# CKM First Row Unitarity from lattice QCD

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**References**
**with F. He, R. Gupta & T. Bhattacharya** in progress
**with T. Bhattacharya, R. Gupta, S. Mondal & B. Yoon:** PRD 108 (2023) 034508
see also: **X. Feng et.al.** PRL 124 (2020) 192002
**P. Ma et.al.** PRL 132 (2024) 191901

# Outline

- Why First row Unitarity
- Experimental Landscape
- Lattice & Theory
- γW-box Contribution to neutron β decay
- Impact on $V_{ud}$ and $\Delta_{CKM}$
- Outlook with upcoming experiments

# CKM Unitarity

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

First Row Unitarity measure $\Delta_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$

$|V_{ub}| \sim 10^{-3}$ , negligible

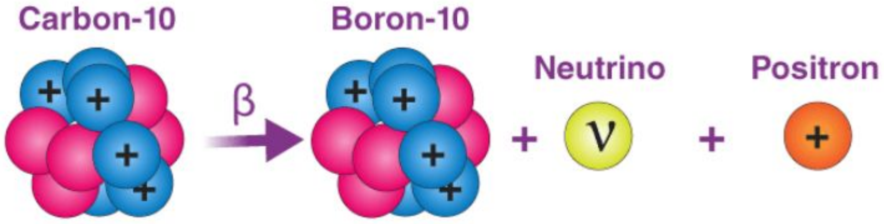
Current $\Delta_{\mathrm{CKM}}$:

−2.8σ deviation when using ($K\ell3$ , $K\mu2/\pi\mu2$)

−3.0σ deviation when using ($K\ell3$ , β-decay)

−1.7σ deviation when using ($K\mu2/\pi\mu2$ , β-decay)

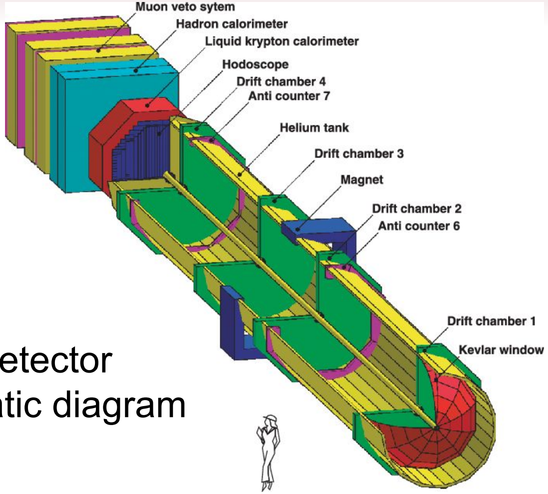
# Experimental inputs on $|V_{ud}|$




- Superallowed $0^+ \rightarrow 0^+$
  - RIBF, CENS, IGISOL
  - *Ft* values → $|V_{ud}|$
- Neutron β-decay
  - bottle method(UCN$\tau$), beam method(PERKEO, J-PARC)
  - A asymmetry → $g_A$ (UCNA, PERKEO)
- Mirror transitions
  - exotic nuclei from RIBF, CENS, BRIF
- π β-decay
  - PIONEER experiment

# Experimental inputs on $|V_{us}|$

- Kℓ3 semileptonic decays
  - NA48/2, KLOE, J-PARC
  - Needs lattice $f_+(0)$
- Kμ2 /πμ2 leptonic decays
  - NA62, KLOE
  - Needs lattice $f_K/f_\pi$ + EM/ISB
- Other (Hyperon decays, $\tau$ decays)
  - J-PARC, Belle II
  - Not at the precision test level




NA48 detector schematic diagram

J-PARC facility

# Theory Inputs on $|V_{ud}|$ & $|V_{us}|$

- $f_+(0)$
  - Form factor calculation on K semileptonic processes on the lattice

    See results by PACS (**T. Yamazaki,** 2412.05778),
- $f_K/f_\pi$
  - Leptonic decay constant

    See FLAG for results by ETM, CalLat, CLQCD, QCDSF, RBC/UKQCD
- $g_A$
  - Nucleon charges calculation

    See FLAG for results by PNDME, NME, PACS, ETM, RQCD, Mainz
- $\gamma$W-box
  - radiative correction to hadron β decay

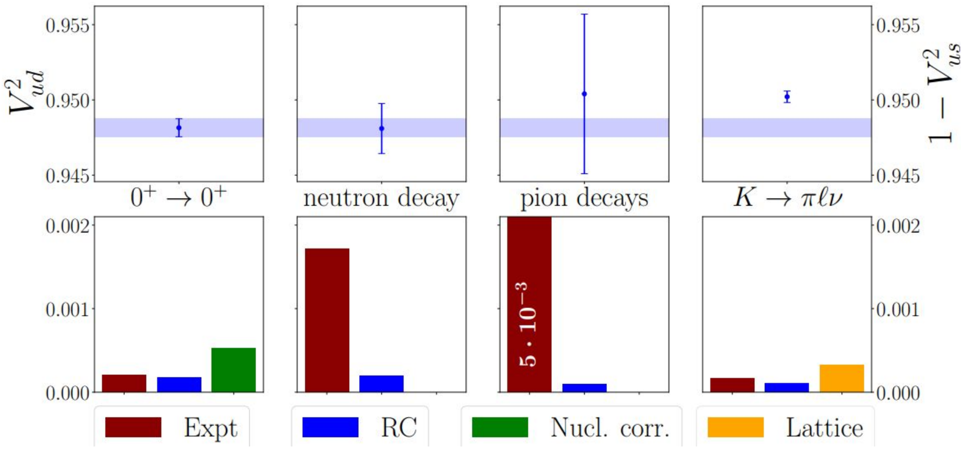
See **X. Feng et.al.** PRL 124 (2020) 192002

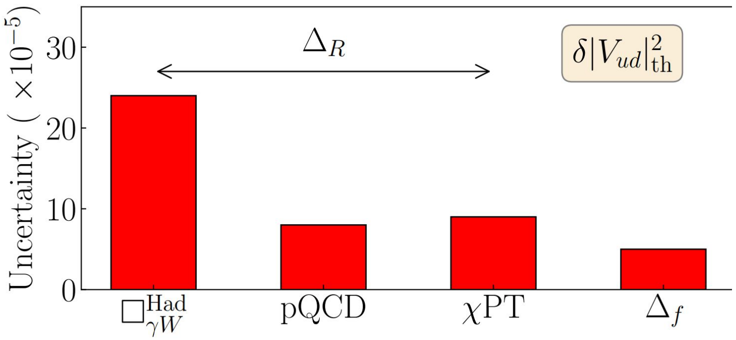
**JY et al. :** PRD 108 (2023) 034508

**P. Ma et.al.** PRL 132 (2024) 191901

and stay tuned for **JY**, **F. He, R. Gupta & T. Bhattacharya** in progress

# Why γW-box?



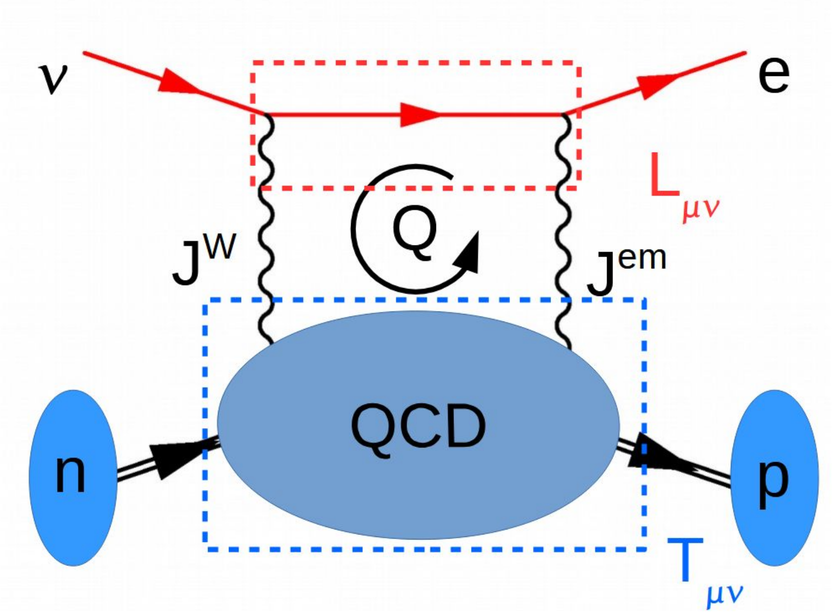

(Top) $|V_{ud}|$ determination error budget

Every determination of $|V_{ud}|$ requires EW radiative correction, $\Delta_R^V$.

(Bottom) Theory Error in $|V_{ud}|$

Dominant theory uncertainty inside $\Delta_R^V$
: hadronic γW-box diagram

# What is γW-box?



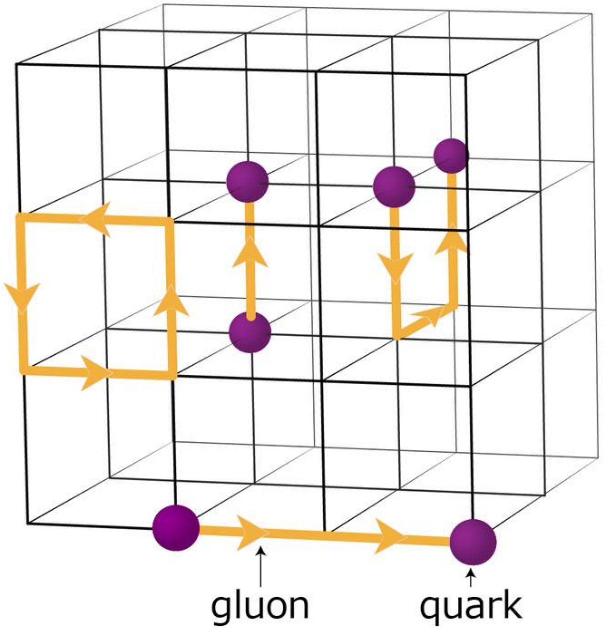

Radiative correction involves hadronic scale

$$\Delta_{\rm np} = \int_0^{+\infty} dQ^2 \int_{-Q}^{Q} dQ_0 \frac{1}{Q^4} \frac{1}{Q^2+m_W^2} L^{\mu\nu}(Q,Q_0) T_{\mu\nu}^{VA}(Q,Q_0)$$

Sensitivity to Hadronic scale → Lattice QCD

$$T_{\mu\nu}^{VA} = \frac{1}{2}\int d^4x\, e^{iQ\cdot x} \langle N_f(p)| T\left[J_\mu^{em}(0,0) J_\nu^{W,A}(\vec{x},t)\right] |N_i(p)\rangle$$

# Lattice QCD (I)



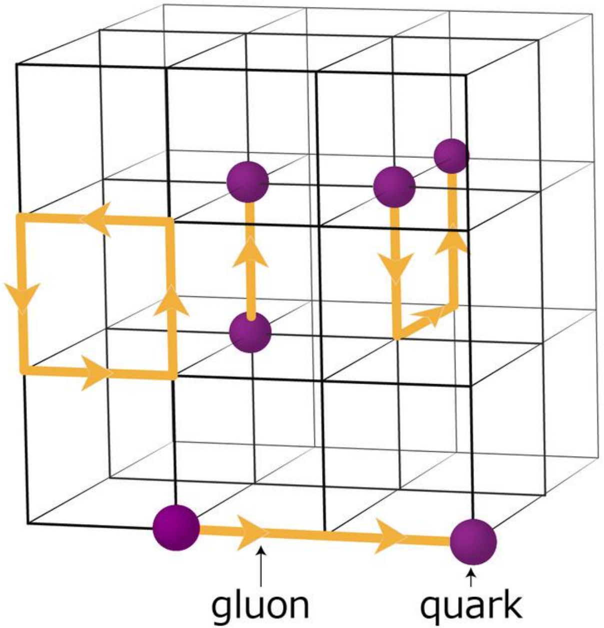

- quark fields $\psi$(x), $\bar{\psi}$(x) on the lattice sites
- gauge link $U_\mu$(x) acts as parallel transporter connecting x $\rightarrow$x+a$\mu$
- Lattice derivative

$$D_\mu\psi(x) = U_\mu(x)\psi(x+a\mu) - \psi(x),$$

- Path integral discretized, Wick rotated

$$\langle\mathcal{O}[U_\mu,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int\prod_{n,\mu} dU_\mu d\psi d\bar{\psi}\,\mathcal{O}[U_\mu,\psi,\bar{\psi}]\exp\{-\sum_n \mathcal{L}_{\text{latt}}[U_\mu,\psi,\bar{\psi}]\}$$

# Lattice QCD (II)




- Fermion integrated out

$$\langle \bar{\mathcal{O}}[U_\mu] \rangle = \frac{1}{Z} \int \prod_{n,\mu} dU_\mu \, \bar{\mathcal{O}}[U_\mu] \; \exp\left\{ -S_{\mathrm{latt}}^{\mathrm{eff}}[U_\mu] \right\}$$

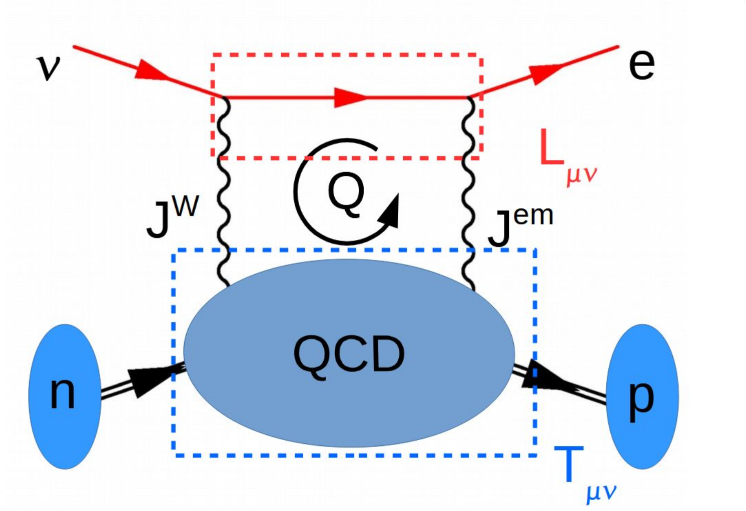
- Monte Carlo Simulation

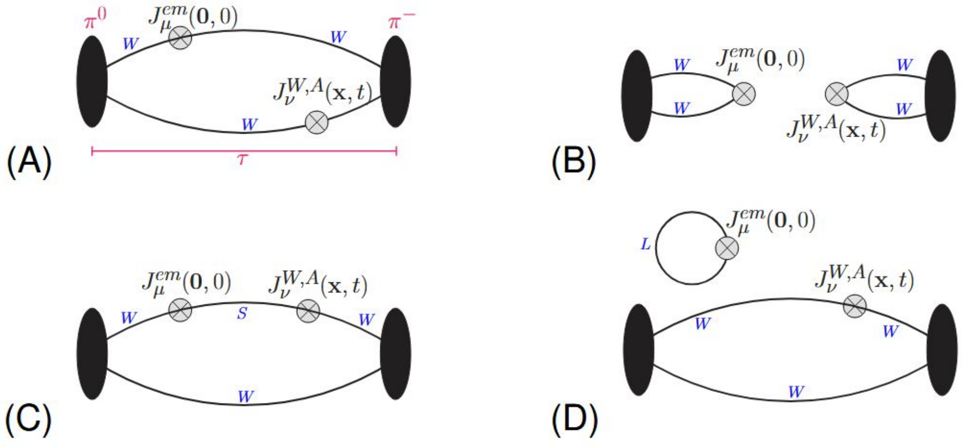
$$\langle \bar{\mathcal{O}}[U_\mu] \rangle = \frac{1}{N} \sum_{i=1}^{N} \bar{\mathcal{O}}[U_\mu^{(i)}] + O\left(\frac{1}{\sqrt{N}}\right)$$

- Typical degree of freedom

$O(10^7 \sim 10^{10})$

# Lattice Calculation

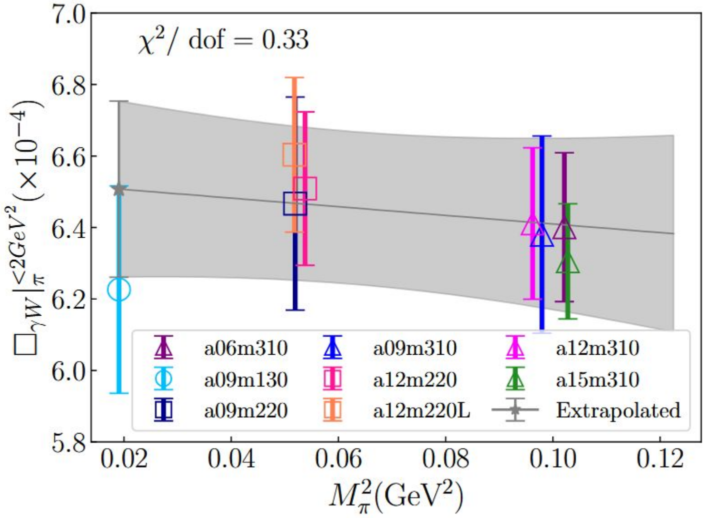


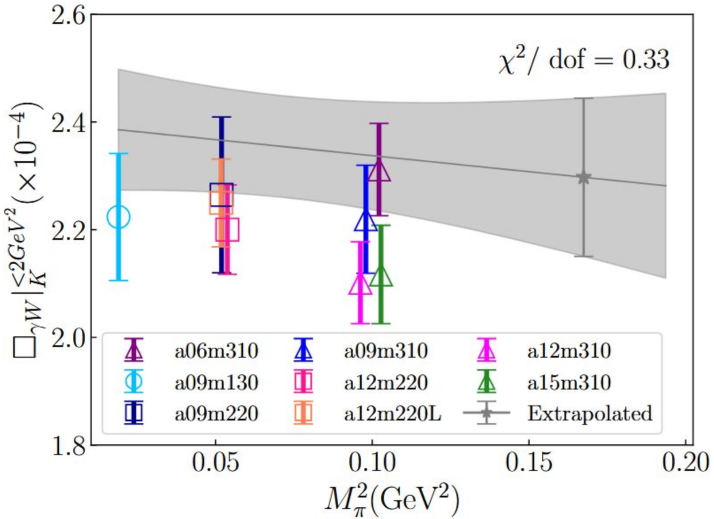




quark line diagram for pion/kaon case

- Diagram (B) does not exist due to γ5 hermiticity
- Diagram (D) provides only a minor contribution since it vanishes at SU(3) symmetry

# Meson Result - π/K box correction



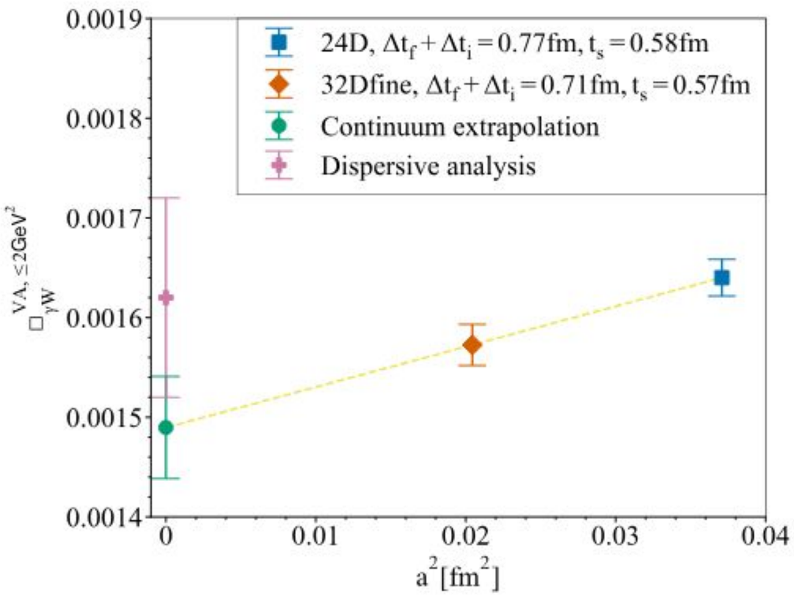

| | $\square_\pi$ | $\square_K$ |
|---|---|---|
| Our result | $2.810(26) \times 10^{-3}$ | $2.389(17) \times 10^{-3}$ |
| X. Feng et al. | $2.830(11)(26) \times 10^{-3}$ | $2.437(44) \times 10^{-3}$ |

# Ongoing: N box correction

- First computation carried out on two ensembles with DWF action

see P. Ma et. al. Phys.RevLett. 132 (2024) 191901

- Agreement to dispersive analysis result
- Superallowed $|V_{ud}|$ gives 1.8σ from Unitarity

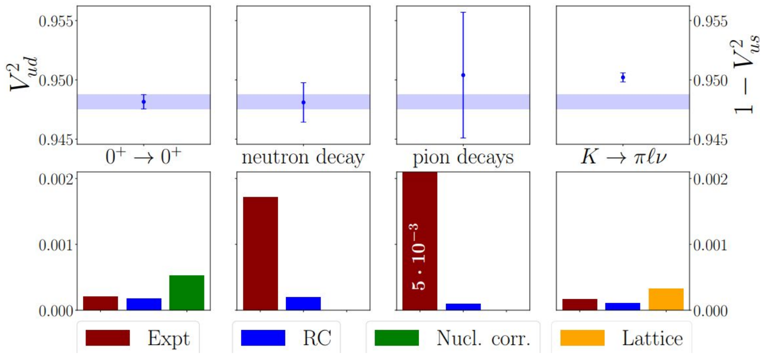


# Ongoing: N box correction

In the next run in UCNA and UCNτ

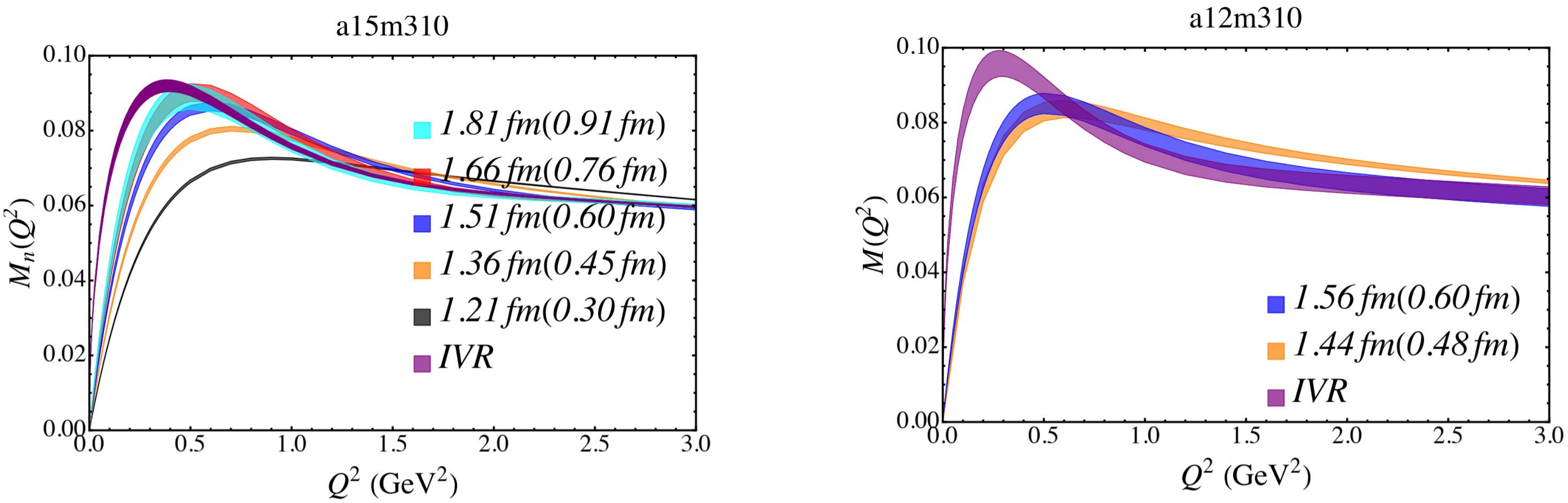
- Error in $g_A$ will decrease down to ~0.01%
- Error in $\tau_n$ will decrease down to ~0.15s

$g_A$ and $\tau_n$ enters precision era



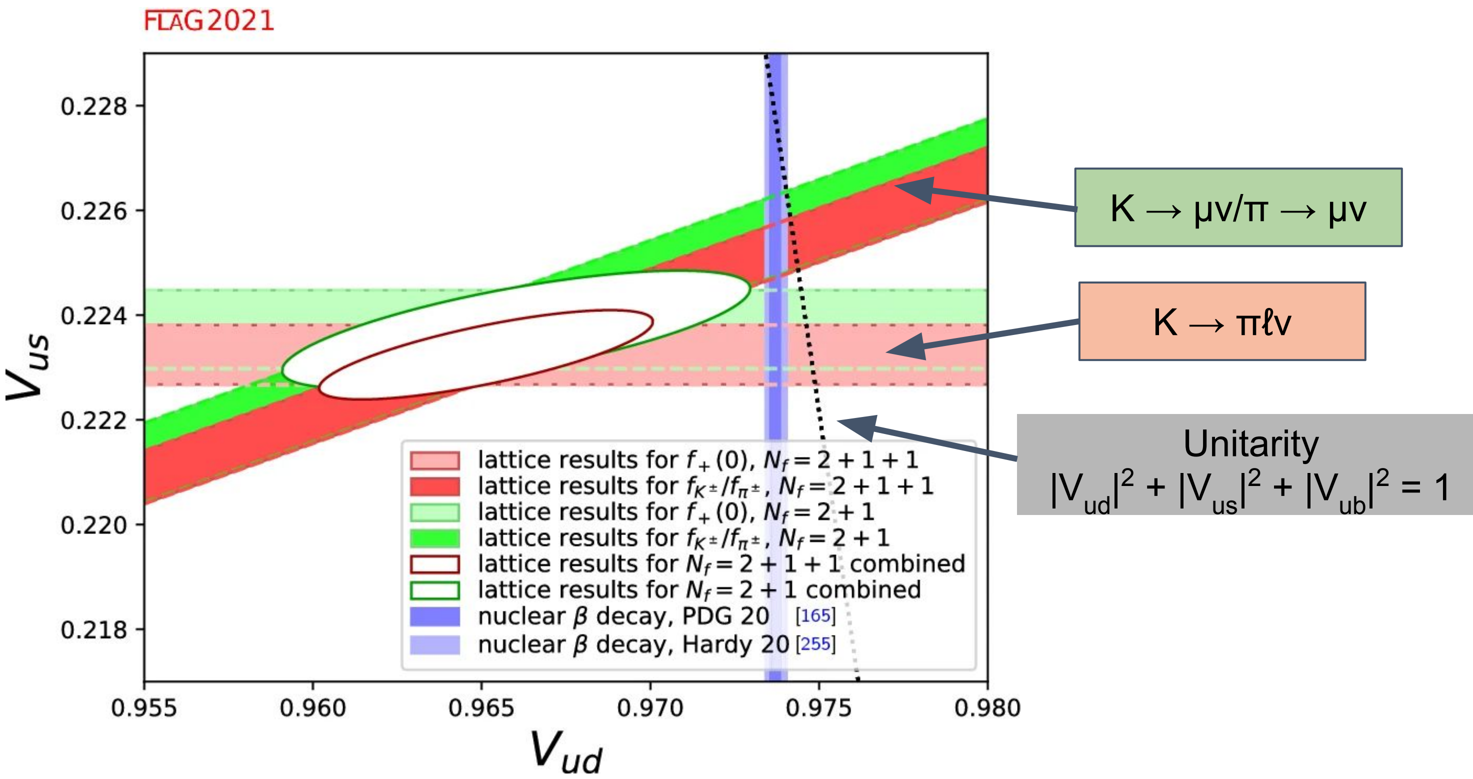

Calls for new precision test on CKM first row unitarity

# Ongoing: N box correction



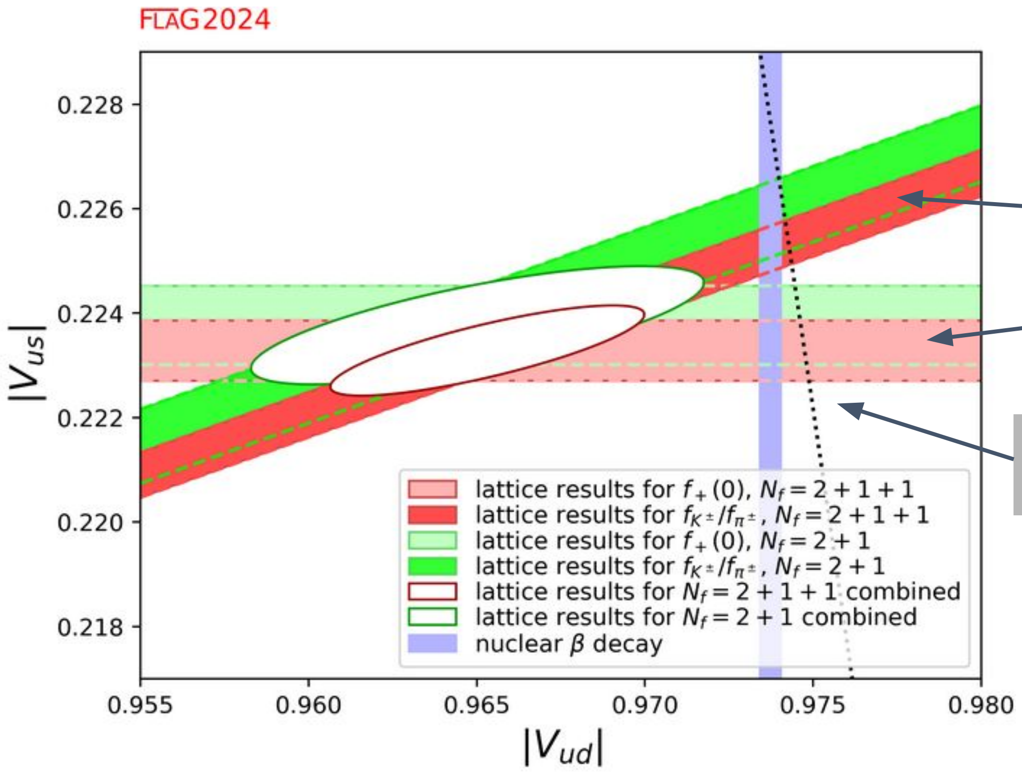

- Ready on two ensembles, three more to go

# First Row CKM Unitarity (I)




- Tension to Unitarity ~ $3\sigma$

# First Row CKM Unitarity (I)




$K \to \mu\nu / \pi \to \mu\nu$

$K \to \pi\ell\nu$

Unitarity

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

- Tension resolved?

# CKM Unitarity (II)

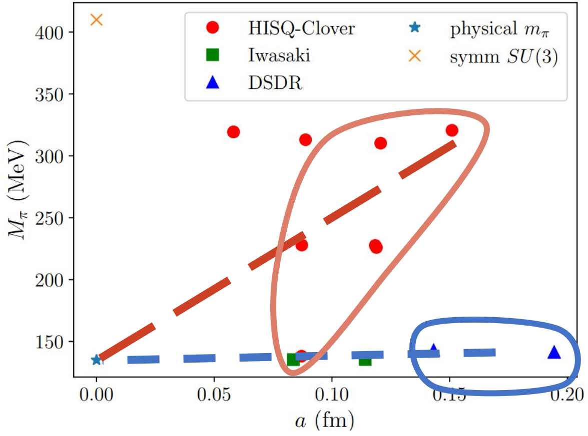
## What happened during that period?

- Unaccounted systematic uncertainty included

$$|V_{ud}| = 0.97370(10)_{exp}(10)_{RC} \;\; \textit{PDG 2020} \rightarrow 0.97367(11)_{exp}(13)_{\Delta}(27)_{NS} \;\; \textit{PDG 2024}$$

- First Lattice calculation on radiative correction appeared

$$|V_{ud}| = 0.9733(3)_{\tau n}(3)_{gA}(1)_{RC} \;\; \textit{PDG 2020} \rightarrow 0.97413(20)_{\tau n}(35)_{gA}(13)_{\Delta} \;\; \textit{PDG 2024}$$

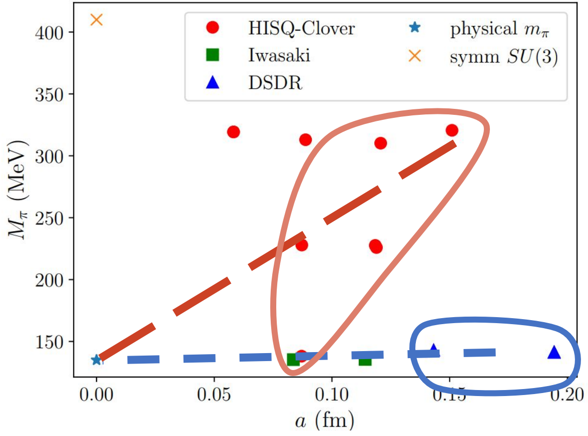



| | X. Feng et al. | Our group (in progress) |
|---|---|---|
| **# of ensemble** | 2 | 5 |
| **Action** | DWF | HISQ/Clover |
| **π Mass (MeV)** | 140 | 135, 220, 310 |

# CKM Unitarity (III)

## What will we get ?

- Most stringent $|V_{ud}|$ determination from neutron beta decay
- First confirmation of lattice calculation on radiative correction

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# Final Takeaways

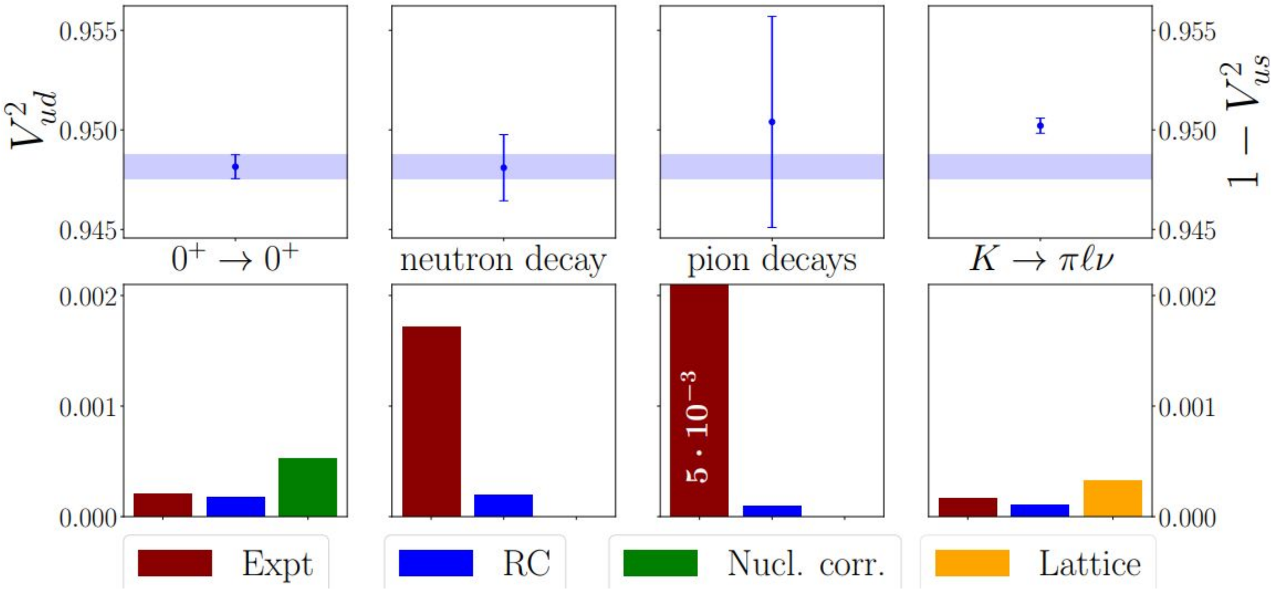
- Lattice $\gamma$W-box correction is entering precision era
- $\Delta_R^V$ remains the dominant source of theory uncertainty
- With future neutron experiments reaching 0.2%, CKM unitarity will be tested again
- A coordinated effort is essential

# Final Takeaways

- Lattice $\gamma$W-box correction is entering precision era
- $\Delta_R^V$ remains the dominant source of theory uncertainty
- With future … be tested again
- A coordinat…

Thank you for your attention

# Why γW-box?




With a precision determination on the horizon in upcoming experiments (UCN next rounds) ,
*Neutron decay* is the most stringent determination!