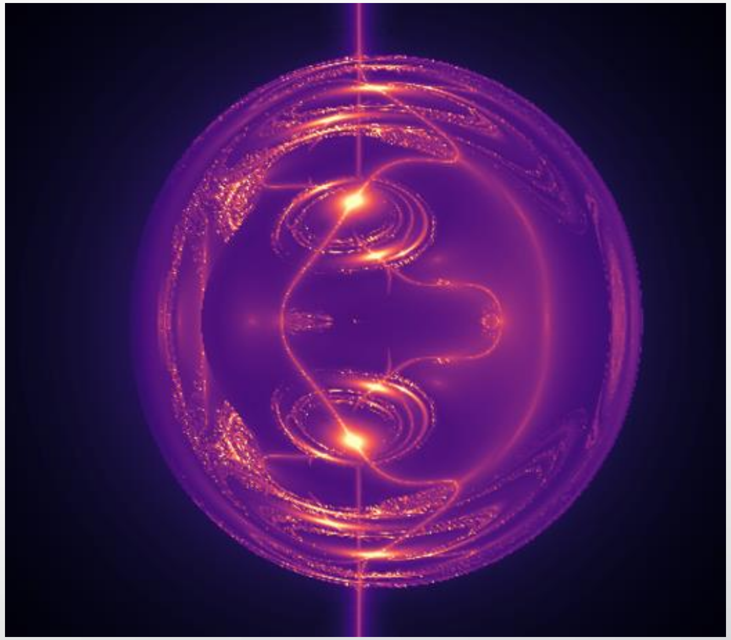
Black Holes Lessons from Multipole Ratios:
A New Window into Black Holes



Iosif Bena, Daniel Mayerson
IPhT, CEA Saclay, France

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Theory Session
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- « Multipole Ratios: A New Window into Black Holes,» *Phys.Rev.Lett.* 125 (2020) 22, 221602 [arXiv:2006.10750 [hep-th]] *(short paper)*
- « Black Holes Lessons from Multipole Ratios, » [arXiv:2007.09152 [hep-th]] *(long paper)*

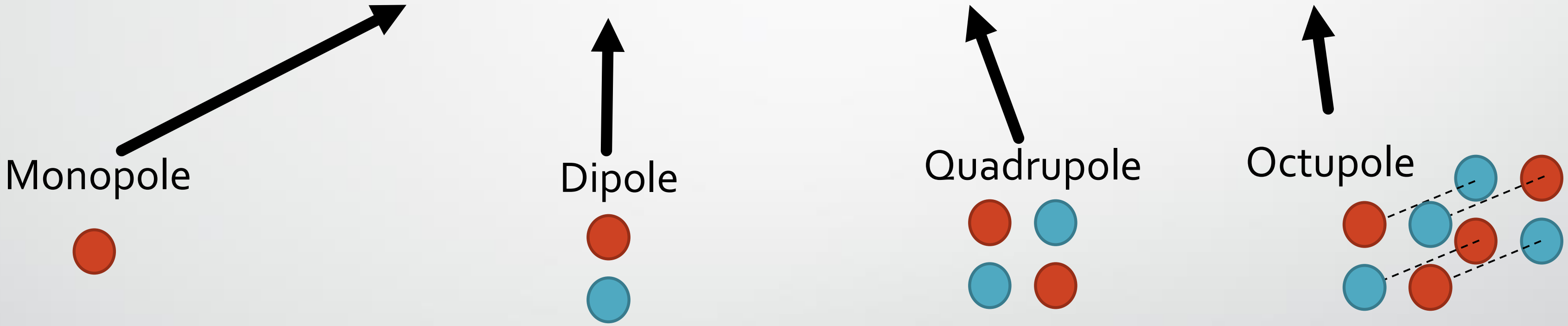
Outline

- Gravitational multipoles
- New Kerr Multipole Ratios
- New SUSY BH Multipole Ratios
- Multipole (Ratios) – A New Window into Black Holes

Gravitational multipoles (1)

- Multipoles in electrodynamics

$$V = \sum_{l \geq 0} \frac{1}{r^{l+1}} M_l P_l(\cos\theta) = \frac{M_0}{r} + \frac{M_1}{r^2}\cos\theta + \frac{M_2}{r^3} P_2(\cos\theta) + \frac{M_3}{r^4} P_3(\cos\theta) + \dots$$



- Problem in GR: coordinate transformations $\leftrightarrow$ multipoles not well-defined?
- Solution: Geroch-Hansen, Thorne formalism

Gravitational multipoles (2)

- Thorne: ACMC-N coordinates
  (« asymptotically Cartesian and mass-centered to order N »)

Mass

Mass multipoles:
Coordinate-invariant

$$g_{tt} = -1 + \frac{2M}{r} + \sum_{l\geq 2}^{N} \frac{2}{r^{l+1}} \left(M_l P_l + \sum_{l'<l} c_{ll'}^{(tt)} P_{l'} \right)$$

$$+ \frac{2}{r^{N+2}} \left(M_{N+1} P_{N+1} + \sum_{l' \neq N+1} c_{(N+1)l'}^{(tt)} P_{l'} \right) + \mathcal{O}\left(r^{-(N+3)} \right),$$

No dipole!

Coordinate-dependent
« harmonics »

- Similar for space-space components and: $g_{t\phi} \sim (\text{current multipoles } S_l)$

Gravitational multipoles (3)

- Mass multipoles M_l , current multipoles S_l

$$g_{tt} \sim \sum_l \frac{M_l}{r^{l+1}} \qquad g_{t\phi} \sim r \sum_l \frac{S_l}{r^{l+1}}$$

  - Mass $M = M_0$ angular momentum $J = S_1$

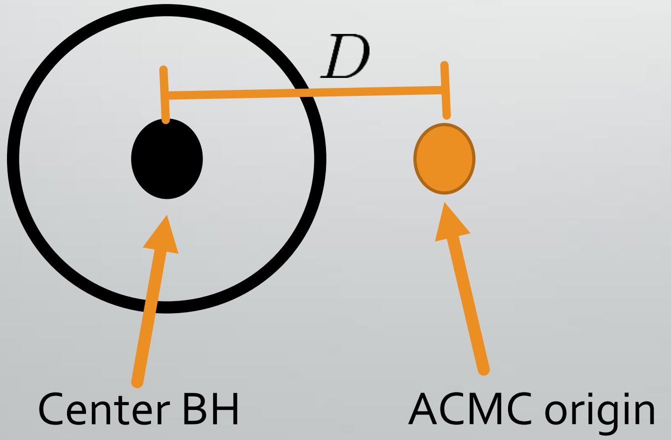
- Basic example: Kerr (M, a)
- Multipoles: $M_{2n} = M(-a^2)^n, \qquad S_{2n+1} = Ma(-a^2)^n$

$$M_{2n+1} = S_{2n} = 0$$

New Kerr Multipole Ratios (1)

- Remember Kerr: $M_{2n} = M(-a^2)^n,\ S_{2n+1} = Ma(-a^2)^n$

- Most general STU black hole
  10 parameters: 4 electric charges, 4 magnetic charges, mass, angular momentum
- Multipoles: all function of 4 parameters (M, J, a, D)
  - Note: $J \neq Ma$
  - D « dipole » parameter $\leftrightarrow$ BH electromagnetic interactions



New Kerr Multipole Ratios (2)

- Remember Kerr: $M_{2n} = M(-a^2)^n, \; S_{2n+1} = Ma(-a^2)^n$

- Most general STU black hole: multipoles (M, J, a, D)
- Reduces back to Kerr:

$$\mathcal{M}_{\text{Kerr}} = \lim_{J\to Ma} \lim_{D\to 0} \mathcal{M}(M, J, a, D)$$

- Multipole ratios, e.g.: $\mathcal{R} \equiv \dfrac{M_2 S_2}{M_3 S_1}$ Vanish for Kerr $\leftrightarrow$ ratio undefined

- General BH: $\mathcal{R} = 1$
- Kerr: $\mathcal{R}_{\text{Kerr}} = \lim_{J\to Ma} \lim_{D\to 0} \mathcal{R}(M, J, a, D) = 1$

Analogy: $\dfrac{\sin 0}{0} = ?$

vs

$\lim_{x\to 0} \dfrac{\sin x}{x} = 1$

New Kerr Multipole Ratios (3)

- Can define any (dimensionless) multipole ratio like this!

$$\mathcal{R}_{\text{Kerr}} = \lim_{J\to M a} \lim_{D\to 0} \mathcal{R}(M, J, a, D)$$

$$\boxed{\mathcal{R}_{\text{Kerr}} = \lim_{(\text{def. BH})\to(\text{Kerr})} \mathcal{R}}$$

- *Note: limit always well-defined (non-trivial!)*

- More examples: $$\frac{M_{l+2}S_l}{M_l S_{l+2}} = 1 - \frac{4}{3+(-1)^l(2l+1)}$$

$$\frac{M_2 S_l}{M_{l+1}S_1} = 1$$

New Kerr Multipole Ratios (4)

$$\mathcal{R}_{\text{Kerr}} = \lim_{J\to Ma} \lim_{D\to 0} \mathcal{R}(M,J,a,D) = \lim_{(\text{def. BH})\to(\text{Kerr})} \mathcal{R}$$

- Constrains all pert. deviations away from Kerr in string theory! $\delta(\text{Kerr}) \sim \epsilon$

$$S_{2n} = -nM(-a^2)^n \epsilon \qquad M_{2n+1} = nMa(-a^2)^n \epsilon$$

$$\delta M_{2n} = -n^2 M(-a^2)^n \left(\frac{2n-3}{4n}\right)\epsilon^2 \qquad \delta S_{2n+1} = -n^2(-a^2)^n Ma \left(\frac{2n+1}{4n}\right)\epsilon^2$$

- Constrains models of (small) deviations of Kerr!

New Kerr Multipole Ratios (5)

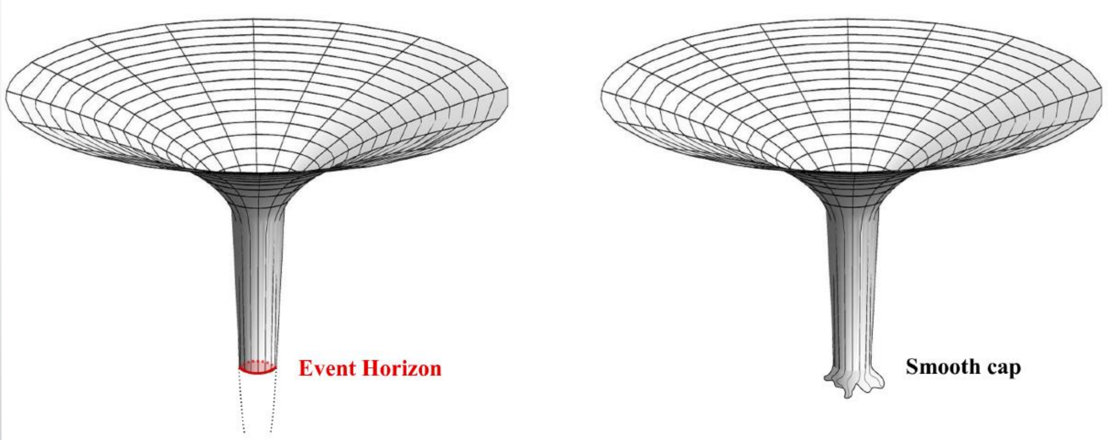
- **String theory prediction** for small deviations away from Kerr
  - Constrains models of (small) deviations of Kerr!

- **Observational consequences…? (more speculative)**
  - Late-time relaxation after BH formation; how different multipole rad. dies out
    - → Relation to quasinormal modes?
  - Measurements find different multipole (ratios)?
    - → Deformations are in theory $\neq$ string theory
    - → OR: BH horizon scale physics very different than GR!

New SUSY BH Multipole Ratios (1)

- Same procedure for (non-rotating) 4D supersymmetric BH
  - non-rotating: all multipoles vanish except $M_0 = M$
- Deform and then $\lim_{(\text{def. BH}) \to (\text{SUSY})} \mathcal{R} := \mathcal{R}_{\text{SUSY}}$ **Indirect**

- OR: use « fuzzballs » or microstate geometries of BH in string theory
  - Supersymmetric
  - Smooth
  - No horizon
  - Same asymptotic charges as BH
  - Extra (string theory) dims: « bubbles » in space



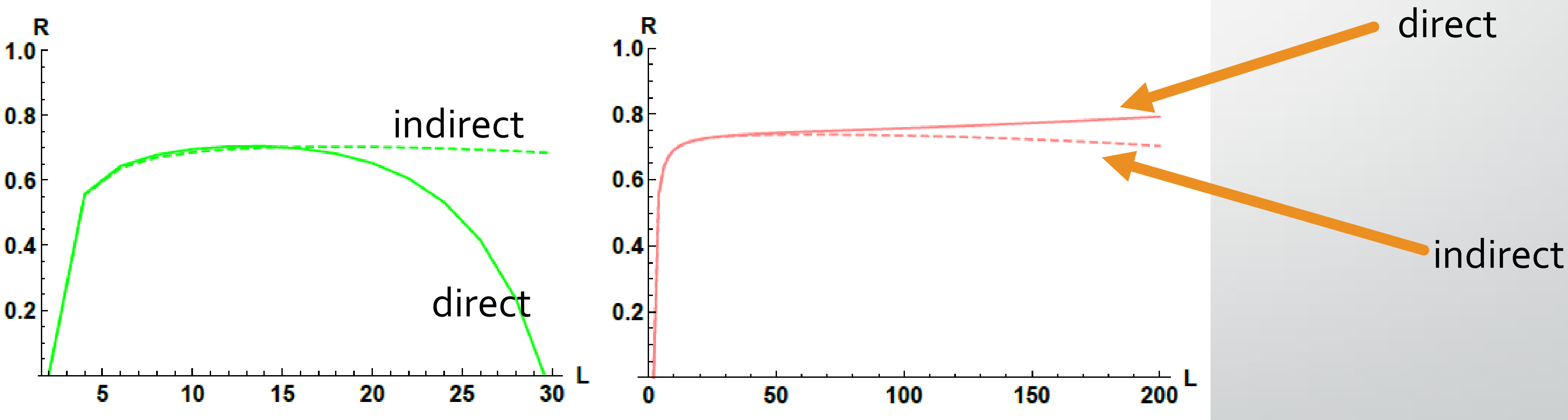
- Second way to calculate ratios: $\mathcal{R}_{(\text{microstate})} := \mathcal{R}_{\text{SUSY}}$ **Direct**

Also: Bianchi, Consoli, Grillo, Morales, Pani 2007.01743 & 2008.01445

New SUSY BH Multipole Ratios (2)

- Remarkable agreement between **indirect** and **direct** for some BHs

$$\lim_{(\text{def. BH})\to(\text{SUSY})} \mathcal{R} := \mathcal{R}_{\text{SUSY}} \qquad \mathcal{R}_{(\text{microstate})} := \mathcal{R}_{\text{SUSY}}$$



(b) $R_2^{(M)}(L, \delta = 1/2)$ as a function of L for geometry A

(c) $R_2^{(M)}(L, \delta = 1/2)$ as a function of L for geometry B

Multipole (Ratios):
A New Window into Black Holes

- **New multipole ratios for Kerr** (and SUSY BH)!
- Kerr = (special) limit of generic 4D BH in string theory
- Define new multipole ratios: $\mathcal{R}_{\text{Kerr}} = \lim_{(\text{def. BH}) \to (\text{Kerr})} \mathcal{R}$
- **String theory prediction** for small deviations away from Kerr
  - Constrains models of (small) deviations of Kerr!
- **Possible observational consequences?**
  - Late-time relaxation (QNMs)?
  - Measurements of multipole (deformations)?

New SUSY BH Multipole Ratios (3)

$$\mathcal{R}^{(\mathrm{direct})}_{\mathrm{SUSY-BH}} = \lim_{(scaling)} \mathcal{R}_{\mathrm{microstate}}$$

$$\mathcal{R}^{(\mathrm{indirect})}_{\mathrm{SUSY-BH}} = \lim_{J,a\to 0} \lim_{D\to D(Q_I)} \mathcal{R}(M,J,a,D)$$

- « Mismatch parameter »: $\mathcal{E}^{(\mathcal{R})} \equiv \left| \frac{\mathcal{R}^{(\mathrm{dir})} - \mathcal{R}^{(\mathrm{ind})}}{\mathcal{R}^{(\mathrm{ind})}} \right|$

- For SUSY BH/microstate: « entropy parameter » $\mathcal{H} = \left(\frac{S(Q_I, P_I)}{S(Q_I, 0)} \right)^2$
  - *Entropy of black hole compared to entropy of black hole with no magnetic charges (and same electric charges)*

New SUSY BH Multipole Ratios (4)

$$\mathcal{R}^{(\text{direct})}_{\text{SUSY-BH}} = \lim_{(scaling)} \mathcal{R}_{\text{microstate}}$$

$$\mathcal{R}^{(\text{indirect})}_{\text{SUSY-BH}} = \lim_{J,a\to 0} \lim_{D\to D(Q_I)} \mathcal{R}(M,J,a,D)$$

- « Mismatch parameter »: $\mathcal{E}$
- « Entropy parameter »: $\mathcal{H}$

AMAZING correlation!

Geometry	$\mathcal{H}$	$\mathcal{E}_{(\text{ave})}$
$(1,0)$	0.28	2.79
$(2,1)$	0.098	15.7
A	7.7×10^{-4}	0.0451
B	7.9×10^{-6}	0.000888
C	0.055	2.31
D	0.24	8.56

« pincers » (1,0) & (2,1)