Gravitational waves from supernova and protoneutron star asteroseismology

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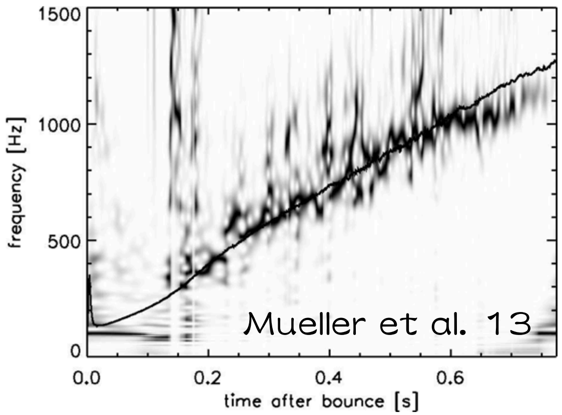
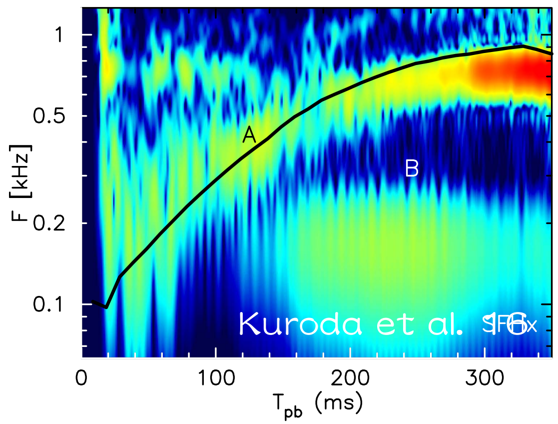
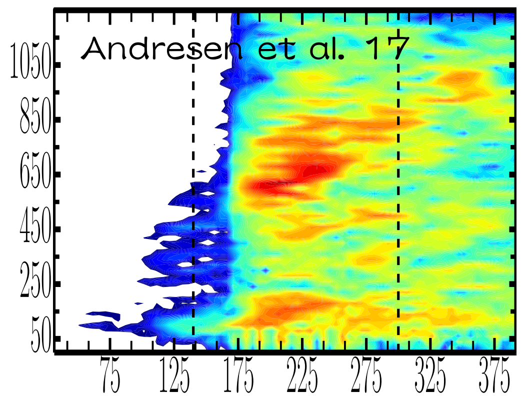
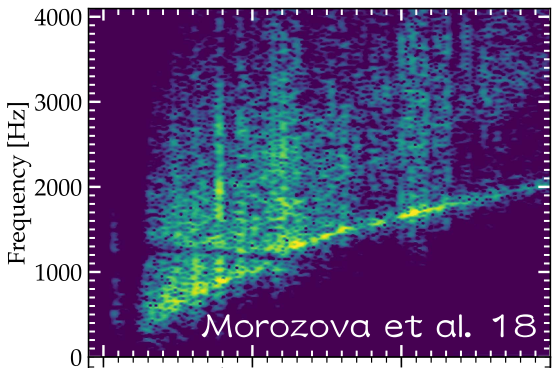
2nd candidate as GW sources

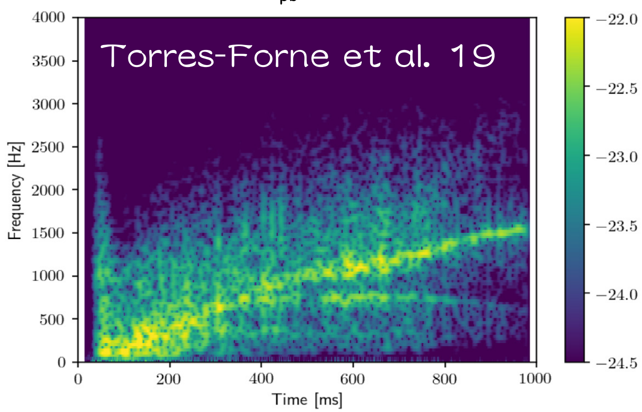
- supernovae
  - event rate : ~1/100 yr in our galaxy
  - compered to binary merger, system is more spherically symmetric
    - less energy of gravitational waves
  - many numerical simulations show the existence of GW signals





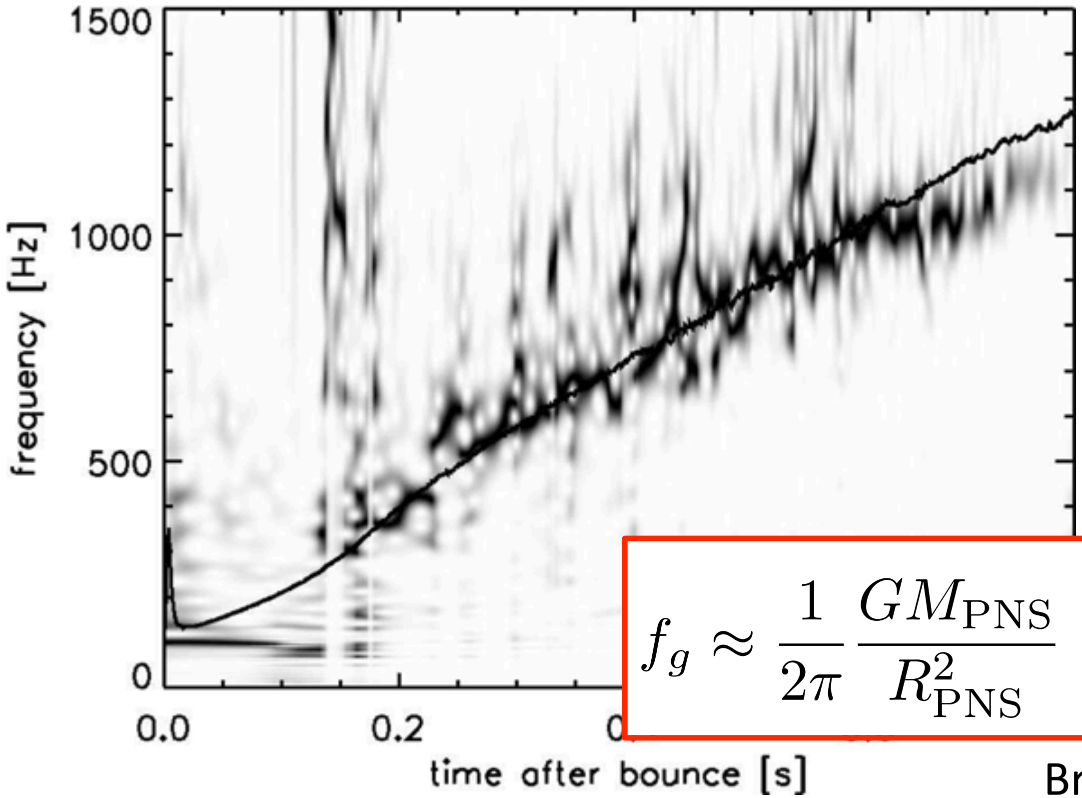


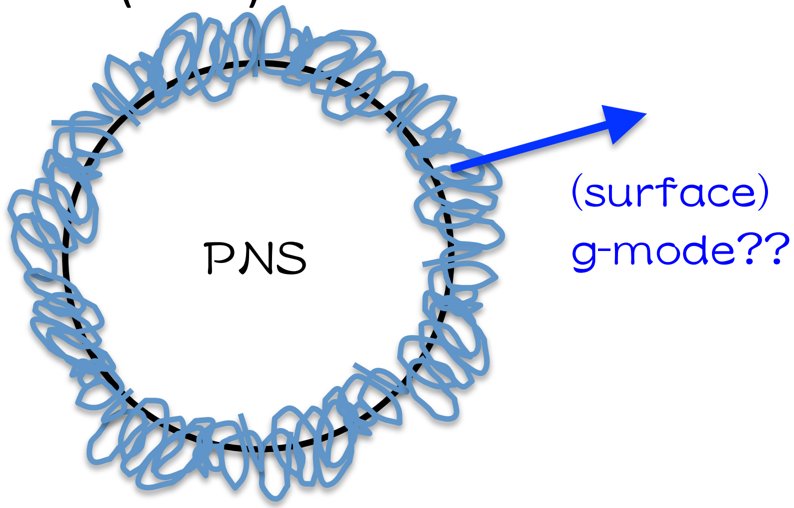




g-mode oscillations?

- 2D non-rotation with convection by Muller et al. (2013)
  → excitations of specific frequency



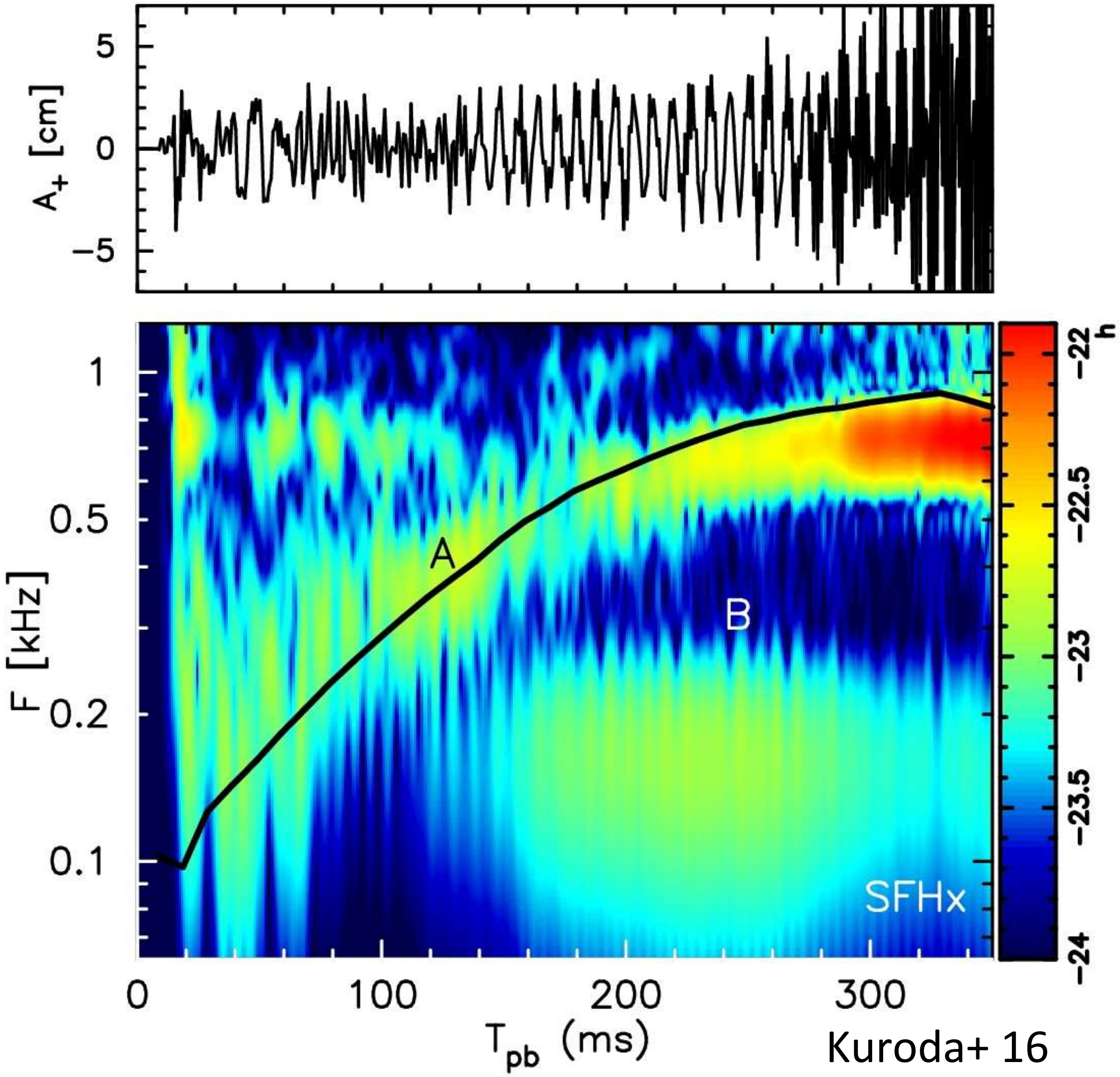


the convection & the standing accretion-shock instability

$$f_g \approx \frac{1}{2\pi}\frac{GM_{\rm PNS}}{R^2_{\rm PNS}}\left(\frac{1.1 m_n}{\langle E_{\bar{\nu}_e}\rangle}\right)^{1/2}\left(1-\frac{GM_{\rm PNS}}{c^2 R_{\rm PNS}}\right)^2$$

Brunt-Vaisala frequency @ PNS surface

GW from PSNs



Kuroda+ 16

- Numerical simulations tell us the GW spectra.
- difficult
  - to extract PNS physics
- We adopt the perturbation approach to see the physic behind the GWs by identifying them with the freq. from PNS.

eigenfrequencies

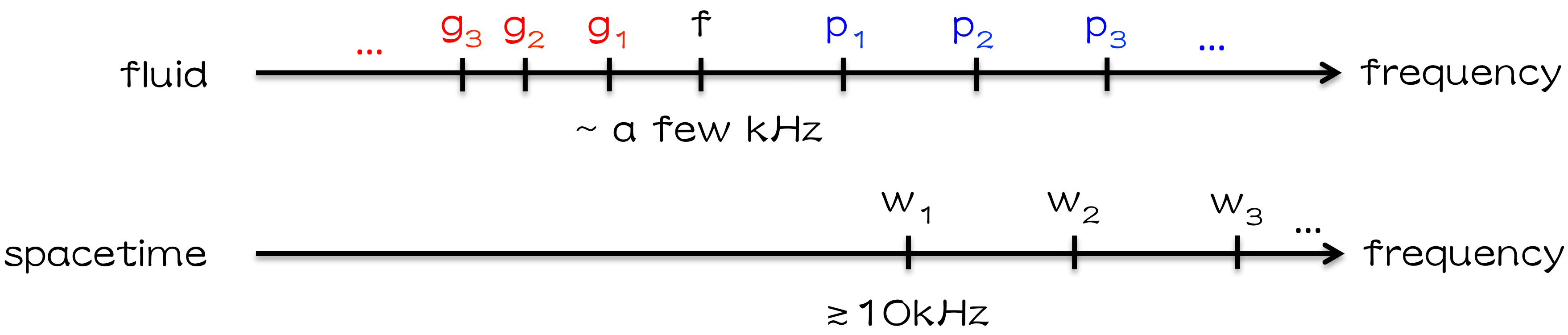
- identify via linear analysis
- variables = background + perturbations

$$f = f_0 + \delta f$$

- expand the perturbed variables

$$\delta f(t,r,\theta,\phi) = \delta f(r) e^{i\omega t} Y_{lm}(\theta,\phi)$$

  - if background is spherically symmetric, the perturbations are independent from *m*
  - ω is an eigenfrequencies of star for each *l*, where f = ω/2π
  - subscript denotes <u>the number of radial nodes</u> in eigenfunction

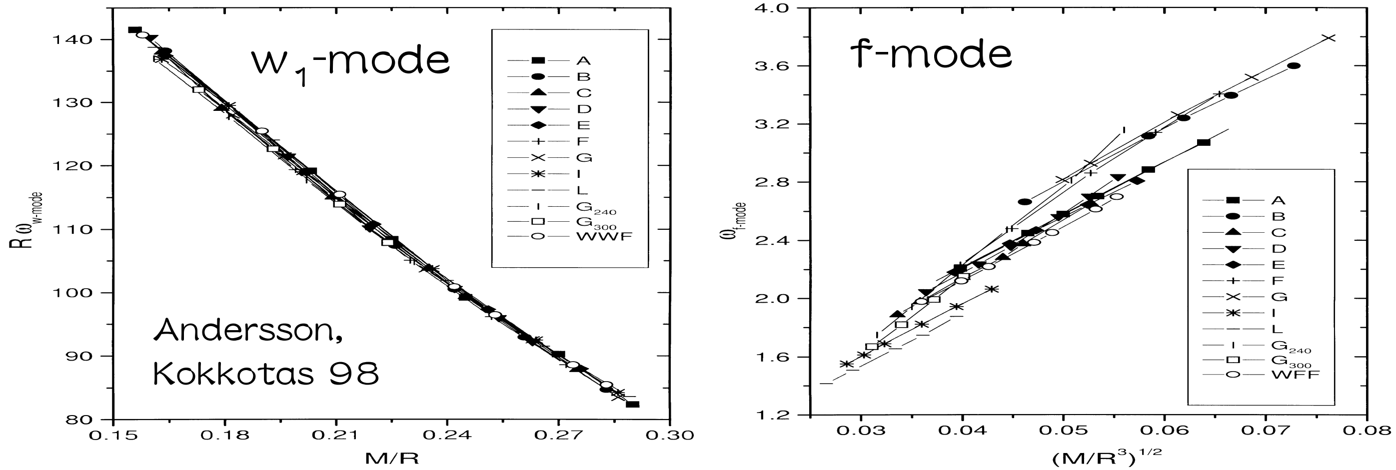


non-radial Oscillations in (proto)-neutron stars

- <u>axial type oscillations</u>
  - no stellar deformation, no density variation
    - w-modes (spactime) : oscillations of specetime itself ~ M/R
- <u>polar type oscillations</u>
  - with density variation & stellar deformation
  - important for considering the GWs emission
    - f-mode (fundamental) ~ $(M/R^3)^{1/2}$
    - p-modes (pressure) : sound speed crossing ~ $(M/R^3)^{1/2}$
    - g-modes (gravity) : thermal/composition gradients ~ B-V frequency
    - Alfven modes
    - inertial modes (effect of rotation)
    - w-modes (spactime) : oscillations of specetime itself ~ M/R

what we learn from GW obs.

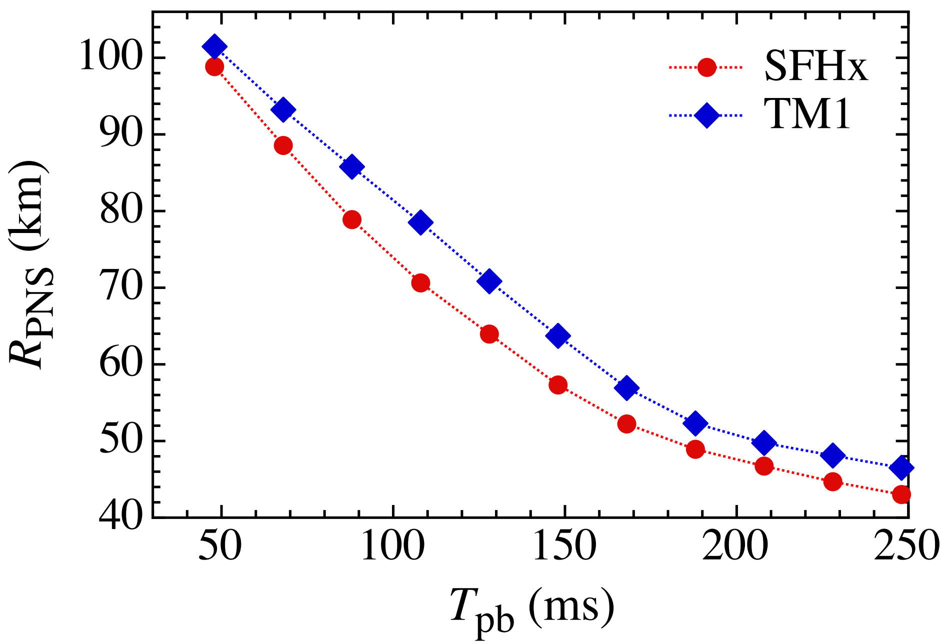
- Via direct observations of GWs, one may extract the PNS or NS properties.
  - Asteroseismology
- In fact, it is known for cold neutron stars that
  - f-mode, which is a acoustic oscillation, is characterized by the stellar average density
  - w-mode, which is a spacetime oscillation, is characterized by the stellar compactness

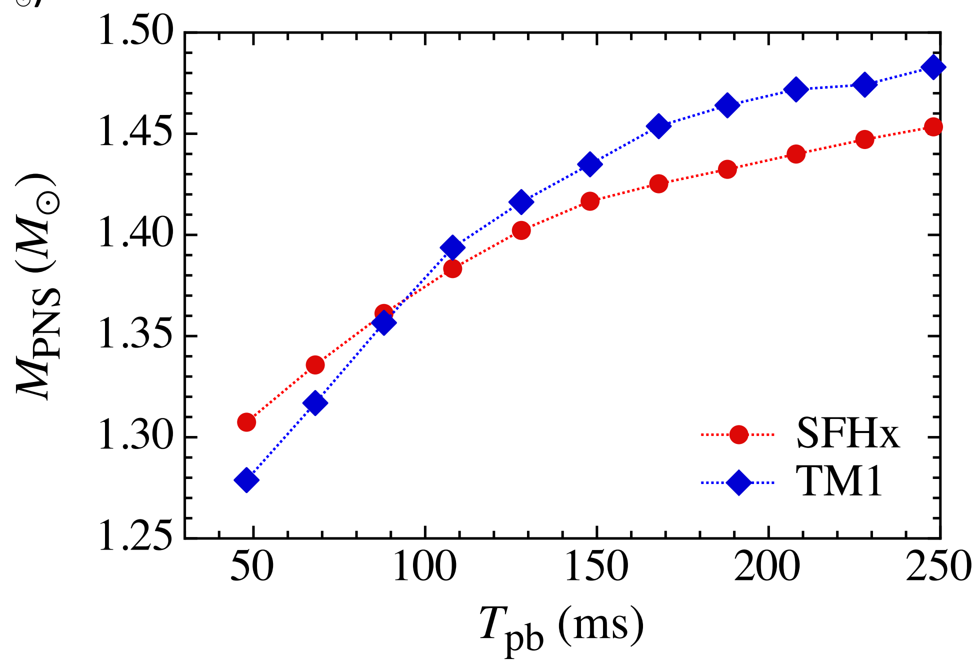


- If similar characterization is possible, one could extract the PNS average density and compactness, via the simultaneous observations of f- and w-modes GWs.

PNS models (HS+17)

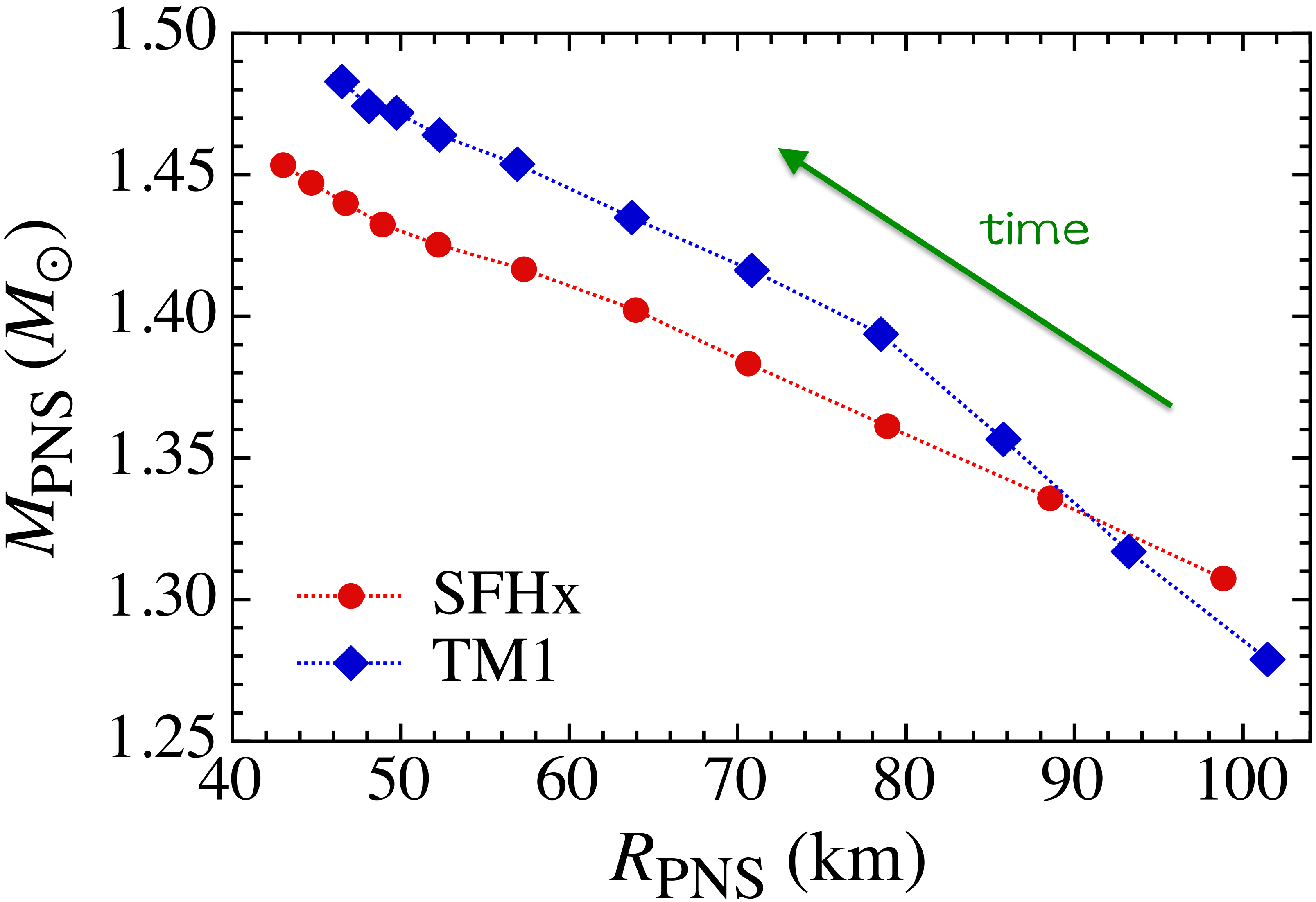
- we adopt the results of 3D-GR simulations of core-collapse supernovae (Kuroda et al. 2016)
  - progenitor mass = $15M_\odot$
  - EOS : SFHx ($2.13M_\odot$) & TM1 ($2.21M_\odot$)





  - R_{PNS} is defined with $\rho_s = 10^{10}$ g/cm^3
  - using the radial profiles as a background PNS model, the eigenfrequencies are determined.

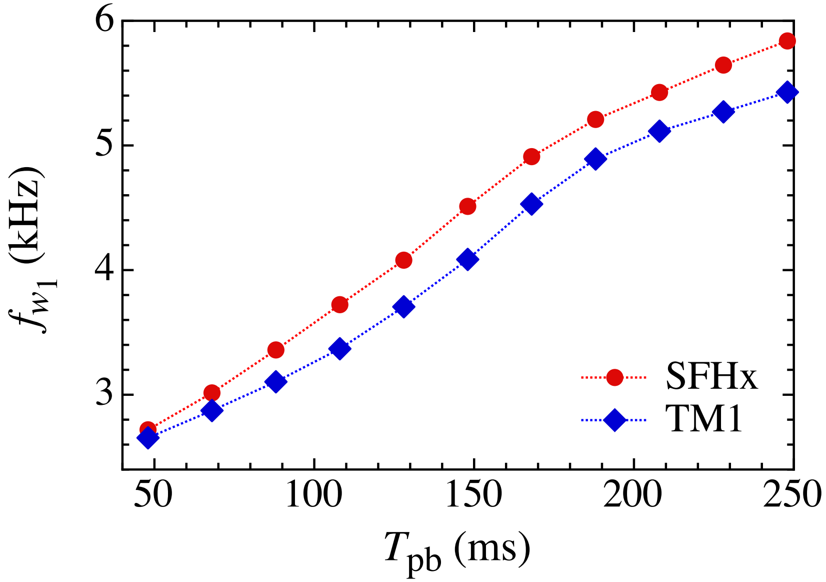
M-R evolution after core-bounce

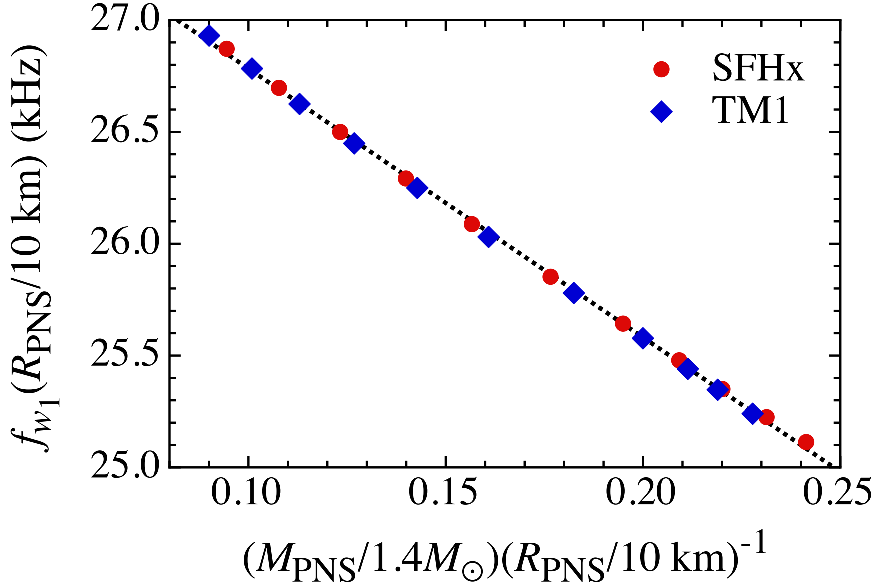


evolution of w_1-modes

- frequencies depend on the EOS.
  - increasing with time
  - can be characterized well by M_{PNS}/R_{PNS}
- as for cold NS, we can get the fitting formula, almost independent from EOS

$$f_{w_1}^{(\mathrm{PNS})}(\mathrm{kHz}) \approx \left[27.99 - 12.02\left(\frac{M_{\mathrm{PNS}}}{1.4\ M_\odot}\right)\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-1}\right] \times \left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-1}$$

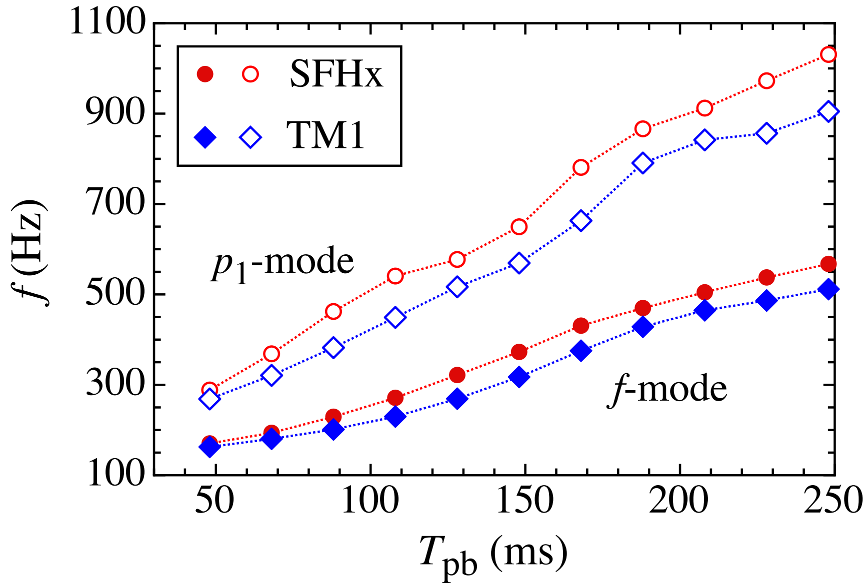


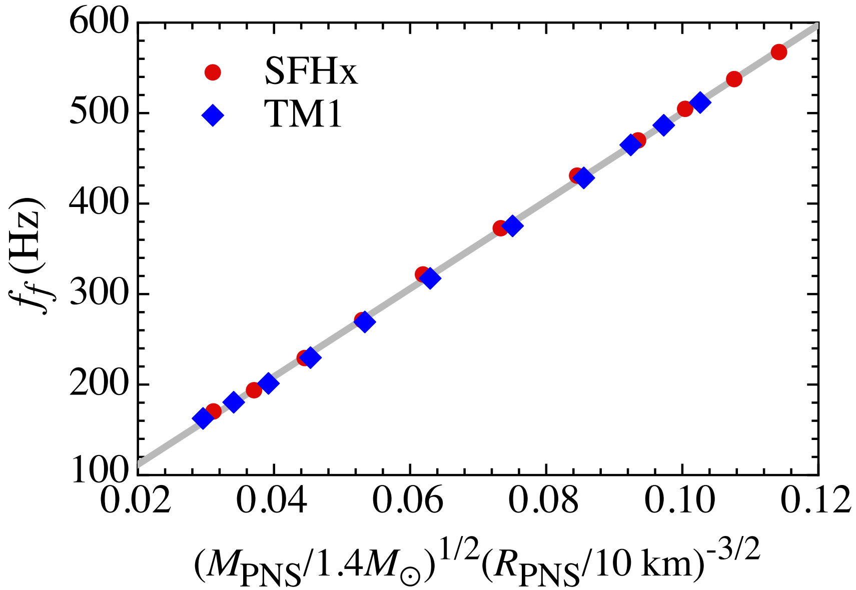


evolution of f-mode

- frequencies can be expressed well by the average density independent of the EOS (and progenitor mass)
- we derive the fitting formula as a function of M_{PNS}/R_{PNS}^3

$$f_f^{(PNS)}(\mathrm{Hz}) \approx 14.48 + 4859 \left(\frac{M_{PNS}}{1.4\ M_\odot}\right)^{1/2} \left(\frac{R_{PNS}}{10\ \mathrm{km}}\right)^{-3/2}$$

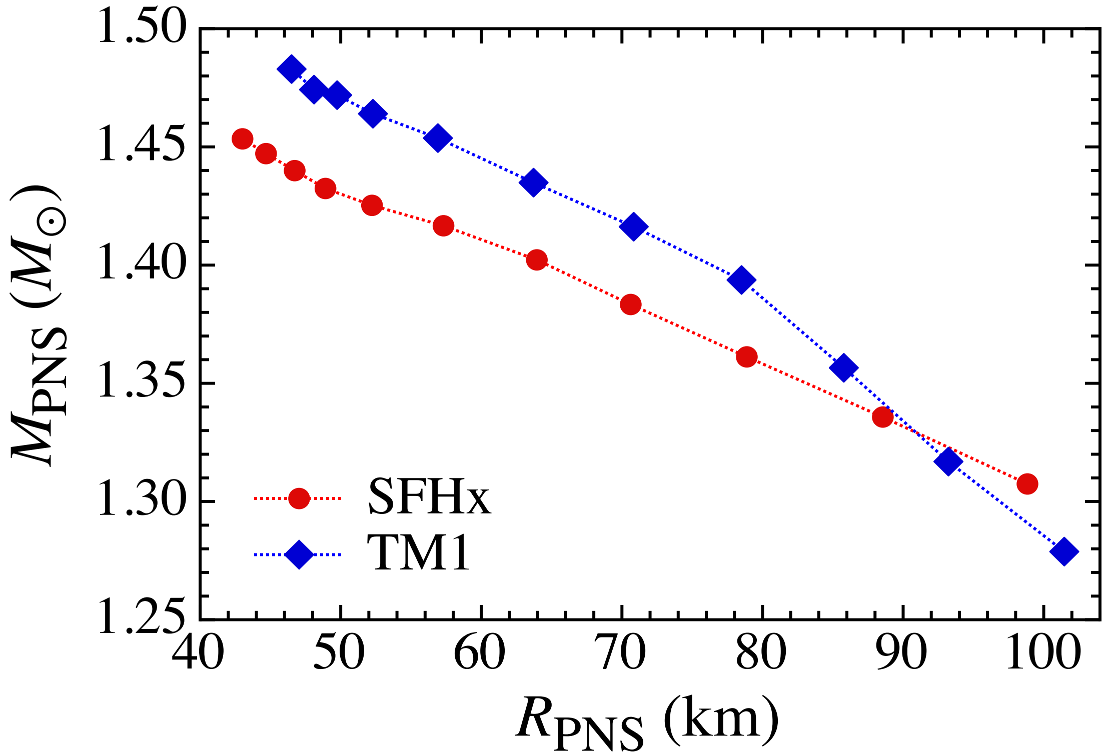




* Note that we neglect the g-mode oscillations in this study

determination of EOS

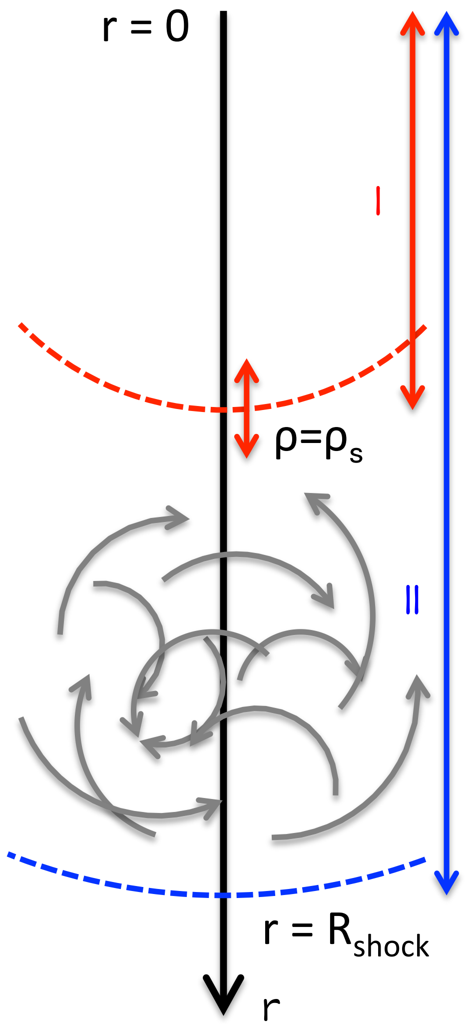
- GW spectra evolutions $f_f(t)$ & $f_{w1}(t)$
  → evolutions of M_{PNS}/R_{PNS}^3 & M_{PNS}/R_{PNS}
- one can determine (M_{PNS}, R_{PNS}) at each time after core bounce
  → determination of the EOS
- unlike cold NS cases, in principle one can determine the EOS even with ONE GW event !



different two approaches

- PNS models, whose surface defined with a specific surface density, ρ_s (Model I)
  - Sotani & Takiwaki 16; 1D-Newton, without rotation
  - Sotani+17; 3D-GR, without rotation
  - Morozova+18; 2D-effective GR, without rotation
  - Radice+19; 3D-effective GR, without rotation
  - Sotani+19; 3D-GR, without rotation
  - Sotani & Sumiyoshi 19,; 1D-GR black hole formation without rotation
  - Sotani & Takiwaki 20a, b, c; 2D-effective GR without rotation
- Numerical region up to the shock radius, R_{shock} (Model II)
  - Torres-Forne+18; 2D-GR, with rotation
  - Torres-Forne+19a; 2D-GR, with rotation/2D-effective GR, without rotation
  - Torres-Forne+19b; 1D-Newton/effective GR/GR, without rotation
- With either I or II, to prepare the background PNS model for linear analysis, the numerical data is averaged in the angular direction, assuming the static solution at each time step.
  - linear analysis on the static, spherically background model.

difference in two approaches

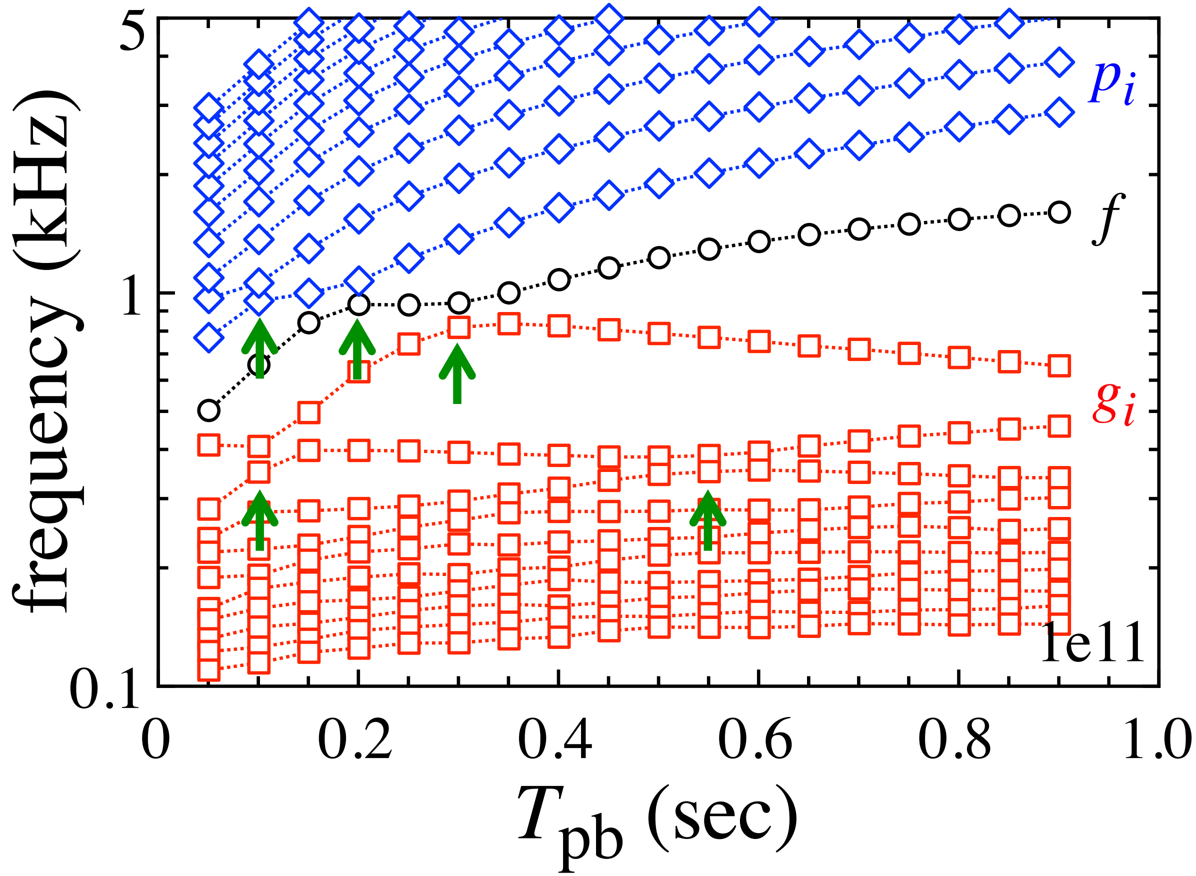


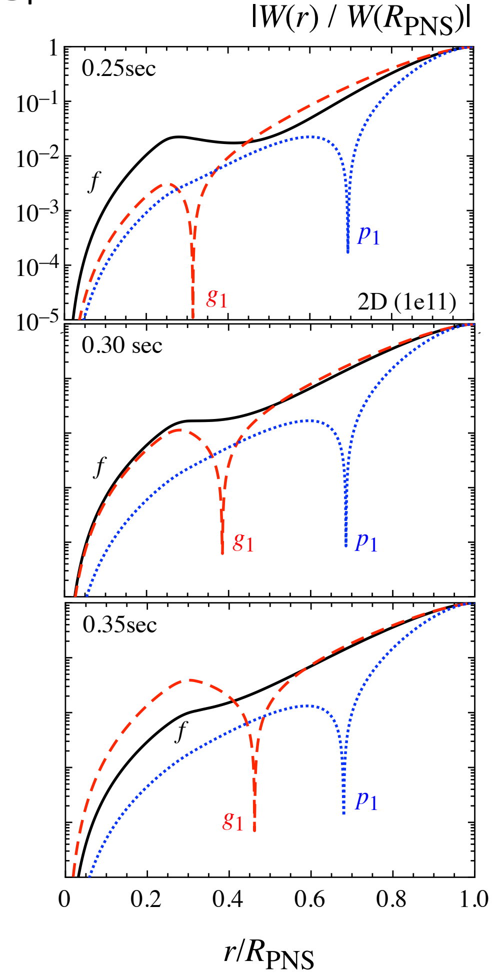
- computational domain
  - Model I : only inside R_{PNS} defined by ρ_s
  - Model II : up to R_{shock}
- Boundary condition for solving the eigenvalue problem
  - Model I : $\Delta p = 0$ @$r = R_{PNS}$
  - Model II : $\delta\xi^r = 0$ @$r = R_{shock}$
  - mathematically, problem to solve is complete different
  - for the both models, the BC is a kind of assumption (not exact one)
- advantage
  - Model I : matter motion is relatively small
    mode classification is as usual
  - Model II : boundary is uniquely determined
- disadvantage
  - Model I : uncertainty in choice of ρ_s
  - Model II : matter motion may not be negligible outside R_{PNS}
    mode classifications is different from the standard one.

avoided crossing in GW frequency

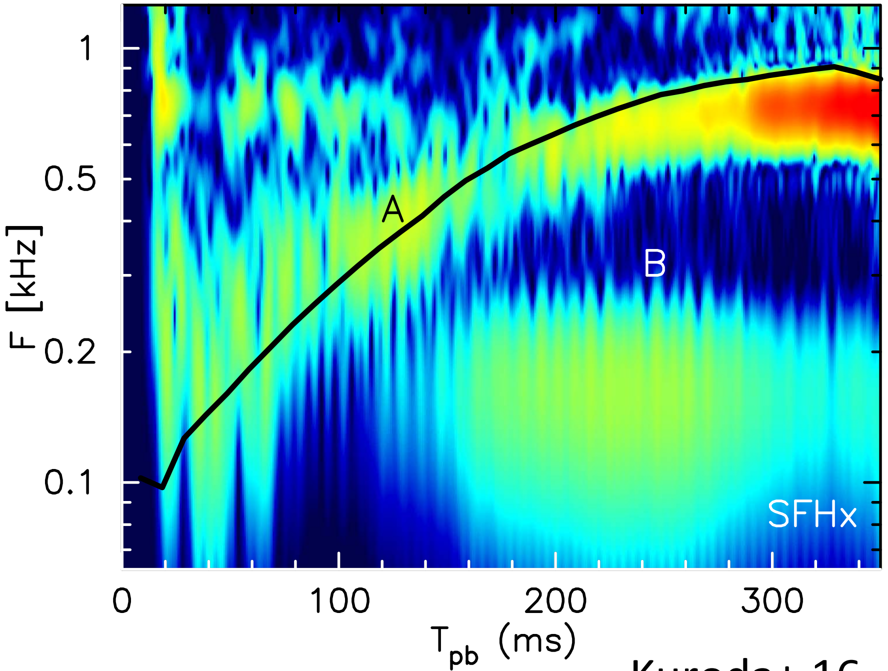
(Sotani&Takiwaki 20b)

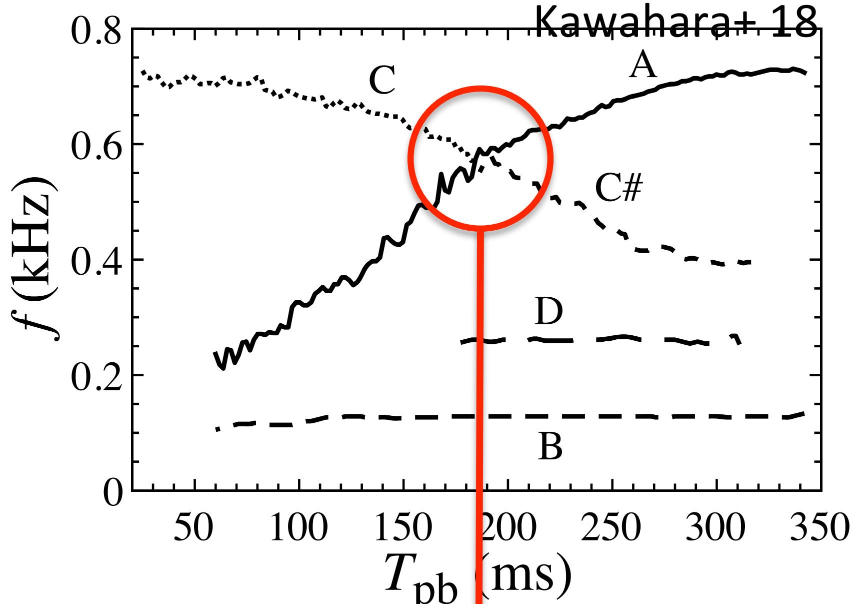
- in the early phase, one can observe the phenomena of avoided crossing between the eigenmodes.
- even in later phase, one can still observe between g_i-modes



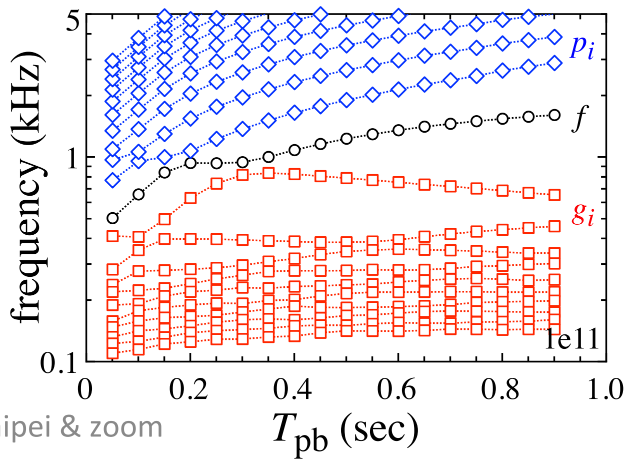


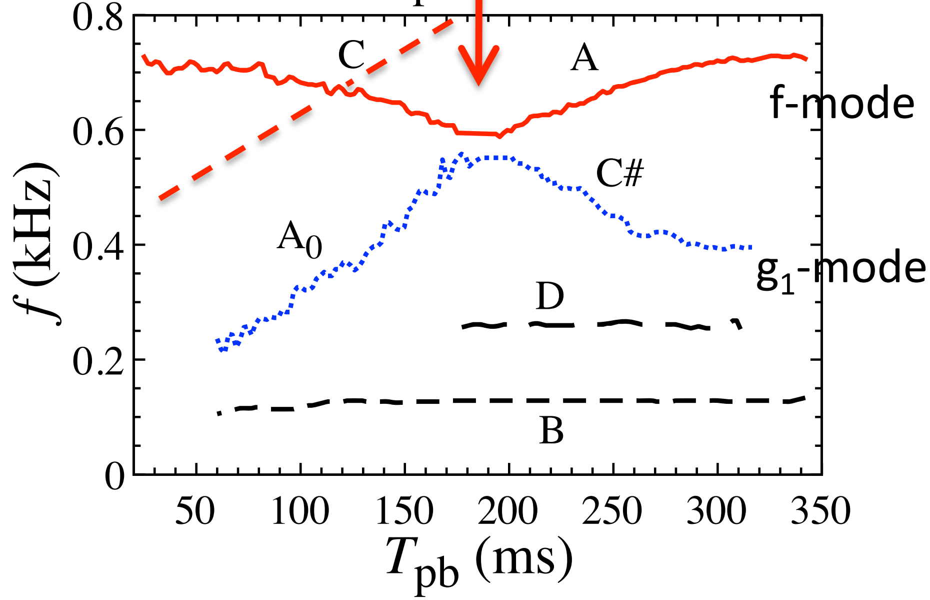
avoided crossing in GW signal?





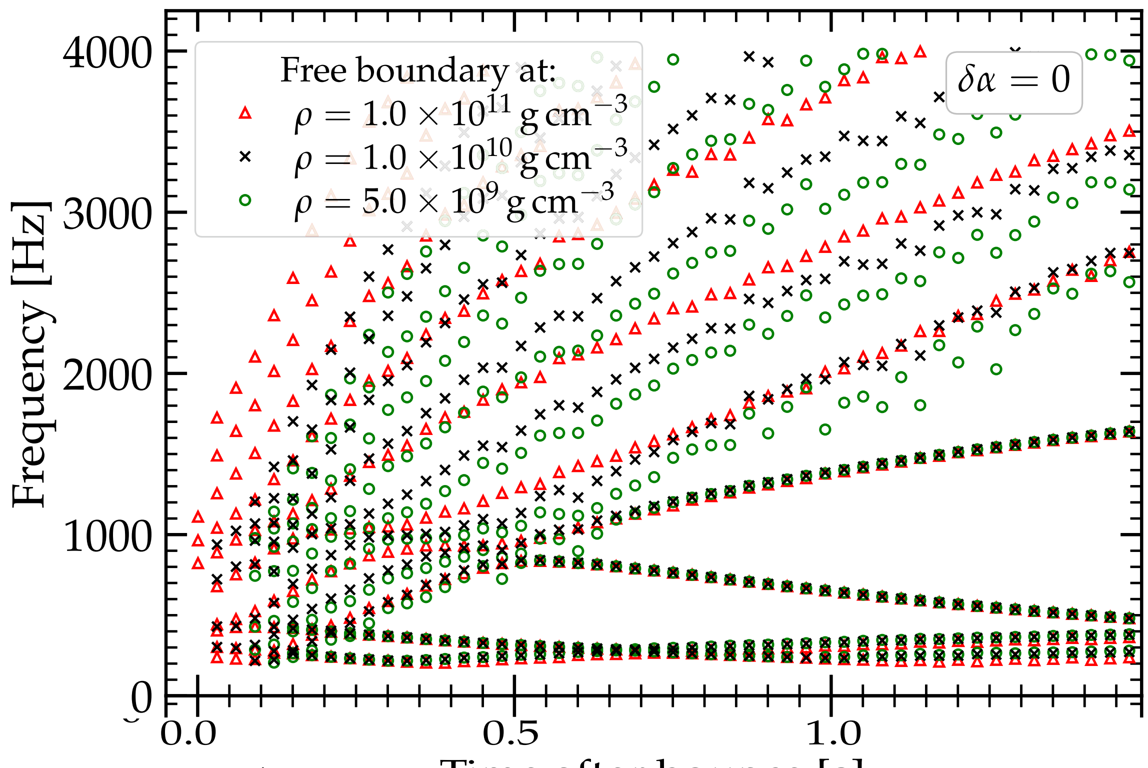
Kuroda+ 16



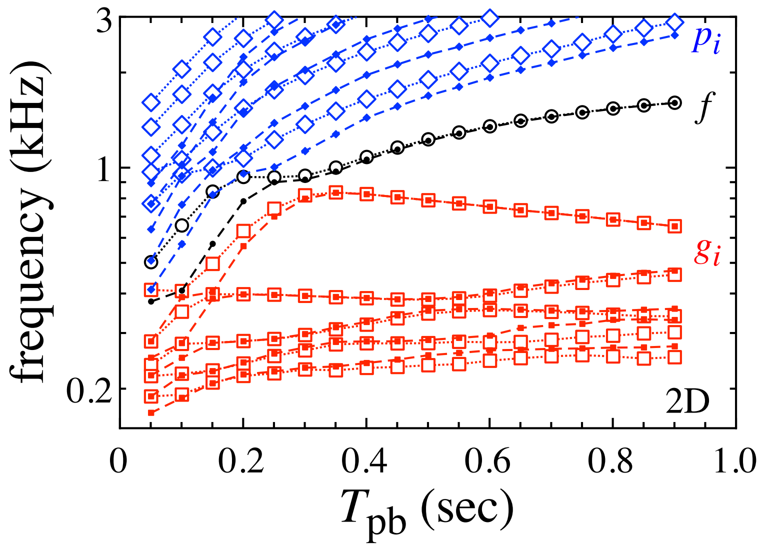


Comment on uncertainty in ρ_s for Model I

- in the late phase after core bounce, e.g., ~ 500ms, f-mode freq. becomes almost independent of the choice of ρ_s (Morozova+18)
- we also confirm this feature, i.e., f- & g_1-modes in later phase are almost independent of ρ_s, where g_1-mode decreases with time (Sotani & Takiwaki 20b).

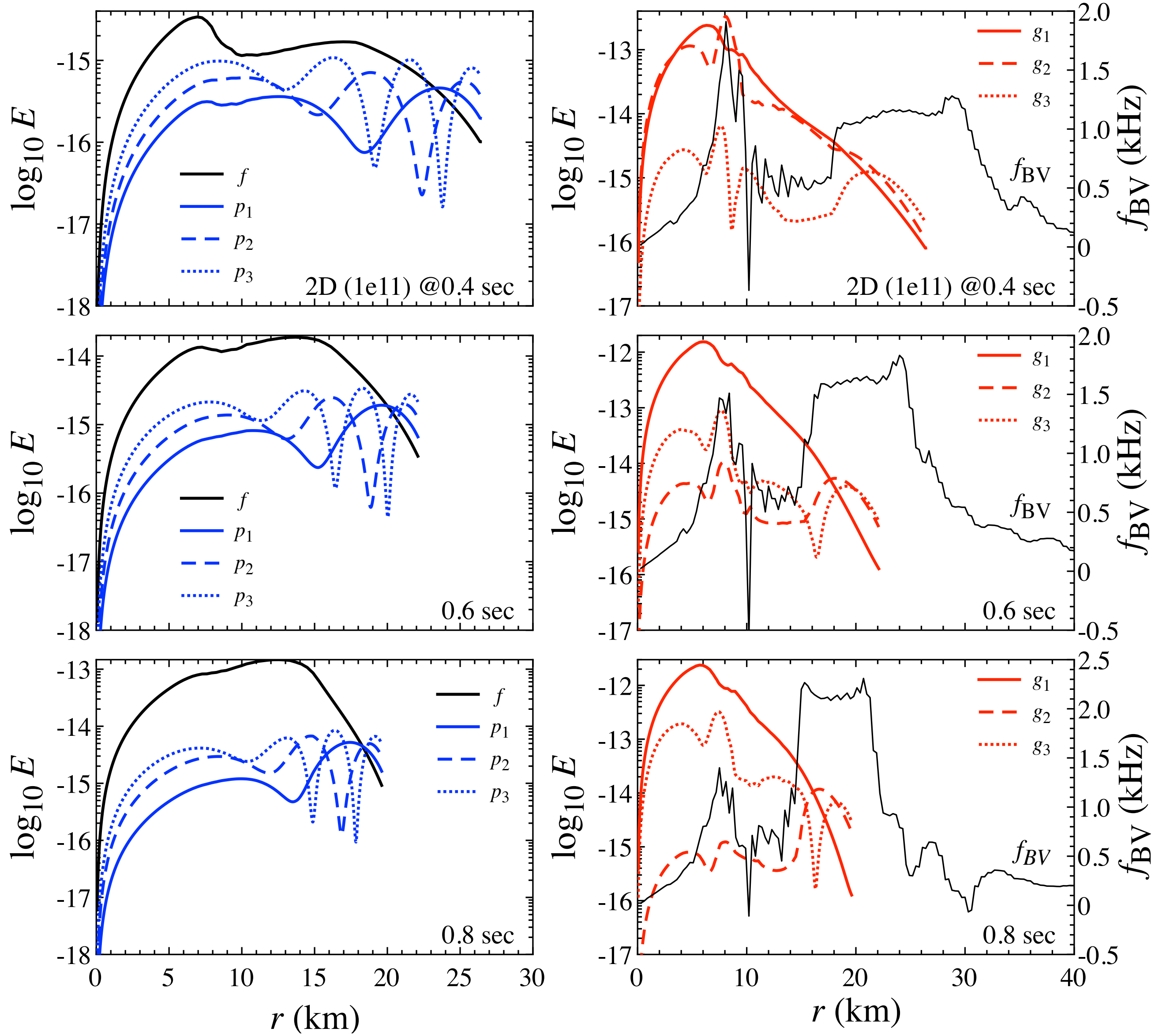


Morozova+18



dotted : 10^{11} g/cm^3
dashed : 10^{10} g/cm^3

pulsation energy density



$$E(r) \sim \frac{\omega^2 \varepsilon}{r^4}\left[W^2 + \ell(\ell+1)r^2 V^2\right]$$

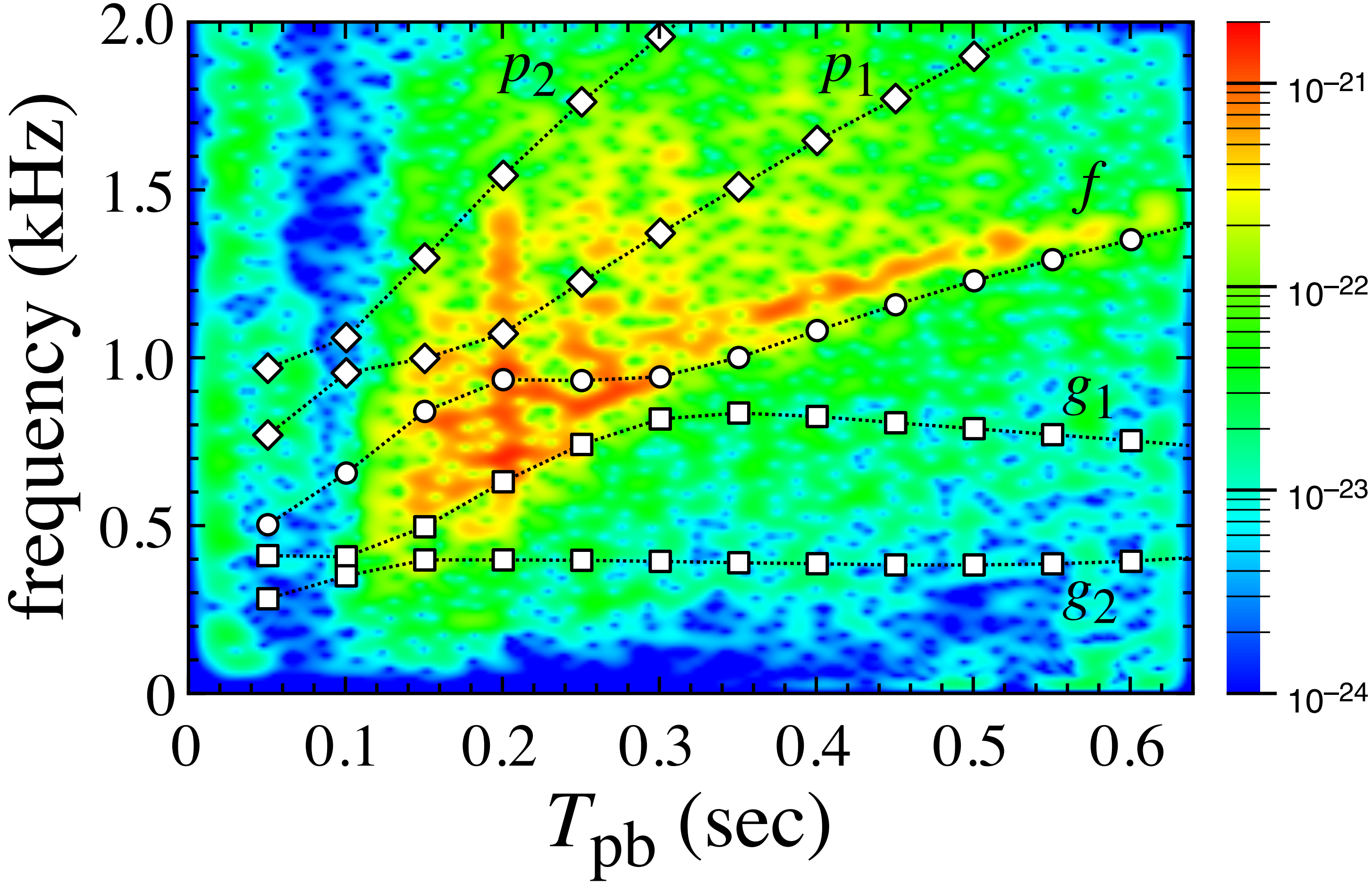
$$f_{\rm BV} = {\rm sgn}(\mathcal{N}^2)\sqrt{|\mathcal{N}^2|/2\pi}$$

$$\mathcal{N}^2 = -e^{2\Phi-2\Lambda}\frac{\Phi'}{\varepsilon+p}\left(\varepsilon' - \frac{p'}{c_s^2}\right)$$

- f- & g_1-modes are not dominant @PNS surface
  → f- & g_1-modes weakly depend on ρ_s
- g_i-modes related to f_{BV}
- g_1-mode is strongly associated with BV freq. @r=8km, which decreases with time
  → decrease of g_1-mode

comparison with GW signals in numerical simulation

- GW signals correspond to g_1-mode in early phase and f-mode after avoided crossing.



GWs in SN simulation is the f-mdoe!

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Three-dimensional core-collapse supernova simulations of massive and rotating progenitors

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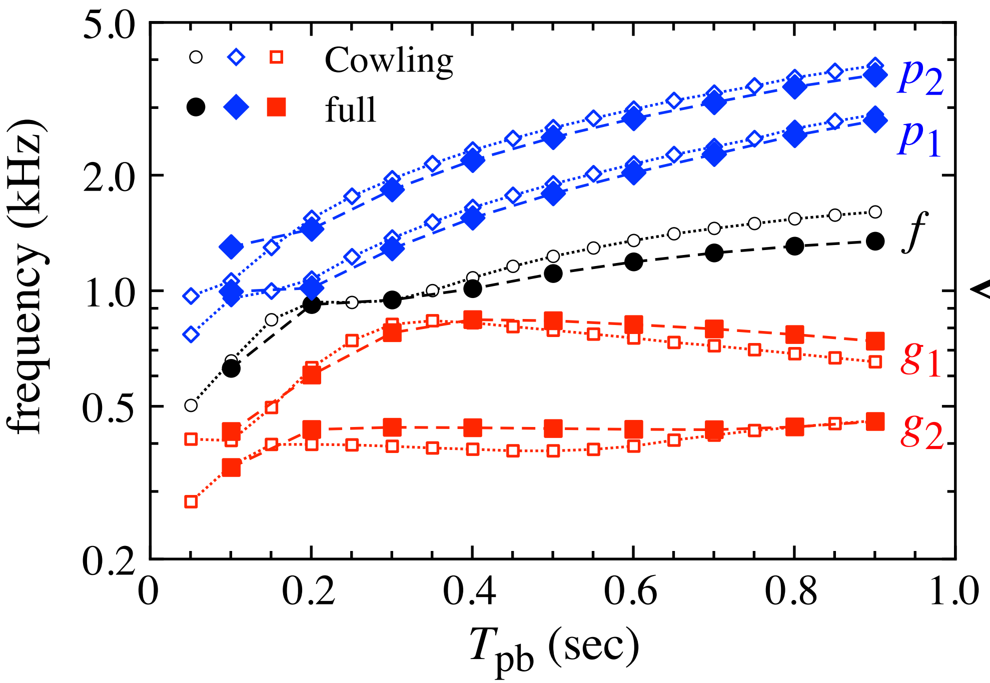
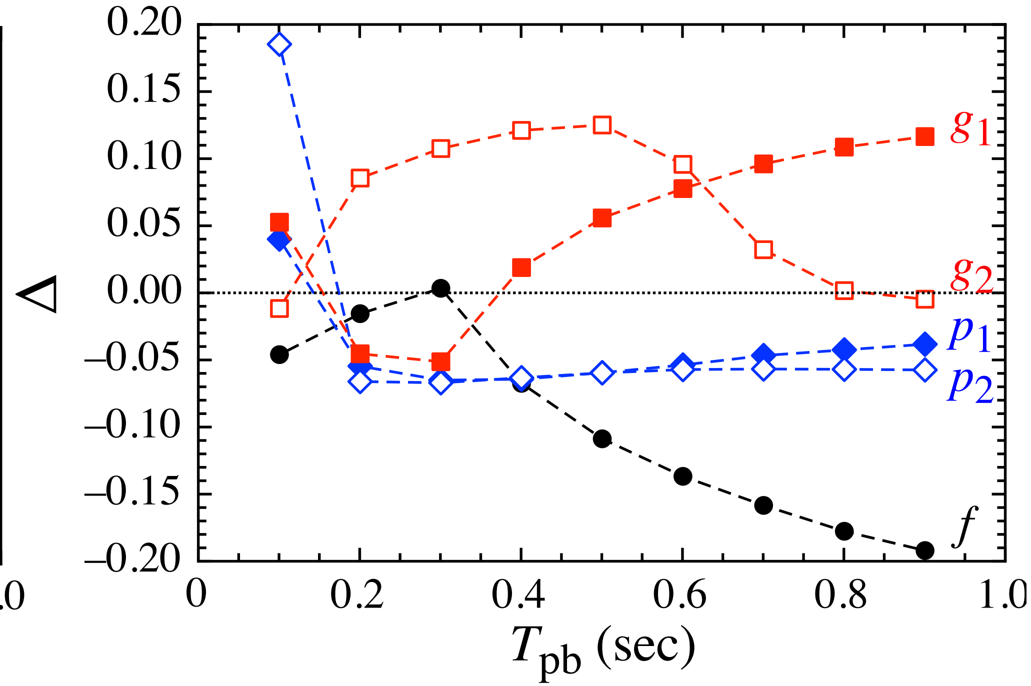
ABSTRACT

We present 3D simulations of the core-collapse of massive rotating and non-rotating progenitors performed with the general relativistic neutrino hydrodynamics code COCONUT-FMT. The progenitor models include Wolf-Rayet stars with initial helium star masses of 39 $M_{\odot}$ and 20 $M_{\odot}$, and an 18 $M_{\odot}$ red supergiant. The 39 $M_{\odot}$ model is a rapid rotator, whereas the two other progenitors are non-rotating. Both Wolf-Rayet models produce healthy neutrino-driven explosions, whereas the red supergiant model fails to explode. By the end of the simulations, the explosion energies have already reached 1.1×10^{51} and 0.6×10^{51} erg for the 39 $M_{\odot}$ and 20 $M_{\odot}$ model, respectively. They produce neutron stars of relatively high mass, but with modest kicks. Due to the alignment of the bipolar explosion geometry with the rotation axis, there is a relatively small misalignment of 30° between the spin and the kick in the rapidly rotating 39 $M_{\odot}$ model. For this model, we find that rotation significantly changes the dependence of the characteristic gravitational-wave frequency of the f-mode on the proto-neutron star parameters compared to the non-rotating case. Its gravitational-wave amplitudes would make it detectable out to almost 2 Mpc by the Einstein Telescope. The other two progenitors have considerably smaller detection distances, despite significant low-frequency emission in the most sensitive frequency band of current gravitational-wave detectors.

accuracy of the Cowling approx.

(Sotani & Takiwaki 20c)





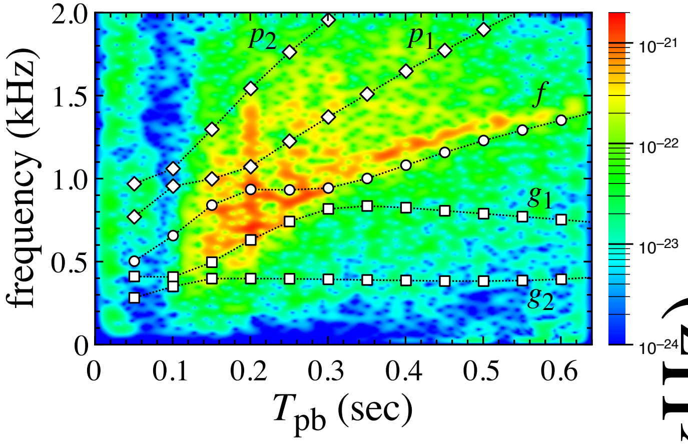
- less than ~20% accuracy
- f-mode with the Cowling approx. is overestimated.

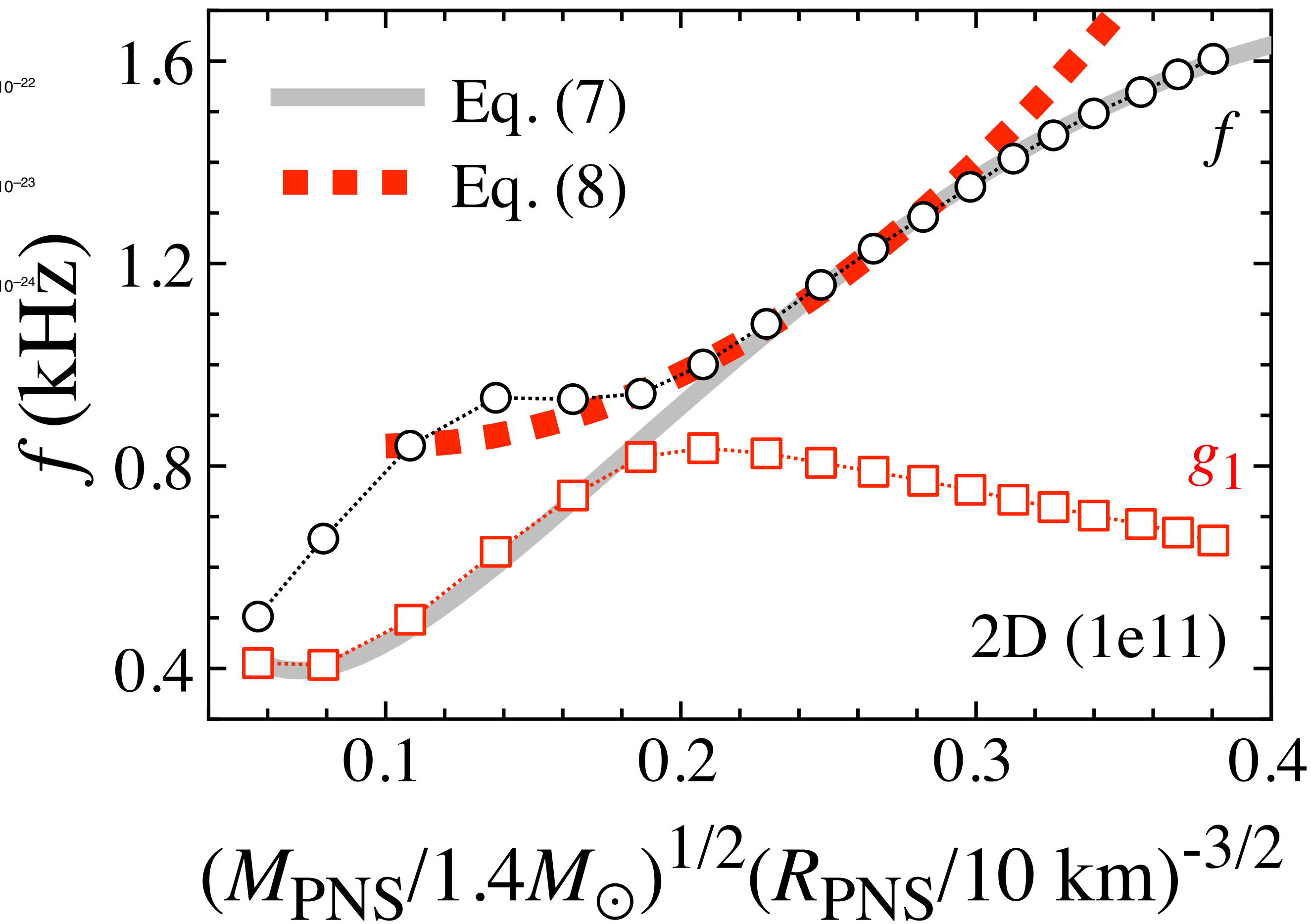
$$\Delta \equiv \frac{f_{\rm full} - f_{\rm Cowling}}{f_{\rm full}}$$

Remark that the accuracy could become better with larger *l*

$$\left(\frac{\omega_{Cow}}{\omega}\right)^2 = \frac{4\pi G\rho\ell}{3} \Big/ \frac{8\pi G\rho}{3}\frac{\ell(\ell-1)}{2\ell+1} = \frac{2\ell+1}{2(\ell-1)}$$

empirical formula for GW signal



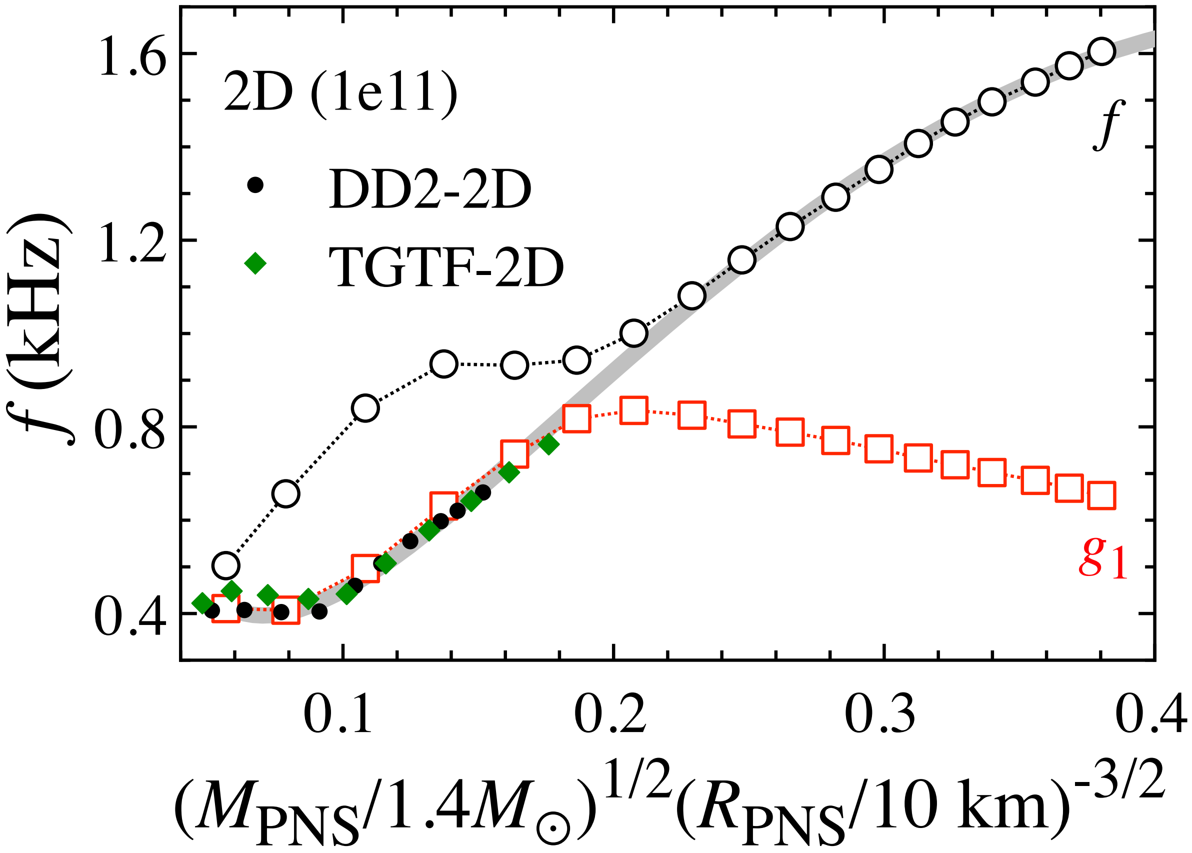


$$\boxed{f\,(\text{kHz}) = -3.250 - 0.978\ln(x) + 15.984x - 15.051x^2}$$

$$x = (M_{\rm PNS}/1.4M_{\odot})^{1/2}(R_{\rm PNS}/10\text{km})^{-3/2}$$

universality

- at least, g_1-modes seem to be well expressed independently of EOS and progenitor models.



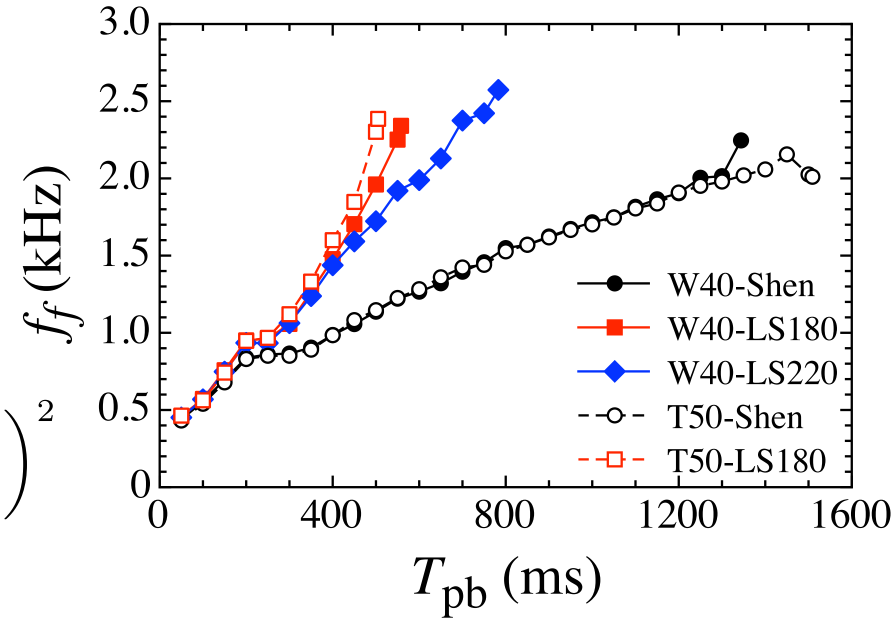
case for BH formation

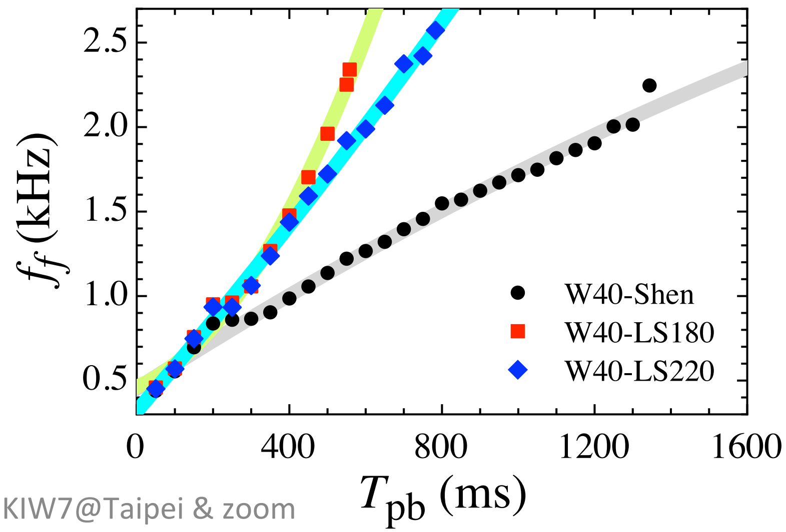
(Sotani & Sumiyoshi 19)

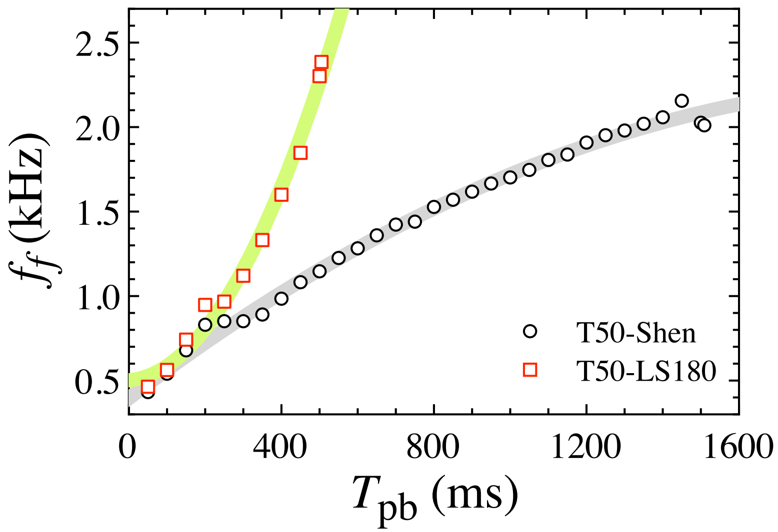
- Time evolution of f-mode GW strongly depends on the progenitor models.
- In any case, it can be well fitted as a function of T_{pb}, such as

$$f_f(\mathrm{kHz}) = c_0 + c_1\left(\frac{T_{\rm pb}}{1000\,\mathrm{ms}}\right) + c_2\left(\frac{T_{\rm pb}}{1000\,\mathrm{ms}}\right)^2$$

  - one can expect high fre. f-mode GW, even though it is not detected directly.





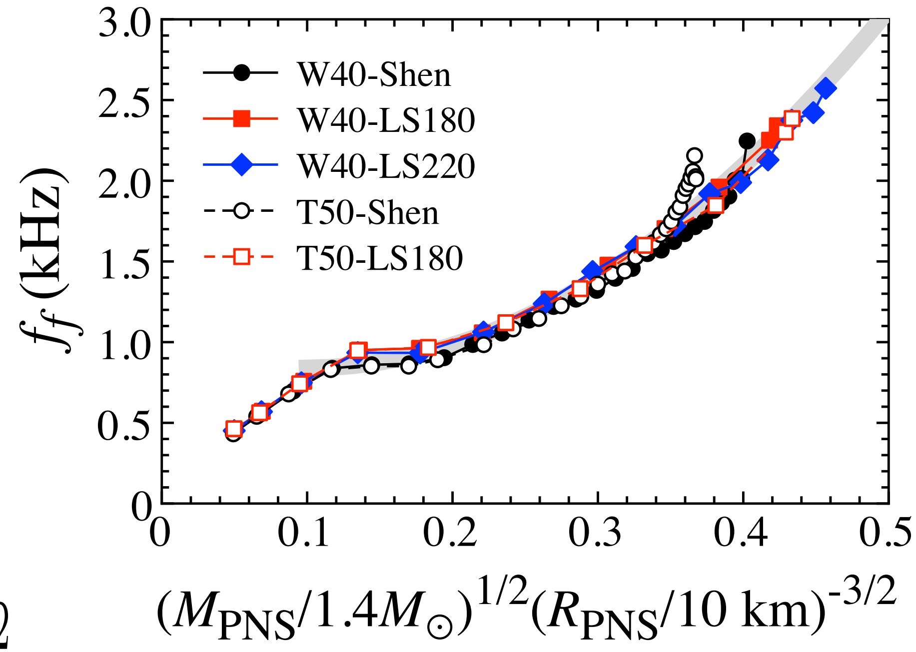


Universality in f-mode GWs

- The f-mode frequencies are well-expressed as a function of stellar average density, <u>independently of progenitor models.</u>

$$f_f(\mathrm{kH}) = 0.9733 - 2.7171X + 13.7809X^2$$

$$X \equiv (M_{\mathrm{PNS}}/1.4M_{\odot})^{1/2}(R_{\mathrm{PNS}}/10\,\mathrm{km})^{-3/2}$$



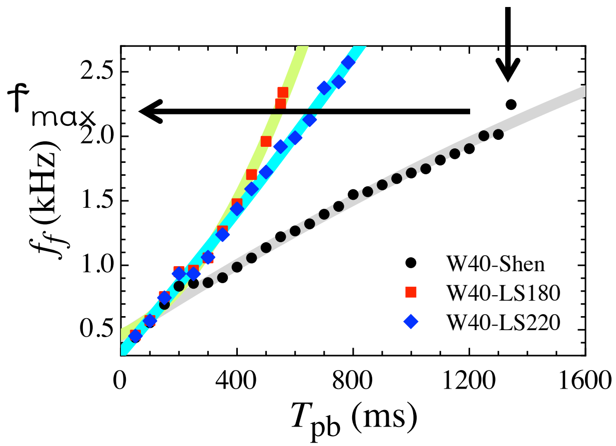
- Through the f-mode GW, one can extract the PNS average density, which leads to the time evolution of PNS average density.

For PNS with maximum mass

- PNS at the moment when it collapses to BH, corresponds to the PNS model with maximum mass.

one can know via neutrino observation

neutrino ob.



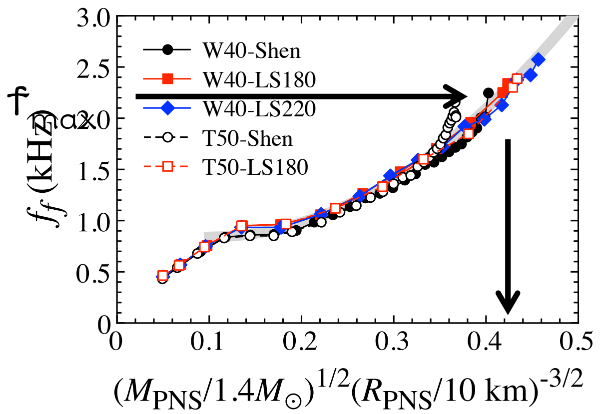
- How to determine the PNS property

① With the data of the f-mode GW, one can fit the time evolution of the f-mode GW

② Owning to the neutrino observation, one can know the moment when PNS collapses to BH

③ The f-mode frequency is expected via ① and ②

④ Via the universal relation of the f-mode, one can extract the average density of PNS with maximum mass



summary

- we examine the GW freq. from PNSs
- one could see the evolution of PNS mass and radius via the simultaneous observations of f- and w_1-modes in GWs
- f- & g_1-modes in later phase are almost independent of ρ_s
- GW signals in numerical simulations correspond to g_1- & f-mode
- for the case of BH formation, one can find the PNS properties with the maximum mass with the help of the neutrino observations