Study of the frequency domain analysis method to estimate calibration errors

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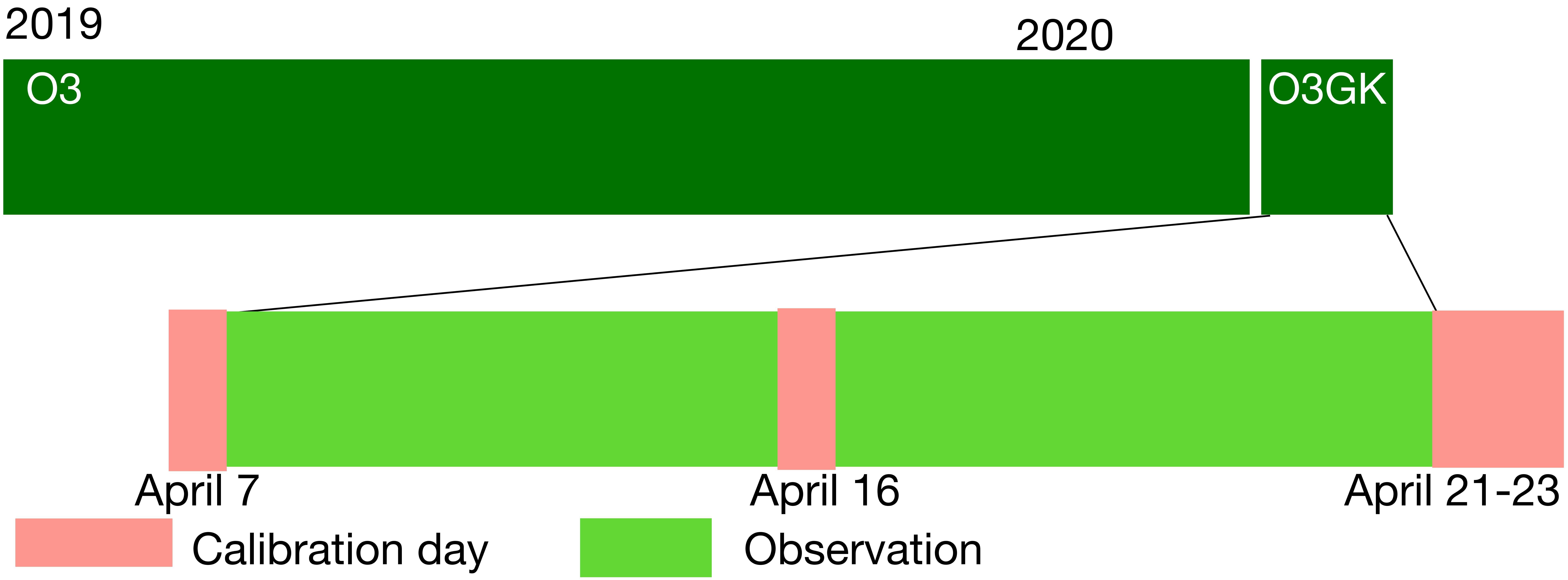
Outline

1. Introduction of KAGRA
2. Introduction of Calibration
3. Motivation
4. Maximum-likelihood method
5. Mask and reason
6. Fitting Result
7. Summary

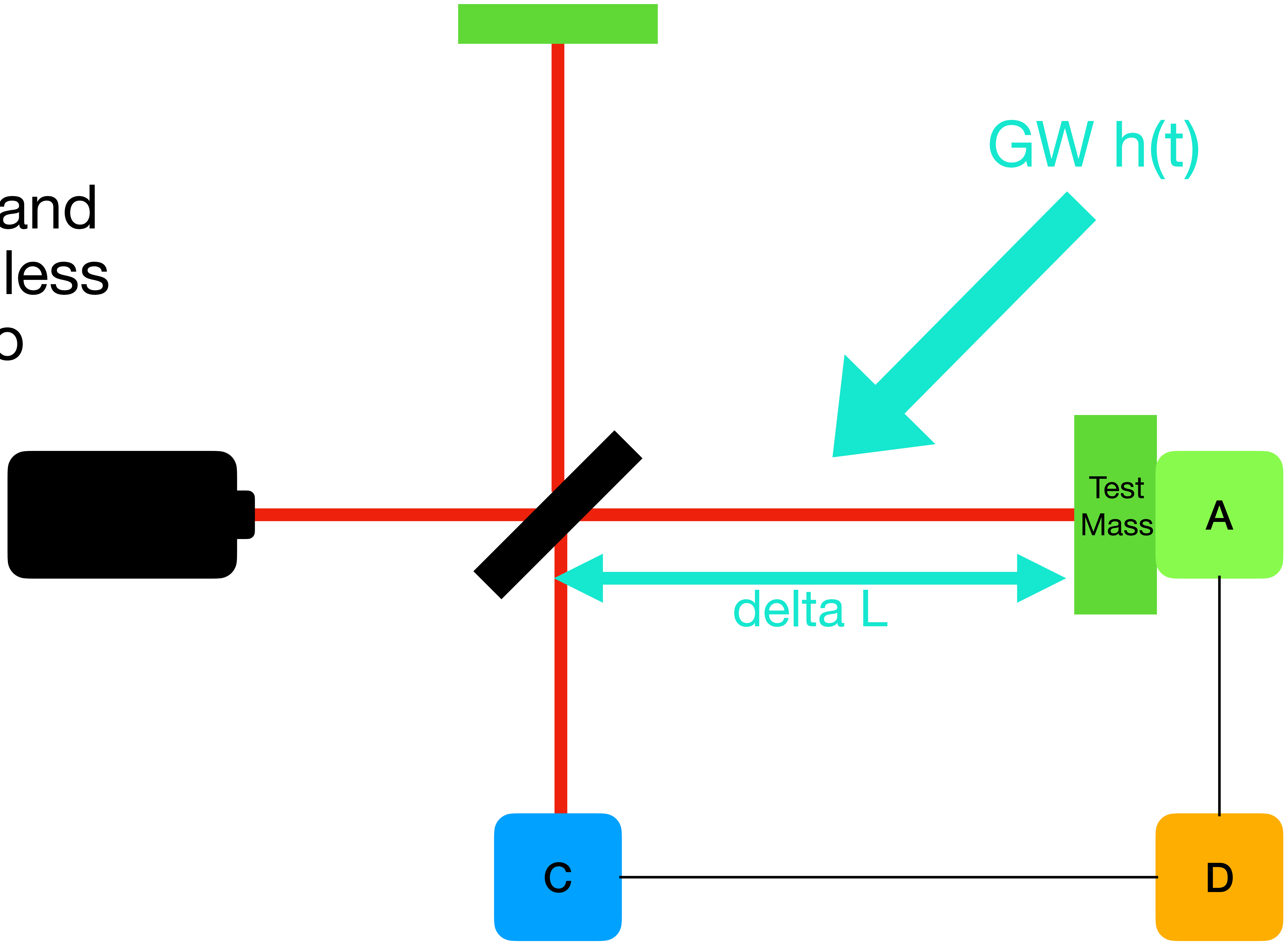
Introduction of KAGRA

Observation3 starts in 2020, when KAGRA(Japan) and GEO600(Germany) joint GW observatory network.



What is Calibration?

We cannot know exactly the relationship between GW strain and detector's signal we received unless we use some external sources to calibrate it.

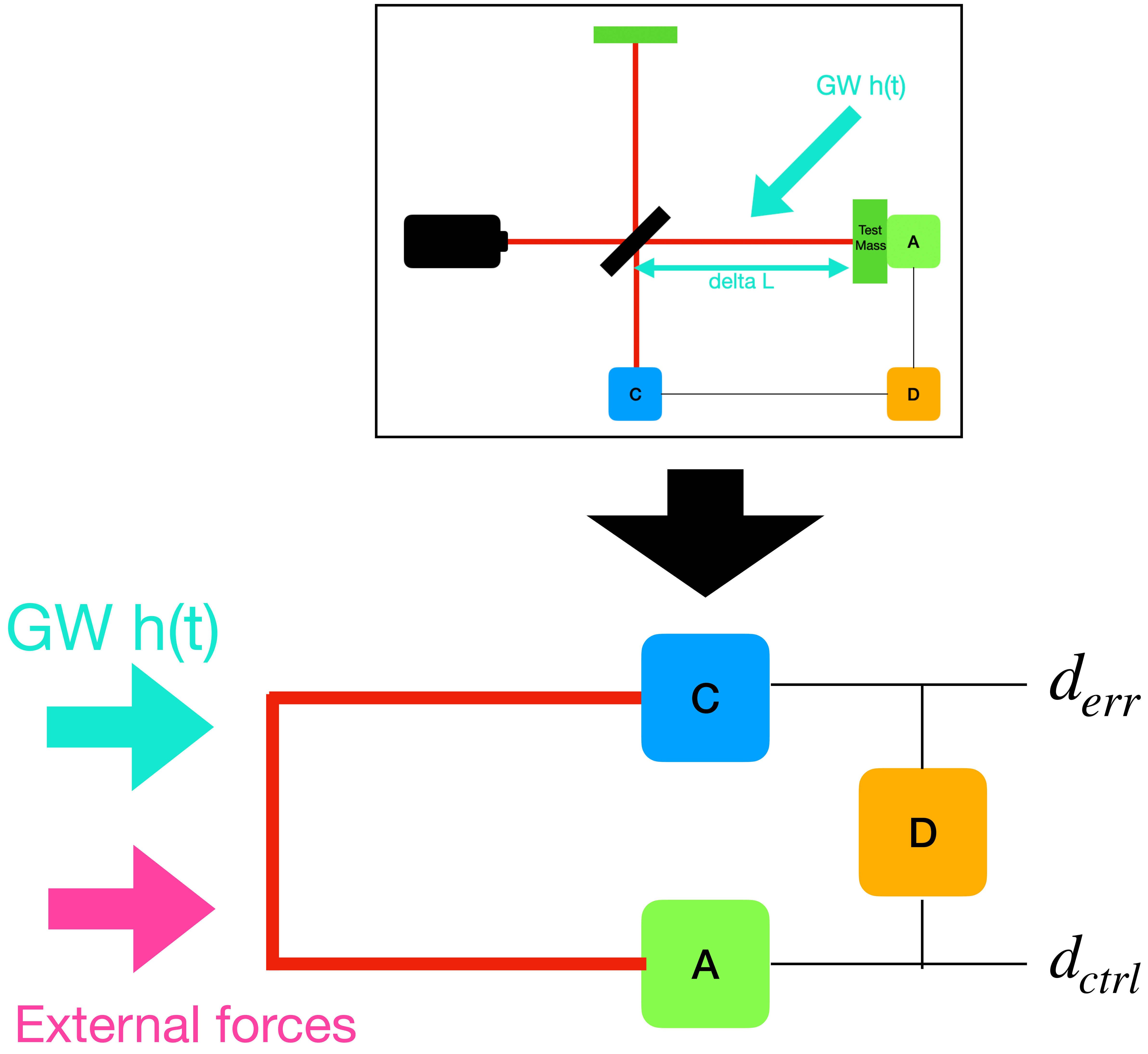


DARM Model

Interferometer is large, Gravitational waves is extremely tiny. Measuring such a small signals requires calibration.

DARM (Differential ARM length) model explains how h(t) from interferometer to signals we detect.

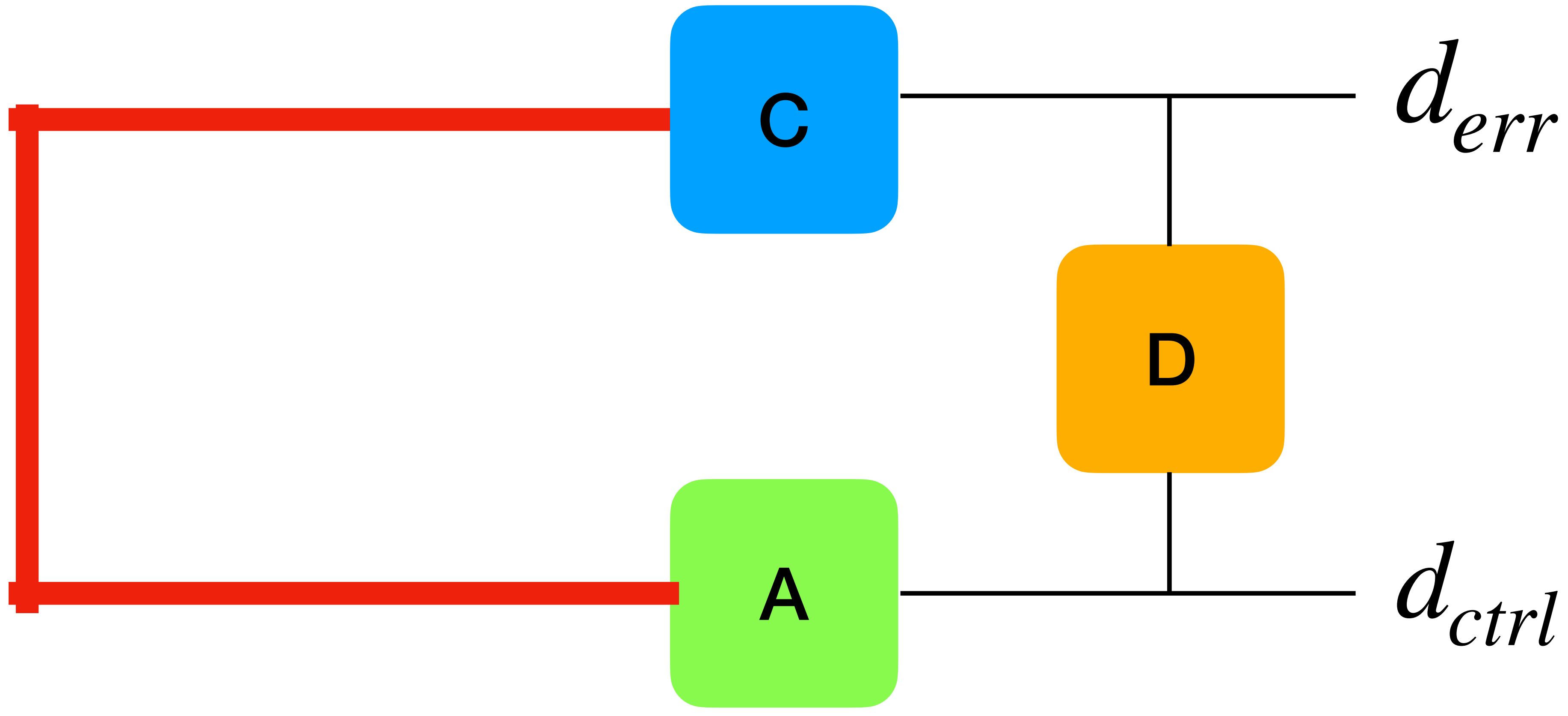
With calibrating this model parameters. We can improve accuracy of it.



Estimate parameters and errors of A,C

Reconstruction

After calibrate parameters, we can obtain Response function, which is crucial for reconstruction GW signals.



$$G = ACD$$

$$R = C^{-1} + AD$$

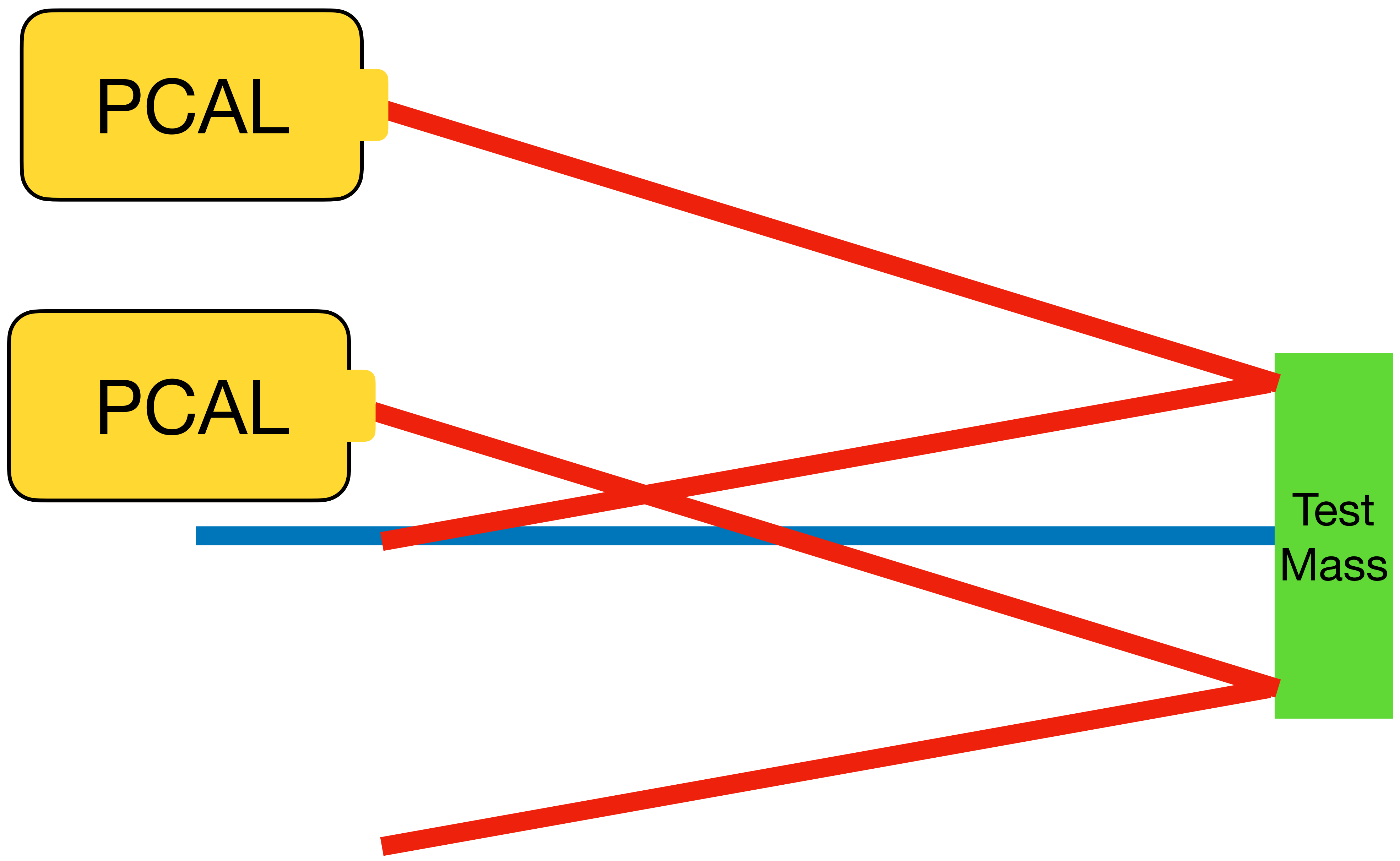
$$h(t) = R * d_{err} = \frac{1}{L}(\frac{1+G}{C})d_{err}$$

$$\frac{\delta h(t)}{h(t)} = 1 - \frac{\delta R}{R} \quad \frac{\sigma h(f,t)}{R^{(model)}} = \frac{\sigma_h(f,t)}{h}$$

Photon Calibrator

The calibration requires forces which is out of the loop, means we should inject signals independently.

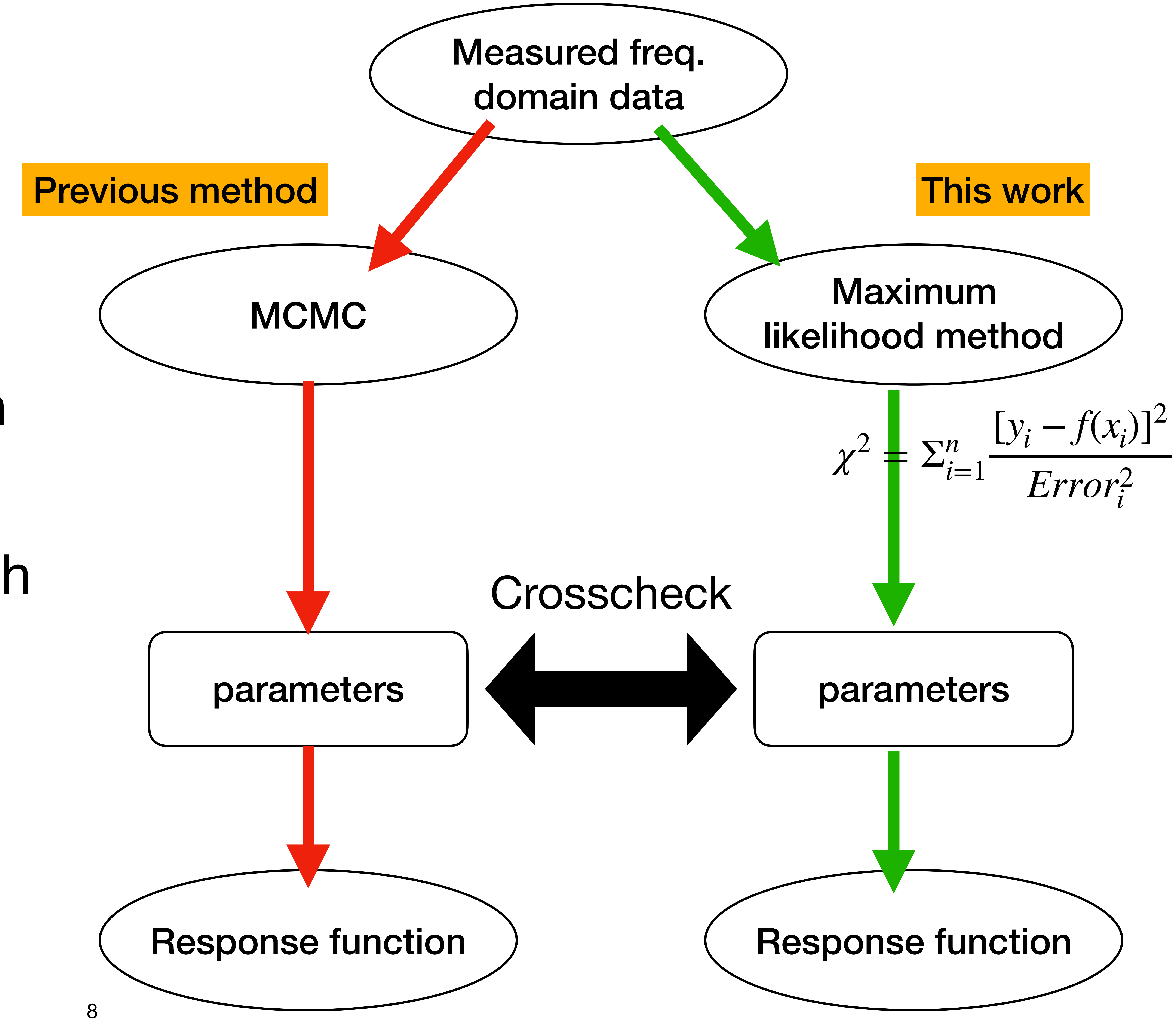
P(Photon) Calibrator injects two laser beams in above and below to generate known signals. Thus, we can realize the relationship of them.



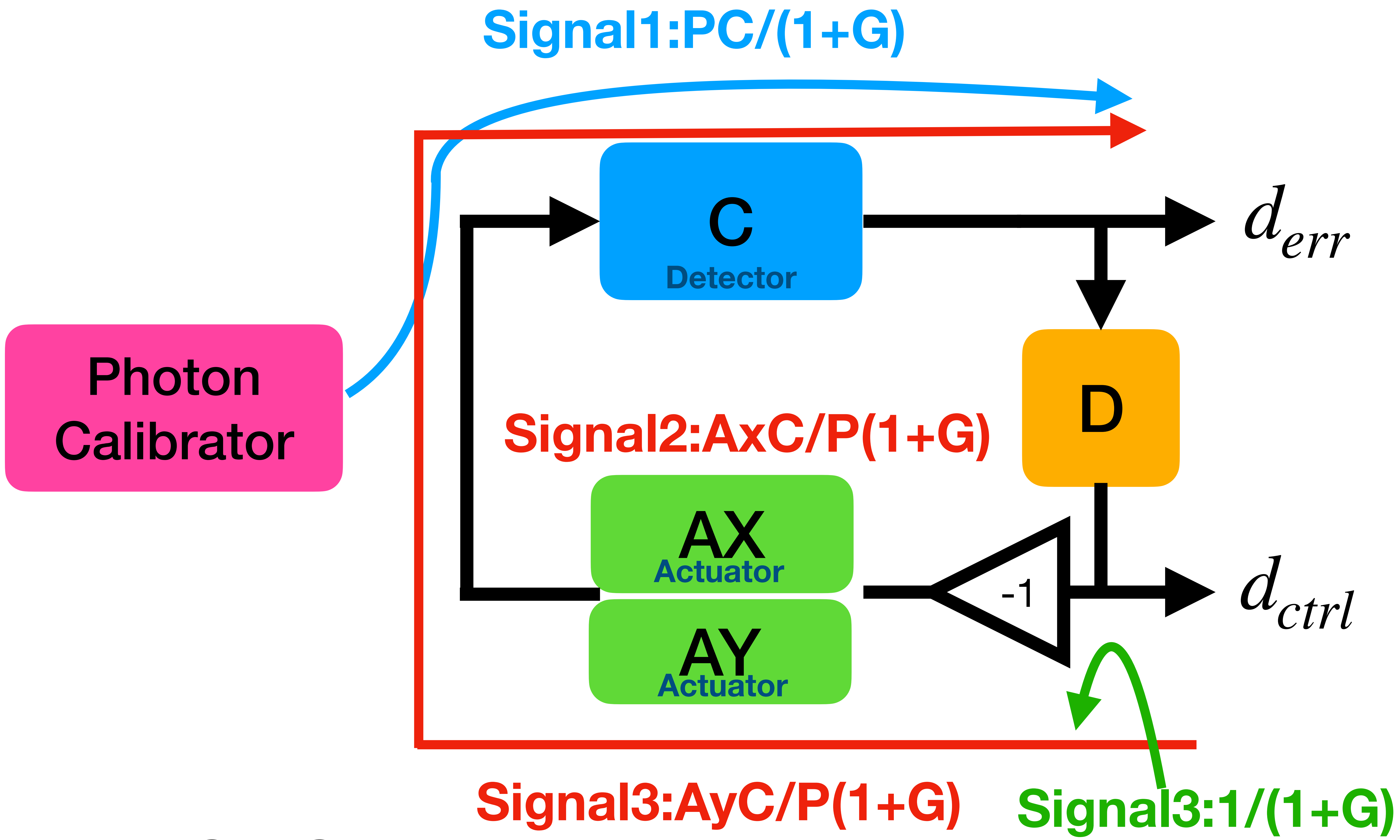
Development of Independent pipeline

Motivation

- Build a new pipeline of error estimation.
- In crosscheck the parameters with Bayesian(MCMC) analysis.
- We can avoid the analysis bias with independent method



Measurement of A and C



Calibration data set

Data-1: Apr. 7
Data-2:Apr.16
Data-3:Apr.21
Data-4:Apr.22
Data-5:Apr.23

G=ACD
Signal1:PC/(1+G)
Signal2:AxC/P(1+G)
Signal3:AyC/P(1+G)
Signal3:1/(1+G)

Analysis procedure

Step.1:Data process

Measured data

Signal1:PC/(1+G)

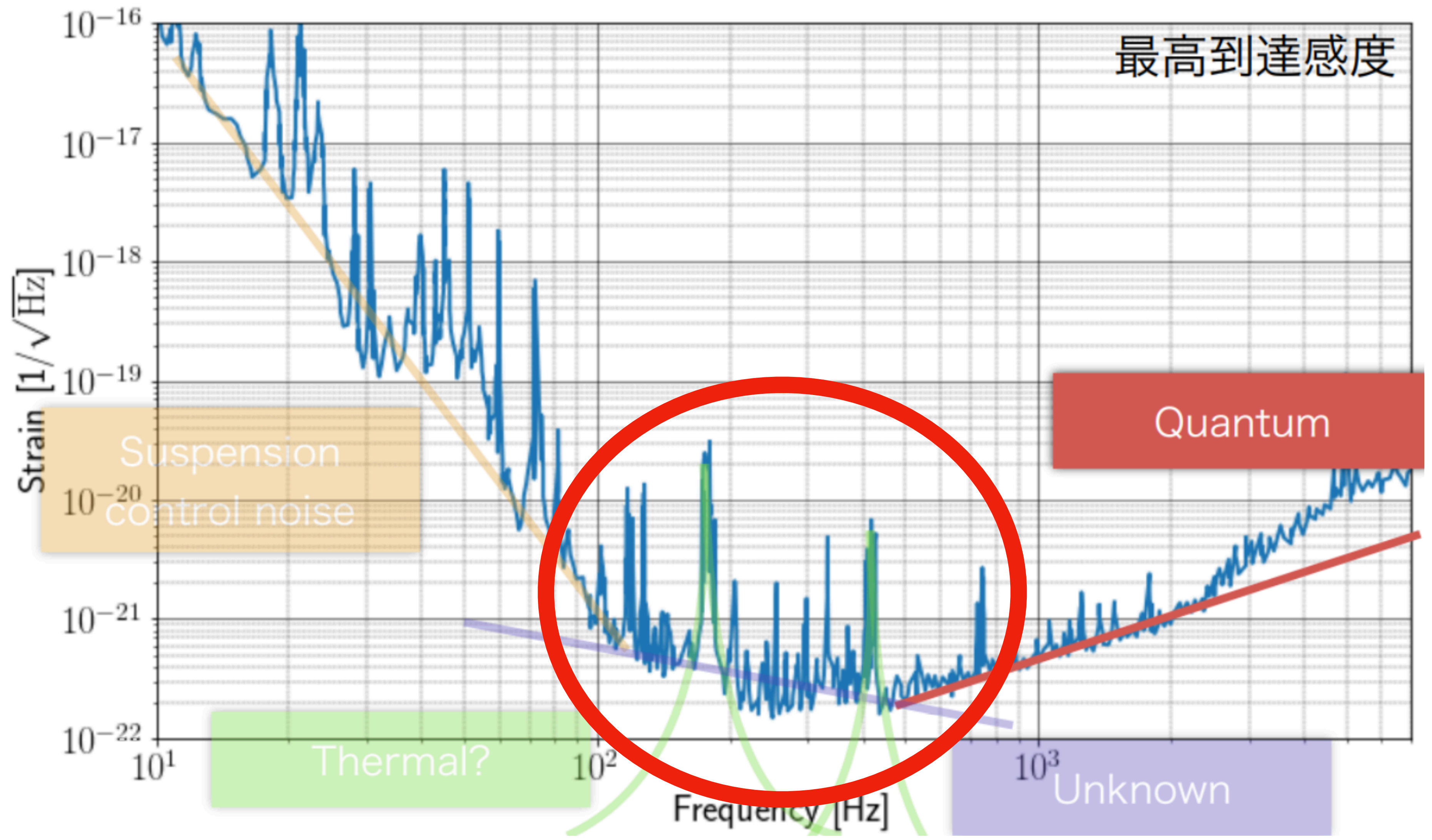
Signal2:AxC/P(1+G)

Signal3:AyC/P(1+G)

Signal3:1/(1+G)

Process of Ax,Ay,C,AC,P function

Step.2:Data quality check



Define the mask to avoid well-known resonance peak

Step.3:Simultaneous fitting and Estimation of Model Parameters

$$\overrightarrow{\theta} = (H_{ax}, \tau_{ax}, H_{ay}, \tau_{ay}, H_c, \tau_c, H_p, \tau_p)$$

Variables	Parameters
Hax	Actuator Efficiency of X arm
tax	Time Delay of Suspension System
Hay	Actuator Efficiency of Y arm
tay	Time Delay of Suspension System
Hc	Optical efficiency
fc	Cavity Pole Frequency
tc	Time Delay of the C
Hp	Pcal Efficiency
tp	Pcal Time Delay

Actuator

$$A_x = H_{ax} e^{-2i\pi f(\tau_{ax} - \tau_p)}$$

$$A_y = H_{ay} e^{-2i\pi f(\tau_{ay} - \tau_p)}$$

Sensing

$$C = \frac{Hc}{1 + i\frac{f}{f_c}} e^{-2\pi f(\tau_c + \tau_p)}$$

Pcal

$$R_x = H_p e^{-2i\pi f \tau_p}$$

A+C

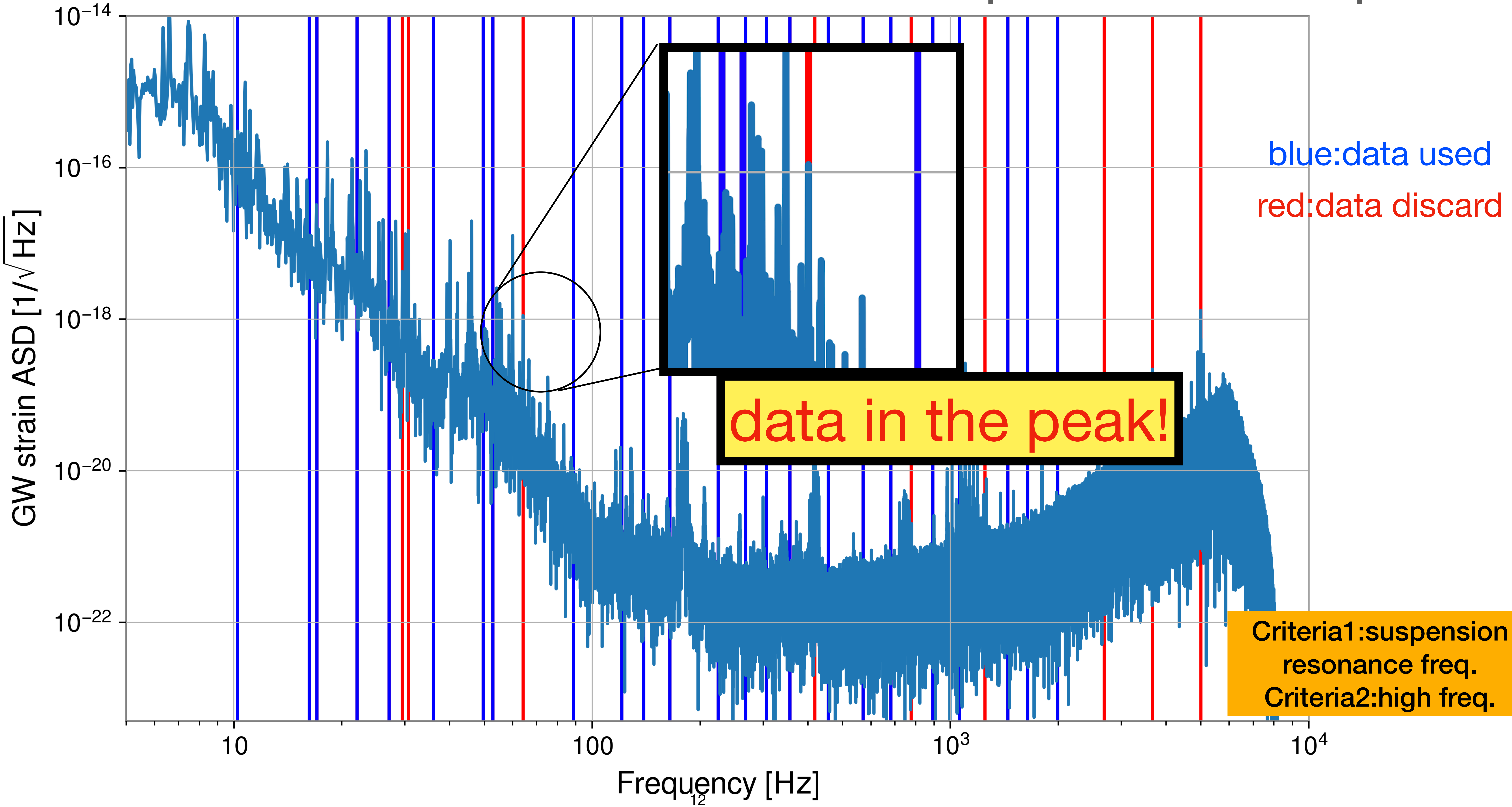
$$AC = (H_{ax} e^{-2i\pi f \tau_{ax}} + H_{ay} e^{-2i\pi f \tau_{ay}}) \frac{H_c}{1 + i\frac{f}{f_c}} e^{-2i\pi f \tau_c}$$

Simultaneously Fitting

➡ Example of Data-1: Apr. 7

Mask

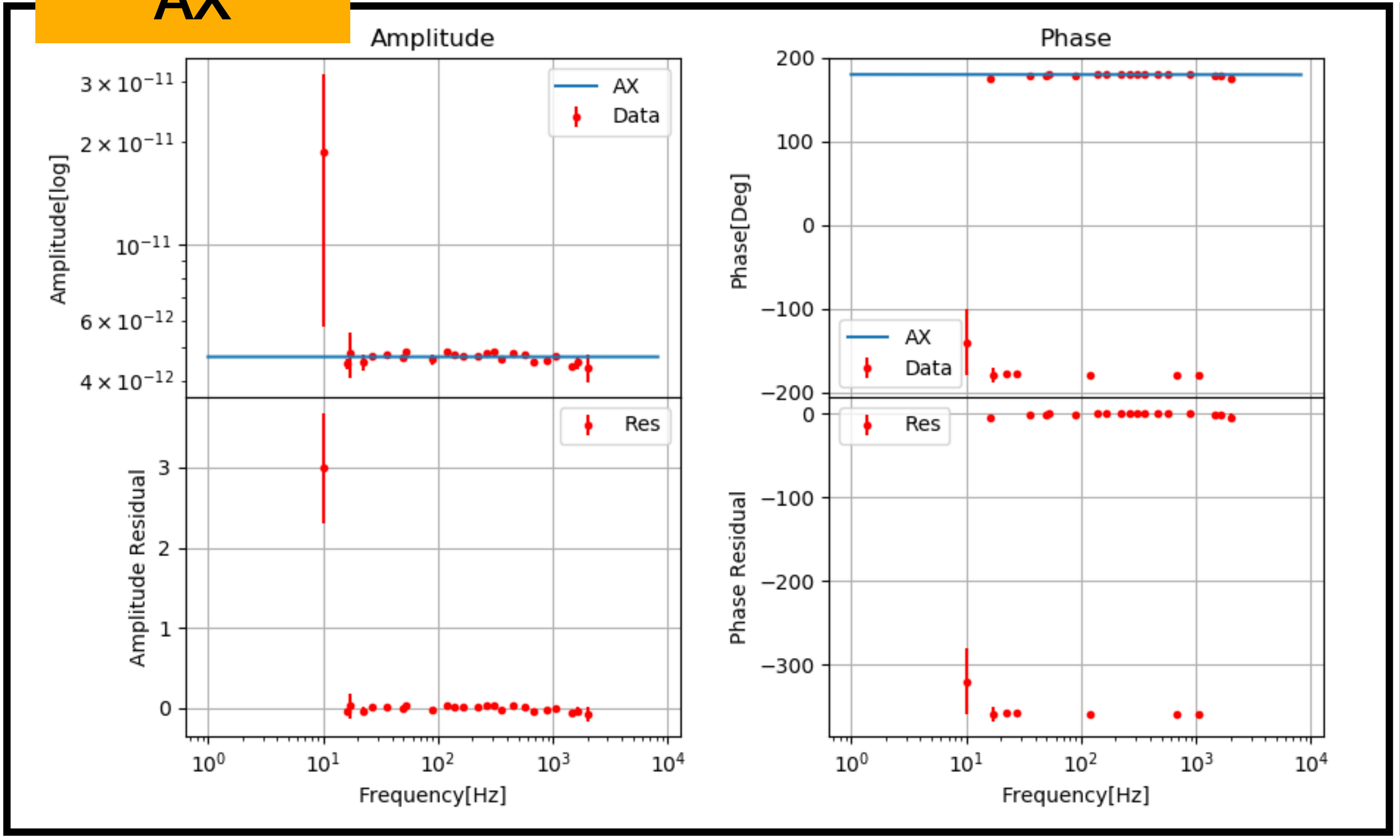
Example of Data-1: Apr. 7



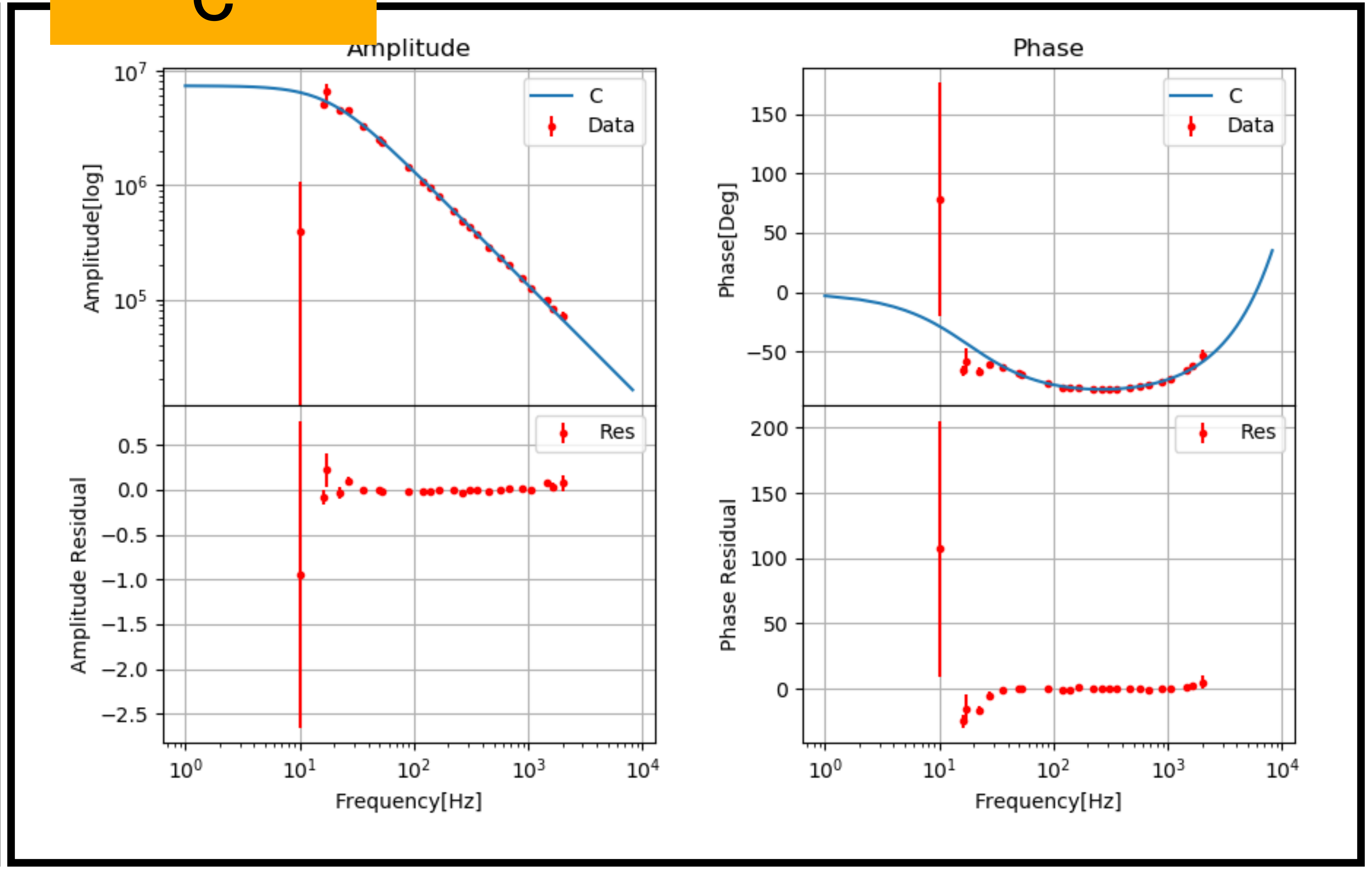
Results of simultaneous fitting

Example of Data-1: Apr. 7

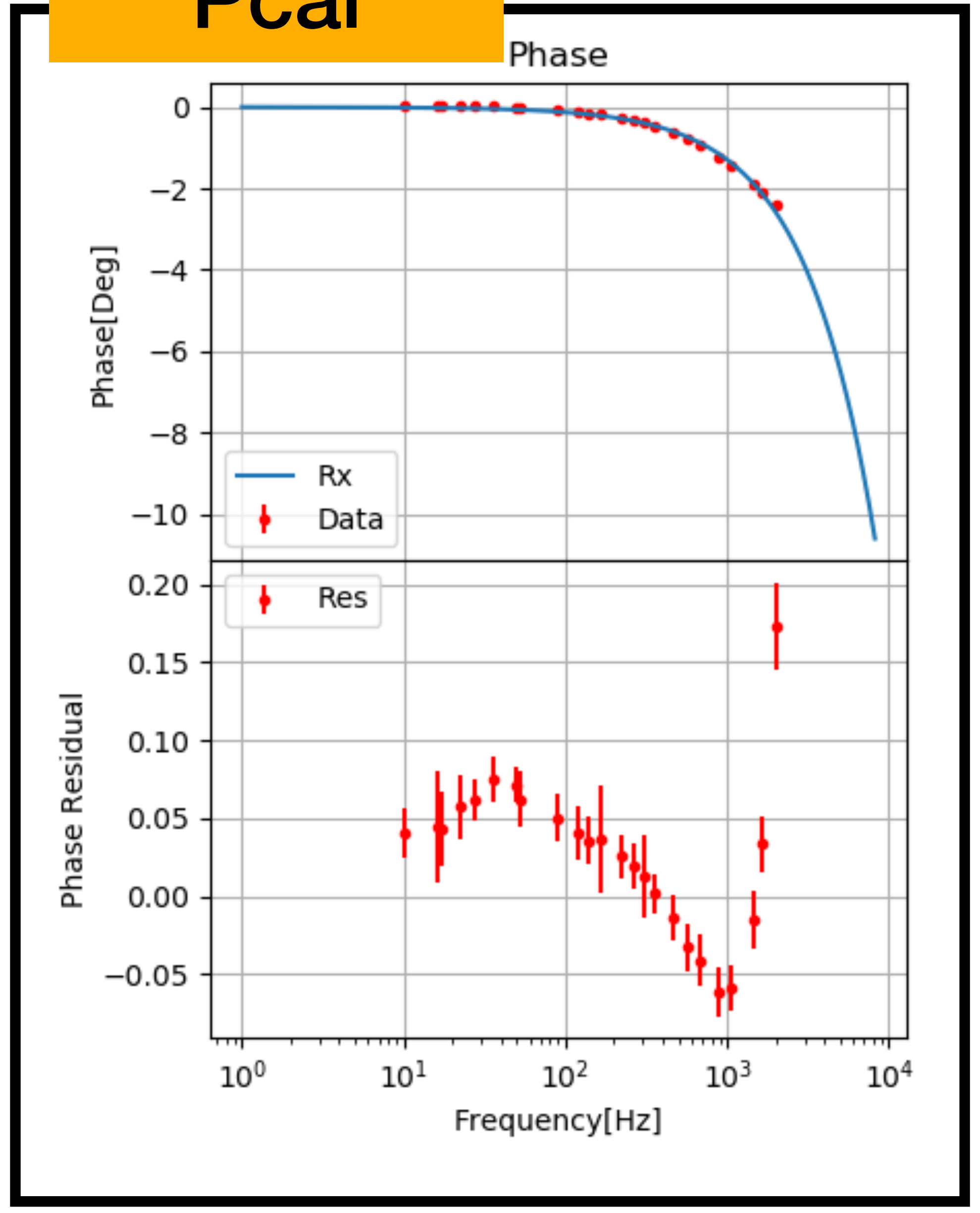
AX



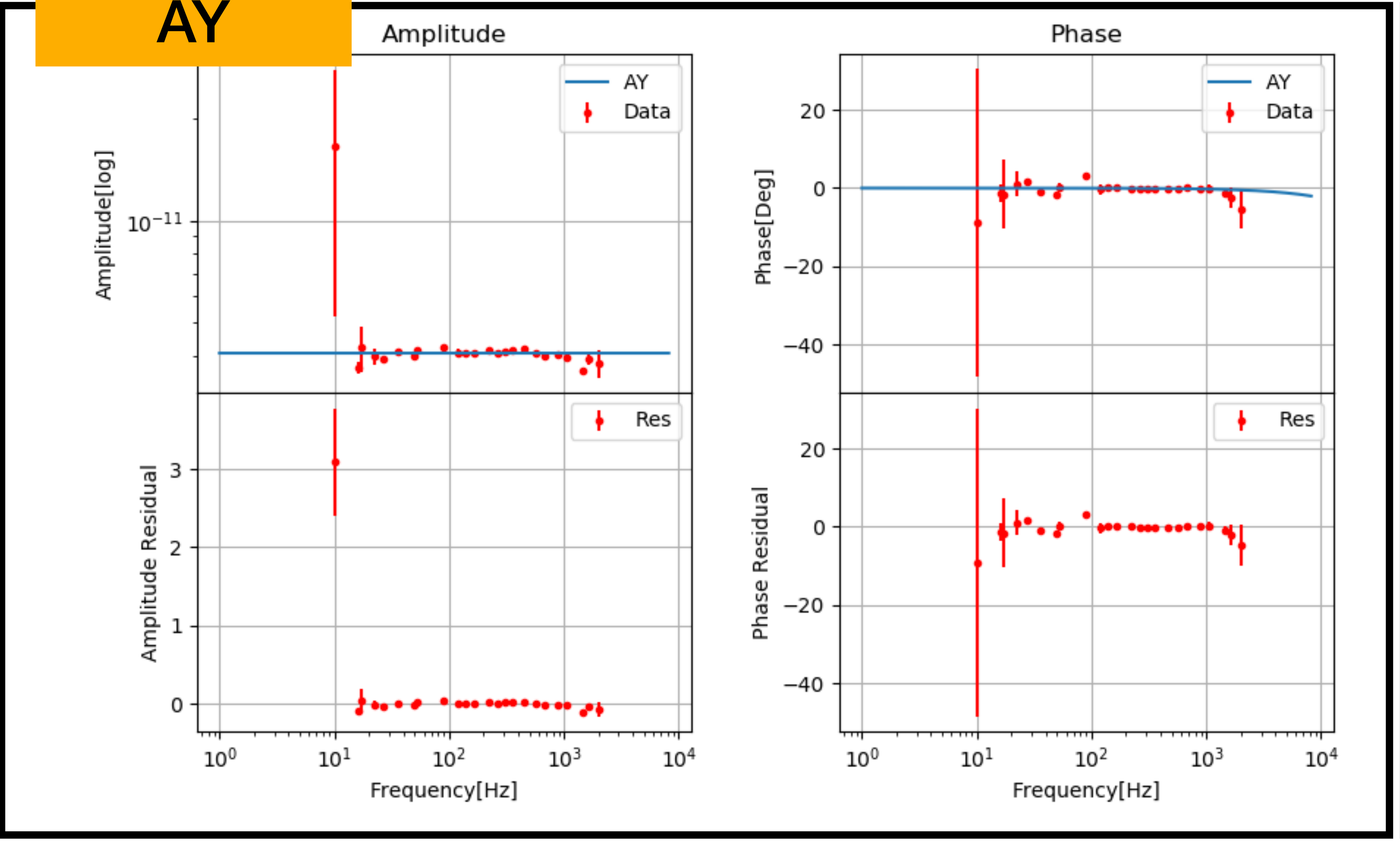
C



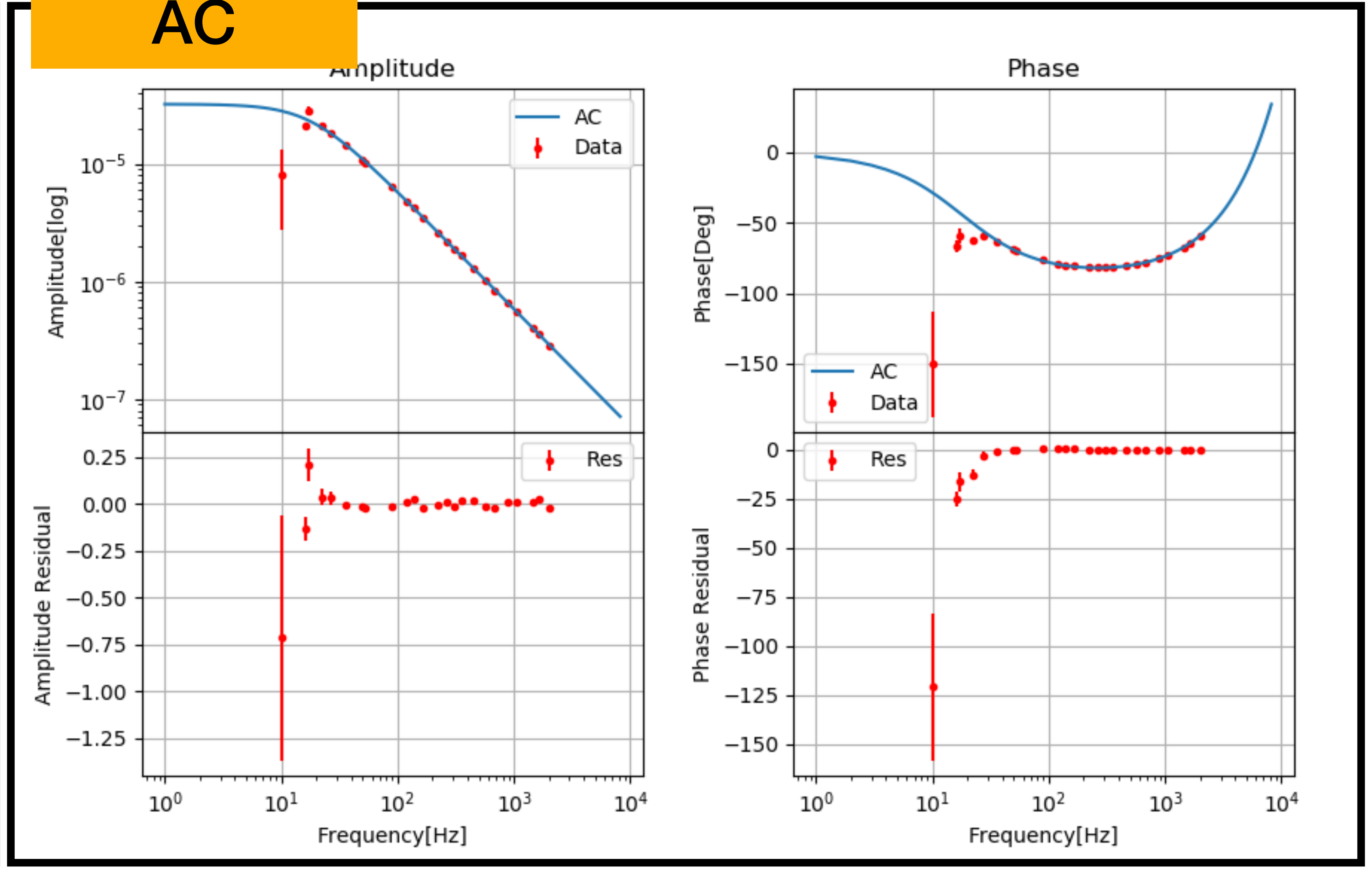
Pcal



AY

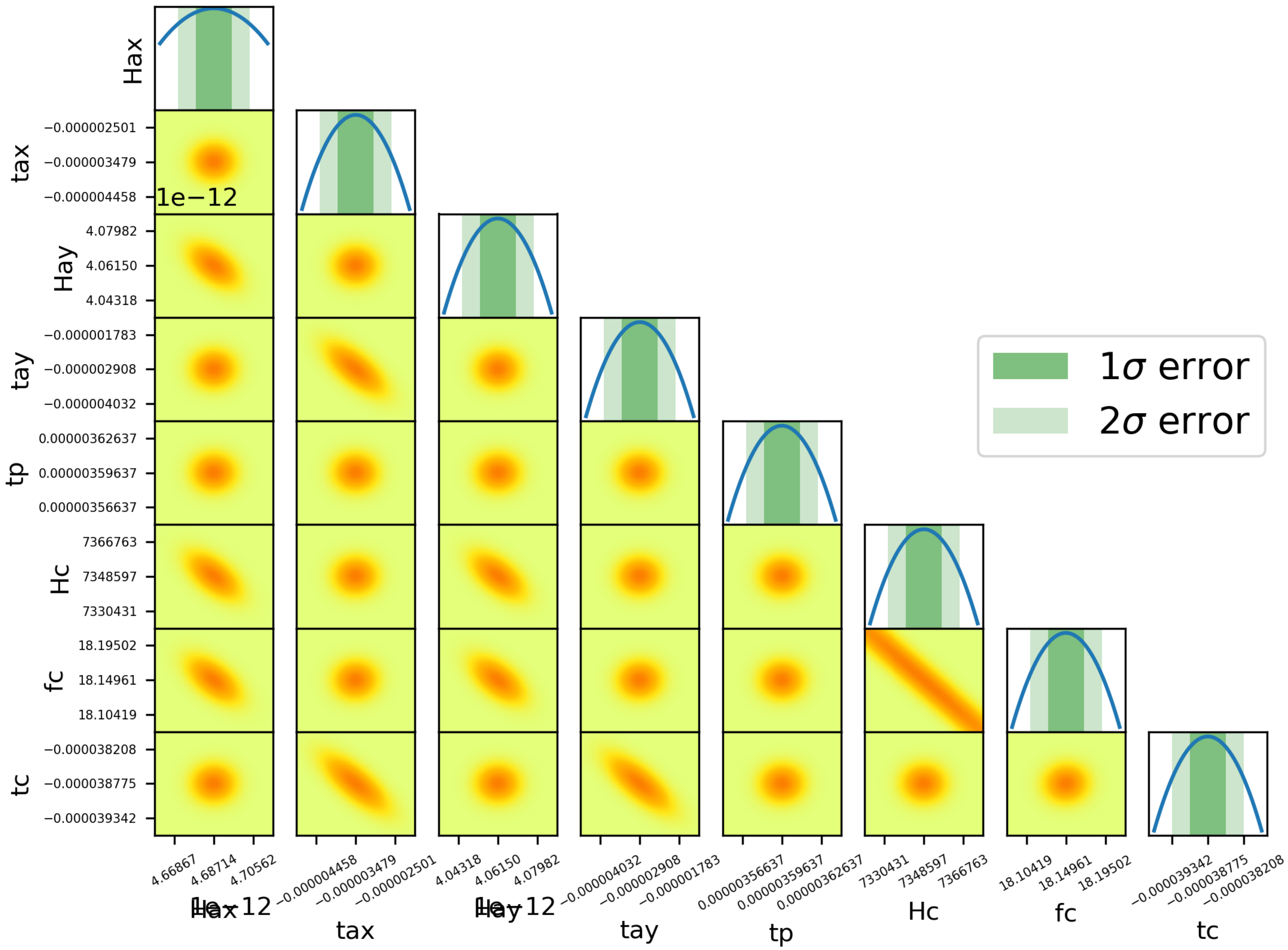


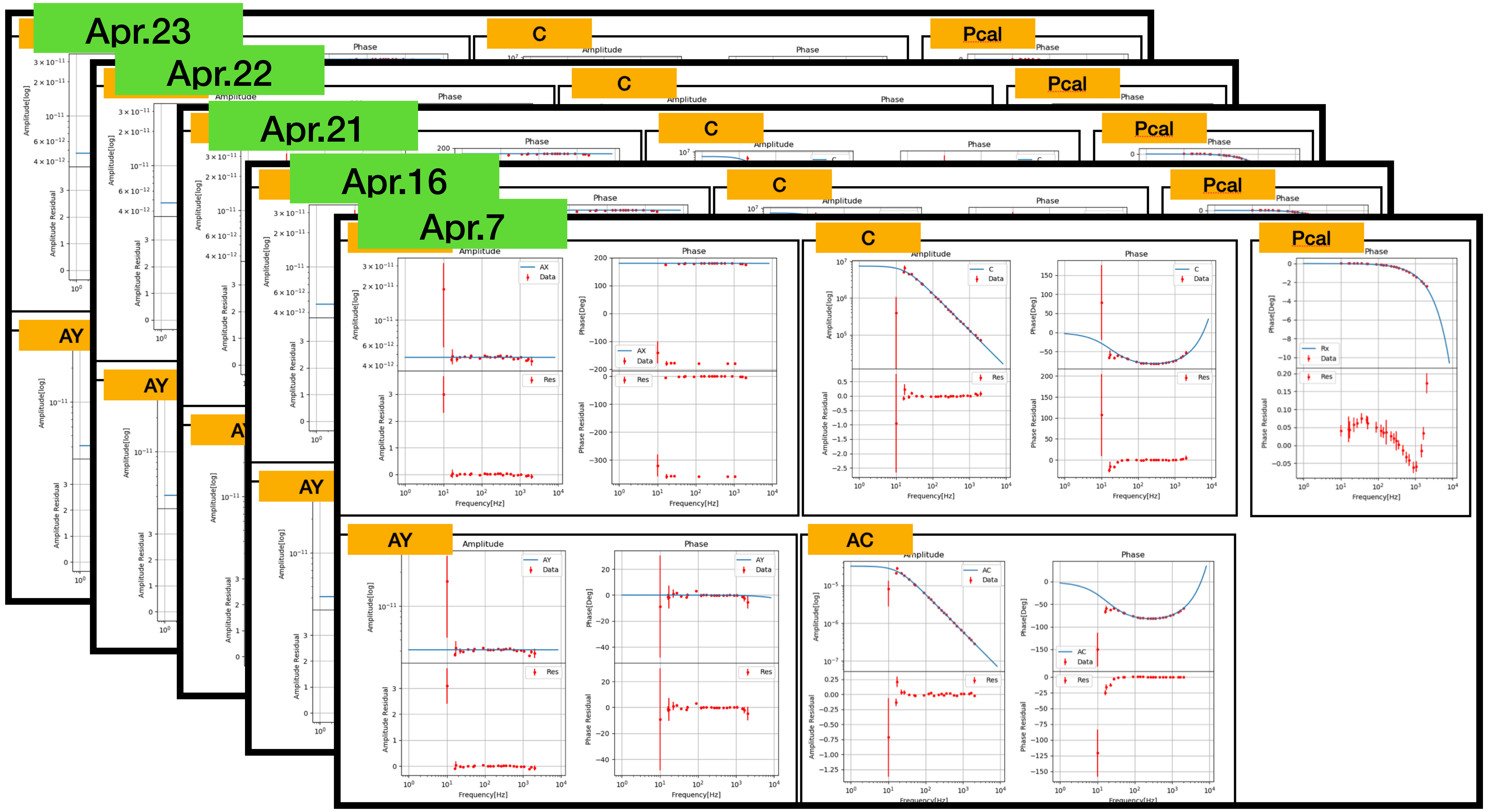
AC



Correlation Result

Example of Data-1: Apr. 7



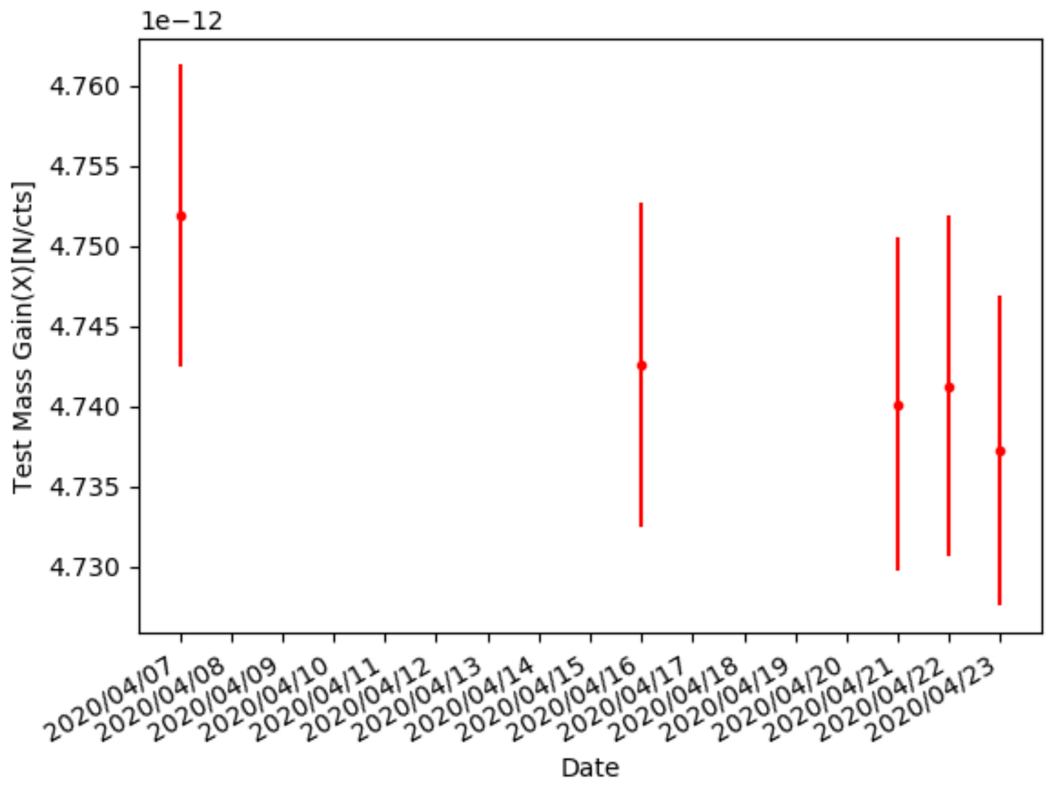


By checking time trends of 2 weeks data

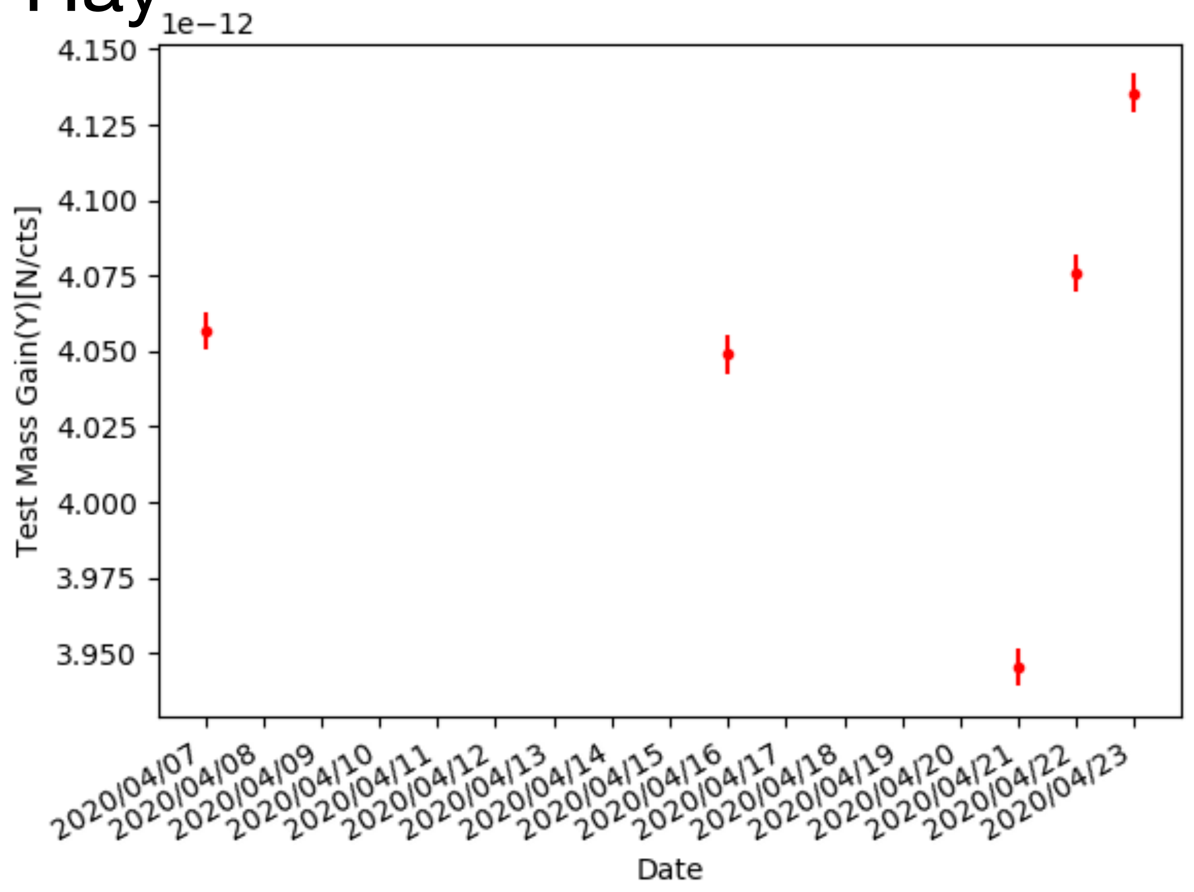
Fitting Result

$$\overrightarrow{\theta} = (H_{ax}, \tau_{ax}, H_{ay}, \tau_{ay}, H_c, \tau_c, H_p, \tau_p)$$

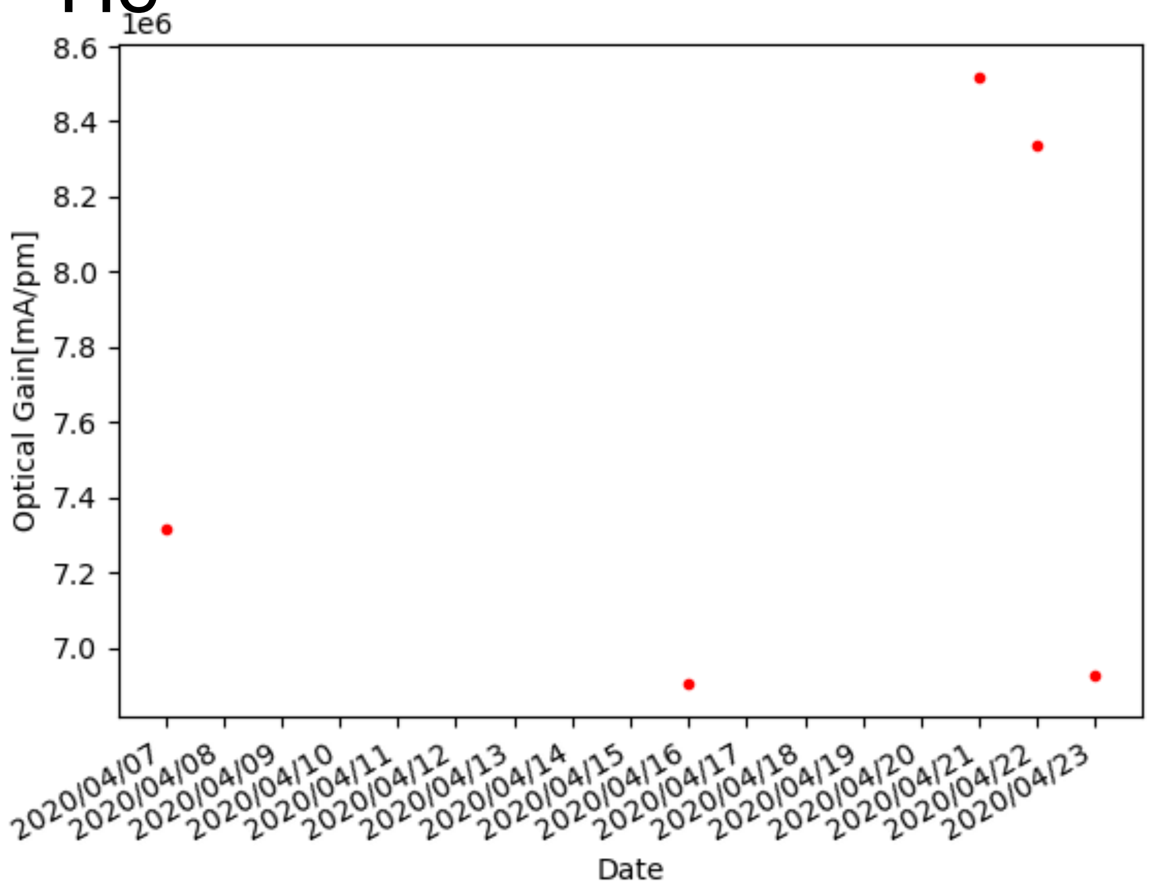
Hax



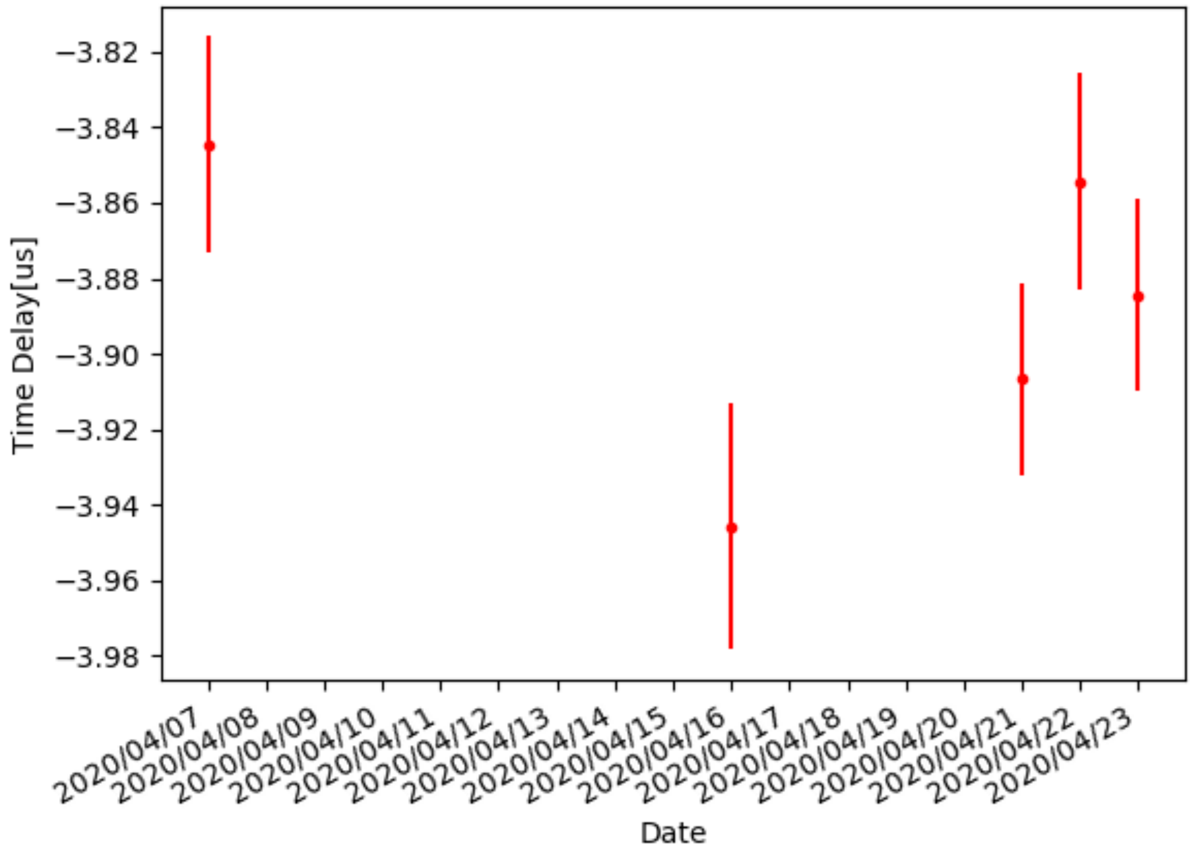
Hay



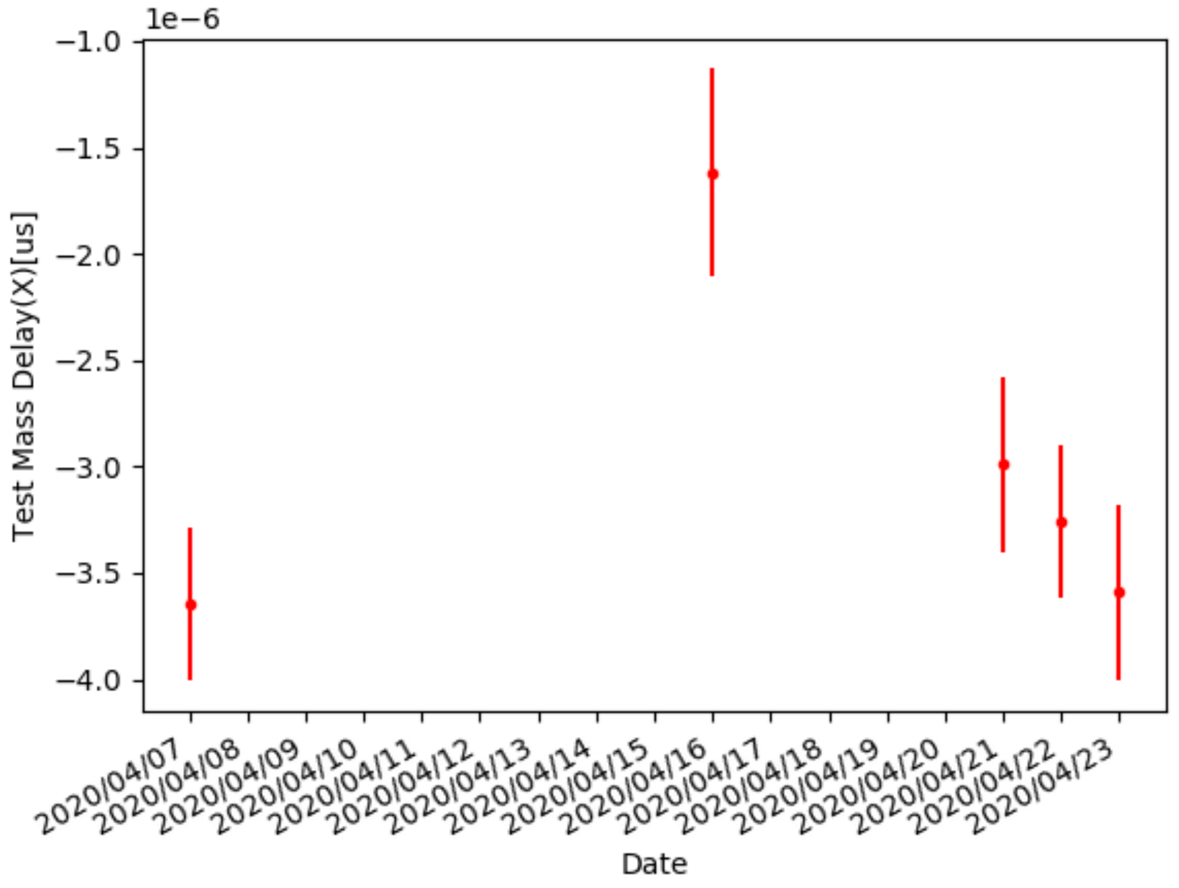
Hc



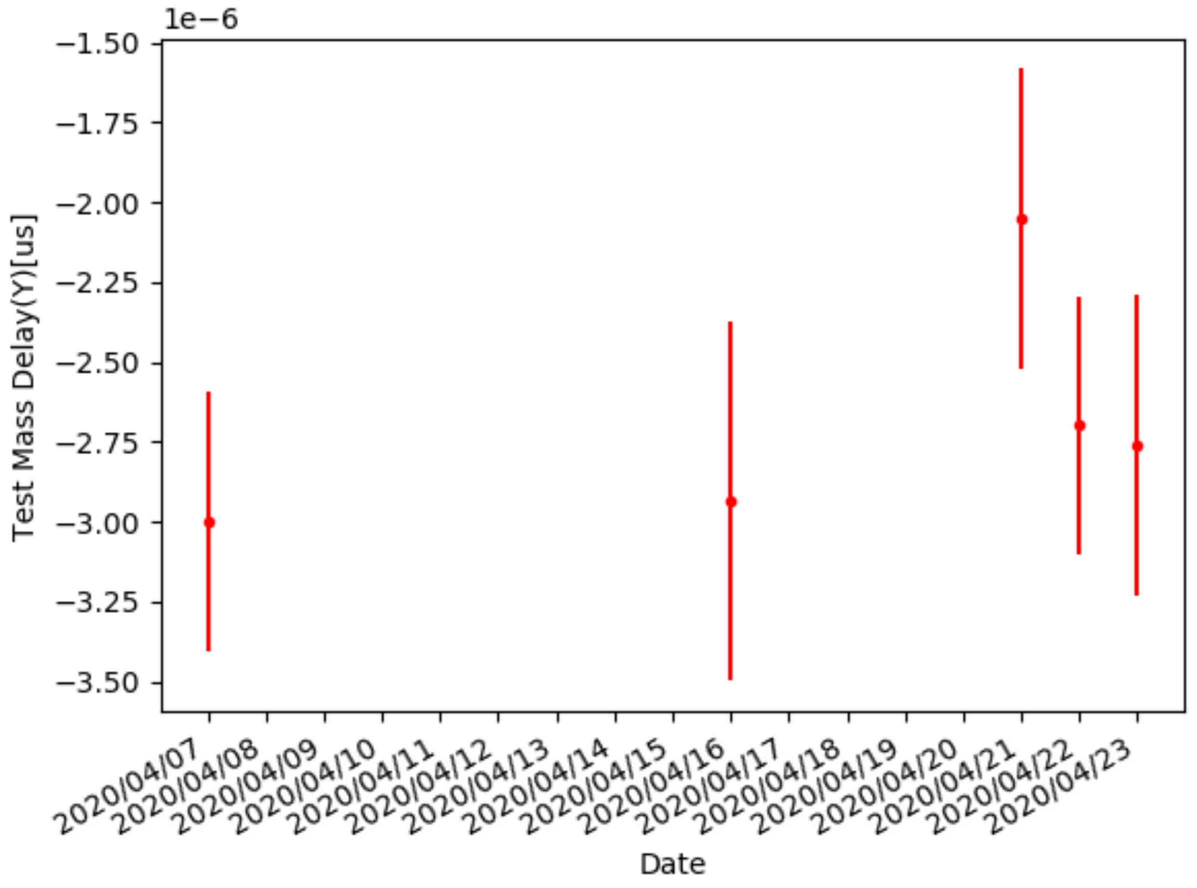
tc



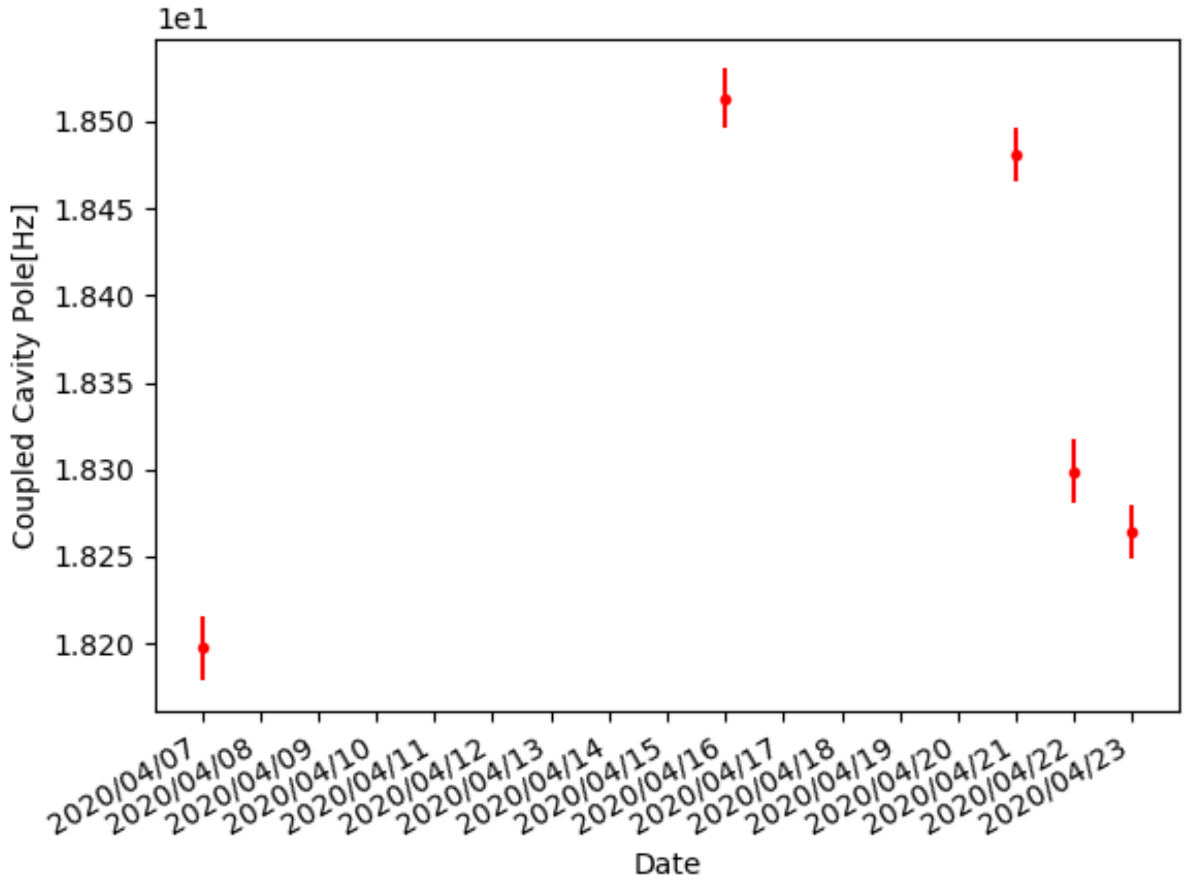
tax



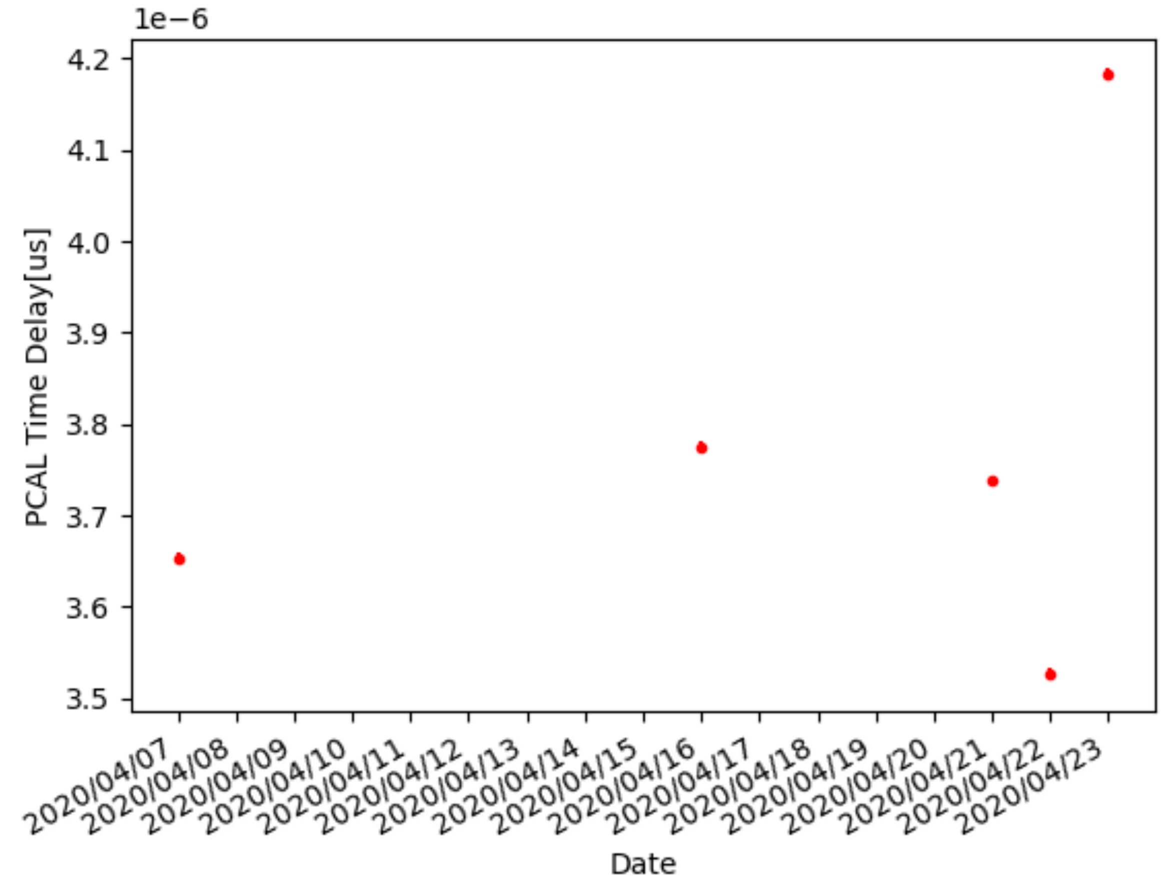
tay



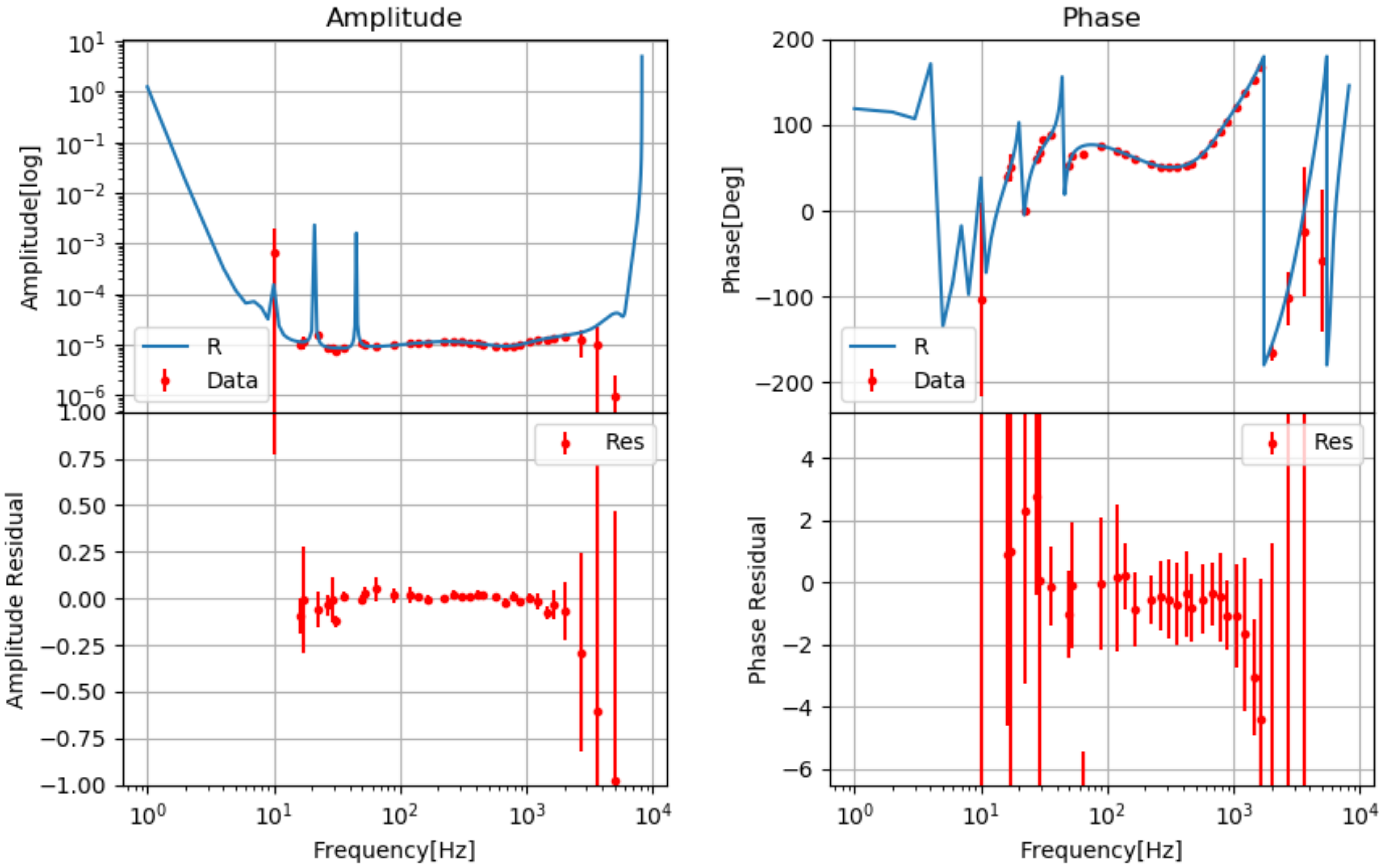
fc



tp



Response function



$$h(t) = R * d_{err} = \frac{1}{L}\left(\frac{1+G}{C}\right)d_{err}$$

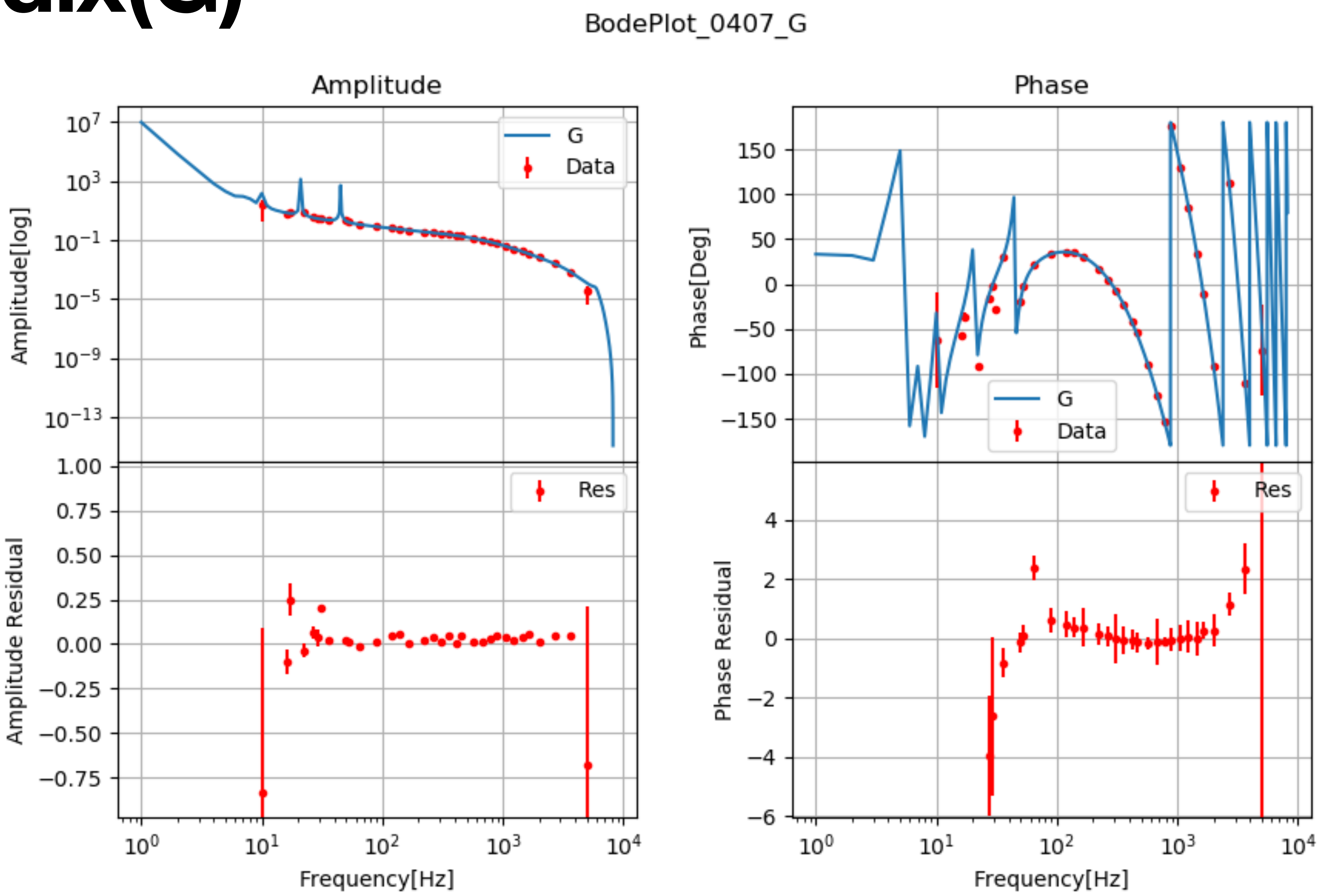
$$\frac{\delta h(t)}{h(t)} = 1 - \frac{\delta R}{R} \quad \frac{\sigma h(f,t)}{R^{(model)}} = \frac{\sigma_h(f,t)}{h}$$

Summary

- DARM model is crucial for improving the accuracy of interferometer.
- We use external sources(Pcal) to calibrate the model.
- Future plan: With another pipeline, we can crosscheck them.

The End

Appendix(G)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G
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Amplitude[log]
Amplitude Residual
Phase[Deg]
Phase Residual
Frequency[Hz]
Frequency[Hz]