Equilibrium sequences of differentially rotating stars with post-merger-like rotational profiles

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Motivation: Why equilibrium modelling?

- Numerical simulations are the primary weapons of choice to study complex phenomena, such as BNS mergers and their remnants.
- Performing realistic simulations comes at a price of several thousand core-hours.
- A thorough exploration of a problem's parameter space can become challenging.

see: Shibata et al (2005), Hotokezaka et al (2011), Sekiguchi et al (2011), Bauswein & Janka (2012), Bauswein et al (2012), Hotokezaka et al (2013), Bernuzzi et al (2014), Dietrich et al (2015), De Pietri et al (2016), Radice et al (2018), and many more ...

Rough outline of equilibrium modelling

- Ignore some aspects of the realistic configuration (e.g. non-axisymmetric deformations, oscillations, time-dependence, thermal structure), to obtain a simplified model.
- What do we get in return?
  - allows for faster and wider exploration of the parameter space
  - configurations can be used as initial data for dynamical simulations
  - provides important insights on stellar stability

Basic framework

The line element for a stationary, axisymmetric star in full GR is

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{\gamma-\rho}r^2\sin^2\theta(d\phi - \omega dt)^2 + e^{2\mu}(dr^2 + r^2 d\theta^2)$$

where γ, ρ, ω and μ are metric functions depending only on the coordinates r and θ.

see: Friedman & Stergioulas (2013), Paschalidis & Stergioulas (2017)

Komatsu, Eriguchi and Hachisu scheme

- Start from an initial guess for the metric potentials γ, ρ, ω, μ, the energy density ϵ and the angular velocity Ω.
- Use the first integral of the hydrostationary equilibrium and the specified EOS to obtain an updated matter distribution.
- Update the γ, ρ, ω and μ distributions
- Resume steps until convergence to a solution is achieved.

see: Komatsu et al (1989), Cook et al (1992), Nozawa et al (1998)

KEH rotation law

- A simple, computationally convenient, differential rotation law

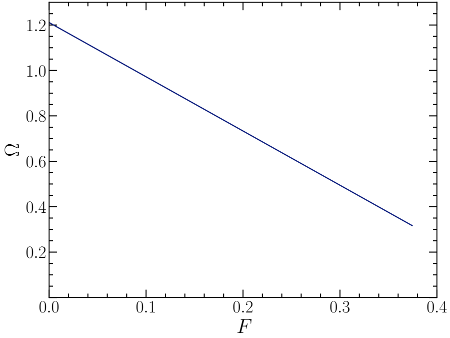
$$F(\Omega) = A^2(\Omega_c - \Omega)$$

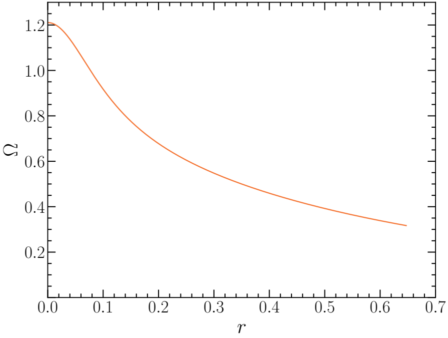
- A is a positive constant that determines the length scale over which the angular velocity Ω changes within the star.
- The KEH law is reduced to uniform rotation for $A \to \infty$ and to the j-constant law for $A \to 0$.

see: Ansorg et al (2009), Espino & Paschalidis (2019), Espino et al (2019)

KEH rotation law

The KEH law is widely used in the literature.There is only a minor problem: for the case of a BNS merger remnant, the rotational profile doesn't look like this...

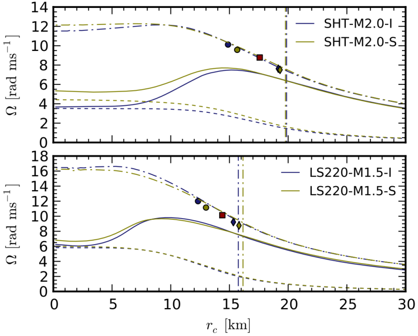




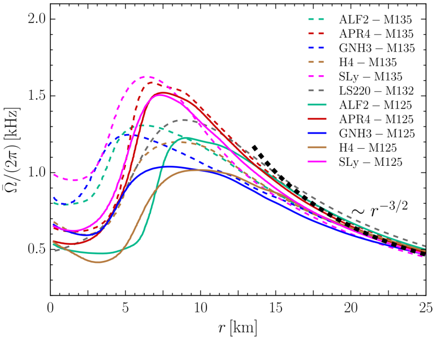
$\Omega(r)$ rotational profile from simulations

But like this...

Kastaun & Galeazzi (2015)



Hanauske et al (2017)



$\Omega(r)$ rotational profile from simulations

- The remnant's rotational profile has been studied extensively in a variety of setups.
- The rotational profile exhibits a maximum away from center and modest EOS dependence.

see: Kastaun & Galeazzi (2015), Bauswein & Stergioulas (2015), Kastaun et al (2016), Endrizzi et al (2016), Kastaun et al (2017), Ciolfi et al (2017), Hanauske et al (2017), Endrizzi et al (2018), Kiuchi et al (2018), Ciolfi et al (2019), East et al (2019), De Pietri et al (2020), ...

Works departing from KEH law

- Galeazzi et al (2012)
- Uryu et al (2016)
- Uryu et al (2017)
- Bauswein & Stergioulas (2017)
- Bozzola et al (2017)

Uryu et al (2017): new differential rotation laws

- A new 4-parameter differential rotation law was proposed in Uryu et al (2017):

$$\Omega = \Omega_c \frac{1 + \left(\frac{F}{B^2\Omega_c}\right)^p}{1 + \left(\frac{F}{A^2\Omega_c}\right)^{q+p}}$$

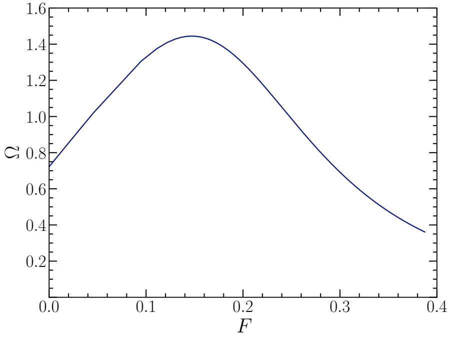
- Uryu et al presented only a few models in their work. This is the first systematic study of equilibrium models with the new rotation law.
- Iosif & Stergioulas (2020), arXiv:2011.10612

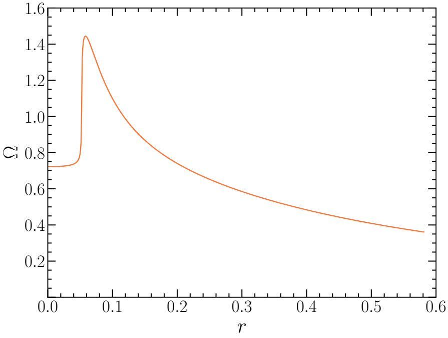
Preparing comparison with KEH law

- We choose a cold, polytropic EOS with N=1 and K=100 (for direct comparison with models constructed with previous rotation law)
- We use dimensionless units of $c = G = M_\odot = 1$.
- We expanded the `RNS` code (that implements the KEH scheme with modifications from Cook+ 1992) for the case of the new Uryu+ law.
- `RNS` code: https://gitlab.com/niksterg/rns1-1

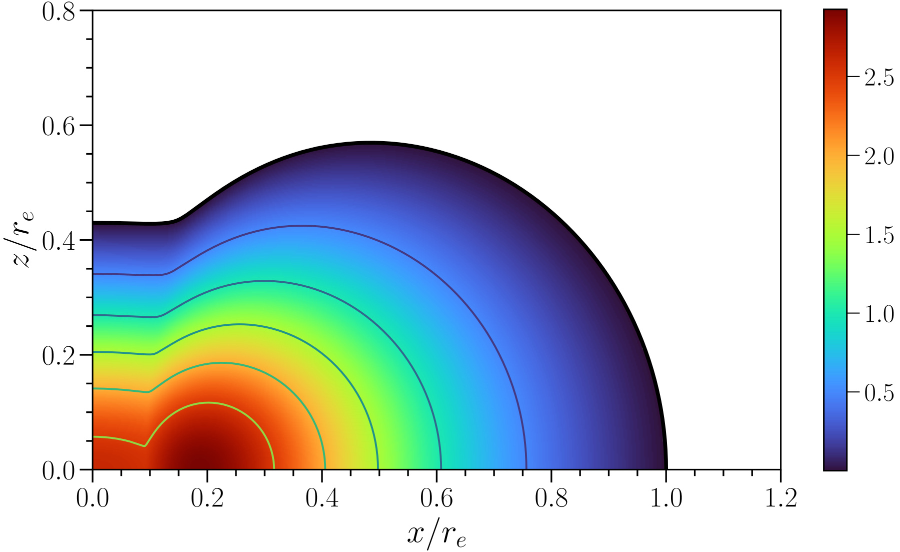
see: Stergioulas et al (2004), Iosif & Stergioulas (2014), Stergioulas & Friedman (1995)

What does this new law look like?

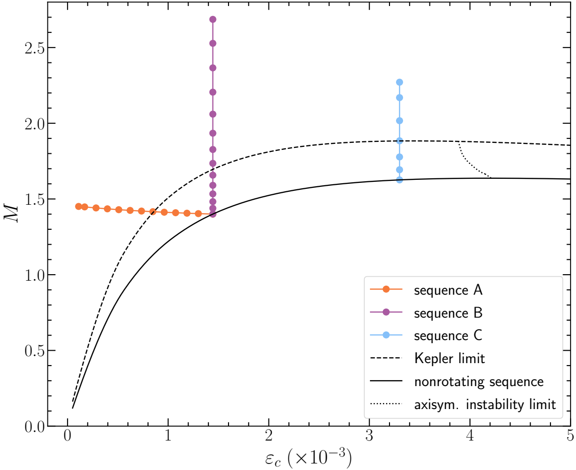




Meridional rest mass density distribution

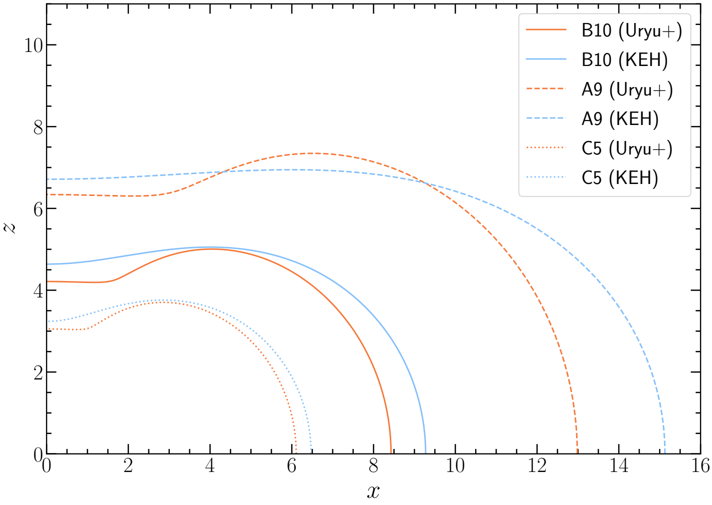


Equilibrium sequences, polytropic EOS (N=1)

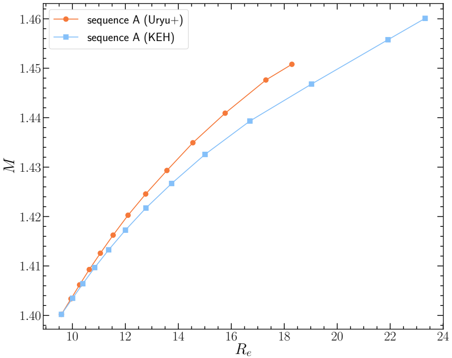


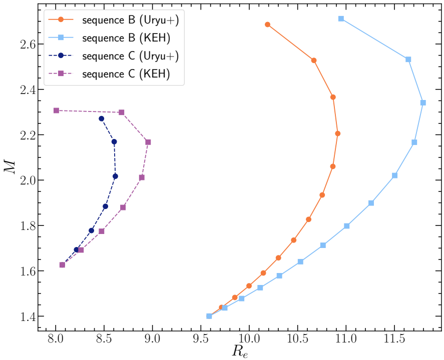
- Sequence A: constant rest mass $M_0 = 1.506$
- Sequence B: constant central energy density $\varepsilon_c = 1.444 \times 10^{-3}$
- Sequence C: constant central energy density $\varepsilon_c = 3.3 \times 10^{-3}$

Stellar surfaces for models with $r_p/r_e \sim 0.5$



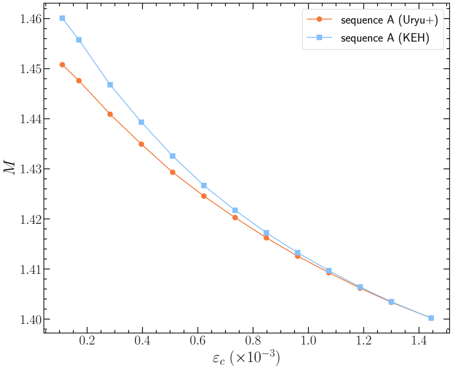
Uryu+ vs KEH law: circumferential radius R_e

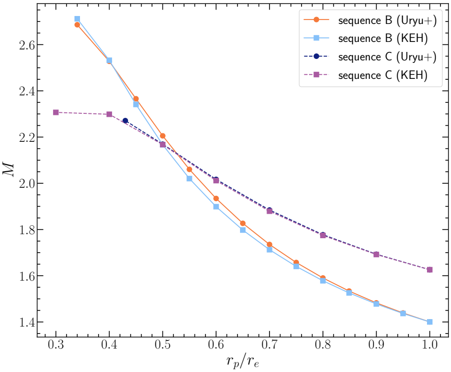




Ω_e was found to be smaller for the Uryu+ models than for the KEH models, resulting in a weaker centrifugal force and thus in a smaller radius.

Uryu+ vs KEH law: gravitational mass





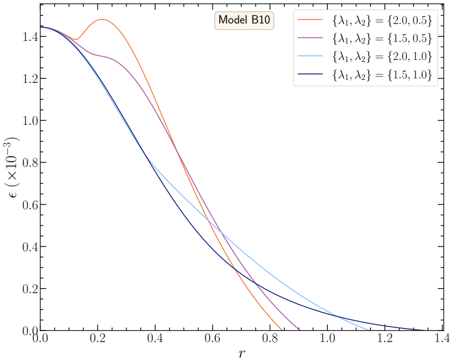
Uryu+ law parameters

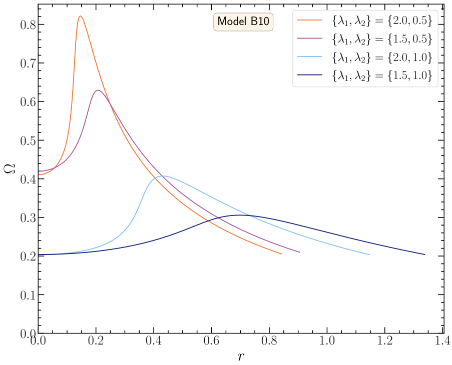
- We employed $p = 1$ and $q = 3$. Generalization to arbitrary non-integer {p, q} values is not straightforward.
- Parameters A and B in the Uryu+ rotation law can be calculated indirectly by fixing the ratios:

$$\lambda_1 = \frac{\Omega_{\mathsf{max}}}{\Omega_c} \qquad , \qquad \lambda_2 = \frac{\Omega_e}{\Omega_c}$$

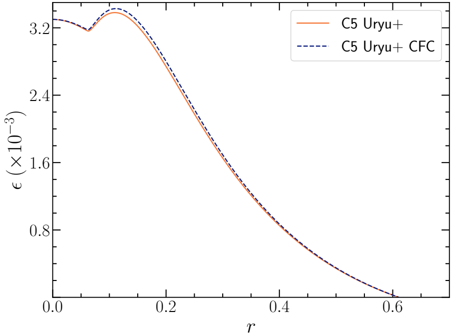
- Solve the system of equations for λ_1 and λ_2 to determine values for A and B in each iteration.

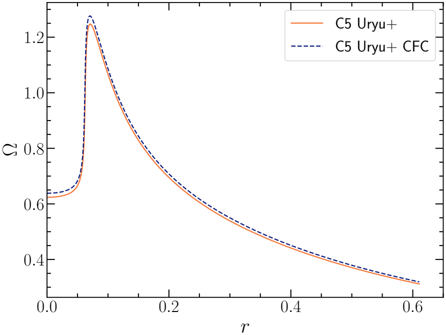
Effect of parameters λ_1, λ_2





GR vs IWM-CFC configurations





For models relevant to merger remnants the IWM-CFC approximation maintains an acceptable accuracy (relative error for local quantities is up to $\sim 2.5\%$ and $\sim 1\%$ for the masses and the ratio $T/|W|$).

see: Isenberg (2008), Wilson et al (1996), Stergioulas et al (2004), Iosif & Stergioulas (2014)

Summary

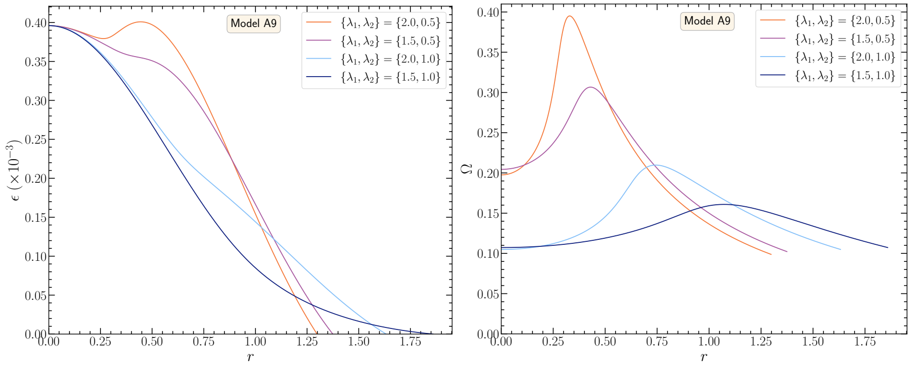
- There has been a shortage of reference models constructed with the newly proposed Uryu+ rotation laws. Our study partially fills this gap, for the case of the new 4-parameter rotation law.
- The versatility of the new law allows us to construct models that have a similar rotational profile and axis ratio as observed for merger remnants, while at the same time being quasi-spherical.
- Through the ratios λ_1 and λ_2, one can easily take advantage of information provided from numerical simulations, in order to fine-tune the parameter values of the rotation law.

Further steps

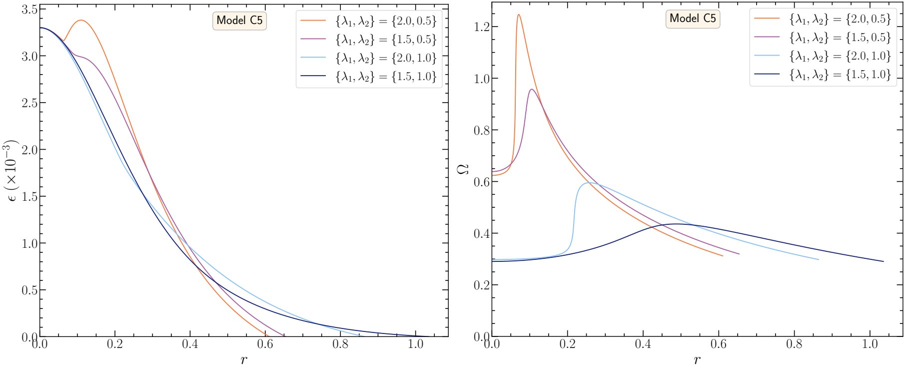
- tabulated EOS
- hot EOS
- T and Ω profiles taken directly from simulations
- perform time evolutions, study oscillations and dynamical collapse

related recent works: Zhou et al (2019), Passamonti & Andersson (2020), Xie et al (2020), Camelio et al (2020) supplementary slides

Effect of parameters λ_1, λ_2: low density



Effect of parameters λ_1, λ_2: high density



Some definitions

- F denotes the gravitationally redshifted angular momentum per unit rest mass and enthalpy

$$F = u^t u_\phi = \frac{v^2}{(1 - v^2)(\Omega - \omega)} = \frac{(\Omega - \omega) r^2 \sin^2 \theta e^{-2\rho}}{\left[1 - (\Omega - \omega)^2 r^2 \sin^2 \theta e^{-2\rho}\right]}$$

- $j = h u_\phi$ is the specific angular momentum, i.e. the angular momentum per unit baryon mass, consistent with the integral expression of the total angular momentum $J = \int j dM_0$.
- j is conserved along an axisymmetric flow, i.e. its Lie derivative vanishes, $\mathcal{L}_u (h u_\phi) = 0$.

A comment on the integrability condition

- Consider the first integral of the hydrostationary equilibrium:

$$H - \ln u^t + \int_{\Omega_{\text{pole}}}^{\Omega} F(\Omega')d\Omega' = \text{constant}$$

- In the case of the transient compact remnant formed after a BNS merger, the numerically extracted $\Omega(r)$ profile in the equatorial plane, implies that $F(\Omega)$ is not a one-to-one function.
- This means: instead of $F = F(\Omega)$, we need to consider $\Omega = \Omega(F)$.

$$\int F d\Omega = \int F \frac{d\Omega}{dF} dF$$

A (very) brief look on the IWM-CFC approximation

- Assume conformal flatness for the spatial metric

$$\gamma_{ij} = \psi^4 \eta_{ij}$$

where ψ is a conformal factor and η_{ij} is the flat metric.

- The line element in the $3+1$ formalism of GR

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

where α is the lapse function and β^i is the shift vector.

see: Isenberg (2008), Wilson et al (1996)

Framework of study

- Consider an stationary, axisymmetric star, in spherical-like coordinates.
- In the absence of meridional circulation, β^ϕ is the only non-zero component of the shift vector β^i.

GR and CFC line elements

- line element in IWM-CFC form

$$ds^2 = -\alpha^2 dt^2 + \psi^4(dr^2 + r^2 d\theta^2) + \psi^4 r^2 \sin^2\theta (d\phi + \beta^\phi dt)^2$$

- line element in full GR

$$ds^2 = -e^{\gamma+\rho} dt^2 + e^{\gamma-\rho} r^2 \sin^2\theta (d\phi - \omega dt)^2 + e^{2\mu}(dr^2 + r^2 d\theta^2)$$

- compare the above to get

$$\alpha = e^{(\gamma+\rho)/2}\,, \qquad \psi = e^{\mu/2} = e^{(\gamma-\rho)/4}\,, \qquad \beta^\phi = -\omega$$

Code implementation

- The line element in full GR can take the IWM-CFC form, if we set:

$$\mu = \frac{\gamma - \rho}{2}$$

- Get a numerical code, that already solves for the full GR metric.
- Convert it to CFC by imposing the above condition instead of solving for the metric potential μ.
- `RNS` code: https://gitlab.com/niksterg/rns1-1

see: Stergioulas & Friedman (1995), Stergioulas et al (2004), Iosif & Stergioulas (2014)