Use of Excess Power Method and Convolutional Neural Network in All-Sky Search for Continuous Gravitational Waves

Takahiro S. Yamamoto (Kyoto Univ.)

collaborator: Takahiro Tanaka (Kyoto Univ.)

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Continuous Gravitational Waves (CGWs)

1) Duration longer than the observational period
2) Small change rate of the frequency
3) Almost constant amplitude

Possible source of CGWs

- Rotating neutron stars (pulsars) e.g. Sieniawska & Bejger (2019)

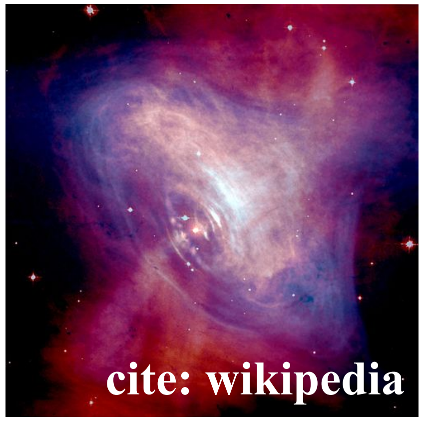
  ➡ **Inner structure of NS**

- Axion cloud around BH ("gravitational atom") e.g. Arvanitaki et al (2010)

  ➡ **Existence of axion particle**

- Small-mass BH binaries e.g. Carr et al (2016)

  ➡ **Existence of primordial BHs**

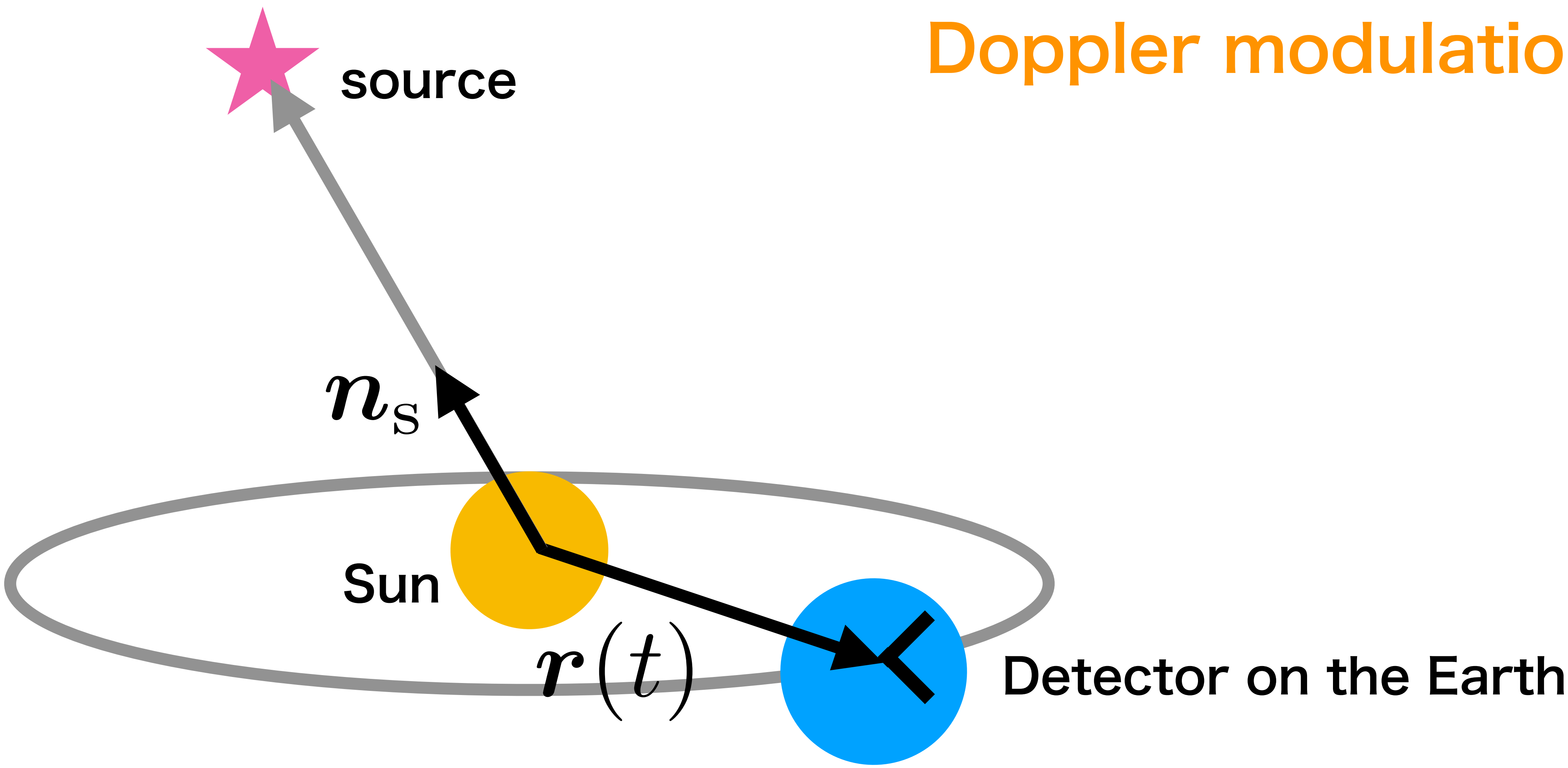


Phase modulation by detector motion

Signal model (monochromatic) $h(t; f_{\rm gw}, \boldsymbol{n}_{\rm s}) = h_0 e^{i\Phi(t; f_{\rm gw}, \boldsymbol{n}_{\rm s})}$

w/ $\Phi(t; f_{\rm gw}, \boldsymbol{n}_{\rm s}) = 2\pi f_{\rm gw} \left(t + \frac{\boldsymbol{r}(t) \cdot \boldsymbol{n}_{\rm s}}{c} \right)$

Doppler modulation



Observed signal depends on the source location.

Matched Filtering (MF)

For all-sky search,
we need calculate SNR for each grid points on the sky.

SNR

$$\rho_{\rm MF}(f_{\rm gw}, \boldsymbol{n}_{\rm g}) \propto \int_0^{T_{\rm obs}} dt \;\; s(t) \cdot h_0 e^{i\Phi(t; f_{\rm gw}, \boldsymbol{n}_{\rm g})}$$

Computational time

$$T_{\rm comp} \sim 2.2 \times 10^8 \, {\rm sec} \left(\frac{P}{\rm 1PFlops} \right)^{-1}$$

(f_{gw}=100Hz, Tobs=10^7sec, sampling rate 1024Hz)

The matched filtering is optimal, but not realistic.

What we did is …

We combine excess power method and deep learning method,
and propose the new detection method of monochromatic waves.

Our method can reduces a computational time
by assisting the coherent method.

Method

Time resampling technique

Jaranowski et al. (1998)

- Use less number of grid points than the case of coherent MF.

$$\Phi(t) = 2\pi f_{\rm gw} t + 2\pi f_{\rm gw} \frac{\boldsymbol{r}(t)\cdot\boldsymbol{n}_{\rm source}}{c}$$

Define the new time coordinate for each grid point

$$\zeta := t + \frac{\boldsymbol{r}(t)\cdot\boldsymbol{n}_{\rm grid}}{c}$$

$$\Phi(t) = 2\pi f_{\rm gw}\zeta + \delta\Phi_{\odot}(t)$$

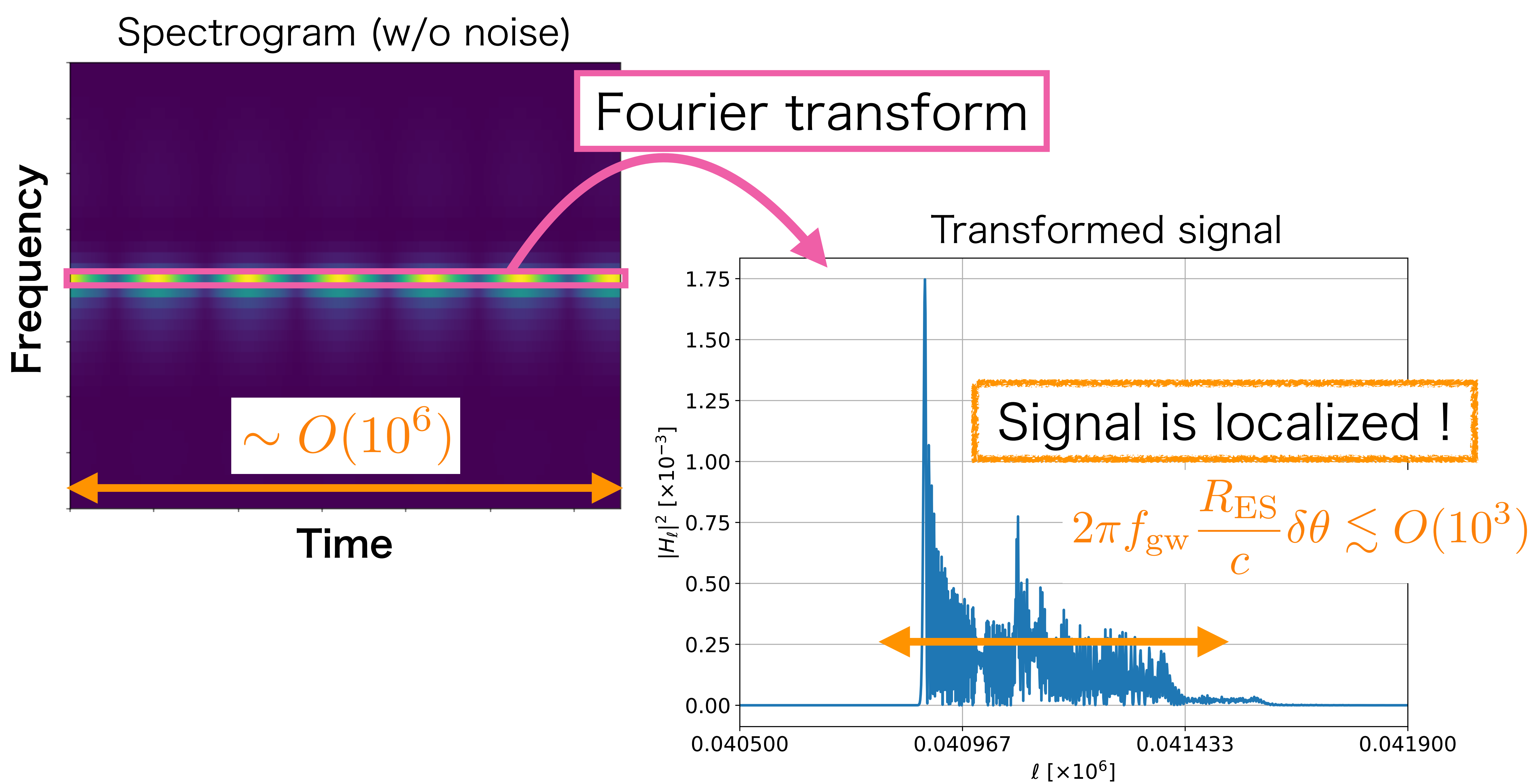
Residual phase

$$\sim 2\pi f_{\rm gw}\frac{R_{\rm ES}\delta\theta}{c}\cos(\Omega_{\odot}\zeta)$$

# of grids $N_{\rm grid} = 352,436$ for $f_{\rm gw} = 100{\rm Hz}$

Method

- Short-time Fourier transform is taken to the resampled strains.
- Fourier transform is carried out for each frequency of STFT.



Method

1. Excess power method

If the signal power is localized within few bins,
one can efficiently find the signal by counting power.

→ **Select candidates**

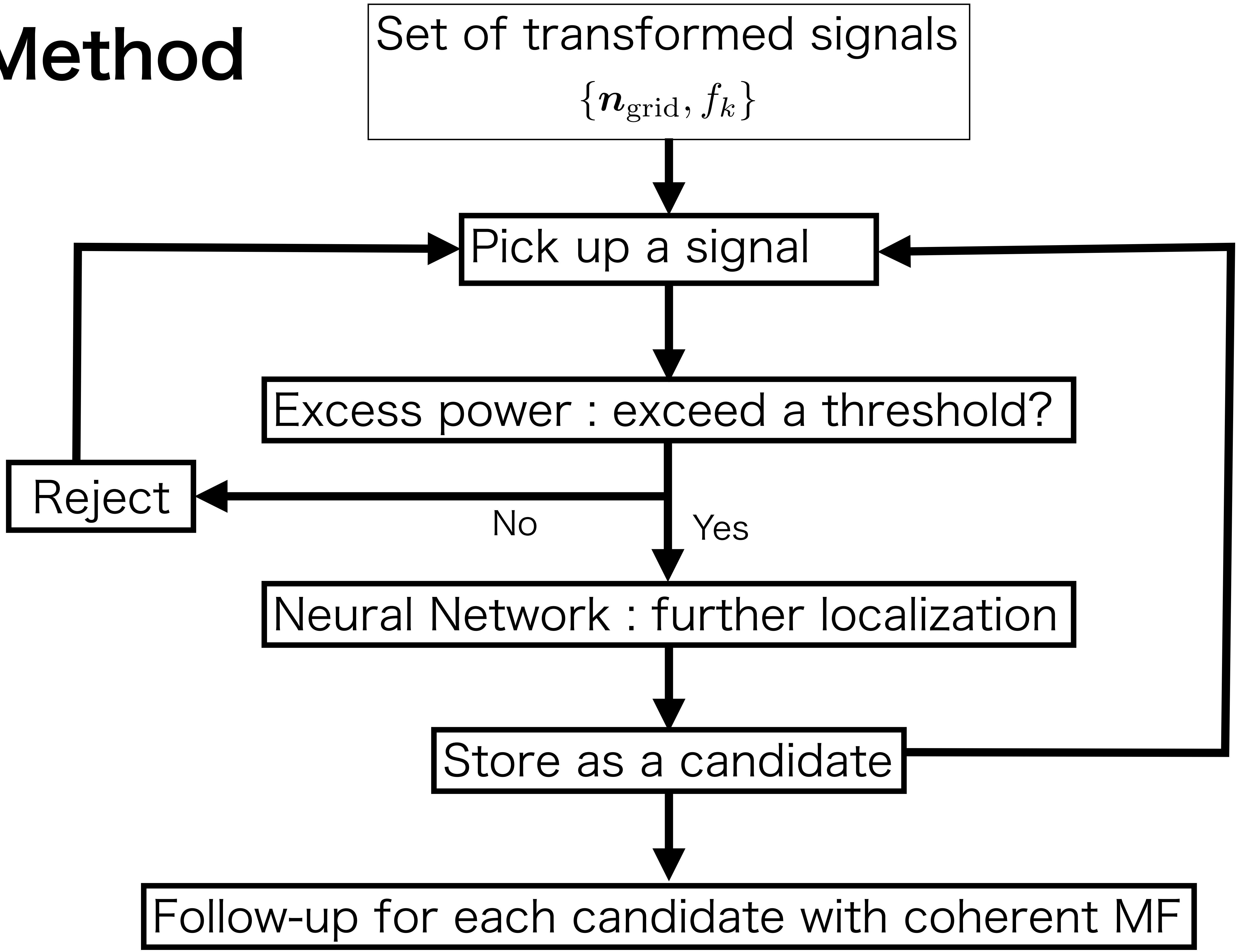
2. Artificial neural network

Using optimized (trained) neural network,
one can find the signal w/o matching with many templates.

→ **Predict the source location**

These methods are computationally cheap.
Combining these methods,
we restrict the possible parameter region
of the source location and the GW frequency
to be searched by the coherent method.

Method


Set of transformed signals
$\{\boldsymbol{n}_{\rm grid}, f_k\}$
Pick up a signal
Excess power : exceed a threshold?
Reject
No
Yes
Neural Network : further localization
Store as a candidate
Follow-up for each candidate with coherent MF

Setup of demonstration

We examine our method for a simplified situation,

- Single detector (LIGO Hanford), 100% duty cycle
- Stationary and Gaussian detector noise
- duration 10^7 sec

The efficiency of the excess power method and the follow-up analysis is examined by analytical calculations.

Neural network is trained and tested with signals of limited parameter range.

- $|f_{\rm gw} - 100\ {\rm Hz}| \leq (T_{\rm seg})^{-1}$
- Source direction is distributed only near the reference grid point.

It's enough to provide the proof-of-principle.
We suppose the assumption that
the neural network is applicable for all-sky search and for f_{gw} < 100 Hz
with the fine-tuning.

Results: Excess power method

- The false alarm probability (FAP) is estimated analytically.

$$\hat{h}_0 := h_0 \left(\frac{S_{\rm n}(f_{\rm ref})}{1{\rm Hz}^{-1}} \right)^{-1/2}$$

- Sensitivity is quantified by $\hat{h}^{95\%}$,
  the minimum amplitude which can be detected with 95% detection probability.

- Benchmark: the coherent MF with # of false positives < O(1).

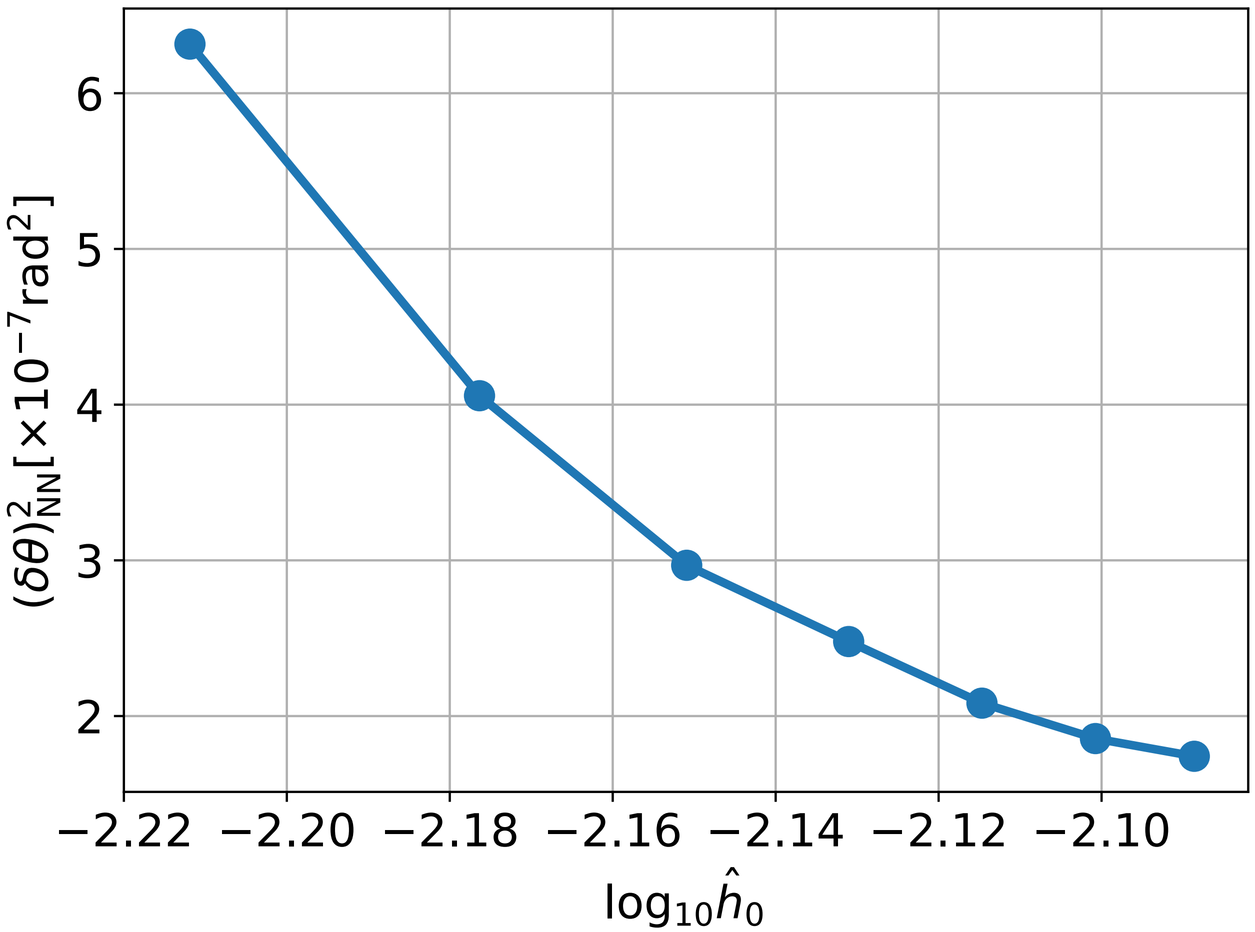
$$\Rightarrow {\rm FAP}_{\rm MF} \sim 4.6 \times 10^{-21}$$

- The sensitivity of the coherent MF $\log_{10} \hat{h}^{95\%}_{\rm MF} = -2.27286$

${\rm FAP}_{\rm EP}$	$\hat{\rho}_{\rm EP}$	$\log_{10} \hat{h}^{95\%}_{\rm EP}$	$\hat{h}^{95\%}_{\rm EP} / \hat{h}^{95\%}_{\rm MF}$
10^{-8}	5.61200	-2.08862	1.52843
10^{-7}	5.19934	-2.10075	1.48632
10^{-6}	4.75342	-2.11468	1.43941
10^{-5}	4.26489	-2.13104	1.38618
10^{-4}	3.71902	-2.15092	1.32416
10^{-3}	3.09023	-2.17635	1.24886
10^{-2}	2.32635	-2.21189	1.15073

$\hat{\rho}_{\rm EP}$: threshold for SNR of excess power

Results: Neural network

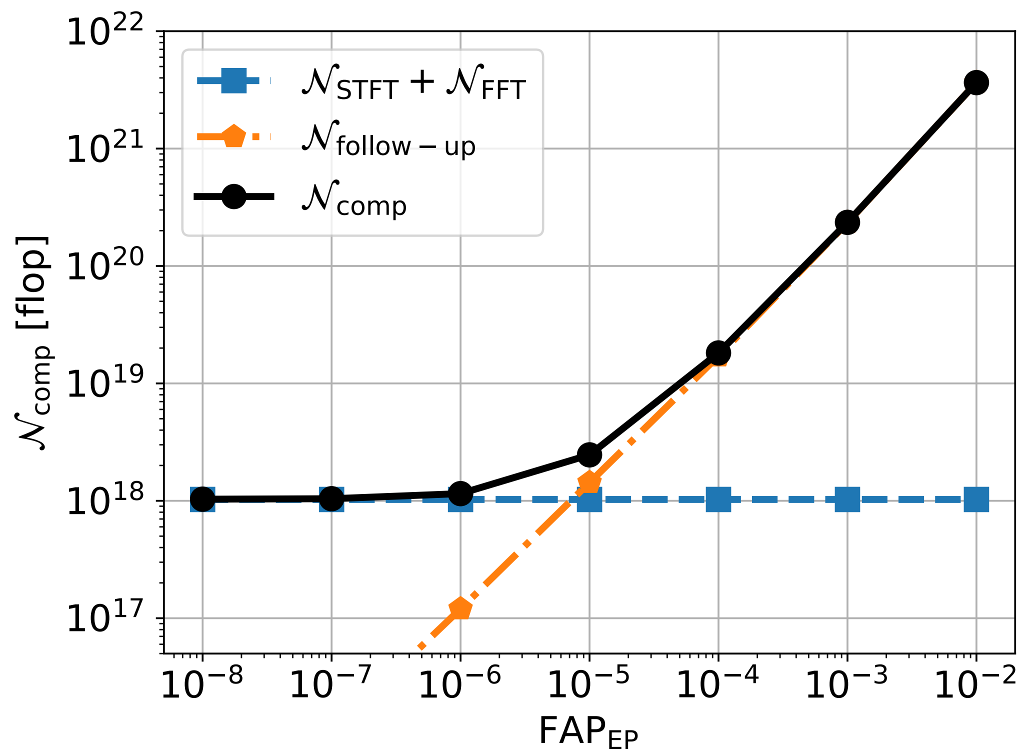


A patch is the region where the reference grid point covers.

Area of a patch ~ 6.35 x 10^{-5} [rad^2]

Our neural network can reduce the computational cost of the follow-up coherent analysis **by 10^{-2}**.

Results: Cost vs Sensitivity

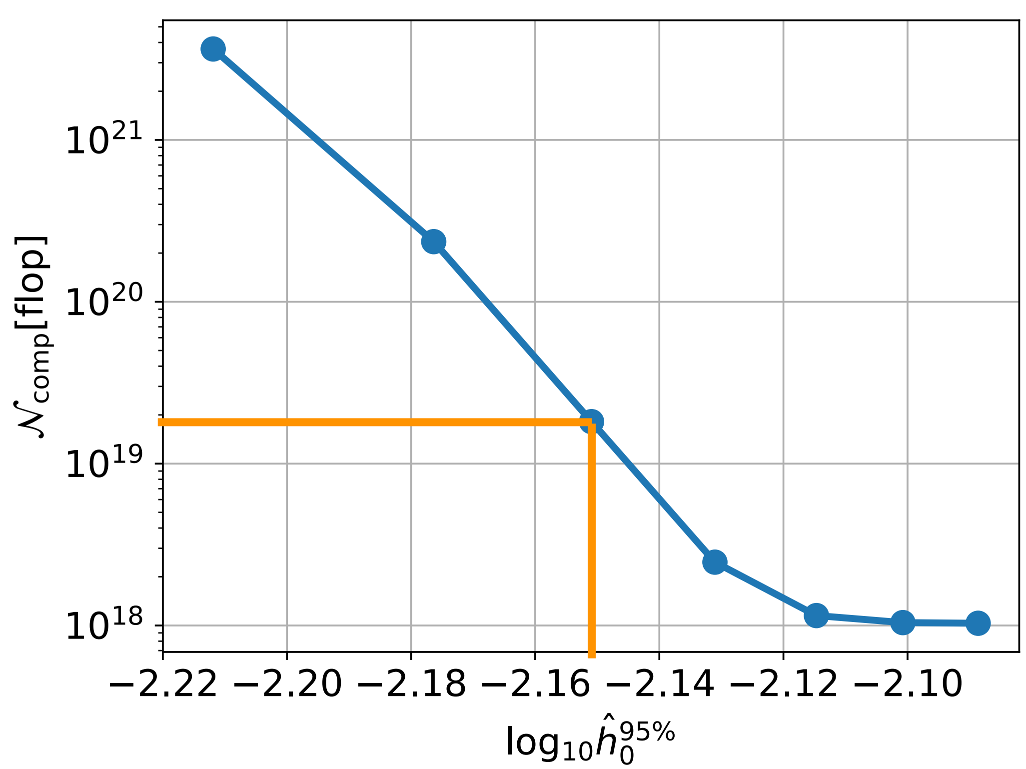


- Total computational cost is estimated by

$$\mathcal{N}_{\rm comp} = \mathcal{N}_{\rm STFT} + \mathcal{N}_{\rm FFT} + \mathcal{N}_{\rm follow\text{-}up}$$

transform the strain follow-up

- Computational costs of excess power and neural network are negligibly small.



- If we have 1TFlops computational power, $N_{comp} \sim 10^{19}$ is acceptable.

- FAP~10^{-4} is reasonable.

Detectable amplitude is only 32% larger than MF.

Conclusion

We proposed the new search method of CGWs.

With some simplification, we assessed the ability of our method.

We show that our method can detect larger signals
than that of the coherent matched filtering only by ~32%
with reasonable computational resources.

Our setup of this work seems to be too simplified,
but it's enough to show a proof-of-principle.

It is expected that our method is applicable to all-sky search
of monochromatic wave of $f_{gw} < 100$ Hz.
To apply for all-sky unknown search,
we need to sophisticate our method (especially ANN).