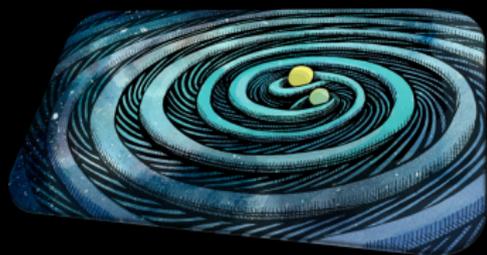




The 7th KAGRA International Workshop

18-20 Dec 2020, Hybrid-style Workshop



Propagation of GWs with CPT violation

Kavli Institute for Astronomy and Astrophysics

Speaker: Lijing Shao (邵立晶)

KAGRA International Workshop

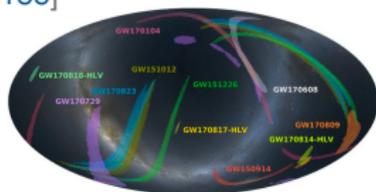
Content

- The **most generic propagation** of GWs depends on
 - GW **frequency**
 - GW **polarization**
 - GW source **direction**
- For the first time, we combined 11 GW events in the catalog **GWTC-1**, and bounded *multiple* coefficients for generic Lorentz/CPT violation

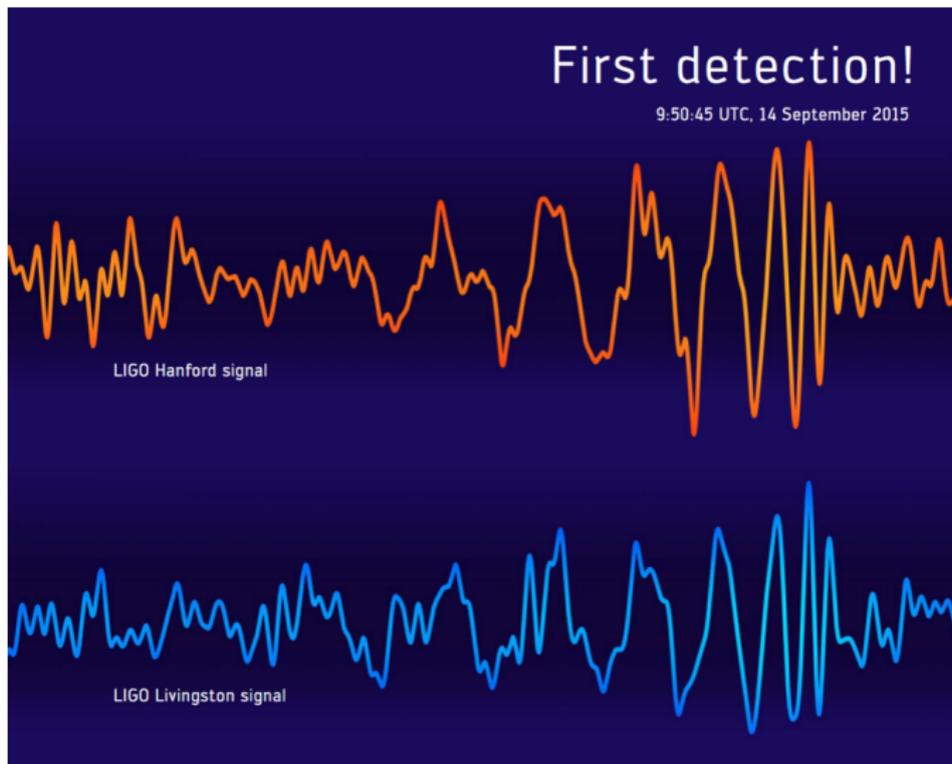


L. Shao, Phys. Rev. D **101** (2020) 104019 [[arXiv:2002.01185](https://arxiv.org/abs/2002.01185)]

“Combined search for anisotropic birefringence in the gravitational-wave transient catalog GWTC-1”



Gravitational-wave Data



Gravitational Waveform (Time Domain)

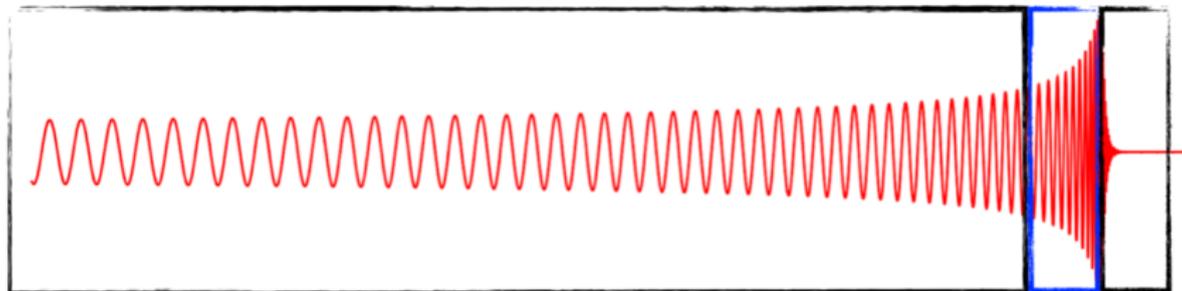
- **Inspiral**: post-Newtonian expansion
- **Merger**: numerical relativity
- **Ringdown**: black hole perturbation

“Inspiral”

post-Newtonian method

“Ringdown”

BH perturbation



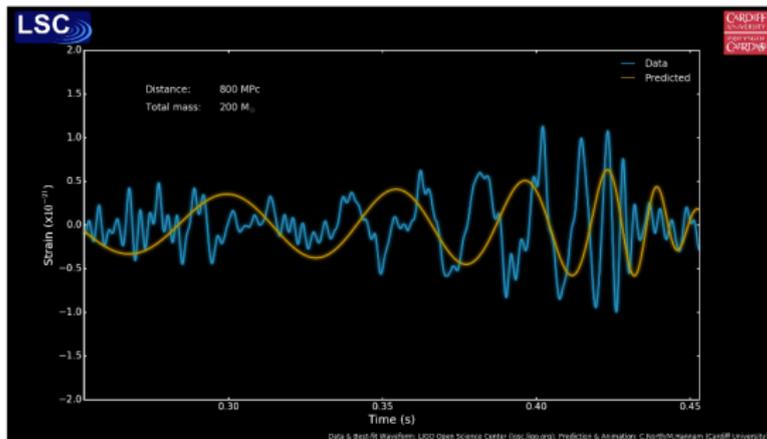
Effective-one body (EOB): Buonanno & Damour 1999, 2000
Bohé, Shao, Taracchini et al. 2017

“Merge”
Numerical relativity

Matched Filter

- **Matched filtering** is a standard analysis method for **wideband** time series data [Finn 1992]

$$(\mathbf{g} | \mathbf{k}) \equiv 2 \int_0^\infty \frac{\tilde{g}^*(f)\tilde{k}(f) + \tilde{g}(f)\tilde{k}^*(f)}{S_n(f)} df$$



Parameter Estimation: GW150914

- GW data encode plenty of information of GW sources
 - Apply **Matched filter** to **data & theory**

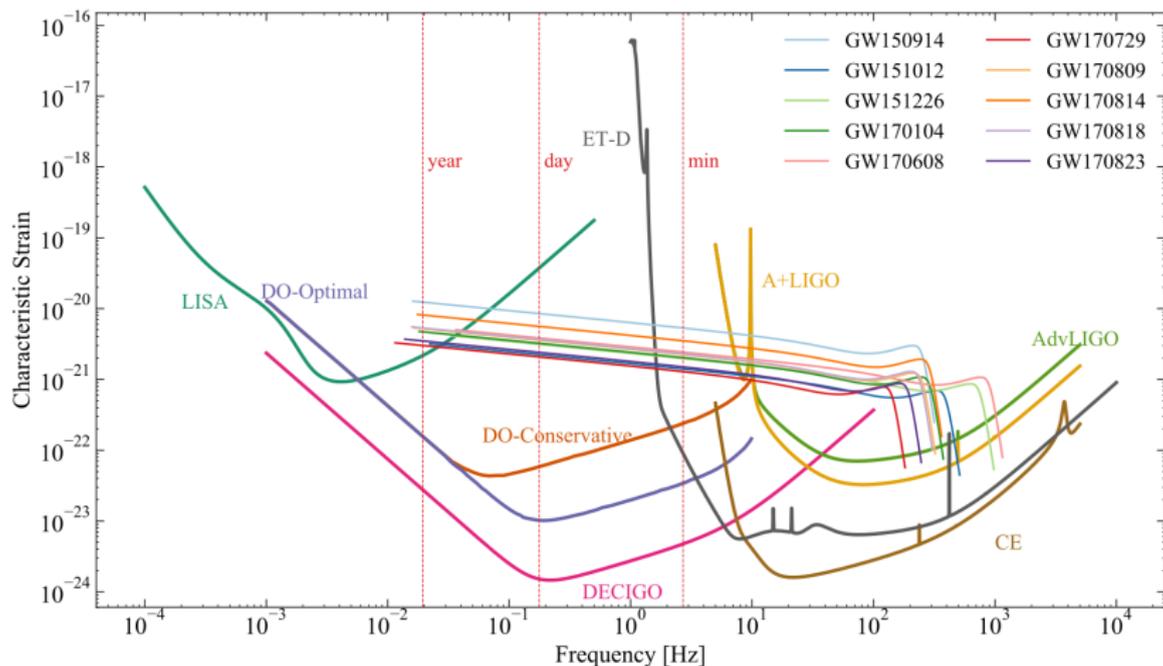
Primary black hole mass	$36^{+5}_{-4} M_{\odot}$
Secondary black hole mass	$29^{+4}_{-4} M_{\odot}$
Final black hole mass	$62^{+4}_{-4} M_{\odot}$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

LIGO/Virgo 2016, PRL

GW Transient Catalog GWTC-1 (LIGO/Virgo 2019)

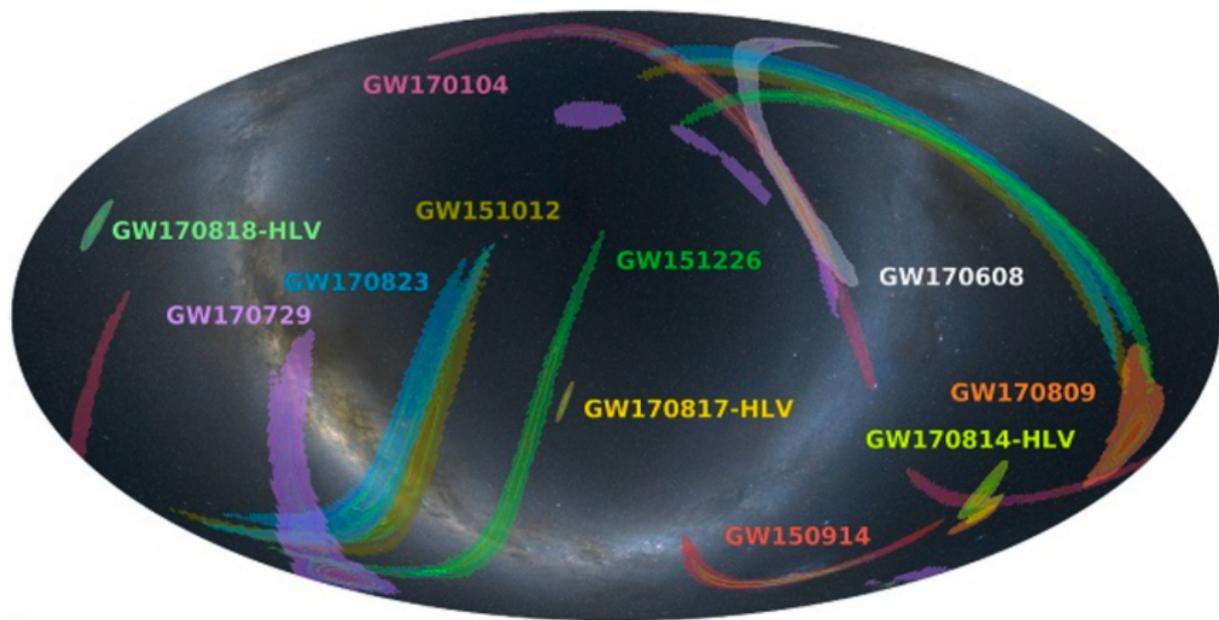
	Type	$m_1 [M_\odot]$	$m_2 [M_\odot]$	d_L [Mpc]	Redshift z
GW150914	BBH	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$
GW151012	BBH	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$
GW151226	BBH	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$
GW170104	BBH	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$
GW170608	BBH	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$
GW170729	BBH	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$
GW170809	BBH	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$
GW170814	BBH	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$
GW170817	BNS	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$
GW170818	BBH	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$
GW170823	BBH	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$

Signals of GW Events



Liu, Shao, Zhao, Gao 2020, MNRAS [[arXiv:2004.12096](https://arxiv.org/abs/2004.12096)]

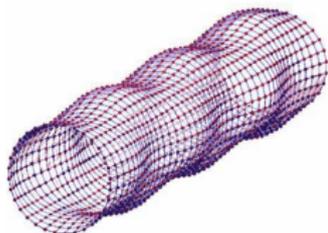
GWTC-1: Sky Position (LIGO/Virgo 2019)



Graviton Dispersion Relation

- **GR**: massless spin-2 metric field $\Rightarrow E = p$
- Lorentz-invariant massive graviton $\Rightarrow E^2 = p^2 + m^2$
 - Both the **phase velocity** E/p and the **group velocity** $\partial E/\partial p$ depend on the energy/frequency of graviton
 - GWs gain **frequency-dependent time delays** when they arrive at the Earth
 - In a FRW spacetime, one has [Will 1998, PRD57:2061]

$$\Delta t_a = (1 + z) \left[\Delta t_e + \frac{D}{2\lambda_g^2} \left(\frac{1}{f_e^2} - \frac{1}{f'_e{}^2} \right) \right]$$

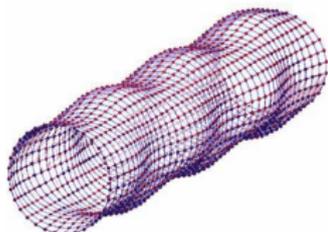


Propagation of GWs

- The extra time delay results in a phase shift in $h(f) \propto e^{i\Psi(f)}$

$$\Psi(f) = \Psi_{\text{GR}}(f) - \frac{\pi^2 D \mathcal{M}}{\lambda_g^2 (1+z)} (\pi \mathcal{M} f)^{-1}$$

- On the other hand, the waveform is *totally calculable* and *deterministic* in GR
- Therefore, GWs provide *an observational window* to the dispersion relation of graviton



Standard-model Extension

- The most generic **linearized gravity** has the Lagrangian

[Kostelecký & Mewes 2018]

$$\mathcal{L}_{\mathcal{K}^{(d)}} = \frac{1}{4} h_{\mu\nu} \hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma}$$

where $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} = \mathcal{K}^{(d)\mu\nu\rho\sigma} i_1 i_2 \cdots i_{d-2} \partial_{i_1} \partial_{i_2} \cdots \partial_{i_{d-2}}$

- It predicts a modified dispersion relation for GWs

$$\omega = \left(1 - \zeta^0 \pm \sqrt{(\zeta^1)^2 + (\zeta^2)^2 + (\zeta^3)^2} \right) p$$

Standard-model Extension

$$\omega = \left(1 - \zeta^0 \pm \sqrt{(\zeta^1)^2 + (\zeta^2)^2 + (\zeta^3)^2} \right) \rho$$

$$\zeta^0 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\mathbf{n}}) k_{(I)jm}^{(d)}$$

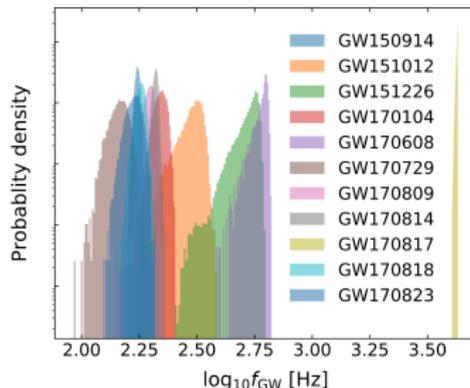
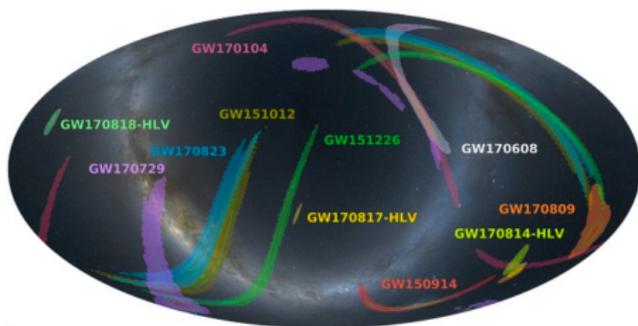
$$\zeta^1 \mp i\zeta^2 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\mathbf{n}}) \left[k_{(E)jm}^{(d)} \pm ik_{(B)jm}^{(d)} \right]$$

$$\zeta^3 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\mathbf{n}}) k_{(V)jm}^{(d)}$$

- Therefore, gravitons of different **polarization** or **frequency**, coming from different **directions** have different velocity

GWTC-1 Events

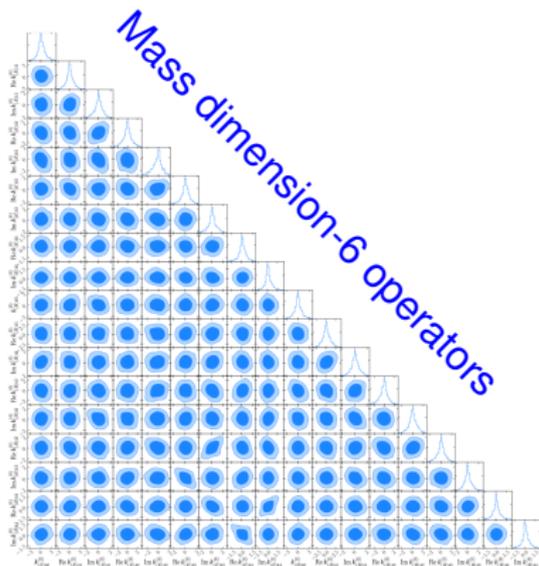
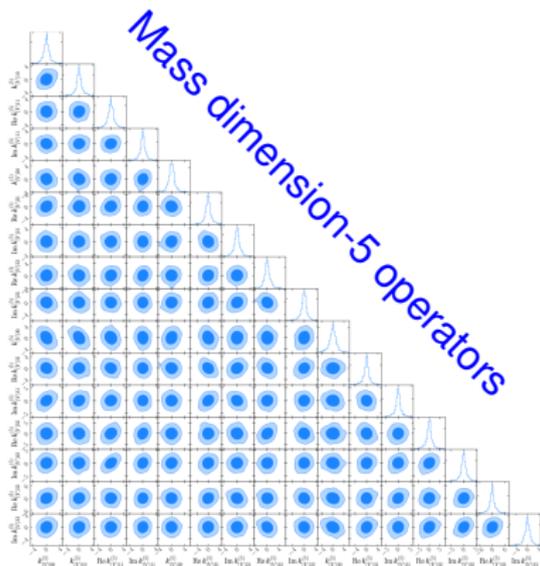
A simplified/naive approach: $|\omega_{\text{GW}}\Delta t| \leq 2\pi/\rho$



We have all the information available to perform the test

Shao 2020, PRD101:104019

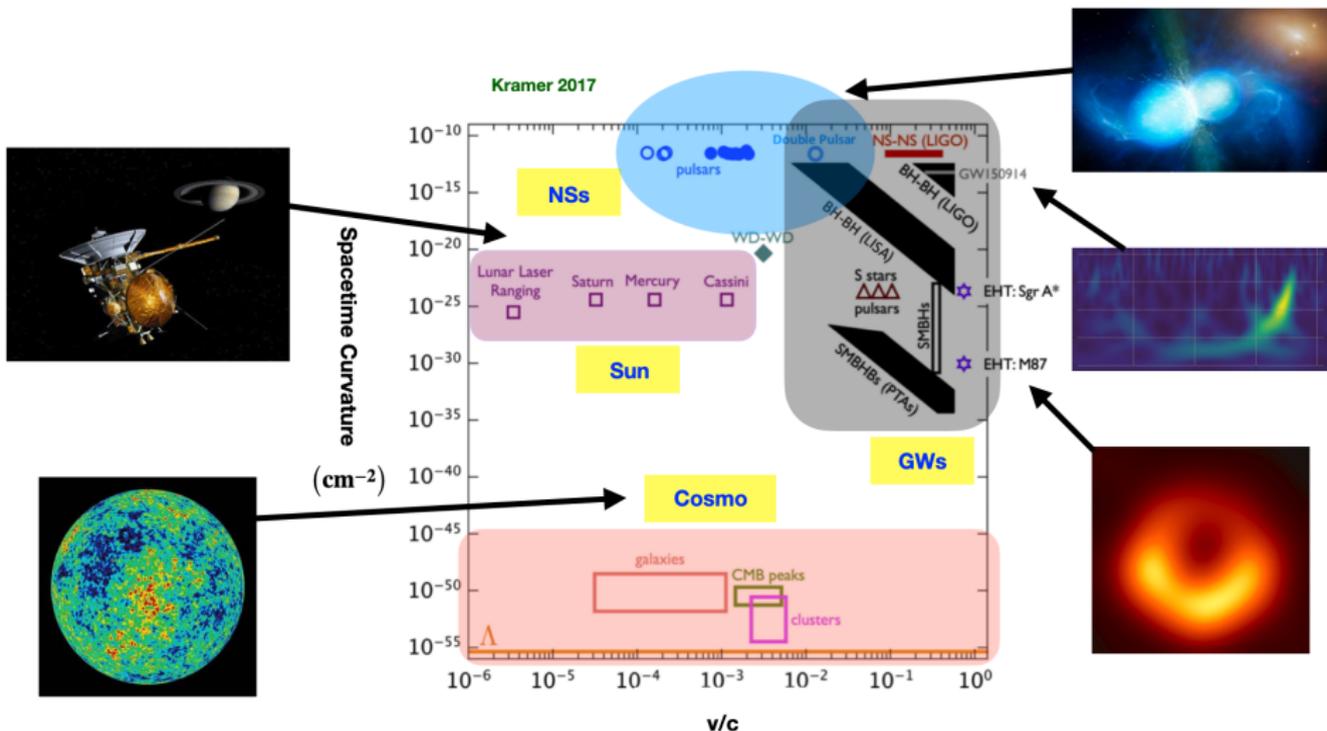
Anisotropic Birefringence Combined Search



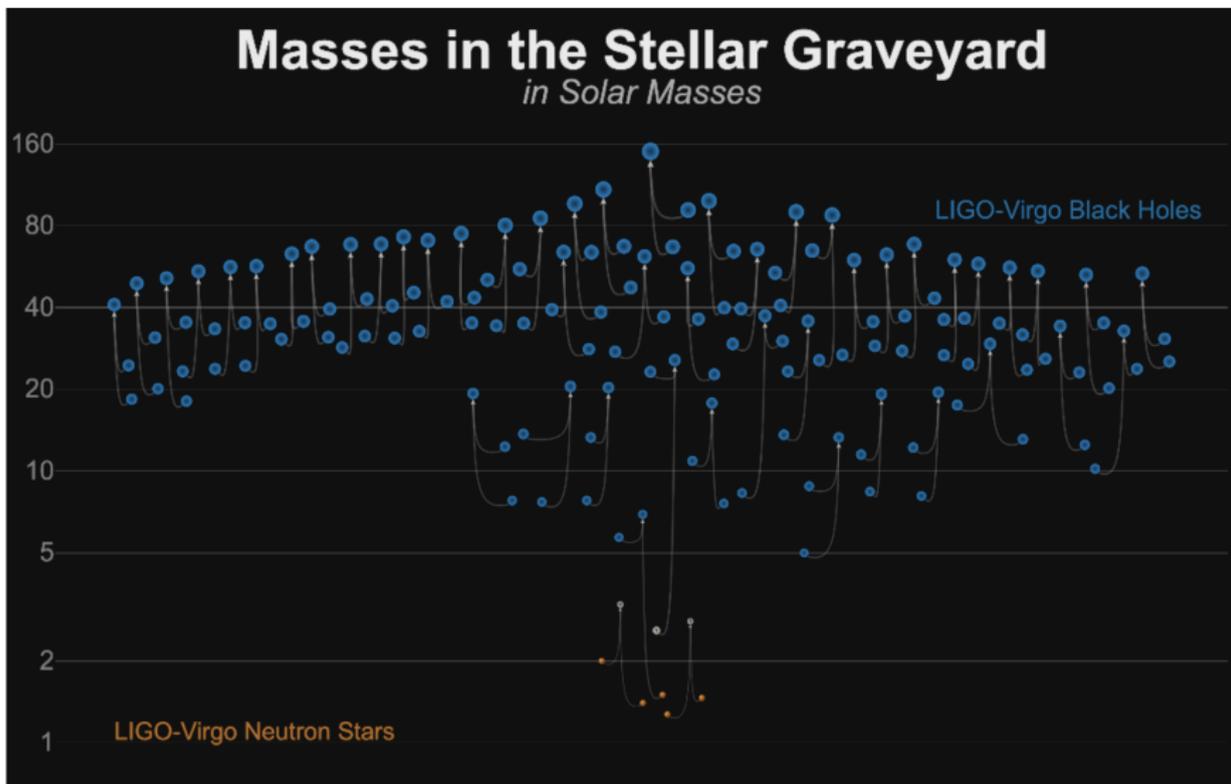
We have all the information available to perform the test

Shao 2020, PRD101:104019

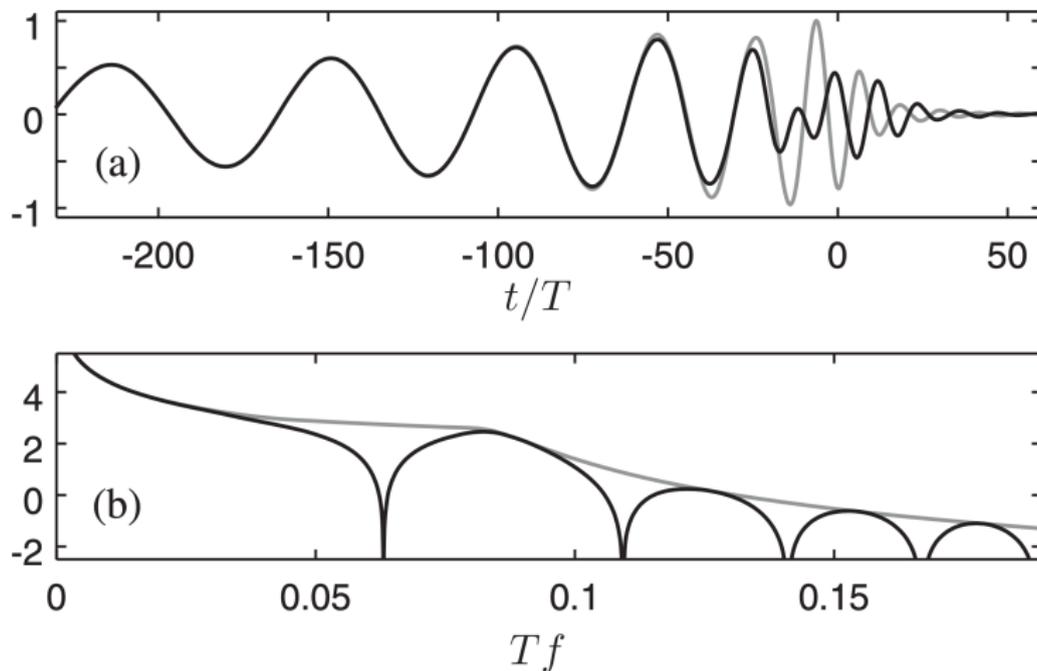
Parameter Space in Gravity Tests



GWTC-2: Released Recently! (LIGO/Virgo 2020)



Matched Filter Analysis



Mewes, PRD99:104062

Thank you!

