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Asian Nuclear Physics Association

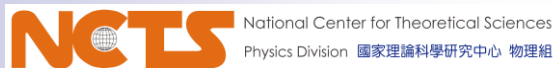
Initial-State-Driven Spin and Polarization Effects in Heavy Ion Collisions



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(ANPhA 2025, Taipei, Nov. 29th, 2025)

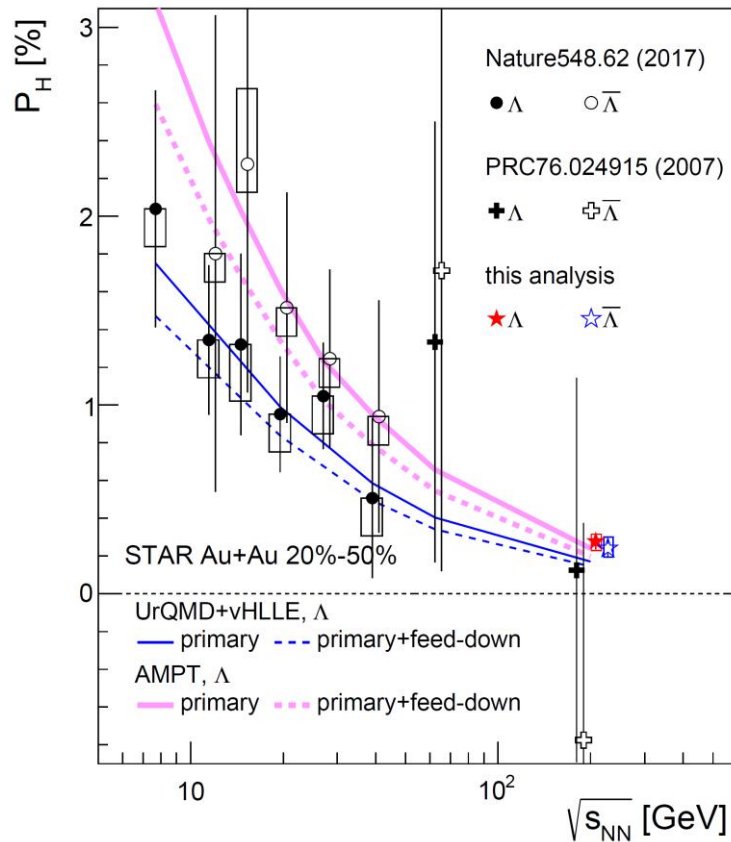


Global Λ polarization in HIC

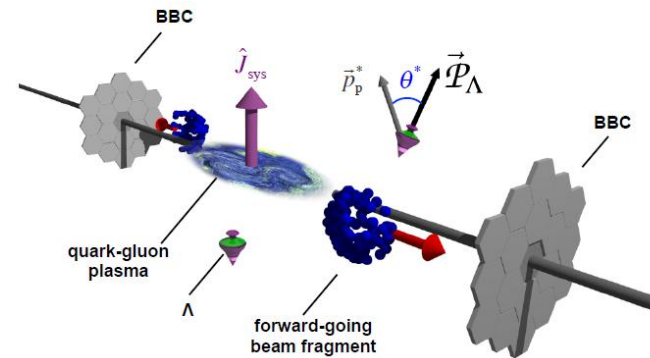
- The measurement of global polarization of Λ hyperons revealed the spin-orbit interaction & strong vorticity in heavy ion collisions. Z.-T. Liang & X.-N. Wang, PRL. 94, 102301 (2005)

(relativistic Barnett effect)

- Self-analyzing via the weak decay : $\Lambda \rightarrow p + \pi^-$



L. Adamczyk et al. (STAR), Nature 548, 62 (2017)



- Successfully described by the modified Cooper–Frye formula in **global equilibrium** : F. Becattini, et al., Ann. Phys. 338, 32 (2013)
R. Fang, et al., PRC 94, 024904 (2016)

$$\mathcal{P}^\mu = \frac{\int d\Sigma \cdot p f_p^{(0)} (1 - f_p^{(0)}) \epsilon^{\mu\nu\rho\sigma} q_\nu \omega_{\rho\sigma}}{8M_\Lambda \int d\Sigma \cdot p f_p^{(0)}}$$

$$\omega_{\rho\sigma} = \frac{1}{2} \left(\partial_\rho \left(\frac{u_\sigma}{T} \right) - \partial_\sigma \left(\frac{u_\rho}{T} \right) \right) \cdot \text{thermal vorticity}$$

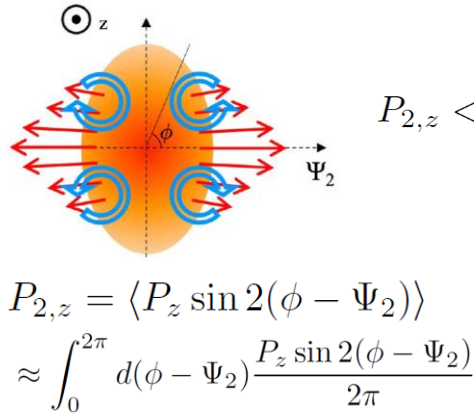
- Global pol. from (average) kinetic vorticity :

$$P_{\Lambda(\bar{\Lambda})} \approx \int_p \mathcal{P}^{-y}(p) \simeq \frac{\omega}{2T} \longrightarrow \omega = \frac{1}{2} |\nabla \times u| \sim 10^{22} s^{-1}$$

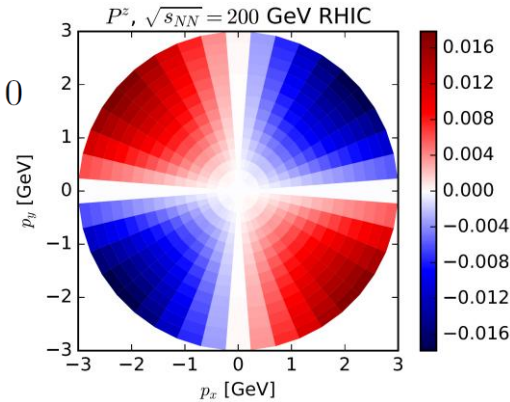
F. Becattini et al., PRC95, 054902 (2017)

Longitudinal (local) polarization

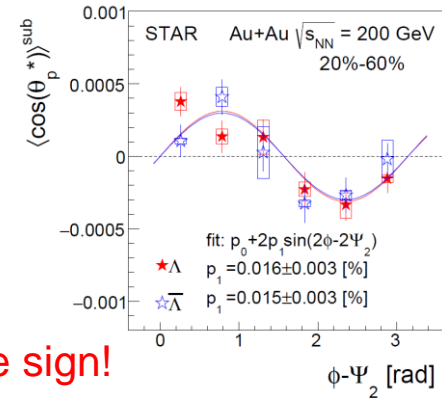
Longitudinal polarization along the beam direction in HIC :



F. Becattini, I. Karpenko, PRL 120, 012302 (2018)

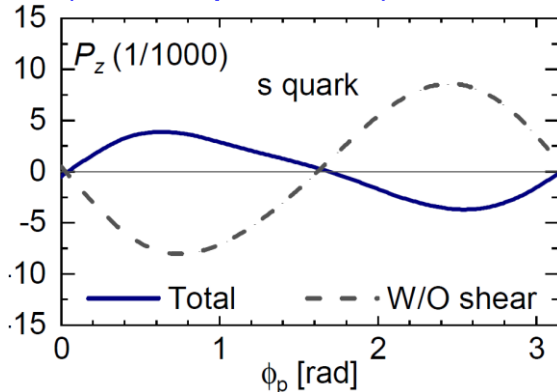


J. Adam et al. (STAR), PRL. 123, 132301 (2019)



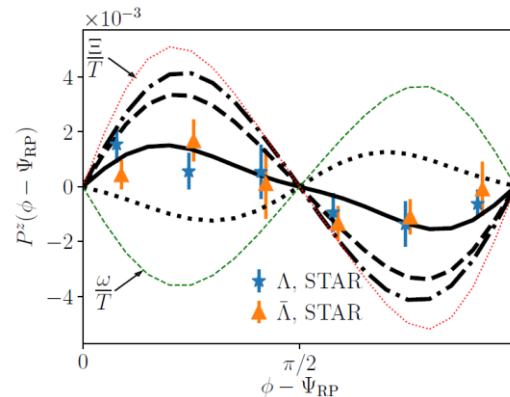
❖ Thermal-shear corrections : $\mathcal{P}^\mu = \frac{\int d\Sigma \cdot p f_p^{(0)} (1 - f_p^{(0)}) \epsilon^{\mu\nu\rho\sigma} q_\nu [\omega_{\rho\sigma} - u_\rho \pi_{\sigma\lambda} p^\lambda / (T p \cdot u)]}{8M_\Lambda \int d\Sigma \cdot p f_p^{(0)}}$

(local equilibrium)



strange-memory scenario :
 $M_\Lambda \rightarrow m_s$

B. Fu et al., PRL 127, 142301 (2021)

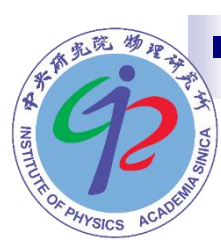


Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018)
 S. Liu, Y. Yin, JHEP 07, 188 (2021)
 F. Becattini, et al., PLB 820,136519 (2021)

isothermal approx. :
 no T gradient correction
 F. Becattini et al., PRL. 127, 272302 (2021)

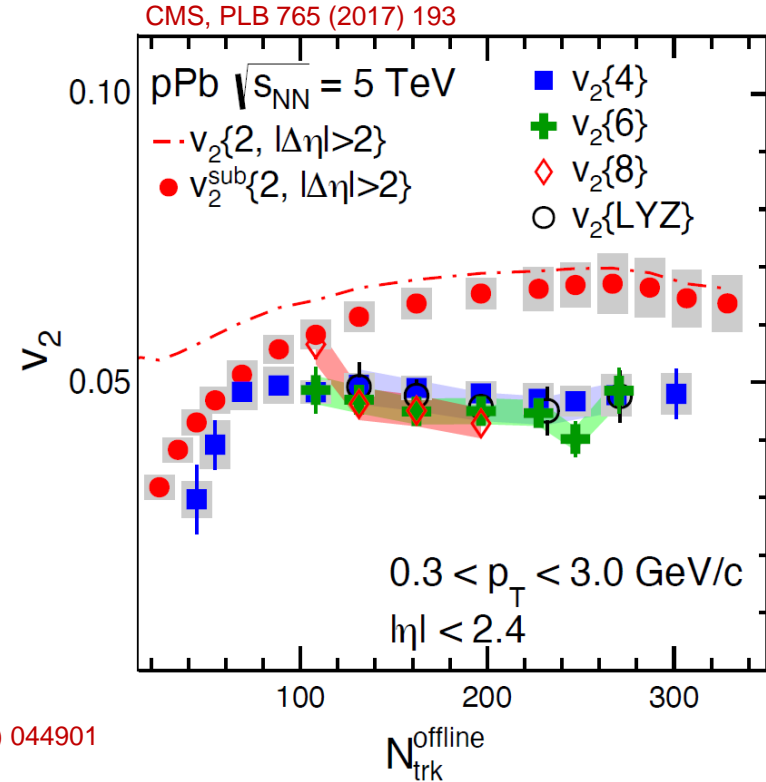
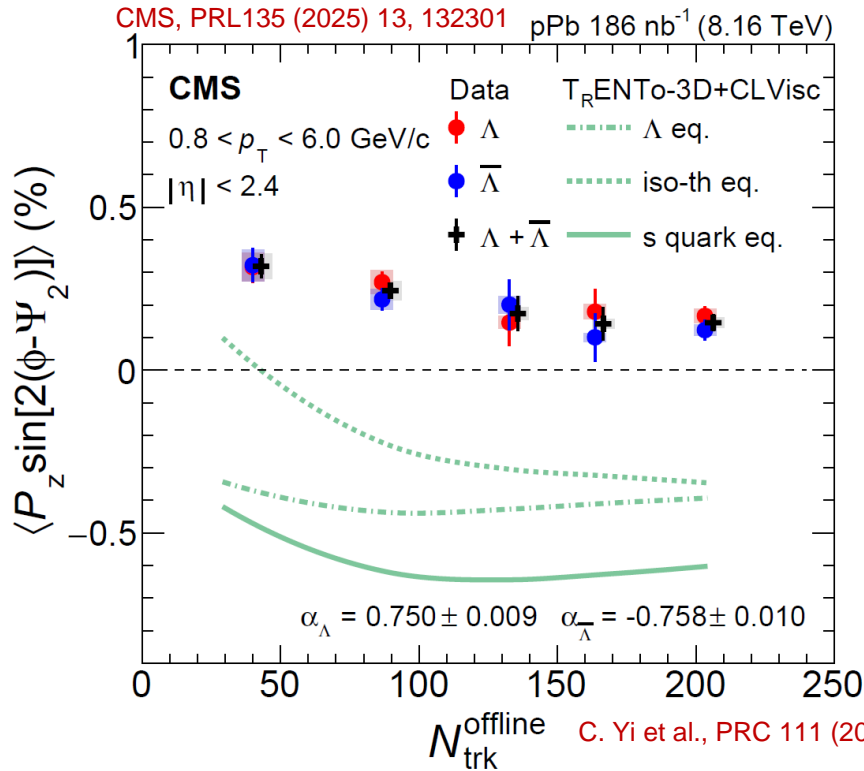
❖ The overall sign depends on adopted approximations.

see also C. Yi et al., PRC 104, 064901 (2021), W. Florkowski et al., PRC 105, 064901(2022)



Small-collision systems

■ Longitudinal polarization in high-multiplicity pA collisions :



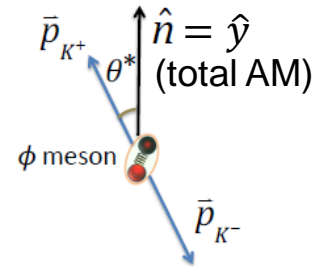
- ❖ Thermal vorticity + thermal shear as the **final-state effect** fails to describe the measurements
- ❖ An opposite trend w.r.t the flow : **initial-state or non-equilibrium effects?**

Spin alignment of vector mesons

- Angular dep. of the decay particle w.r.t the spin quantization axis :

$$\frac{dN}{d \cos \theta^*} \propto [1 - \rho_{00} + \cos^2 \theta^* (3\rho_{00} - 1)]$$

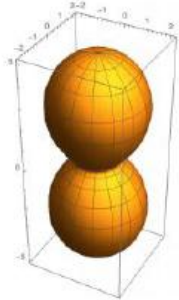
$$\rho_{00} = \frac{1 - \langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}{3 + \langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}$$



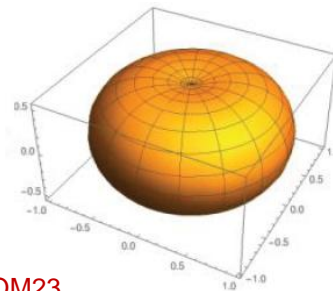
Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)

$\rho_{00} \neq 1/3$: spin correlation

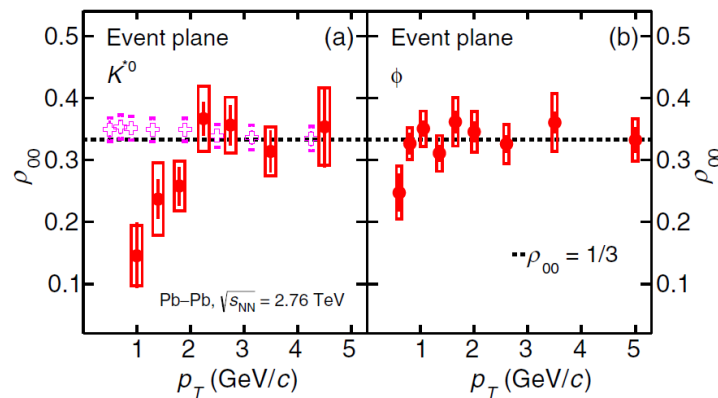
$\rho_{00} > 1/3$:



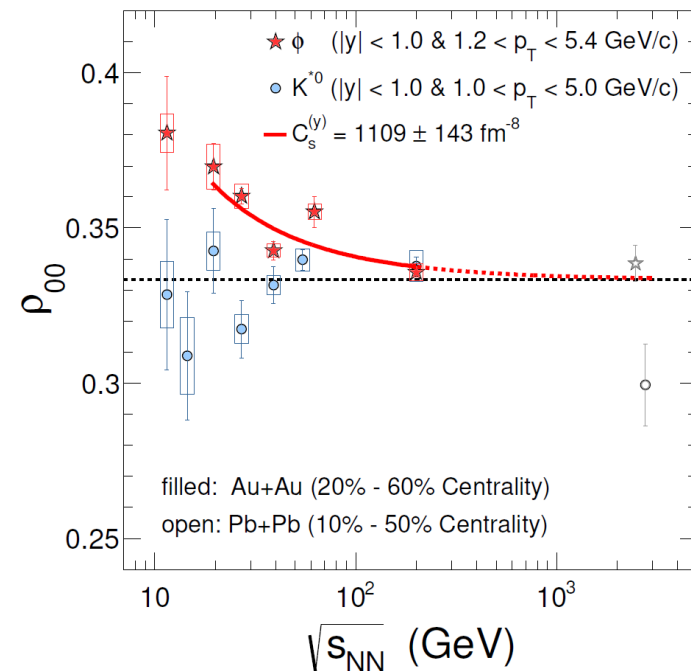
$\rho_{00} < 1/3$:



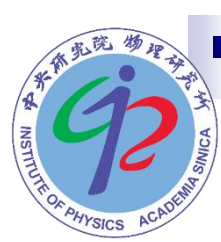
B. Xi, QM23



S. Acharya et al. (ALICE), PRL.125, 012301 (2020)



M.S. Abdallah et al. (STAR), Nature 614 (2023) 7947, 244-248,



Spin polarization beyond subatomic swirls?

- Spin alignment puzzle : the deviation of ρ_{00} from $1/3$ is unexpectedly large
e.g. $\rho_{00} \approx \frac{1}{3} - \left(\frac{\omega}{T}\right)^2$, $\frac{\omega}{T} \sim 0.1\%$ at LHC energy. (from Λ pol. \sim s quark pol.)

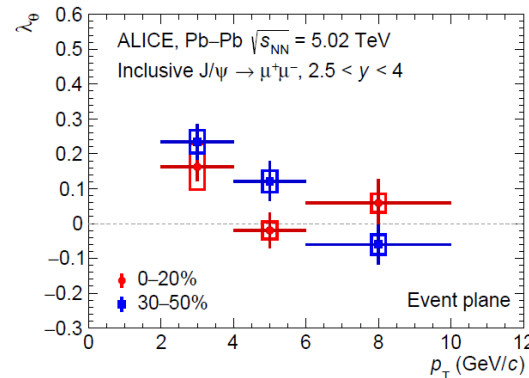
- Flavor & collision energy dep. :

	ϕ	K^{*0}
ALICE	$\rho_{00} < 1/3$ ($p_T \leq 1$ GeV)	$\rho_{00} < 1/3$
STAR	$\rho_{00} > 1/3$	$\rho_{00} \approx 1/3$

- Spin alignment for J/ψ :

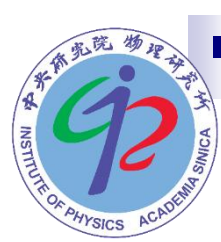
S. Acharya et al. (ALICE), PRL 131,042303 (2023)

$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}} > 0 \implies \rho_{00} < \frac{1}{3}$$



- Other sources for the spin correlation (alignment) beyond vorticity?
- Small spin pol. & large spin correlation imply that the source may be **fluctuating**.
- Electromagnetic fields can polarize the spin. How about gluon fields in QCD matter? $\mathcal{P} \propto \mathbf{B} - \mathbf{u} \times \mathbf{E}$

magnetic pol. spin Hall effect with momentum anisotropy



An intuitive picture

- An intuitive construction of **color-singlet** spin observables :

- ❖ Building blocks for quark transport :

$$\mathbf{F}^a = g(\mathbf{E}^a + \mathbf{u} \times \mathbf{B}^a), \quad \mathcal{P}^a = g(\mathbf{B}^a - \mathbf{u} \times \mathbf{E}^a).$$

chromo-Lorentz force

chromo-magnetic polarization & spin Hall effect

- ❖ Parity-even correlators only :

$$\langle E^i(x) E^j(x) \rangle = \delta^{ij} \langle E^i(x) E^i(x) \rangle, \quad \langle B^i(x) B^j(x) \rangle = \delta^{ij} \langle B^i(x) B^i(x) \rangle, \quad \langle E^i(x) B^j(x) \rangle = 0.$$

- spin alignment (without flow) : $\delta\rho_{00} = \rho_{00} - \frac{1}{3} \sim \langle \mathcal{P}^a \cdot \mathcal{P}^a \rangle \sim \langle \mathbf{B}^a \cdot \mathbf{B}^a \rangle$
- spin polarization of Λ (strange-equilibrium scenario) : p – even scalar

$$\mathcal{P} \sim \langle (\mathbf{p} \cdot \mathbf{F}^a) \mathcal{P}^a \rangle \quad \text{flow-induced polarization}$$

$$\sim \langle (\mathbf{p} \times \mathbf{u}) (\langle \mathbf{B}^a \cdot \mathbf{B}^a \rangle + \langle \mathbf{E}^a \cdot \mathbf{E}^a \rangle) \rangle \quad p \text{ – even axial-vector}$$

- Such effects as **non-equilibrium** spin transport can be systematically derived from the **quantum kinetic theory**, as the generalization of the Boltzmann equation, with spin & color degrees of freedom.

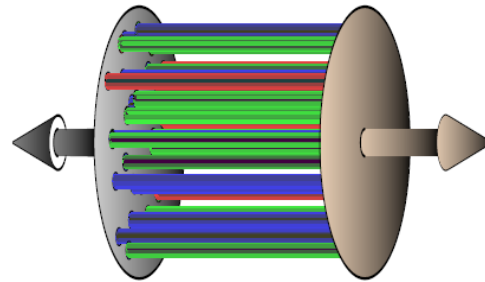
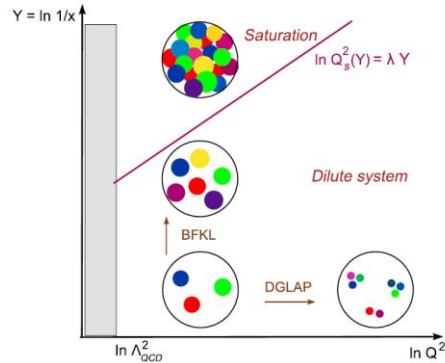
Review : Y. Hidaka S. Pu, Q. Wang, DY, PPNP 127, 103989 (2022)

DY, JHEP 06, 140 (2022)

B. Müller, DY, PRD 105, L011901 (2022)

Color-field induced spin alignment

- Initial-state gluons may form the **glasma** phase characterized by predominantly longitudinal chromo-electromagnetic fields from CGC.



(at top RHIC & LHC energies)

Reviews : F. Gelis et al., Ann.Rev.Nucl.Part.Sci.60:463-489,2010

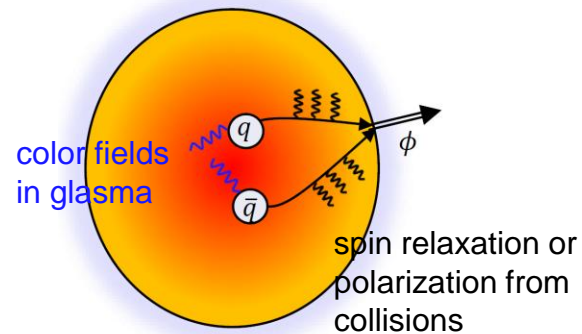
J. Berges et al., Rev. Mod. Phys. 93 (2021) 3, 035003

- Assuming the early quark production in glasma

N. Tanji, J. Berges, PRD, 97, 034013 (2018)

Why glasma fields for spin alignment?

- (1) intrinsic saturation scale $Q_s \gg \omega$
- (2) fluctuating (no effect on global Λ pol.)
- (3) intrinsic anisotropy (need not be along \hat{n})



Updated coalescence model :

$$\rho_{00}(q) \approx \frac{1 - \text{Tr}_c \langle \hat{\mathcal{P}}_q^y(q/2) \hat{\mathcal{P}}_{\bar{q}}^y(q/2) \rangle_{q=0}}{3 - \sum_{i=x,y,z} \text{Tr}_c \langle \hat{\mathcal{P}}_q^i(q/2) \hat{\mathcal{P}}_{\bar{q}}^i(q/2) \rangle_{q=0}}$$

glasma effect :

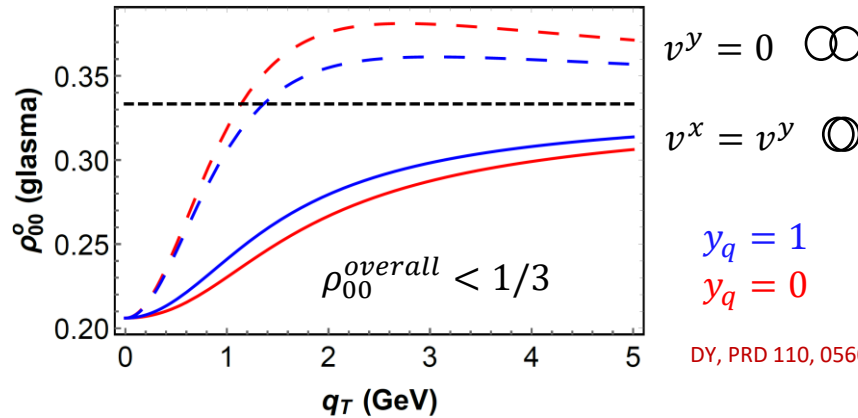
$$\text{Tr}_c \langle \hat{\mathcal{P}}_q^z(q/2) \hat{\mathcal{P}}_{\bar{q}}^z(q/2) \rangle_{q=0} < 0$$

\longrightarrow $\delta\rho_{00} = \rho_{00} - \frac{1}{3} \propto -\langle B^{az}(x) B^{az}(x') \rangle e^{-2\Delta t/\tau_R} < 0$
A. Kumar, B. Müller, DY, PRD 107, 076025 (2023)

glasma effect
relaxation in QGP

Transverse spin alignment spectra

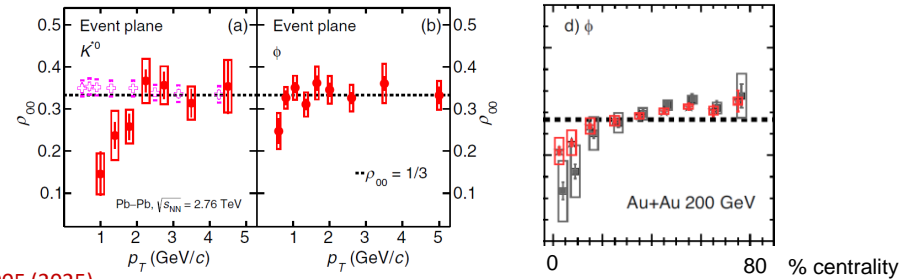
- Out-of-plane spin alignment (ϕ mesons) :
- ❖ Initial-state effect (glasma) : **high-energy**



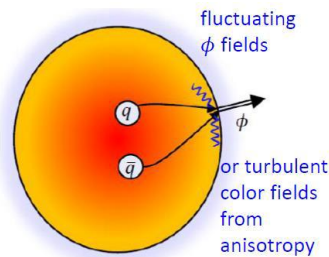
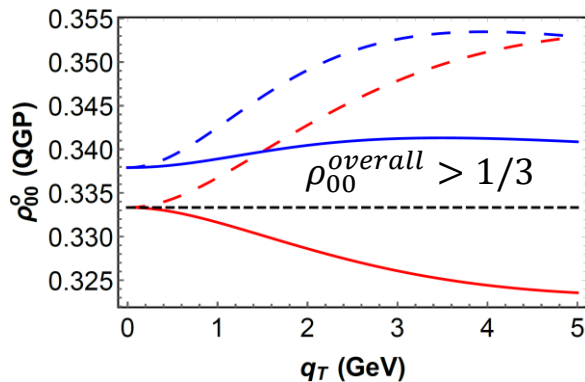
DY, PRD 110, 056005 (2025)

boost color fields for momentum dep.

weighting needed : $\langle \rho_{00}(q_T) \rangle = \frac{\int d\phi_q dy_q [\rho_{00}(\mathbf{q}) \mathcal{N}]}{\int \phi_q dy_q \mathcal{N}}$

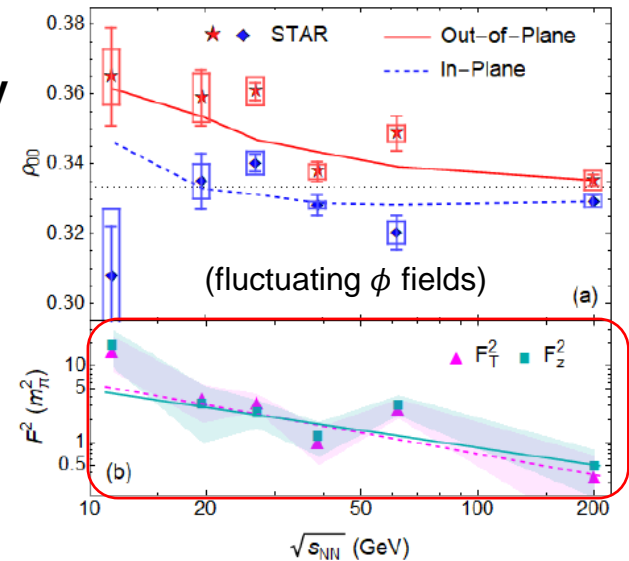


- ❖ Final-state effect (strong-force fields in QGP) : **low-energy**



X.-L. Sheng et al., PRD 109, 036004, (2024)
PRL 131, 042304 (2023)
B. Müller, DY, PRD 105, L011901 (2022)
DY, JHEP 06, 140 (2022)

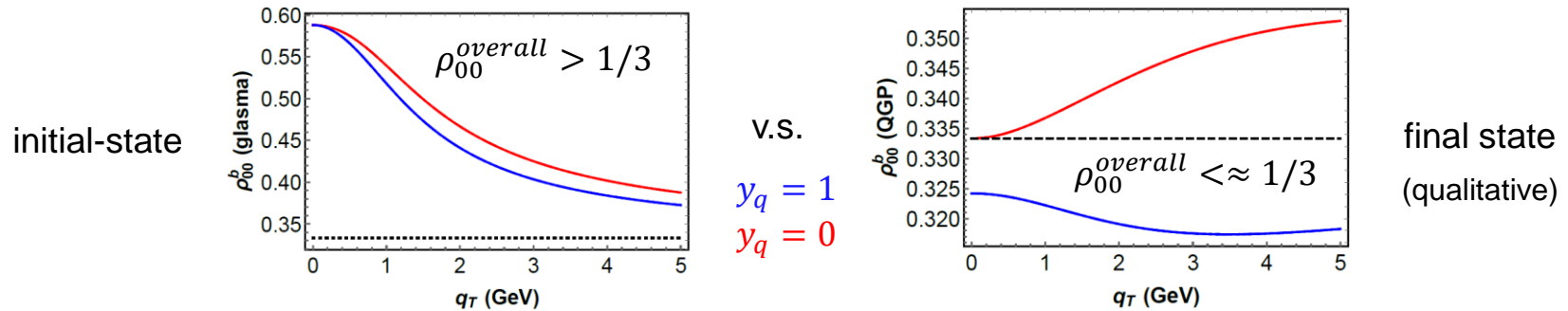
(isotropic color fields from QGP : qualitative)



- Collision-energy dependence may come from initial v.s. final state effects

Observables for future measurements

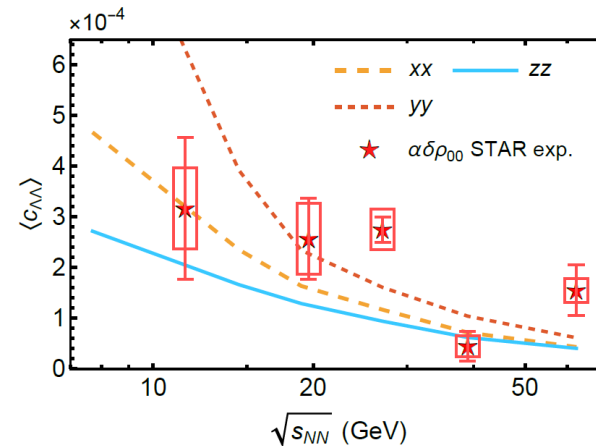
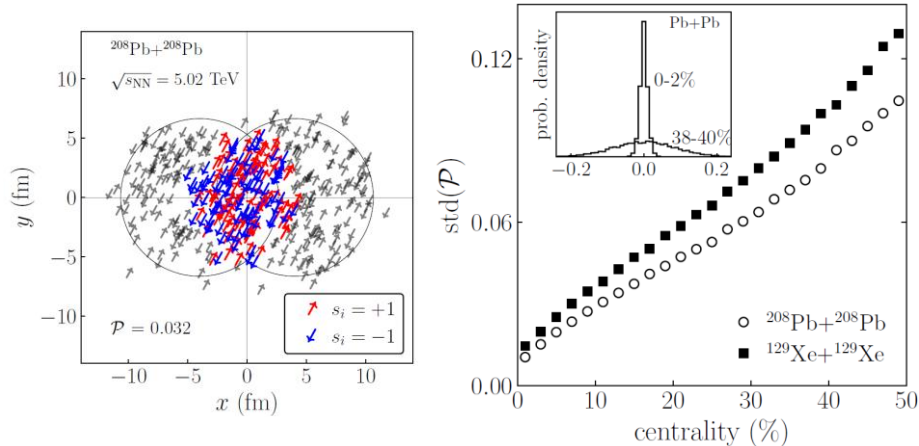
- Longitudinal spin alignment along the beam direction : DY, PRD 110, 056005 (2025)



- Spin-spin correlation of hyperons : $C_{12}^{\mu\nu}(p_1, p_2) = \frac{1}{N_{\text{event}}} \sum_{\text{event}} \frac{\int d\Sigma(x_1) \cdot p_1 \int d\Sigma(x_2) \cdot p_2 P_1^\mu P_2^\nu f_1 f_2}{[\int d\Sigma(x_1) \cdot p_1 f_1] [\int d\Sigma(x_2) \cdot p_2 f_2]}$
D. Shen, Ji. Chen, A. Tang, arXiv: 2407.21291
J.-p. Lv et al., PRD 109, 114003 (2024)

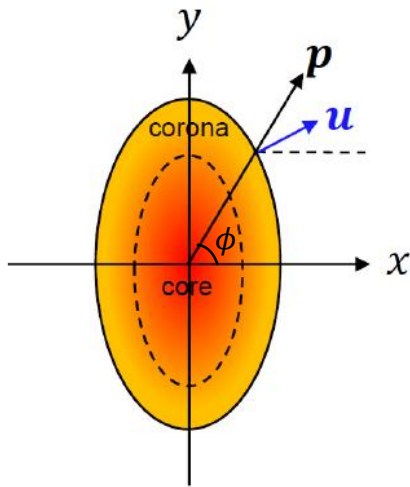
initial-state : nucleon spin fluct., glasma fields?

final-state : meson fields, thermal-gluon fields?



Longitudinal Λ polarization (focused on pA)

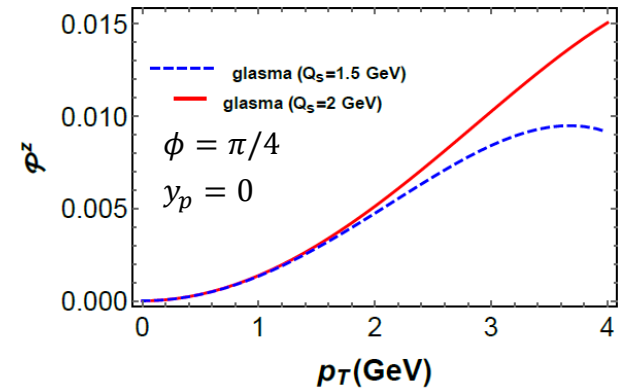
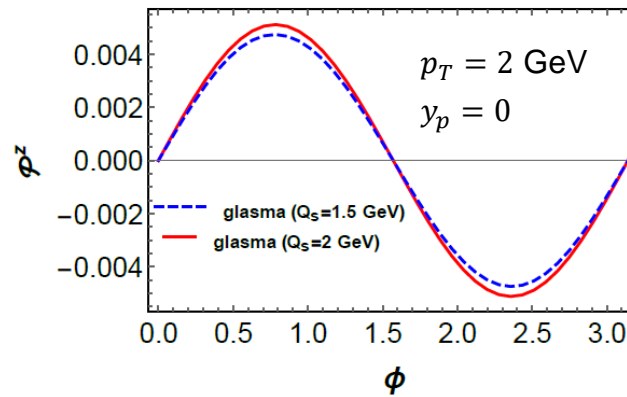
- Longitudinal polarization from the **corona of glasma** (early hadronization) :



weak initial anisotropic flow from pressure gradient

H. Sung, B. Müller, DY, arXiv: 2507.23210

$$\mathcal{P}^z \approx \frac{\hbar(N_c^2 - 1)Q_s^2 \Delta t}{16M_\Lambda N_c^2 \epsilon_p} \int d\Sigma \cdot p (p^x u^y - p^y u^x) \mathbf{p} \times \mathbf{u} \text{ structure} \\ \times (u^0 \partial_{p \cdot u / Q_s} - Q_s / \epsilon_p) \partial_{p \cdot u / Q_s} f_V^S, \quad f_V^S(p \cdot u / Q_s) = e^{-\pi(p \cdot u)^2 / |gE^\alpha|}$$

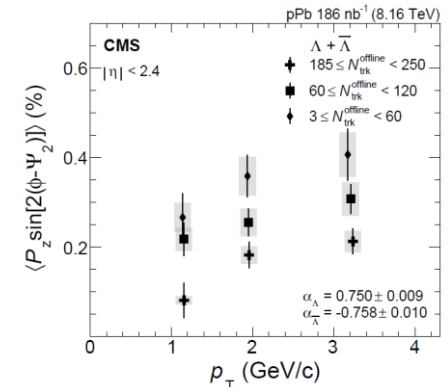


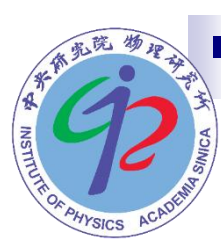
- ✓ consistent sign & comparable order of magnitude with observations

For smaller systems & high p_T

For larger systems & low p_T

$$\diamond \text{ Full pol. : } \mathcal{P}^z = \frac{N_{GL} \mathcal{P}_{GL}^z + N_{QGP} \mathcal{P}_{QGP}^z}{N_{GL} + N_{QGP}}$$





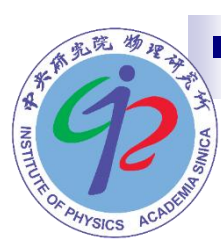
Conclusions & outlook

□ Conclusions :

- ✓ Initial-state effects such as **glasma fields** could play a significant role on **local spin polarization** and **spin alignment**, while there exist competing final-state effects.
- ✓ The **size** and **energy** dependence of collision systems may be helpful to disentangle the competing effects.
- ✓ New observables like longitudinal spin alignment & spin-spin correlations of hyperons will be also useful.

□ Outlook :

- More sophisticated modeling & simulations are needed.
e.g., more accurate estimation on spin relaxation in QGP
- (Collective) spin transport for heavy quarks : more sensitive to the initial-state effects.



Thank you!



QKT for relativistic massive fermions

- **Quantum kinetic theory (QKT)** : tracking the spin evolution in phase space

Review : Y. Hidaka S. Pu, Q, Wang, DY, PPNP 127, 103989 (2022)

- **Axial kinetic theory (AKT)** : scalar/axial-vector kinetic eqs. (SKE/AKE)

➤ SKE : $p \cdot \Delta f_V = \mathcal{C}[f_V]$, $\Delta_\mu = \partial_\mu + eF_{\nu\mu}\partial_p^\nu$.

K. Hattori, Y. Hidaka, DY, PRD 100 (2019), 096011
DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)

standard Boltzmann (Vlasov) eq.

➤ AKE : $p \cdot \Delta \tilde{a}^\mu + eF^{\nu\mu}\tilde{a}_\nu - \frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma eF_{\beta\nu})\partial_p^\beta f_V = q_\nu \widehat{\Sigma_V^\nu} \tilde{a}^\mu + \dots + \frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}q_\nu(\Delta_\sigma \widehat{\Sigma_V^\rho}) f_V$,

($\tilde{a}^\mu(p, x)$: effective spin four vector)

entangled f_V & \tilde{a}^μ

$$\widehat{AB} = A^<B^> - A^>B^<.$$

(\hbar : gradient corrections in phase space)

□ Spin pol. spectra : $\mathcal{P}^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^\mu(\mathbf{p}, x)}{2m \int d\Sigma \cdot \mathcal{N}(\mathbf{p}, x)}$, $\mathcal{J}_5^\mu(\mathbf{p}, x) \propto \int dp_0 \mathcal{A}^\mu(p, x)$

$$\mathcal{A}^\mu = 2\pi \left(\delta(p^2 - m^2) \tilde{a}^\mu + \hbar \delta'(p^2 - m^2) e \tilde{F}^{\mu\nu} p_\nu f_V \right)$$

(dynamical)

(non-dynamical)

- Spin alignment from quark coalescence :

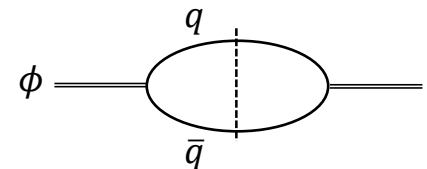
KE for vector mesons : $q \cdot \partial f_\lambda^\phi \approx \epsilon_\mu^*(\lambda, \mathbf{q}) \epsilon_\nu(\lambda, \mathbf{q}) \mathcal{C}_{\text{coal}}^{\mu\nu}(q, x)$

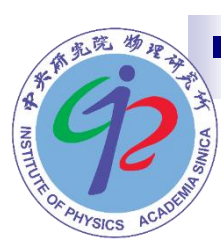
quark-meson int. :

$$\mathcal{L}_{\text{int}} = g_\phi \bar{\psi} \gamma^\mu V_\mu \psi$$

($\rho_{\lambda\lambda} \propto f_\lambda^\phi$, $\lambda = 0, \pm 1$)

➔ $\rho_{00}(q) = \frac{\int d\Sigma_X \cdot q f_0^\phi(q, X)}{\int d\Sigma_X \cdot q (f_0^\phi(q, X) + f_{+1}^\phi(q, X) + f_{-1}^\phi(q, X))}$





Axial kinetic theory with color fields

- Incorporation of background color fields into Wigner functions and kinetic equations.

- Color decomposition : $O = O^s I + O^a t^a$

U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)

H. T. Elze, M. Gyulassy, D. Vasak, Nucl. Phys. B276, 706 (1986).

e.g., $\mathcal{A}^\mu(\mathbf{p}, x) = \mathcal{A}^{s\mu}(\mathbf{p}, x)I + \mathcal{A}^{a\mu}(\mathbf{p}, x)t^a$, $f_V(\mathbf{p}, x) = f_V^s(\mathbf{p}, x)I + f_V^a(\mathbf{p}, x)t^a$,
 $\tilde{a}^\mu(\mathbf{p}, x) = \tilde{a}^{s\mu}(\mathbf{p}, x)I + \tilde{a}^{a\mu}(\mathbf{p}, x)t^a$.

- Kinetic equations : DY, JHEP 06, 140 (2022)

SKEs : $p^\rho \left(\partial_\rho f_V^s + \frac{g}{2N_c} F_{\nu\rho}^a \partial_p^\nu f_V^a \right) = \mathcal{C}_s$, $p^\rho \left(\partial_\rho f_V^a + g F_{\nu\rho}^a \partial_p^\nu f_V^s + \frac{d^{bca}}{2} g F_{\nu\rho}^b \partial_p^\nu f_V^c \right) = \mathcal{C}_o^a$,

diffusion

dynamical spin polarization

AKEs : $p^\rho \partial_\rho \tilde{a}^{s\mu} + \frac{g}{2N_c} \left(p^\rho F_{\nu\rho}^a \partial_p^\nu \tilde{a}^{a\mu} + F^{a\nu\mu} \tilde{a}_\nu^a \right) - \frac{\hbar}{4N_c} \epsilon^{\mu\nu\rho\sigma} p_\rho (\partial_\sigma g F_{\beta\nu}^a) \partial_p^\beta f_V^a = \mathcal{C}_s^\mu$,

$p^\rho \partial_\rho \tilde{a}^{a\mu} + g \left(p^\rho F_{\nu\rho}^a \partial_p^\nu \tilde{a}^{s\mu} + F^{a\nu\mu} \tilde{a}_\nu^s \right) + \frac{d^{bca}}{2} g \left(p^\rho F_{\nu\rho}^b \partial_p^\nu \tilde{a}^{c\mu} + F^{b\nu\mu} \tilde{a}_\nu^c \right) - \frac{\hbar}{2} \epsilon^{\mu\nu\rho\sigma} p_\rho (\partial_\sigma g F_{\beta\nu}^a) \partial_p^\beta f_V^s = \mathcal{C}_o^{a\mu}$.

Axial Wigner functions : $\mathcal{A}^{s\mu}(\mathbf{p}, x) = \frac{1}{2\epsilon_{\mathbf{p}}} \left[\tilde{a}^{s\mu} - \frac{\hbar}{4N_c} \tilde{F}^{a\mu\nu} \left(\partial_{p\nu} f_V^a - \frac{\epsilon_{\mathbf{p}}}{2} \partial_{p_\perp\nu} (f_V^a / \epsilon_{\mathbf{p}}) \right) \right]_{p_0=\epsilon_{\mathbf{p}}}$,

$\mathcal{A}^{a\mu}(\mathbf{p}, x) = \frac{1}{2\epsilon_{\mathbf{p}}} \left[\tilde{a}^{a\mu} - \frac{\hbar}{2} \tilde{F}^{a\mu\nu} \left(\partial_{p\nu} f_V^s - \frac{\epsilon_{\mathbf{p}}}{2} \partial_{p_\perp\nu} (f_V^s / \epsilon_{\mathbf{p}}) \right) \right]_{p_0=\epsilon_{\mathbf{p}}}$.

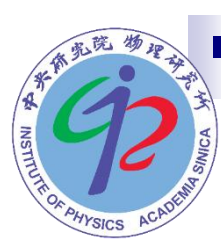
dynamical (w/ memory effect)

non-dynamical (w/o memory effect)

Spin

polarization:

$\mathcal{P}^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p \text{Tr}_c \mathcal{A}^\mu(\mathbf{p}, x)}{2m \int d\Sigma \cdot p (2\epsilon_{\mathbf{p}})^{-1} f_V^s(\mathbf{p}, x)} = \frac{\int d\Sigma \cdot p \mathcal{A}^{s\mu}(\mathbf{p}, x)}{2m \int d\Sigma \cdot p (2\epsilon_{\mathbf{p}})^{-1} f_V^s(\mathbf{p}, x)}$.



Spin alignment from glasma

- Generalized AKT with color fields : $\tilde{a}^\mu(\mathbf{p}, x) = \tilde{a}^{s\mu}(\mathbf{p}, x)I + \tilde{a}^{a\mu}(\mathbf{p}, x) t^a$.
DY, JHEP 06, 140 (2022)
 B. Müller, DY, PRD 105, L011901 (2022) (more dominant in the perturbative approach)

- Dynamical spin polarization from glasma fields : A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)

$$\tilde{a}^{a\mu}(\mathbf{p}, x) \approx -\frac{\hbar g}{2} e^{-(t_f - t_i)/\tau_R^0} (B^{a\mu}(t_i) \partial_{\epsilon_{\mathbf{p}}} f_V^s(\epsilon_{\mathbf{p}}, t_i) - B^{a\mu}(t_f) \partial_{\epsilon_{\mathbf{p}}} f_V^s(\epsilon_{\mathbf{p}}, t_f))$$

suppressed

- Correlation of initial color magnetic fields : $g^2 \langle B^{az}(x) B^{az}(x) \rangle_{x_0=t_i} \sim \frac{Q_s^4 (N_c^2 - 1)}{2N_c}$
K. J. Golec-Biernat and M. Wusthoff, Phys. Rev. D 59, 014017 (1998)
 P. Guerrero-Rodríguez and T. Lappi, Phys. Rev. D 104, 014011 (2021)

- Initial quark distribution function : $f_V^s(\epsilon_{\mathbf{p}}, t_i) = 1 / (e^{\epsilon_{\mathbf{p}}/Q_s} + 1)$

- spin correlation : $\langle \mathcal{P}_q^z \mathcal{P}_{\bar{q}}^z \rangle \sim \frac{Q_s^2}{m_q m_{\bar{q}}} e^{-2(t_f - t_i)/\tau_R^0}$

- Order-of-magnitude estimation (for ϕ) : $\rho_{00} \sim \frac{1}{3 + 10 e^{-2(t_f - t_i)/\tau_R^0}} < \frac{1}{3}$
 $Q_s \approx 1 \sim 2$ GeV
glasma effect relaxation effect

- Heavy-quark approx. : $\tau_R^0 \approx \left(\frac{g^2 C_2(F) m_D^2 T}{6\pi m^2} \ln g \right)^{-1} \approx 5 \text{ fm}/c \Rightarrow \rho_{00} \approx 0.24$

Spin correlations from color fields

- Spin density matrix can be directly related to Wigner functions of the coalesced quark and antiquark through the quark-meson interaction.

❖ Kinetic theory of vector mesons :

$$q \cdot \partial f_\lambda^\phi = \epsilon_\mu^*(\lambda, \mathbf{q}) \epsilon_\nu(\lambda, \mathbf{q}) [C_{\text{coal}}^{\mu\nu}(q, x)(1 + f_\lambda^\phi) - C_{\text{diss}}^{\mu\nu}(q, x)f_\lambda^\phi] \approx \boxed{\epsilon_\mu^*(\lambda, \mathbf{q}) \epsilon_\nu(\lambda, \mathbf{q}) C_{\text{coal}}^{\mu\nu}(q, x)}$$

quark-meson int. :

$$\mathcal{L}_{\text{int}} = g_\phi \bar{\psi} \gamma^\mu V_\mu \psi$$

➔

$$\rho_{00}(q) = \frac{\int d\Sigma_X \cdot q f_0^\phi(q, X)}{\int d\Sigma_X \cdot q (f_0^\phi(q, X) + f_{+1}^\phi(q, X) + f_{-1}^\phi(q, X))}$$

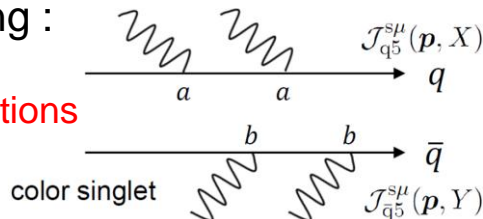
$$= \frac{1 - \text{Tr}_c \langle \hat{\mathcal{P}}_q^y(\mathbf{q}/2) \hat{\mathcal{P}}_{\bar{q}}^y(\mathbf{q}/2) \rangle_{\mathbf{q}=0}}{3 - \sum_{i=x,y,z} \text{Tr}_c \langle \hat{\mathcal{P}}_q^i(\mathbf{q}/2) \hat{\mathcal{P}}_{\bar{q}}^i(\mathbf{q}/2) \rangle_{\mathbf{q}=0}}$$

❖ Spin correlations in the non-relativistic limit :

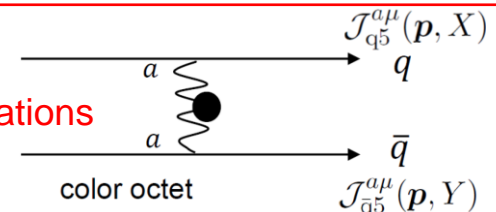
$$\text{Tr}_c \langle \hat{\mathcal{P}}_q^i(\mathbf{p}) \hat{\mathcal{P}}_{\bar{q}}^i(\mathbf{p}) \rangle \approx \frac{4 \int d\Sigma_X \cdot p (\langle \mathcal{A}_q^{\text{si}}(\mathbf{p}, X) \mathcal{A}_{\bar{q}}^{\text{si}}(\mathbf{p}, X) \rangle + \langle \mathcal{A}_q^{\text{oi}}(\mathbf{p}, X) \mathcal{A}_{\bar{q}}^{\text{oi}}(\mathbf{p}, X) \rangle) / (2N_c)}{\int d\Sigma_X \cdot p f_{\text{V}q}^{\text{si}}(\mathbf{p}, X) f_{\text{V}\bar{q}}^{\text{si}}(\mathbf{p}, X)}$$

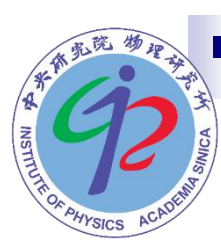
➤ Weak coupling :

4-field correlations
 $\propto g^4$



2-field correlations
 $\propto g^2$





Color fields from the glasma

- Solving linearized Yang-Mills eqs. : $[D_\mu, F^{\mu\nu}] = J^\nu$

P. Guerrero-Rodriguez, T. Lappi, PRD 104, 014011 (2021)

- Color-field correlators in the glasma :

evolution in time

$$\text{e.g. } \langle E_T^{ai}(X') E_T^{aj}(X'') \rangle = -\bar{N}_c \epsilon^{in} \epsilon^{jm} \int_{\perp; q, u}^{X'} \int_{\perp; l, v}^{X''} \Omega_-(u_\perp, v_\perp) \frac{q^n l^m}{ql} \times J_1(qX'_0) J_1(lX''_0),$$

$$\langle B_T^{ai}(X') B_T^{aj}(X'') \rangle = -\bar{N}_c \epsilon^{in} \epsilon^{jm} \int_{\perp; q, u}^{X'} \int_{\perp; l, v}^{X''} \Omega_+(u_\perp, v_\perp) \frac{q^n l^m}{ql} \times J_1(qX'_0) J_1(lX''_0),$$

$$\bar{N}_c \equiv \frac{1}{2} g^2 N_c (N_c^2 - 1),$$

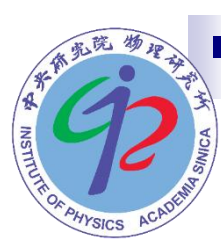
$$\Omega_{\mp}(u_\perp, v_\perp) = [G_1(u_\perp, v_\perp) G_2(u_\perp, v_\perp) \mp h_1(u_\perp, v_\perp) h_2(u_\perp, v_\perp)],$$

$$\int_{\perp; q, u}^{X'} \equiv \int \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 u_\perp e^{iq_\perp(X' - u)_\perp}.$$

unpolarized & linearly polarized
Gluon distribution functions

- Golec-Biernat Wusthoff (GBW) distribution : K. J. Golec-Biernat and M. Wusthoff, PRD 59, 014017 (1998)

$$\Omega_\pm(u_\perp, v_\perp) = \Omega(u_\perp, v_\perp) = \frac{Q_s^4}{g^4 N_c^2} \left(\frac{1 - e^{-Q_s^2 |u_\perp - v_\perp|^2 / 4}}{Q_s^2 |u_\perp - v_\perp|^2 / 4} \right)^2$$



More on spin polarization from color fields

- Isochronous freeze-out in 3+1 D : $\tilde{\tau} \equiv \sqrt{\tau^2 - x^2 - y^2}$

$$d\Sigma \cdot p = dr d\Phi d\eta r \sqrt{1 - \epsilon^2} (G_1 \cosh(y_p - \eta) + G_2), \quad G_1 = \sqrt{(m^2 + p_T^2)} \tau_f,$$

$$G_2 = -(xp^x + yp^y)$$

- Manifestation of $\sin 2\phi$: when $|\mathbf{p} \cdot \mathbf{u}| \ll Q_s$

$$\int d\Sigma \cdot p (p^x u^y - p^y u^x) \xrightarrow{G_2 \text{ dominates}} \int d\Sigma \cdot p u^{[y} p^{x]} u^0 \approx u_T p_T^2 \pi \sin 2\phi \int \frac{dr d\eta r^3 \tau \delta}{r_m^2}$$

$$u^\mu = \frac{1}{N_u} (t, x\sqrt{1+\delta}, y\sqrt{1-\delta}, z)$$

- QGP case : when $|\mathbf{p} \cdot \mathbf{u}| \gtrsim T \xrightarrow{\text{yellow arrow}} G_1 \text{ dominates}$

H. Sung, B. Müller, DY, arXiv: 2507.23210

single freeze-out thermal model at $\sqrt{s_{NN}} = 130$ GeV

