Master thesis :

Parametric Instability in KAGRA large scale cryogenic gravitational wave telescope

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UNIVERSITY OF TOYAMA

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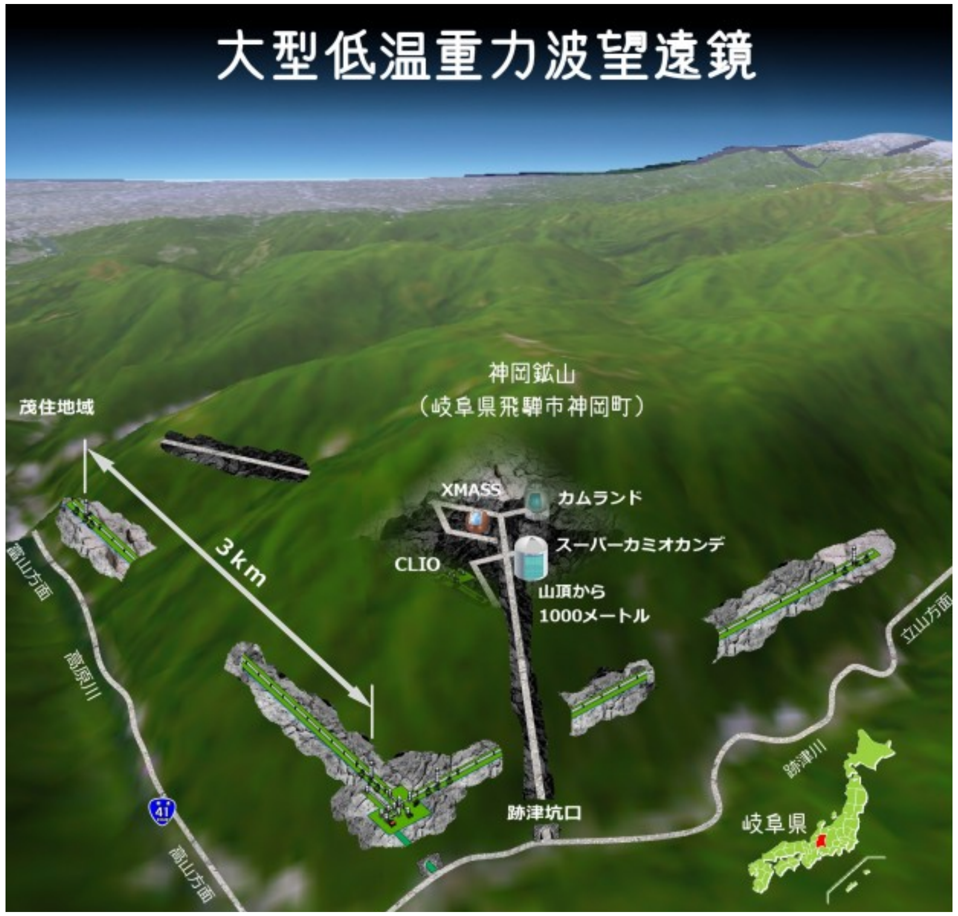
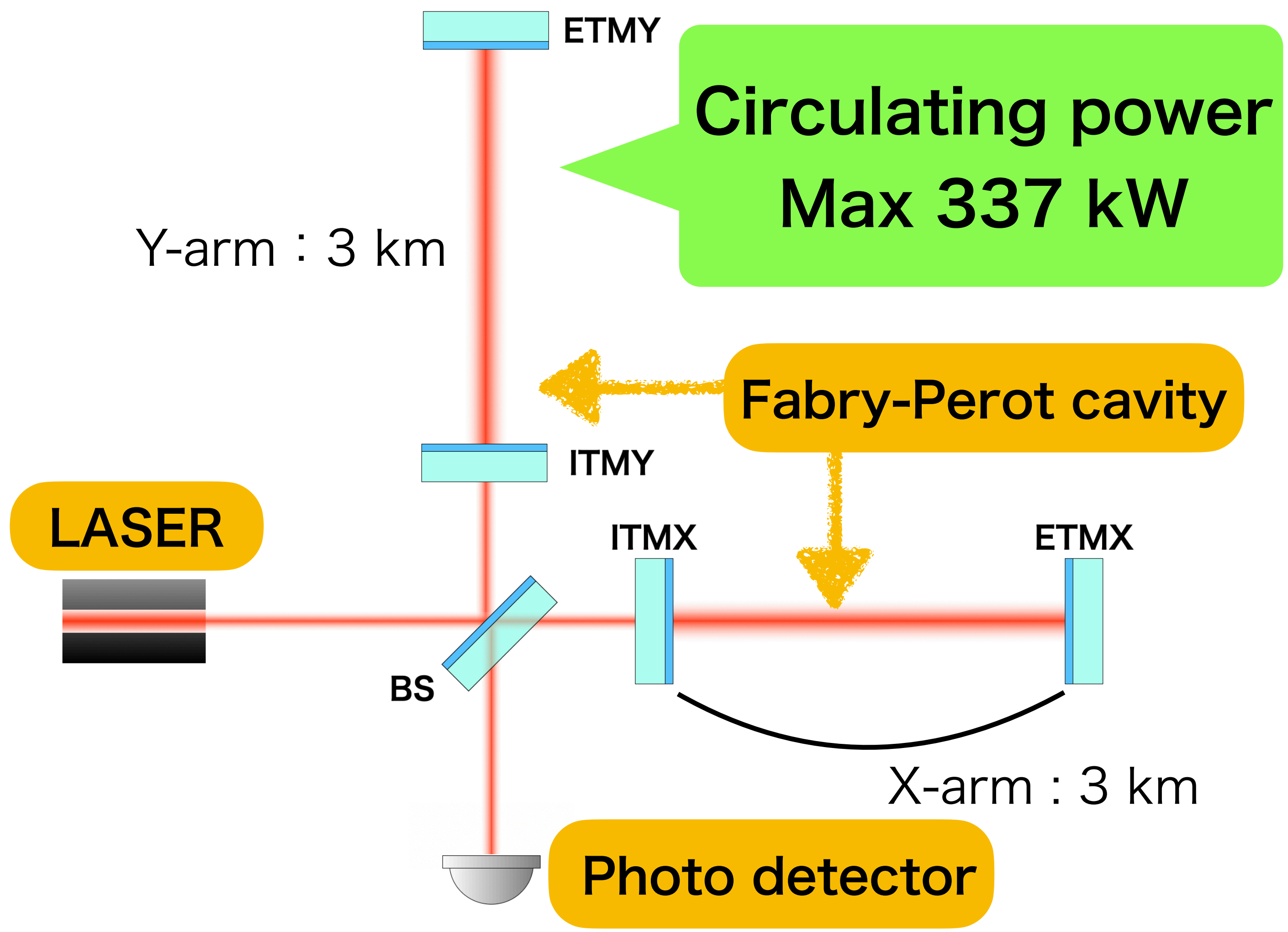
Outline of KAGRA



Gifu, Japan

- Located in underground
- Sapphire mirrors are cooled down to the cryogenic temperature (20 K)



Michelson interferometer

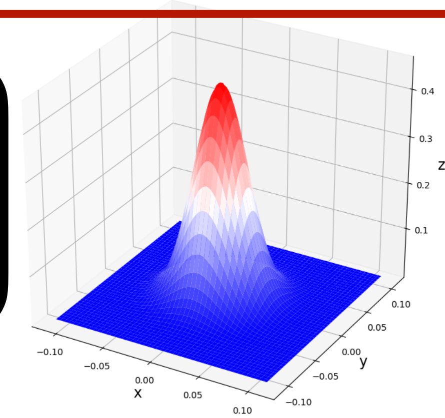
+Fabry-Perot cavity

Cavity length 3 km

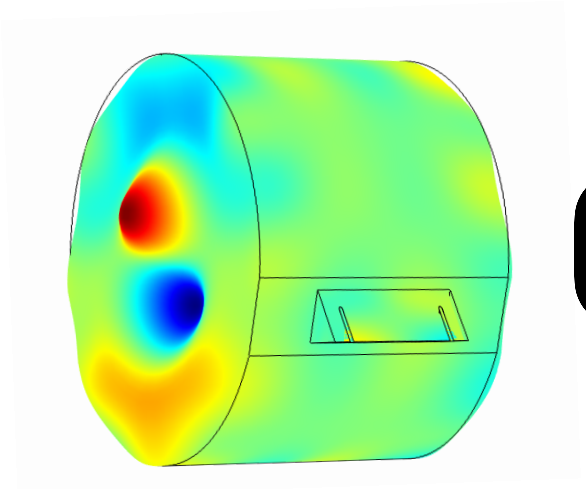
Contents

What is Parametric Instability ?

Main mode
Gaussian (TEM$_{00}$)



Elastic modes



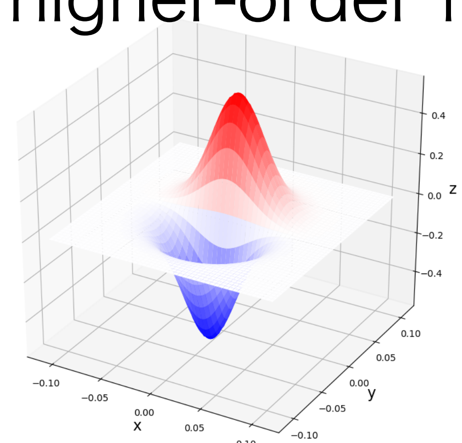
①Light other than Gaussian beam is generated by the elastic modes of the mirror.

Excite each other

③Radiation pressure of higher-order mode excites the elastic modes.

②Light other than Gaussian beam resonates when the frequency and mode shape of this light match that of the higher-order mode of the cavity.

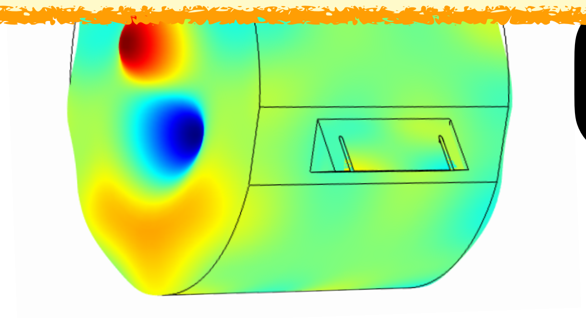
Higher order modes of cavity



What is Parametric Instability ?

When the elastic modes of mirror and higher-order mode of the cavity excite each other largely, the stability of the interferometer operation is affected.

→ **Parametric Instability (PI)**



Elastic modes

①Light other than Gaussian beam is generated by the elastic modes of the mirror.

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②Light other than Gaussian beam resonate when the frequency and mode shape of this light match that of the higher-order mode of the cavity.

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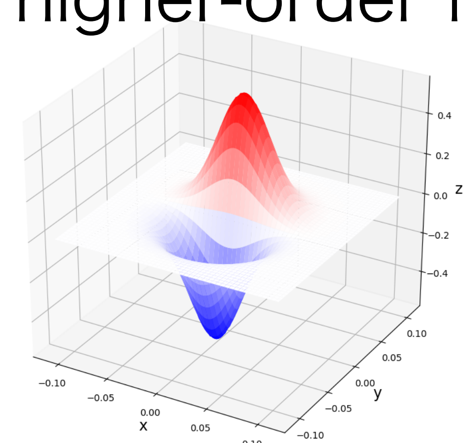
Higher order modes of cavity



What is Parametric Instability ?

The gain of this loop is called parametric gain.

Parametric gain

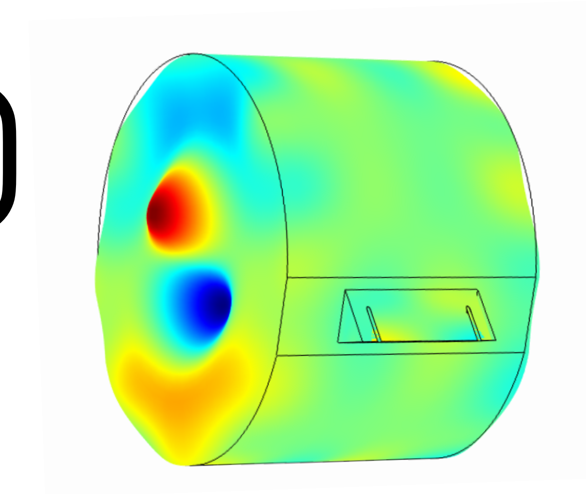
$$\mathscr{R} = \frac{4PQ_mQ_o}{McL\omega_m^2}\frac{\Lambda}{1+\Delta\omega^2/\delta_0^2}$$

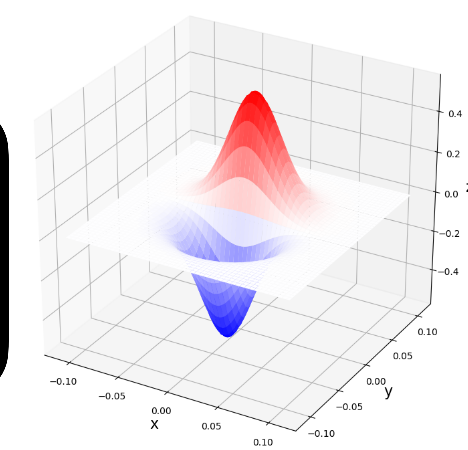
- L : Cavity length
- M : Mass of the mirror
- P :Circulating power
- Q_m :Q-value of mirror
- Q_o :Q-value of cavity
- ω_m : Elastic mode angular frequency of mirror

Elastic modes

Excite each other

Higher order modes of cavity





If it is larger than unity, the resonance state is unstable.

What is Parametric Instability ?

Parametric gain

$$\mathscr{R} = \frac{4PQ_mQ_o}{McL\omega_m^2}\frac{\Lambda}{1+\Delta\omega^2/\delta_0^2}$$

Overlap factor

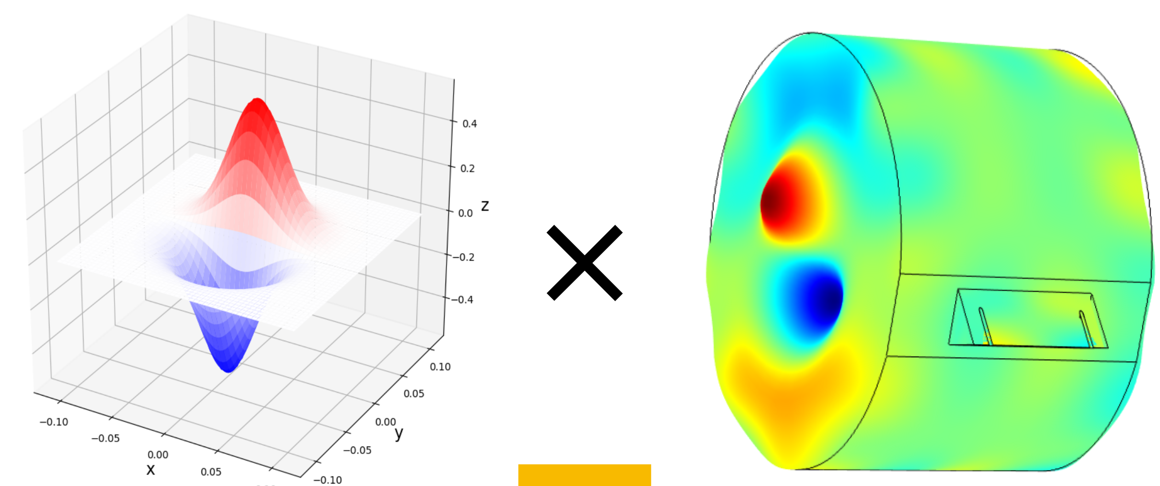
The overlap between the shapes of the elastic mode of the mirror and the higher-order mode of light.

Circulating power of Fabry-Perot cavity

In the interferometric gravitational wave detector, high power laser is adopted in order to suppress the shot noise.

→ Parametric gain increases

Higher-order mode Elastic mode



Overlap factor

? Λ

What is Parametric Instability ?

Parametric gain

$$\mathscr{R} = \frac{4PQ_mQ_o}{McL\omega_m^2}\frac{\Lambda}{1+\Delta\omega^2/\delta_0^2}$$

The parametric gain is large when the detuning between the frequency of light due to elastic vibration and the resonance frequency of the higher-order mode of the cavity is small.

$$\Delta\omega = \omega_0 - \omega_{\mathrm{HG}} - \omega_m$$

ω_0 : Resonant angular frequency in main mode of the cavity

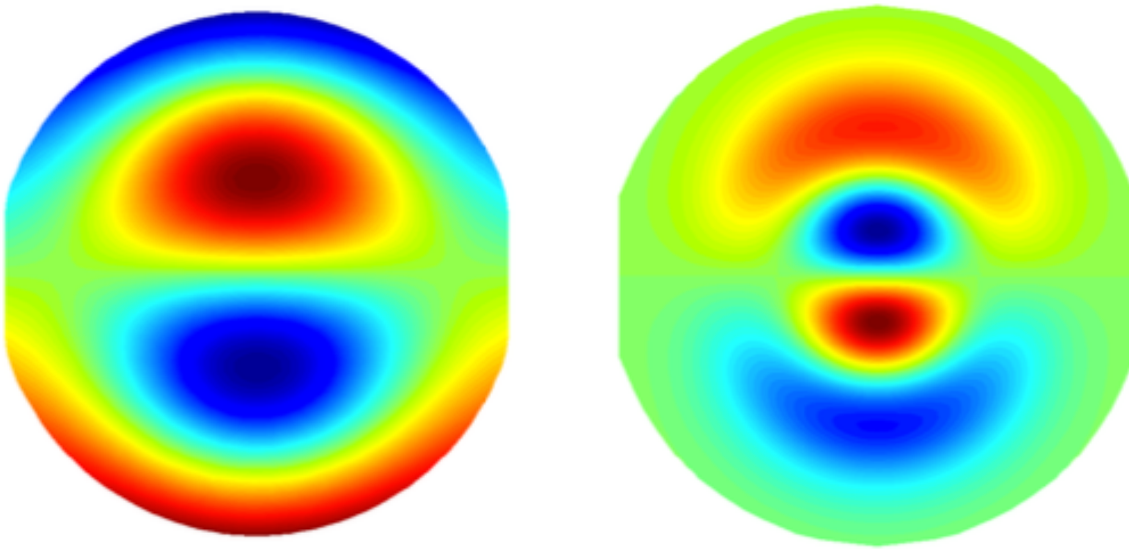
ω_{HOM} : Resonant angular frequency in higher-order mode of the cavity

ω_m : Resonant angular frequency of elastic mode

What is Parametric Instability ?

In Advanced LIGO, Parametric Instability actually appeared.

Elastic mode　Higher-order mode



In Advanced LIGO, the elastic(left panel) and optical(right panel) modes causes PI.

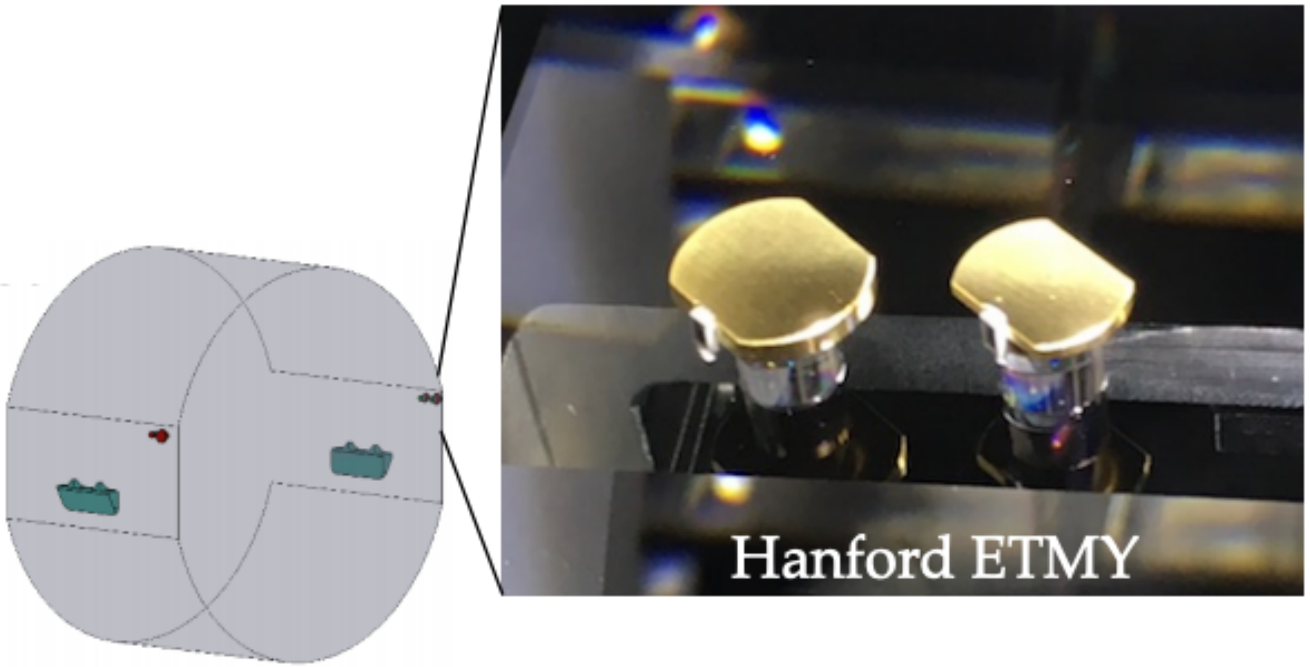
Frequency of elastic vibration
→**15.5 kHz**
Higher-order mode
→**Third order mode**

$$\Lambda = 0.1$$

$$\mathscr{R} = 2$$

Solution

A damper is attached to the mirror in order to suppress elastic vibration.



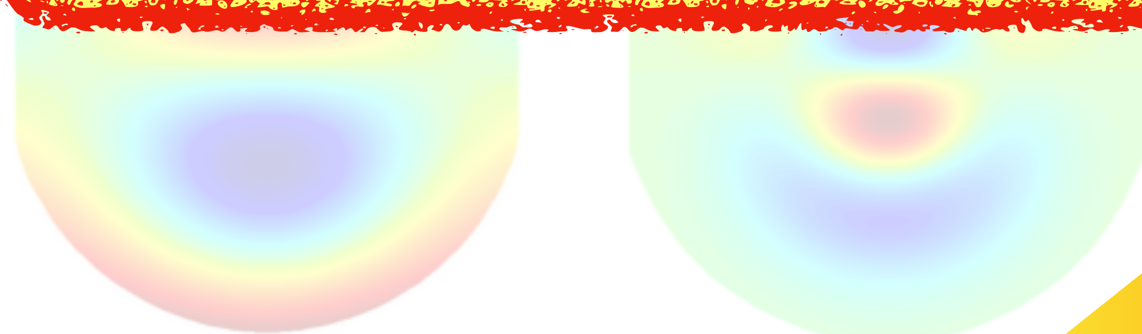
Reference : GWADW, 23 May 2019 LIGO-G1900963

What is Parametric Instability ?

In Advanced LIGO, Parametric Instability actually appeared.

Advanced LIGO overcame PI.

In 2015, First detection of the gravitational wave



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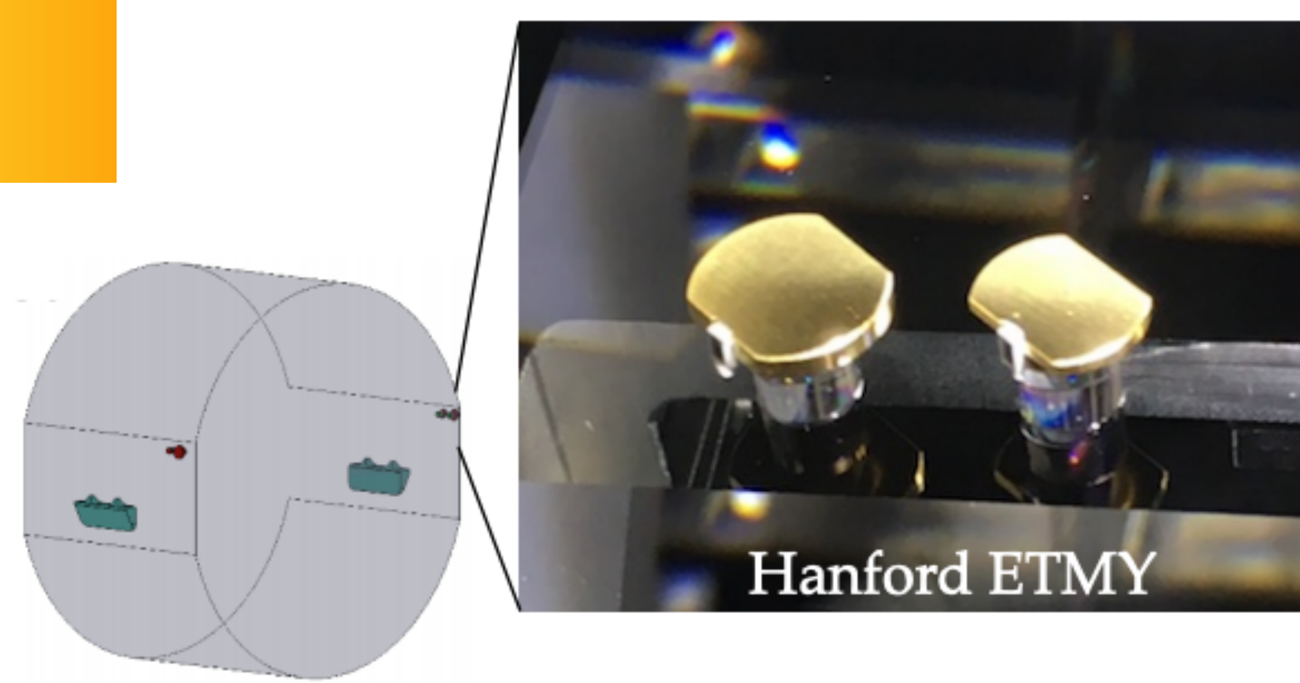


Reference : GWADW, 23 May 2019 LIGO-G1900963

What is Parametric Instability ?

In KAGRA

Since KAGRA and LIGO have different parameters and conditions, parametric instability does not occur at similar frequencies and mode shapes.

Parameter	LIGO	KAGRA
Cavity length	**4000 m**	**3000 m**
Mirror material	Fused silica	Sapphire
Mass of mirror	40 kg	23 kg
Beam radius at the mirrors	**60 mm**	**35 mm**

What is Parametric Instability ?

In KAGRA

In KAGRA, Parametric Instability was discussed 12 years ago with design values at the planning stage.

(Journal of Physics: Conference Series 122 (2008) 012015)

As a result, the estimated total number of unstable modes of KAGRA is about one tenth that of LIGO.

What is Parametric Instability ?

In KAGRA

However, the countermeasures for KAGRA were not considered.
In addition, we changed mirror design in last 12 years.
Therefore, we start to investigate KAGRA parametric instability again.

	12 years ago		Current
Mass of mirror	**30 kg**	→	**23 kg**

In our study, we take the measured KAGRA cavity lengths into account.

X-arm ： 2999.990 m
Y-arm ： 3000.015 m
(Reference:klog-7332,9505)

Contents

Simulation and results

It is necessary to know the elastic mode of the mirror in order to find the mode which are unstable in KAGRA.

→We simulate the elastic mode with Finite Element Method software COMSOL

Parameter

- Mass of mirror $M = 23\ \mathrm{kg}$
- Radius of curvature mirror $R = 1900\ \mathrm{m}$
- Wave length of light $\lambda = 1064 \times 10^{-9}\ \mathrm{m}$
- Finesse of cavity $\mathcal{F} = 1500$
- Circulating power of cavity $P = 337\ \mathrm{kW}$
- Mechanical Q of mirror $Q_m = 1.0 \times 10^{8}$
- Optical Q of cavity $Q_o = \dfrac{2L\mathcal{F}}{\lambda} = 8.46 \times 10^{12}$

Simulation and results

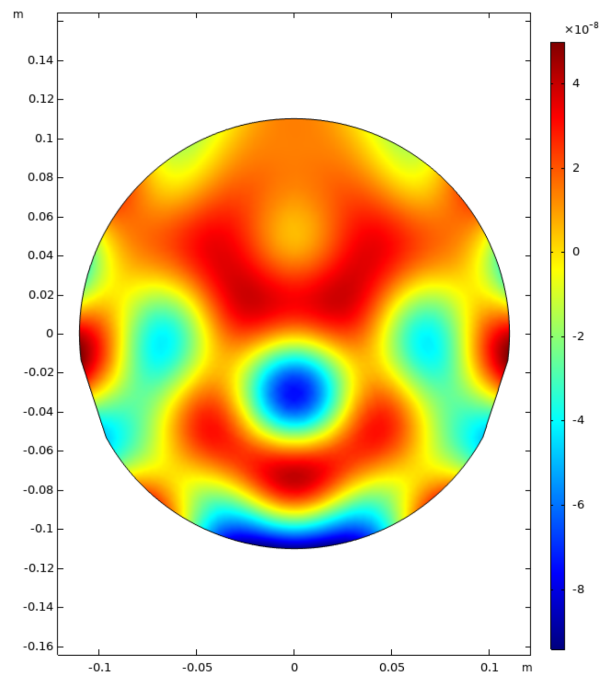
Results **Number of unstable mode**

The number of unstable modes in KAGRA is one for X-arm and one for Y-arm.

The number of unstable modes is comparable with the result shown 12 years ago.

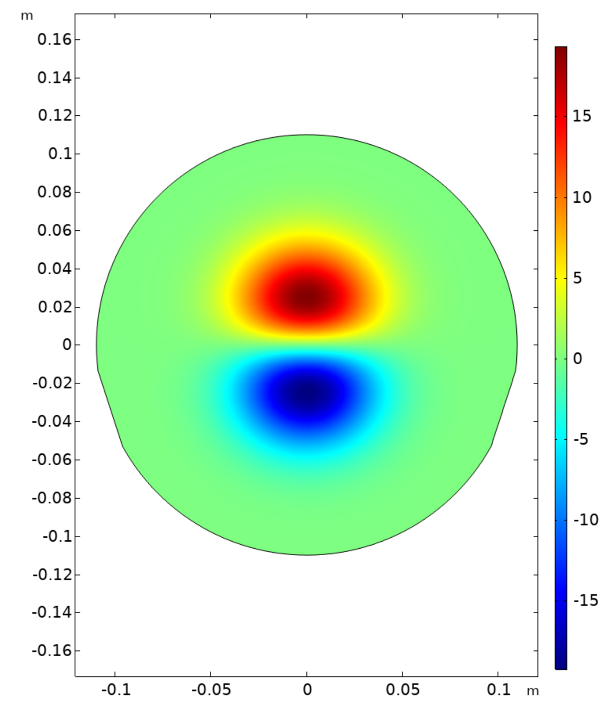
Simulation and results

Results



Elastic mode

84.746 kHz



Higher-order mode

TEM_{01}

at 84.75 kHz in X-arm

at 84.78 kHz in Y-arm

X-arm

$$\Lambda = 0.4$$

$$\mathscr{R} = 66.19$$

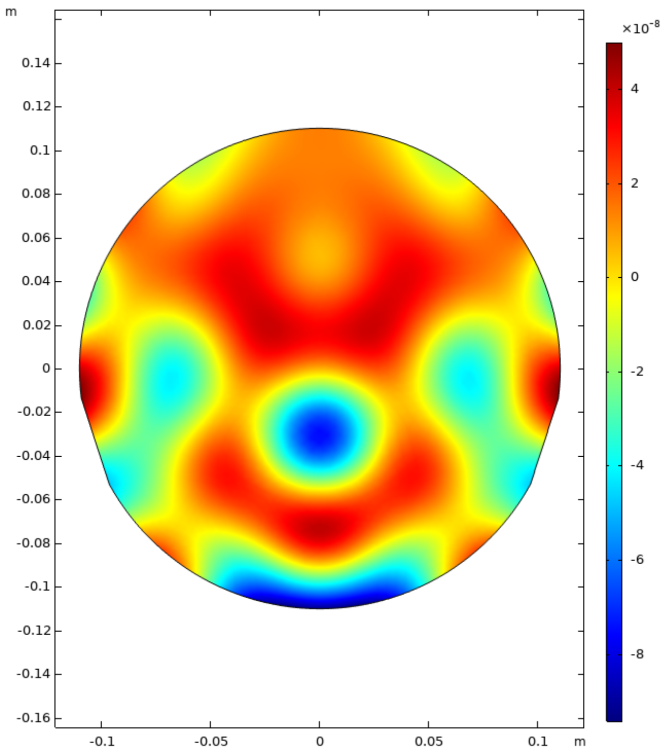
Y-arm

$$\Lambda = 0.4$$

$$\mathscr{R} = 13.49$$

Simulation and results

Results



Elastic mode frequency of mirror
84.746 kHz

the difference in cavity length of about 2.6 cm

×1/5

X-arm

Parametric gain

$$\mathscr{R} = 66.19$$

Y-arm

Parametric gain

$$\mathscr{R} = 13.49$$

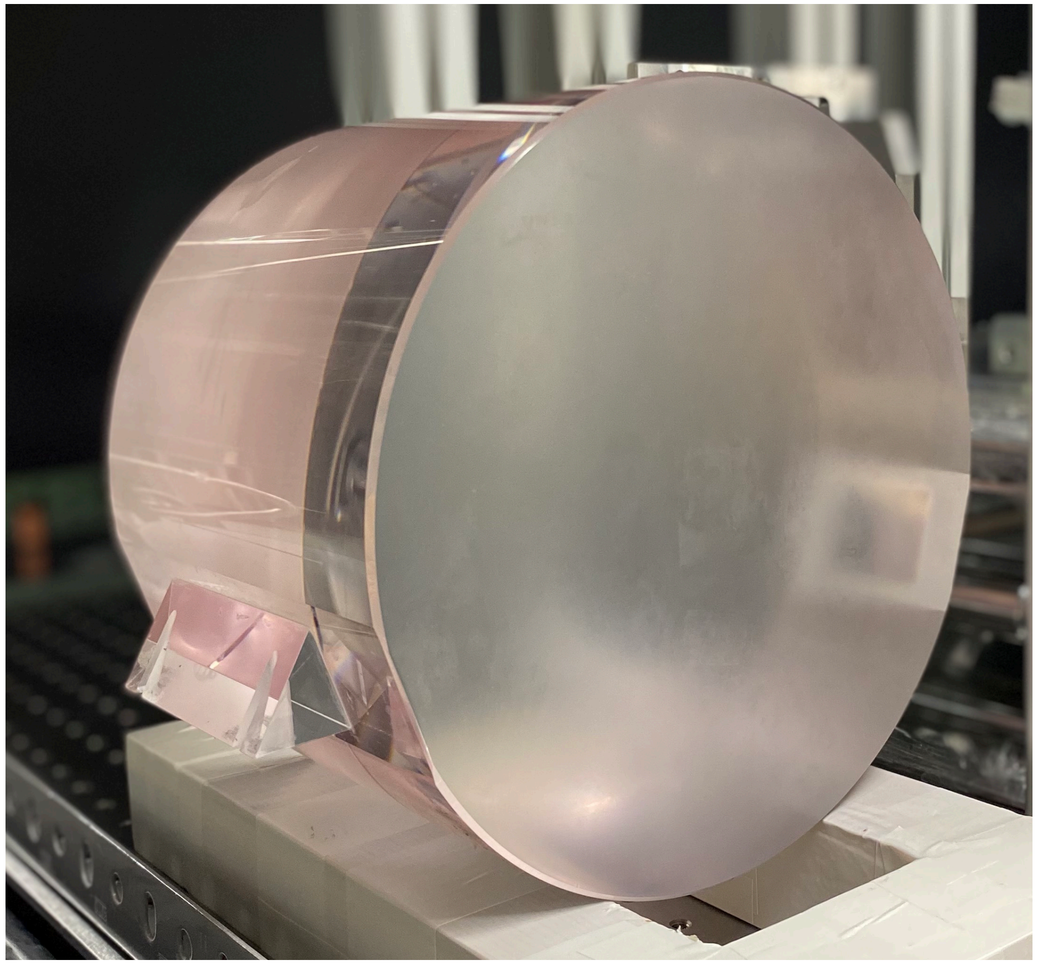
Contents

Experiment

Set up

Experiment to check
the mode shape derived from simulation

Sapphire dummy bulk
for testing in KAGRA



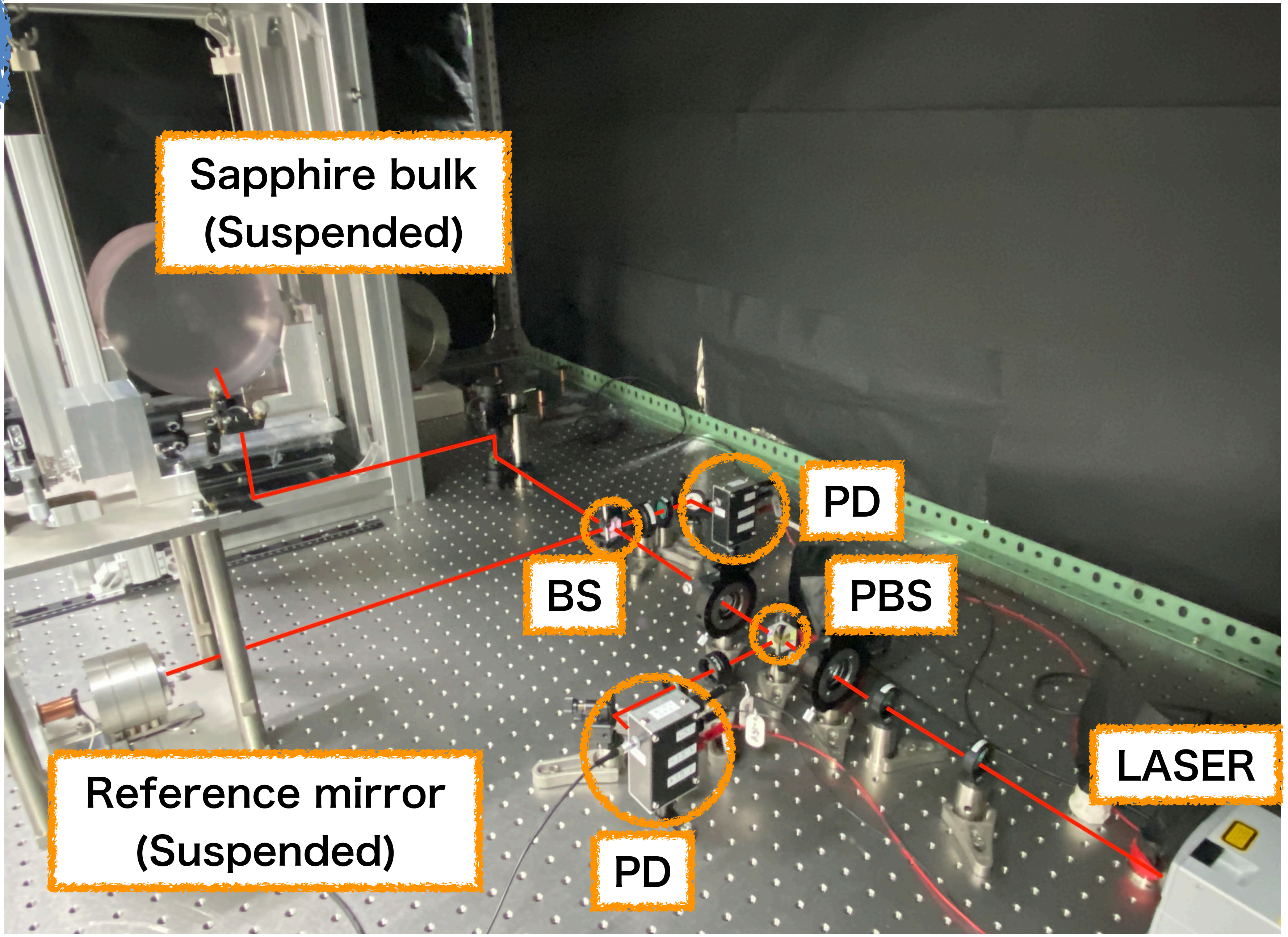
This bulk has the same size
as KAGRA mirror

↓

It is possible to measure the
same elastic mode as those of
the mirror installed in KAGRA.

Experiment

Set up

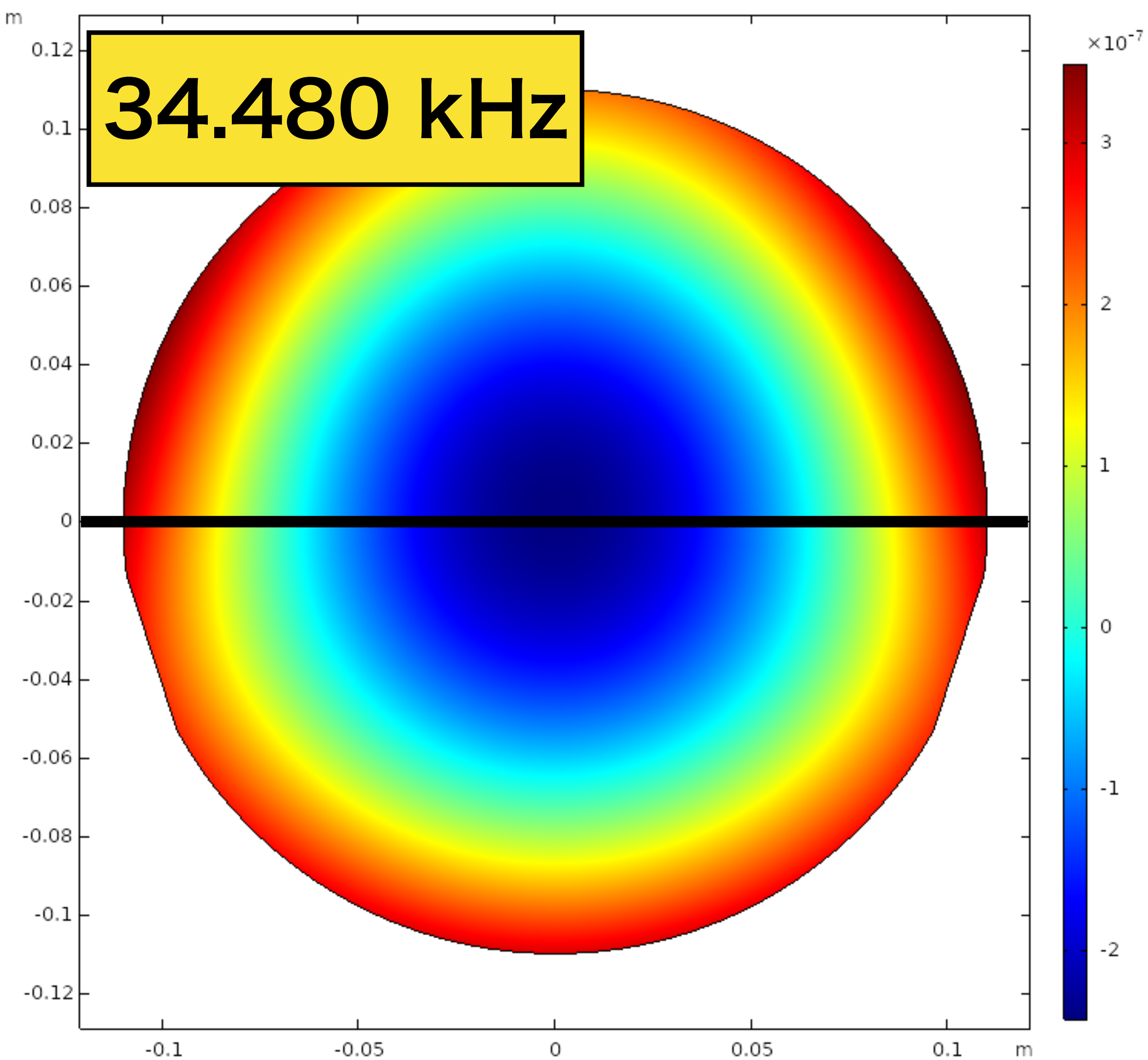


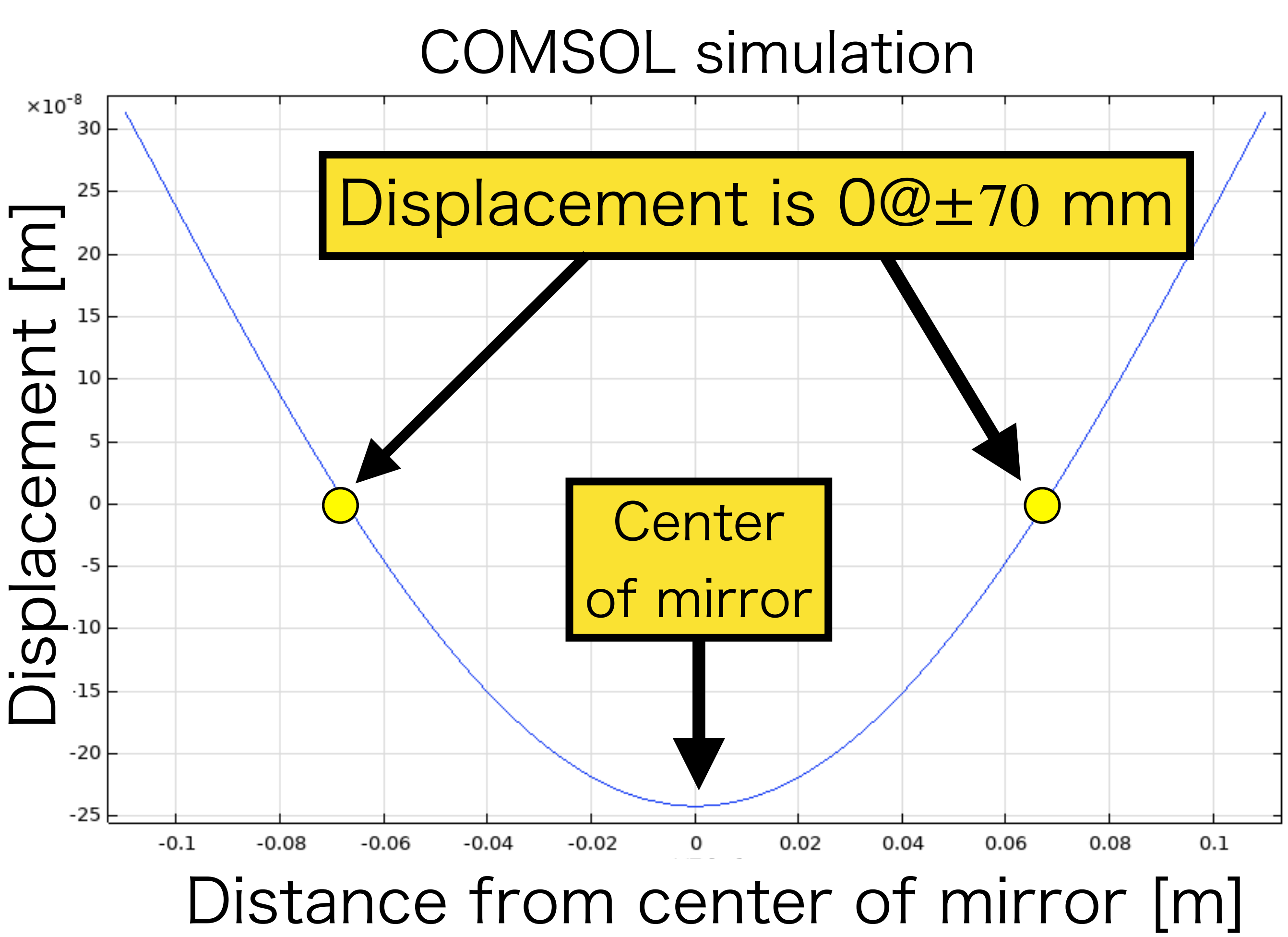
The elastic mode of the bulk is excited, and the shape of the bulk is measured with Michelson interferometer.

Experiment

Measurement

As the first step,
we measured the axisymmetric mode
to check mode shape measurement.

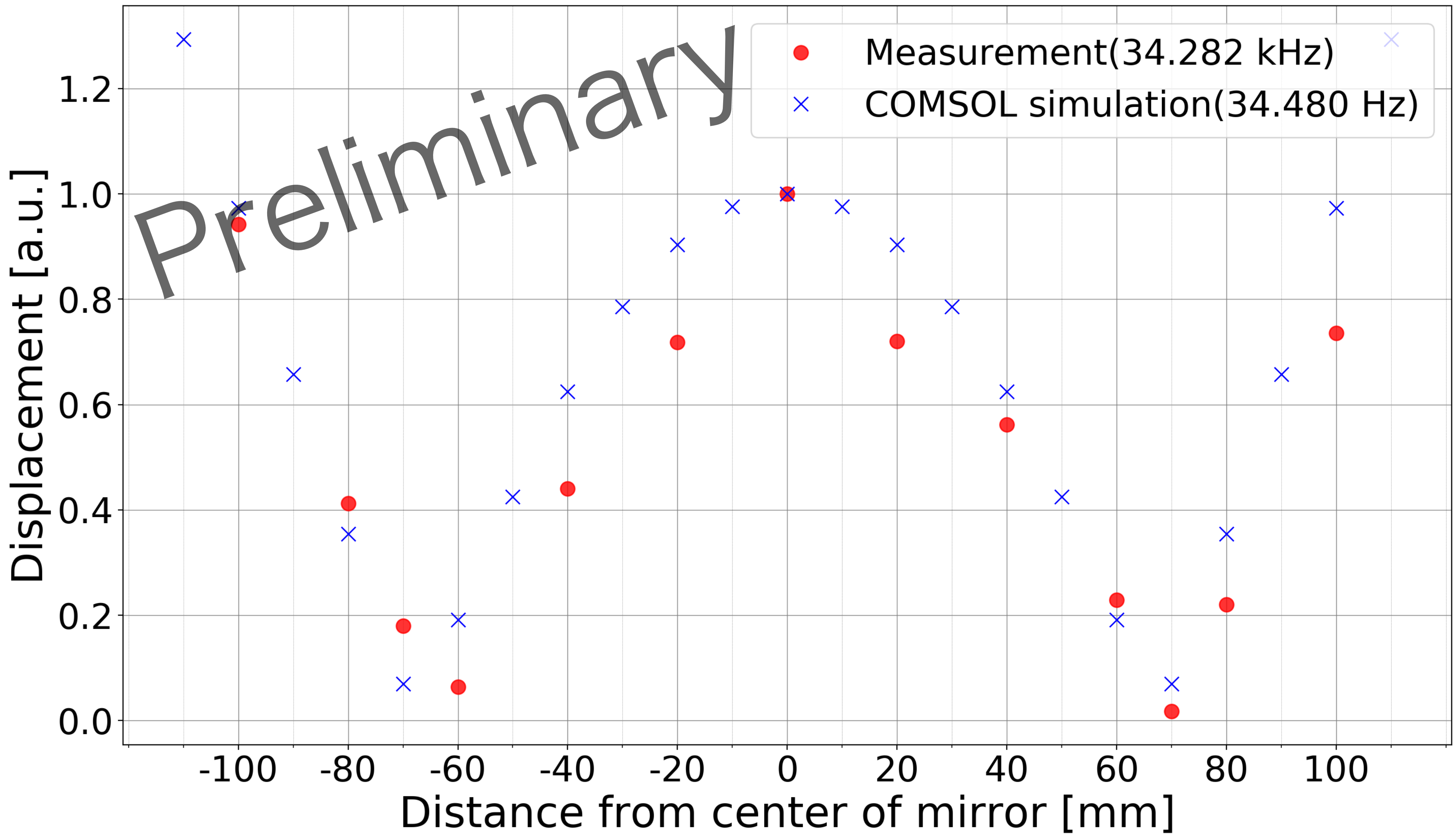




Experiment

Results

As the first step, we measured the axisymmetric mode to check mode shape measurement.



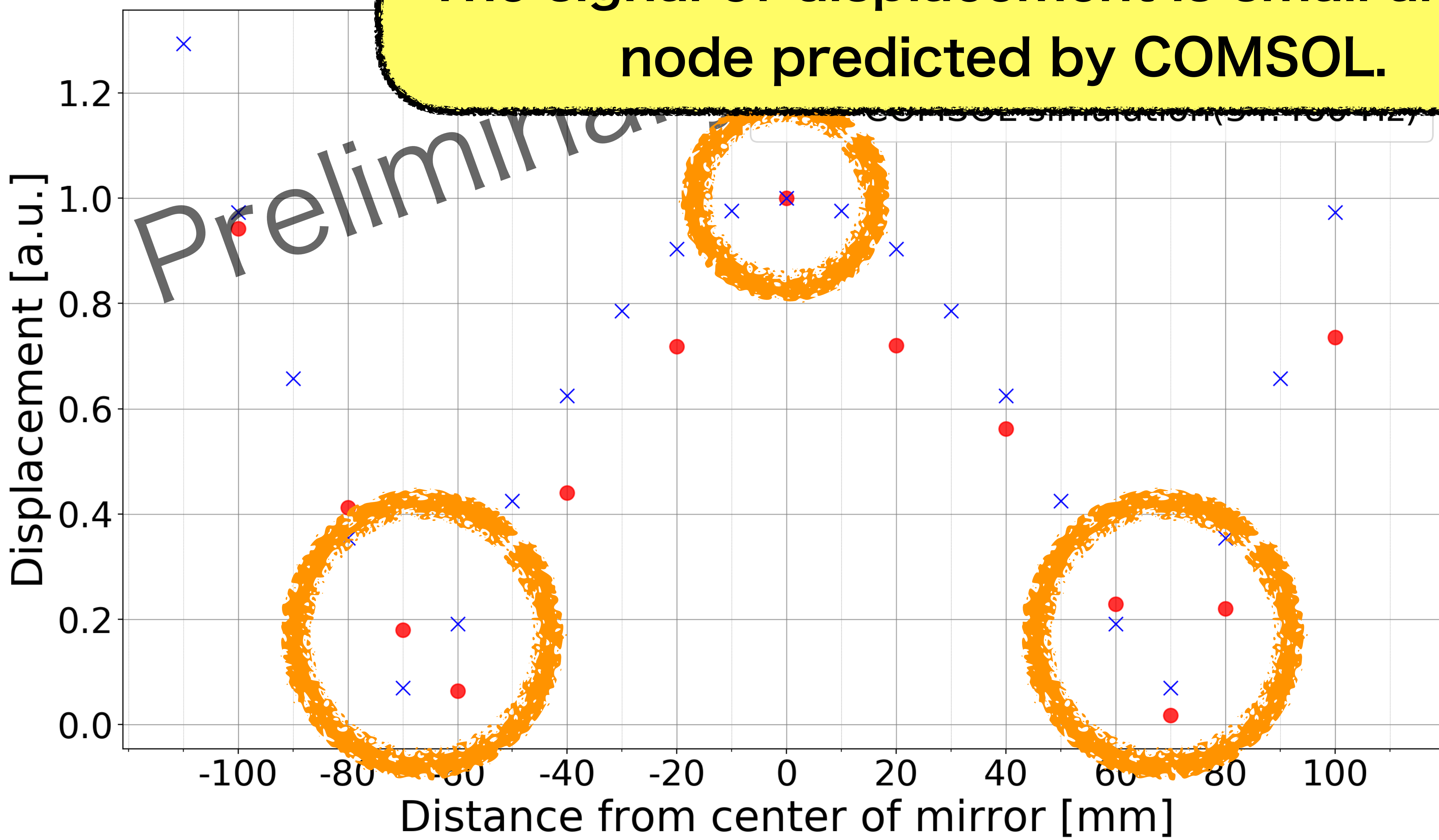
Experiment

Results

As the first step,

we mea[...]

to chec[...]

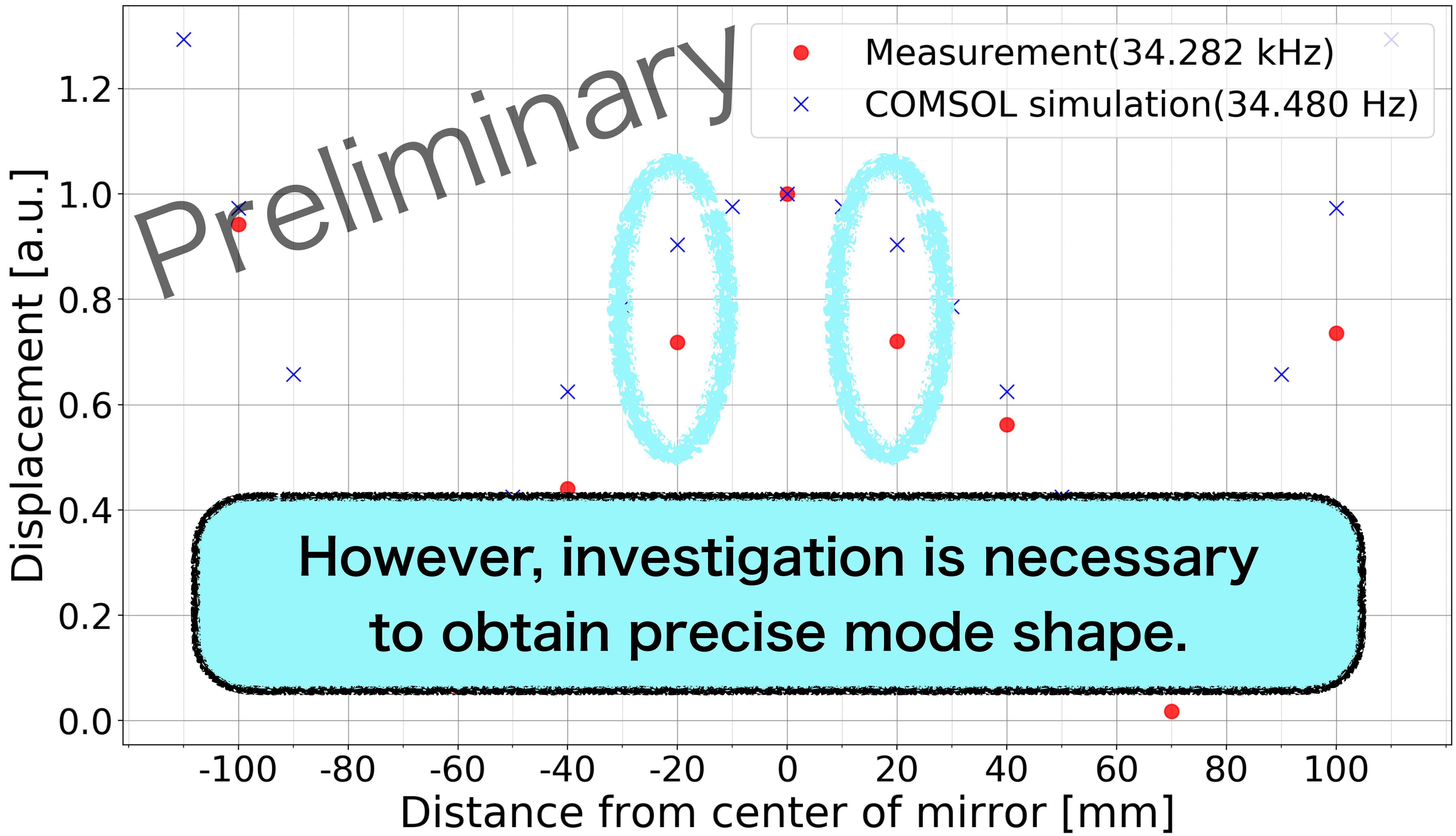


Our result shows that the displacement at the center of mirror is local maximum.
The signal of displacement is small around node predicted by COMSOL.

Experiment

Results

As the first step,
we measured the axisymmetric mode
to check mode shape measurement.



However, investigation is necessary
to obtain precise mode shape.

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Summary and future plan

Summary

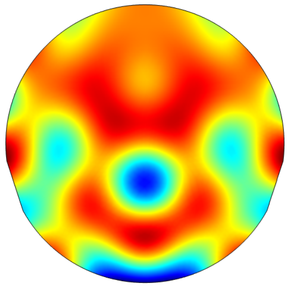
Parametric instability is one of serious issues in stable interferometer operations.

- **We simulated PI with current KAGRA parameters.**

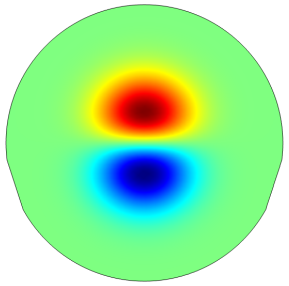
→ The number of unstable modes in KAGRA is one for X-arm and one for Y-arm.

→ The elastic mode of 84.746 kHz and the light of TEM_{01} cause Parametric instability.

Their parametric gain is about 66 and 13 in X and Y-arm, respectively.



Elastic mode



Higher-order mode

- **We proceed with experiment to check simulated elastic mode shape.**

Future plan

How to suppress Parametric Instability in KAGRA.

Thank you for your attention !

Backup slides

- **Shot noise**

Quantum fluctuations
in the number of photons in a photodetector

The amount of noise corresponding
to the displacement of the mirror

$$\delta x = \sqrt{\frac{\hbar c \lambda}{4\pi P}}$$

→ Increasing the power
of the light
reduces the noise.

Transverse mode spacing

$$\nu_{\rm tms} = \frac{\cos^{-1}\sqrt{g_1 g_2}}{\pi} \times f_{\rm FSR} \quad , \quad g_1 = 1 - \frac{L}{R_1} \quad , \quad g_2 = 1 - \frac{L}{R_2}$$

KAGRA

$L = 3000$ m

$R_1 = R_2 = 1900$ m

$f_{\rm FSR} = 49.97$ kHz

$\nu_{\rm tms} = 15.16$ kHz

LIGO

$L = 4000$ m

$R_1 = 1934$ m, $R_2 = 2245$ m

$f_{\rm FSR} = 37.53$ kHz

$\nu_{\rm tms} = 5.072$ kHz

- **Difference in the speed of sound inside the mirror**

  →**The resonance frequency of the mirror changes.**

Fused silica	**6 km/s**
Sapphire	**10 km/s**

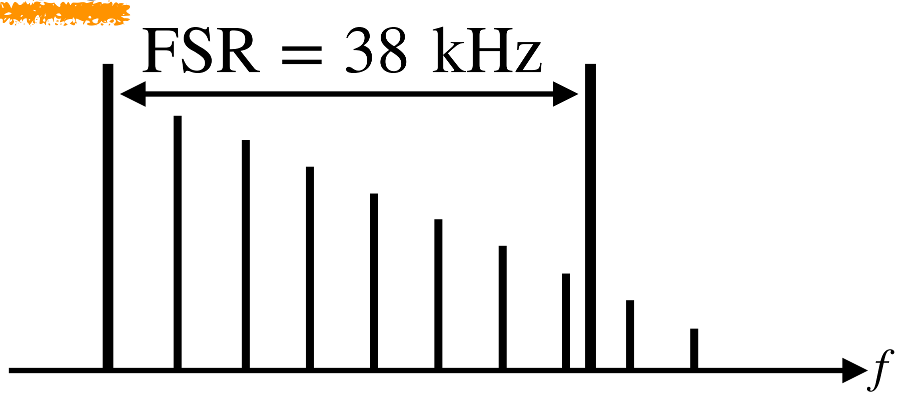
Elastic mode density is inversely proportional to the cube of the speed of sound

$$\frac{\text{KAGRA}}{\text{LIGO}} = \frac{\frac{1}{10^3}}{\frac{1}{6^3}} = \frac{216}{1000} \sim \frac{1}{5}$$

If the thickness of the mirror is the same, the wavelength of the sound wave is the same.
→The resonant frequency is lower the slower the speed of sound.

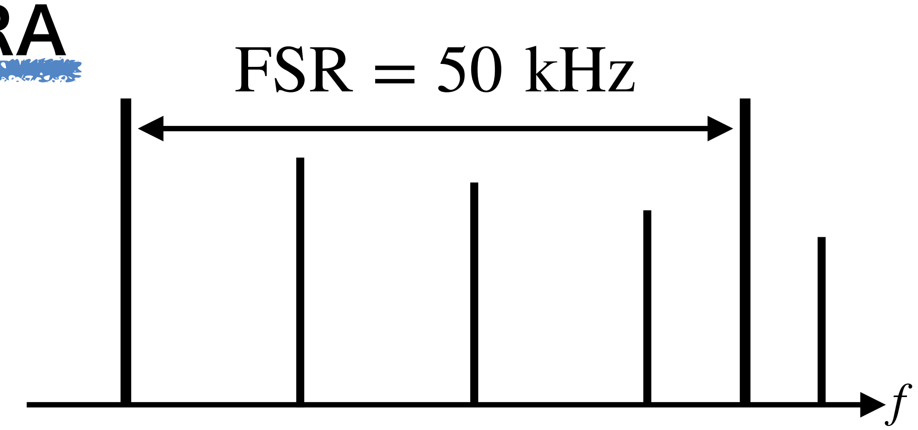
- **Difference in transverse mode spacing**

LIGO



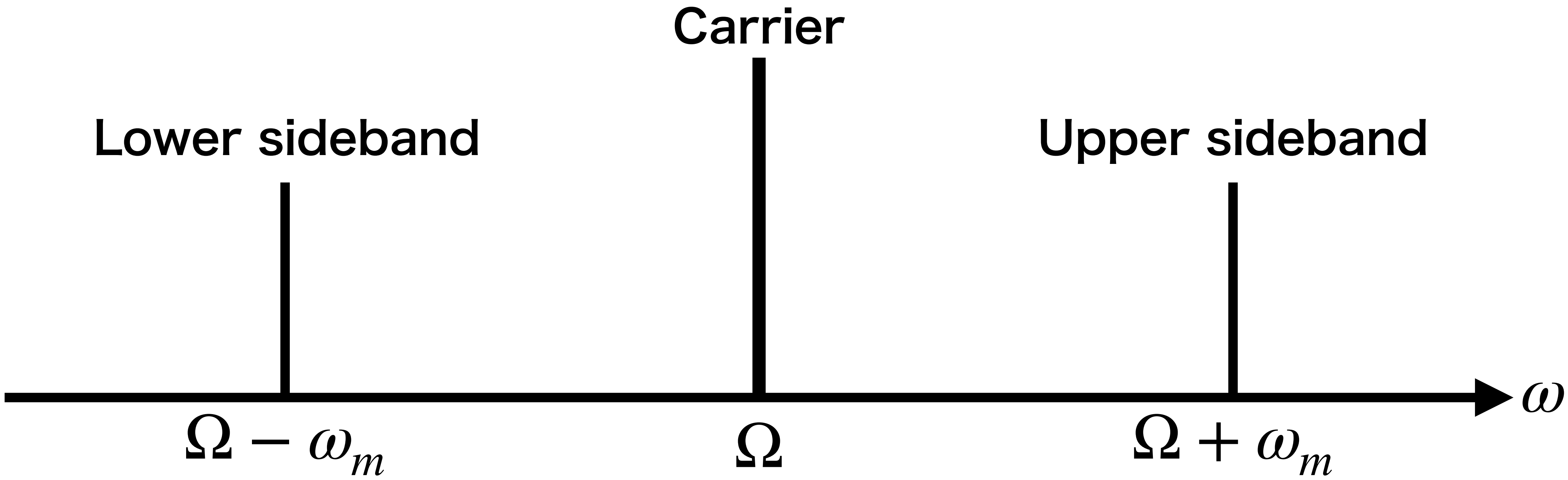
7 transverse modes in FSR

KAGRA

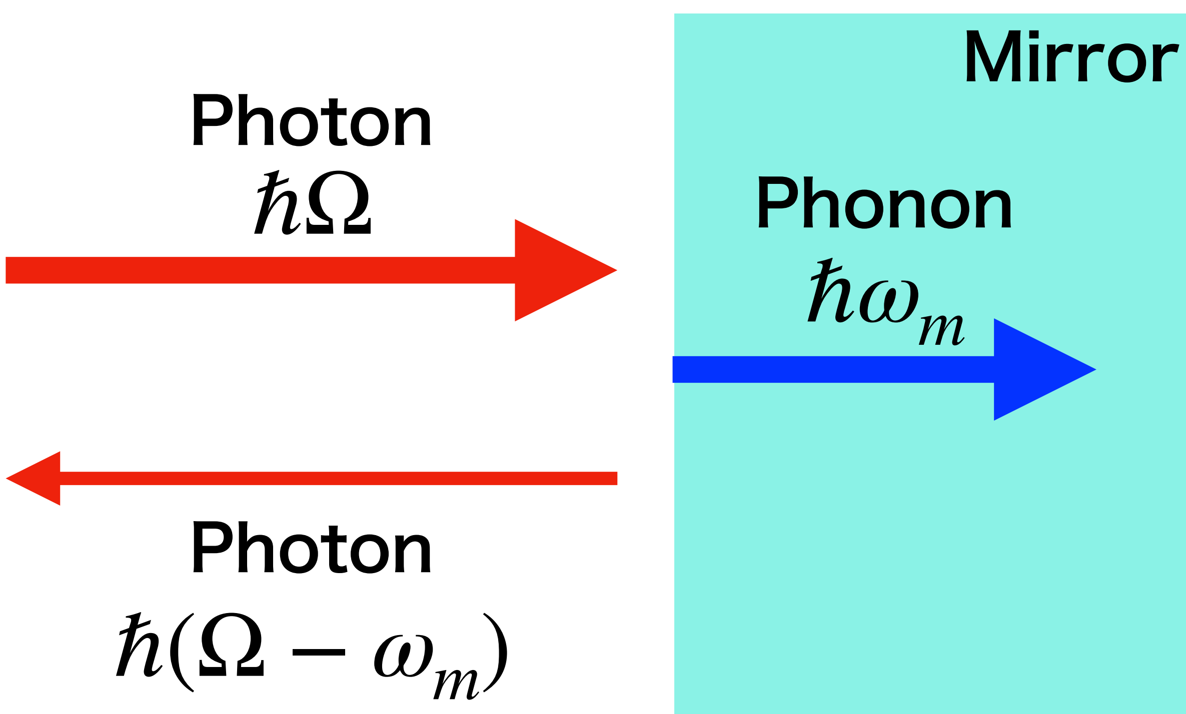


3 transverse modes in FSR

$$\frac{\text{KAGRA}}{\text{LIGO}} \sim \frac{1}{2}$$

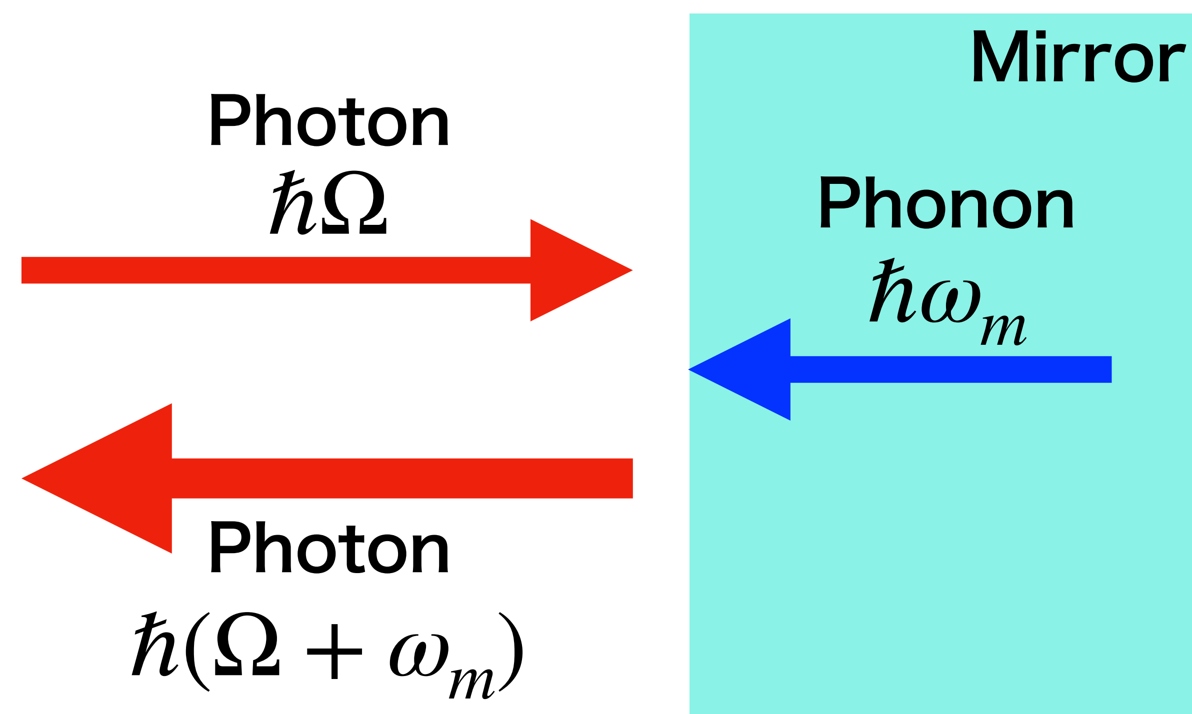


Lower sideband



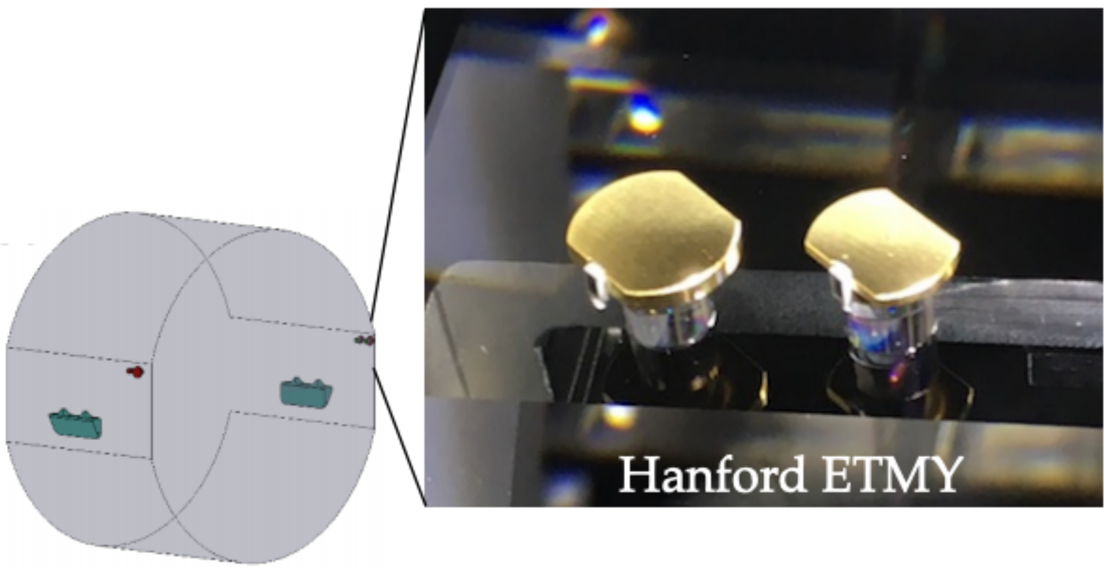
Giving energy to the mirror

Upper sideband

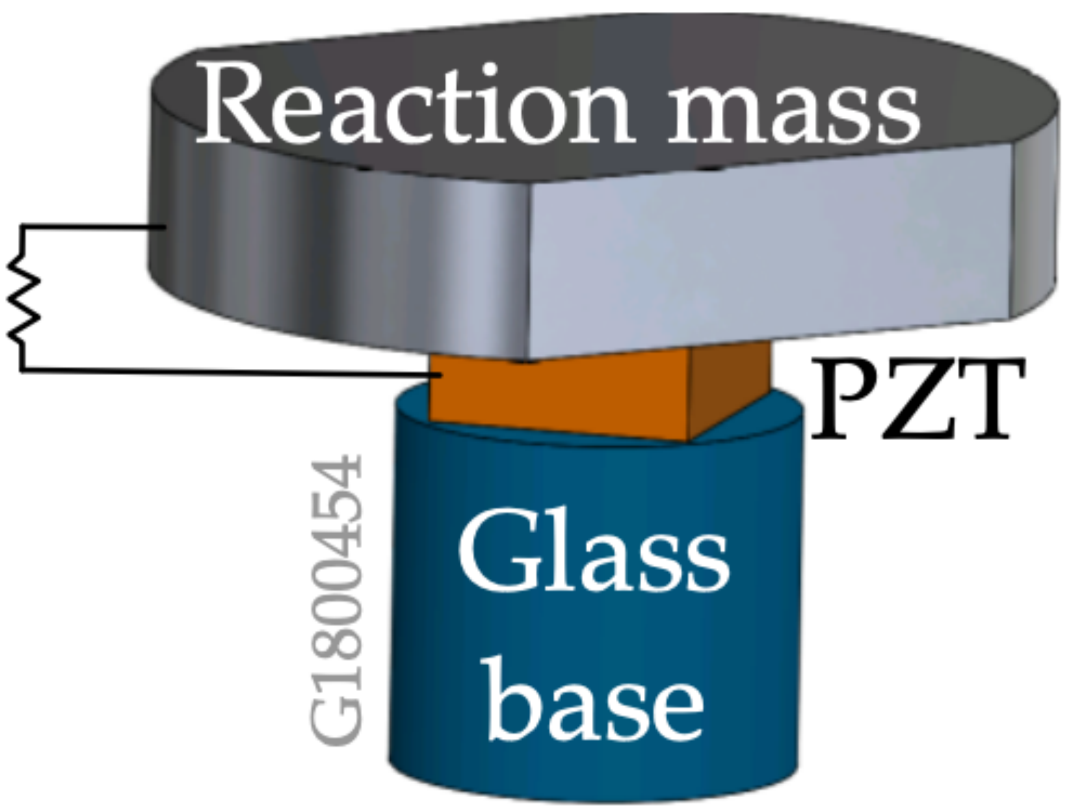


Drains energy from the mirror

The damper used in Advanced LIGO



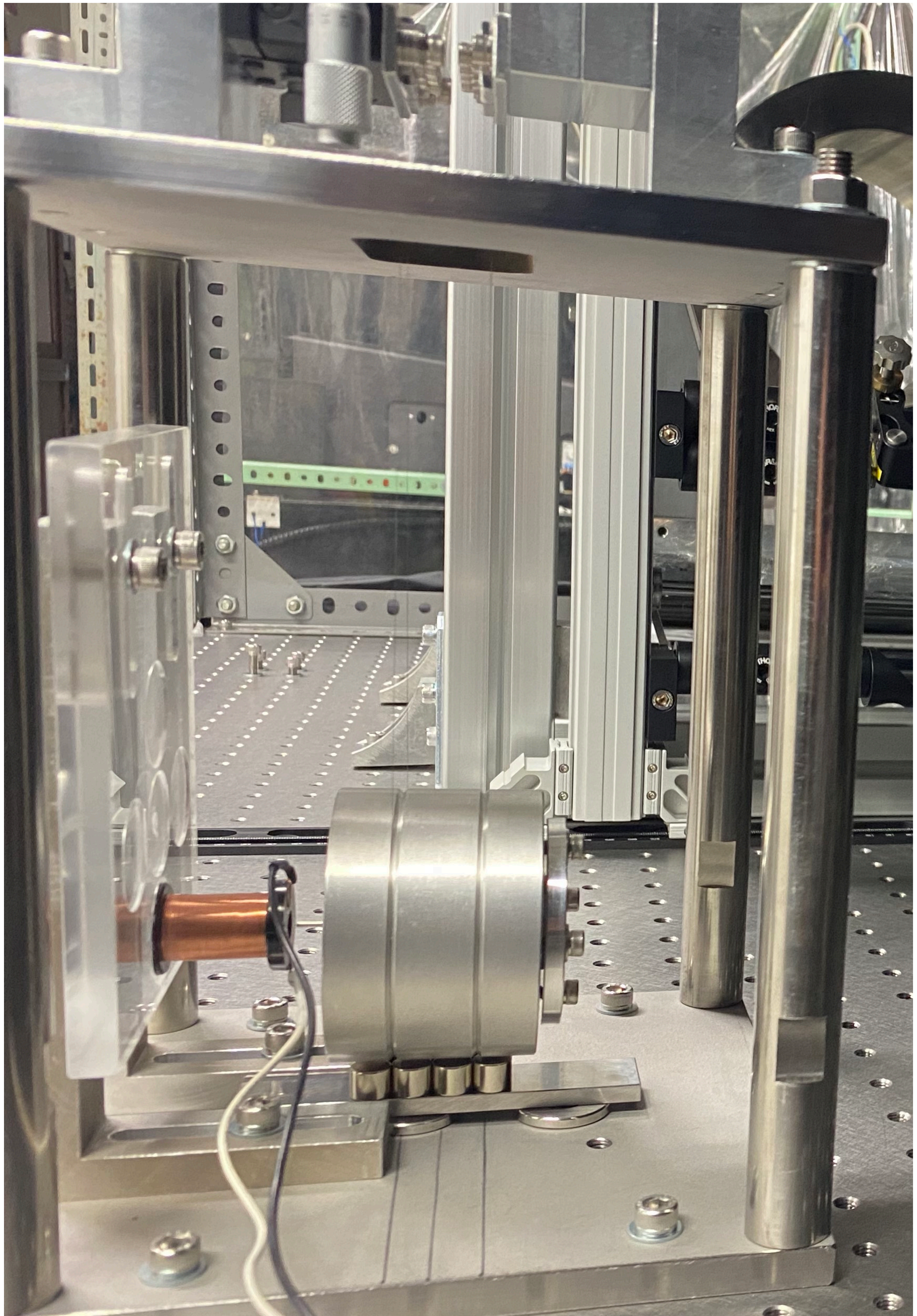
The Reaction mass moves
by Elastic vibration,
and the PZT generates a voltage.

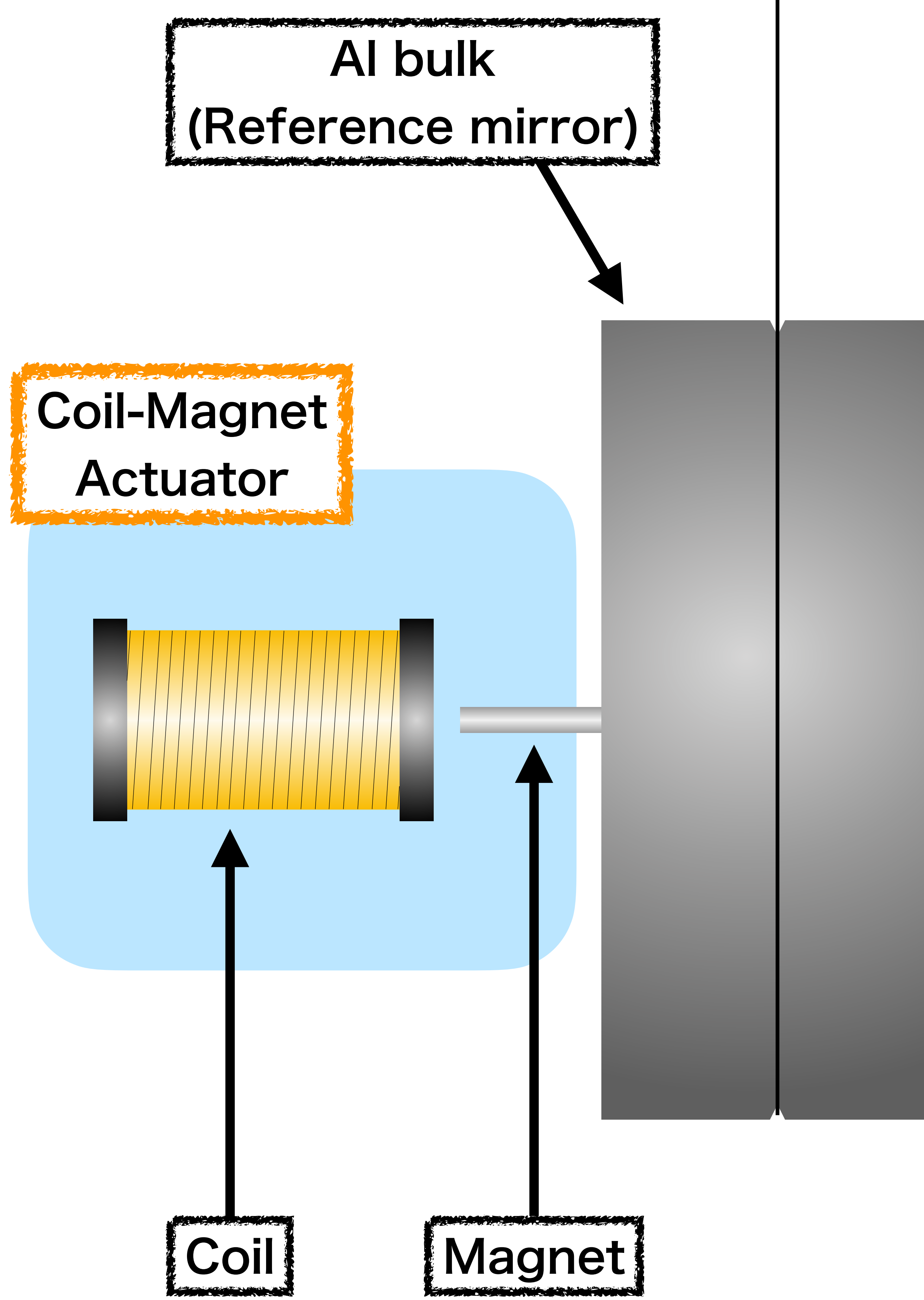


Electrical energy is released from the resistance as heat energy.

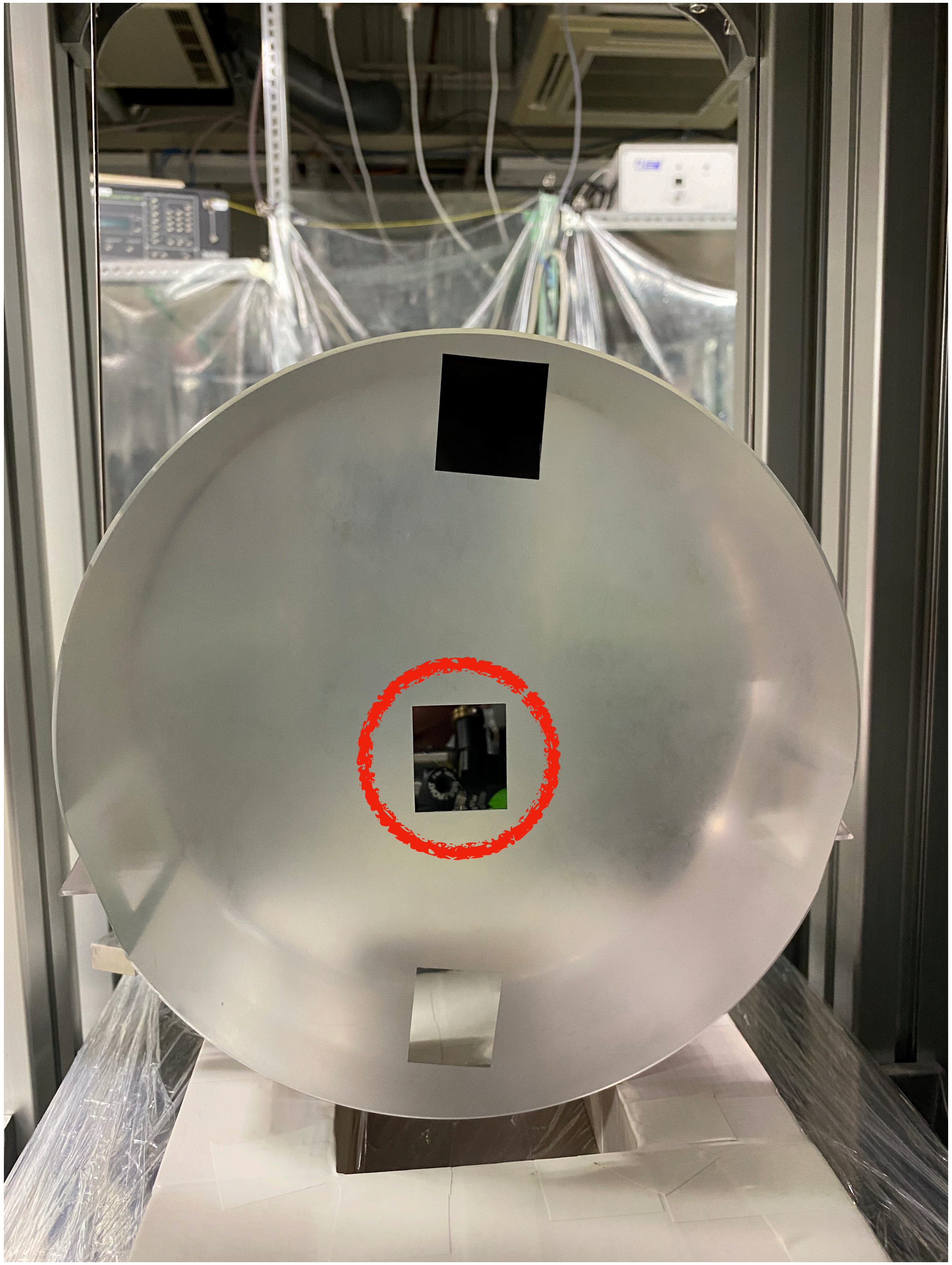
→ Elastic vibration is damped.

Reference
: GWADW, 23 May 2019 LIGO-G1900963





Al bulk
(Reference mirror)
Coil-Magnet
Actuator
Coil
Magnet



Sheet mirror

Thickness **0.16 mm**