Improvement of calibration error method with higher order harmonics

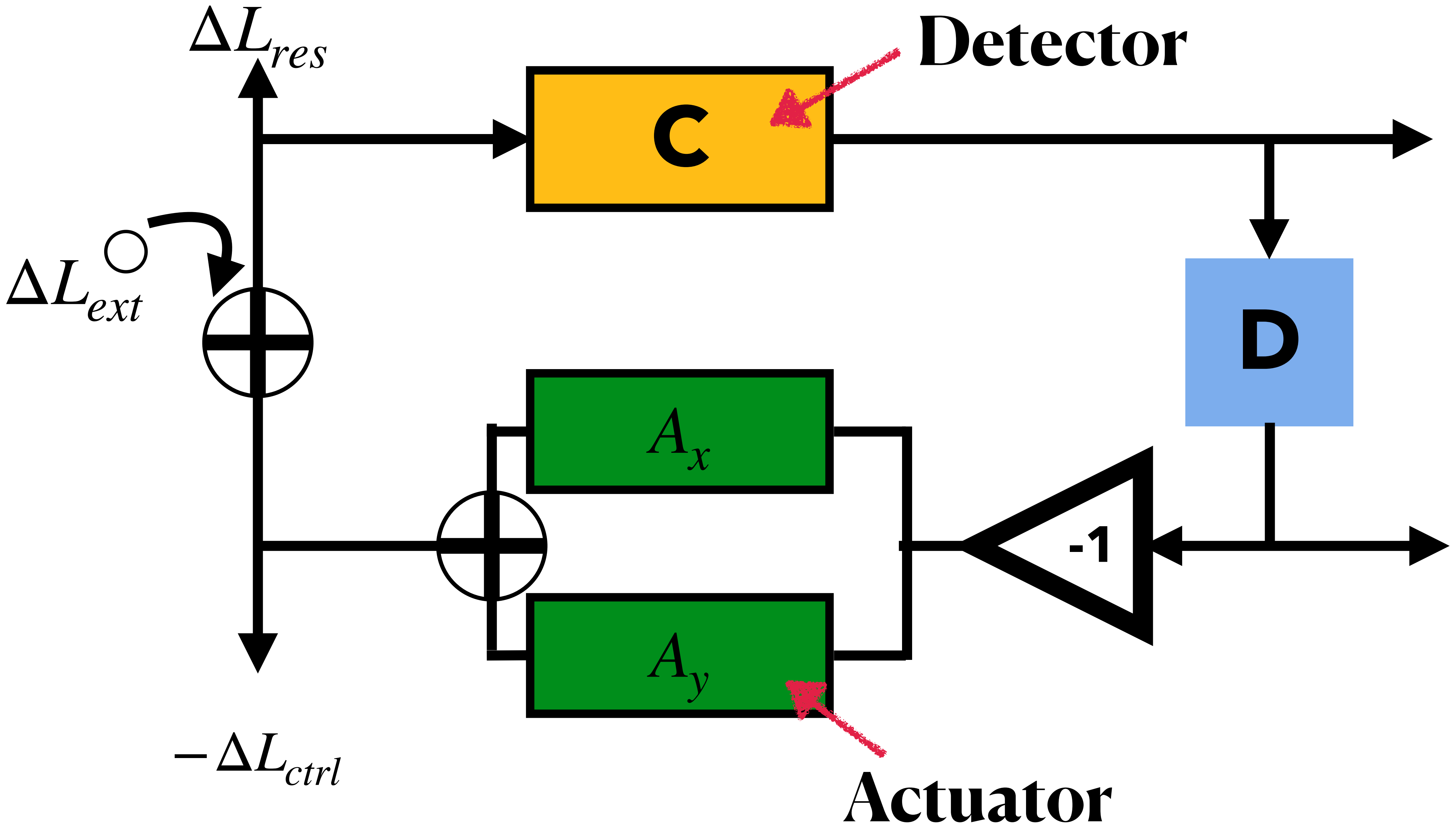
Hsiang-Yu Huang on behalf of KAGRA collaboration
National Taiwan Normal University
Academia Sinica
Supervisor : Prof. Henry Wong, Academia Sinica
Collaborator : Prof. Yuki Inoue, National Central University

12/19/2020 The 7th KAGRA International Workshop

Outline

- Overview of calibration
- Calibrator
- Calibration measurement
- Analysis procedure
- Analysis demonstration
- Simulation Result
- Summary

Modeling the interferometer



- Measure the changes of arm length between Detector(C) and Actuator(A) . Based on these information , we can separate the C model and A model by estimating parameter of C and A.

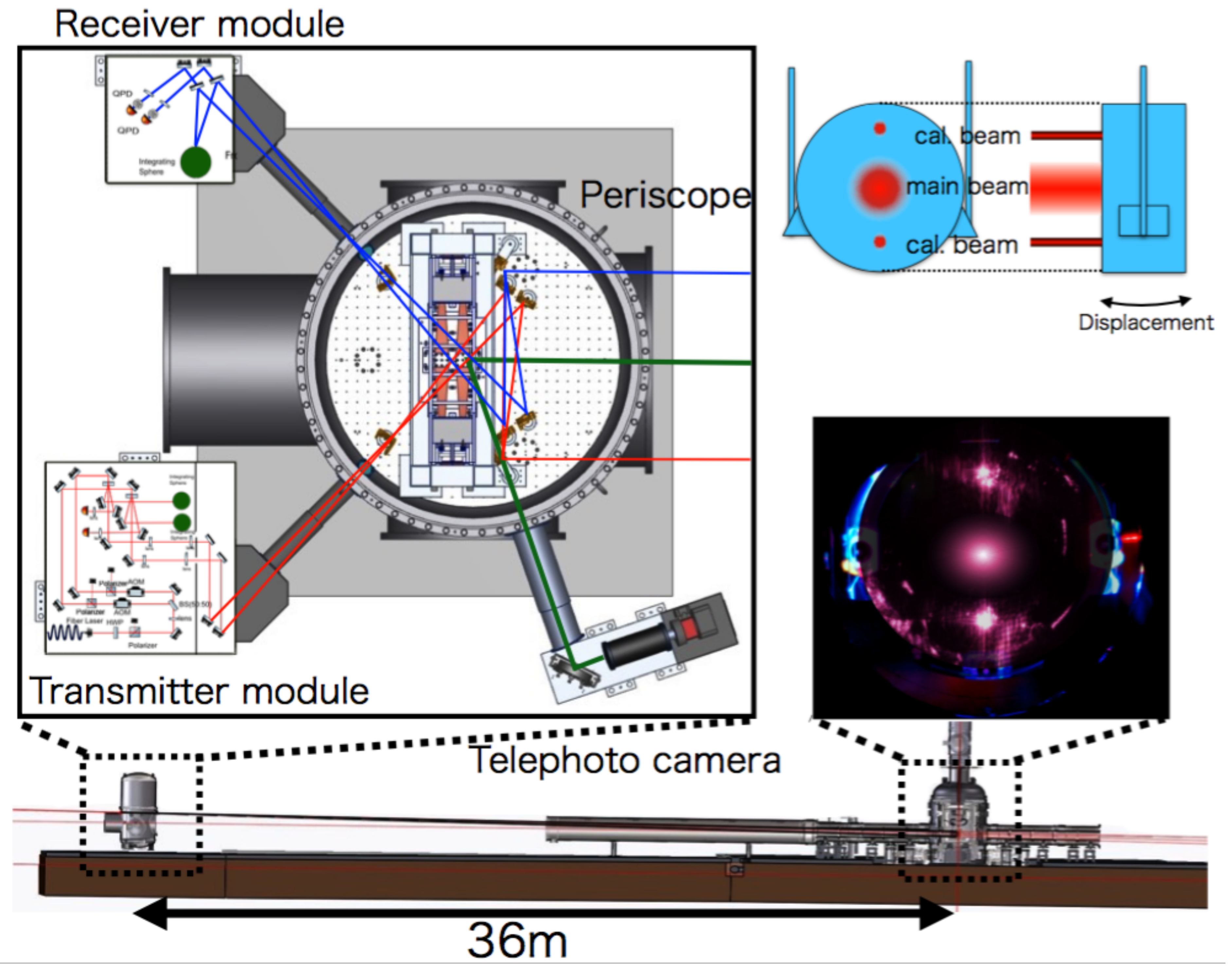
Definition of Calibration

Parameter estimation of C and A

Definition of Reconstruction

Calculate interferometer response

Photon Calibrator



- Push the mirror (test mass) by photon pressure.
- We inject the signal from outside to separate and estimate C and A.

$$dx = \frac{2P\cos\theta}{c} s(f)\left(1 + \frac{M}{I}\vec{a}\cdot\vec{b}\right)$$

Force

Geometrical factor

$$= \Gamma V_{signal} s(f)$$

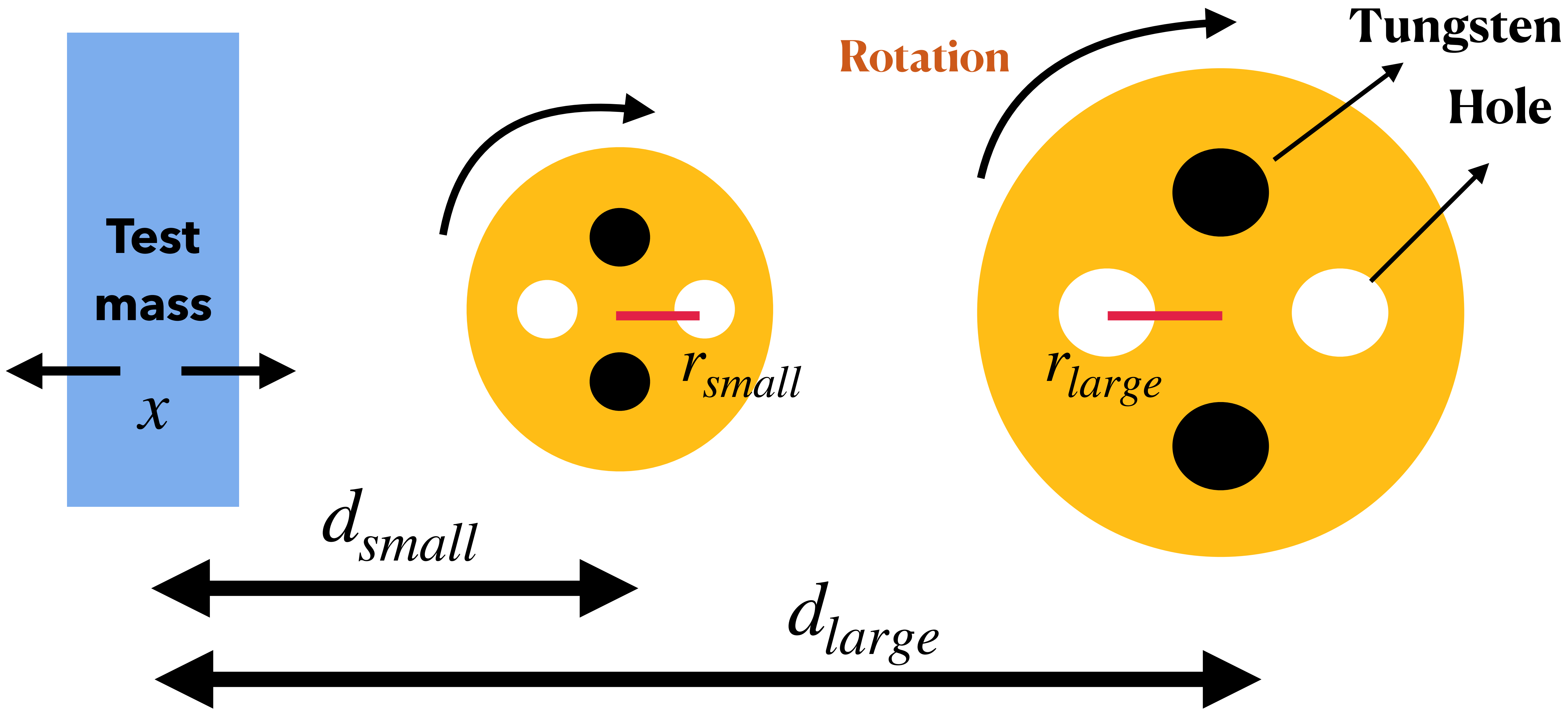
Γ : Force coefficient(m/V)

Free mass motion : $s(f) = \frac{1}{M(2\pi f)^2}$

Error source

- Absolute calibration limit ~3 %

Gravity field Calibrator



Analytical Solution

$$F_{small} = \frac{2GMm}{d_{small}^2}\epsilon_{small}^n(n+1)P_n(\cos\theta)$$

$$F_{large} = \frac{2GMm}{d_{large}^2}\epsilon_{large}^n(n+1)P_n(\cos\theta)$$

$$\longrightarrow \quad x = \frac{1}{Mw^2}(F_{small} + F_{large})$$

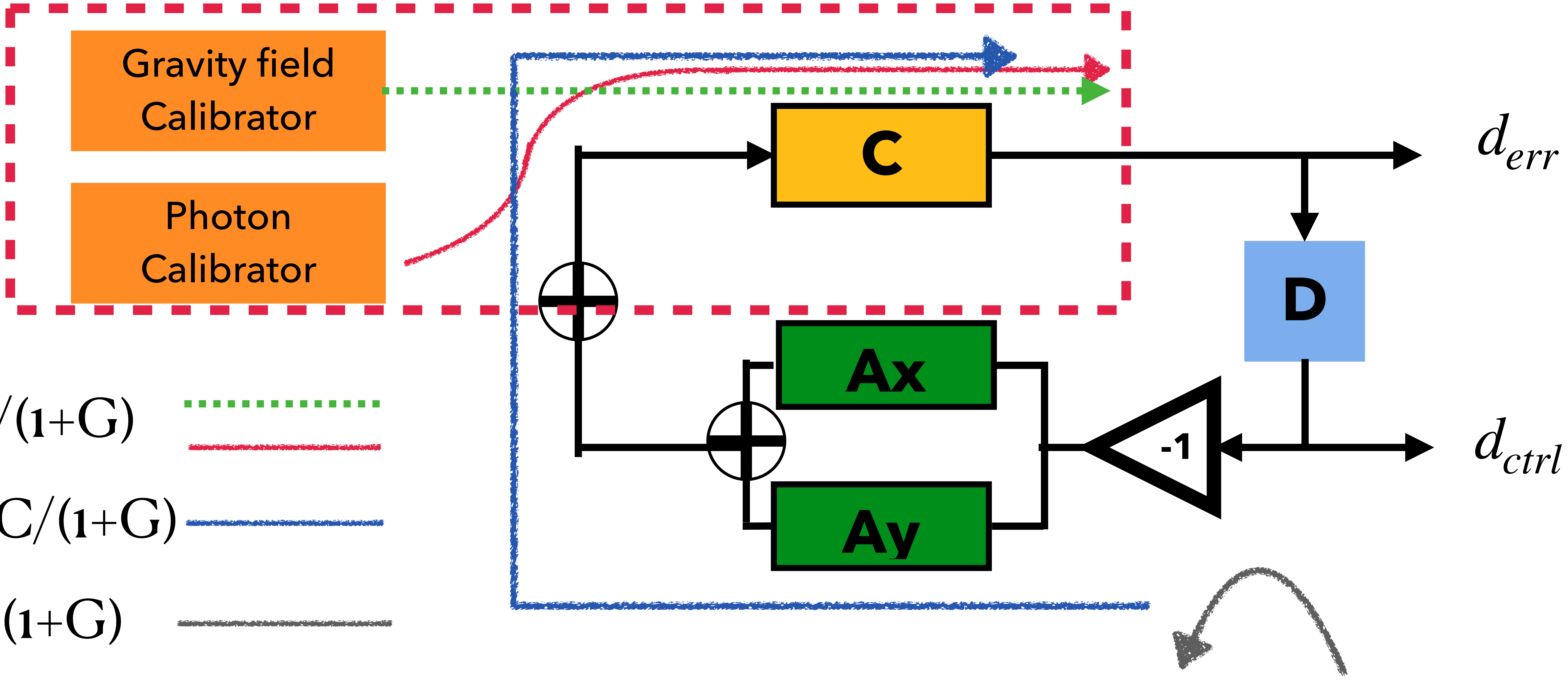
$$\epsilon = \frac{r_s}{d_s}, \quad s = small, large$$

P_n : Legendre Polynomial $\quad w : 2\pi f$

Error source

- higher order harmonics

Calibration measurement



- Signal 1 : C/(1+G)
- Signal 2 : AC/(1+G)
- Signal 3 : 1/(1+G)

$G = ACD$

Error source

- Parameter estimation of C model

Analysis procedure

Idea demonstration

Measurement data accumulation / processed

↓

Mean and standard deviation estimation

↓

Estimate the ratio of PCal signal to GCal signal

PCal

GCal

Demodulation

Science target

PCal measurement

$$d_{err}^{PCal} = \eta x_{PCal}^{Model} \frac{C}{(1+G)}$$

GCal measurement

$$d_{err}^{GCal} = x_{GCal}^{2f,4f,6f} \frac{C}{(1+G)}$$

GCal/PCal

C/(1+G) cancel out !

$$\frac{1}{\eta} = \frac{d_{err}^{GCal}}{d_{err}^{PCal}} \frac{x_{PCal}^{2f,model}}{x_{GCal}^{2f,model}} = \frac{(x_{GCal}^{2f} + x_{GCal}^{4f} + x_{GCal}^{6f})}{x_{PCal}{}^{2f}} \frac{x_{PCal}^{2f,model}}{x_{GCal}^{2f,model}}$$

By estimating and subtracting the ratio of high order term of GCal and PCal signal in specific frequency

$$\left\{ \begin{array}{l} \dfrac{x_{GCal}^{4f}}{x_{PCal}^{2f}} \\ \dfrac{x_{GCal}^{6f}}{x_{PCal}^{2f}} \end{array} \right.$$

→ $\eta = x_{PCal}^{2f} / x_{GCal}^{2f}$

Subtraction of higher order terms

- Expansion of Legendre polynomial

$$x_{small} = \frac{2Gm}{w^2d^2}[(\frac{9}{4}\epsilon^2 + \frac{25}{16}\epsilon^4 + \frac{15}{256}\epsilon^6)\cos 2wt + (\frac{175}{64}\epsilon^4 + \frac{-21}{128}\epsilon^6)\cos 4wt + \frac{273}{256}\epsilon^6 \cos 6wt]$$

$$F_{2f} = f_{2f}^2 + f_{2f}^4 + f_{2f}^6 \qquad F_{4f} = f_{4f}^4 + f_{4f}^6 \qquad F_{6f} = f_{6f}^6$$

$$F_{2f} - k_{2,6}F_{6f} = f_{2f}^{(2)} + f_{2f}^{(4)} \quad \longrightarrow \quad k_{2,6} = \frac{f_{2f}^{(6)}}{f_{6f}^{(6)}}$$

$$F_{4f} - k_{4,6}F_{6f} = f_{4f} \quad \longrightarrow \quad k_{4,6} = \frac{f_{4f}^{(6)}}{f_{6f}^{(6)}}$$

$$f_{2f}^{(2)} + f_{2f}^{(4)} - k_{2,4}f_{4f}^{(4)} = f_{2f}^{(2)} \quad \longrightarrow \quad k_{2,4} = \frac{f_{2f}^{(4)}}{f_{4f}^{(4)}}$$

Analytical parameter

$$k_{4,6} = \frac{-2}{13}$$

$$k_{2,6} = \frac{15}{273}$$

$$k_{2,4} = \frac{4}{7}$$

Simulation parameter input

- Monte Carlo Simulation

Small GCal

Parameter	Value	Relative Uncertainty
G	$6.67408 \times 10^{-11} m^3/kgs^{-1}$	4.7×10^{-5}
m_{s1}	1.8842 kg	4.8×10^{-5}
m_{s2}	1.8842 kg	4.8×10^{-5}
d_s	2.9 m	1×10^{-5}
r_s	0.08 m	1×10^{-3}
f	32	1×10^{-5}
θ_s	0°	

Large GCal

Parameter	Value	Relative Uncertainty
G	$6.67408 \times 10^{-11} m^3/kgs^{-1}$	4.7×10^{-5}
m_{l1}	1.8842 kg	4.8×10^{-5}
m_{l2}	1.8842 kg	4.8×10^{-5}
d_l	$2.9\sqrt{2}m$	1×10^{-5}
r_l	0.08 m	1×10^{-3}
f	32	1×10^{-5}
θ_l	0°	

Simulation parameter input

PCal

Parameter	Value
P	6W
$\cos\theta$	1
C	$3 \times 10^8 m/s$
M	22.95 kg
f	32 Hz
$1 + \frac{M}{I}\vec{a} \cdot \vec{b}$	1
η	0.97

Set parameter from PCal calibration limit

Simulation result - subtraction prove

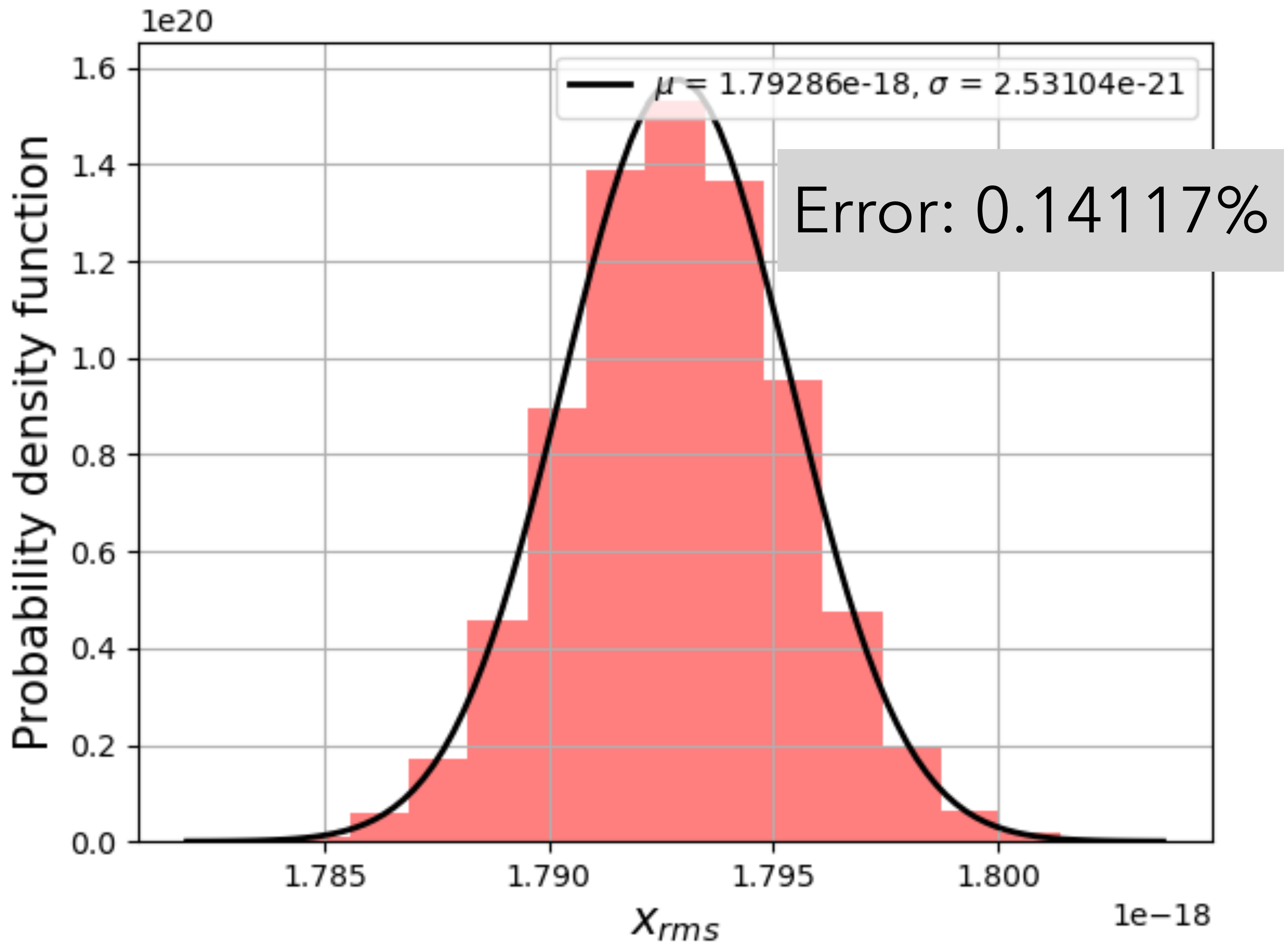
- Monte Carlo simulation

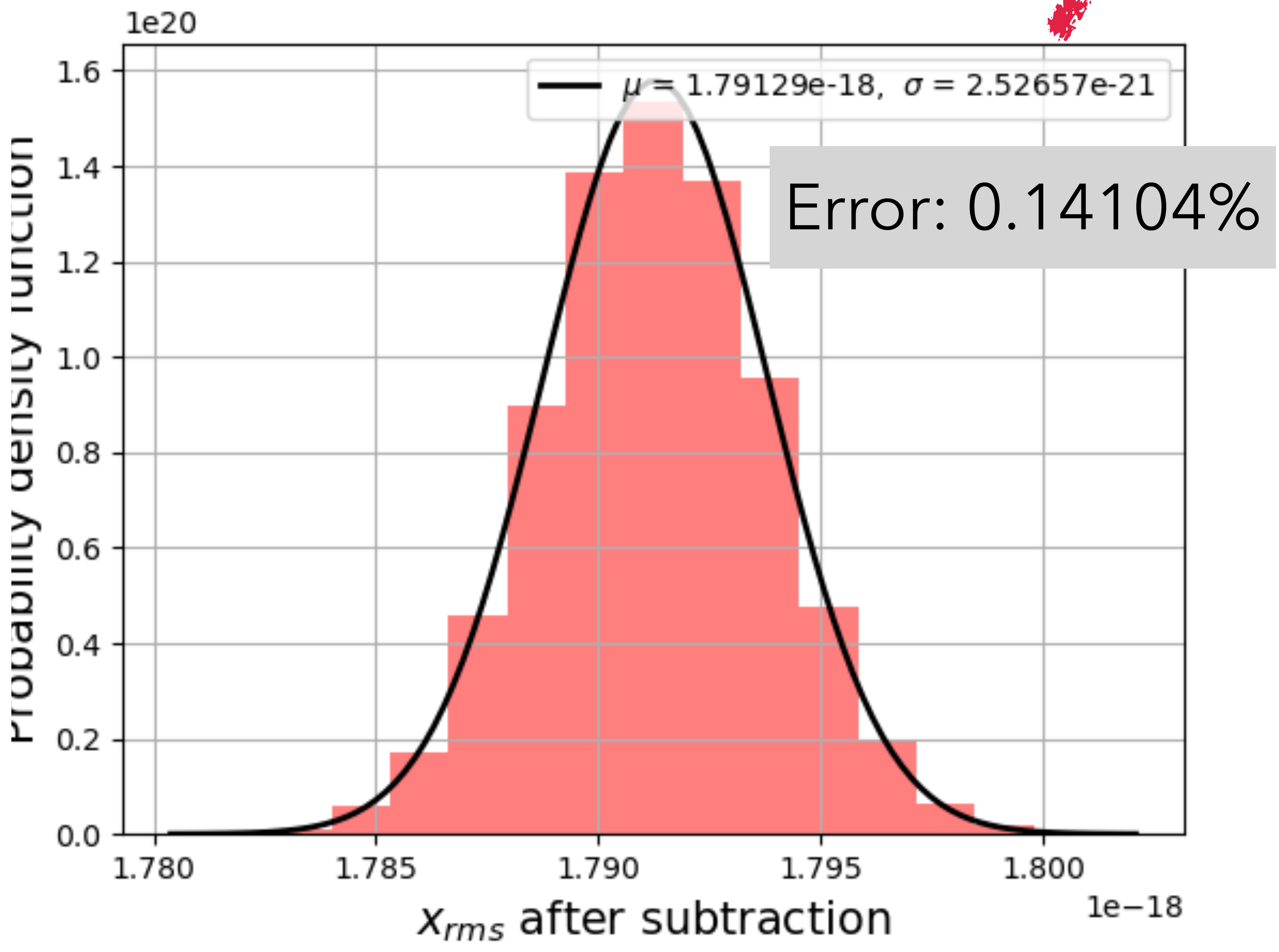
Consistent !

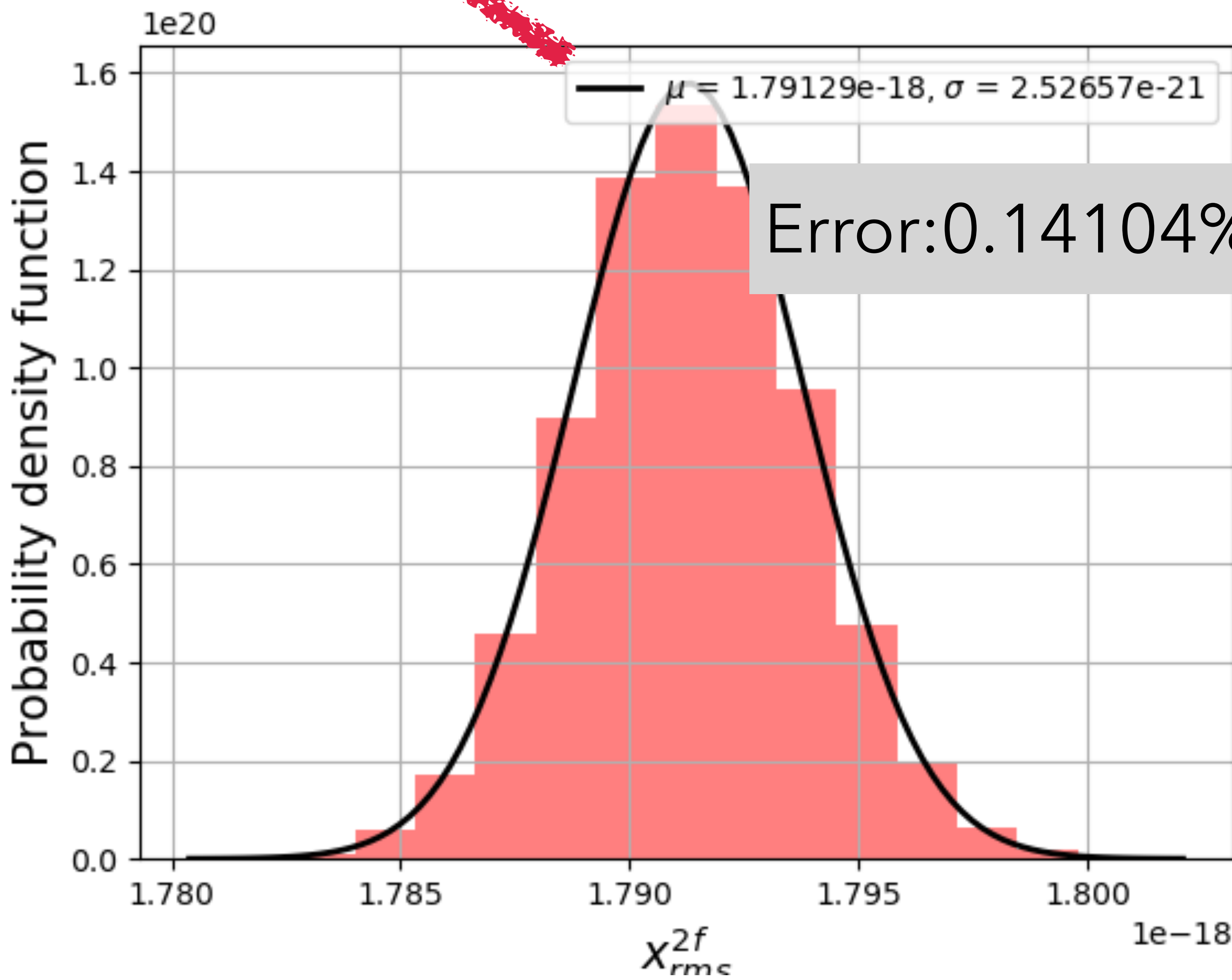
Before

After

2f formula

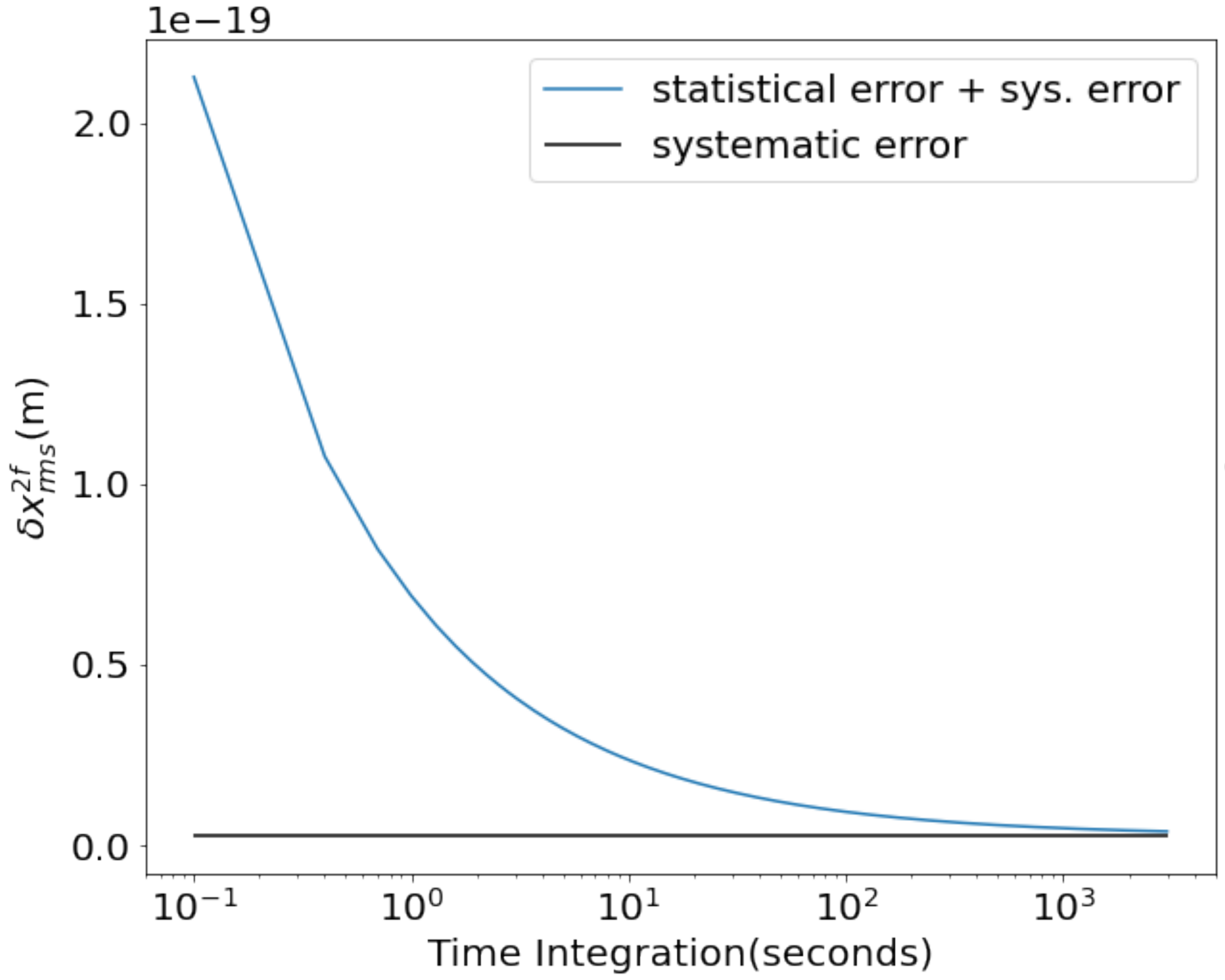




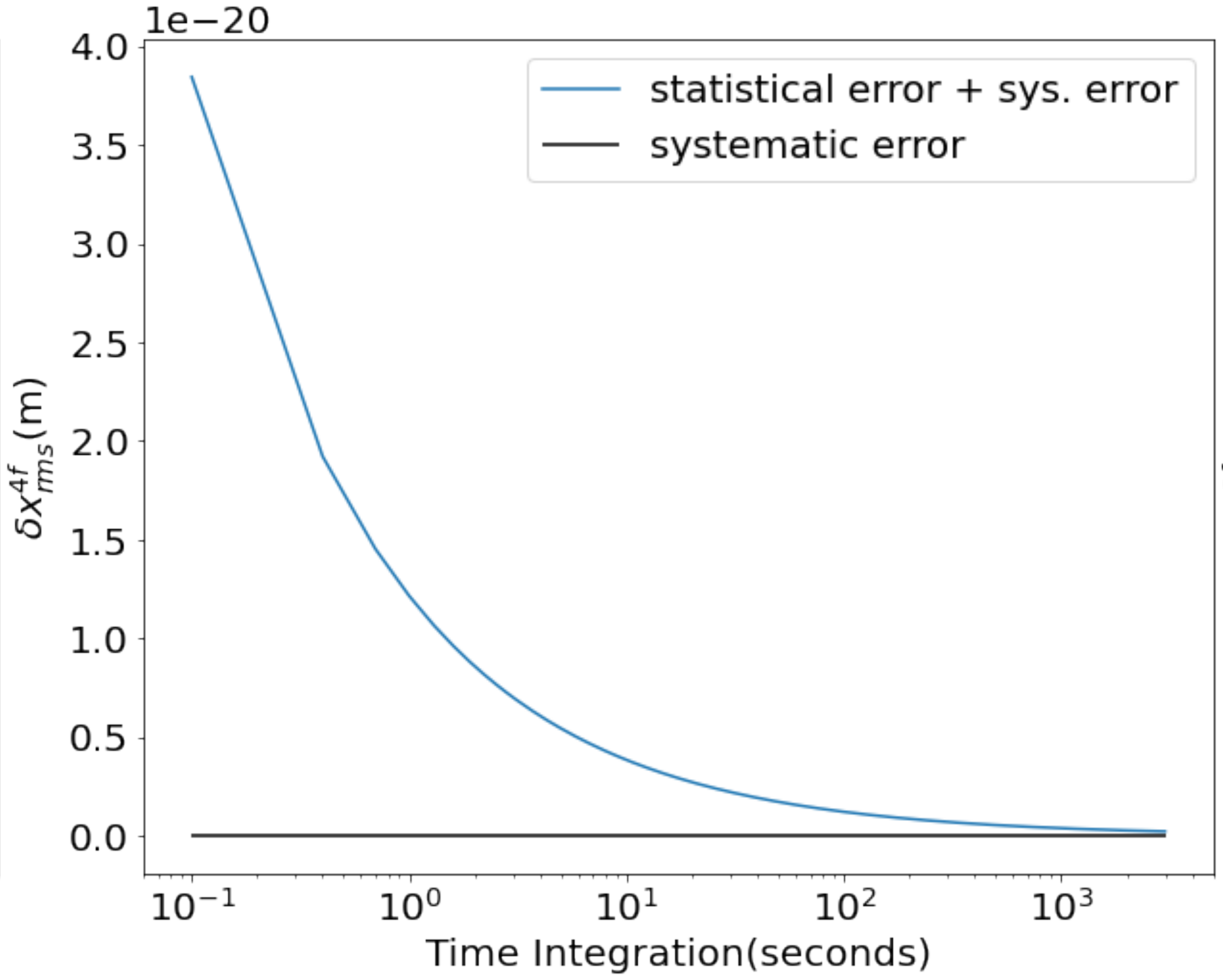


Simulation result - Integration time estimation

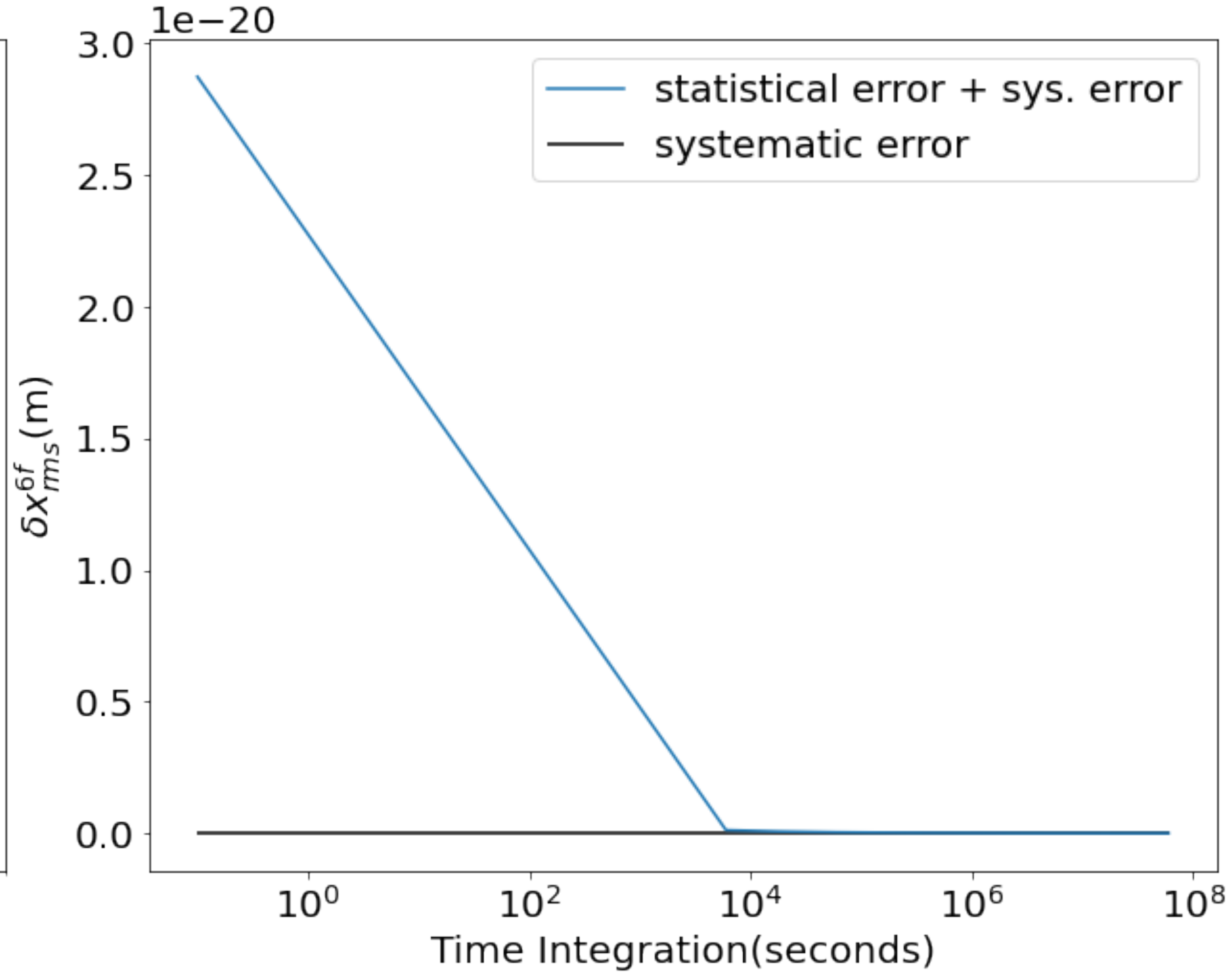
2f



4f



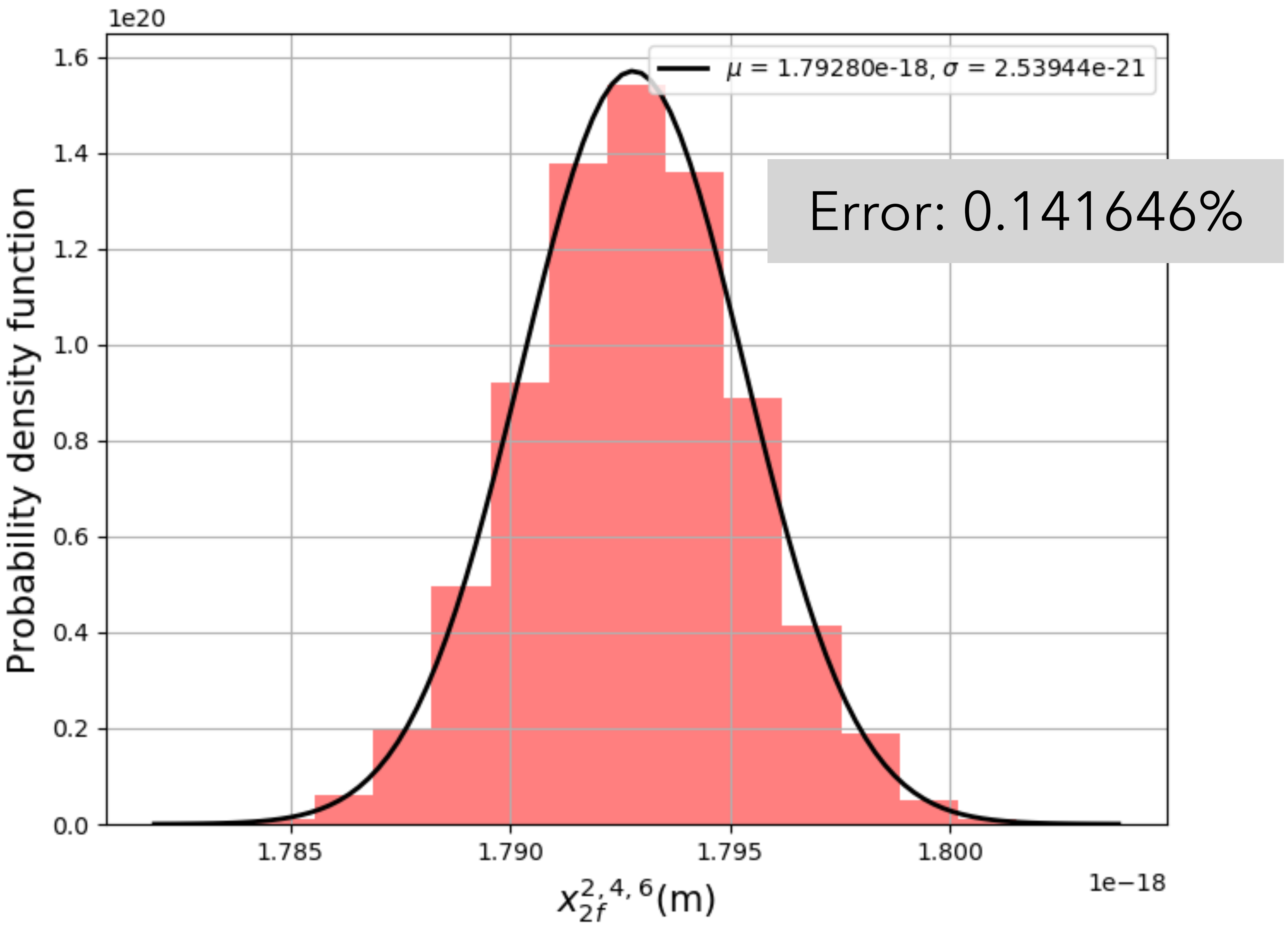
6f



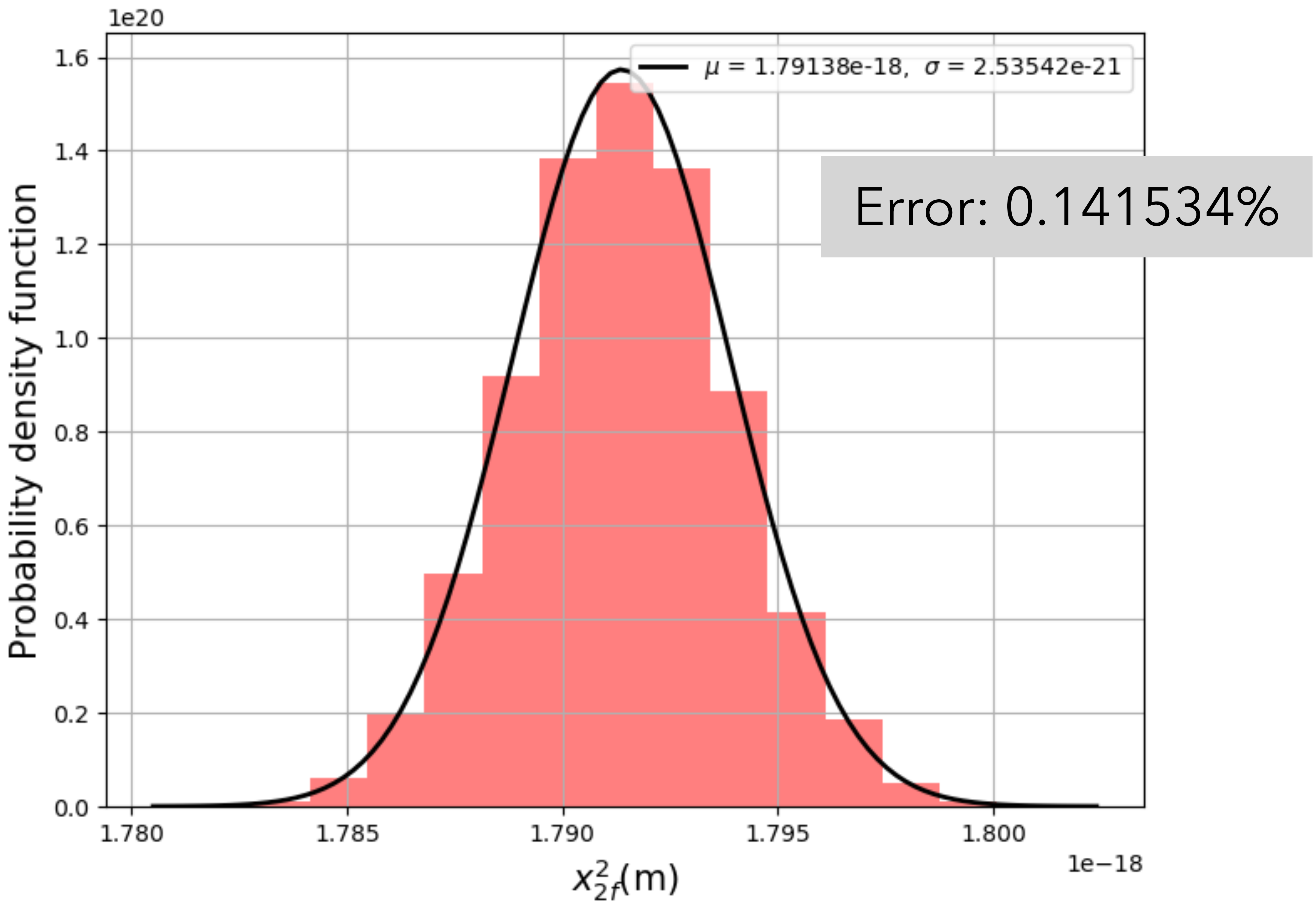
- In typical calibration measurement, we take 1 ~ 10 minutes

Before and After subtraction with integration time

Before



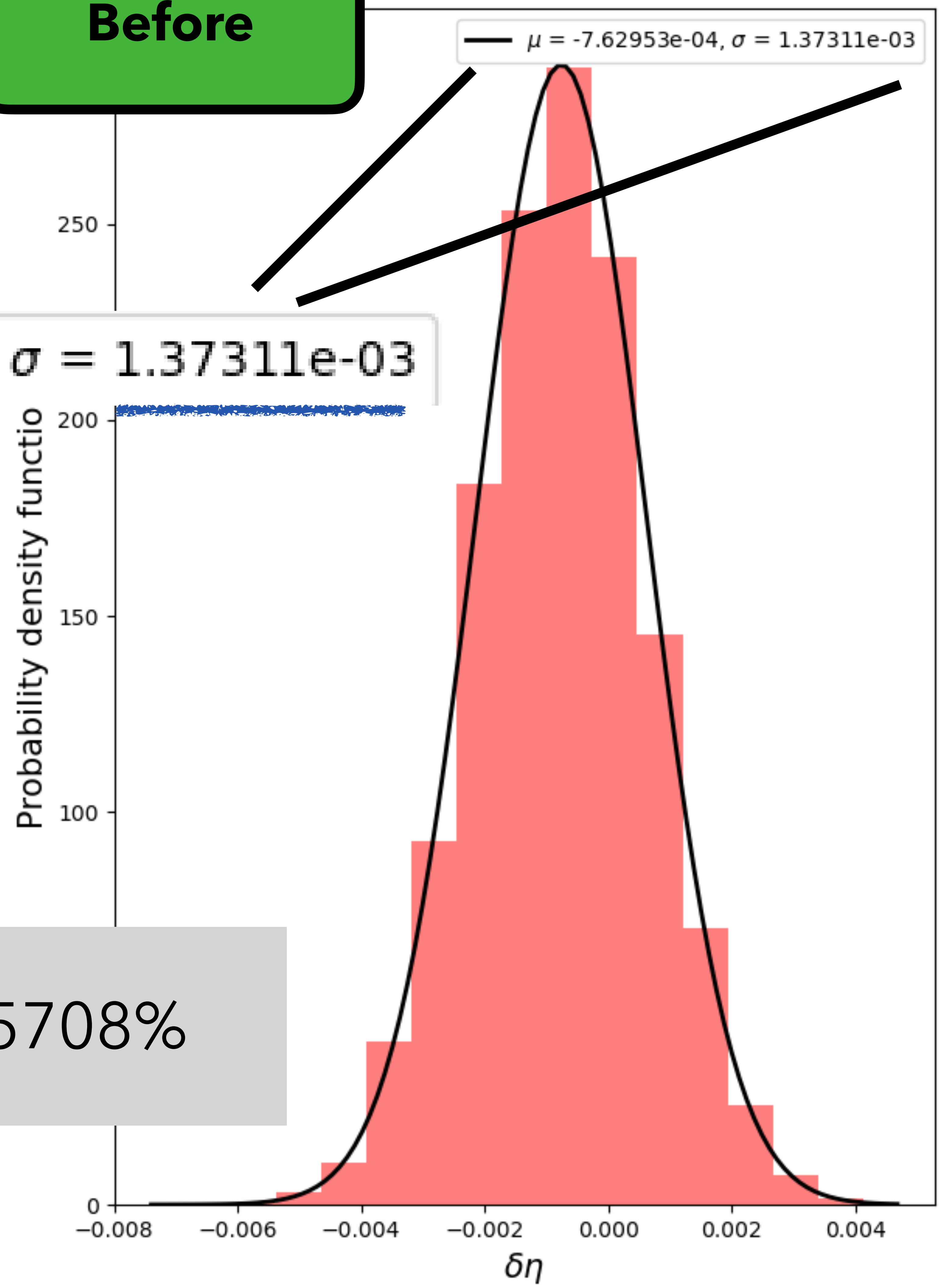
After



Integration time : 1500 seconds

Simulation Result - η estimation

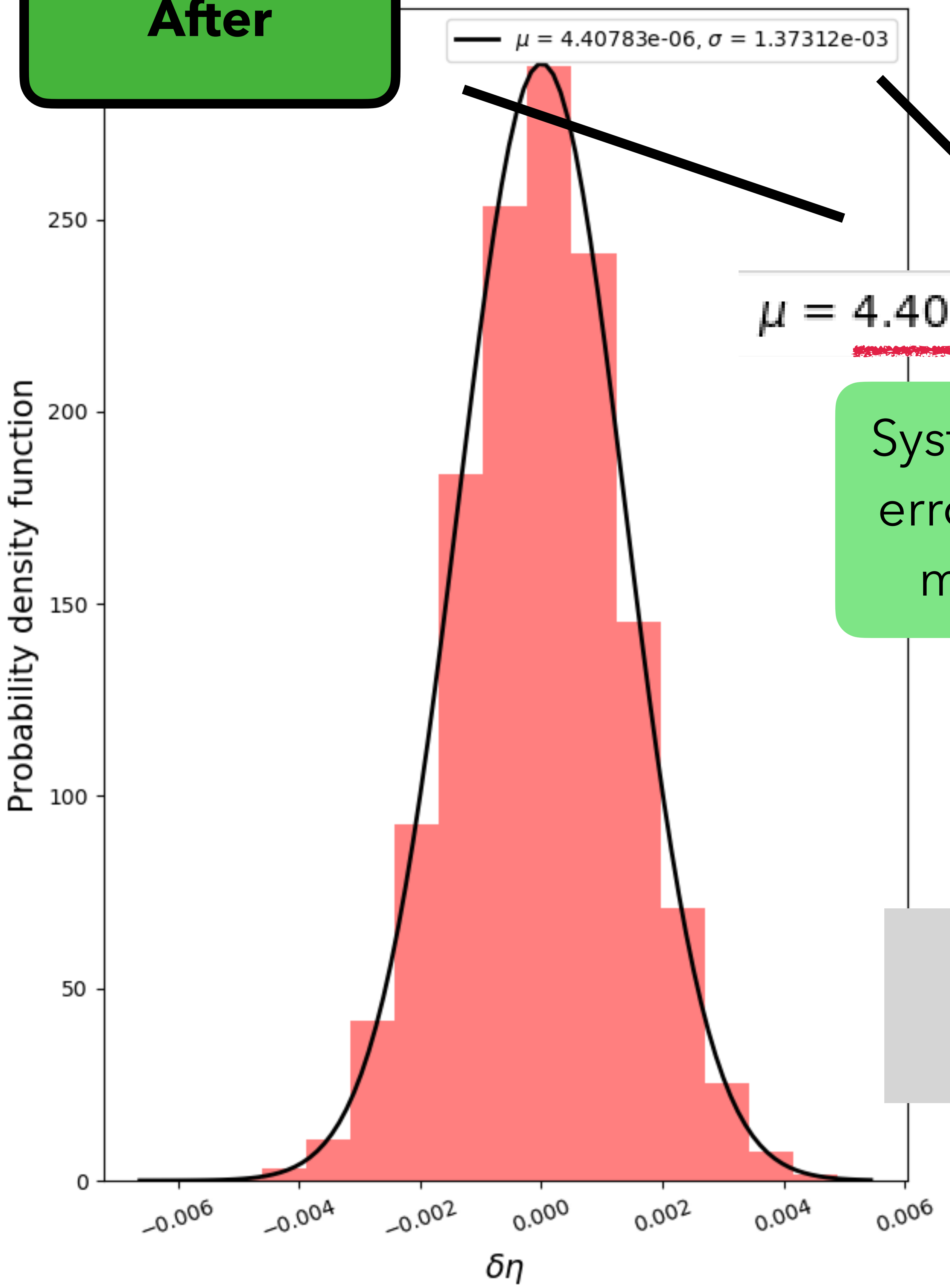
Before



μ = -7.62953e-04, σ = 1.37311e-03

σ : 0.15708%

After

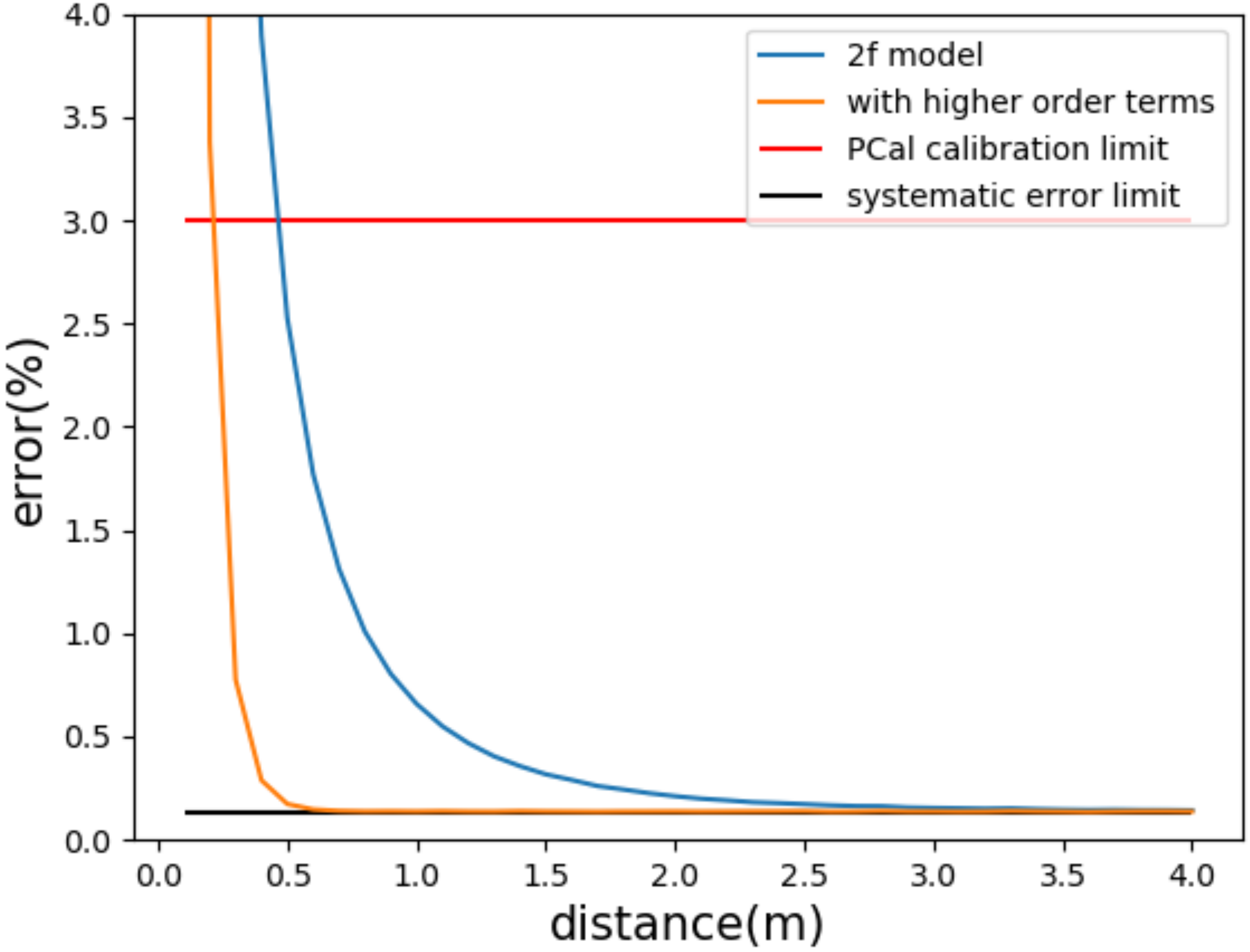


μ = 4.40783e-06, σ = 1.37312e-03

Systematic error of 2f model

Systematic error of GCal

σ : 0.13731%



2f model
with higher order terms
PCal calibration limit
systematic error limit
error(%)
distance(m)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

Summary

- We demonstrate and propose a new method to improve the calibration error.
- By this method, we reduce systematic error of gravity field calibrator with higher orders and moderate the error from calibration measurement

Next Step

- Simulation include calibration measurement — time-dependent parameter

PCal measurement

$$d_{err}^{PCal} = \eta x_{PCal}^{Model} \frac{C}{(1+G)}$$

GCal measurement

$$d_{err}^{GCal} = x_{GCal}^{2f,4f,6f} \frac{C}{(1+G)}$$

GCal/PCal

C/(1+G)
cancel out !

$$\frac{1}{\eta} = \frac{d_{err}^{GCal}}{d_{err}^{PCal}} \frac{x_{PCal}^{2f,model}}{x_{GCal}^{2f,model}} = \frac{(x_{GCal}^{2f} + x_{GCal}^{4f} + x_{GCal}^{6f})}{x_{PCal}{}^{2f}} \frac{x_{PCal}^{2f,model}}{x_{GCal}^{2f,model}}$$

$$\frac{x_{GCal}^{4f}}{x_{PCal}^{2f}}, \quad \frac{x_{GCal}^{6f}}{x_{PCal}^{2f}}$$

Include the effect of time-dependent parameter

By estimating and subtracting the ratio of high order term of GCal and PCal signal in specific frequency

→ $\eta = x_{PCal}^{2f}/x_{GCal}^{2f}$

Thank you for your listening!

Appendix

1. Photon Calibrator(PCal)

- Force coefficient leads to 3% uncertainty
- Estimation error from transfer function

2. Gravity field Calibrator(GCal)

- Systematic error from high order harmonic oscillation term of GCal
- Estimation error from transfer function

Preparation of analysis

GCal

PCal

Simulation

Demodulation Method

Pipeline- C2X FIR

d_{err}^{GCal}

d_{err}^{PCal}

$$TF = \frac{C}{1 + G}$$

PCal reconstruction

- Demodulation method
- Low pass filter
- Bandpass filter
- GWpy package : demodulate