Simulation studies of signal characteristics due to gravity field calibrator(GCal) in gravitational wave detectors

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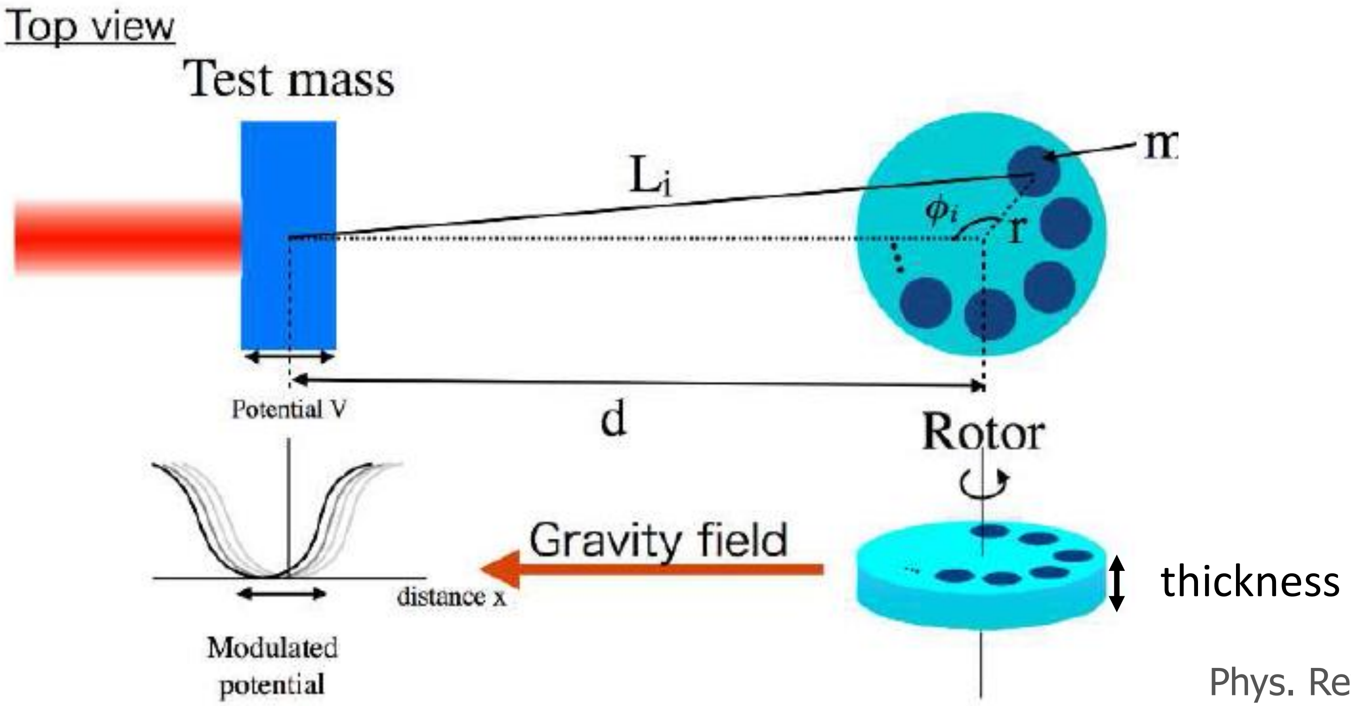
Calibration Systematic Uncertainty

- To reduce the calibration systematic uncertainty, we need some calibration sources for monitoring the time variation of the response of the IFO
- Primary tools:
  - PCal (**P**hoton **Cal**ibrator) : Advance LIGO, Advance Virgo and KAGRA → Periodic force on interferometer mirrors (power modulated laser beam)
  - **Limit**: a few% absolute calibration uncertainty → the uncertainty on the laser power standard of the metrology institutes
- New Candidate:
  - Gcal(**G**ravity field **Cal**ibrator) : Ncal in Virgo

Use the variation of the Newtonian gravitational field produced by moving masses to induce a known displacement of the test mass

GCal(Gravity field Calibrator)

- ☐ Moving mass → Rotor
- ☐ Place the rotor at the same height and the distance of d away from test masses.
- ☐ Multipole mass generate the gravitational potential at the test mass position.



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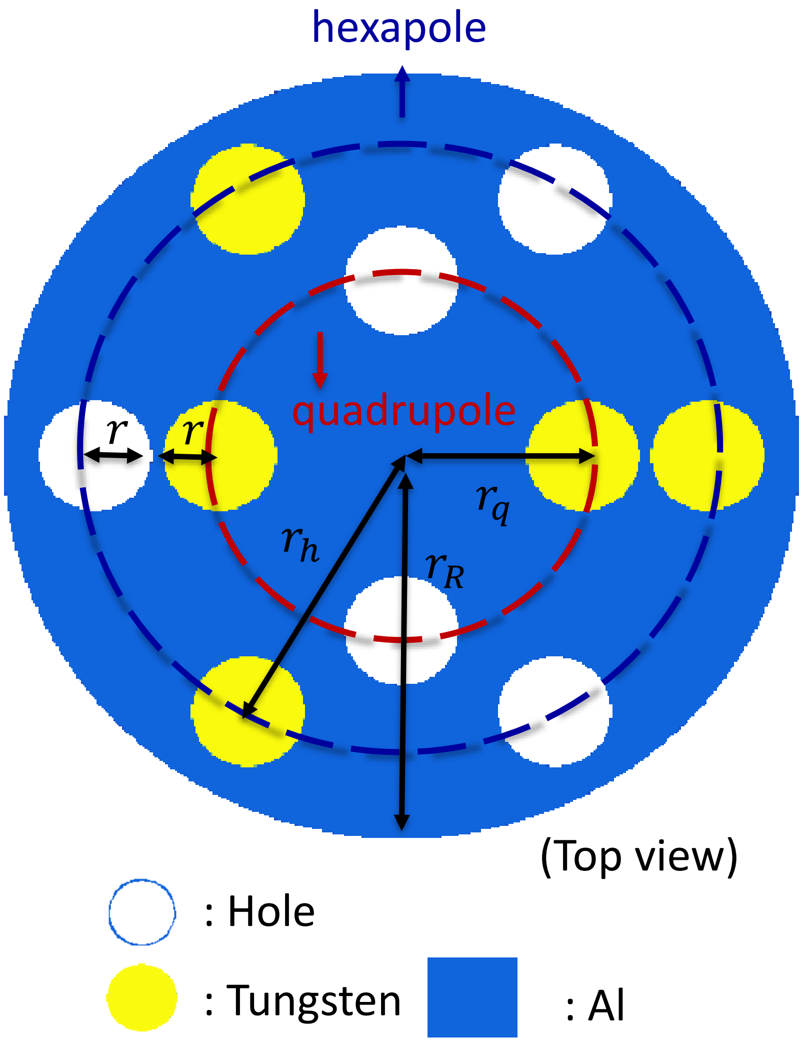
Configuration of the rotor

- with quadrupole and hexapole mass distributions

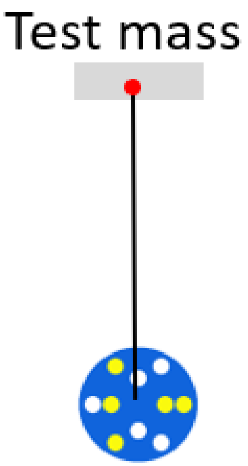


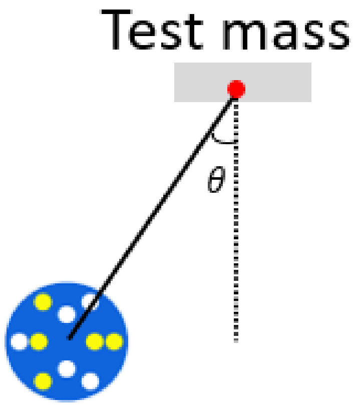
Tungsten density	$19.25 \times 10^3 \ \frac{kg}{m^3}$
Al density	$2.7 \times 10^3 \ \frac{kg}{m^3}$
thickness	$0.05\ m$
r_q	$0.08\ m$
r_h	$0.135\ m$
r_R	$0.175\ m$
r	$0.025\ m$
hexapole	60°
quadrupole	90°
Test mass(KAGRA)	23 kg

Purpose

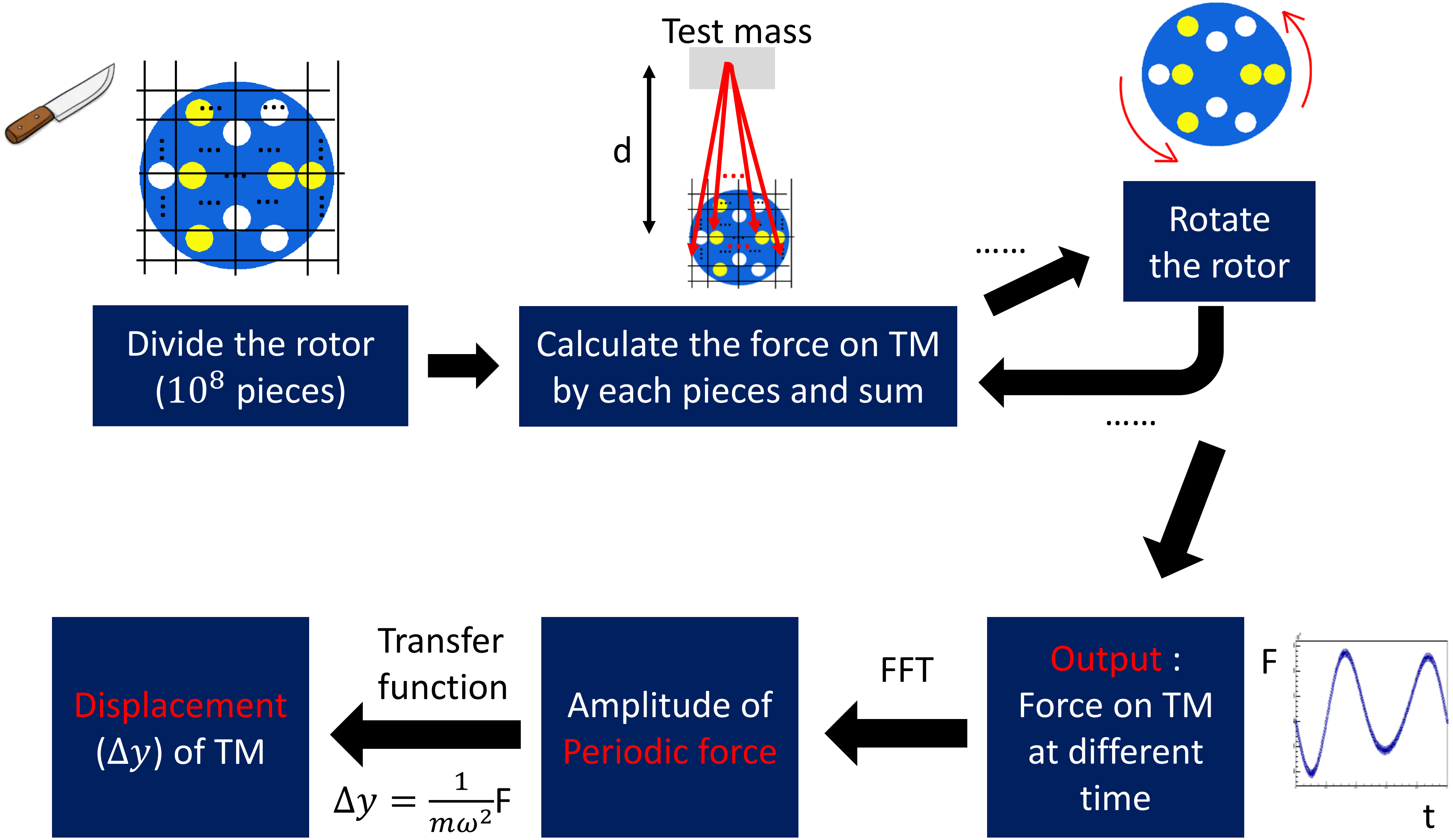
- Would like to simulate(numerical) the force and displacement on test mass in different case

- Single rotor:
  - Simple case : rotor is aligned on the beam axis
  - More general case : rotor is off axis by an angle θ relative to the beam axis





Procedure of Simulation(numerical)



Divide the rotor (10^8 pieces)

Calculate the force on TM by each pieces and sum

Rotate the rotor

Output : Force on TM at different time

FFT

Amplitude of Periodic force

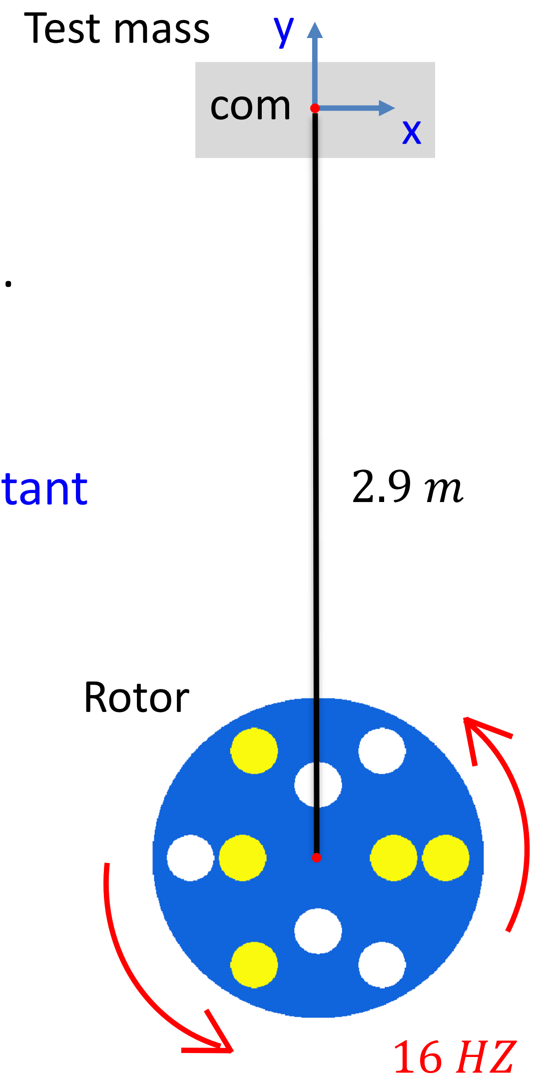
Transfer function

$\Delta y = \frac{1}{m\omega^2}$F

Displacement (Δy) of TM

Single rotor-setup

- ☐ Top view



- ☐ Perfect case:
  - The density of each material is constant.
  - There is no deviation in the position of the configuration.
  - The angular momentum of rotor is constant throughout the rotation.

- ☐ Simulation:
  - Sampling rate : 4096 per second
  - Lasting time : 1 second
  - Output: F_y

Analytical model

- For this configuration:

$$V = \sum_{i=0}^{N} V_i = -GMm_q \sum_{i=0}^{N} {L_i}^{-1} = -\frac{GMm_q}{d} \sum_{i=0}^{N} \sum_{n=0}^{\infty} \left(\frac{r}{d}\right)^n P_n\left[cos\left(\omega_{rot} + \frac{2\pi}{N} i\right)\right]$$

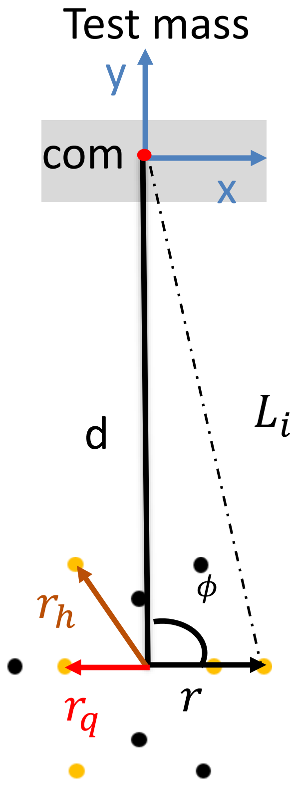
$$\Rightarrow F = \left|\frac{\partial V}{\partial d}\right|$$

$$= \frac{GMm}{d^2} \times \{ \text{ DC term}$$

$$+ \left[\frac{9}{2}\varepsilon^2 + \frac{25}{8}\varepsilon^4 - \frac{735}{256}\varepsilon^6 + \frac{1715175}{1024}\varepsilon^8\right] \times \cos(\mathbf{2}\omega_{rot} t)$$

$$+ \left[\frac{15}{2}\bar{\varepsilon}^3 + \frac{315}{64}\bar{\varepsilon}^5 + \frac{567}{128}\bar{\varepsilon}^7\right] \times \cos(\mathbf{3}\omega_{rot} t)$$

$$+ \text{ other high order terms } \}$$

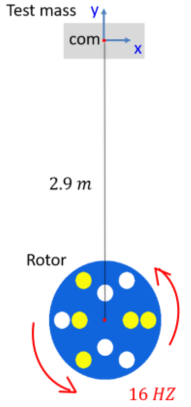


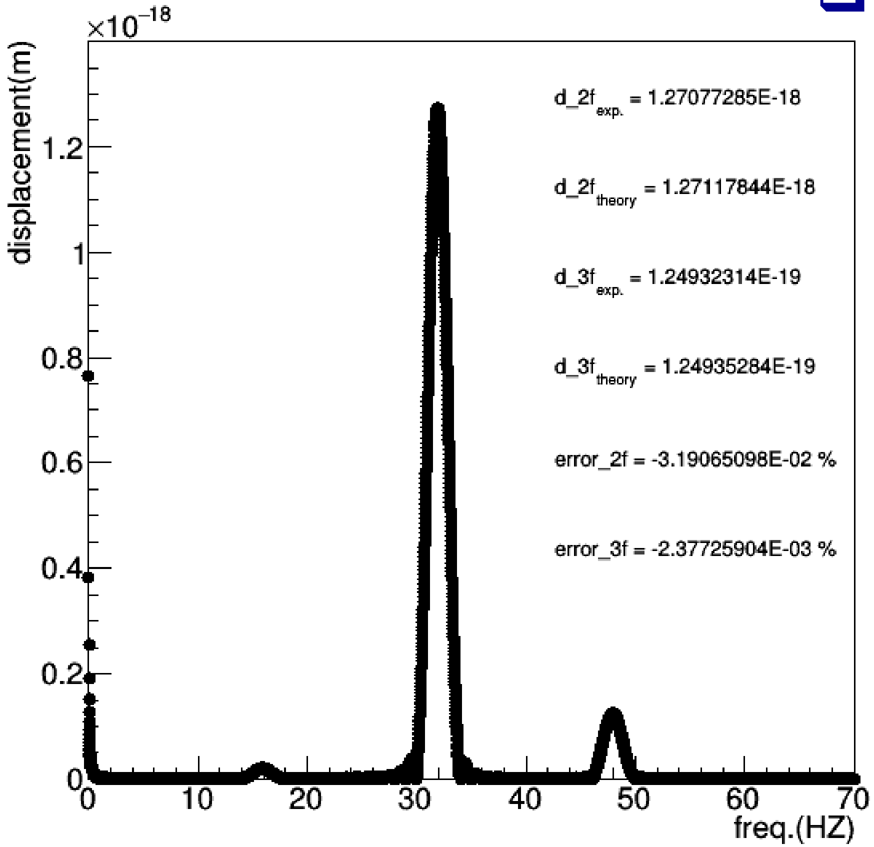
Where $\varepsilon = \frac{r_q}{d}$: from quadrupole

$\bar{\varepsilon} = \frac{r_h}{d}$: from hexapole

Single rotor(rotor)-output

- ❑ The displacement along y axis on test mass (com) :
- ❑ FFT: there are peaks at 32 Hz and 48 Hz.





- ❑ displacement :
  - 2f : 32 HZ : from quadrupole

$$(\Delta y_{2f})_{numerical} = 1.27078 \times 10^{-18}(m)$$

$$(\Delta y_{2f})_{analytical} = 1.27118 \times 10^{-18}(m)$$

Error : 0.032%

  - 3f : 48 HZ : from hexapole

$$(\Delta y_{3f})_{numerical} = 1.249323 \times 10^{-19}(m)$$

$$(\Delta y_{3f})_{analytical} = 1.249353 \times 10^{-19}(m)$$

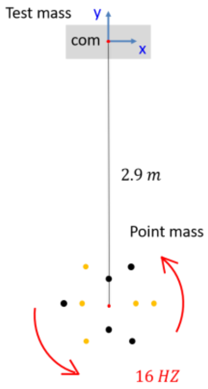
Error : 0.0024%

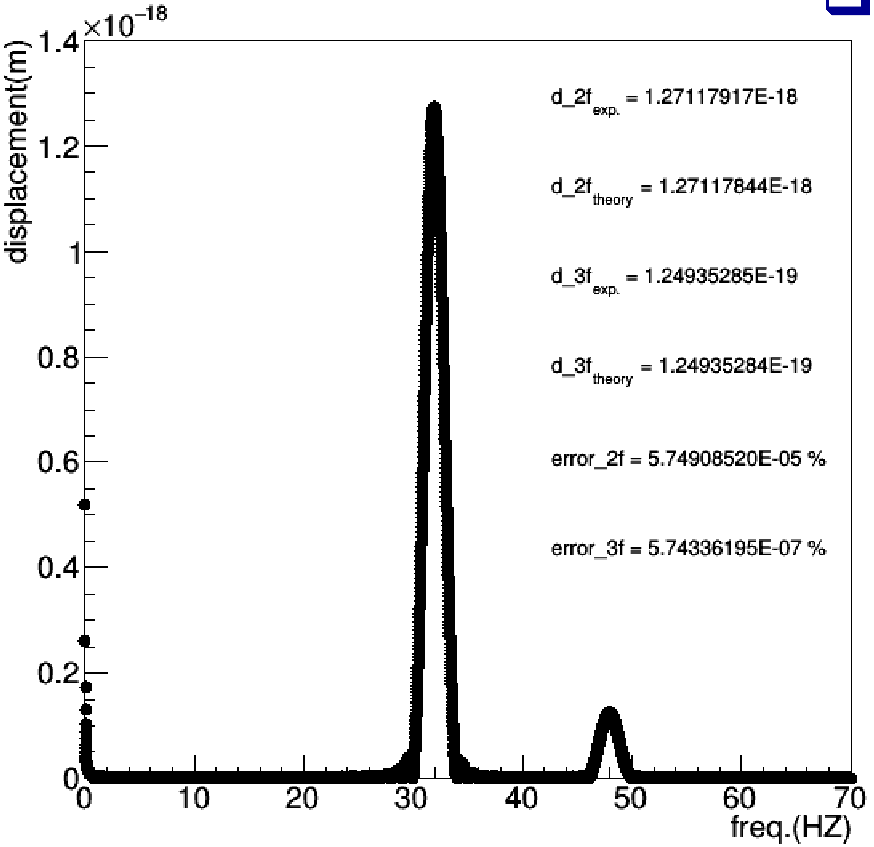
  - Ratio :

$$\left(\frac{\Delta y_{2f}}{\Delta y_{3f}}\right)_{num} = 10.17169 \quad \left(\frac{\Delta y_{2f}}{\Delta y_{3f}}\right)_{anal} = 10.17469$$

Single rotor(point mass)-output

- The displacement along y axis on test mass (com) :
- FFT: there are peaks at 32 Hz and 48 Hz.





- displacement :
  - 2f : 32 HZ : from quadrupole

$$(\Delta y_{2f})_{numerical} = 1.27118 \times 10^{-18} (m)$$

$$(\Delta y_{2f})_{analytical} = 1.27118 \times 10^{-18} (m)$$

Error : $5.75 \times 10^{-5} \%$

  - 3f : 48 HZ : from hexapole

$$(\Delta y_{3f})_{numerical} = 1.249353 \times 10^{-19} (m)$$

$$(\Delta y_{3f})_{analytical} = 1.249353 \times 10^{-19} (m)$$

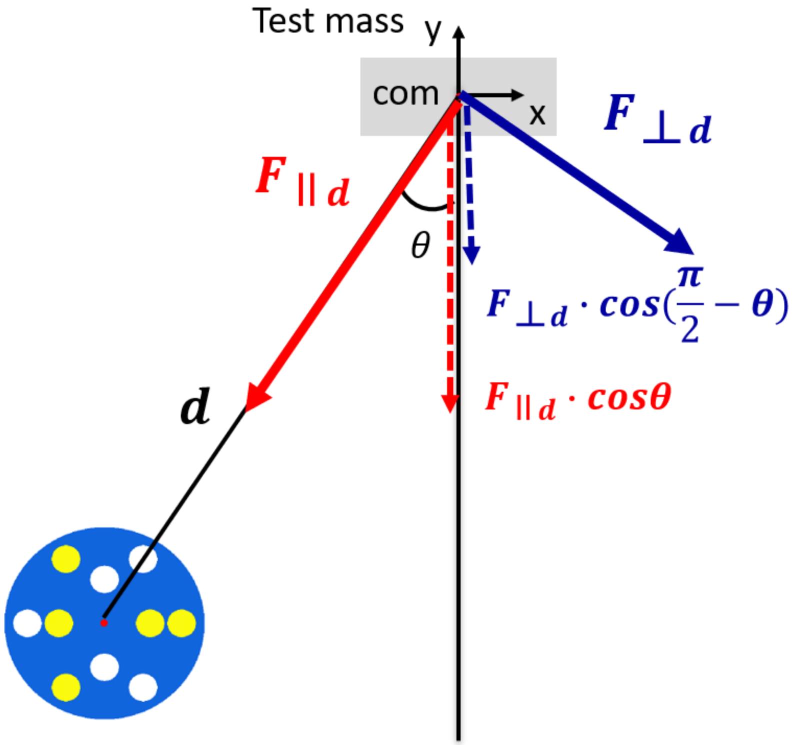
Error :$5.75 \times 10^{-5} \%$

  - Ratio :

$$\left(\frac{\Delta y_{2f}}{\Delta y_{3f}}\right)_{num} = 10.17470 \quad \left(\frac{\Delta y_{2f}}{\Delta y_{3f}}\right)_{anal} = 10.17469$$

More general model(off axis)

- ☐ $F_{y(Net)} = \boldsymbol{F_{\parallel d} \cdot cos\theta} + \boldsymbol{F_{\perp d} \cdot cos\left(\frac{\pi}{2} - \theta\right)}$
- ☐ Parallel force : can be describe by analytical model and match to numerical simulation result

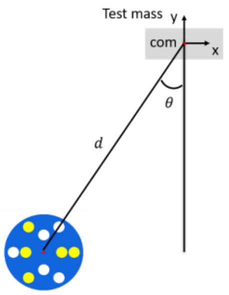


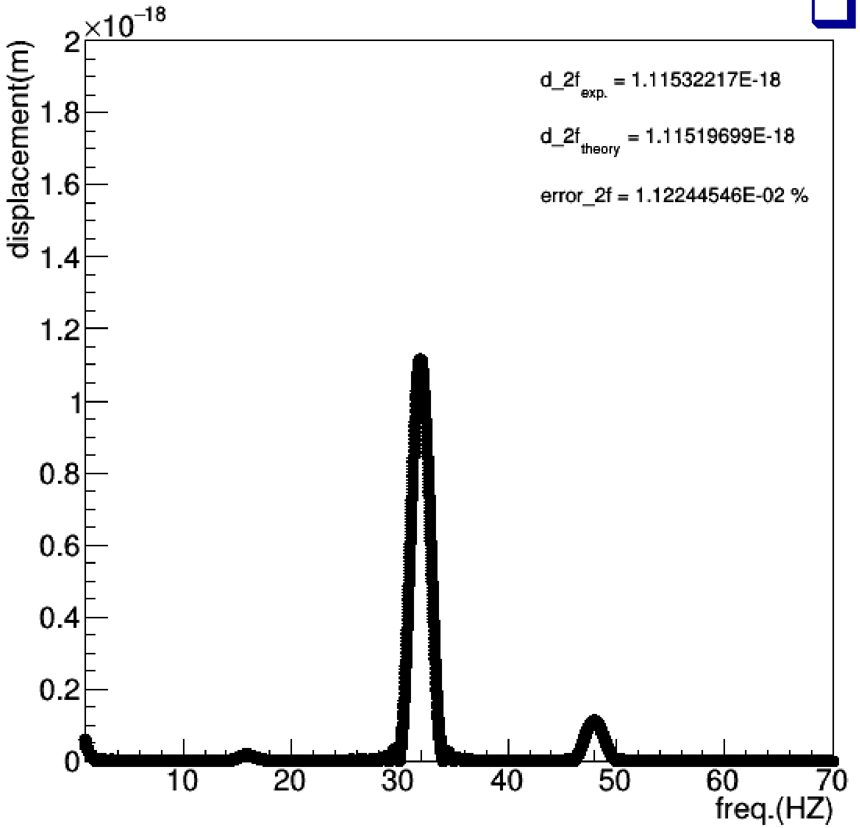
- ☐ Perpendicular force :

  can be **numerically simulated**, but need to derive an **analytical equation** to verify and describe the result

More general model(off axis)-output

- ☐ The displacement along y axis on test mass (com) :
- ☐ FFT: there are peaks at 32 Hz and 48 Hz.





- ☐ displacement :
  - 2f : 32 HZ : from quadrupole

$$(\Delta y_{2f})_{numerical} = 1.115322 \times 10^{-18}(m)$$

$$\boxed{(\Delta y_{2f})_{analytical}} = 1.115196 \times 10^{-18}(m)$$

Error : 0.01122%

$$\boxed{F = \frac{1}{2}\frac{GMm}{d^2} \times [\underbrace{(9cos\theta)^2}_{\boldsymbol{F_{\parallel d}}} + \underbrace{(6sin\theta)^2}_{\boldsymbol{F_{\perp d}}}] \times \varepsilon^2}$$

VIRGO arXiv:1806.06572

Future :

1. The higher order terms of 2f force
2. The 3f force

Summary

- We have developed a point-like model to describe the force(displacement) of test mass(interferometer mirrors) by the rotor
- Simple case(rotor is aligned on the beam axis):
  - Can be predicted and described well by the analytical model
  - There is ~0.04% error between the value from model and rotor simulation
- Future :
  - Derive the analytical model describing the off axis term
  - 2f force with higher order term
  - 3f force

Backup

Gcal signal in Sensitivity limit of KAGRA

- ☐ $(\Delta y_{32Hz})_{numerical} \quad = 1.27078 \times 10^{-18} (m)$
- ☐ $(\Delta y_{48Hz})_{numerical} \quad = 1.249323 \times 10^{-19} (m)$
- ☐ SNR : 32Hz : ~ 19.134 ; 48Hz : ~ 5.141