Hadron form factor from lattice QCD on large volumes at the physical quark mass

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Outline

- PACS10 project
- Nucleon form factor
- Meson form factor
- Kaon semileptonic decay form factor
- Summary

Members of PACS Collaboration

Univ. of Tsukuba : N. Ishizuka, Y. Kuramashi, T. Taniguchi, N. Ukita, T. Yamazaki, T. Yoshié

Hiroshima Univ. : K.-I. Ishikawa; Tohoku Univ. : S. Sasaki, M. Nagatsuka

Univ. of Tokyo : E. Shintani; Riken-CCS : Y. Aoki, Y. Nakamura; Fukuyama Univ. : N. Namekawa

KEK : R. Tsuji; Keio Univ. : H. Watanabe; Seikei Univ. : K. Sato

Lattice QCD

Powerful tool for non-perturbative QCD calculation
based on Monte Carlo simulation with large-scale computer

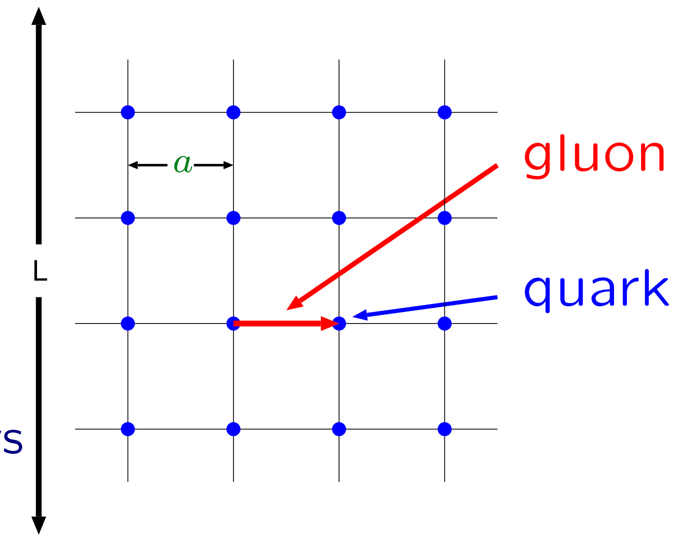
Understanding of hadron property at least quantitatively

Lattice QCD = discretrized QCD

by lattice spacing a

on finite volume $L^3 \times T$

with arbitrary m_q (m_π)



Real world $= a \to 0$, $L, T \gg 1/\Lambda_{\rm QCD}$, $m_\pi = m_\pi^{\rm phys}$

PACS10 project by PACS Collaboration

aims to remove main three uncertainties in lattice QCD

1. large m_π, 2. finite V, 3. finite a

three lattice spacings at physical m_π on (10 fm)^4 volume

$\Rightarrow$ PACS10$_{(c)}$ configuration

PACS10 project since 2016

$N_f = 2+1$	PACS10 configuration		
$L^3 \cdot T$	128^4	160^4	256^4
L [fm]	10.9	10.2	10.5
a [fm]	0.08	0.06	0.04
m_π [GeV]	0.135	0.137	0.142
m_K [GeV]	0.497	0.501	0.514
Machine	OFP	OFP	Fugaku
Node	512	512	16384
Status	done	done	done

$N_f = 2+1+1$	PACS10$_c$ configuration		
$L^3 \cdot T$	128^4	168^4	256^4
L [fm]	10.7	10.5	10.5
a [fm]	0.08	0.06	0.04
m_π [GeV]	0.137	0.137	$\sim$0.137
m_K [GeV]	0.498	0.497	$\sim$0.497
Machine	Fugaku	Fugaku	Fugaku
Node	1024	2744	16384
Status	done	done	running

OFP: Oakforest-PACS (KNL machine); Fugaku: since 2020

PACS10$_{(c)}$ configuration: More than (10 fm)^4 at the physical point

aims to remove main three uncertainties in lattice QCD

$N_f = 2+1(+1)$ nonperturbatively $O(a)$ improved Wilson clover quark action

with 6-stout smeared link + Iwasaki gauge action

same actions as HPCI Field 5 project using K computer [PoS LATTICE2015 (2016) 075]

Fugaku co-design outcome:

a^{-1} determined from Ξ baryon mass

QCD Wide SIMD (QWS) Library for Fugaku

[Ishikawa *et al.*:CPC(2023)]

Resources: Fugaku in HPCI System Research Project

hp200062, hp200167, hp210112, hp220079, hp230199, hp230199, hp240207, hp250218

PACS10 project

precise determination of physical quantity
w/o main systematic uncertainties in lattice QCD

I. quantitatively understand property of hadrons
reproduce experimental values in high accuracy

- Hadron spectrum
- Nucleon form factor — Tsuji, Nagatsuka, Sasaki, Shintani
- Light meson form factor — Sato

II. search for new physics beyond the standard model
discrepancy between theoretical calculation and experiment

- Proton decay matrix element
- Hadron vacuum polarization
- Kaon semileptonic decay form factor

$p^2 = (2\pi/L)^2 \cdot n$

Advantage of PACS10/PACS10$_c$: Form factor in tiny q^2 region

Results obtained mainly from PACS10 ($N_f = 2+1$) configurations

Nucleon form factor

Nucleon isovector form factor

Nucleon matrix element

Vector current

$$\langle N(p')|V_\mu(q)|N(p)\rangle = \overline{u}(p')\left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{q_\nu}{2M}F_2(q^2)\right]u(p)$$

Electoric form factor $G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2}F_2(q^2)$

Magnetic form factor $G_M(q^2) = F_1(q^2) + F_2(q^2)$

Charge radius $\langle r_E^2\rangle = -6\left.\frac{dG_E(q^2)}{dq^2}\right|_{q^2=0}$ related to proton size puzzle

Axial-vector current

$$\langle N(p')|A_\mu(q)|N(p)\rangle = \overline{u}(p')\left[\gamma_\mu\gamma_5 F_A(q^2) + iq_\nu\gamma_5 F_P(q^2)\right]u(p)$$

Axial-vector form factor $F_A(q^2)$

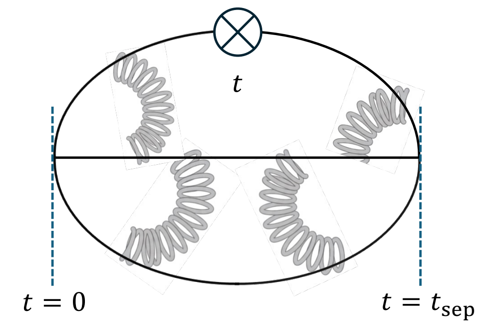
Induced pseudoscalar form factor $F_P(q^2)$

$g_A = F_A(0)$: axial charge

Axial radius $\langle r_A^2\rangle = -\frac{6}{g_A}\left.\frac{dF_A(q^2)}{dq^2}\right|_{q^2=0}$

important for neutrino-nucleon scattering in neutrion experiment (T2K, $\cdots$)

Extraction of matrix element



3-point function

$$C_3(t, t_{\mathrm{sep}}, \vec{q}) = \mathrm{Tr}\left[P_+ \langle 0|N(t_{\mathrm{sep}}; \vec{p} = 0) V_\mu(t; \vec{q}) \overline{N}(0; \vec{q})|0\rangle\right]$$

2-point function

$$P_+ = (1 + \gamma_4)/2$$

$$C_2(t, \vec{p}) = \mathrm{Tr}\left[P_+ \langle 0|N(t_{\mathrm{sep}}; \vec{p}) \overline{N}(0; \vec{p})|0\rangle\right]$$

$$\frac{C_3(t, t_{\mathrm{sep}}, \vec{q})}{C_2(t_{\mathrm{sep}} - t, 0) C_2(t, \vec{q})} = \alpha \langle N(0)|V_\mu(q)|N(q)\rangle + (t, t_{\mathrm{sep}} \text{ dependent parts})$$

excited state contributions

$$\xrightarrow[t, t - t_{\mathrm{sep}} \gg 1]{} \alpha \langle N(0)|V_\mu(q)|N(q)\rangle$$

α : known factor

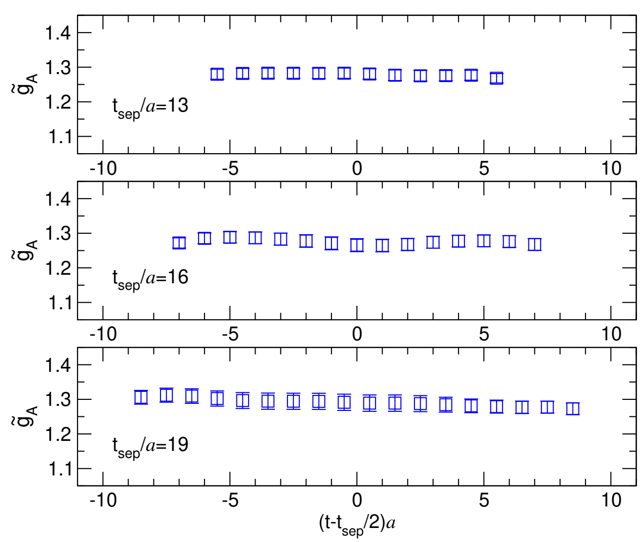
Some techniques to improve signal

- Highly tuned smearing operator
- All-mode average

[Blum *et al.*, PRD88:094503(2013)]

Reasonably flat behavior

e.g. ratio for g_A, but not for F_P



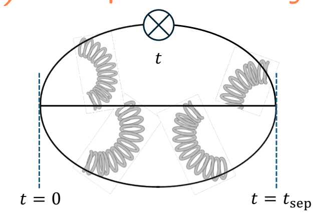
Axial charge g_A

Figures from R. Tsuji

PACS10/L128 [PRD99:1,014510(2020)]; PACS10/L160 [PRD109:9,094505(2024)]

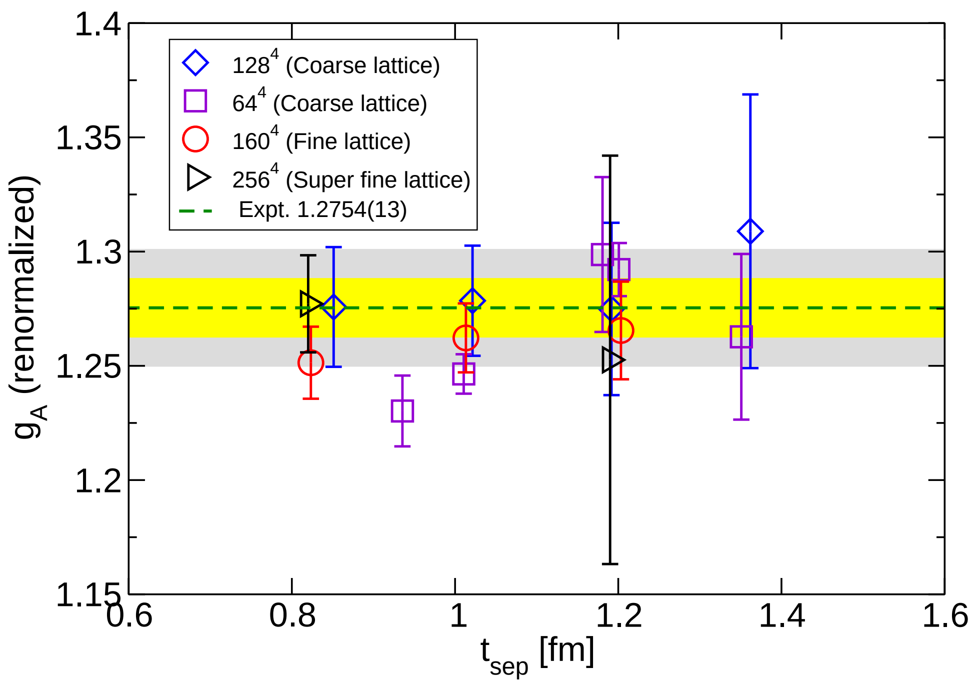
PACS10/L256 (denoted by super fine lattice) are preliminary.

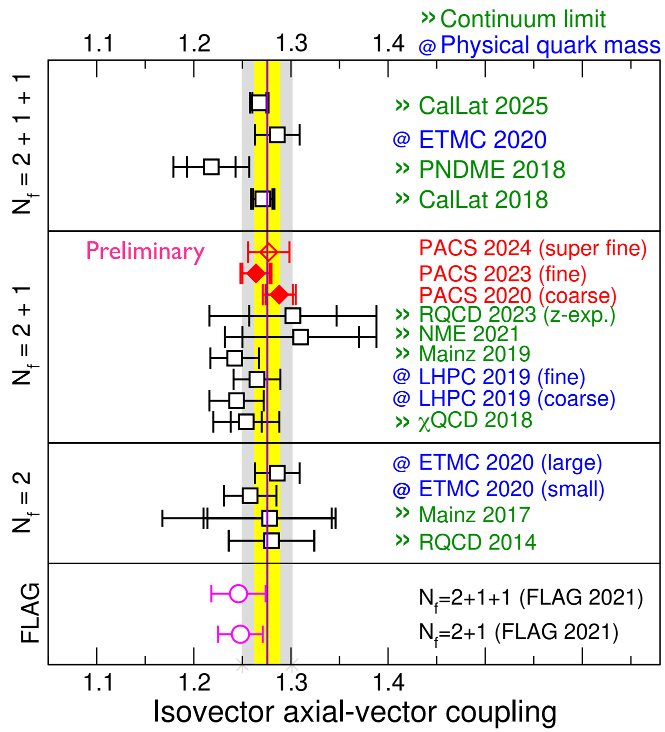
Benchmark of nucleon form factor from lattice QCD



Precisely measured in experiment

Easiest calculation in nucleon matrix elements





Mild t_{sep} and a dependences

Consistent well with experiment within roughly 2%

Future work : $a \to 0$ limit after finishing PACS10/L256 calculation

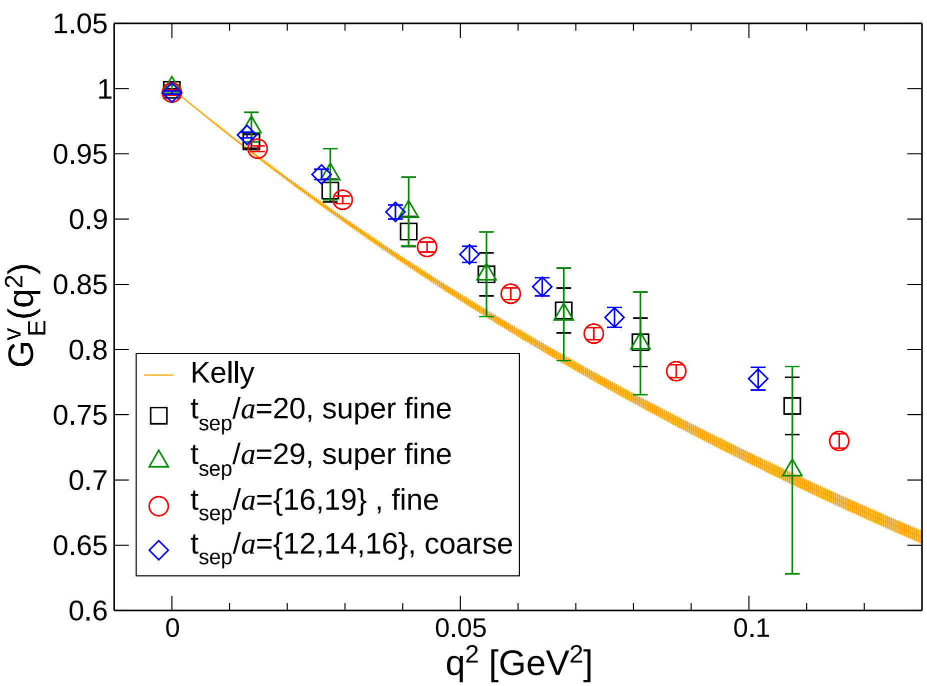
Isovector electoric form factor $G_E(q^2)$

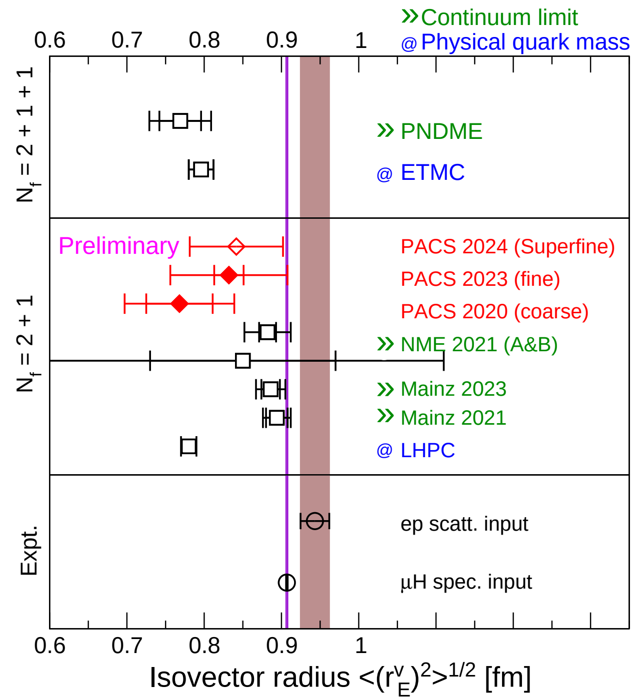
Figures from R. Tsuji

PACS10/L128 [PRD99:1,014510(2020)]; PACS10/L160 [PRD109:9,094505(2024)]

PACS10/L256 (denoted by super fine lattice) are preliminary.

Related to proton size puzzle, but isovector $G_E(q^2)$





experimental $\langle (r_E^V)^2 \rangle = \langle (r_E^p)^2 \rangle - \langle (r_E^n)^2 \rangle$

Comparable to experimental form

Approaching to experiment as $a \to 0$

Similar trend seen in $\langle (r_E^V)^2 \rangle$

Axial-vector form factor $F_A(q^2)$

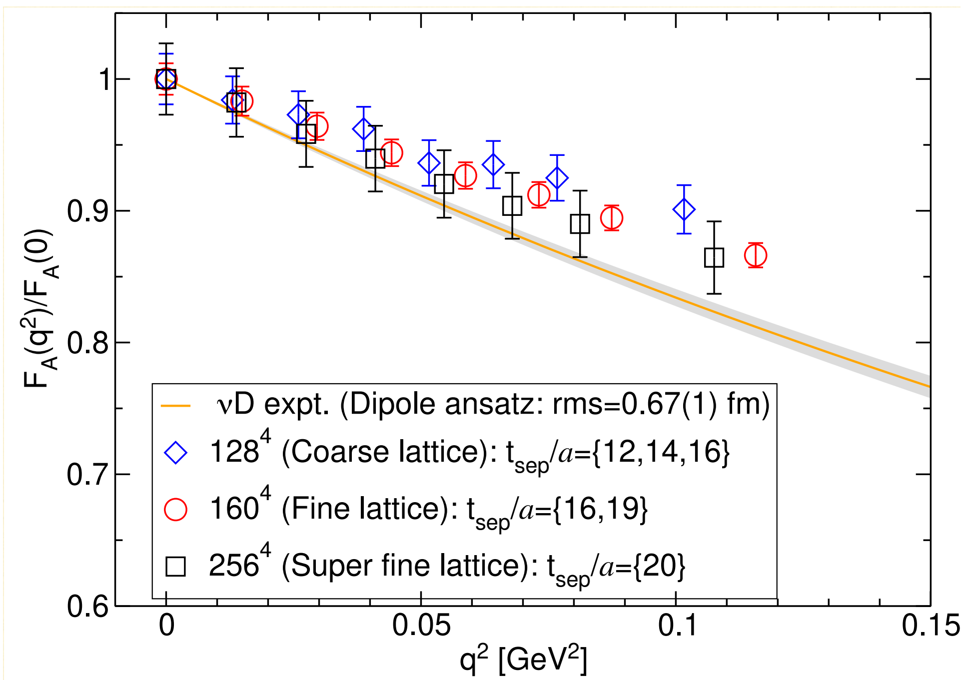
Figures from R. Tsuji

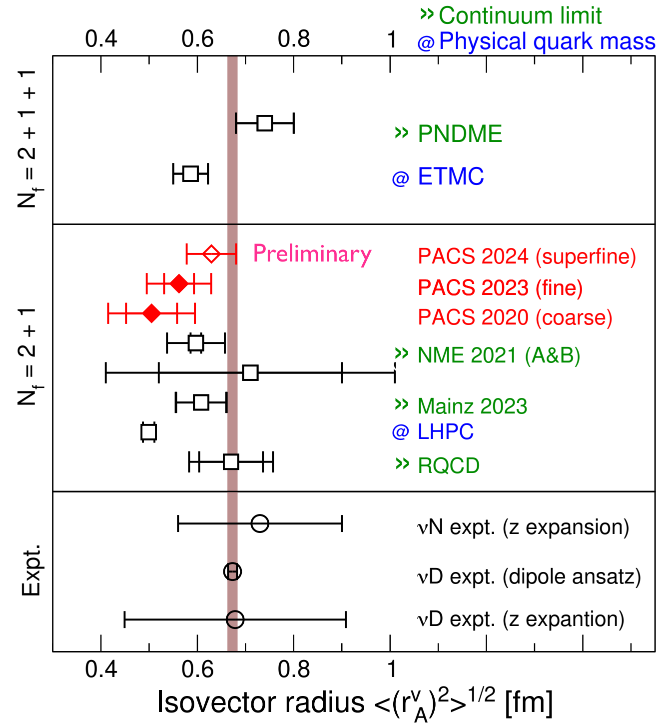
PACS10/L128 [PRD99:1,014510(2020)]; PACS10/L160 [PRD109:9,094505(2024)]

PACS10/L256 (denoted by super fine lattice) are preliminary.

Important to understand neutrino-nucleon scattering

in neutrio experiment





Slightly smaller $\langle (r_A)^2 \rangle$ than νD expt. insisted by rencent experiment

Comparable to experimental dipole form

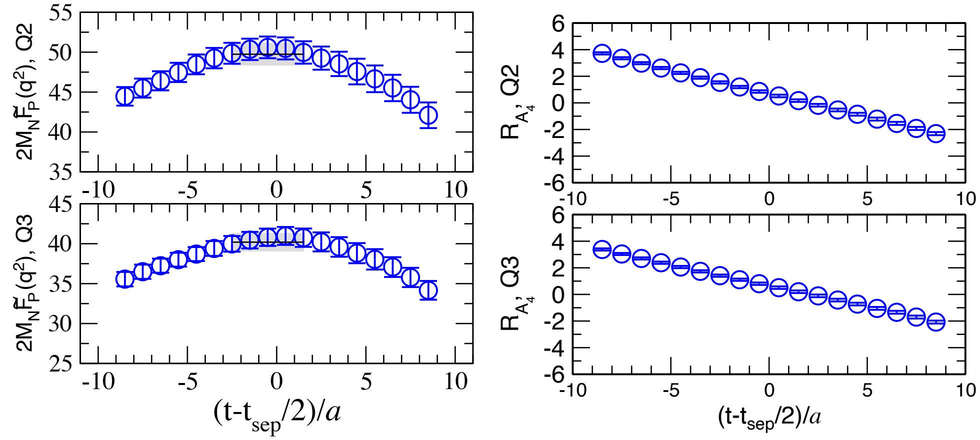
Approaching to dipole form as $a \to 0$

Similar trend seen in $\langle (r_A)^2 \rangle$

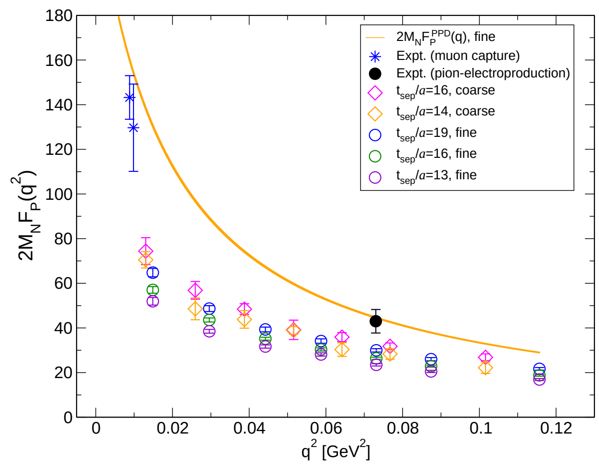
Reduction of leading πN contribution in F_P, G_P [PRD112:7,074510(2025)]

Large πN contribution in F_P, G_P

[Bär, PRD99:054506(2019); Meyer *et al.*, PoS(LAT2018)062; RQCD, JHEP5:126(2020), $\cdots$]



clear t dependence = large excited state



F_P: largely underestimated and t_{sep} dependent

Simple assumption of leading πN contribution inspired by Bär's results

$$R_{A_i}(t,\vec{q}) = K^{-1}[(E_N + M_N)F_A(q^2)\delta_{i3} - q_3 q_i(F_P(q^2) - \Delta_+(t,\vec{q}))]$$

$$R_{A_4}(t,\vec{q}) = iq_3 K^{-1}[F_A(q^2) - \Delta E_N F_P(q^2) + E_\pi \Delta_-(t,\vec{q})]$$

$$\Delta_\pm(t,\vec{q}) = B(q)e^{-(E_\pi - \Delta E_N)t} \pm C(q)e^{-(E_\pi + \Delta E_N)(t_{\text{sep}} - t)}$$

R_A: ratio of 3-pt function with A_μ current to 2-pt functions, $K = \sqrt{2E_N(E_N + M_N)}, \Delta E_N = E_N - M_N$,
$B(q), C(q)$: unknown coefficients, t_{sep}: source-sink separation

well describe t dependence of our data

Reduction of leading πN contribution in F_P, G_P [PRD112:7,074510(2025)]

Simple assumption of leading πN contribution inspired by Bär's results

$$R_{A_i}(t,\vec{q}) = K^{-1}[(E_N+M_N)F_A(q^2)\delta_{i3} - q_3 q_i(F_P(q^2) - \Delta_+(t,\vec{q}))]$$

$$R_{A_4}(t,\vec{q}) = iq_3 K^{-1}[F_A(q^2) - \Delta E_N F_P(q^2) + E_\pi \Delta_-(t,\vec{q})]$$

$$\Delta_\pm(t,\vec{q}) = B(q)e^{-(E_\pi-\Delta E_N)t} \pm C(q)e^{-(E_\pi+\Delta E_N)(t_{\rm sep}-t)}$$

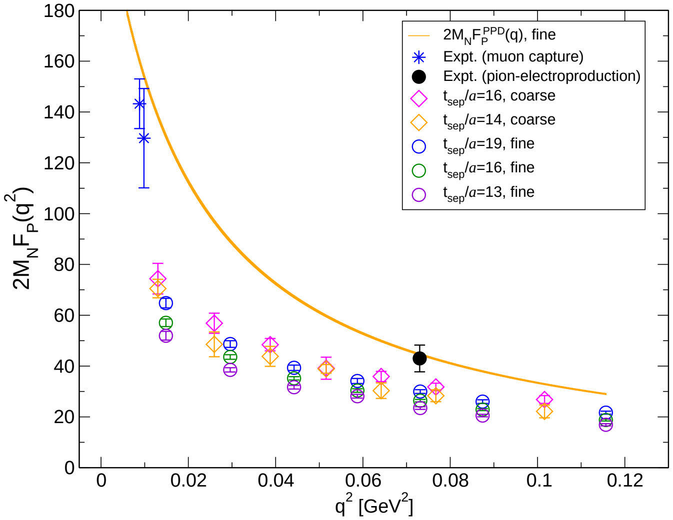
R_A: ratio of 3-pt function with A_μ current to 2-pt functions, $K=\sqrt{2E_N(E_N+M_N)}, \Delta E_N = E_N - M_N$,
$B(q), C(q)$: unknown coefficients, $t_{\rm sep}$: source-sink separation

$\Delta_\pm(t,\vec{q})$ can be removed by linear combination with R_A and $\partial_t R_A$

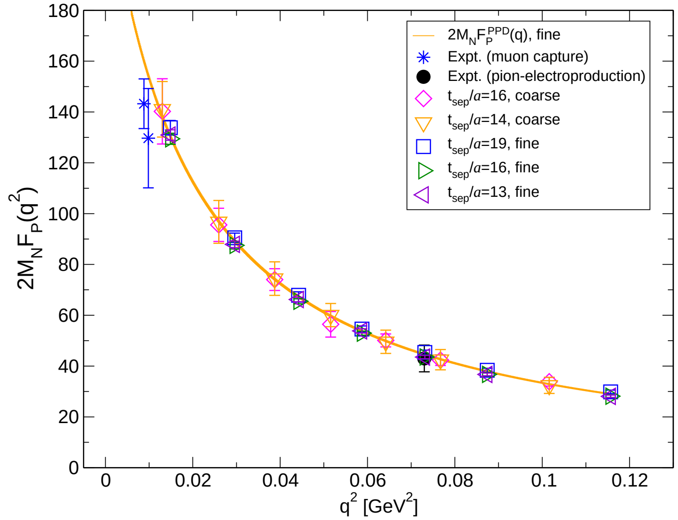
$$F_P(q^2) = \alpha R_{A_i} + \beta\partial_t R_{A_i} + \gamma\partial_t R_{A_4}$$

α, β, γ: appropriate coefficients

standard



new



independent of $t_{\rm sep}$, consistent with pion-pole dominance model

Similar analysis performed with G_P data

Light meson form factor

Electromagnetic form factor

Figures from K. Sato's Doctral Thesis

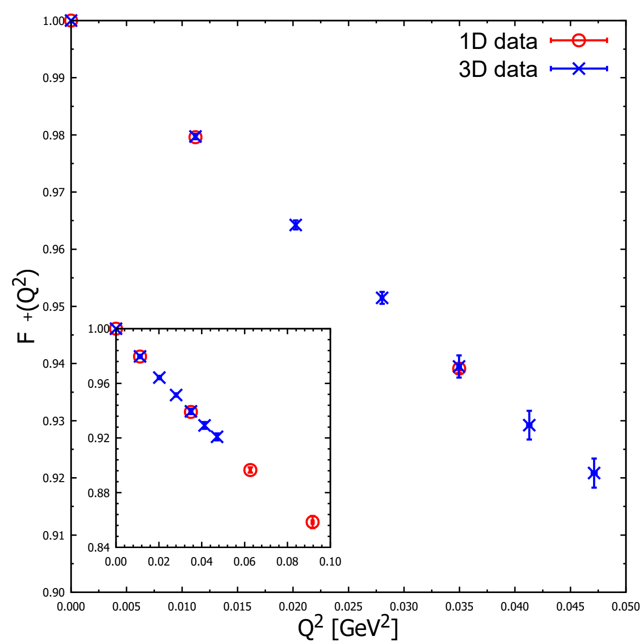
Light meson electromagnetic form factor

calculated by similar way to nucleon form factor; ratio of 3-,2-point functions

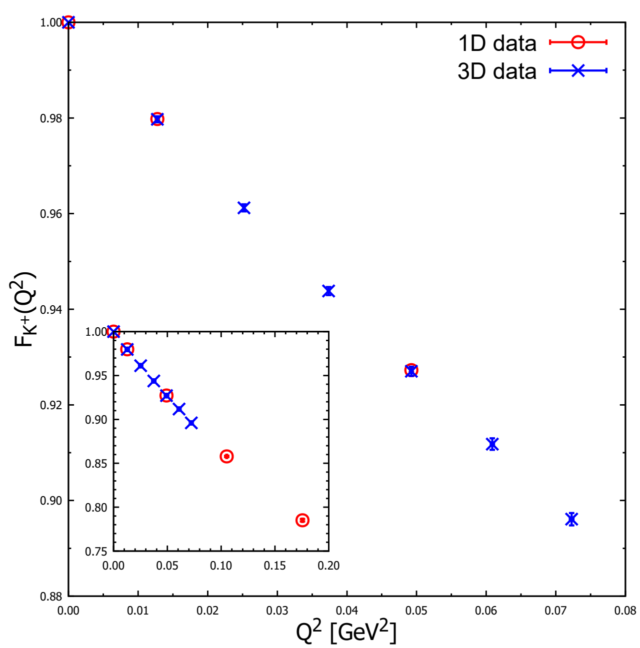
ex) π matrix element : $\langle \pi(0)|V_4(Q)|\pi(Q)\rangle = E_\pi F_\pi(Q^2)$

Preliminary results from PACS10/L128 configurations ($a = 0.08$ fm)

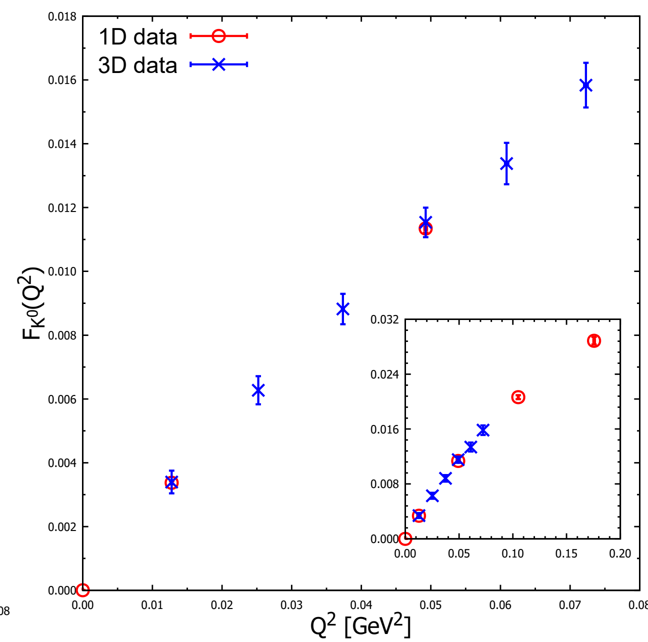
π^+



K^+



K^0



Clear data in tiny Q^2 region

Charge radius obtained from fit with fit assumption

Better to remove systematic error from fit assumption

Charge radius w/o fit assumption cf. Sato *et al.*, PoS(LATTICE2024):324(2025)

Model-independent method w/o fit assumption of form factor

Basic idea in infinite volume

$$\left.\frac{d\widetilde{C}(p)}{dp^2}\right|_{p=0} = -\frac{1}{2}\int dx\, x^2 C(x), \quad C(x) = \int \frac{dp}{2\pi}\widetilde{C}(p)e^{ipx}$$

Lagre finite volume effect observed in lattice QCD calculation

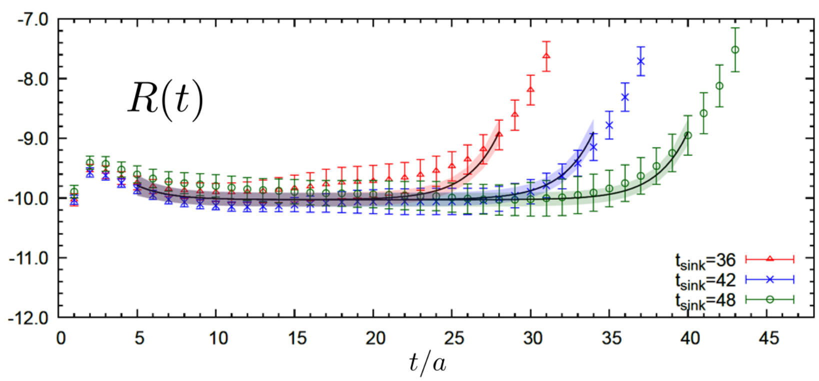
UKQCD, NPB444:401(1995), $\cdots$

Significant reduction of finite volume effect in $\langle r_\pi^2\rangle$ calculation

Feng *et al.*, PRD101:051502(R)(2020)

$$R(t) = \alpha_0 + \alpha_1 C^{(2)}(t) + \alpha_2 C^{(4)}(t) \propto \langle r_\pi^2\rangle + \cdots$$

$C^{(k)}(t) = \sum_x x^{2k} C_{\rm 3pt}(x,t)$, $\widetilde{C}_{\rm 3pt}(p,t) \propto F_\pi(Q^2)$, known parameters $\alpha_0, \alpha_1, \alpha_2$



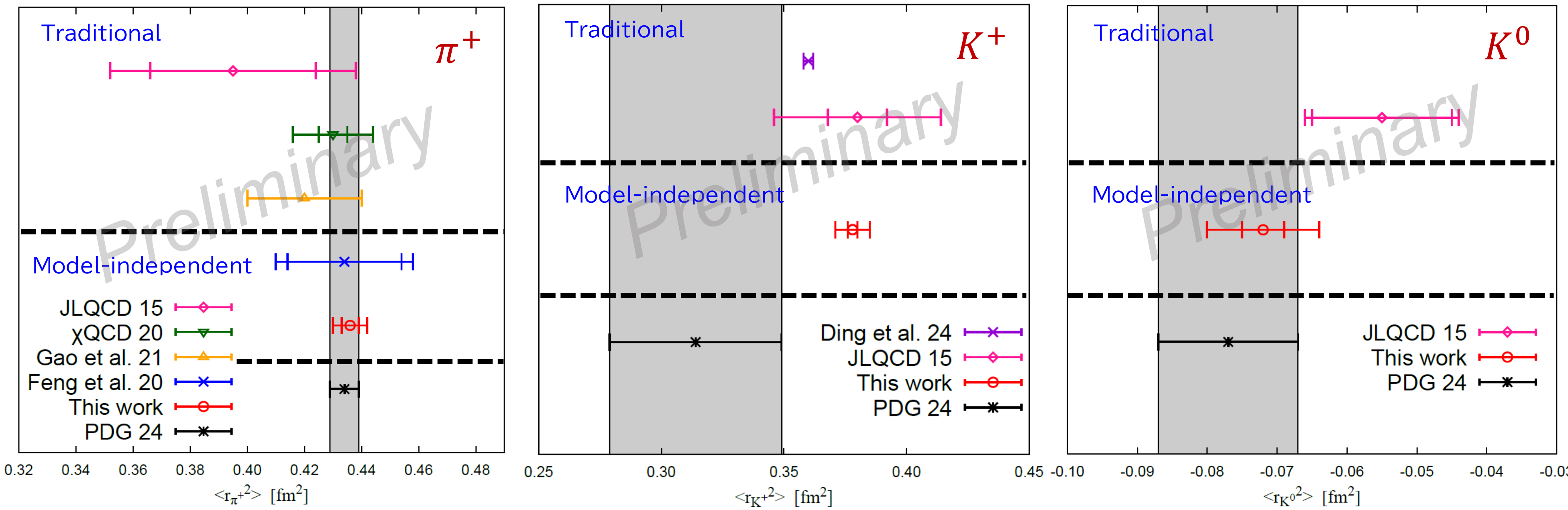
$\langle r_\pi^2\rangle$ obtained from flat region

w/o fit assumption of form factor

figures from Sato's slide at EINN2025; https://agenda.infn.it/event/45343

Charge radius

Preliminary results from PACS10/L128,L160 configurations



figures from Sato's slide at EINN2025; https://agenda.infn.it/event/45343

Reasonably consistent with experiments less than 2σ

K^+ : smaller error than experiment

Comparison with precise experimental value in near future

Phase-2 of AMBER experiment at CERN: kaon charge radius with factor 10 better precision

Kaon semileptonic ($K_{\ell 3}$) decay form factor

$|V_{us}|$ through kaon decay

Urgent task in particl physics

Search for signal beyond standard model (BSM)

Muon $g-2$ @ FNAL, White paper 2025 : ~~5σ away from SM~~(?)

important contribution from lattice QCD

$|V_{us}|$: a candidate of BSM signal

Three determinations of $|V_{us}|$

1. CKM unitarity $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

   constraint in SM (grey band)

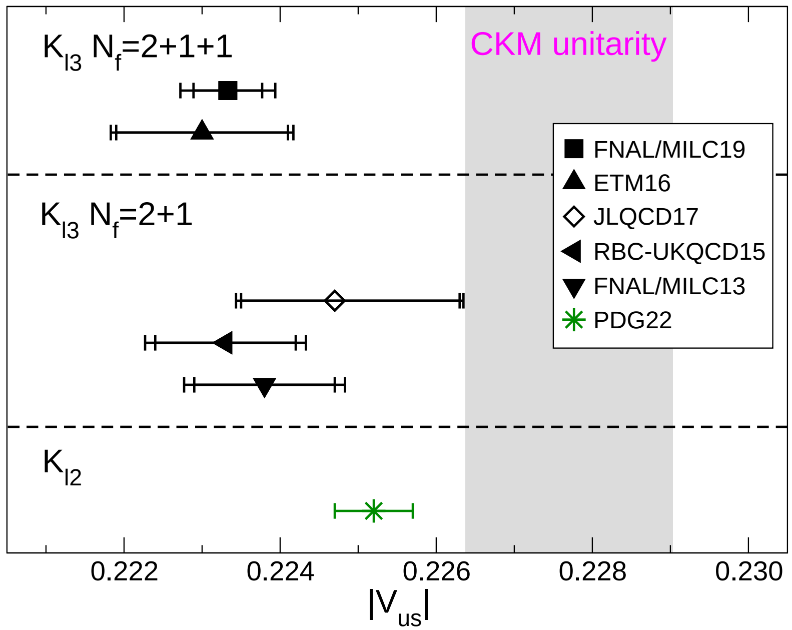
   $|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$ w/ new $|V_{ud}|$

   ['18 Seng *et al.*, '20 Hardy, Towner]

2. $K_{\ell 2}$ $K \to \ell\nu$ decay

3. $K_{\ell 3}$ $K \to \pi\ell\nu$ decay

   Most accurate value ['19 FNAL/MILC]



$|V_{us}|^{1.} \neq |V_{us}|^{2.} \neq |V_{us}|^{3.} \lesssim 3\sigma$ differences

Important to confirm by several independent calculations

Calculation using PACS10 and PACS10$_c$ configuraionts

Simulation parameters

PACS10/L128[PRD101,9,094504(2020)]; PACS10/L160[PRD106,9,094501(2022)]

	PACS10			PACS10$_c$	
$L^3 \cdot T$	128^4	160^4	256^4	128^4	168^4
L [fm]	10.9	10.2	10.5	10.7	10.5
a [fm]	0.08	0.06	0.04	0.08	0.06
m_π [GeV]	0.135	0.138	0.141	0.137	0.137
m_K [GeV]	0.497	0.505	0.514	0.498	0.497
N_{conf}	20	20	20	40	40
N_{meas}	< 2560		1280	5120	

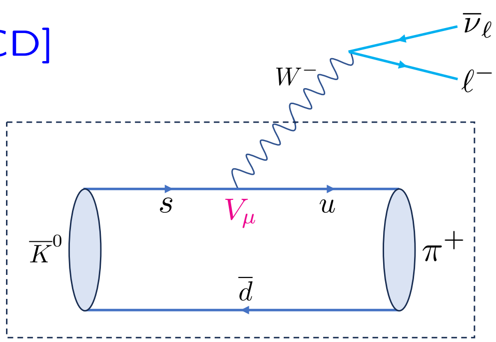
PACS10/L256, PACS10$_c$/L128,L168 are preliminary.

- 2-, 3-point functions w/ $Z(2) \otimes Z(2)$ random source ['08 RBC-UKQCD]

$$C_{\pi,K}(t,p) = \langle 0|O_{\pi,K}(t,\mathbf{p})O^\dagger_{\pi,K}(0,\mathbf{p})|0\rangle$$

$$C_{V_\mu}(t,t_{\mathrm{sep}},p) = \langle 0|O_K(t_{\mathrm{sep}},\mathbf{0})V_\mu(t,\mathbf{p})O^\dagger_\pi(0,\mathbf{p})|0\rangle$$

V_μ: Local vector current renormalized by Z_V
Conserved vector current



- $\langle \pi(p)|V_\mu|K(0)\rangle$ extracted from ratio of 3- to 2-point functions

$K_{\ell 3}$ form factors $f_+(q^2), f_0(q^2)$

$$\langle \pi(p)|V_\mu|K(0)\rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2) \qquad p_K = (M_K, \mathbf{0}), p_\pi = (E_\pi, \vec{p}),\ q^2 = -(M_K - E_\pi)^2 + p^2$$

$f_+(0) \to |V_{us}|$ through $|V_{us}|f_+(0) = 0.21654(41)$ [Moulson:PoS(CKM2016)033(2017)]

q^2 interpolation and $a \to 0$ extrapolation

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ [PACS:PRD106,9,094501(2022)]

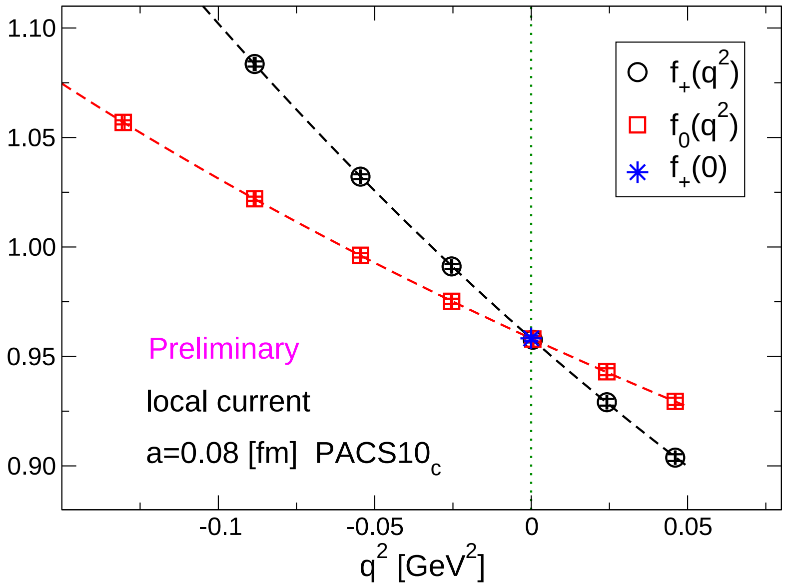
$$f_+(q^2) = 1 - \frac{4}{F_0^2} L_9(\mu) q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^+ q^4$$

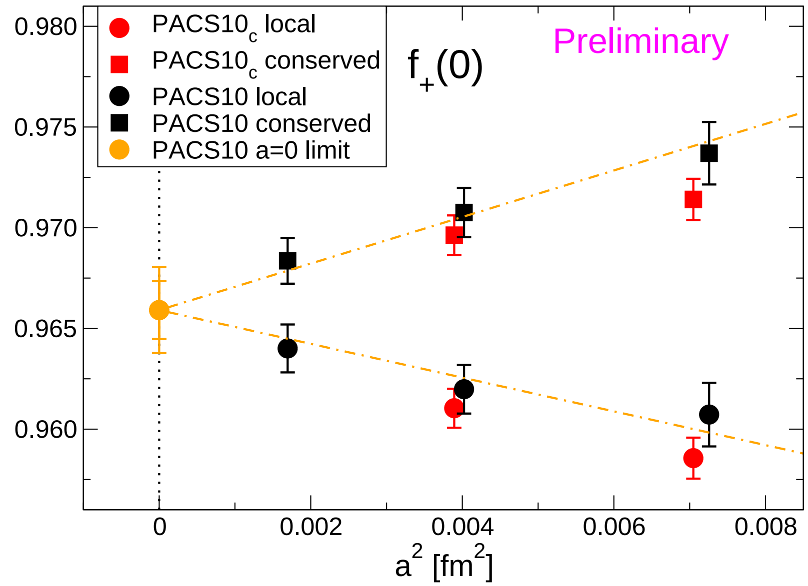
$$f_0(q^2) = 1 - \frac{8}{F_0^2} L_5(\mu) q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^0 q^4$$

K_+, K_0: known functions ['85 Gasser, Leutwyler] with $\mu = 0.77$ GeV, $F_0 = 0.11205$ GeV

free parameters: $L_5(\mu), L_9(\mu), c_2^+, c_2^0, c_0$

F_0 estimated from FLAG $F^{\mathrm{SU(2)}}/F_0$ w/ $F^{\mathrm{SU(2)}} = 0.129$ GeV

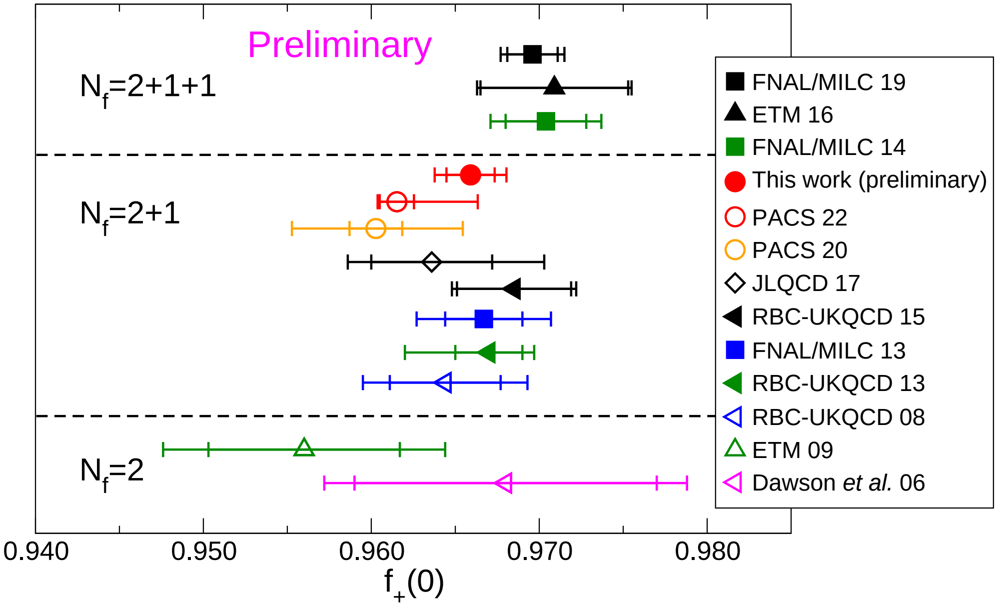


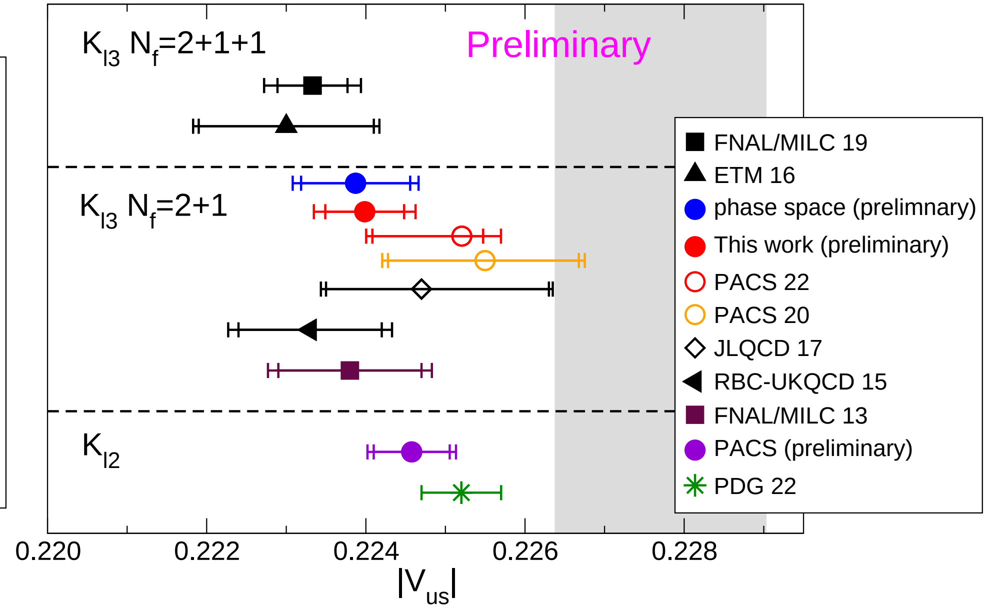


Simultaneous fit for (f_+, f_0) works well.

$a \to 0$ limit only for PACS10 result

$f_+(0)$ and $|V_{us}|$ from PACS10 configs.





inner, outer = statistical, total(stat.+sys.) inner, outer = lattice, total(lat.+exp.)

Standard model (SM) prediction using $|V_{us}| \approx \sqrt{1-|V_{ud}|^2}$, grey band: ['20 Hardy, Towner]

$f_+(0)$: Reasonably agree with previous lattice calculations $\lesssim 2\sigma$

$|V_{us}|$ using $|V_{us}|f_+(0) = 0.21654(41)$ ['17 Moulson]

agree with $|V_{us}|$ from K_{l2} using f_K/f_π $\quad \dfrac{|V_{us}|}{|V_{ud}|}\dfrac{f_K}{f_\pi} = 0.27683(35)$ ['19 Di Carlo *et al.*]

$2 \sim 3\sigma$ difference from CKM unitarity (grey band)

Future work: $a = 0$ limit with PACS10$_c$ results + IB correction

Summary

PACS10 Project

calculation w/o three main systematic uncertainties in lattice QCD

PACS10/PACS10$_c$ configuration

$V \gtrsim (10\ \mathrm{fm})^4$ in physical point at three lattice spacings

Various physical quantity calculations

Nucleon form factor

Presise calculation of isovector form factors obtained at physical m_π

Calculation method of $F_P(q^2)$ and $G_P(q^2)$

Light meson form factor and charge radius

Presise calculation of electromagnetic form factors for π^+, K^+, K^0

Charge radii for π^+, K^+, K^0 using model-independent method

Kaon semileptonic decay form factor

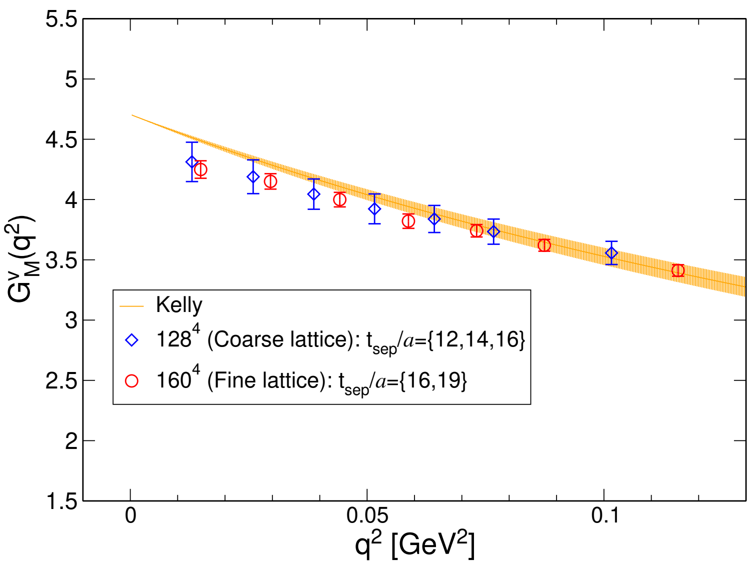
Precise calculation for $f_+(q^2)$ and $f_0(q^2)$ to determine $|V_{us}|$

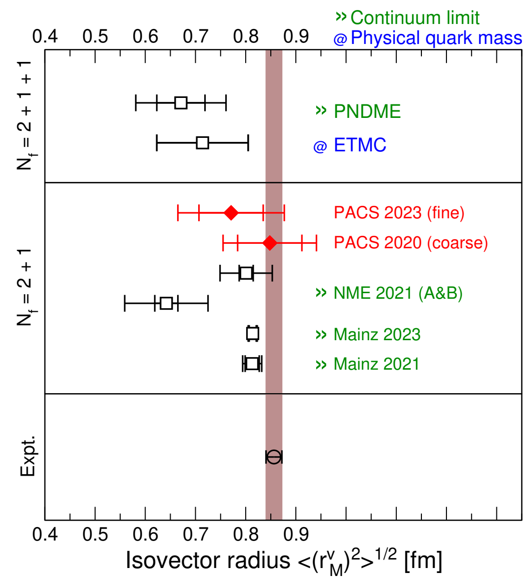
Comparable result of $|V_{us}|$ with those from other groups

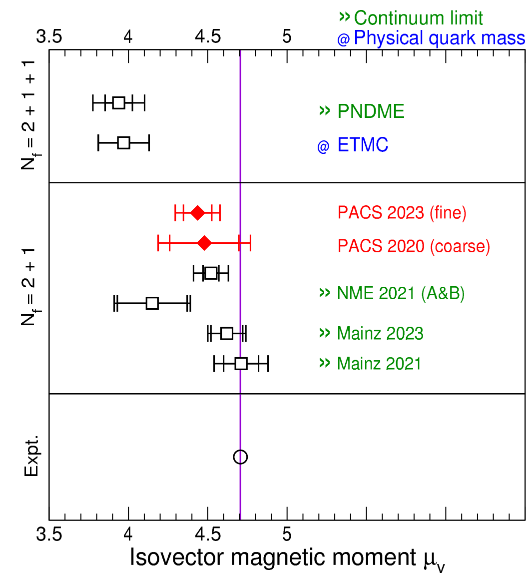
Back up

Isovector magnetic form factor $G_M(q^2)$

PACS10/L128 [PRD99:1,014510(2020)]; PACS10/L160 [PRD109:9,094505(2024)]







Reduction of leading πN contribution in F_P, G_P [PRD112:7,074510(2025)]

Simple assumption of leading πN contribution inspired by Bär's results

$$R_{A_i}(t,\vec{q}) = K^{-1}[(E_N+M_N)F_A(q^2)\delta_{i3} - q_3 q_i(F_P(q^2) - \Delta_+(t,\vec{q}))]$$

$$R_{A_4}(t,\vec{q}) = iq_3 K^{-1}[F_A(q^2) - \Delta E_N F_P(q^2) + E_\pi \Delta_-(t,\vec{q})]$$

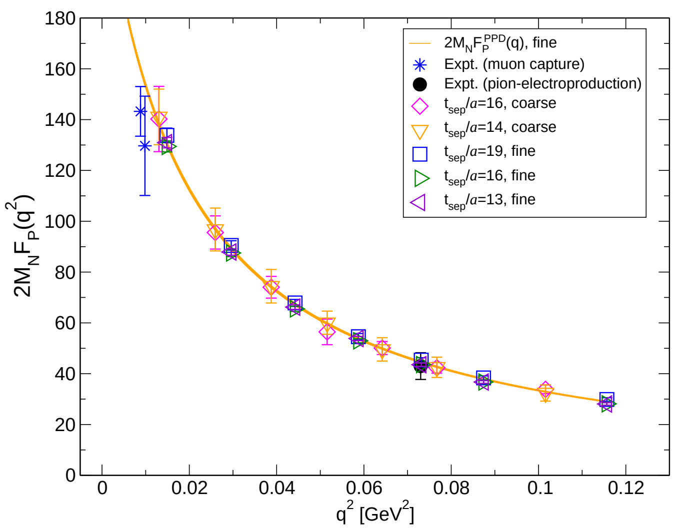
$$\Delta_\pm(t,\vec{q}) = B(q)e^{-(E_\pi - \Delta E_N)t} \pm C(q)e^{-(E_\pi+\Delta E_N)(t_{\mathrm{sep}}-t)}$$

R_A: ratio of 3-pt function with A_μ current to 2-pt functions, $K=\sqrt{2E_N(E_N+M_N)}$, $\Delta E_N = E_N - M_N$, $B(q), C(q)$: unknown coefficients, t_{sep}: source-sink separation

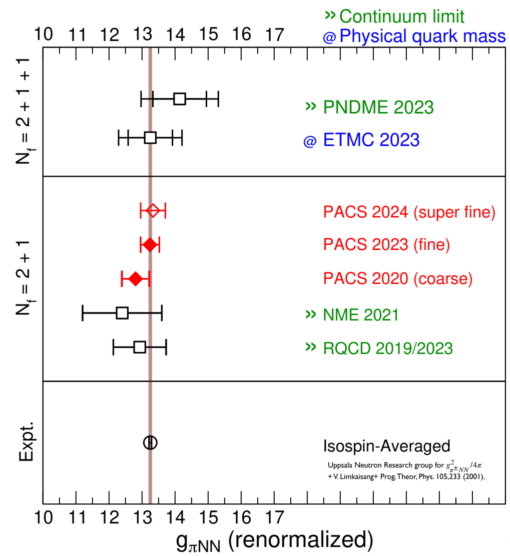
$\Delta_\pm(t,\vec{q})$ can be removed by linear combination with R_A and $\partial_t R_A$

$$F_P(q^2) = \alpha R_{A_i} + \beta \partial_t R_{A_i} + \gamma \partial_t R_{A_4}$$

α, β, γ: appropriate coefficients



$g_{\pi NN}$ coupling



$$g_{\pi NN} = \lim_{q^2 \to -m_\pi^2} \frac{m_\pi^2 + q^2}{2F_\pi} F_P(q^2)$$