Parton physics one the lattice: present and future activities in Taiwan

C.-J. David Lin

National Yang Ming Chiao Tung University
國立陽明交通大學

ANPhA 2025 workshop, Taipei
29/11/2025

Parton physics one the lattice: *selected* present and future activities in Taiwan

(mostly in Hsinchu)

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I will only discuss parton-physics projects with significant Taiwan-community involvement

(research groups, not individual researchers)

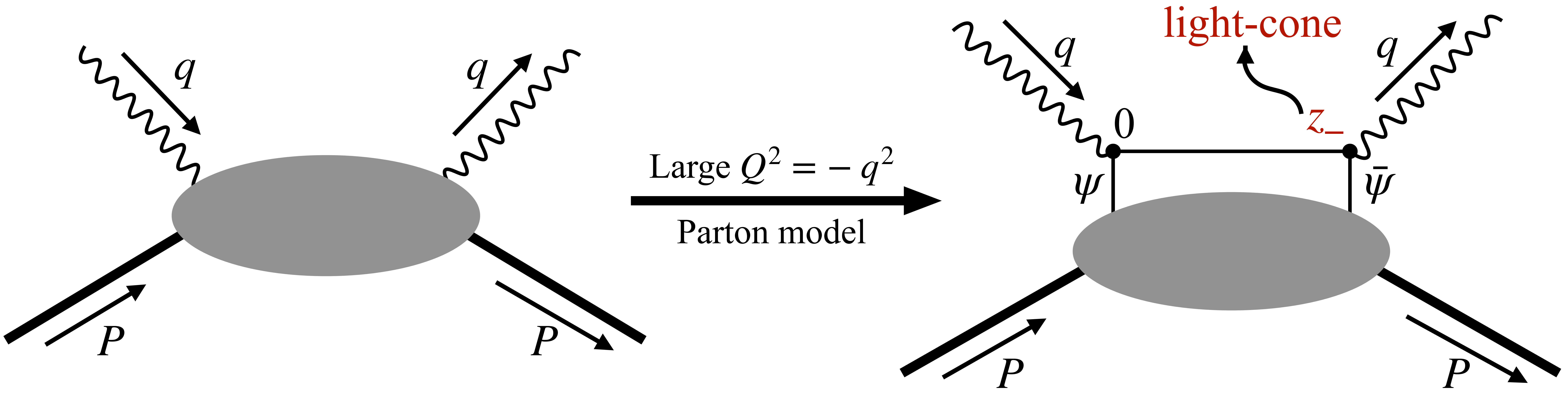
Outline

- The key issue of parton physics on the lattice
- Present: higher moments from HOPE method
- Present & future: Collins-Soper kernel from the soft function

And if there is time…..

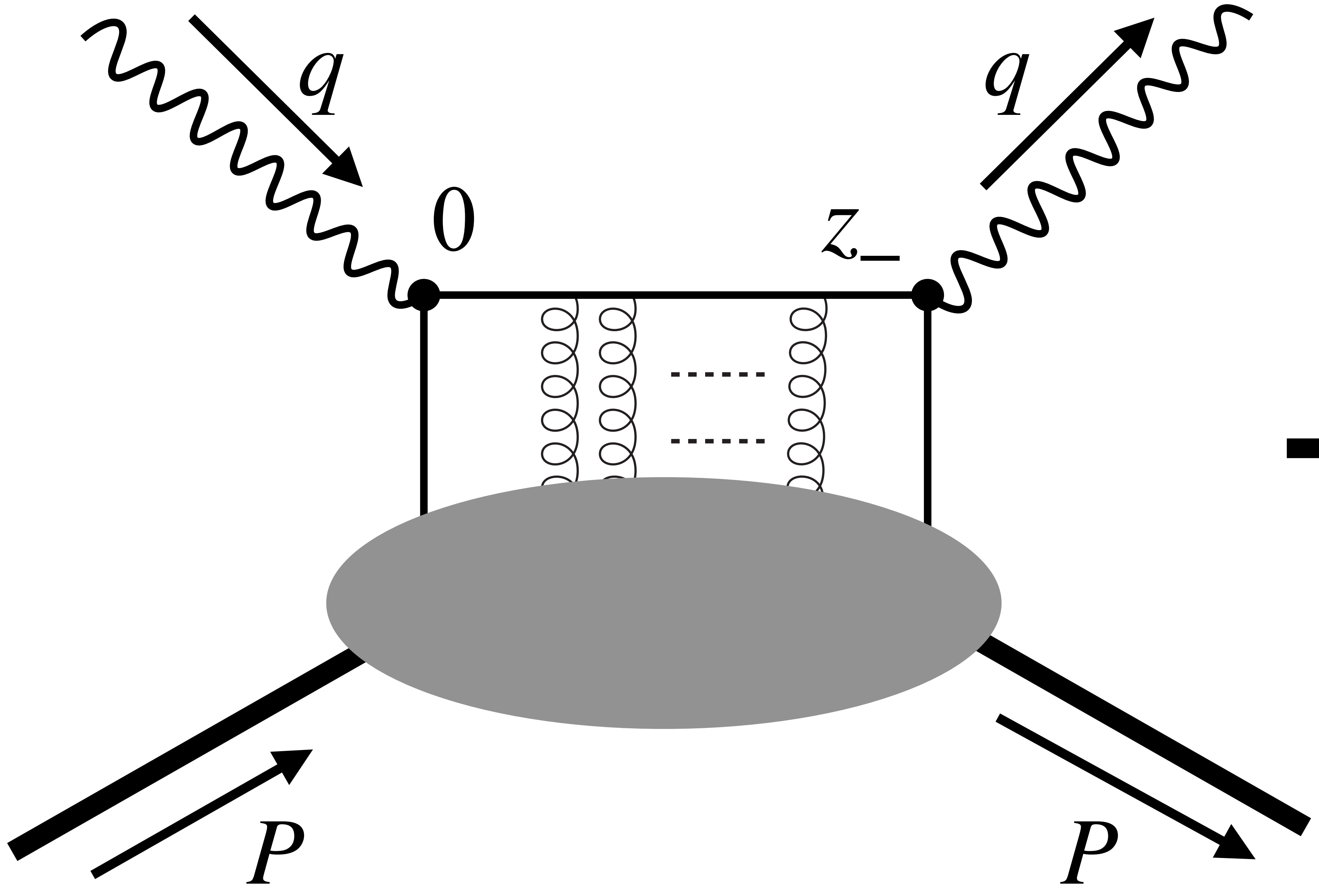
- Future: Hamiltonian formalism and direct access to light-front dynamics

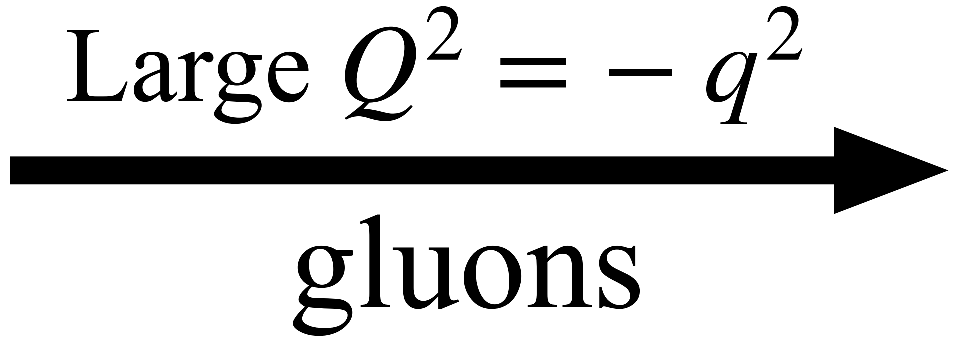
Hadron structure in parton picture

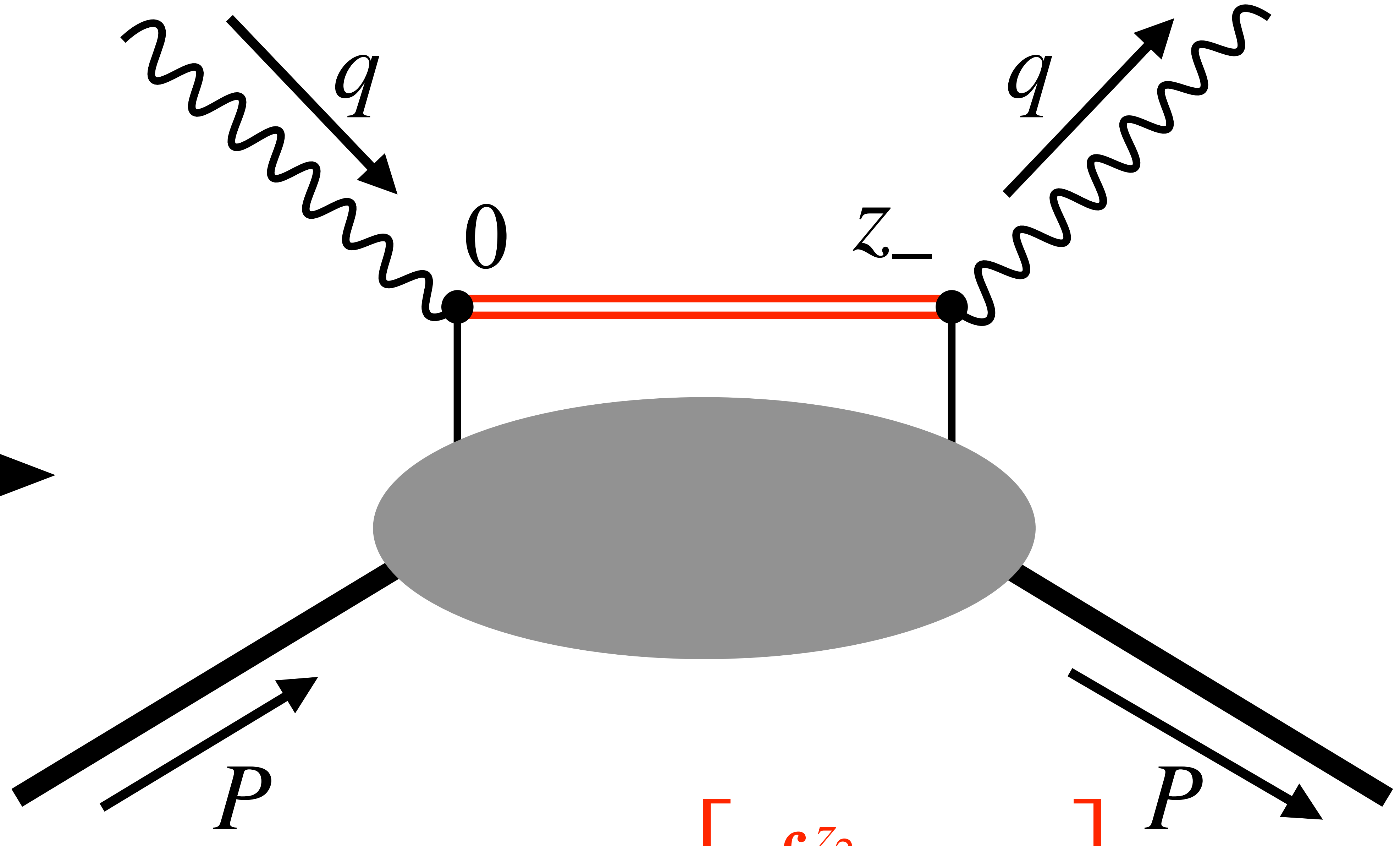


$$f(x) = \int \frac{dz^-}{4\pi} \, \mathrm{e}^{-ixP^+z^-} \langle P, S \,|\, \bar{\psi}(z^-)\Gamma\psi(0) \,|\, P, S \rangle$$

Parton distribution function and Wilson line in QCD



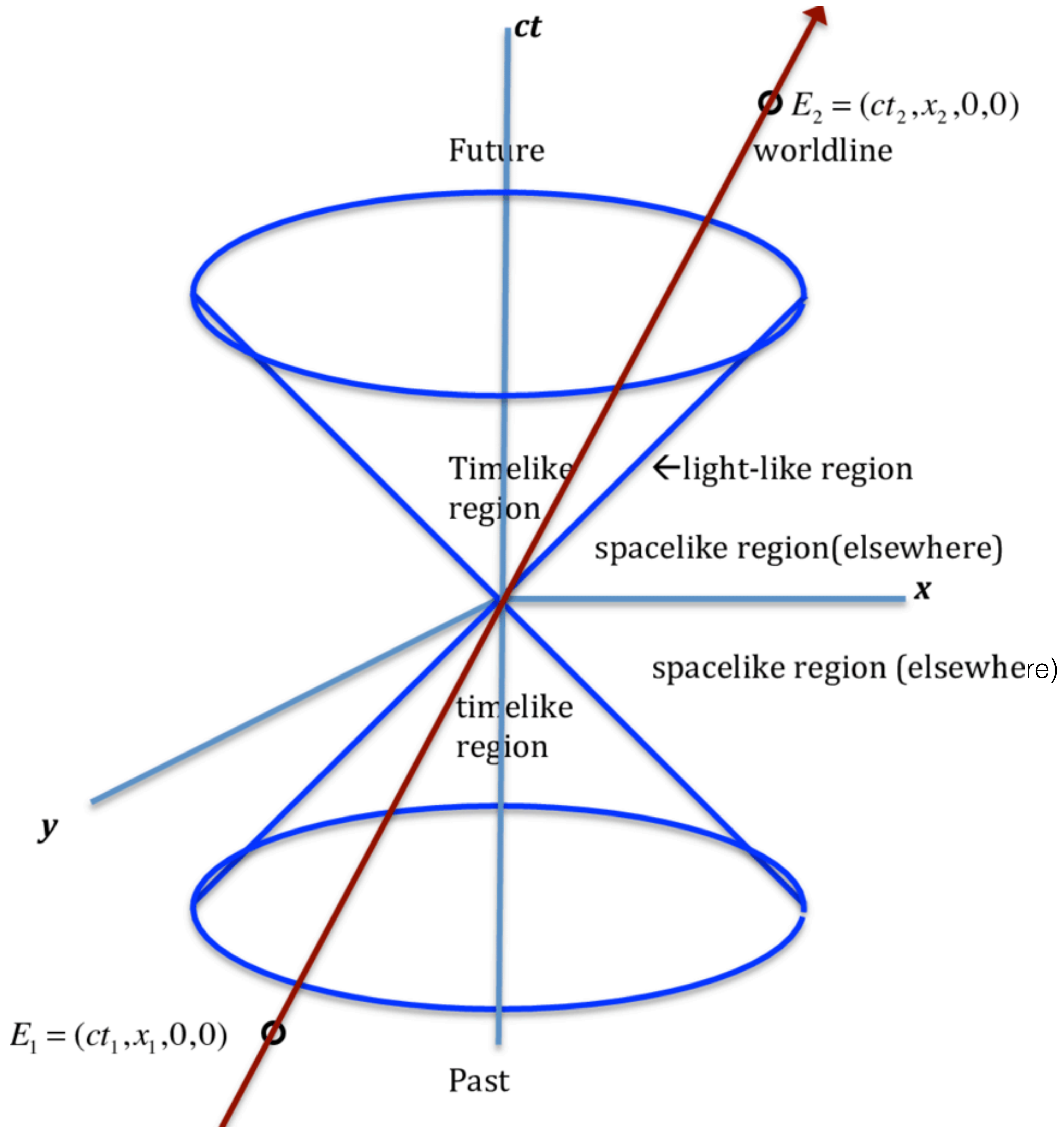




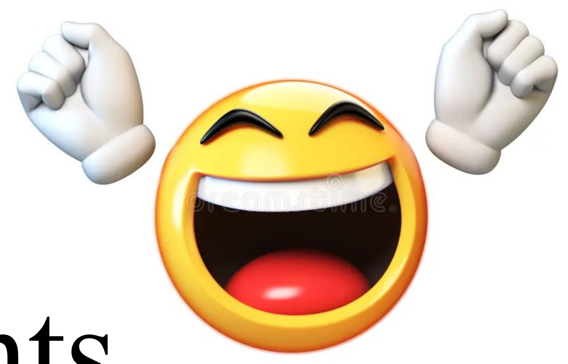
$$W[z_1, z_2] = P\exp\left[i\int_{z_1}^{z_2} dz^\mu A_\mu\right]$$

$$f(x) = \int \frac{dz^-}{4\pi} \mathrm{e}^{-ixP^+z^-} \langle P, S | \bar{\psi}(z^-)\Gamma W[z_-,0]\psi(0) | P, S\rangle$$

Minkowski space info from Euclidean computations



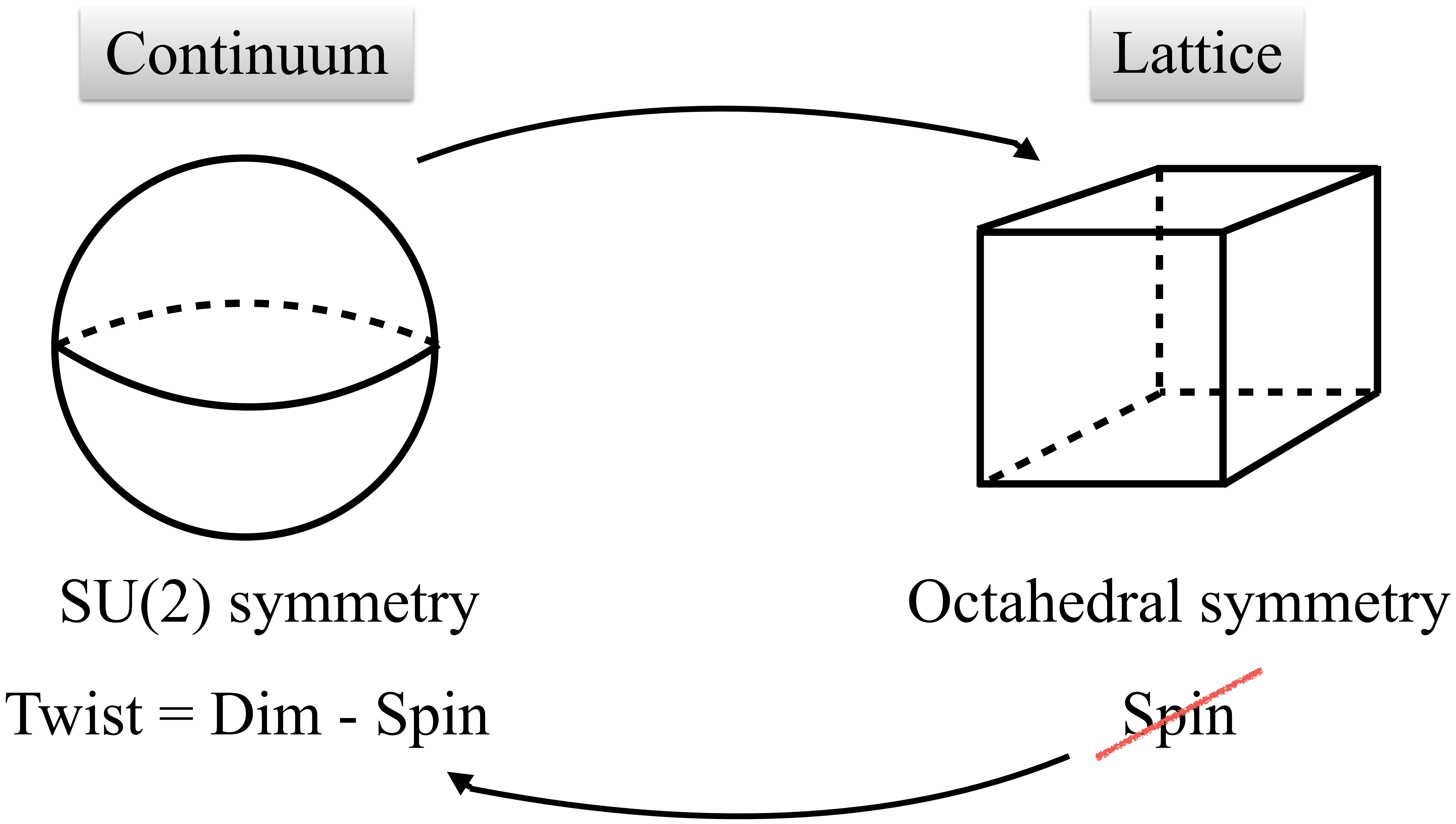
- Space-like regime ⇒ accessible directly
  - ➞ local matrix elements
  - ➞ non-local equal-time matrix elements



- Time-like regime ⇒ not directly accessible
  - ➞ Indirect access to a few quantities
  - ➞ Challenging in general

- Light cone ⇒ shrunk to a point
  - ➞ Indirect, limited access
  - ➞ Challenging in general

Issue with the conventional method



- Lattice breaks rotation symmetry
  - ➞ Spin not a good quantum number
- For a continuum twist-2 $\mathcal{O}_i$
  - ➞ Obtained *via* $\mathcal{O}_i = \sum_j C_{ij}^{\text{latt}}(a)\mathcal{O}_j^{\text{latt}}$
- Often $C_{ij}^{\text{latt}}(a) \sim 1/a^n$
  - ➞ Diverges as $a \to 0$
- Extremely difficult numerically

Only applicable for $\mathcal{O}_{0,1,2,3}$!

Higher moments from the method of heavy-quark operator product expansion (HOPE)

THE HOPE COLLABORATION

Alex Chang
(NYCU)

William Detmold
(MIT)

Matias Escobar
(MIT)

Anthony Grebe
(MIT ⇒ Fermilab ⇒ U of Maryland)

Issaku Kanamori
(NYCU ⇒ Hiroshima U. ⇒ RIKEN RCCS)

CJDL
(NYCU)

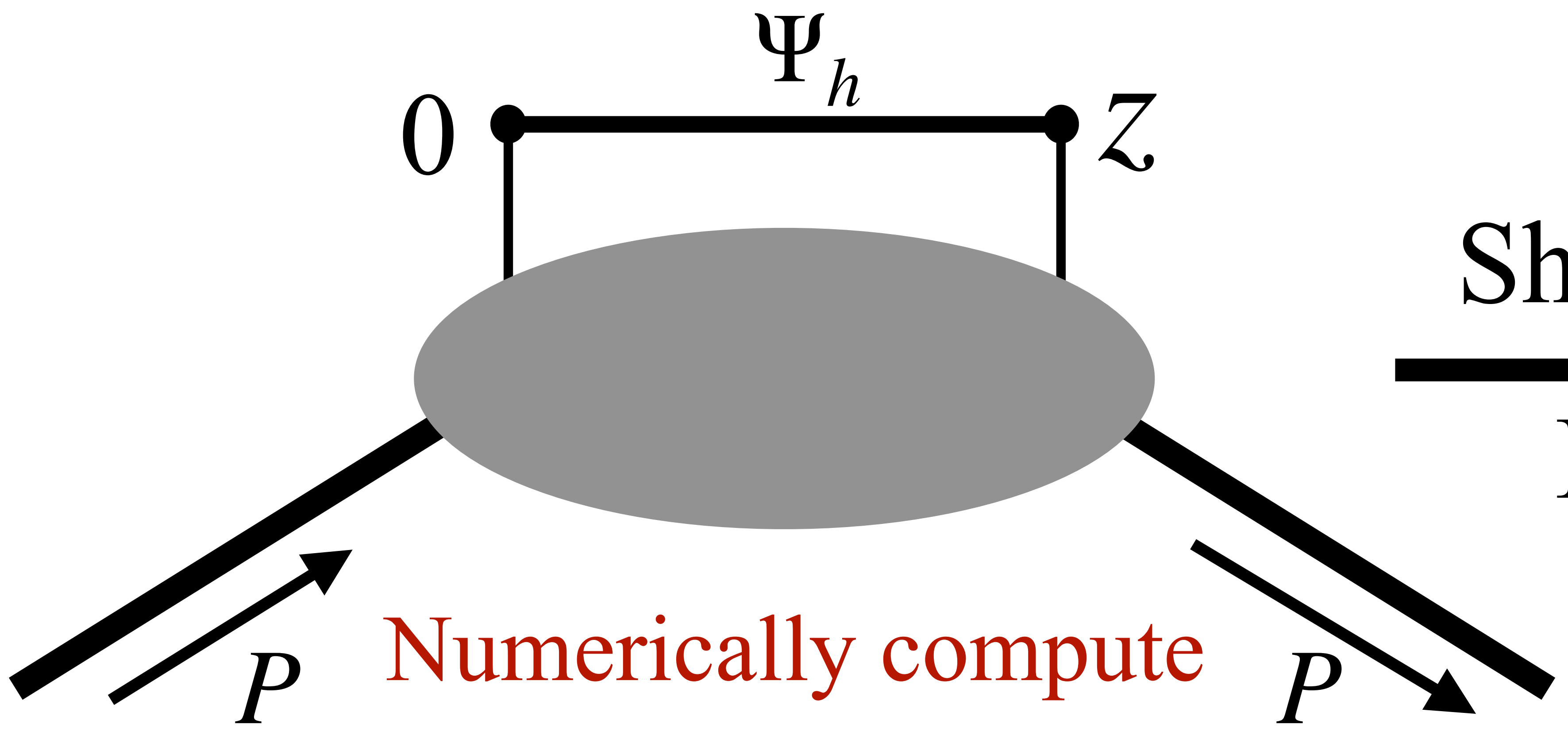
Robert Perry
(NYCU ⇒ U of Barcenona ⇒ MIT)

Yong Zhao
(MIT ⇒ Argonne Nat'l Lab)

The HOPE method for higher Mellin moments

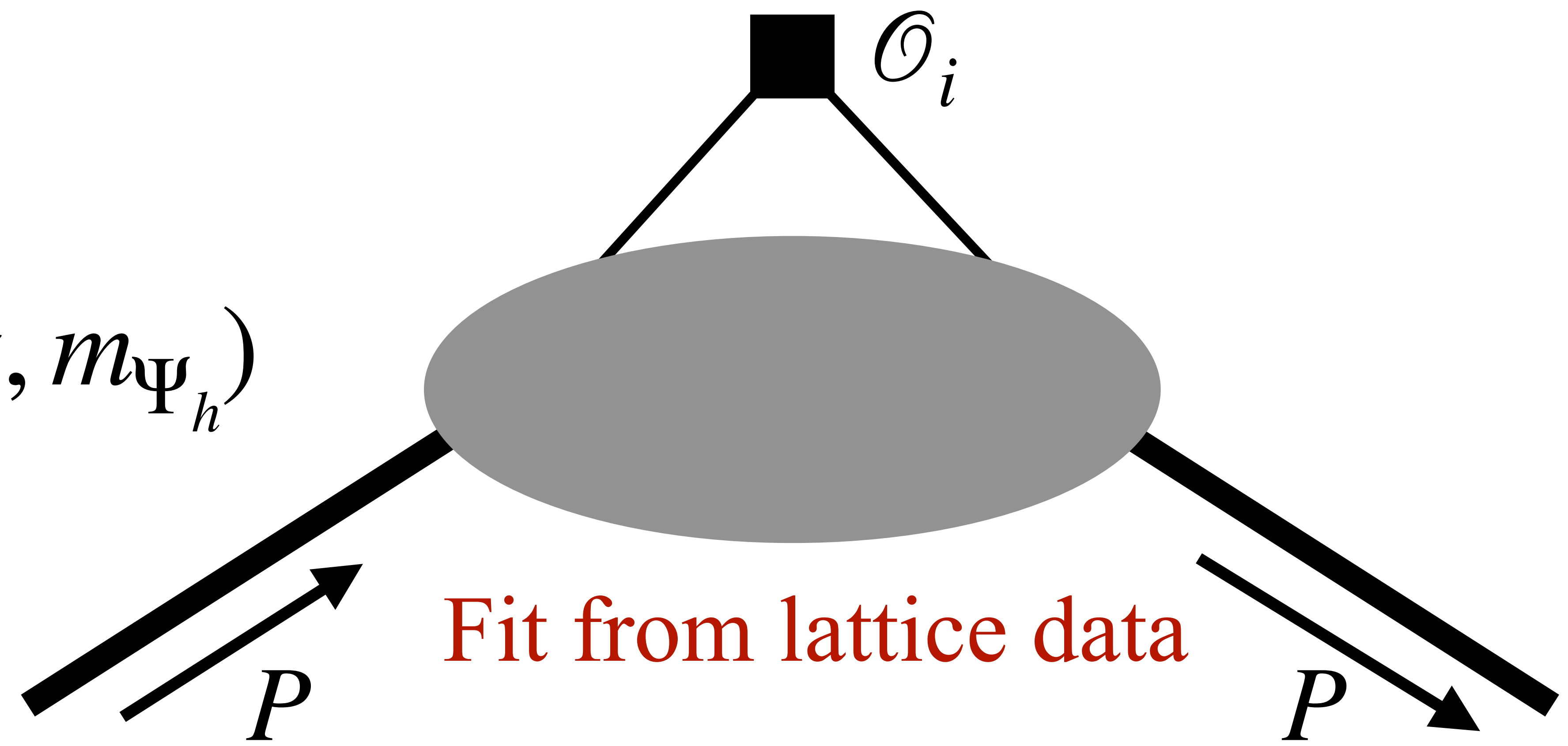
W. Detmold and CJDL, PRD 73 (2006) 014501

HOPE Collaboration, PRD104 (2021) 7, 074511

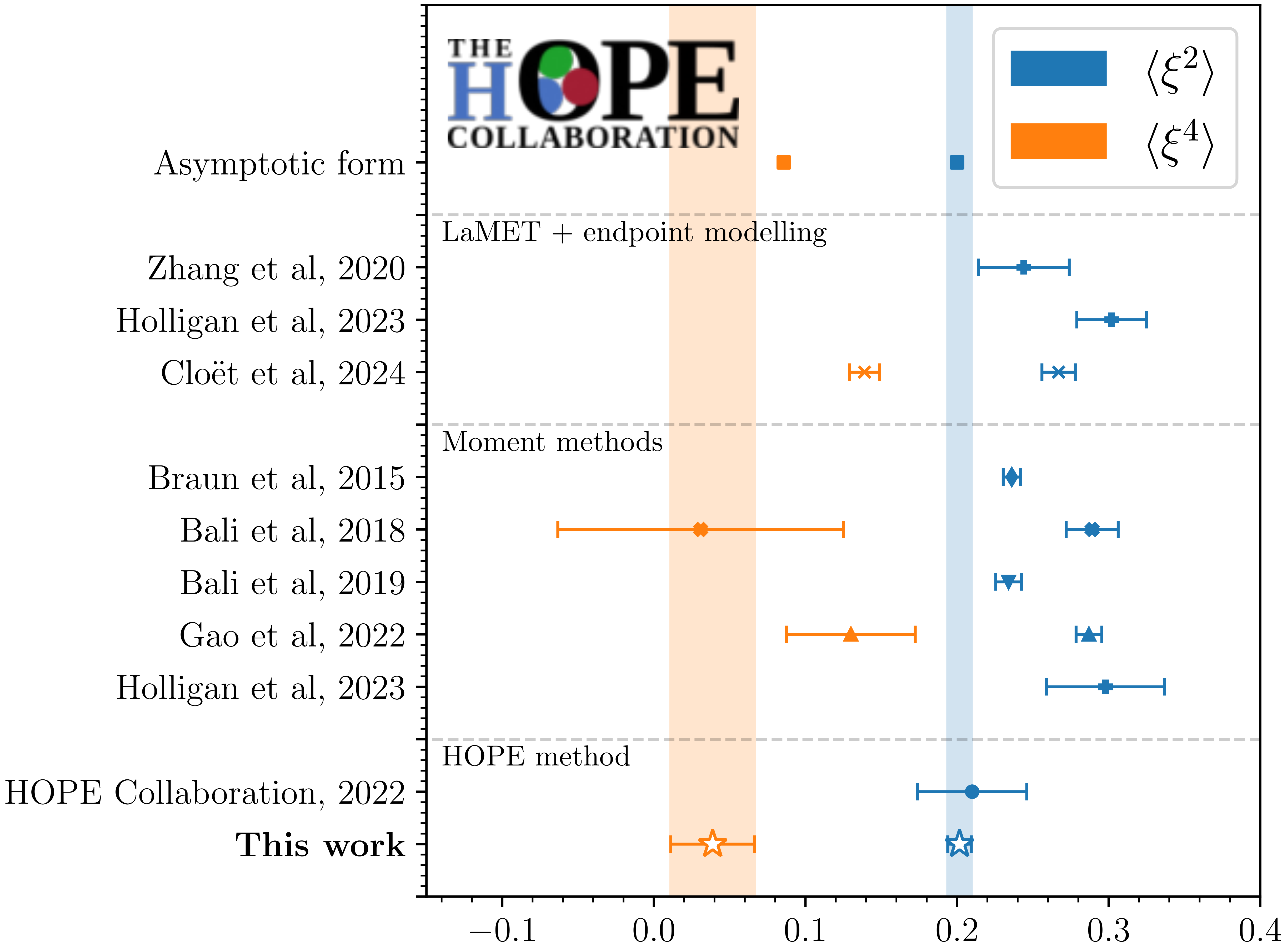


Short-distance → Large m_{Ψ_h}

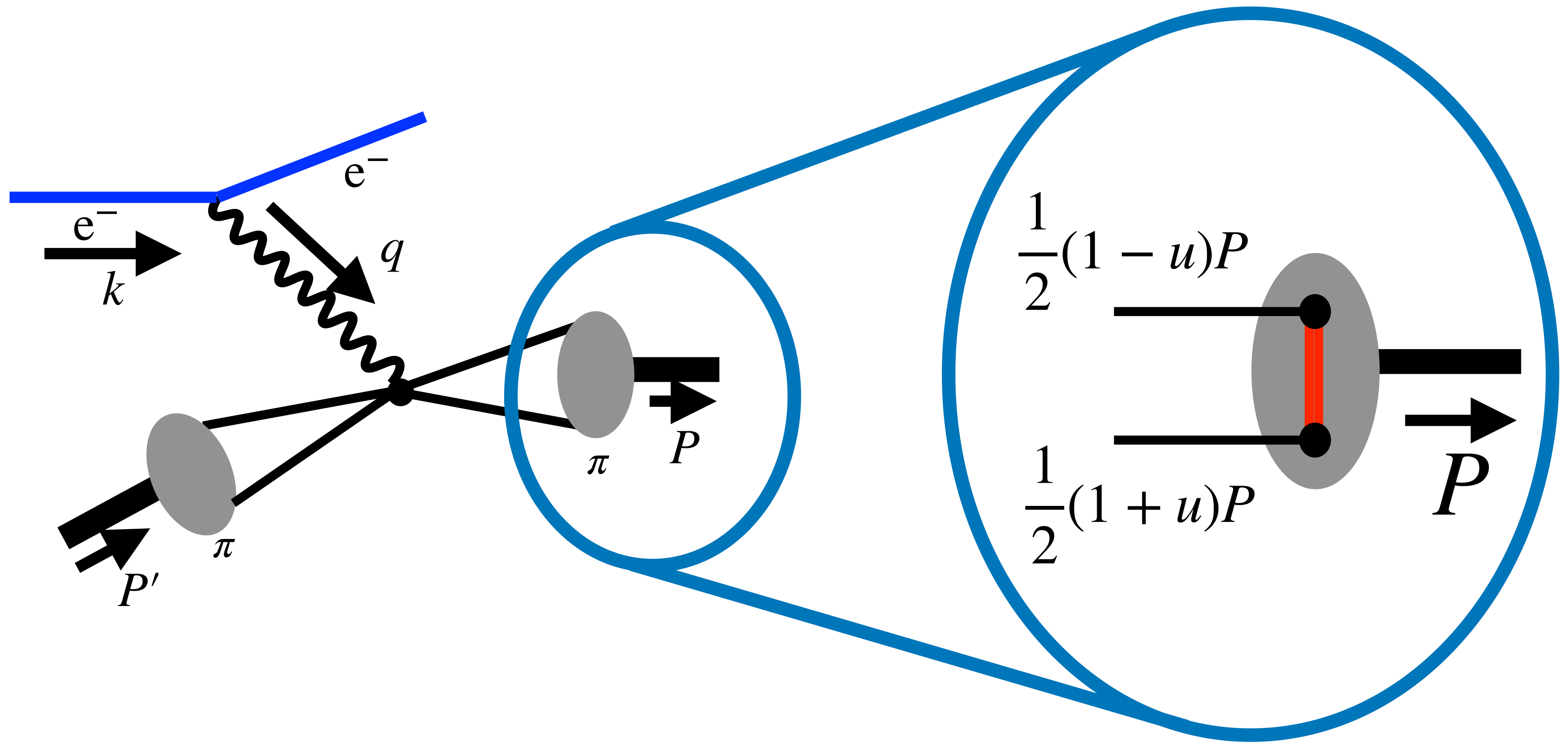
$$\sum_{i=0}^{\infty} C_i^{\mathrm{HOPE}}(z, m_{\Psi_h})$$



The HOPE method for higher Mellin moments



arXiv:2509.04799 [hep-lat]



$$F_\pi(Q^2) = (\text{hard kernel}) \otimes \phi_\pi(\xi) \otimes \phi_\pi(\xi')$$

$$\langle 0 \,|\, \bar{\psi}(z_-)\, \gamma_\mu \gamma_5 W[z_-, -z_-]\, \psi(-z_-) \,|\, \pi(\mathbf{p})\rangle = i p_\mu f_\pi \int d\xi\, e^{ip_+ z_-}\, \phi_\pi(\xi)$$

$$\langle \xi^n \rangle \equiv = \int d\xi\, \xi^n \phi_\pi(\xi)$$

Physics of the electron-ion collider:
The Collins-Soper kernel from the soft function

The collaboration

Anthony Francis
(NYCU)

CJDL
(NYCU)

Wayne Morris
(NYCU)

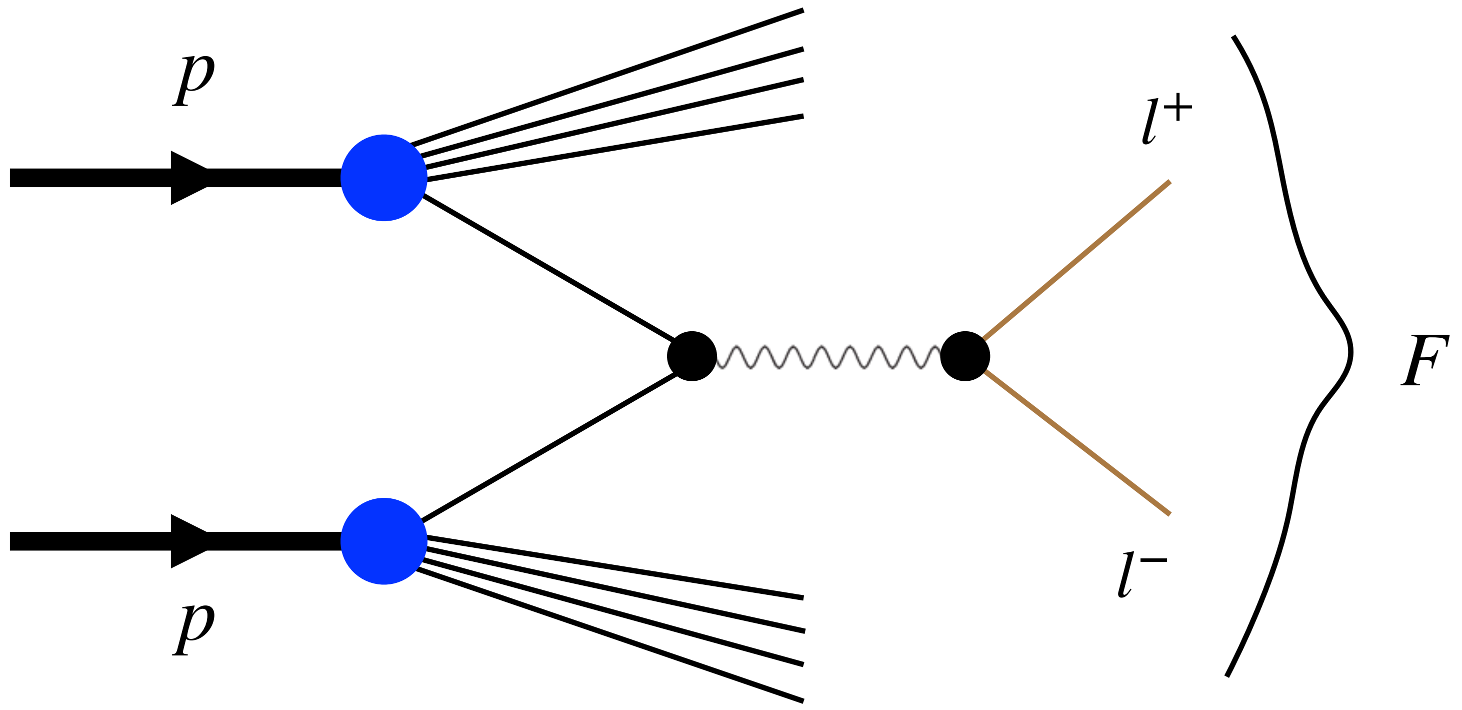
Yong Zhao
(Argonne Nat'l Lab)

arXiv: 2312.04315. [hep-lat] (Lattice 2023 proc.)

arXiv: 2412.12645 [hep-lat] (Lattice 2024 proc.)

& journal paper to appear soon

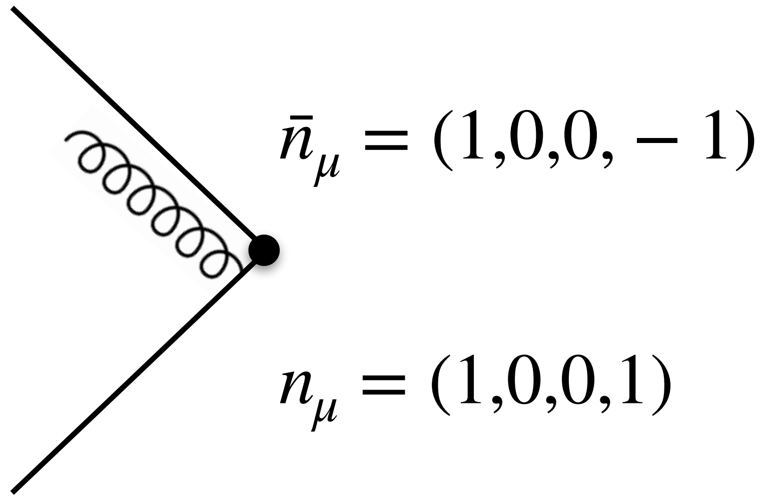
Drell-Yan factorisation and TMDPDF



$$\frac{d\sigma}{dQdYd^2q_T} = \sum_{ij} H_{ij}(Q,\mu) \int d^2b_T\ e^{i\vec{b}_T\cdot\vec{q}_T}\ f_i^{\rm TMD}(x_i,\vec{b}_T,\mu,\zeta_i)\ f_j^{\rm TMD}(x_j,\vec{b}_T,\mu,\zeta_j)$$

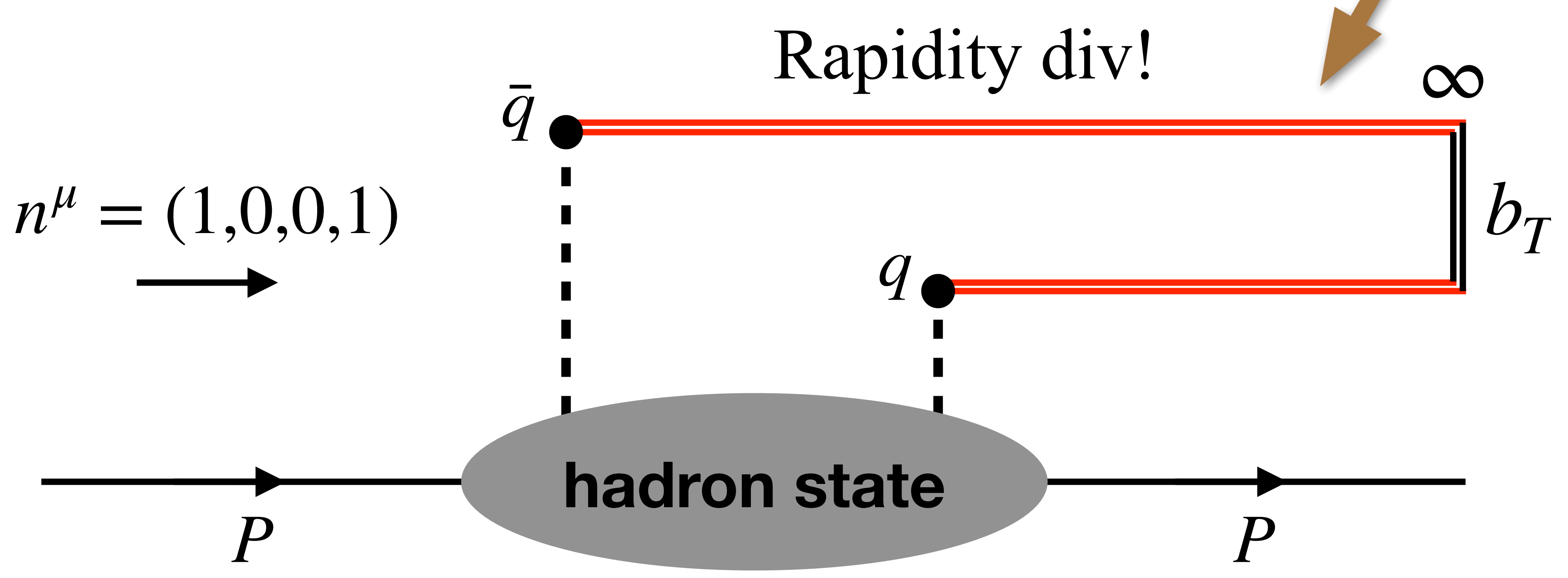
$$\times\left[1+\mathcal{O}\left(\frac{q_T^2}{Q^2},\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)\right]$$

$\zeta_{i,j}$ from "rapidity divergence" and $\zeta_i\zeta_j = Q^4$

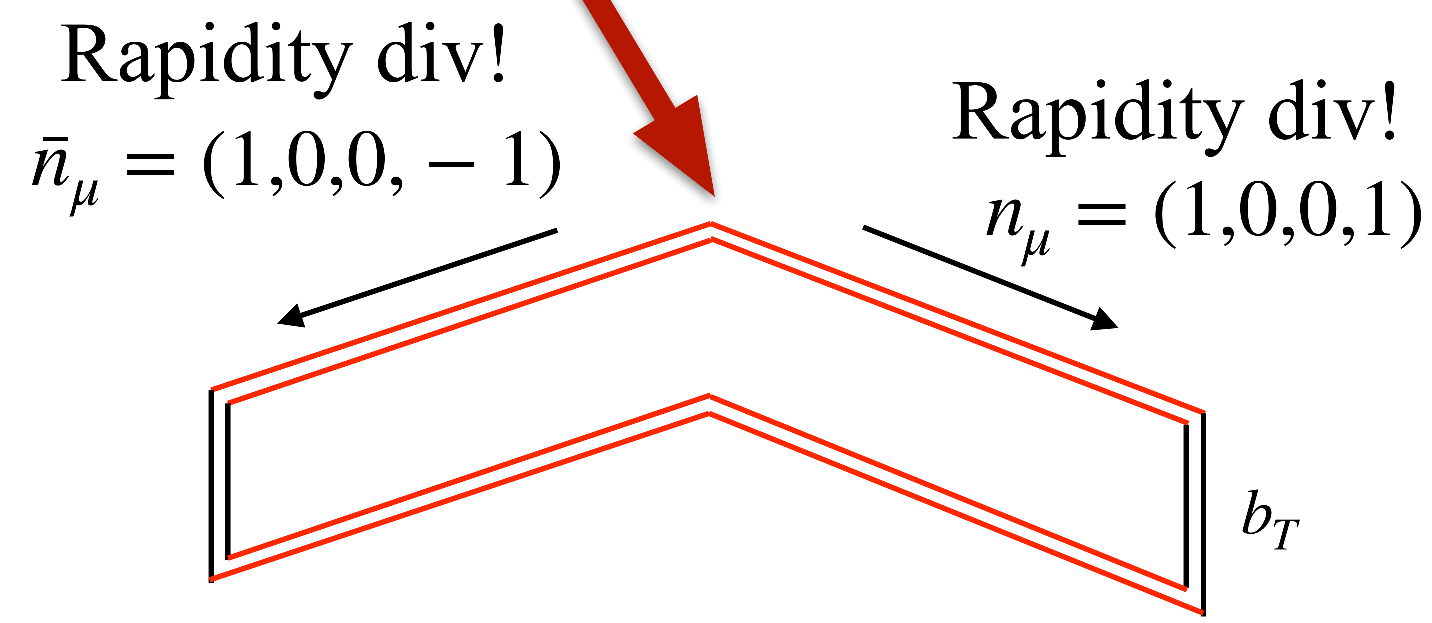


Drell-Yan factorisation and TMDPDF

$$f^{\text{TMD}}(x, \vec{b}_T, \mu, \zeta) = B(x, \vec{b}_T, \mu, \zeta/\nu^2)\,\sqrt{\mathcal{S}(b_T, \mu, \nu)}$$



"Beam function"



"Soft function"

And the "Collins-Super (CS) kernel" for evolution in ν (ζ)

$$\mathcal{S}(b_T, \mu, \nu) \Rightarrow \mathcal{S}_I(b_T, \mu), K(b_T, \mu) \Rightarrow \text{both are } \textit{universal}$$

Relating quasi-TMDPDF to TMDPDF

M.A. Ebert, S.T. Schindler, I.W. Stewart, Y. Zhao, JHEP 04 (2022) 178

$$\underline{\tilde{f}^{\mathrm{TMD}}(x, \vec{b}_T, \mu, P^z)} = \underbrace{C^{\mathrm{TMD}}(\mu, xP^z)}_{\text{pertub. theo.}}\, \underline{g_S(b_T, \mu)} \exp\left[\frac{1}{2}\underline{K(b_T, \mu)} \log\frac{(2xP^z)^2}{\zeta}\right]$$

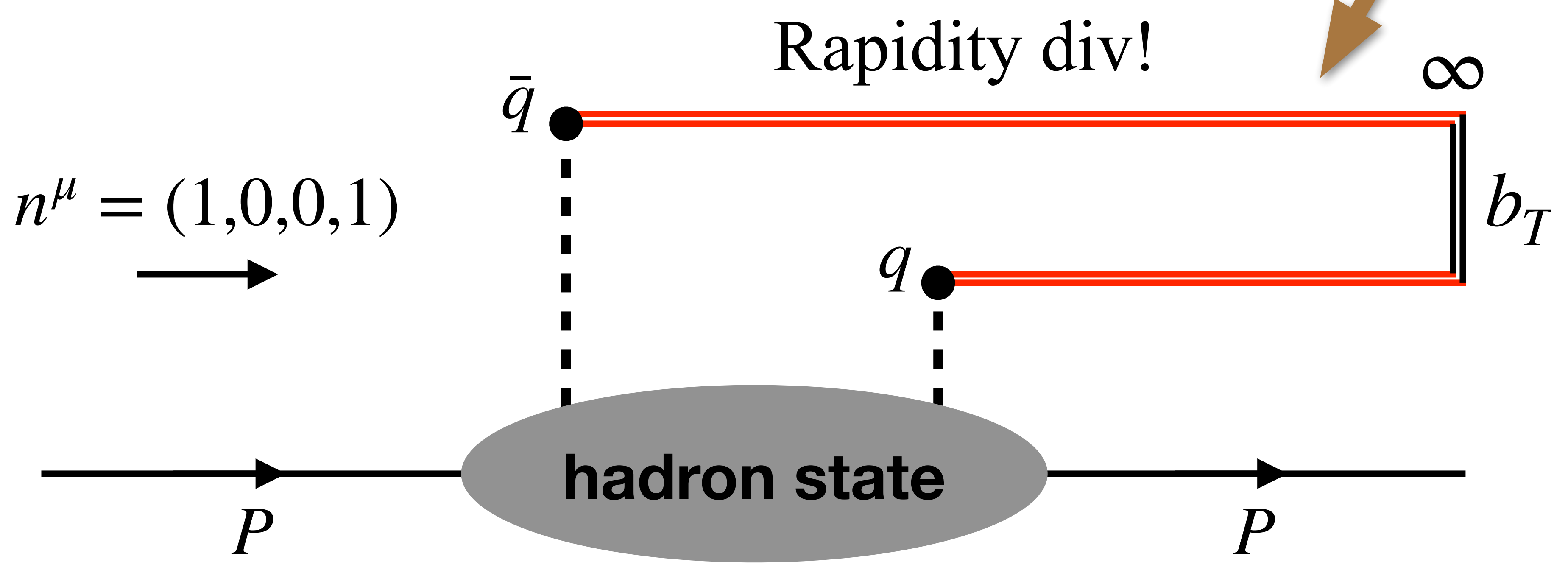
$$\times \underline{f^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta)} + \mathcal{O}\left(\frac{q_T^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right)$$

- To obtain f^{TMD}, one computes $\tilde{f}^{\mathrm{TMD}}$ with lattice QCD
- Also need Collins-Soper kernel, $K(b_T, \mu)$, and the soft function, $g_S(b_T, \mu) \sim \sqrt{S_I(b_T, \mu)}$

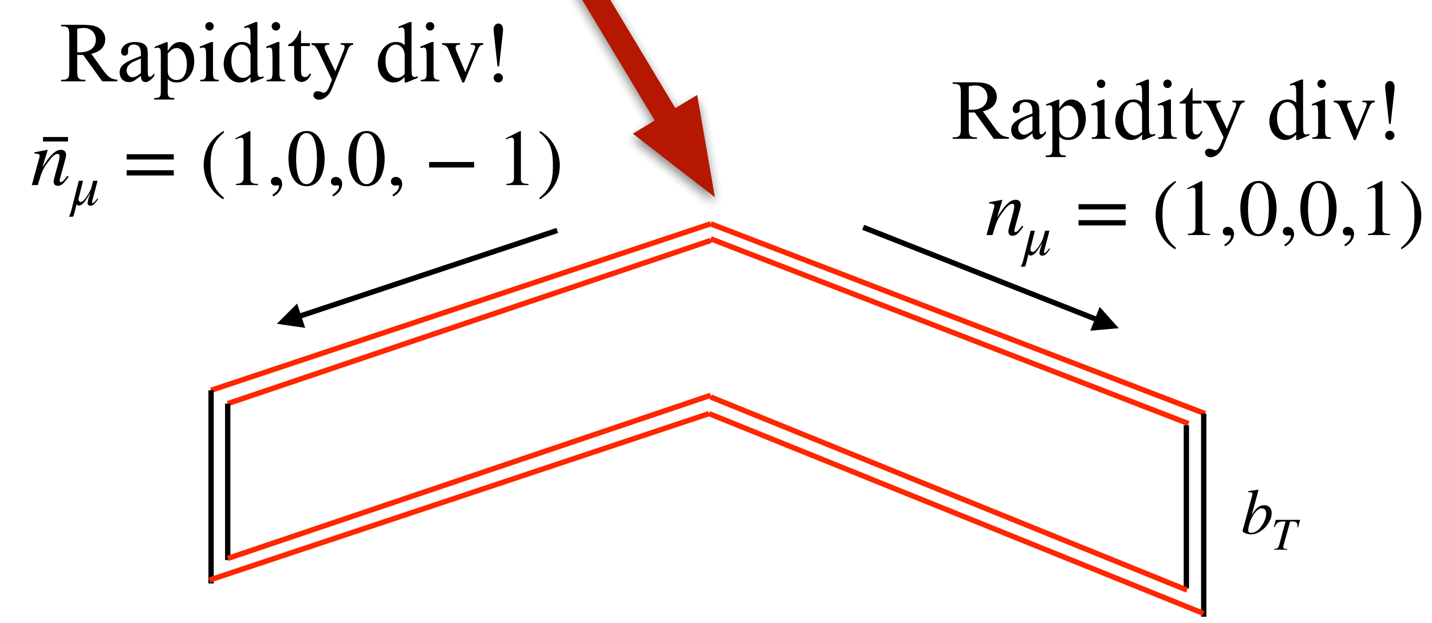
  $\Rightarrow$ Both non-perturbative and universal

A reminder…..

$$f^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = B(x, \vec{b}_T, \mu, \zeta/\nu^2)\,\sqrt{\mathcal{S}(b_T, \mu, \nu)}$$



"Beam function"



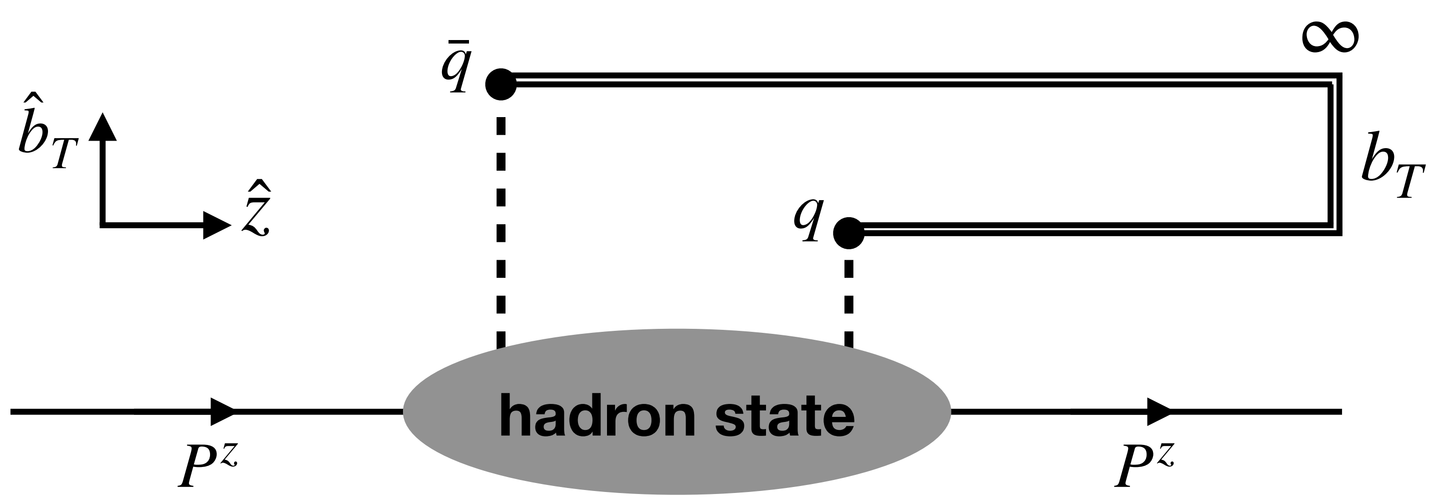
"Soft function"

Rapidity divergence appears in both the beam and the soft functions

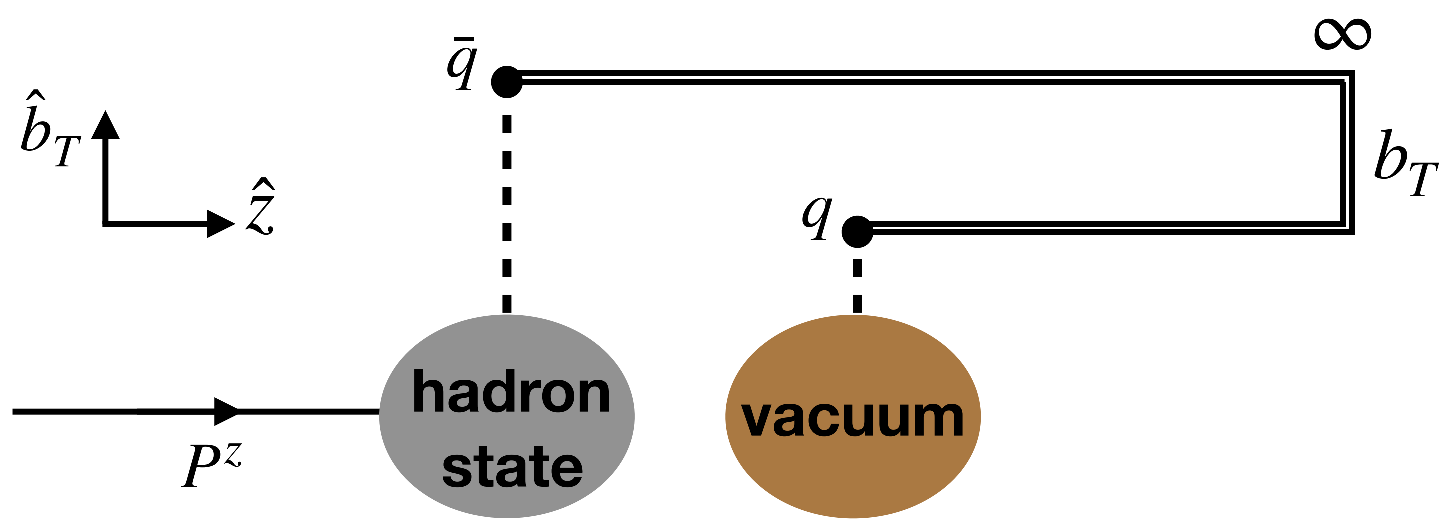
CS kernel from the beam function

M. Ebert, I. Stewart, Y. Zhao, Phys. Rev., **D99** (2019) 034505

- Compute qTMDPDF ($\tilde{f}^{\rm TMD}$) or qTMDWF ($\tilde{\Phi}^{\rm TMD}$)



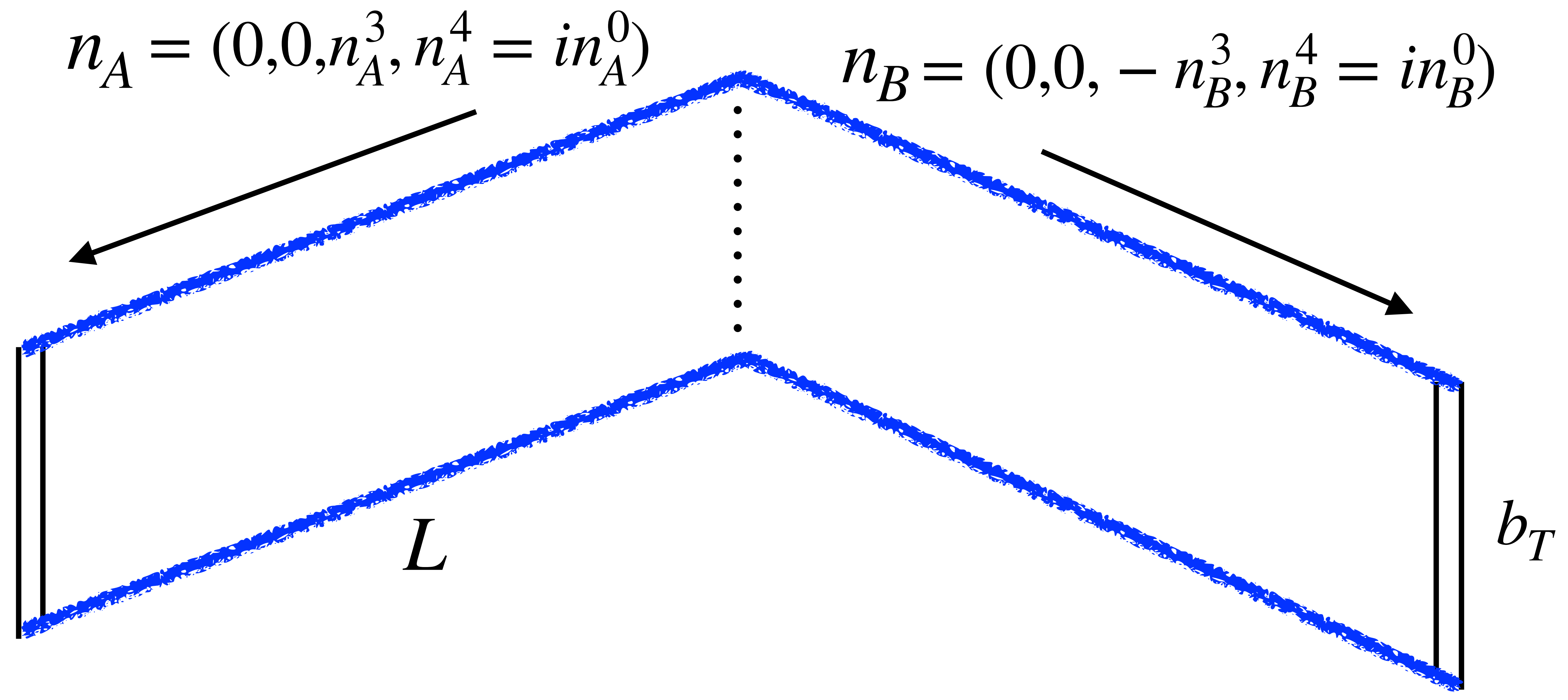
or



- Determine the CS kernel from the ratio (at large P^z)

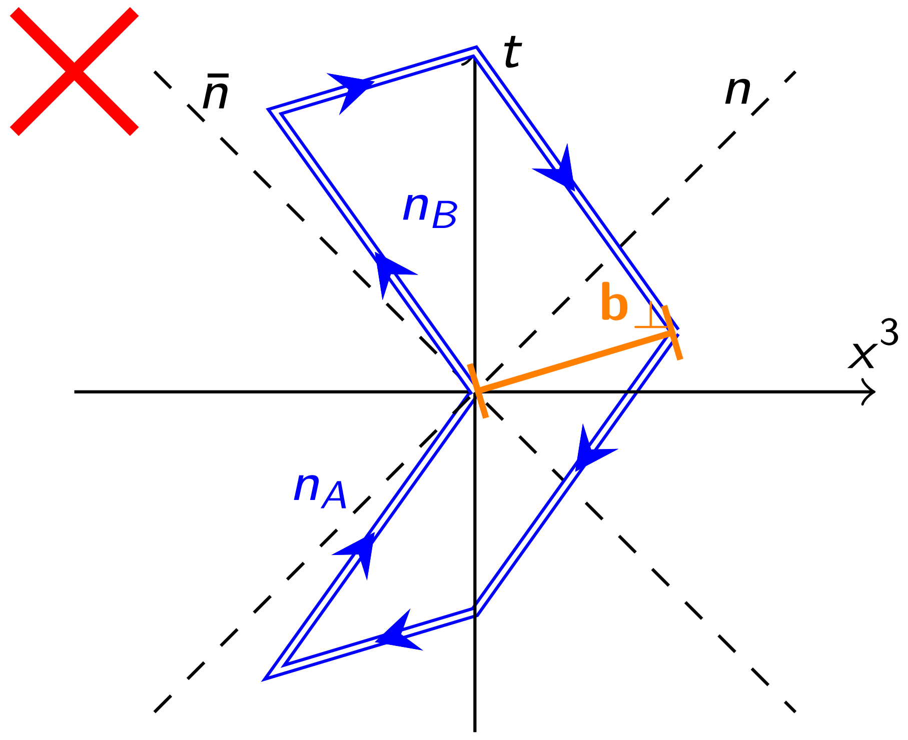
$$K(\mu, b_T) = \frac{1}{\log(P_1^z/P_2^z)} \log \frac{\underbrace{C^{\rm TMD}(\mu, xP_2^z)}\ \tilde{\Phi}^{\rm TMD}(x, \vec{b}_T, \mu, P_1^z)}{\underbrace{C^{\rm TMD}(\mu, xP_1^z)}_{\text{perturbative}}\ \tilde{\Phi}^{\rm TMD}(x, \vec{b}_T, \mu, P_2^z)}$$

Our approach:
CS kernel from the soft function

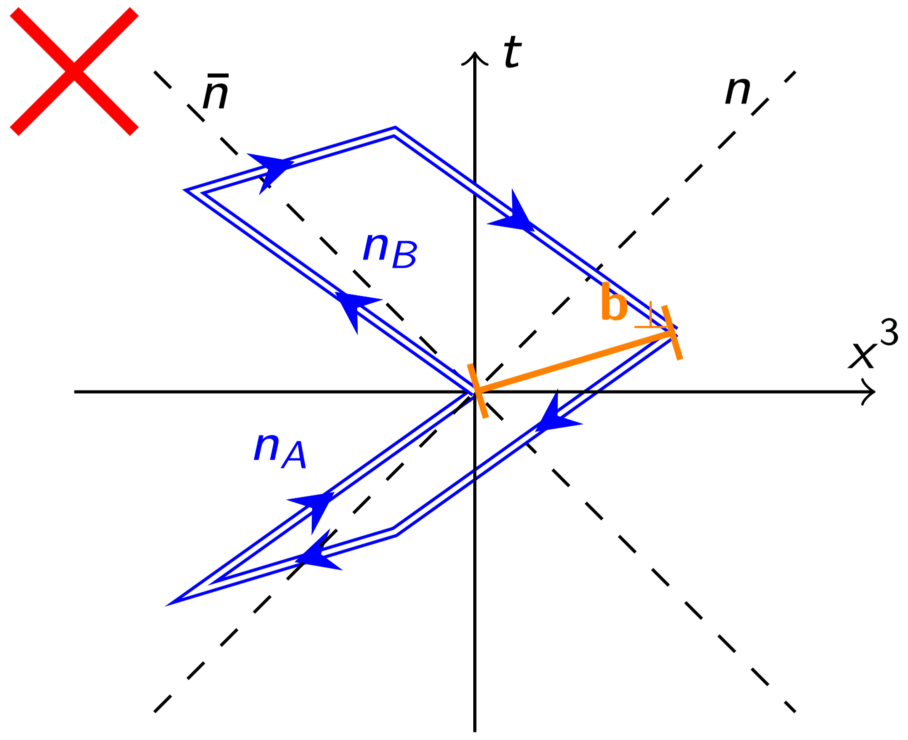


Basic building block: *complex-directional* Wilson loops in Euclidean space

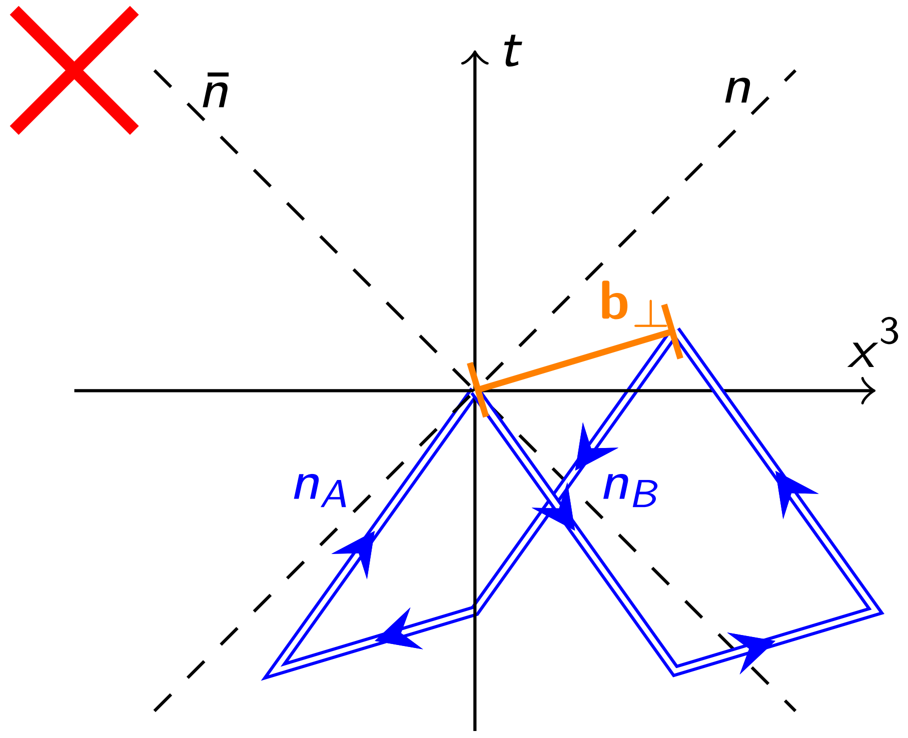
Our approach:
CS kernel from the soft function



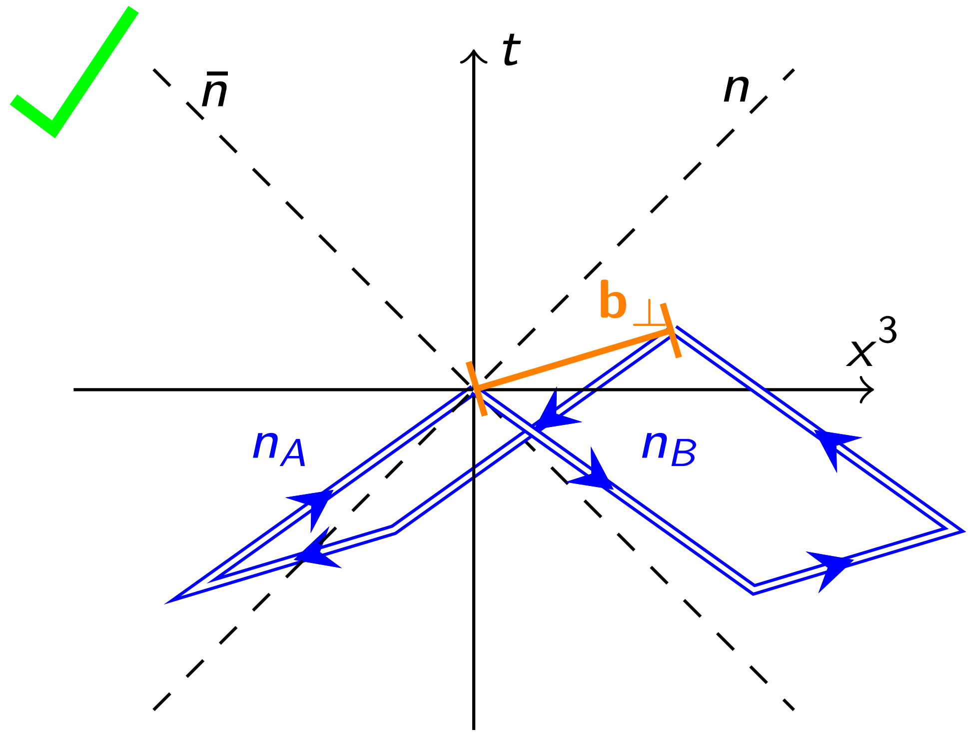
$|r_a| < 1, \quad |r_b| < 1, \quad n_A^0 n_B^0 (r_a r_b + 1) < 0$



$|r_a| > 1, \quad |r_b| > 1, \quad n_A^0 n_B^0 (r_a r_b + 1) < 0$



$|r_a| < 1, \quad |r_b| < 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$



$|r_a| > 1, \quad |r_b| > 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$

- Does not work in “SIDIS-type”
- Only feasible in space-like regime

  ⇒ Minkowski = Euclidean

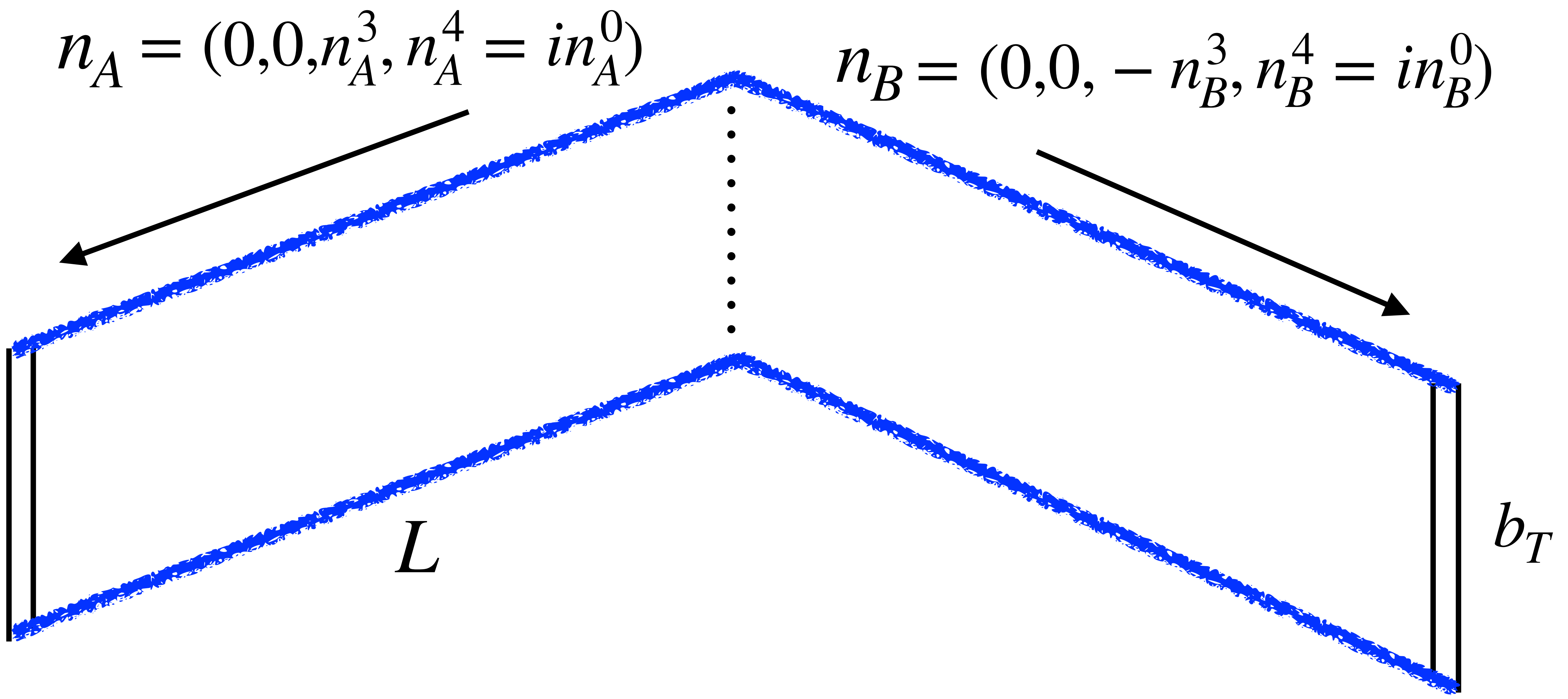
  ⇒ Gives Collins’s soft function

$$S_C(b_T, \mu, y_A, y_B) = S_I(b_T, \mu)\, e^{2K(b_T,\mu)\times(y_A - y_B)}$$

$$r_{a,b} \equiv \frac{n^3_{A,B}}{n^0_{A,B}} = \frac{1 + e^{\pm y_{A,B}}}{1 - e^{\pm y_{A,B}}}$$

Our approach:

The double ratio method



- Choose $r_a = \frac{n_A^3}{n_A^0} = \frac{n_B^3}{n_B^0} = r_b = r$

  $\Rightarrow$ the Wilson loop = $W(b_T, r, L)$

- $R_{\text{single}}(b_1, b_2, r) = \lim_{L\to\infty} \frac{W(b_1, r, L)}{W(b_2, r, L)}$

- $R_{\text{double}}(b_1, b_2, r_1, r_2) = \frac{R_{\text{single}}(b_1, b_2, r_1)}{R_{\text{single}}(b_1, b_2, r_2)}$

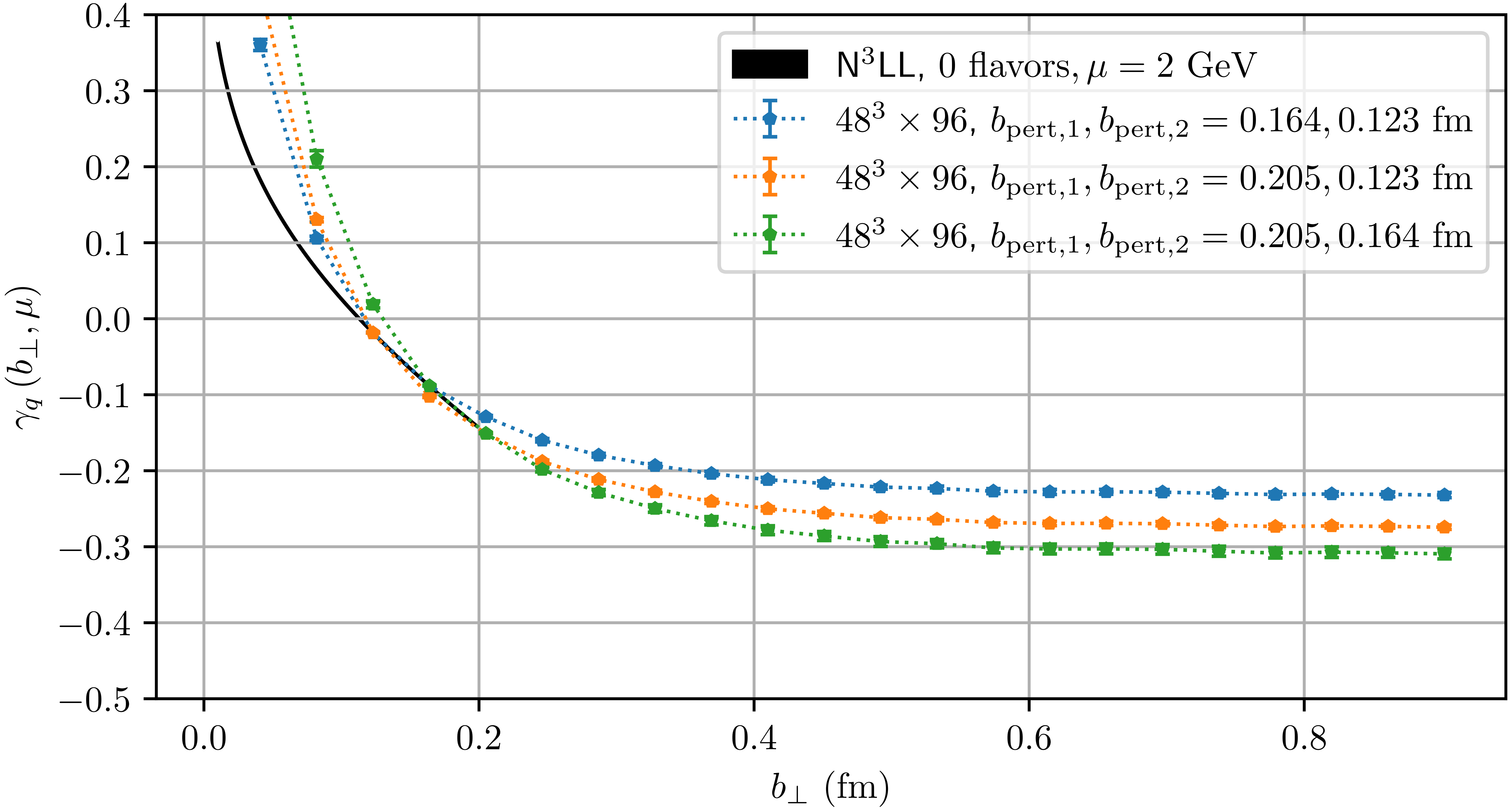
$$\gamma_q(b_1,\mu) = \gamma_q(b_2,\mu) + \frac{\log\left[R_{\text{double}}(b_1, b_2, r_1, r_2, \mu)\right]}{2\log\left(\frac{r_1+1}{r_1-1}/\frac{r_2+1}{r_2-1}\right)}$$

$\Rightarrow$ note: rapidity renormalisation!

$\Rightarrow$ "match to a perturbative result"

Our approach:

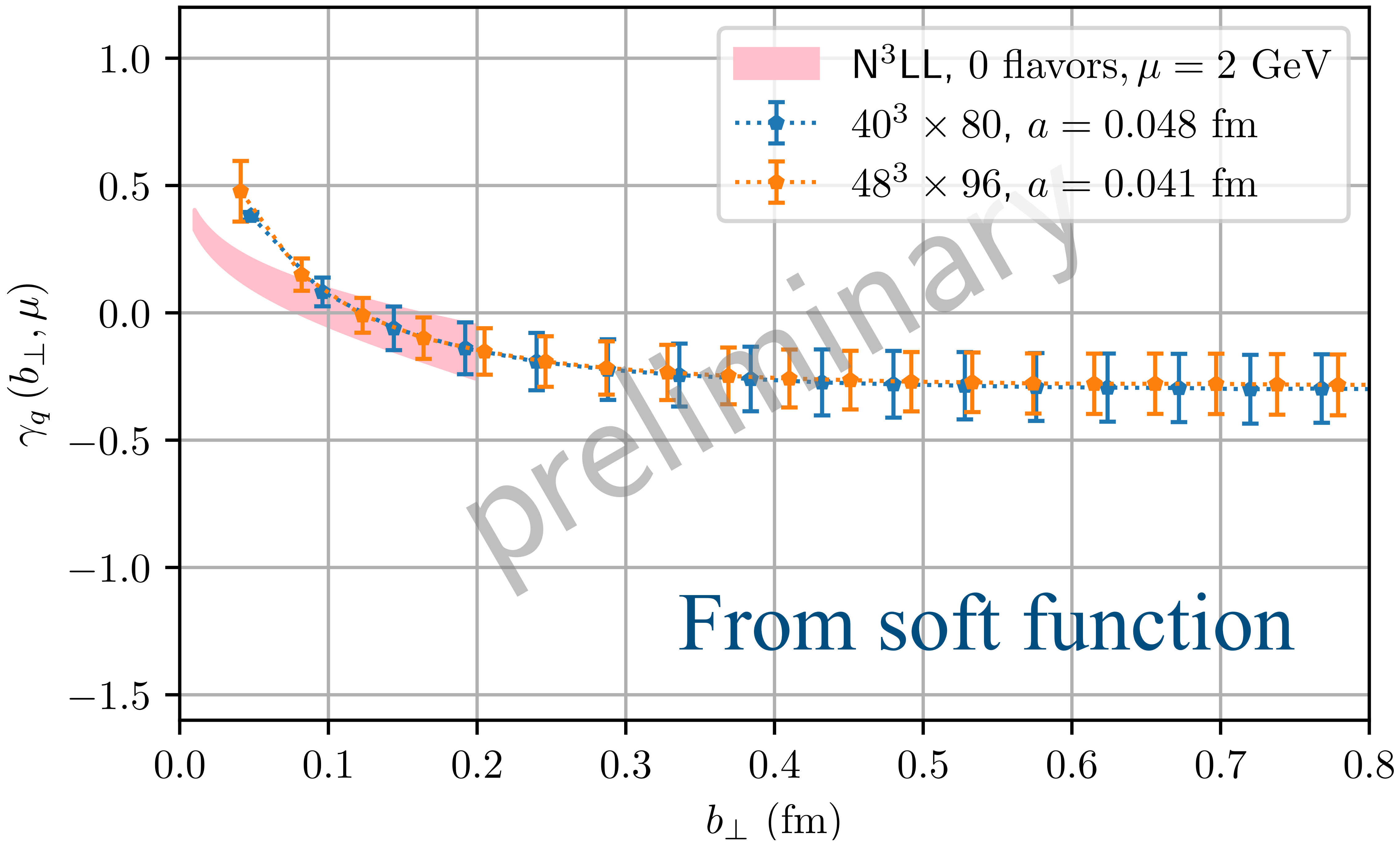
Preliminary numerical results

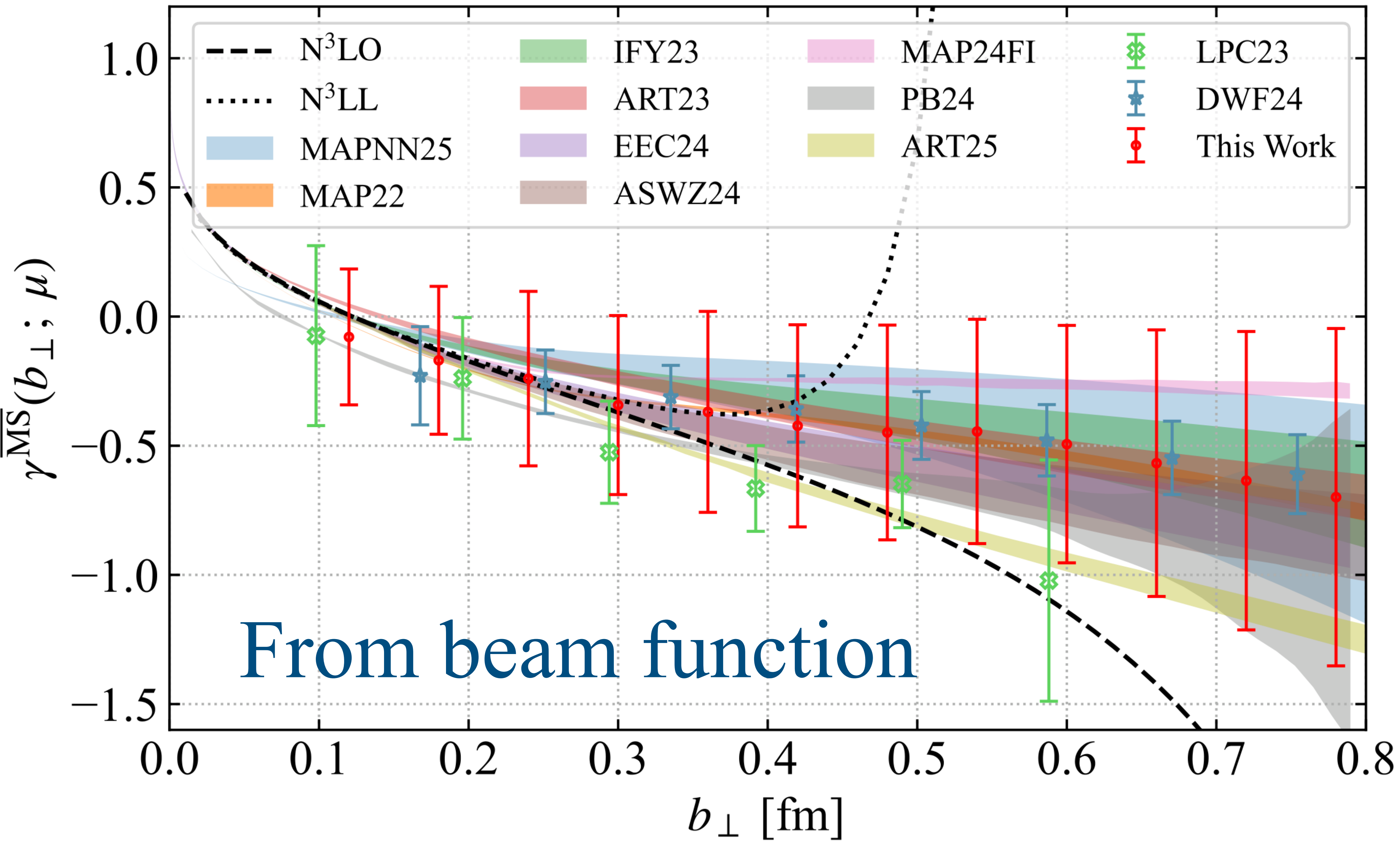


CS kernel extracted at different $b_{\mathrm{pert},1}$, $b_{\mathrm{pert},2}$ within matching window

Our approach:

Preliminary numerical results





Our preliminary result (systematics still overestimated)

Recent results from other groups (figure from arXiv: 2504.04625)

Note: argument for the behaviour $\gamma_q(b, \mu)$ at large b by Collins and Rogers, PRD **91** (2015) 074020

The Hamiltonian formalism

The Hamiltonian formalism: the 1970's

PHYSICAL REVIEW D VOLUME 19, NUMBER 2 15 JANUARY 1979

Quantum electrodynamics on a lattice: A Hamiltonian variational approach to the physics of the weak-coupling region

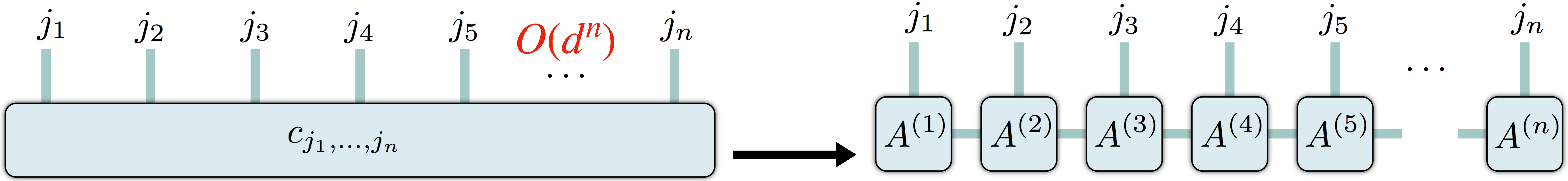
Sidney D. Drell, Helen R. Quinn, Benjamin Svetitsky, and Marvin Weinstein
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 9 June 1978)

We develop and apply a Hamiltonian variational approach to the study of quantum electrodynamics formulated on a spatial lattice in both $2+1$ and $3+1$ dimensions. Two lattice versions of QED are considered: a noncompact version which reproduces the physics of continuum QED, and a compact version constructed in correspondence with lattice formulations of non-Abelian theories. Our focus is on photon dynamics with charged particles treated in the static limit. We are especially interested in the nonperturbative aspects of the solutions in the weak-coupling region in order to clarify and establish aspects of confinement. In particular we find, in accord with Polyakov, that the compact QED leads to linear confinement for any nonvanishing coupling, no matter how small, in $2+1$ dimensions, but that a phase transition to an unconfined phase for sufficiently weak couplings occurs in $3+1$ dimensions. We identify and describe the causes of confinement.

Modern Hamiltonian formalism: tensor networks

Consider a 1-d system of n sites, with local Hilbert space of d dimensions on each site

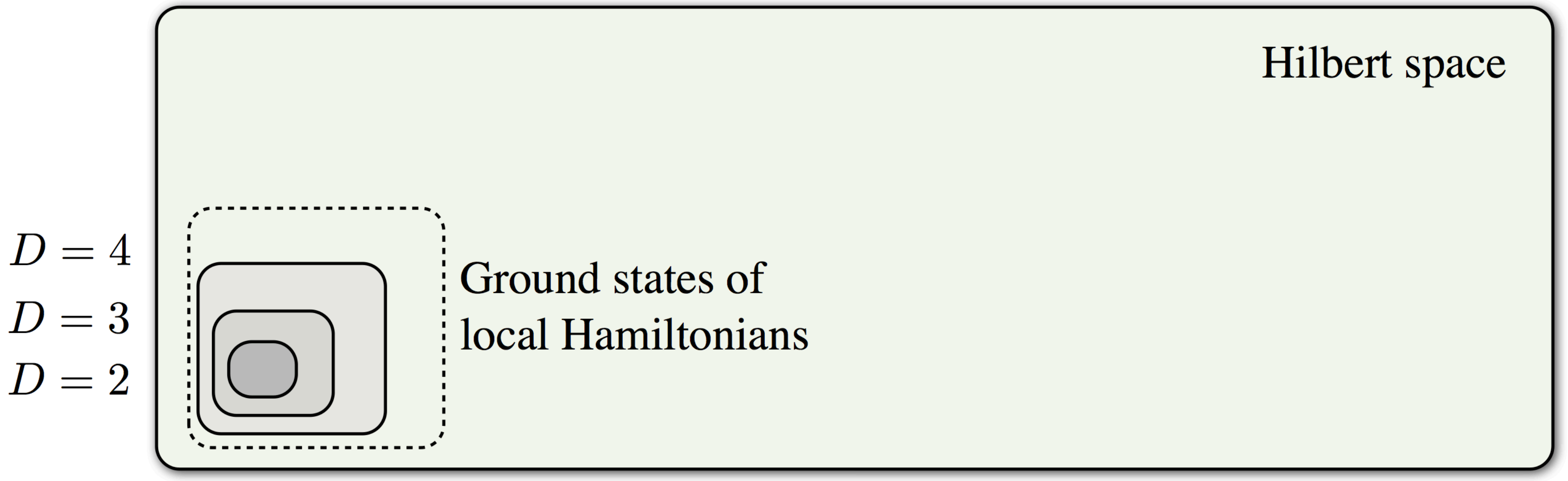
$$|\psi\rangle = \sum_{j_1,\ldots,j_n=1}^{d} c_{j_1,\ldots,j_n} |j_1,\ldots,j_n\rangle = \sum_{j_1,\ldots,j_n=1}^{d} c_{j_1,\ldots,j_n} |j_1\rangle \otimes \cdots \otimes |j_n\rangle$$



Matrix product state

$O(ndD^2)$

D is the "bond dimension"



The collaboration

Mari Carmen Banuls
(MPQ Munich)

Krzysztof Cichy
(Adam Mikiewicz U., Poznań)

CJDL
(NYCU)

Manuel Schneider
(NYCU)

arXiv: 2409.16996 [hep-lat] (Lattice 2024 proc.)

arXiv: 2504.07508 [hep-lat] (under review)

& work in progress

The massive Schwinger model
Gauss's law and temporal gauge

- QED in 1+1 dimensions, $\mathscr{L} = \bar{\psi}[\gamma_\mu(i\partial^\mu - gA^\mu) - m]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu$

- $A_\mu = (\rho, 0) \Rightarrow \mathscr{H}_{\text{Sch}} = -i\bar{\psi}\gamma^1(\partial_1 + igA_1)\psi + m\bar{\psi}\psi + \frac{1}{2}E^2 + (\partial_1 A_0)E + A_0(g\psi^\dagger\psi + \rho)$

  $\Rightarrow$ constraint $\partial_1 E = g\psi^\dagger\psi + \rho$ (Gauss's law)

- Work in temporal gauge, $A_0 = 0$, with Gauss's law $\Rightarrow$ remove gauge field d.o.f.

The massive Schwinger model
Canonical quantisation: gauge link and Gauss's law

- Canonical quantisation ($L_m = E(na)/g$) $\Rightarrow \left[\theta_n, L_m\right] = i\delta_{n,m} \Rightarrow \mathrm{e}^{-i\theta_n} L_n \,\mathrm{e}^{i\theta_n} = L_n + 1$

$$\text{If } L_n |l\rangle = l|l\rangle \Rightarrow \mathrm{e}^{\pm i\theta_n}|l\rangle = |l \pm 1\rangle$$

- Gauss's law $\partial_1 E = g\psi^\dagger\psi + \rho$

$$\Rightarrow \text{Lattice version } L_n - L_{n-1} = Q_n \equiv \phi_n^\dagger\phi_n - \frac{1}{2}[1-(-1)^n] + q_n$$

Fermion

Vacuum

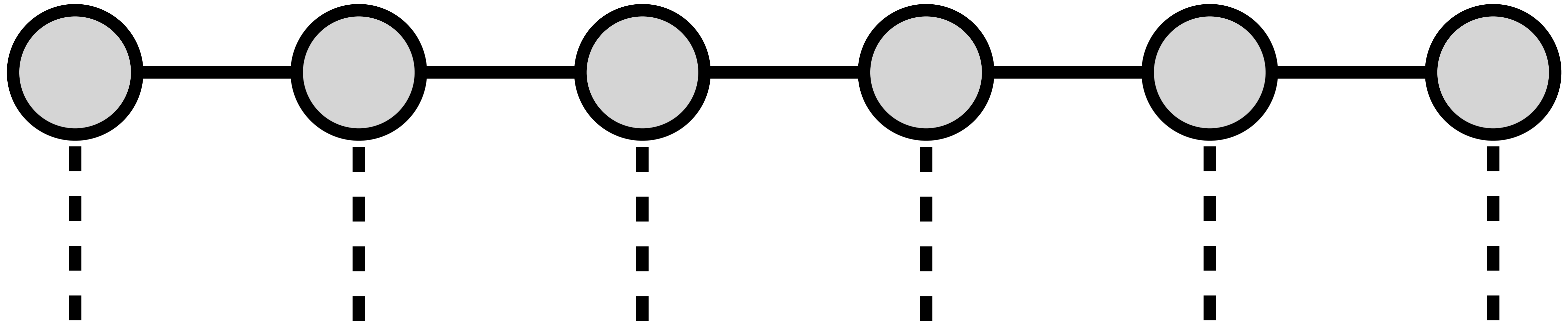
Static charge at site n

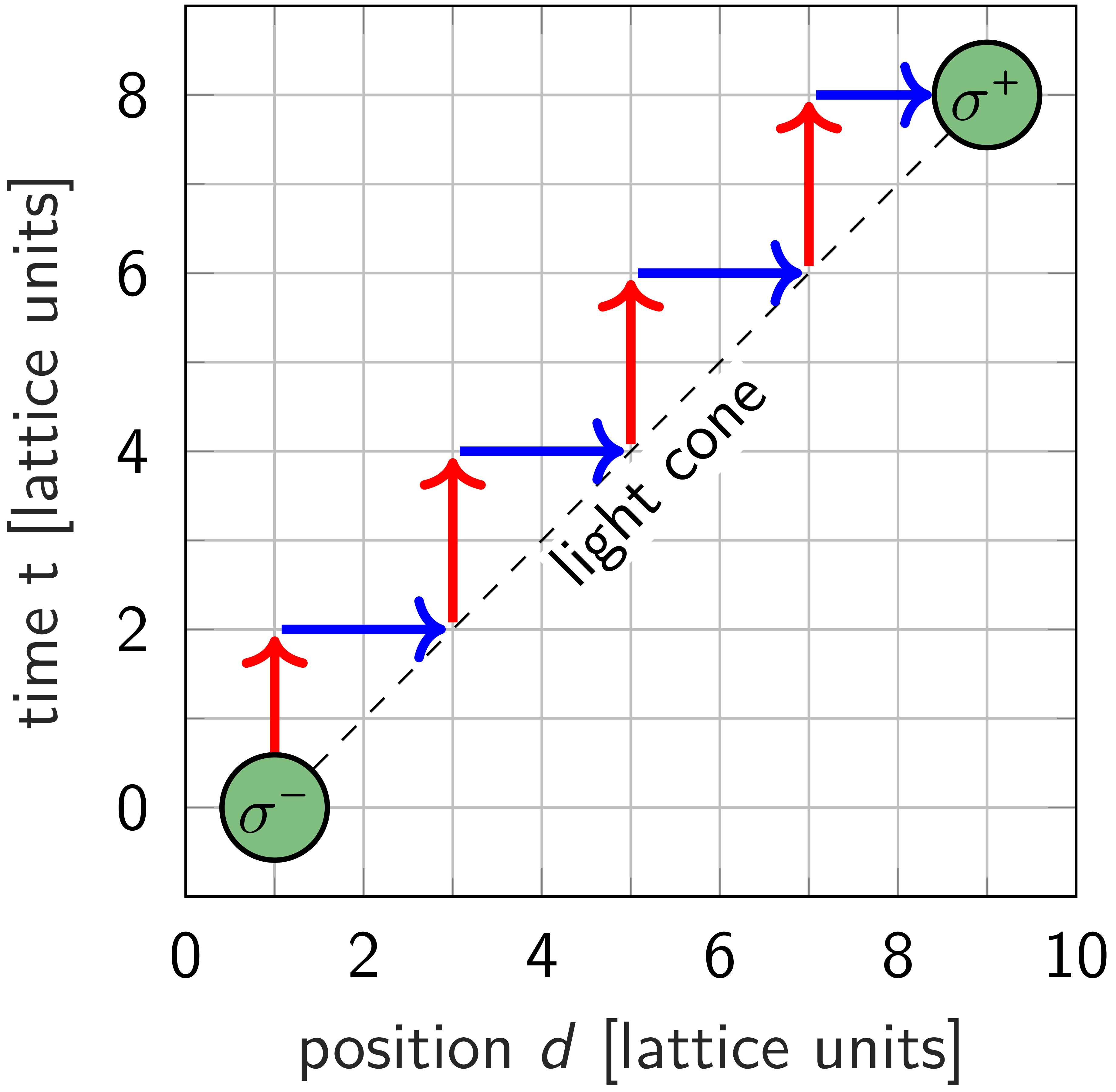
- Then use the Jordan-Wigner transformation to turn it into a spin model

Zigzagging the Wilson line along the light cone

- Temporal direction: $A_0 = 0$, Heisenberg pic
  $\Rightarrow$ Time evolution of the hadronic state

- Spatial direction: $e^{iA_1(na)} = e^{i\theta_n}$ changing $\mathbf{E}$
  $\Rightarrow$ Moving static charge (Gauss' law constraint)





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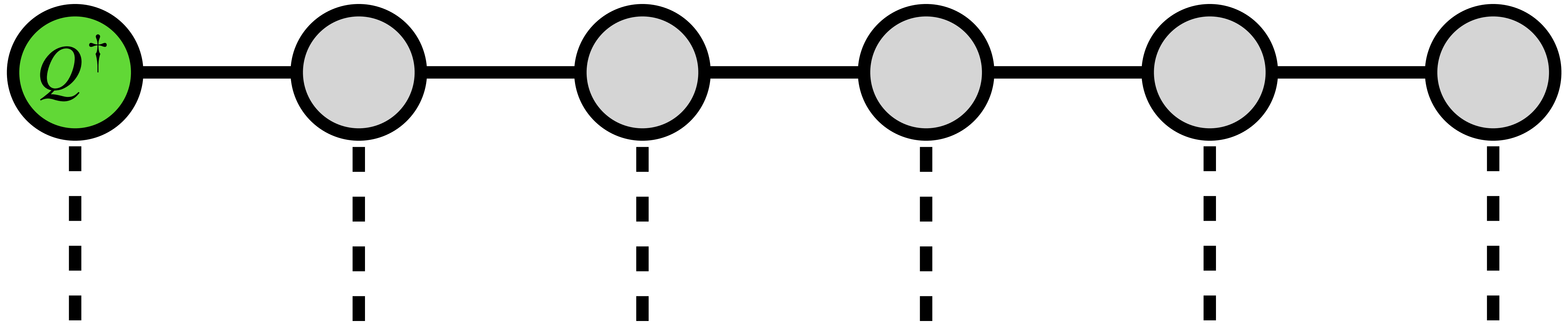

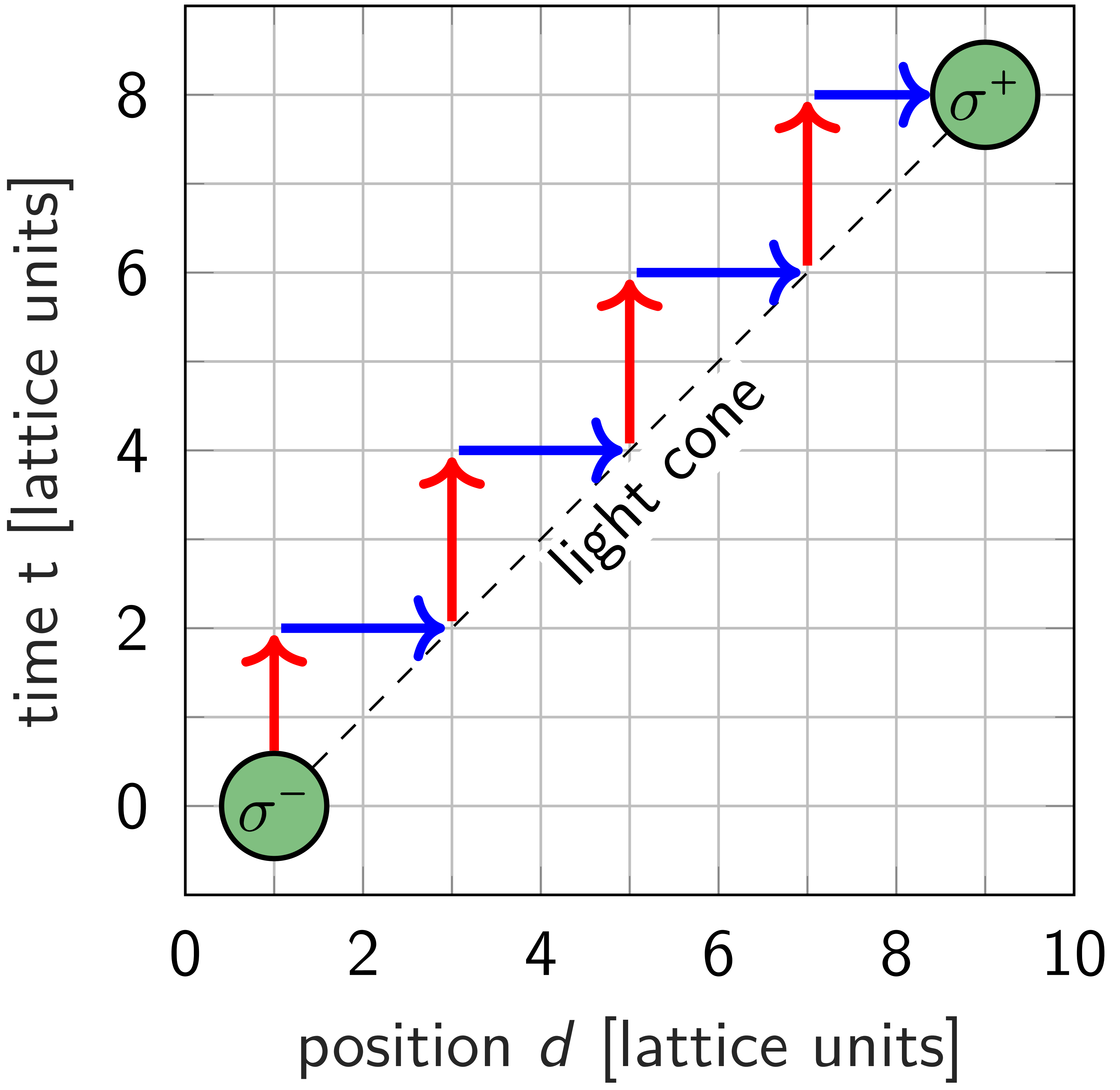

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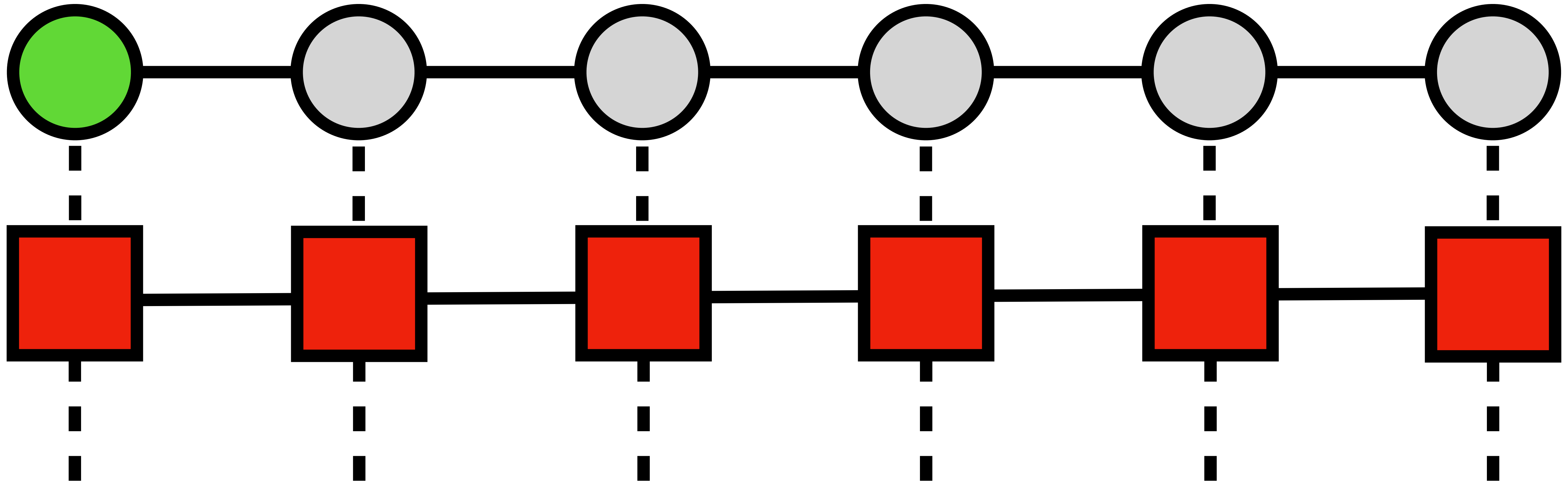
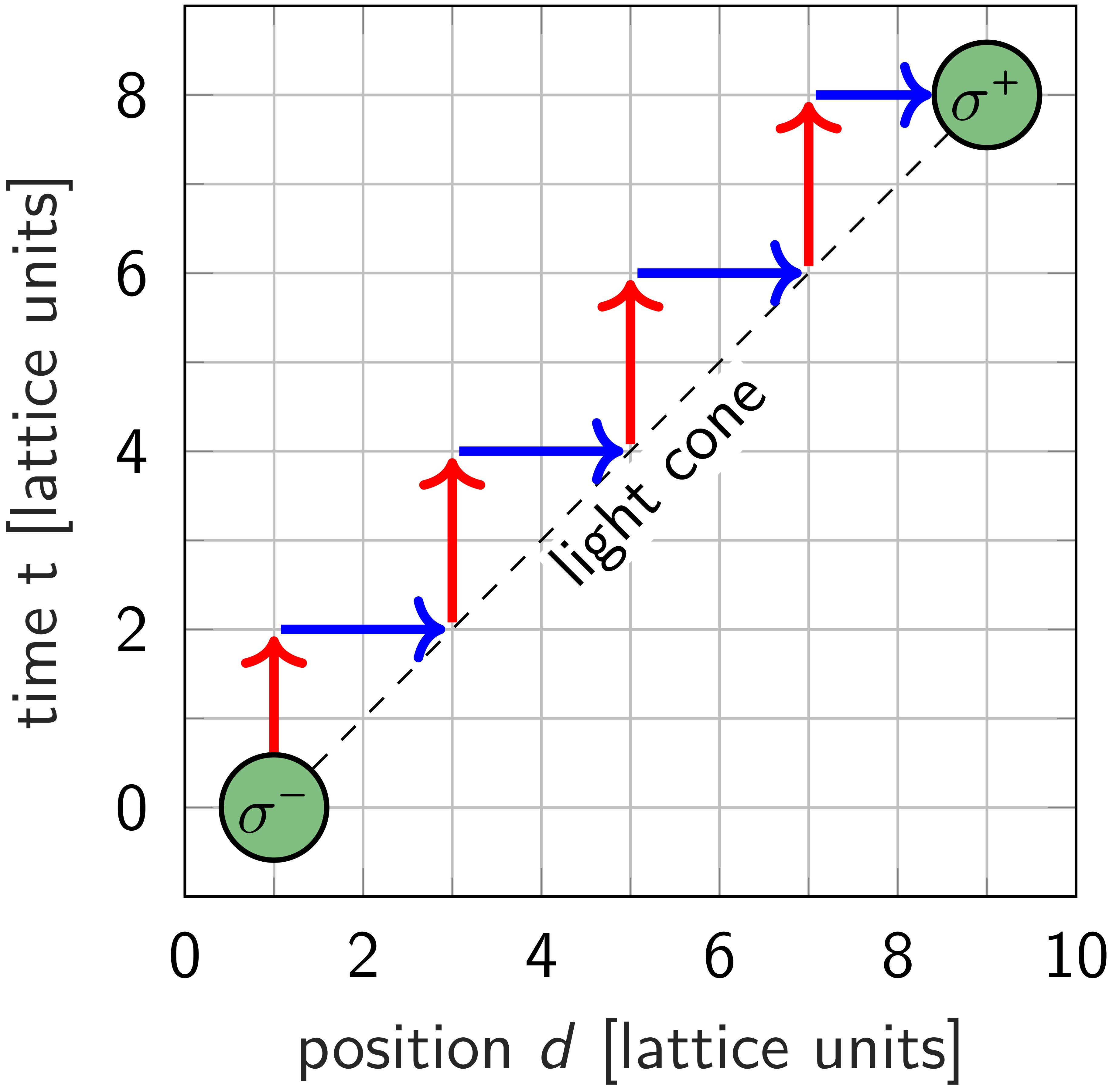




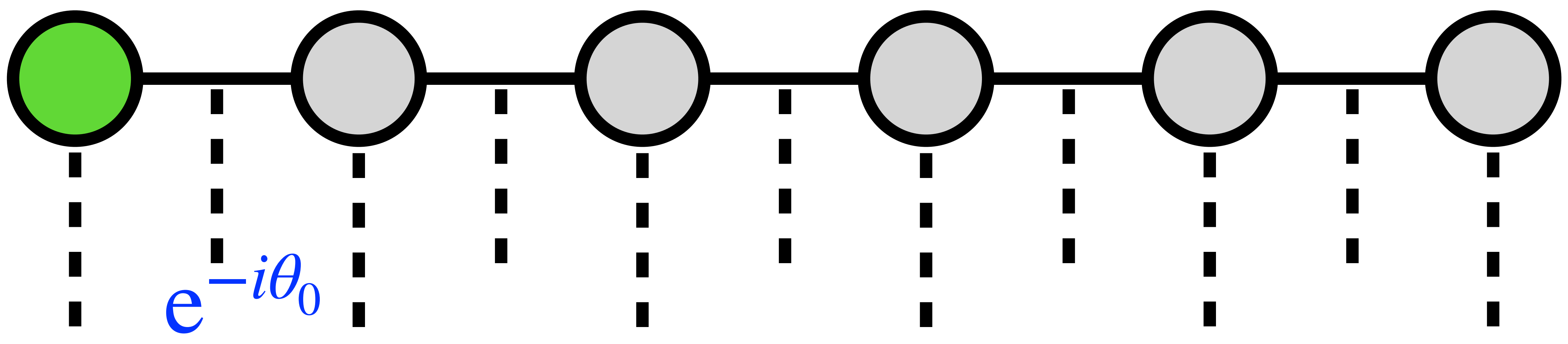
Zigzagging the Wilson line along the light cone

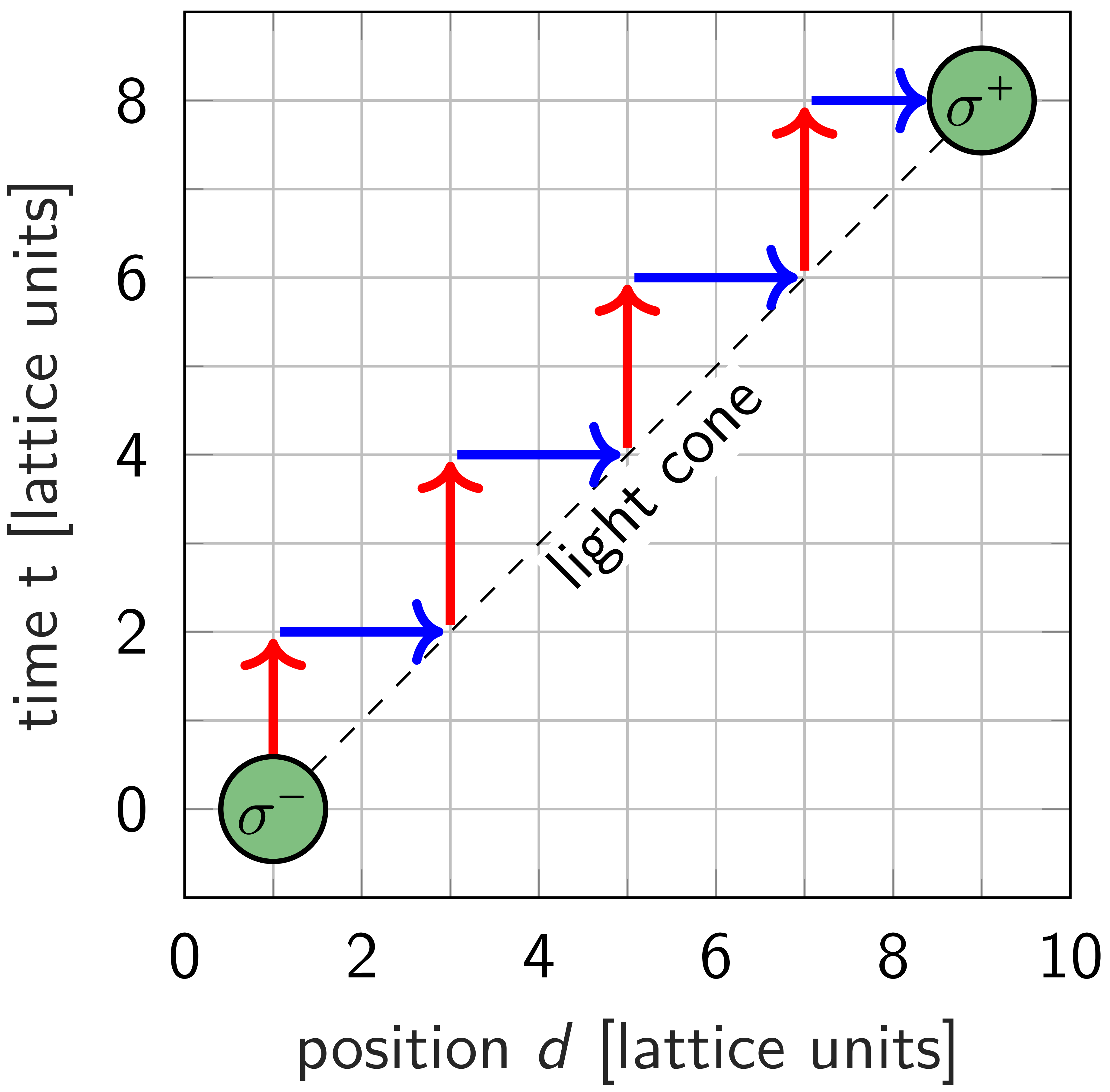
- Temporal direction: $A_0 = 0$, Heisenberg pic

  $\Rightarrow$ Time evolution of the hadronic state

- Spatial direction: $e^{iA_1(na)} = e^{i\theta_n}$ changing $\mathbf{E}$

  $\Rightarrow$ Moving static charge (Gauss' law constraint)





Zigzagging the Wilson line along the light cone

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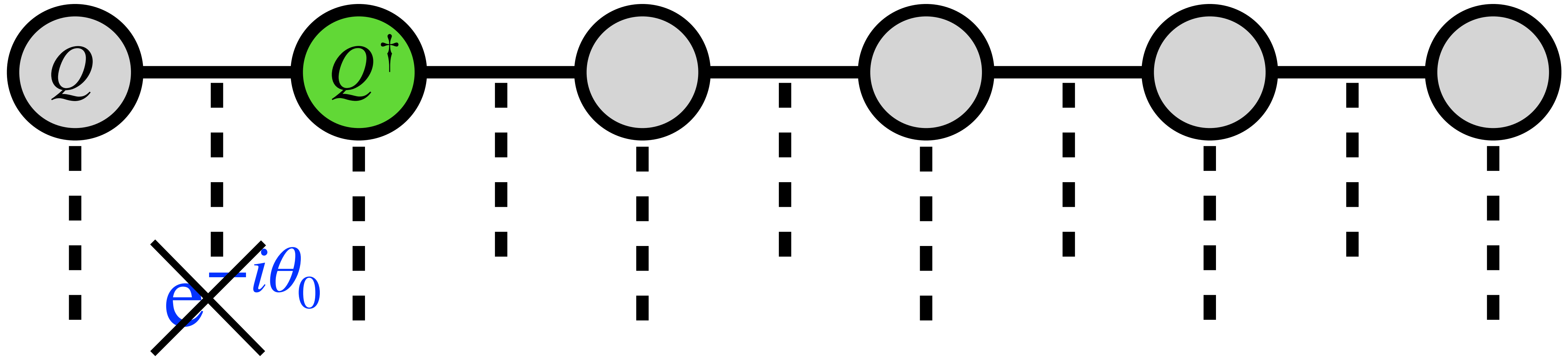

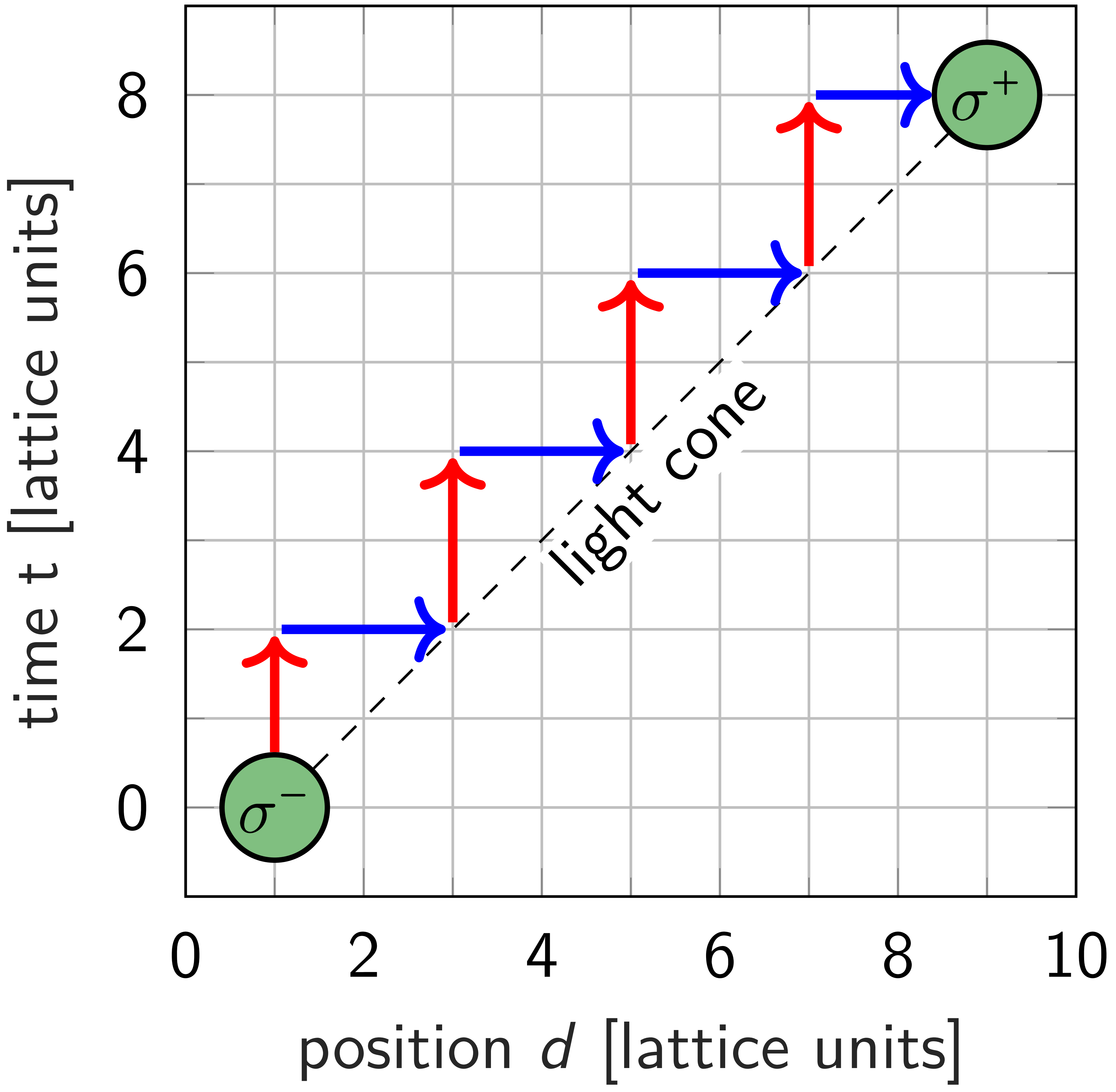

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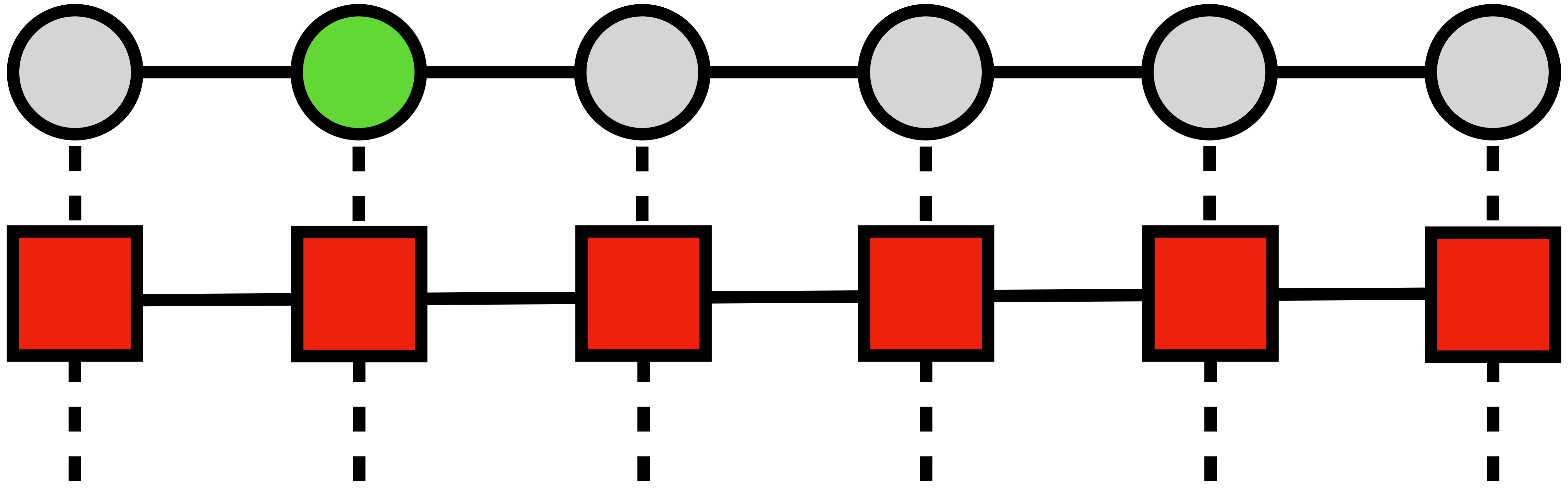

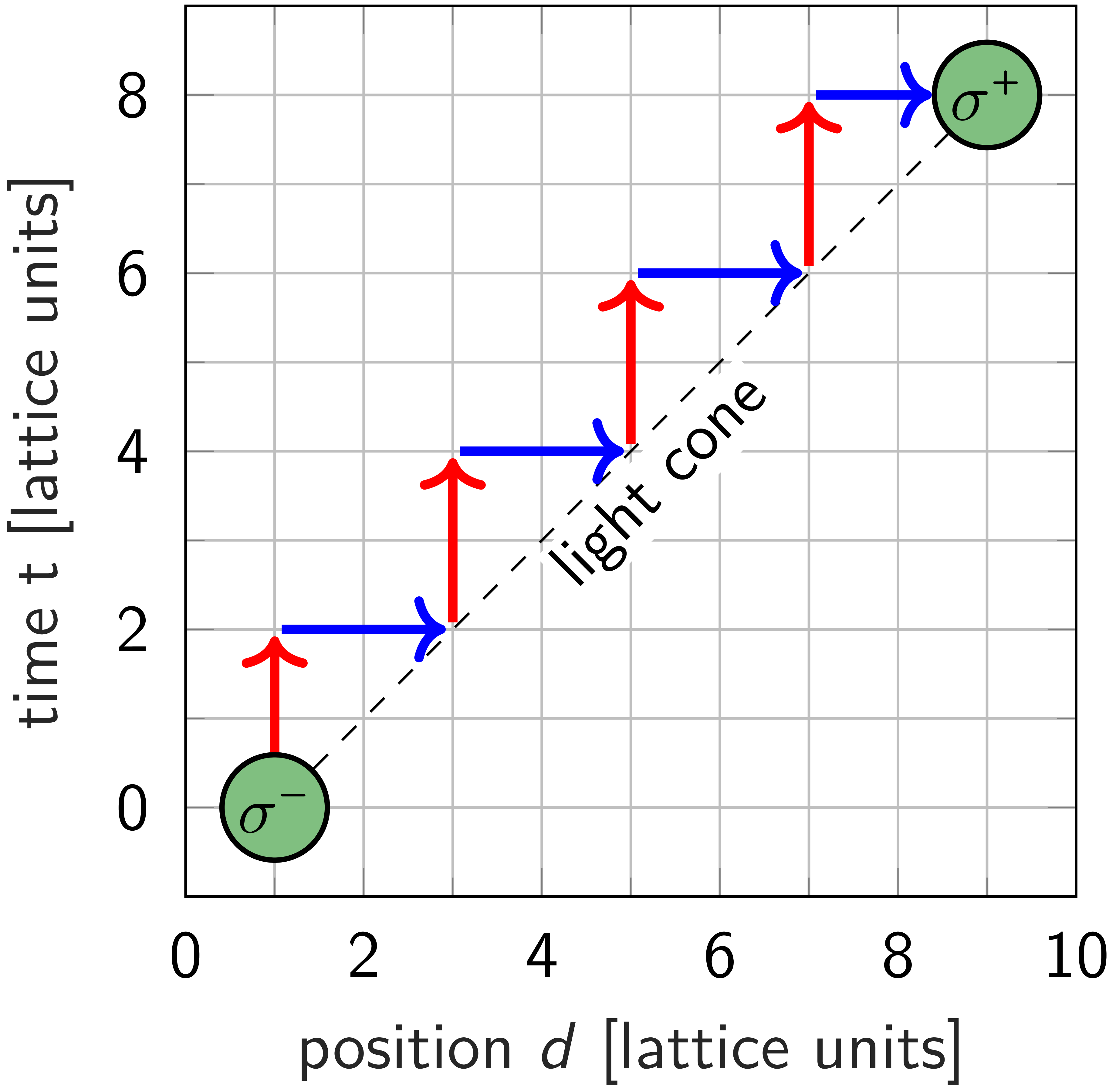

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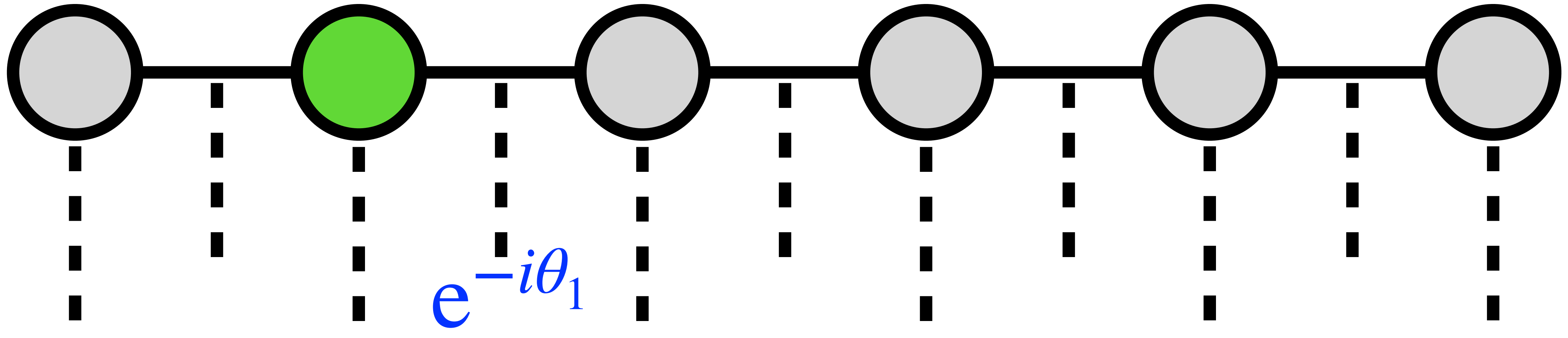

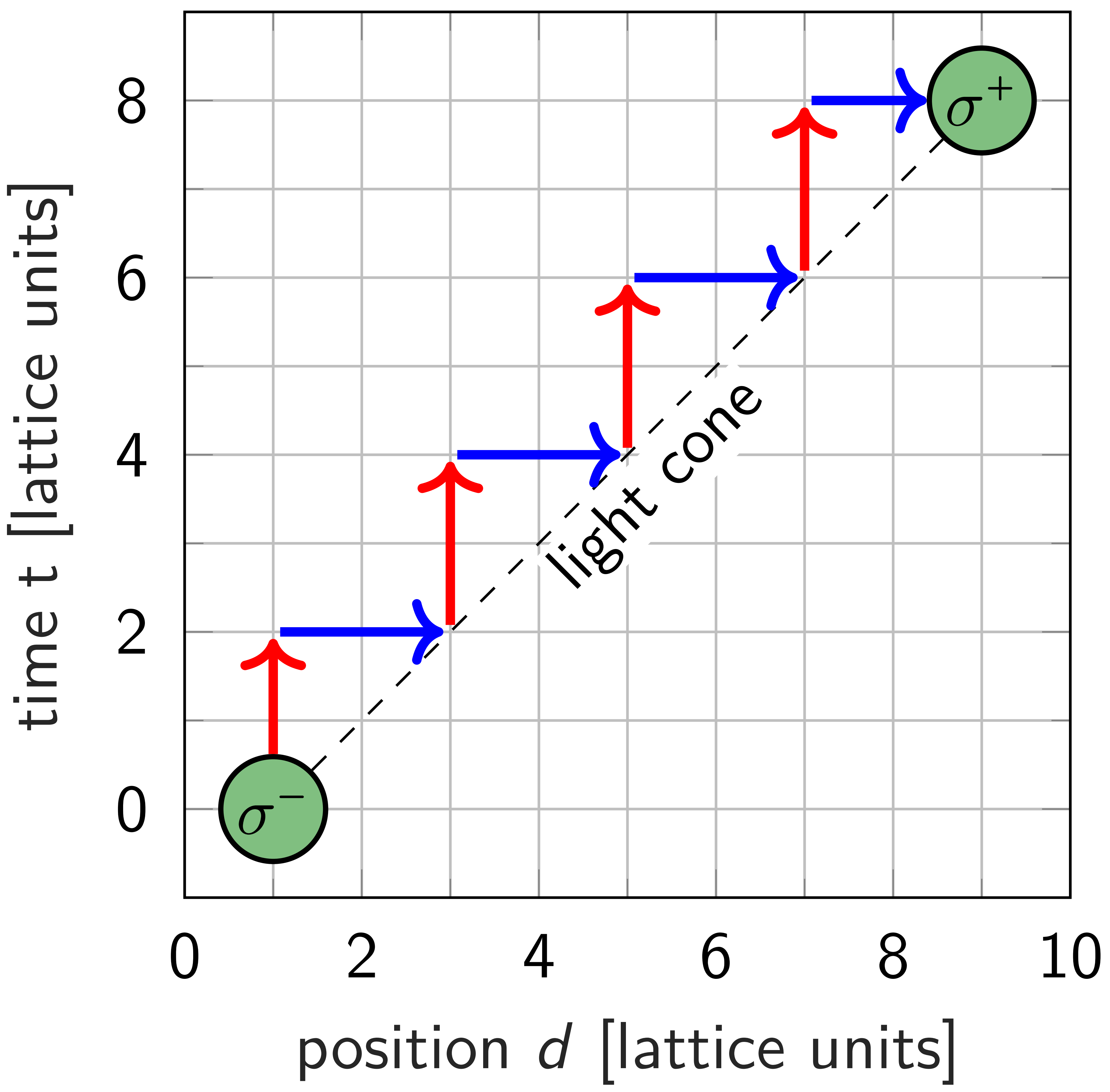

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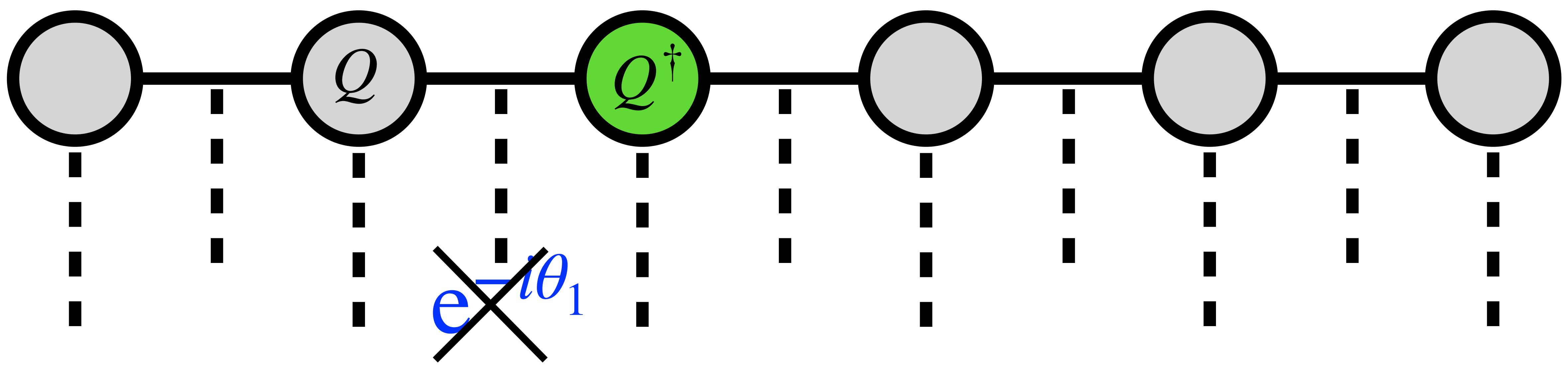

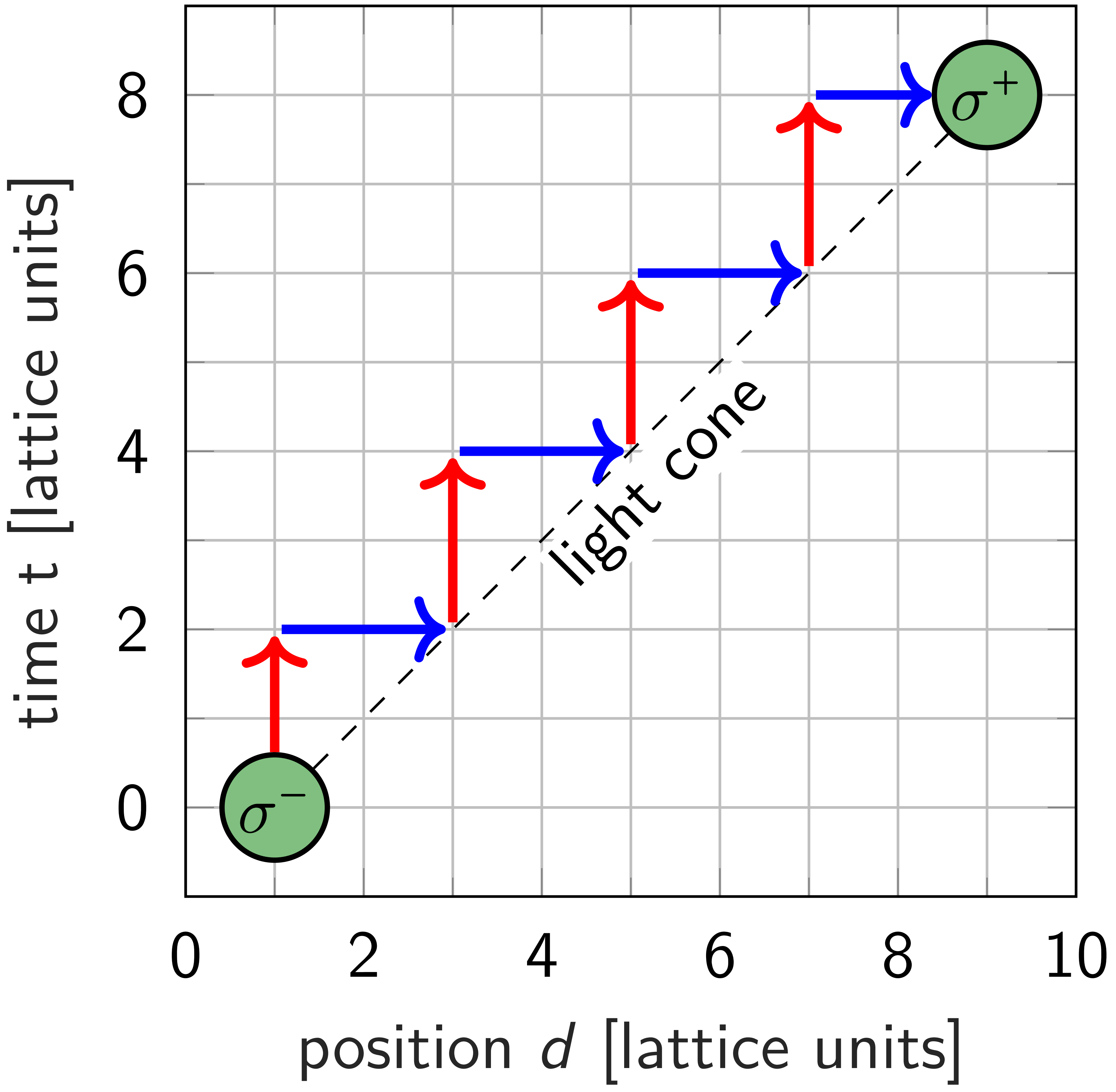

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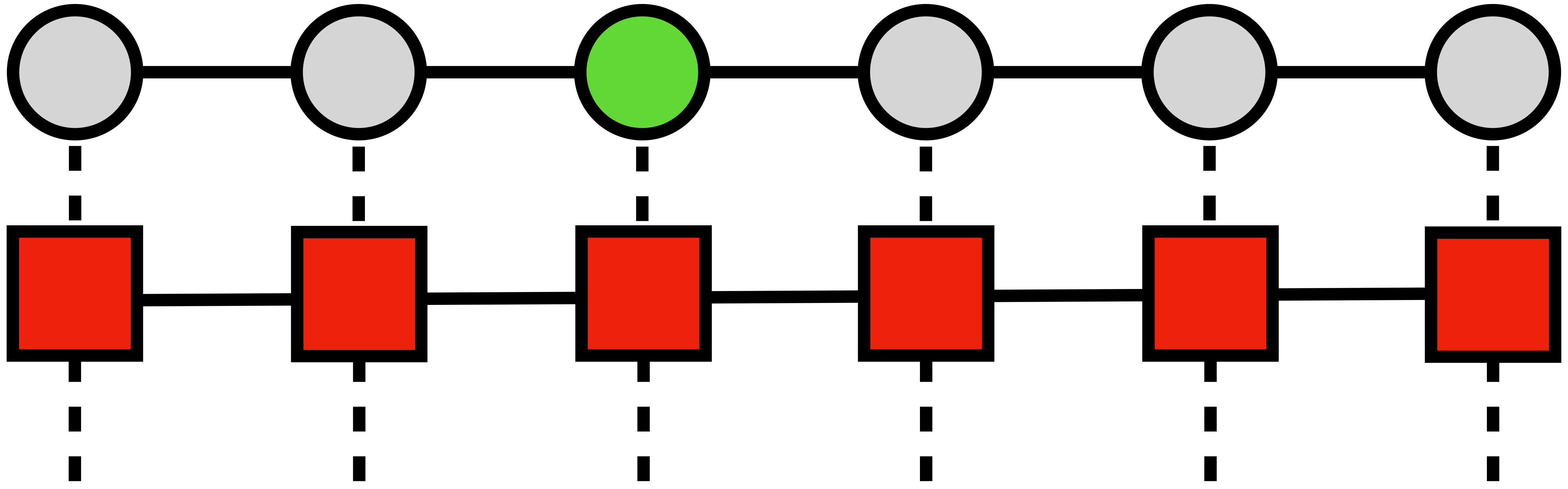
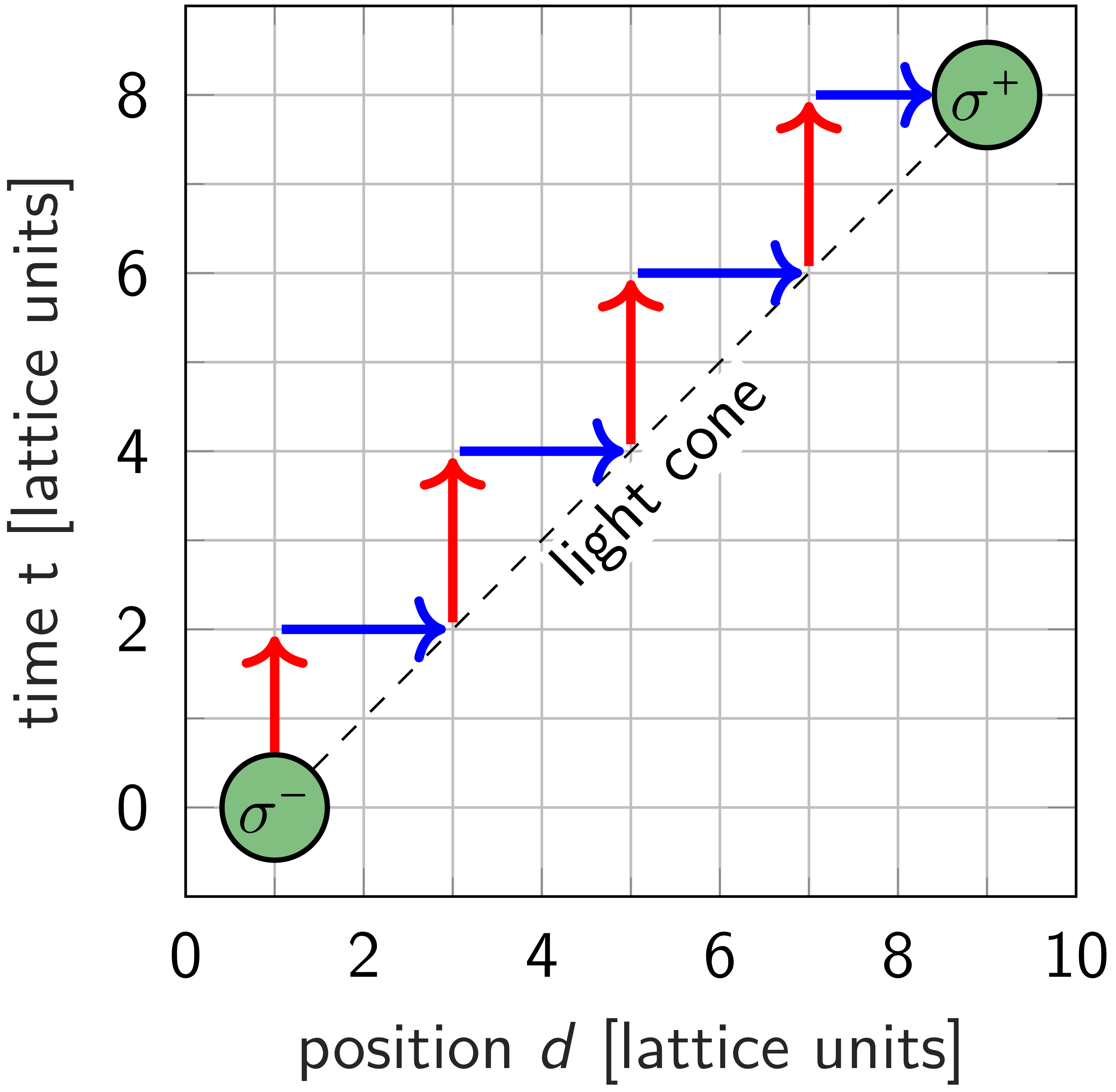
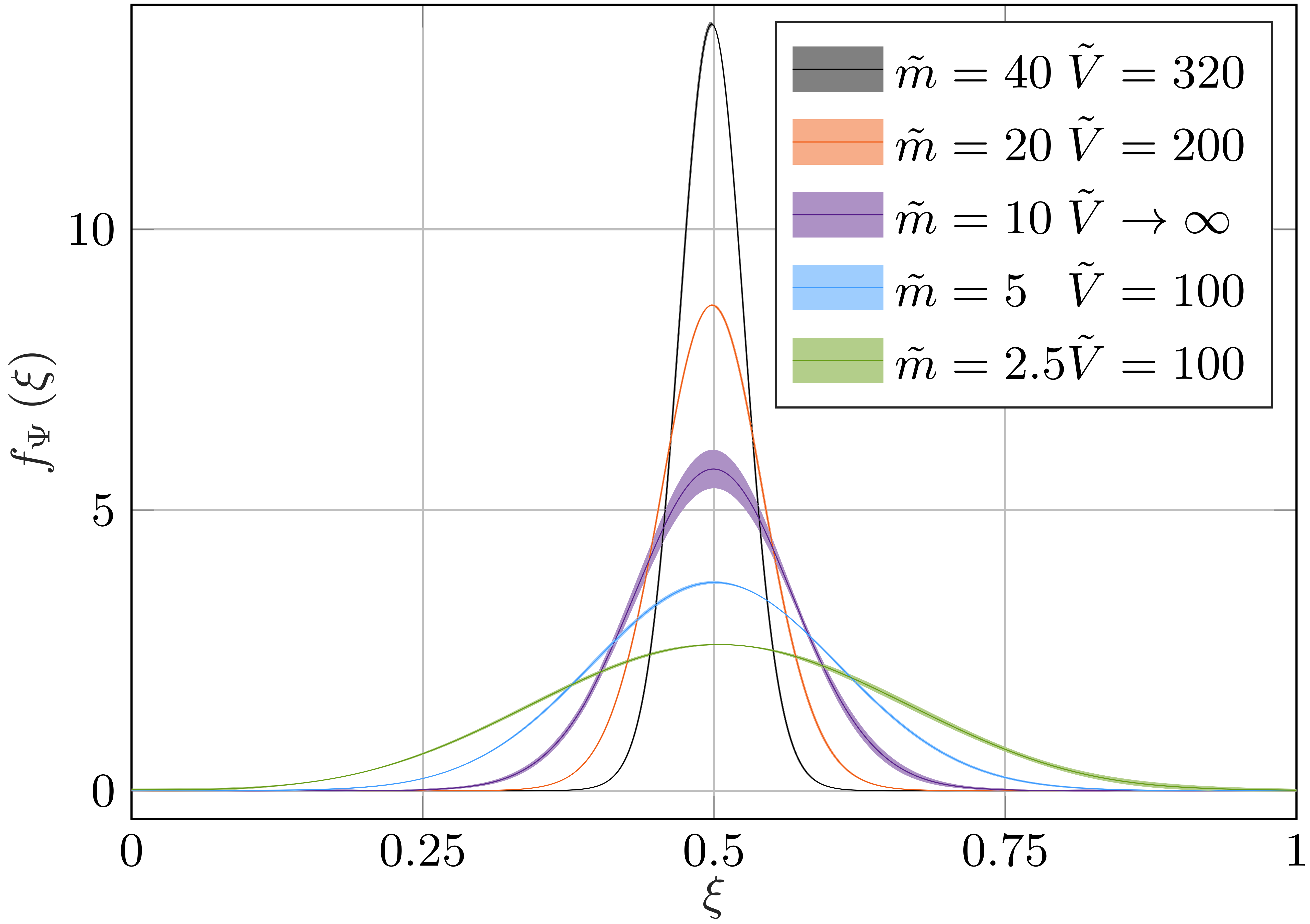
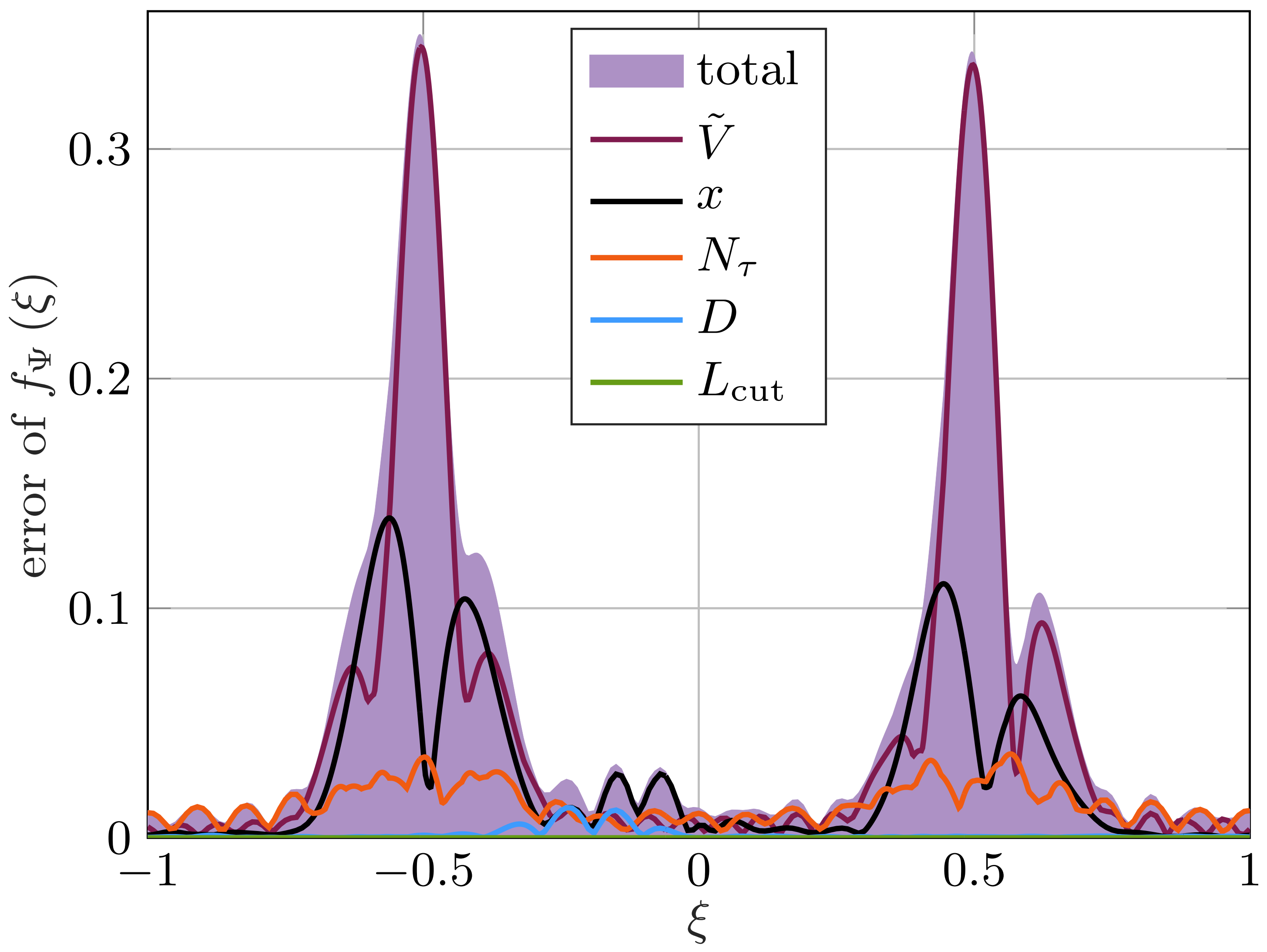




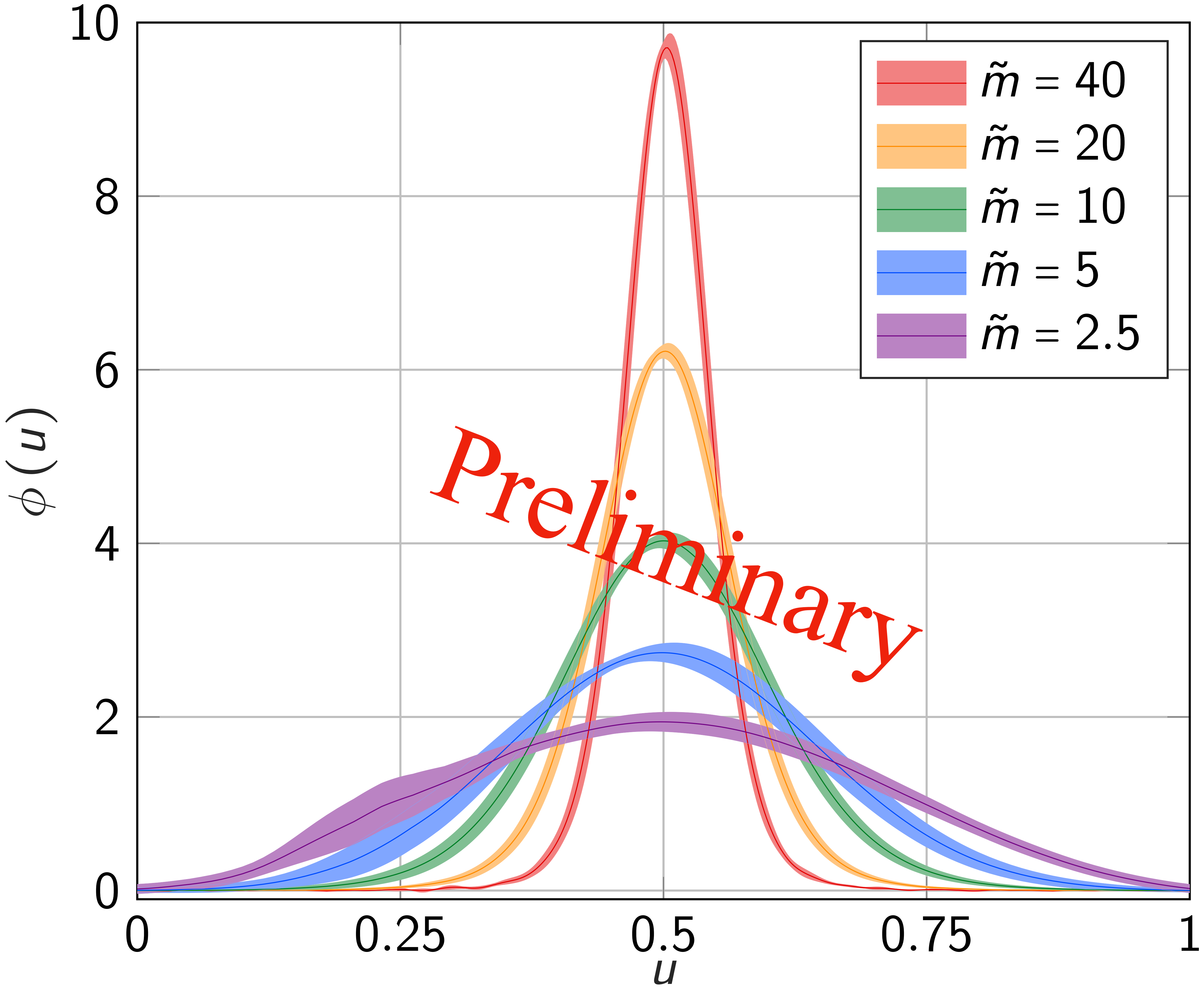
Vector meson PDF



- Results obtained at $a \to 0$
- All systematic errors under control



Meson LCDA

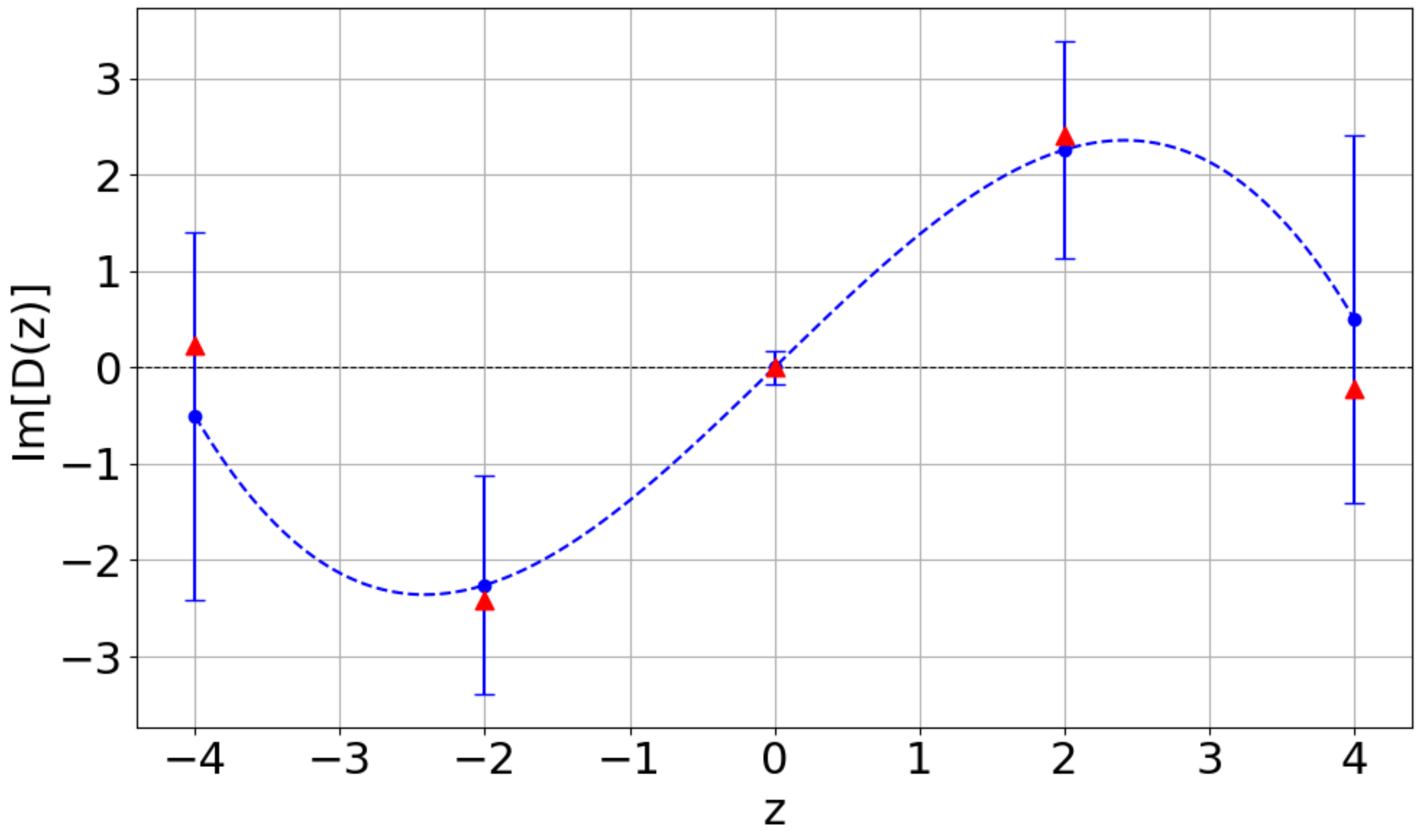


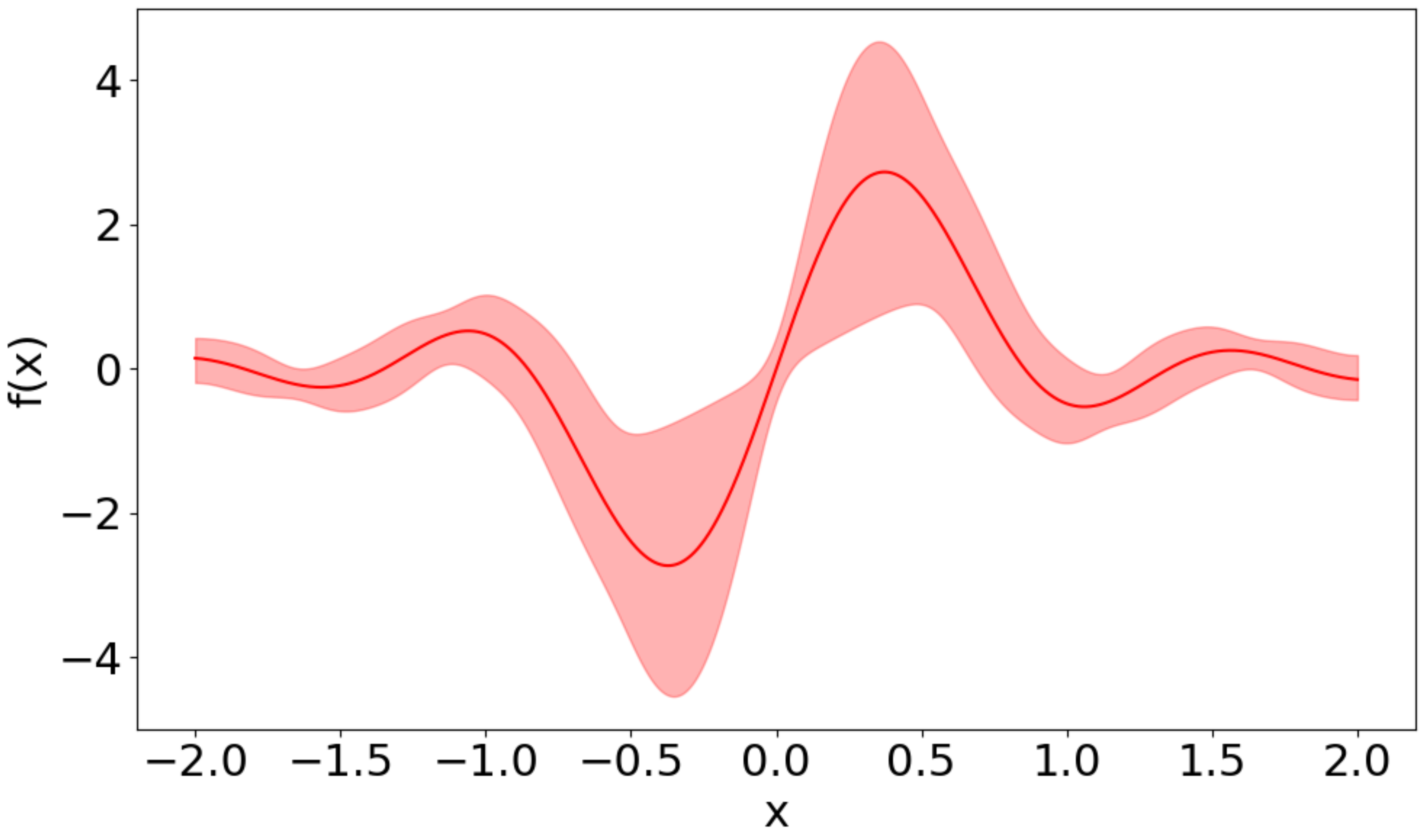
- Results obtained at $a \to 0\,,\ N \to \infty$

Implementation of our method on a quantum computer

Our approach is applicable for quantum simulations

J.-W. Chen, Y.-T. Chen, Meher, arXiv:2506.16829 [hep-lat]





But the error is still large (~100% !), as expected

Concluding remarks

- A small but active lattice-PDF community in Taiwan
- We have our own original ideas and will implement these ideas
- Projects highly relevant to the EIC era