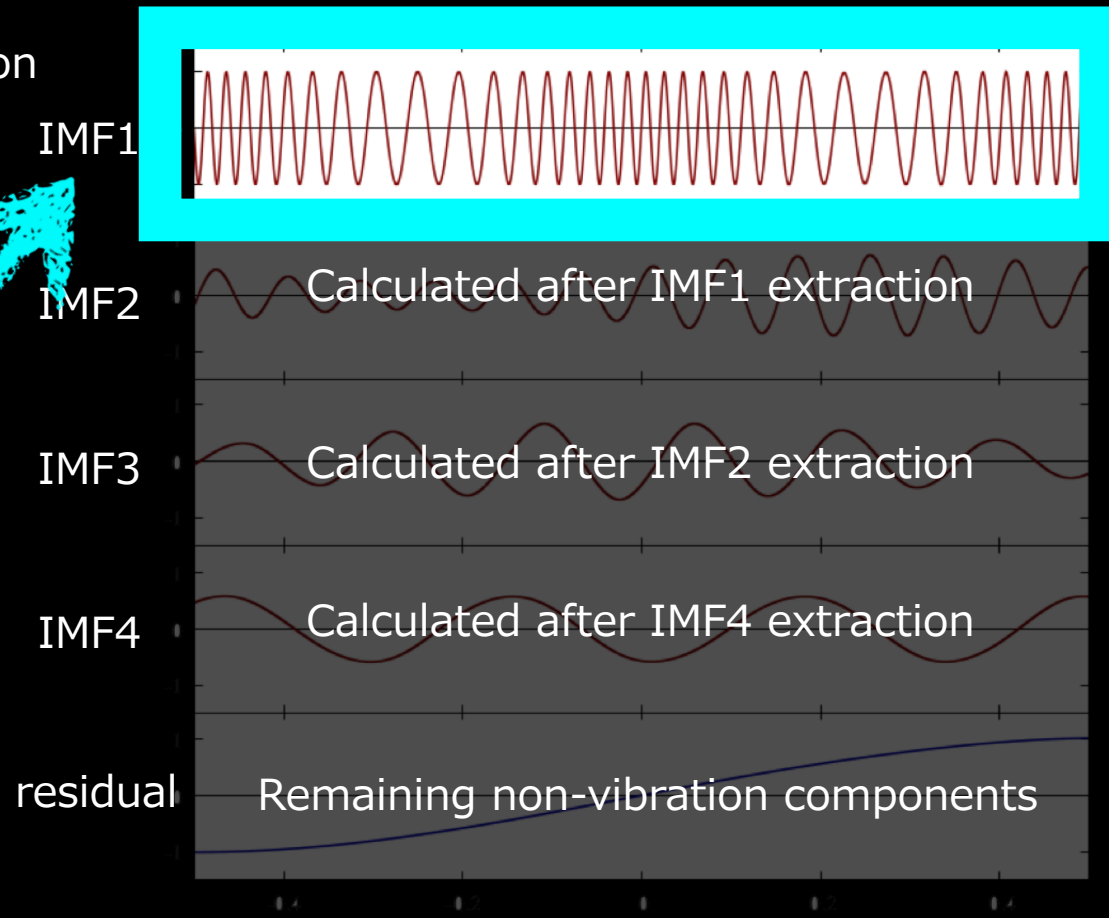

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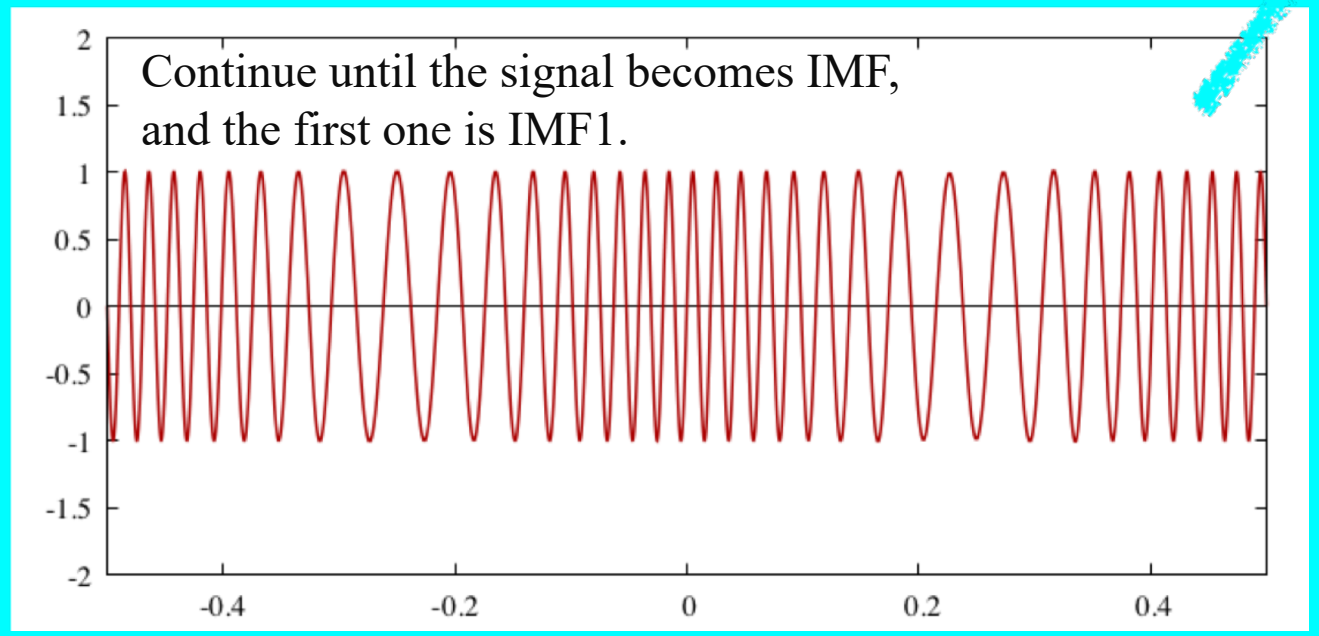

Hilbert-Huang Transform (HHT)

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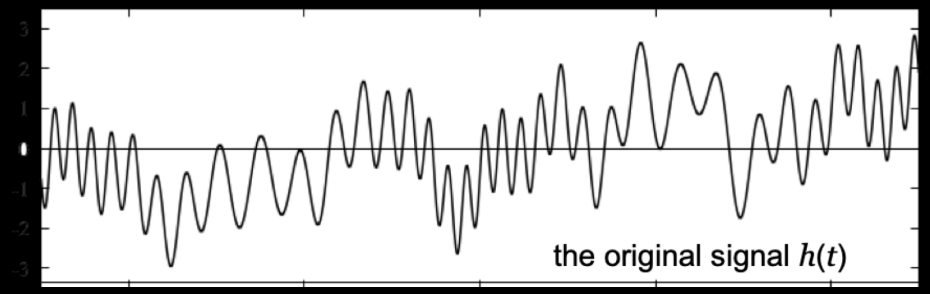

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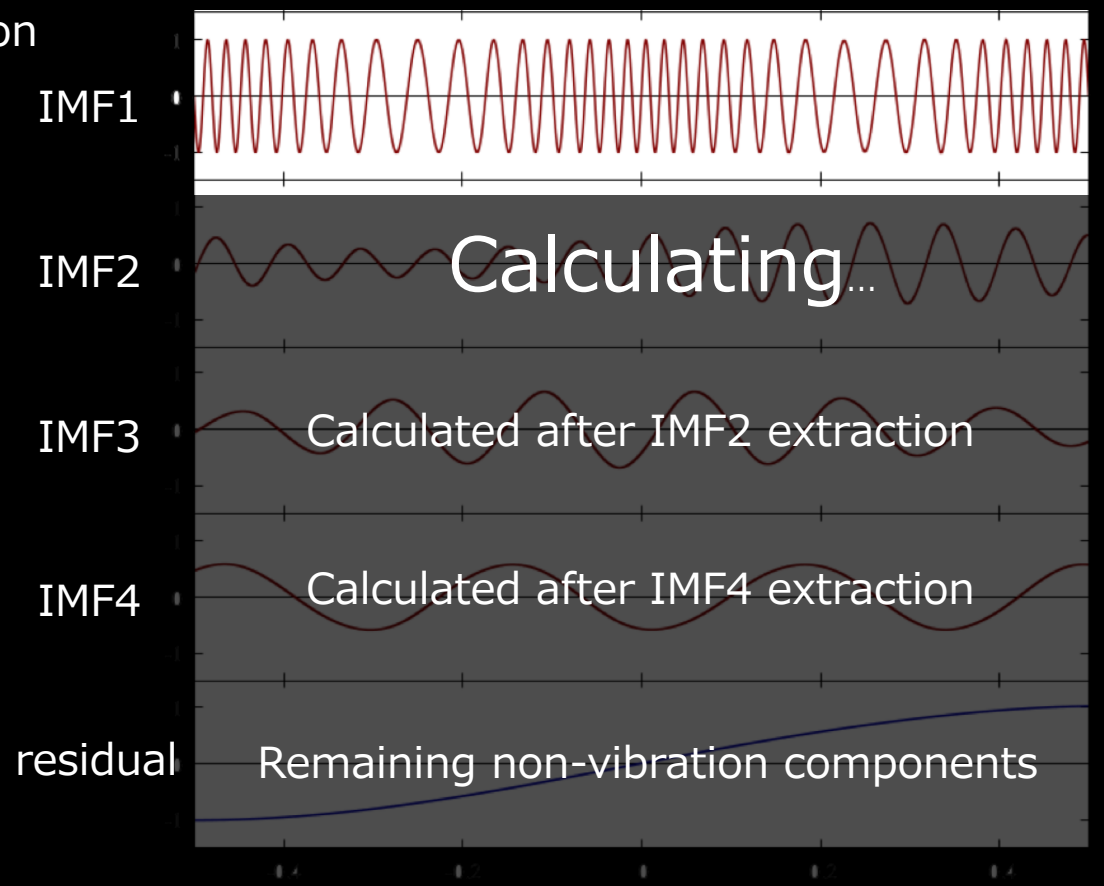

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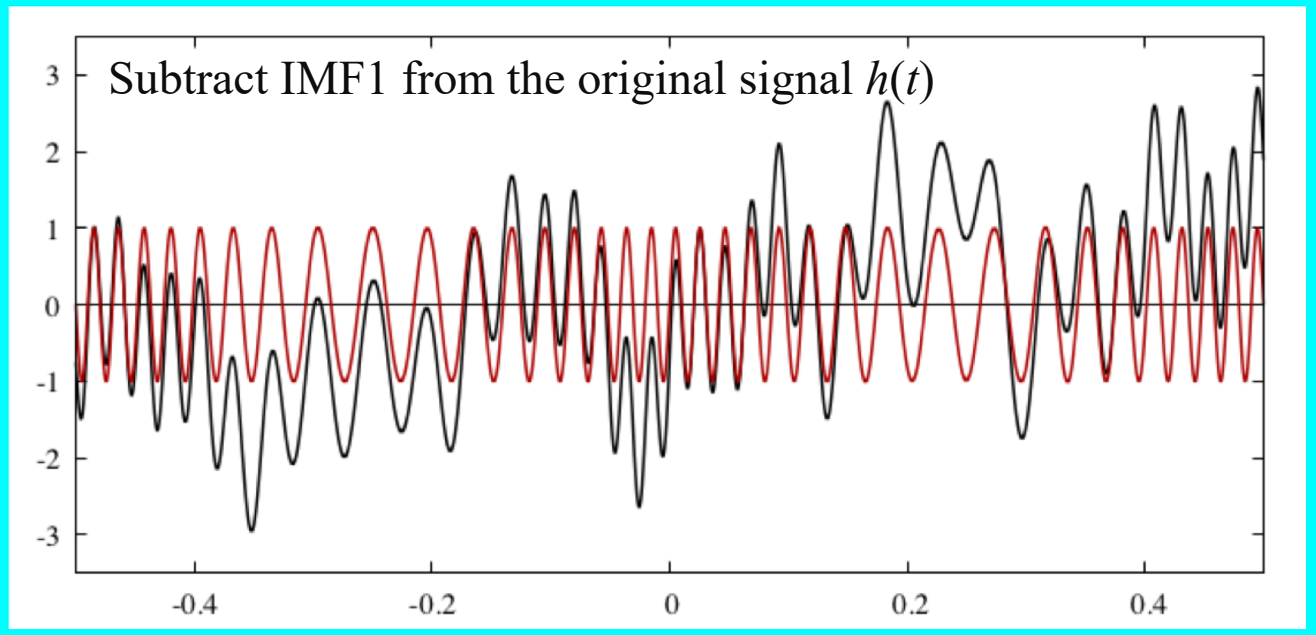

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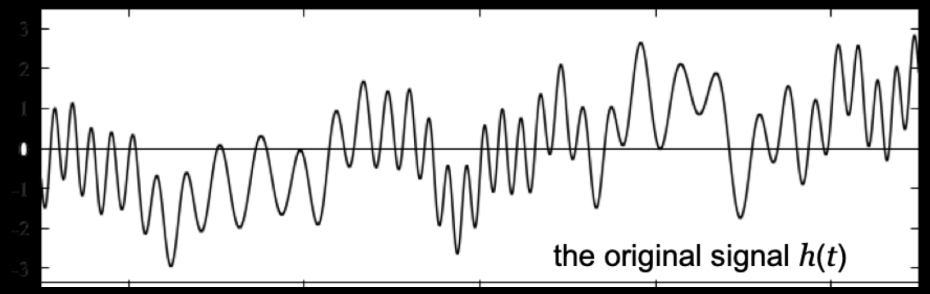

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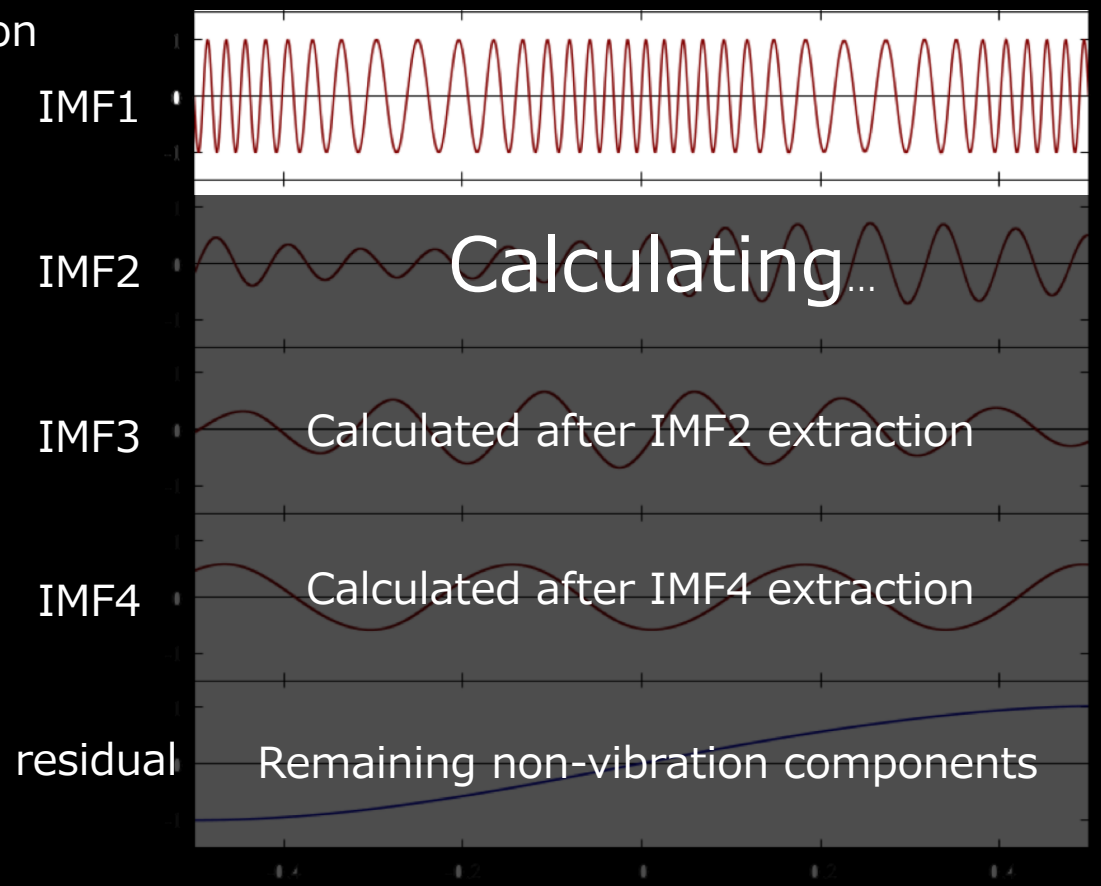

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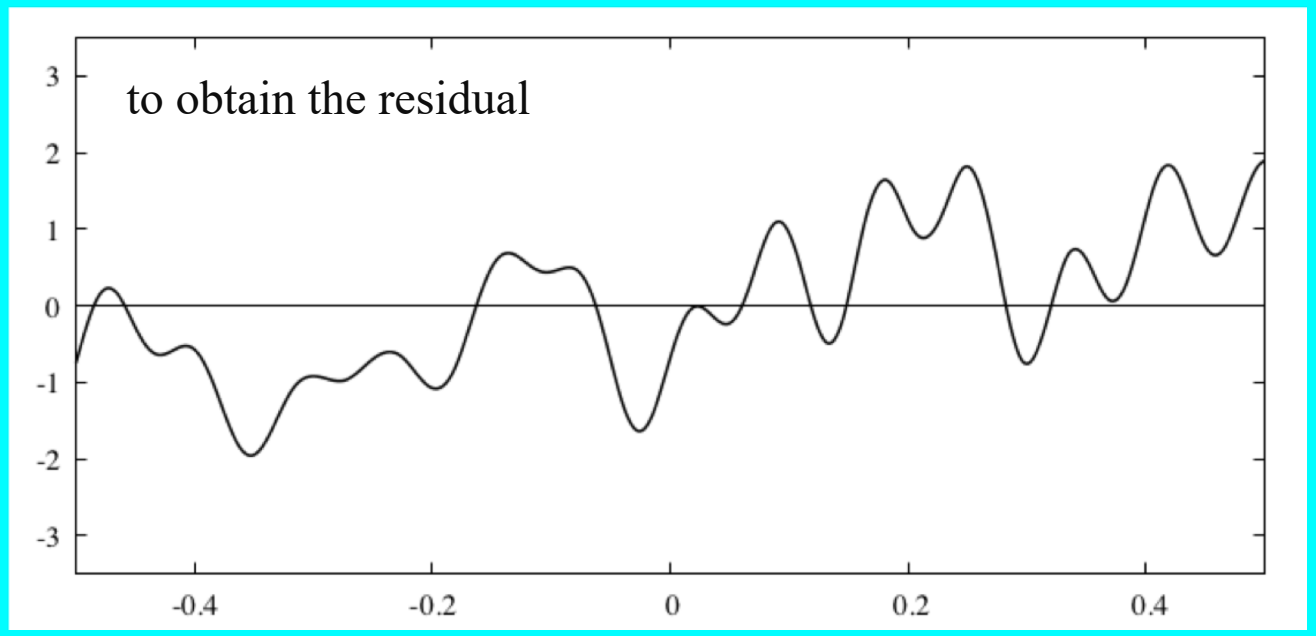

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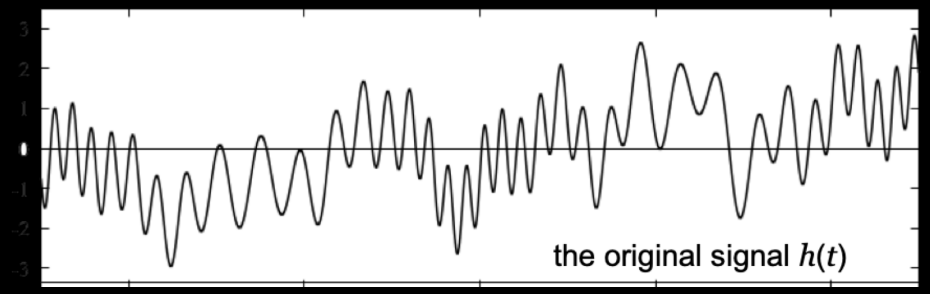

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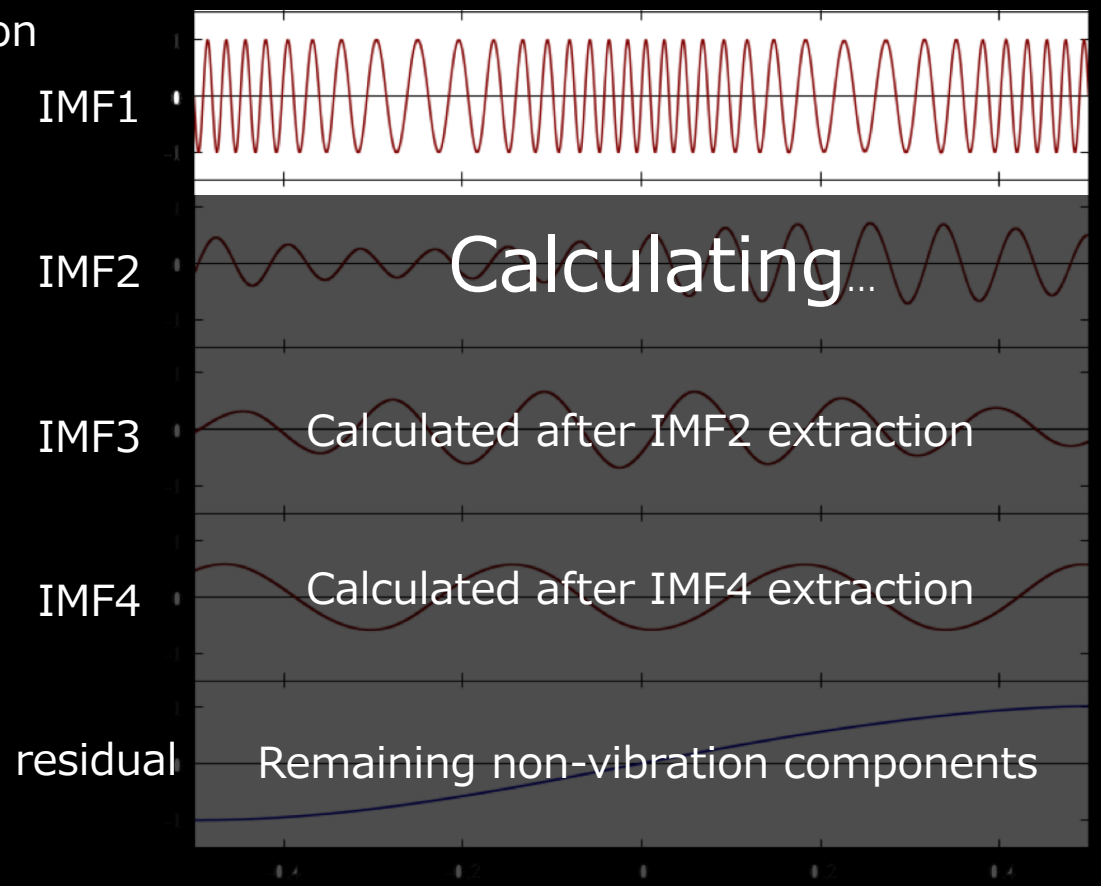

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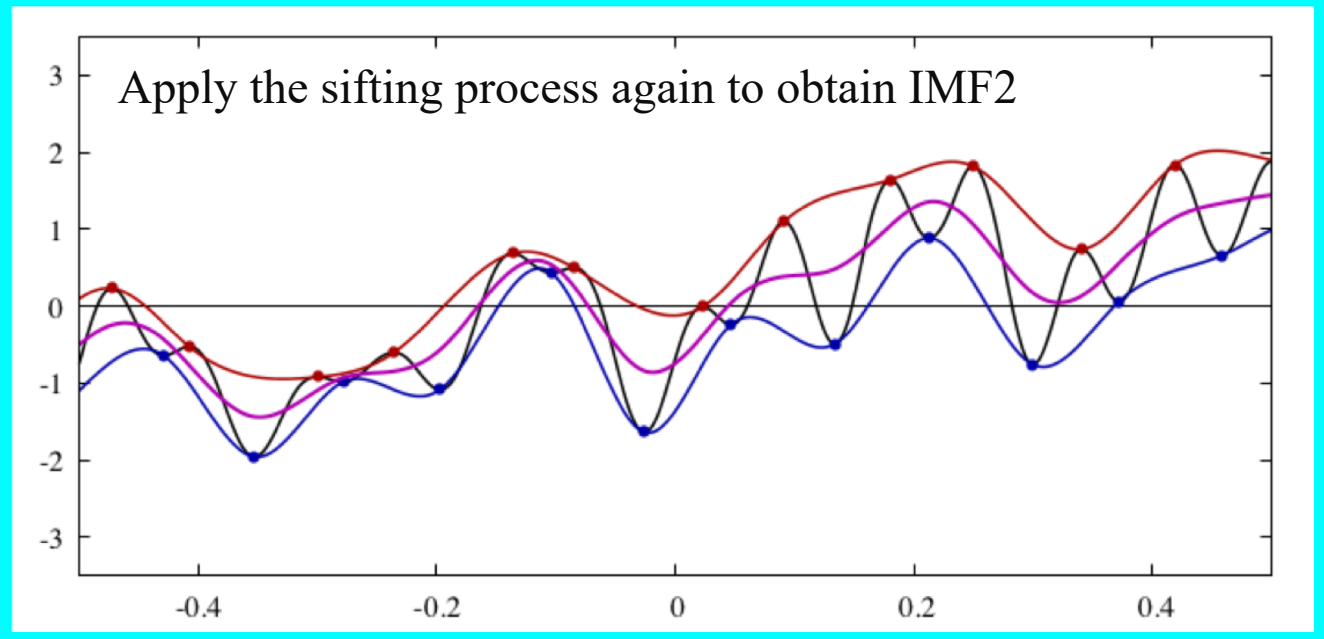
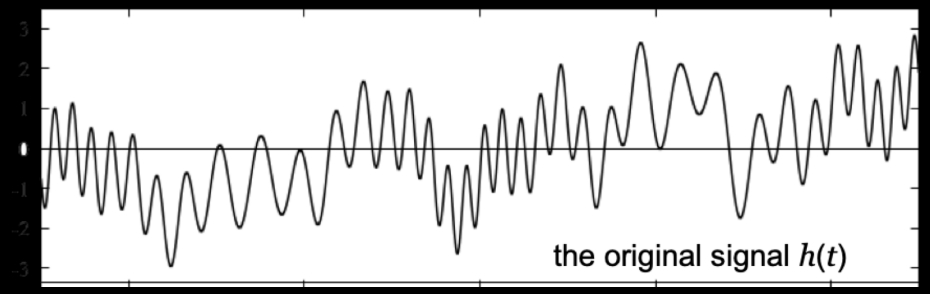


Hilbert-Huang Transform (HHT)

Decompose the signal into the IMFs



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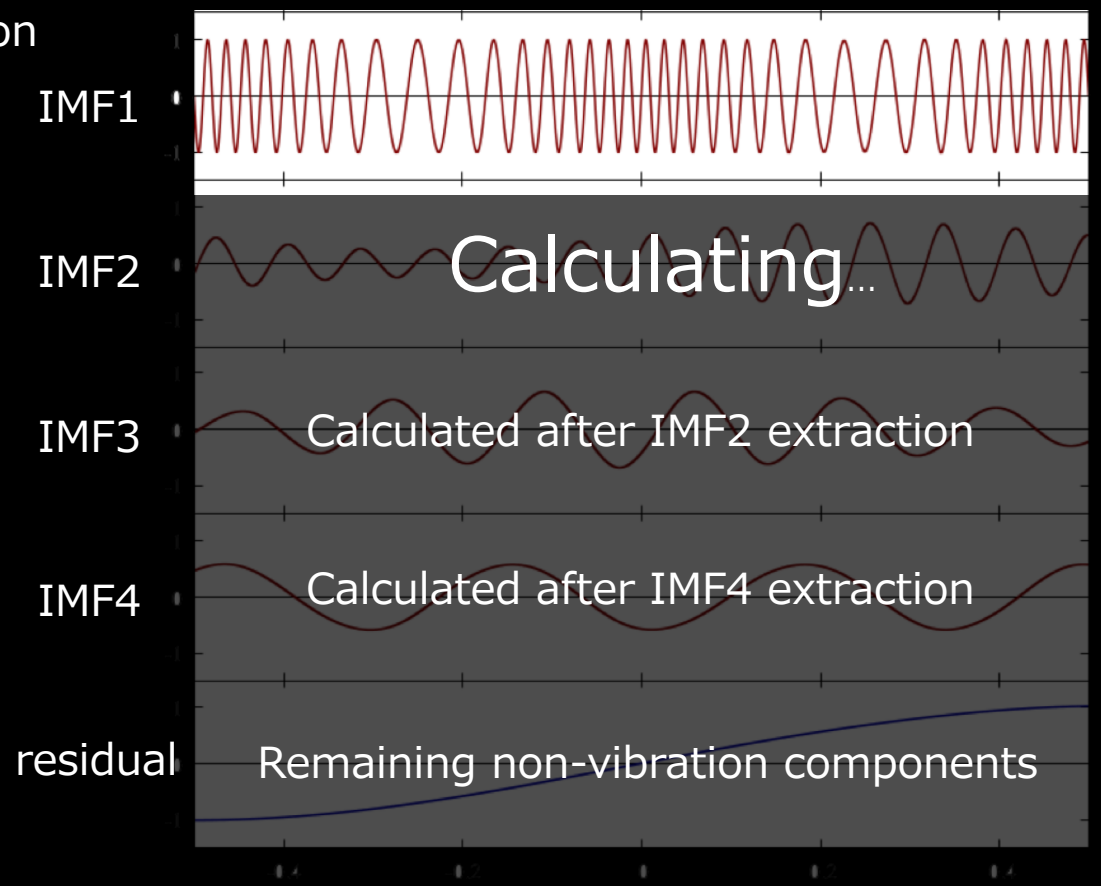
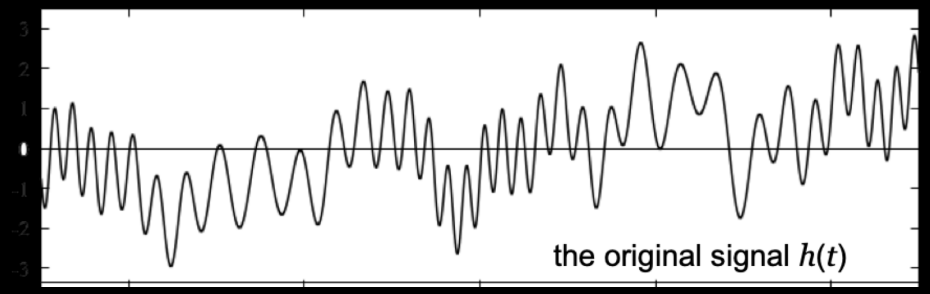


EMD : Empirical mode decomposition

Repeat the procedure

Hilbert-Huang Transform (HHT)

Decompose the signal into the IMFs



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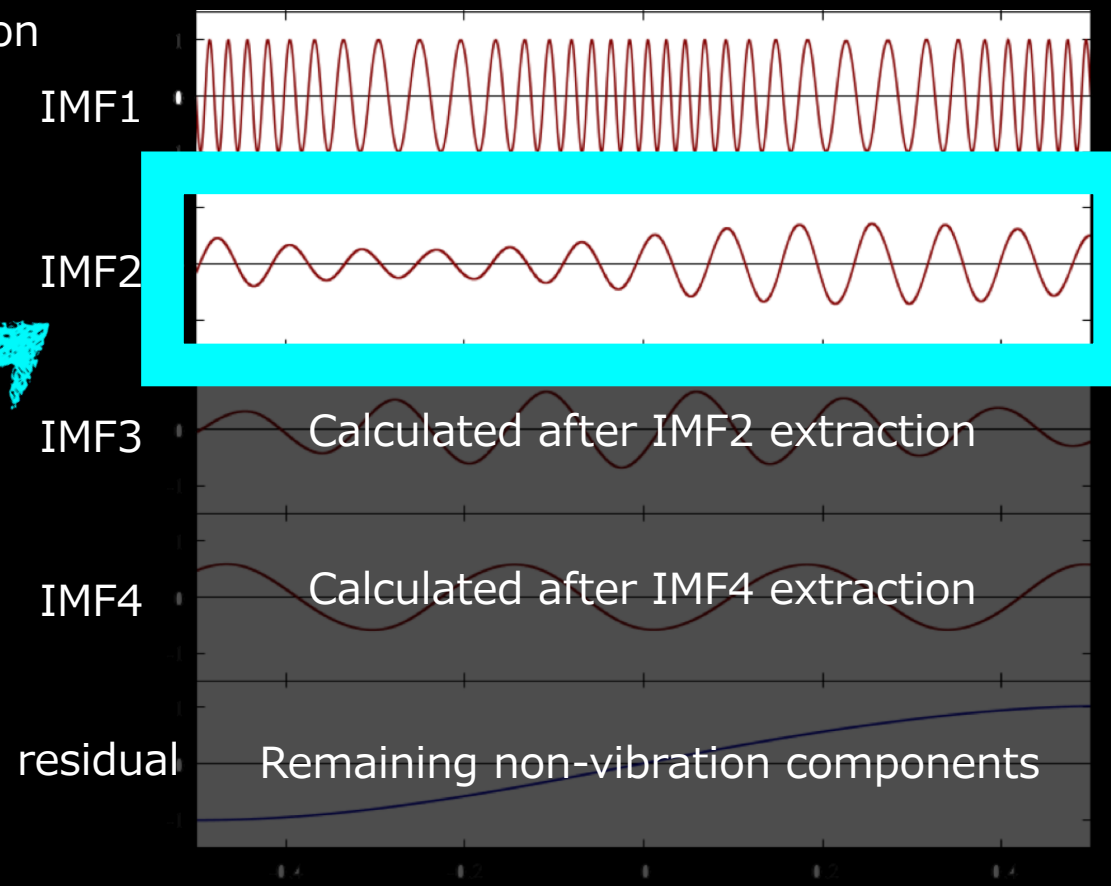

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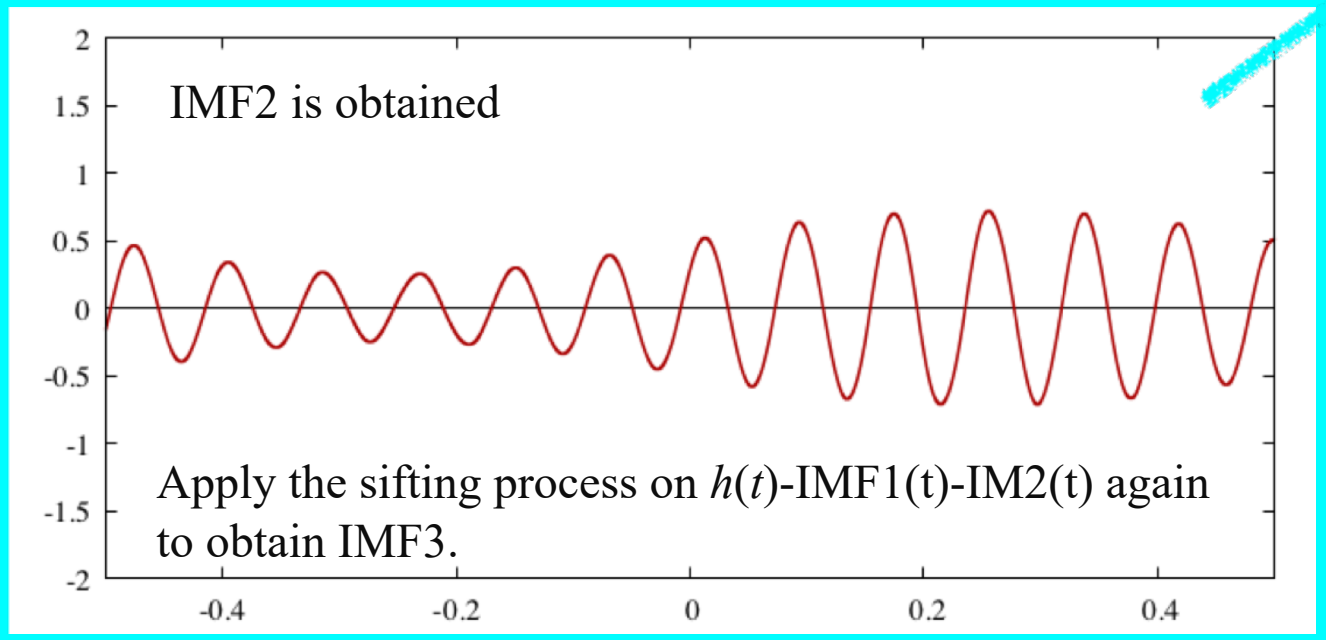
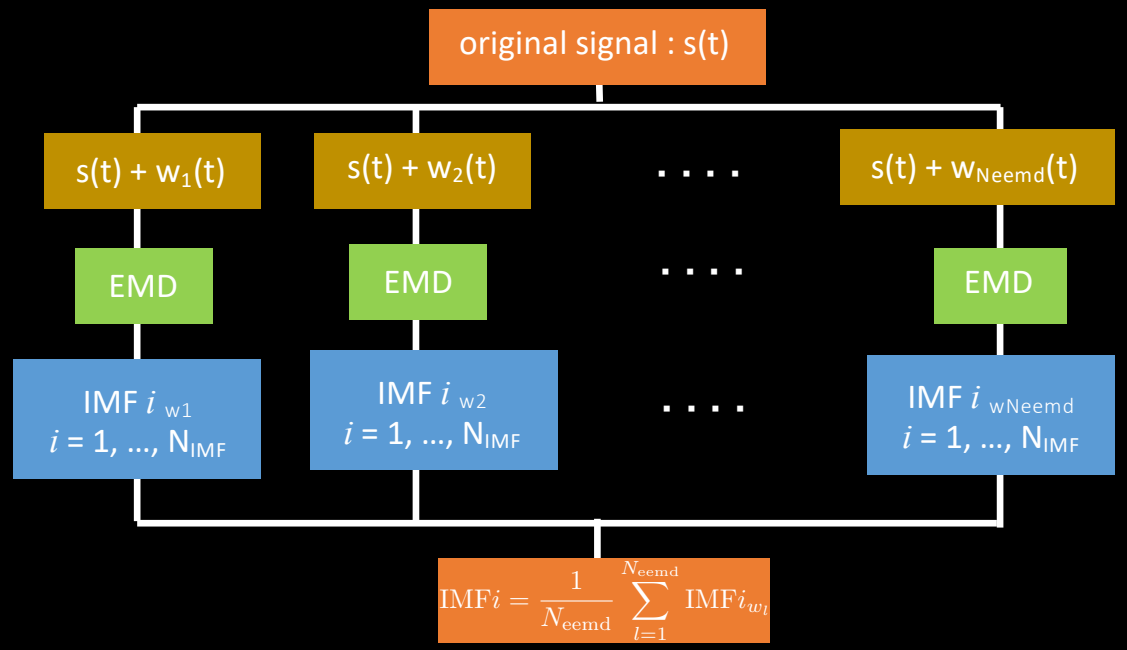


Hilbert-Huang Transform (HHT)

Ensemble empirical mode decomposition (EEMD)

- In the EEMD,
  there is a process of preparing
  Neemd white Gaussian noises
  and repeating the same operation for
  each of them.

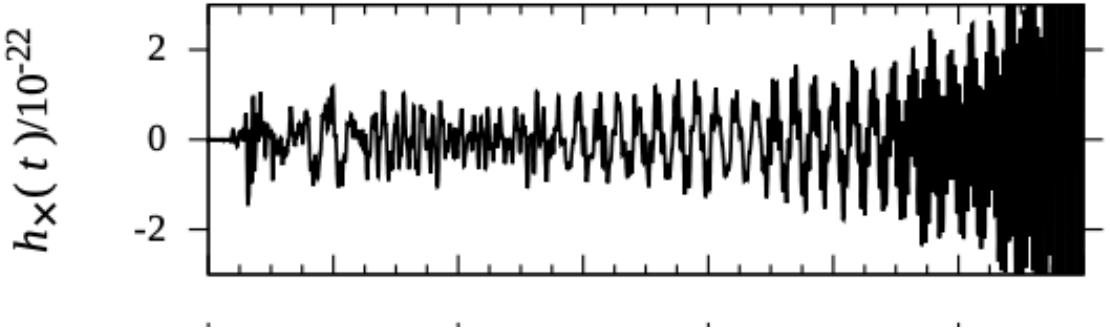
- Adding noise and making it oscillate
  in all frequency bands has the effect
  of preventing the generation of false
  signals,
  which can be caused by
  frequency gaps in the constituent
  waves or sudden oscillating waves.



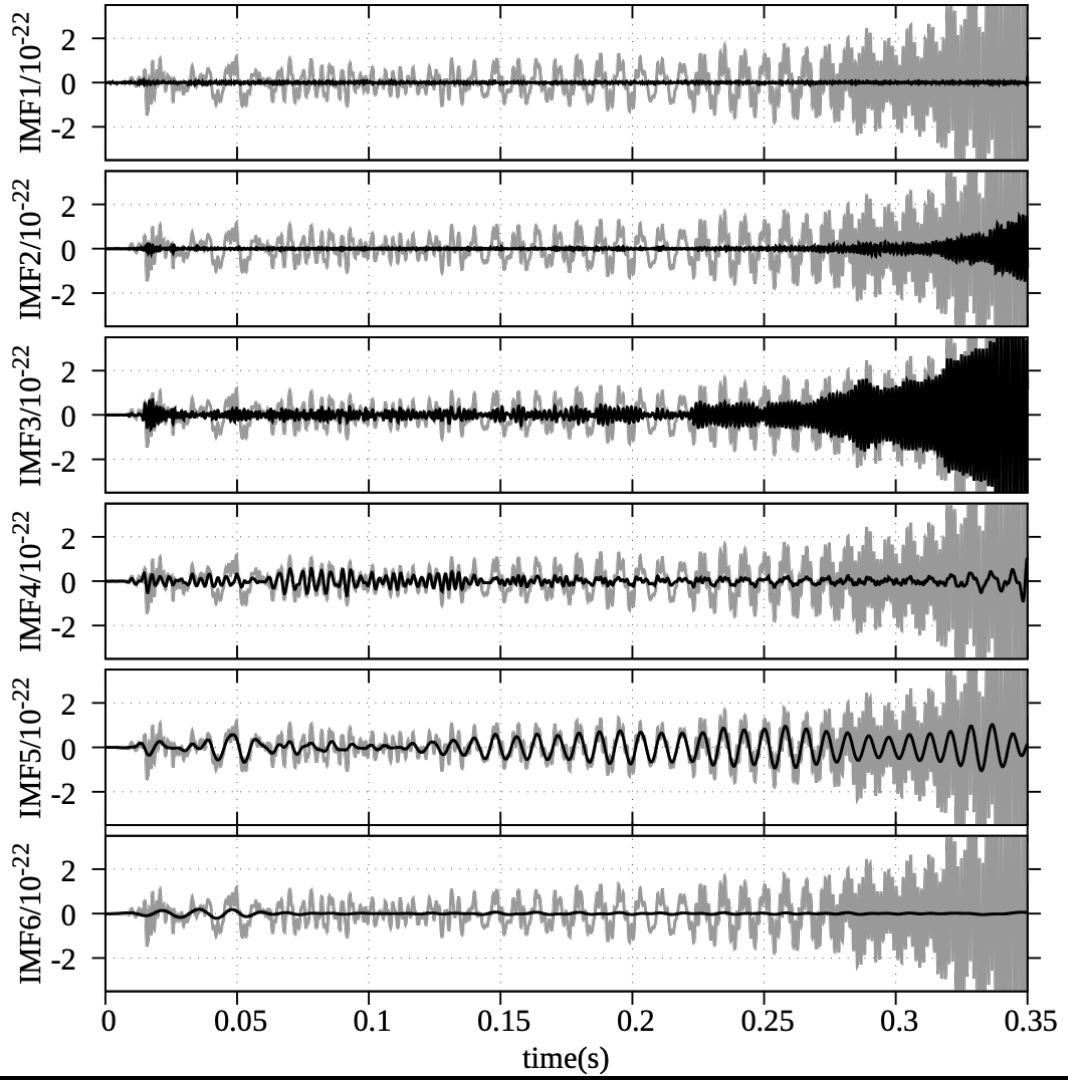
results

grey line. : GW
black lines : IMFs

the GW from the CCSN simulation

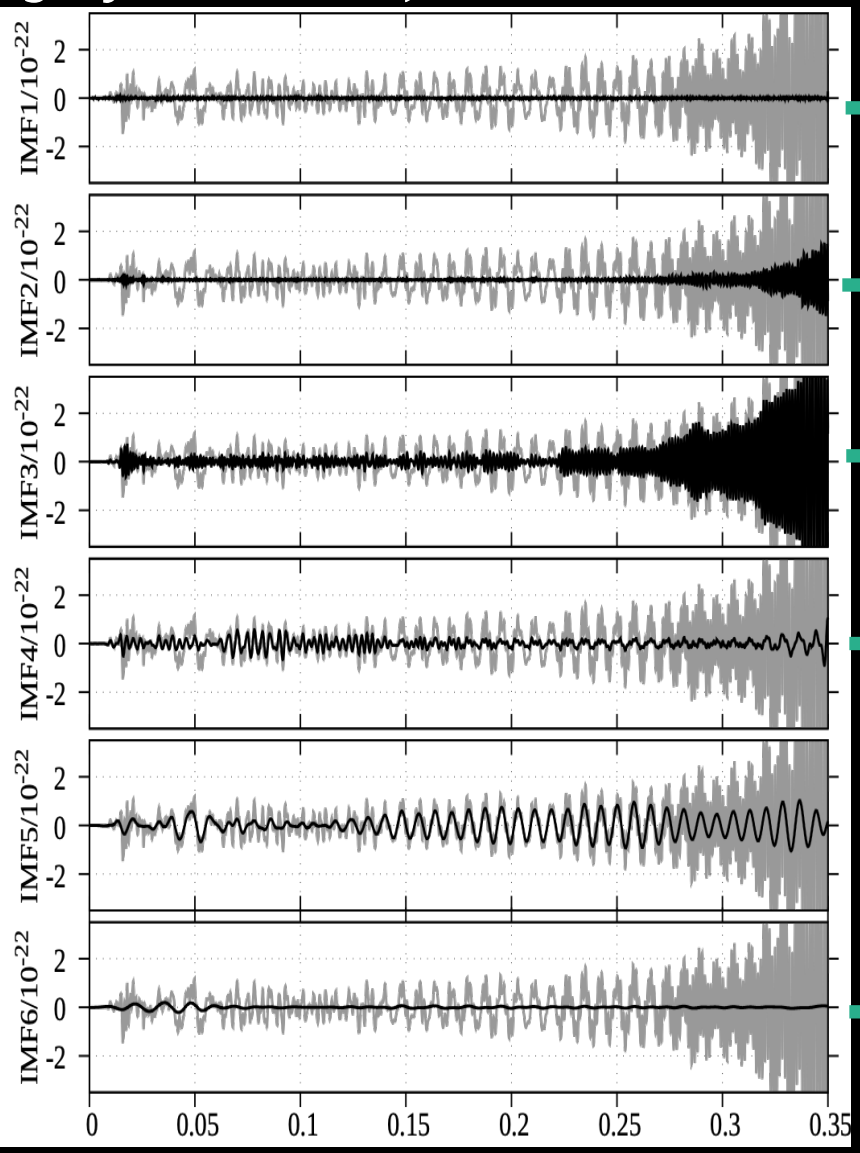


EEMD



results

grey lines : GW, black lines : IMFs



HSA → IA $a_1(t)$, IF $f_1(t)$

HSA → IA $a_2(t)$, IF $f_2(t)$

HSA → IA $a_3(t)$, IF $f_3(t)$

HSA → IA $a_4(t)$, IF $f_4(t)$

HSA → IA $a_5(t)$, IF $f_5(t)$

HSA → IA $a_6(t)$, IF $f_6(t)$

Hilbert spectrum

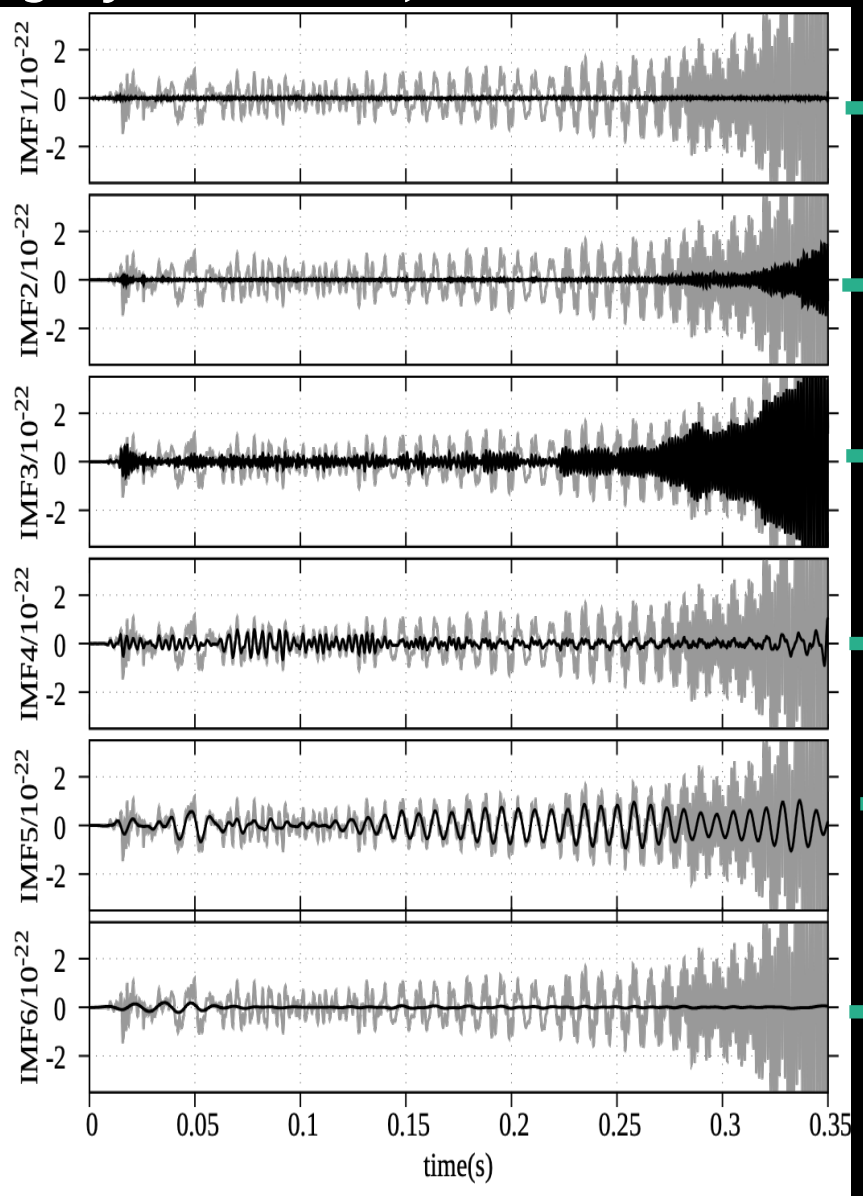
$$H(t,f) = \sum_{k:f_k(t)=f} a_k(t)$$

Time – Frequency diagram

H(t, f)

results

grey lines : GW, black lines : IMFs



HSA → IA $a_1(t)$, IF $f_1(t)$

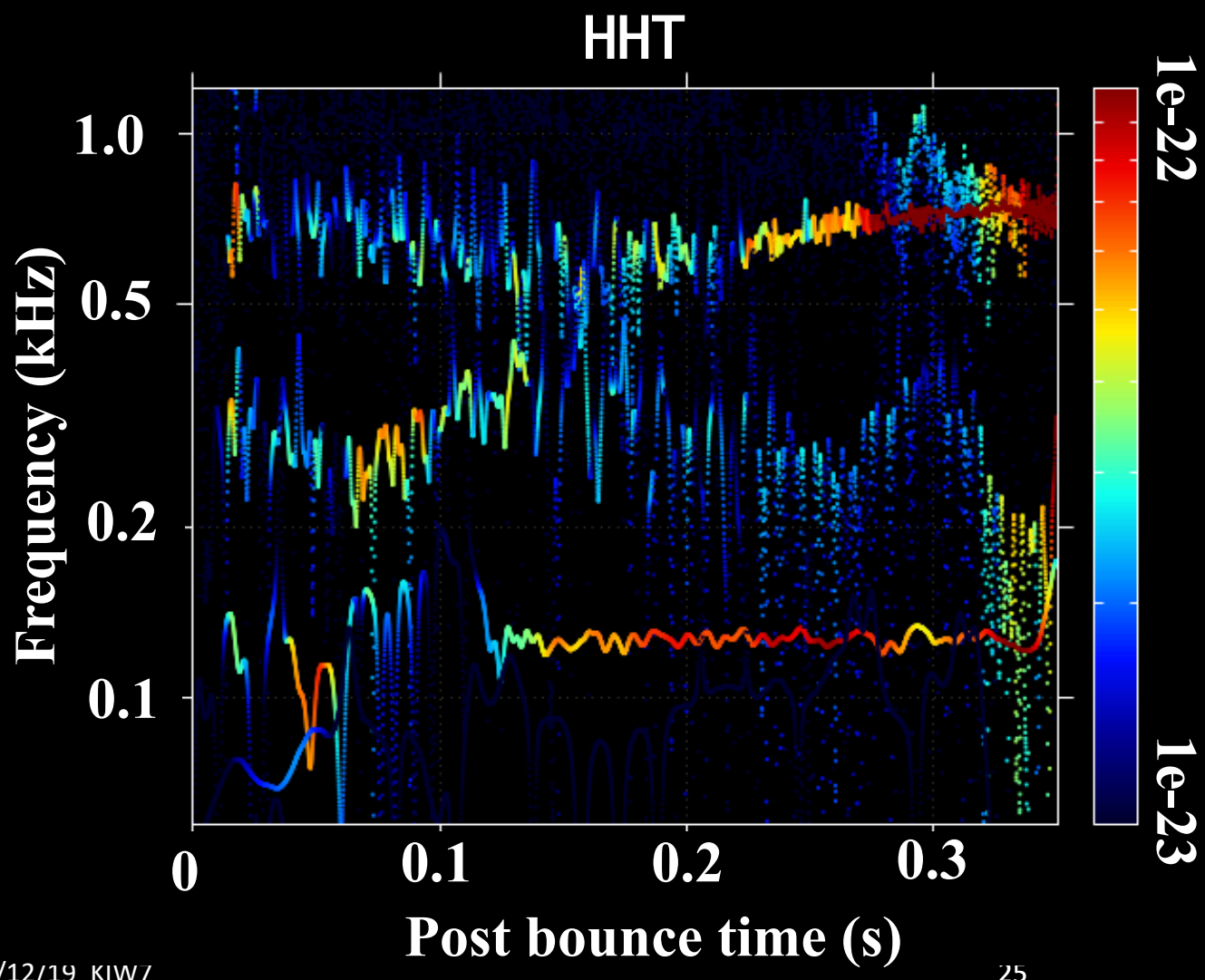
HSA → IA $a_2(t)$, IF $f_2(t)$

HSA → IA $a_3(t)$, IF $f_3(t)$

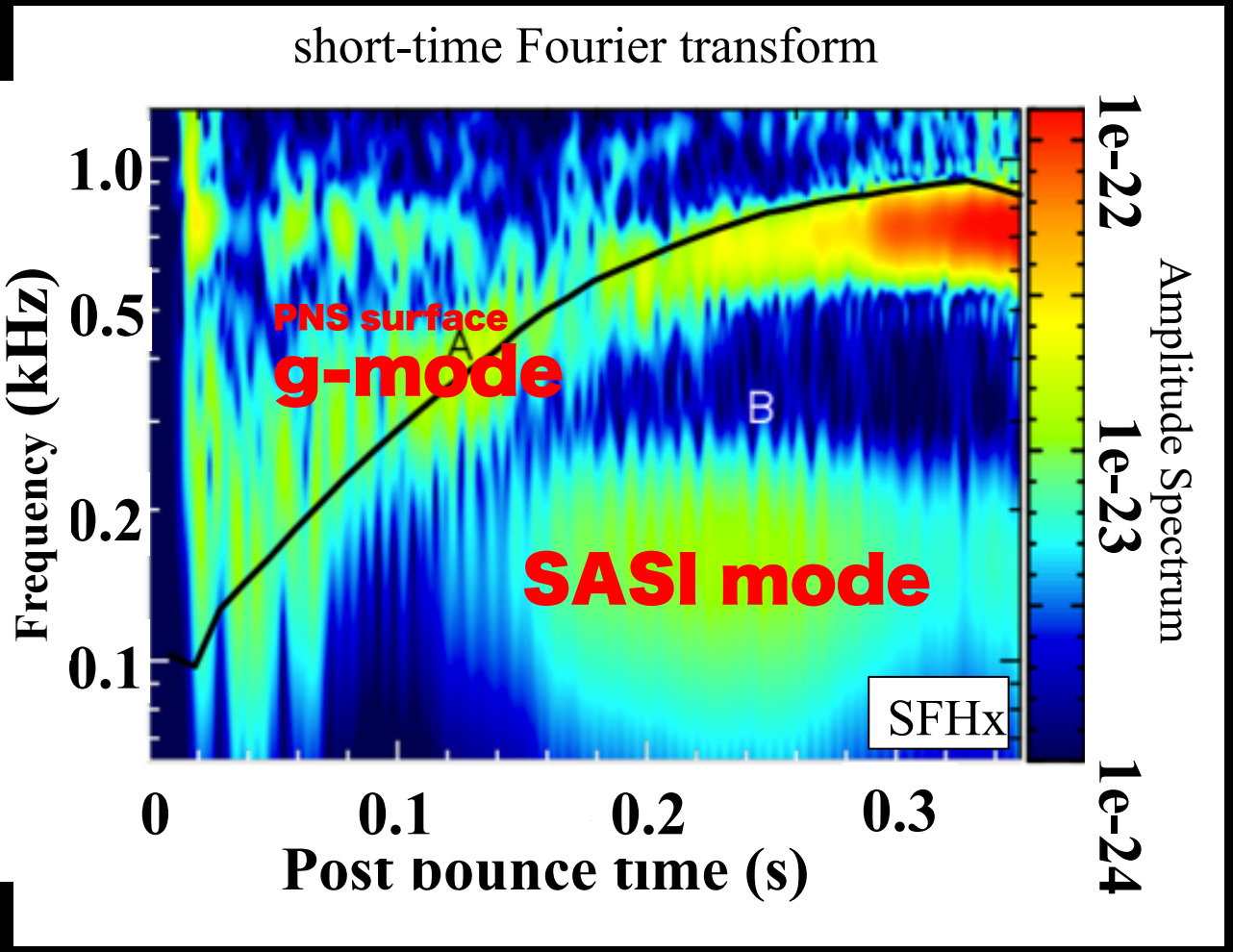
HSA → IA $a_4(t)$, IF $f_4(t)$

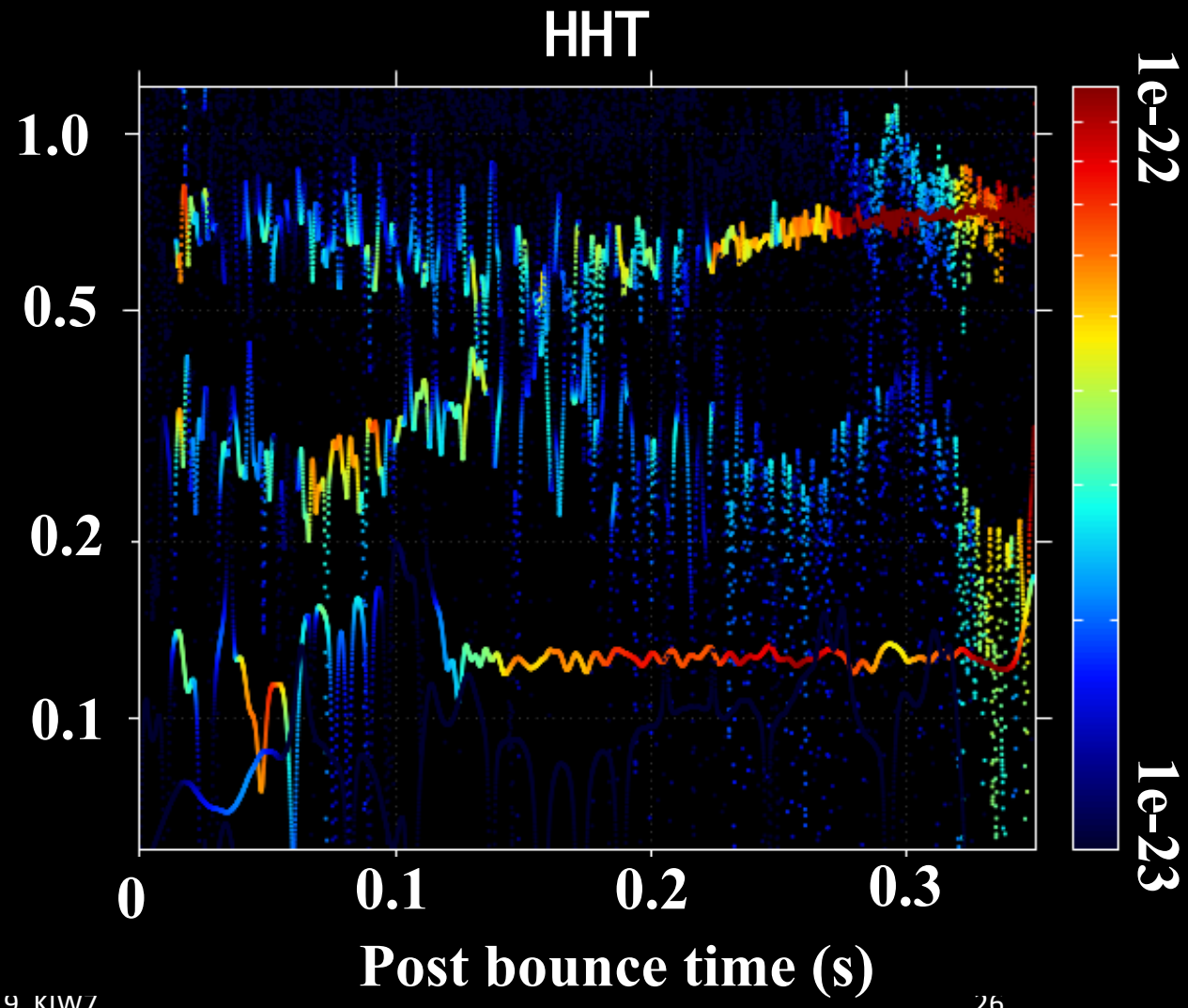
HSA → IA $a_5(t)$, IF $f_5(t)$

HSA → IA $a_6(t)$, IF $f_6(t)$

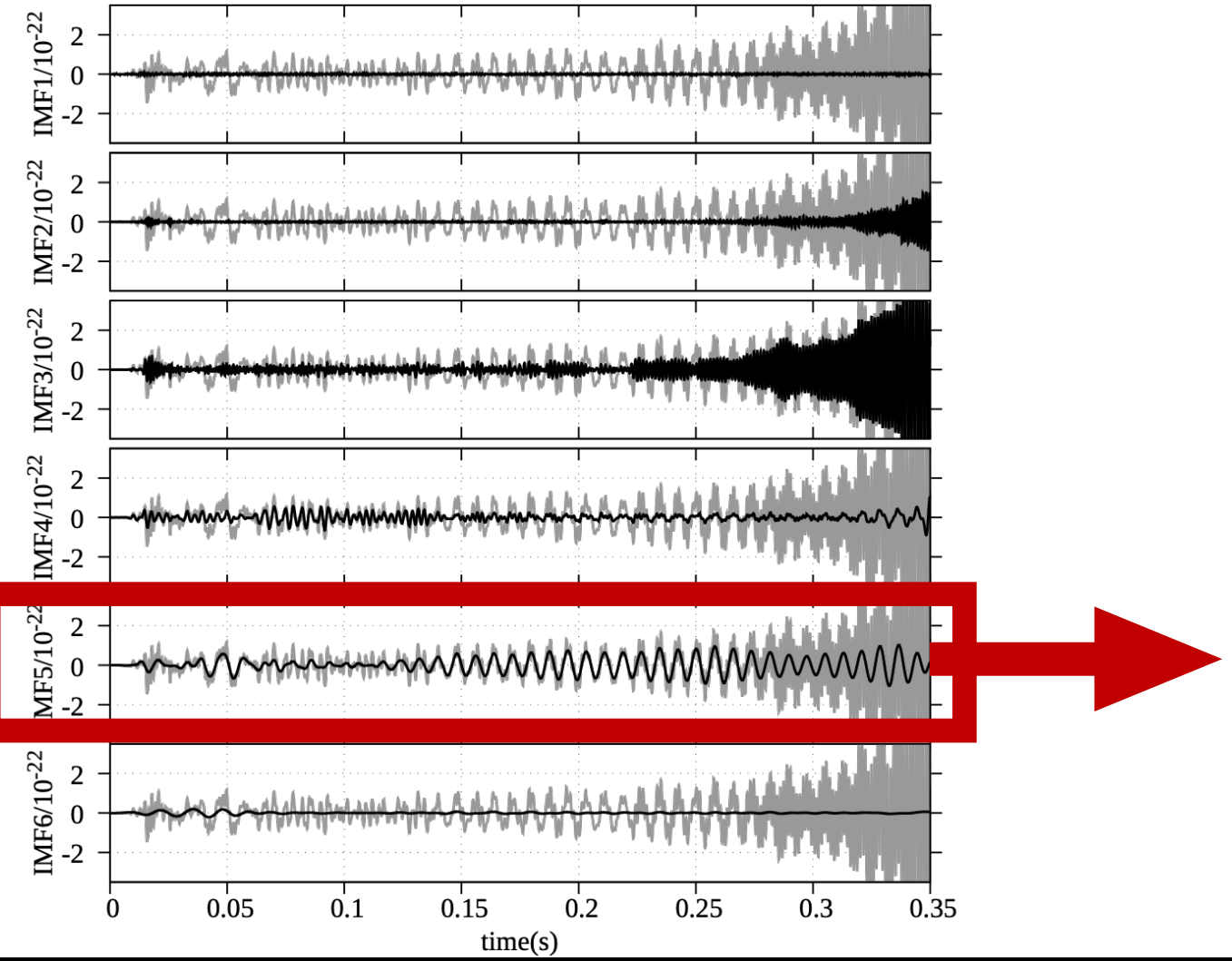


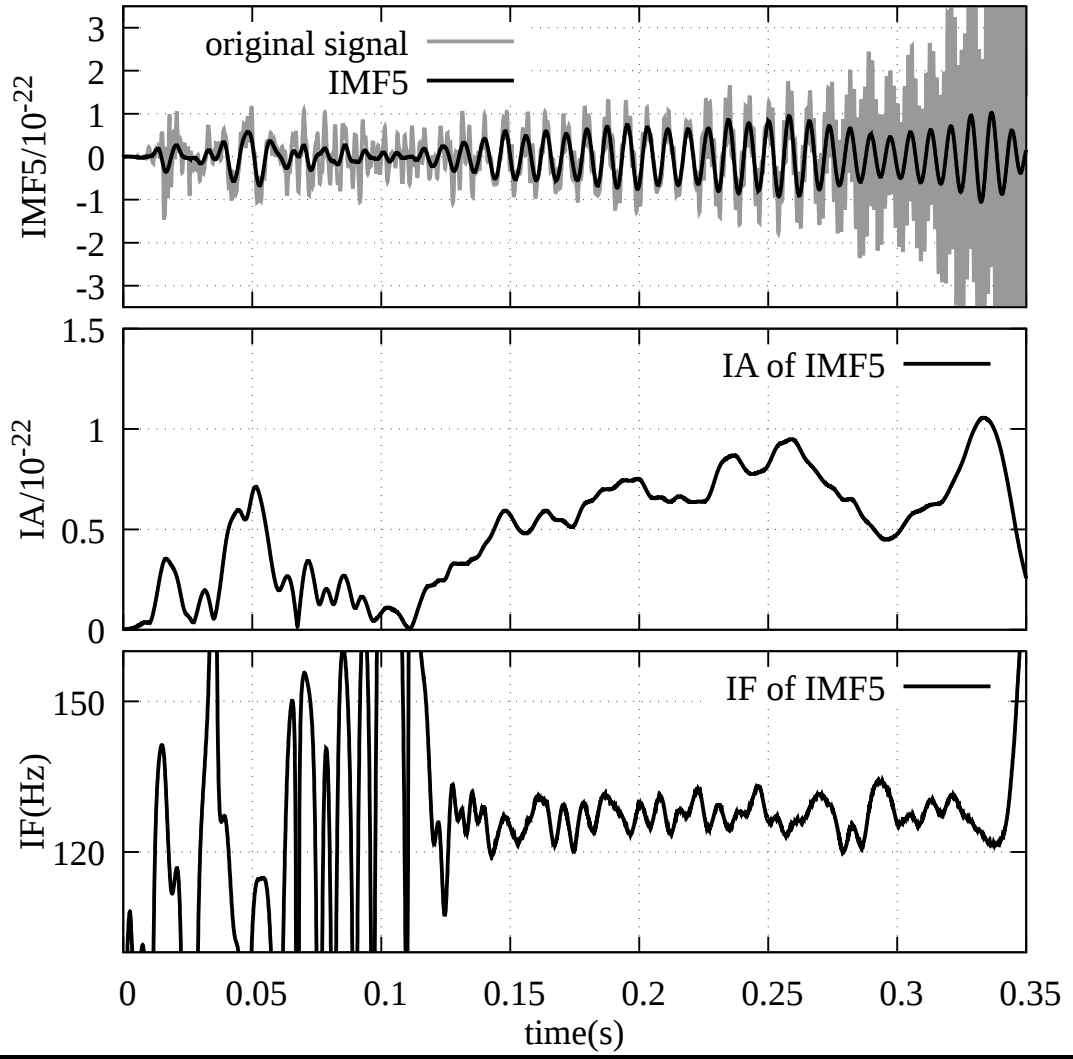
results


short-time Fourier transform
PNS surface
g-mode
A
B
SASI mode
SFHx
Frequency (kHz)
1.0
0.5
0.2
0.1
0
0.1
0.2
0.3
Post bounce time (s)
1e-22
1e-23
1e-24
Amplitude Spectrum


HHT
1.0
0.5
0.2
0.1
0
0.1
0.2
0.3
Post bounce time (s)
1e-22
1e-23

results


IMF1/10^-22
IMF2/10^-22
IMF3/10^-22
IMF4/10^-22
IMF5/10^-22
IMF6/10^-22
2
0
-2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
time(s)


original signal
IMF5
IMF5/10^-22
3
2
1
0
-1
-2
-3
IA of IMF5
IA/10^-22
1.5
1
0.5
0
IF of IMF5
IF(Hz)
150
120
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
time(s)

results

we investigated

(1) the time interval during which SASI was active

→ ?

(2) the time variability of SASI frequency during that interval.

→ ?

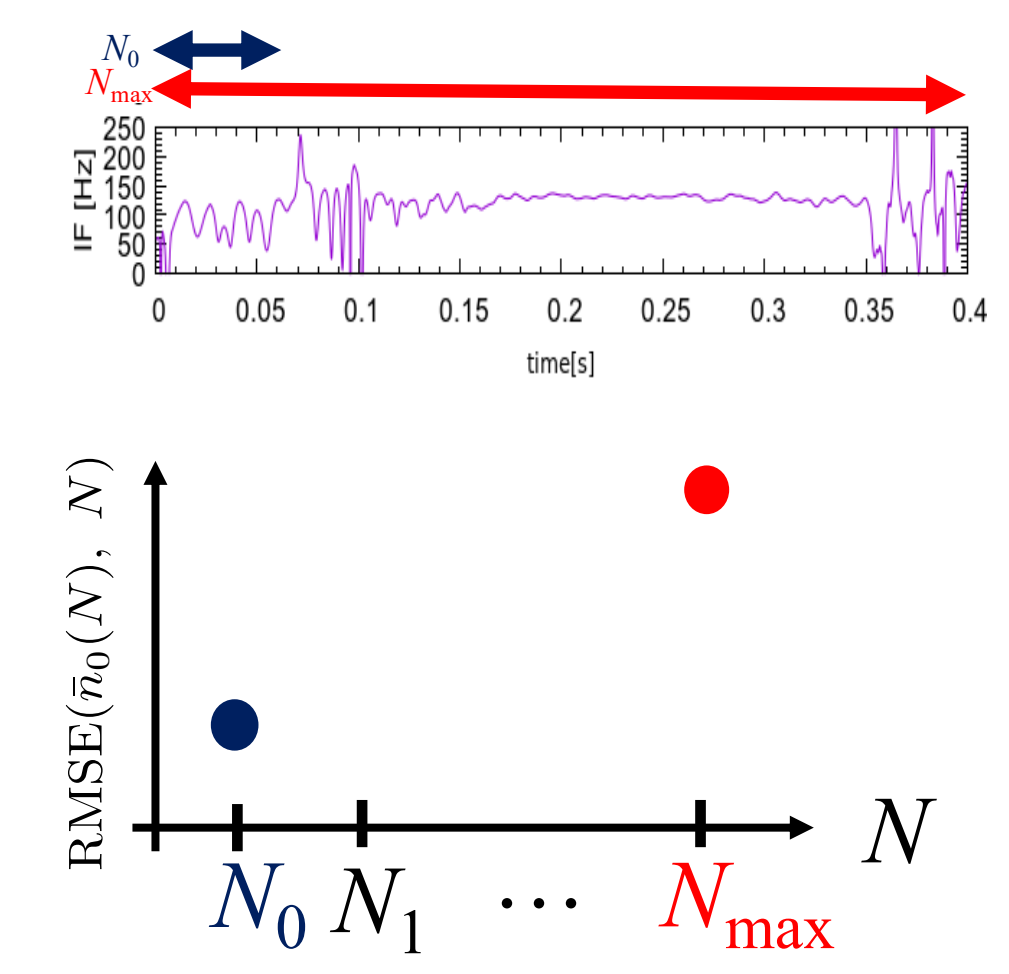
results

Estimating the starting point of the target mode

- First, to find the time interval where SASI is active, we looked for the best start and end time for each time interval length N.
- Specifically, we assumed the frequency to be constant and looked for the one that minimized the following root mean squared error

$$\mathrm{RMSE}(n_0, N_0) = \sqrt{\frac{1}{N_0} \sum_{n=n_0}^{n_0+N_0-1} (f[n] - < f >)^2}$$

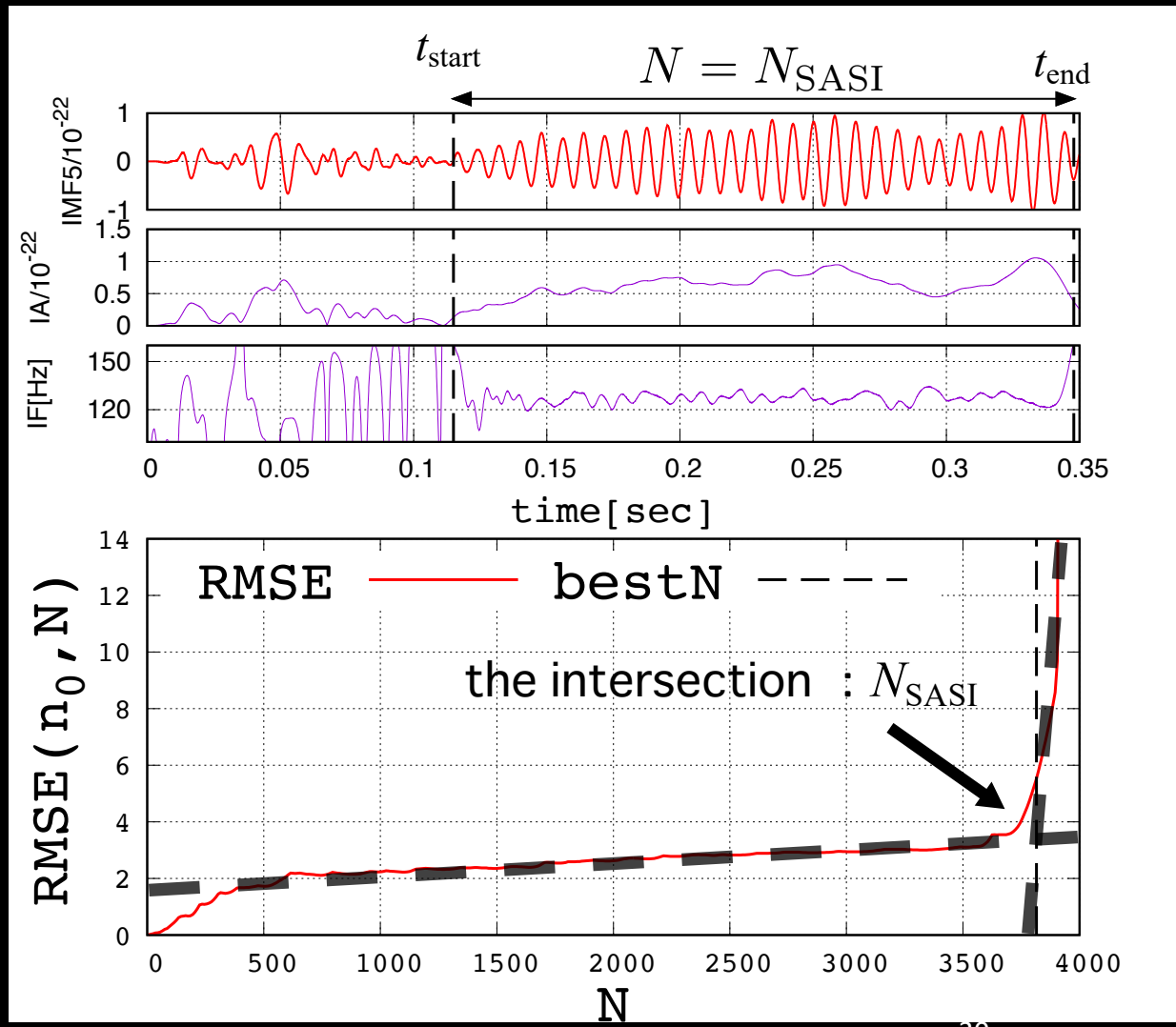
$$\bar{n}_0(N_0) = \mathrm{argmin}_{n_0} \mathrm{RMSE}(n_0, N_0)$$



results

Estimating the starting point of the target mode

- we assumed that the change in the slope of this figure is a sign of the change in the dominant component.
- In other words, the borderline between the noise-dominated interval and the SASI-dominated interval.



results

we investigated

(1) the time interval during which SASI was active

→ $0.11s \lesssim t \lesssim 0.35s$ $< f >= 127.0 \pm 3.7$ Hz

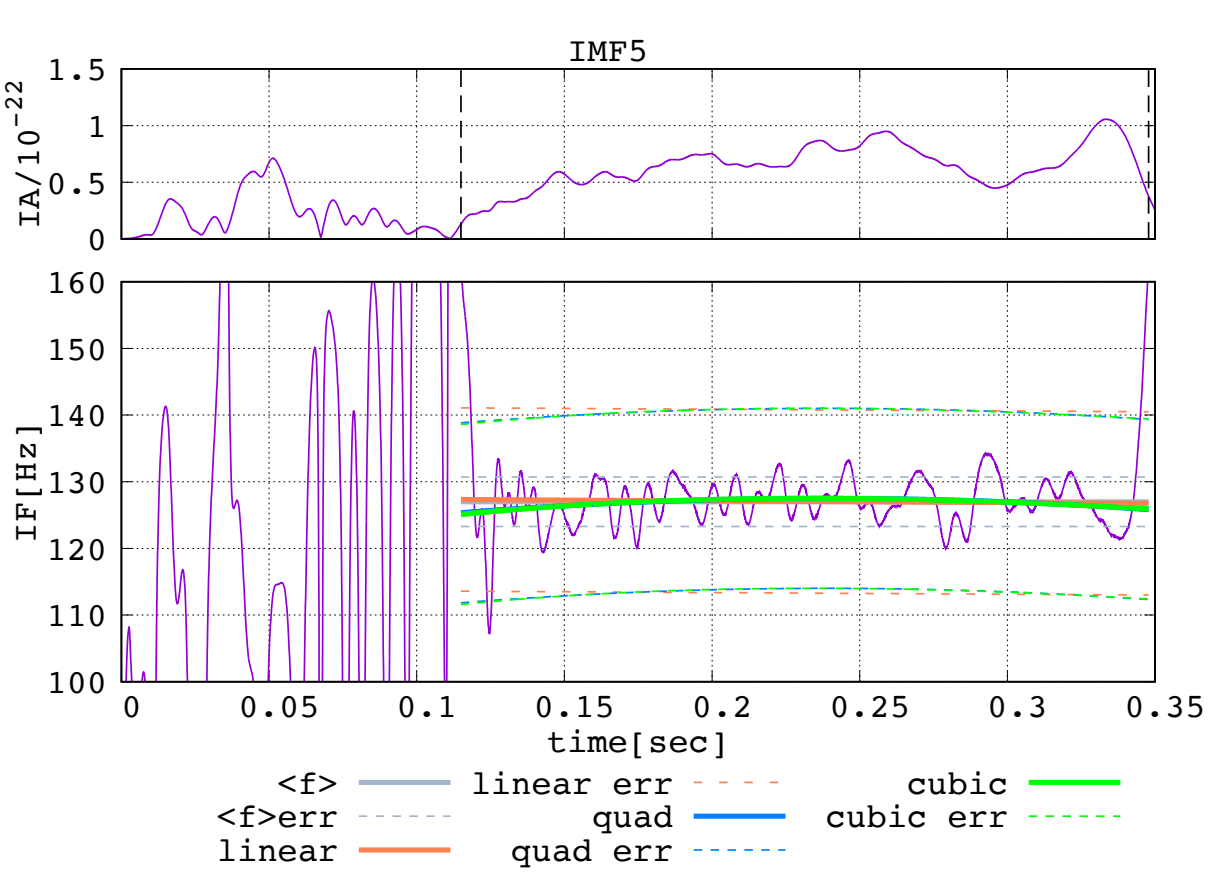
(2) the time variability of SASI frequency during that interval.

→ ?

results Analysis of the frequency trend

- The time variation of the SASI frequency can be considered as a sign of an explosion.

- We propose to perform a weighted least squares method on the instantaneous frequency and use the value of the fitting coefficient to determine if it is constant or not.



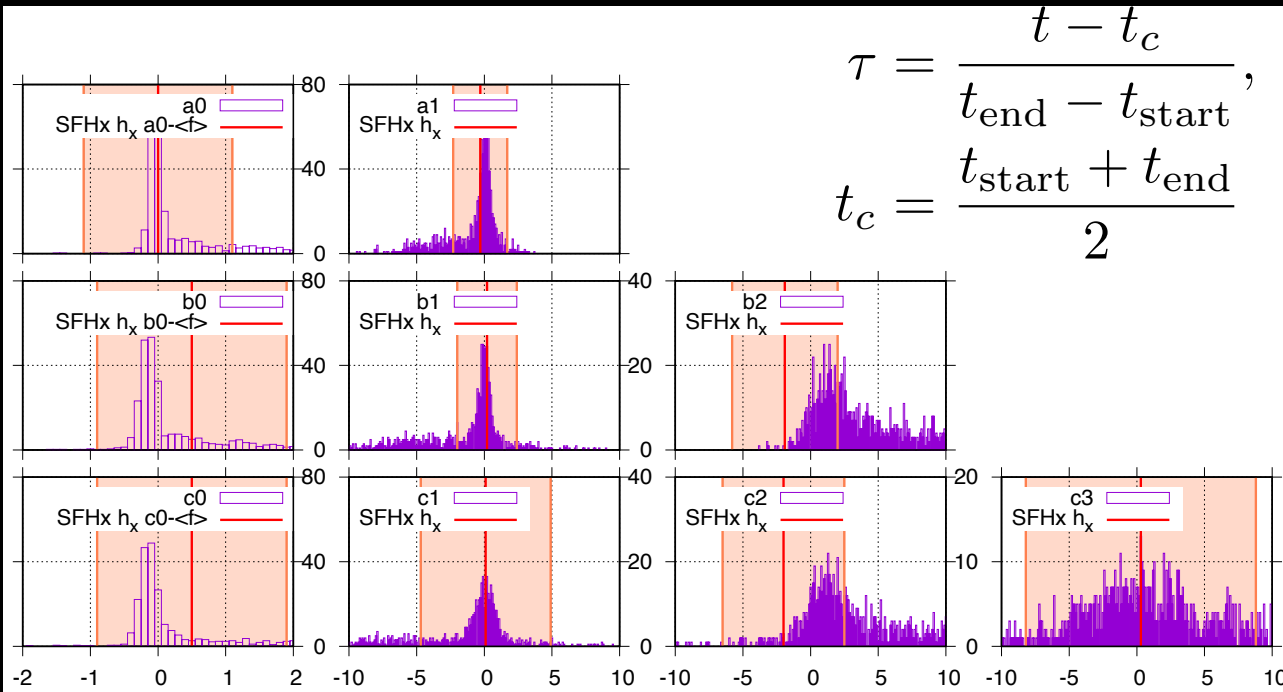
Weight function $$w(t_i) = \frac{\mathrm{IA}^2(t_i)}{\sum_i \mathrm{IA}^2(t_i)}$$

results Analysis of the frequency trend

- The red lines are the values of the fitting coefficients
- The orange squares are the 1 sigma error of the coefficients.
- The histogram is the result of the same analysis for 1000 test waveforms of constant frequency.

✓ The width of the histogram can be regarded as the error range of the method,

⇨ the SASI-derived GW signal has a constant frequency in the range of 1 sigma.



$$\tau = \frac{t - t_c}{t_{\rm end} - t_{\rm start}},$$

$$t_c = \frac{t_{\rm start} + t_{\rm end}}{2}$$

$$f_{\rm lin}(t) = a_0 + a_1\tau + f_{\rm lin,err}$$
$$f_{\rm quad}(t) = b_0 + b_1\tau + b_2\tau^2 + f_{\rm quad,err}$$
$$f_{\rm cubic}(t) = c_0 + c_1\tau + c_2\tau^2 + c_3\tau^3 + f_{\rm cubic,err}$$

results

we investigated

(1) the time interval during which SASI was active

→ $0.11s \lesssim t \lesssim 0.35s$ $< f >= 127.0 \pm 3.7$ Hz

(2) the time variability of SASI frequency during that interval.

→ constant

Summary

- We have shown that the HHT can extract SASI modes from gravitational waves obtained from a CCSN numerical simulations.

- We also proposed a method to check whether the frequency of SASI-derived gravitational waves is constant or not.

- According to the results, the waveforms we analyzed were constant in the range of 1 sigma.

✓ Future work
  - ☐ Analyzing other modes of the gravitational waves with the HHT
  - ☐ Analyzing the signal by adding the detector noise and the simulated waveform.