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Three-nucleon force and continuum effect in nuclei around driplines

Furong Xu

I. Chiral three-nucleon force (3NF)

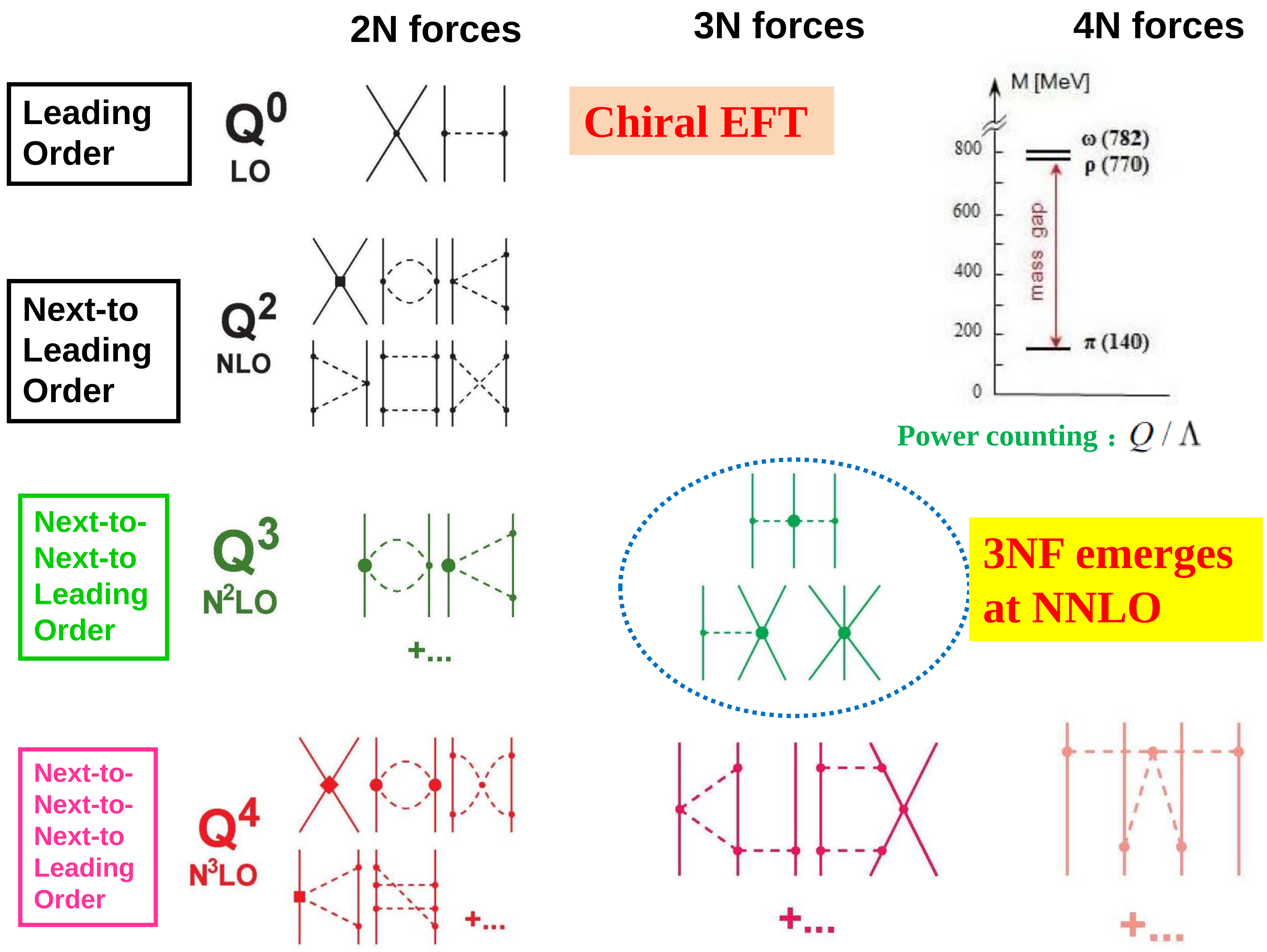
II. The coupling to the continuum

III. Combined effects of 3NF and continuum coupling in dripline regions

seen through *ab initio* many-body Gamow shell-model calculations

IV. Conclusions

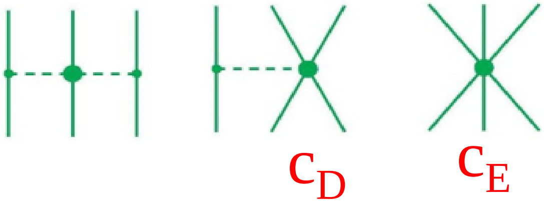
2025 ANPhA Symposium (Taipei, Nov. 27-29)


2N forces
3N forces
4N forces
Leading Order
Q0
LO
Chiral EFT
M [MeV]
800
600
400
200
0
ω (782)
ρ (770)
mass gap
π (140)
Next-to Leading Order
Q2
NLO
Power counting : Q / Λ
Next-to-Next-to Leading Order
Q3
N2LO
+...
3NF emerges at NNLO
Next-to-Next-to-Next-to Leading Order
Q4
N3LO
+...
+...
+...

A-body intrinsic Hamiltonian

$$\hat{H} = \frac{1}{A}\sum_{i<j}^{A}\frac{(\boldsymbol{p}_i - \boldsymbol{p}_j)^2}{2m} + \sum_{i<j}^{A}\hat{V}_{\mathrm{NN}}^{ij} + \sum_{i<j<k}^{A}\hat{V}_{\mathrm{3N}}^{ijk}$$

$$\hat{V}_{\mathrm{3N}} = \hat{V}_{\mathrm{3N}}^{2\pi} + \hat{V}_{\mathrm{3N}}^{1\pi} + \hat{V}_{\mathrm{3N}}^{\mathrm{ct}}$$



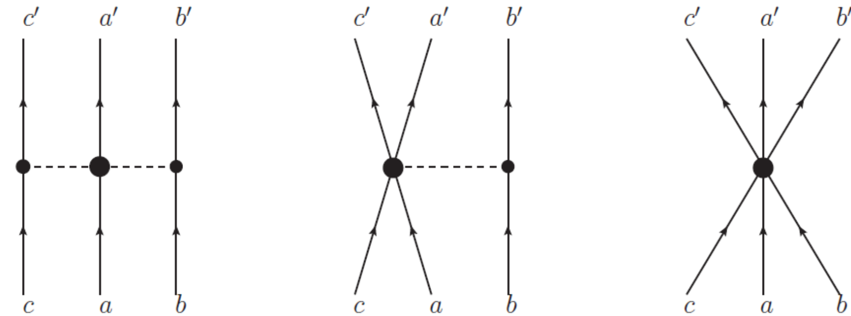
How to optimize 3NF LECs c_D and c_E

using ^{3}H energy and ^{4}He radius, but still many discussions remain.

For many-body calculations, normal-ordered approximation by neglecting residual 3NF

$$\hat{H} = E_0 + \sum_{pq}[t_{pq} + \sum_{r=1}^{A}(V_{prqr}^{\mathrm{NN}} + \frac{1}{2}W_{prqr}^{\mathrm{2B}})] : \hat{a}_p^{\dagger}\hat{a}_q :$$
$$+ \frac{1}{4}\sum_{pqrs}(V_{pqrs}^{\mathrm{NN}} + W_{pqrs}^{\mathrm{2B}}) : \hat{a}_p^{\dagger}\hat{a}_q^{\dagger}\hat{a}_s\hat{a}_r :,$$

3NF



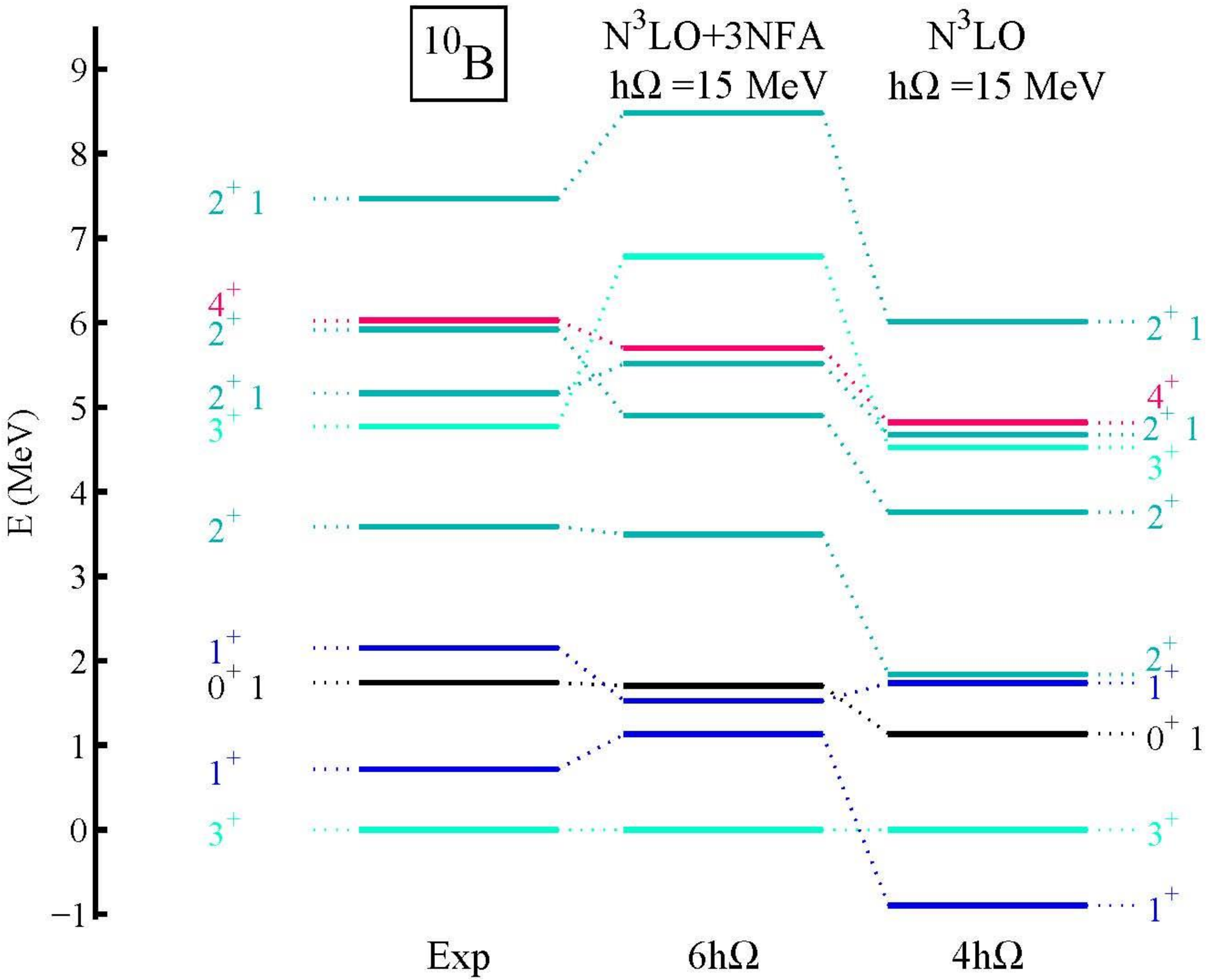
$$v_{3N}^{2\pi} = \frac{1}{(2\pi)^6}\frac{g_A^2}{8f_\pi^2}\sum_{i\neq j\neq k}\frac{(\boldsymbol{\sigma}_i\cdot\boldsymbol{q}_i)(\boldsymbol{\sigma}_j\cdot\boldsymbol{q}_j)}{(q_i^2+M_\pi^2)(q_j^2+M_\pi^2)}F_{ijk}^{\alpha\beta}\tau_i^\alpha\tau_j^\beta$$

$$v_{3N}^{1\pi} = -\frac{1}{(2\pi)^6}\frac{g_A c_D}{8f_\pi^4\Lambda_\chi}\sum_{i\neq j\neq k}\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{q}_j}{q_j^2+M_\pi^2}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(\boldsymbol{\sigma}_i\cdot\boldsymbol{q}_j)$$

$$v_{3N}^{cont} = \frac{1}{(2\pi)^6}\frac{c_E}{2f_\pi^4\Lambda_\chi}\sum_{j\neq k}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left[-\frac{4c_1m_\pi^2}{f_\pi^2}+\frac{2c_3}{f_\pi^2}\boldsymbol{q}_i\cdot\boldsymbol{q}_j\right] + \frac{c_4}{f_\pi^2}\varepsilon^{\alpha\beta\gamma}\tau_k^\gamma\boldsymbol{\sigma}_k\cdot[\boldsymbol{q}_i\times\boldsymbol{q}_j],$$

Calculating the properties of light nuclei using chiral 2N and 3N forces



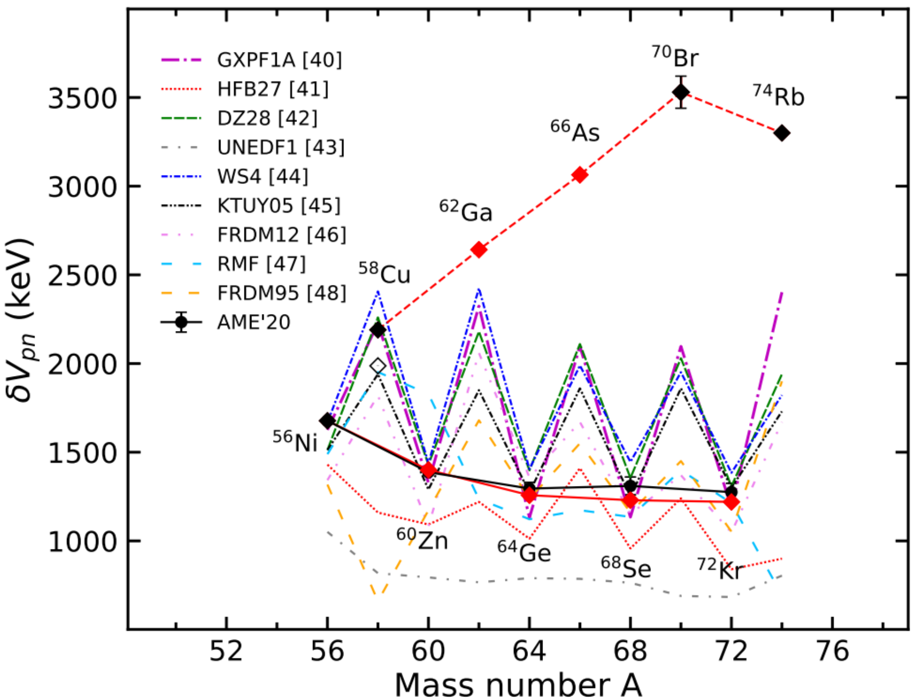
NCSM calculation by P. Navratil et al.

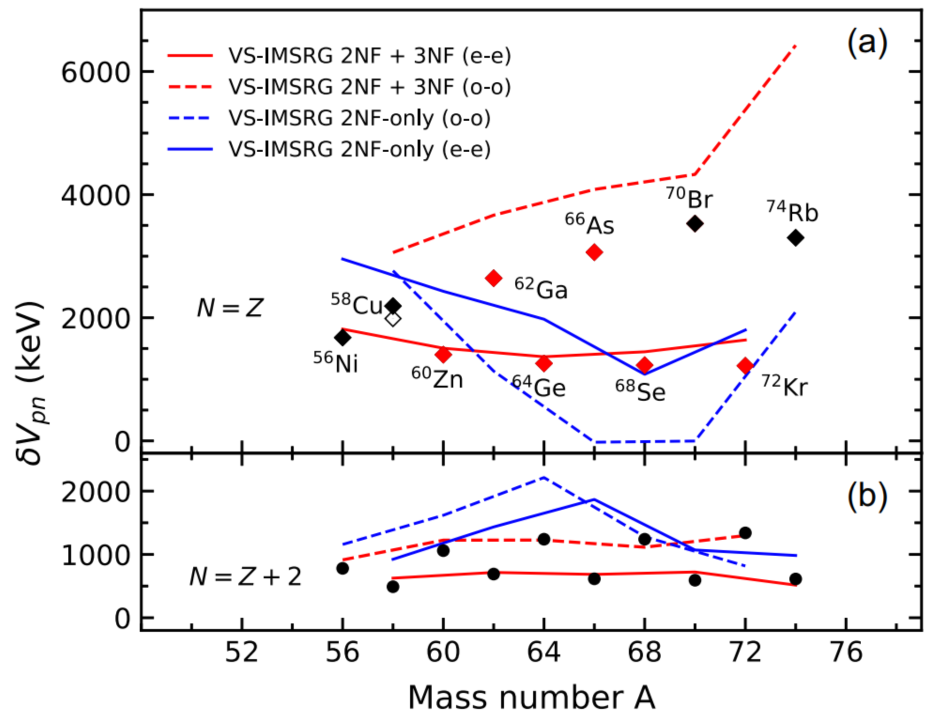
2N (N3LO) +3N (N2LO)

2N (N3LO) only

M. Wang, Y.H. Zhang,…, **FRX**,… PRL 130, 192501 (2023)

PHYSICAL REVIEW LETTERS **130,** 192501 (2023)

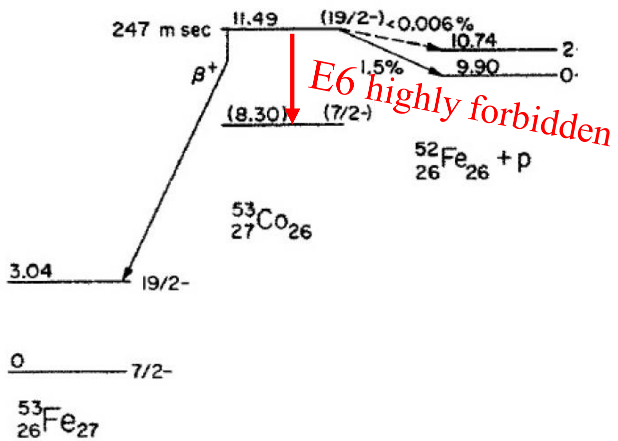


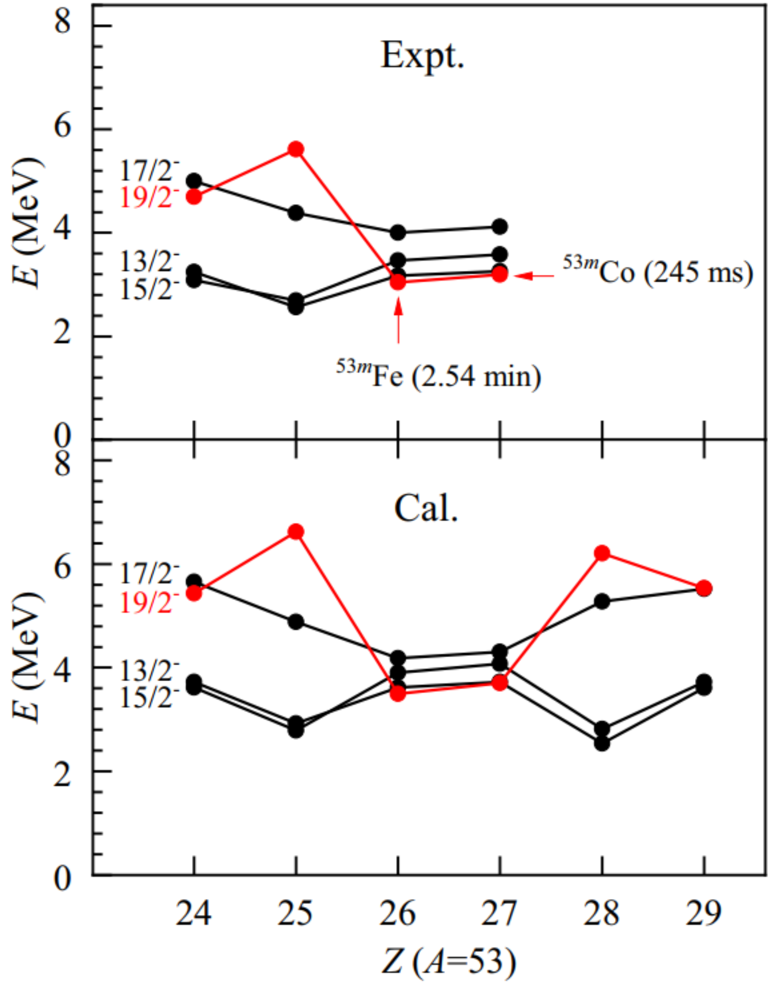


We find that 3NF enhances the *pn* pairing interaction which is strong in odd-odd nuclei.

3NF importance to explaining the isomerism in A=52 chain

1970 first proton decay from an isomer - in ^{53}Co





VS-IMSRG with chiral 2NF plus 3NF named EM1.8/2.0

Q. Yuan, B.S. Hu, X.Y. Xu, J.G. Li, **FRX**, and P.M. Walker, PRC 111, 034309 (2025)

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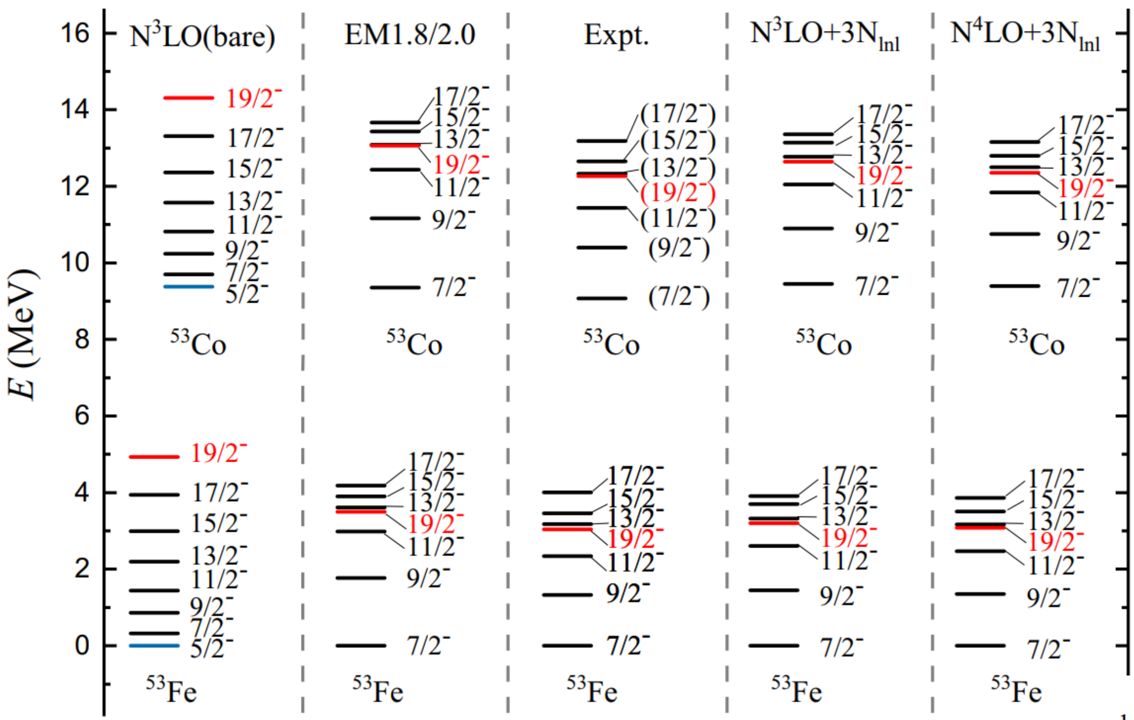


J = 19/2 is the maximum angular momentum obtained by coupling one proton $f_{7/2}$ hole and two $f_{7/2}$ neutron holes.

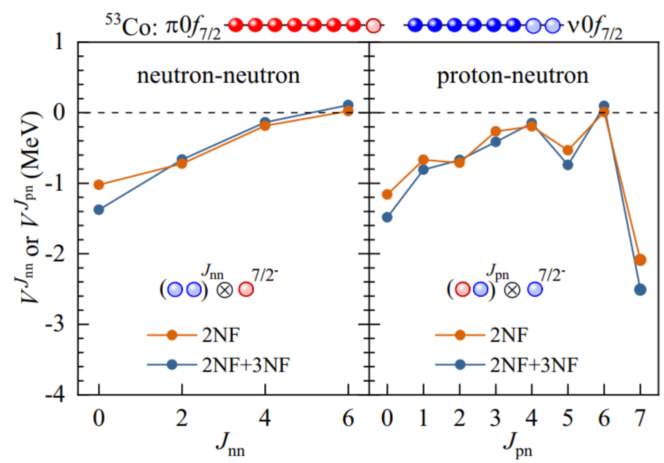
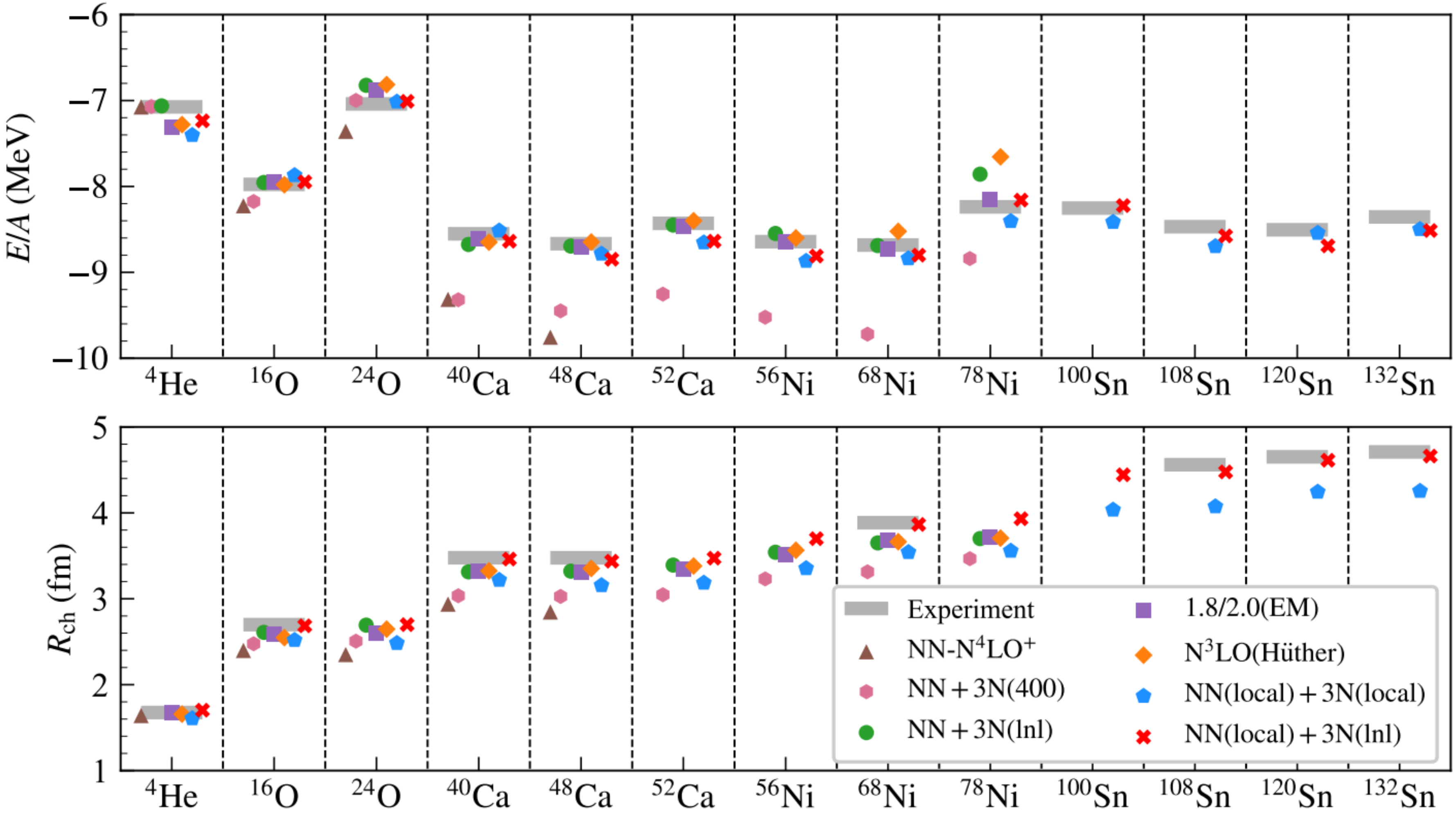


FIG. 4. Two-body interaction matrix elements between two $0f_{7/2}$ holes coupled to different angular momenta, with the interactions 2NF N^{3}LO [45] and 3NF-included EM1.8/2.0 [33,34]. A filled ball indicates a particle in the $0f_{7/2}$ shell (red for proton and blue for neutron), while a hollow faded ball indicates a $0f_{7/2}$ hole.

TABLE II. Fermi (F) and Gamow-Teller (GT) strengths of the β decays from ^{53m}Co ($19/2^-$) to ^{53m}Fe ($19/2^-$) or to ^{53}Fe ($17/2^-$), and the total β-decay half-life, calculated using EM1.8/2.0 [33,34], $N^3LO + 3N_{lnl}$ [43], and $N^4LO + 3N_{lnl}$ [44] interactions.

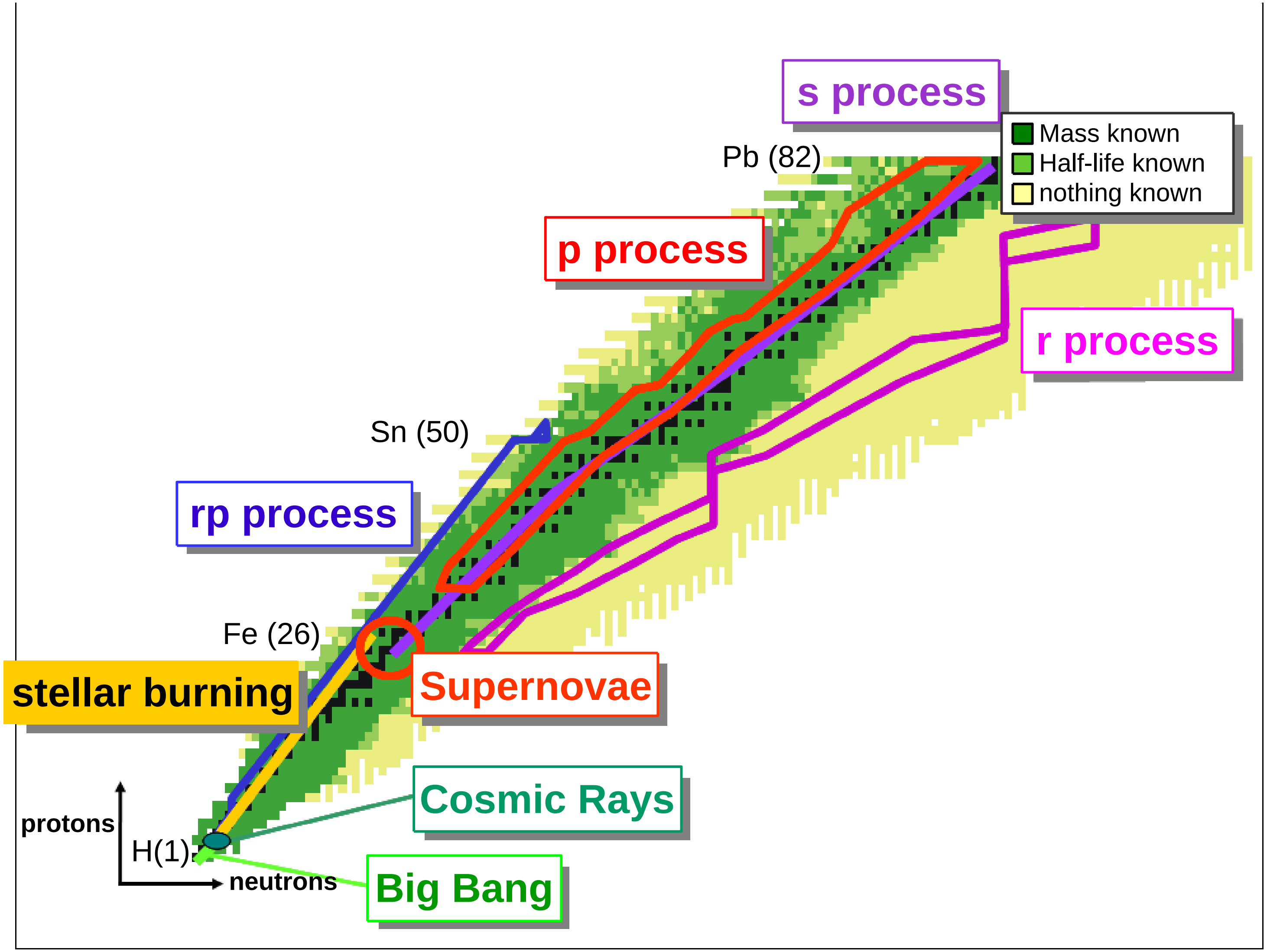
Interactions	B(F)	B(GT)		$t_{1/2}^{\mathrm{Total}}(\beta)$ (s)	
	$19/2^- \rightarrow 19/2^-$	$19/2^- \rightarrow 19/2^-$	$19/2^- \rightarrow 17/2^-$	Cal.	Expt. [5]
EM1.8/2.0	0.997	0.169	0.104	0.327	0.251(12)
$N^3LO + 3N_{lnl}$	1.001	0.144	0.076	0.340	0.251(12)
$N^4LO + 3N_{lnl}$	1.000	0.142	0.087	0.331	0.251(12)

Q. Yuan, B.S. Hu, X.Y. Xu, J.G. Li, **FRX**, and P.M. Walker, PRC 111, 034309 (2025)



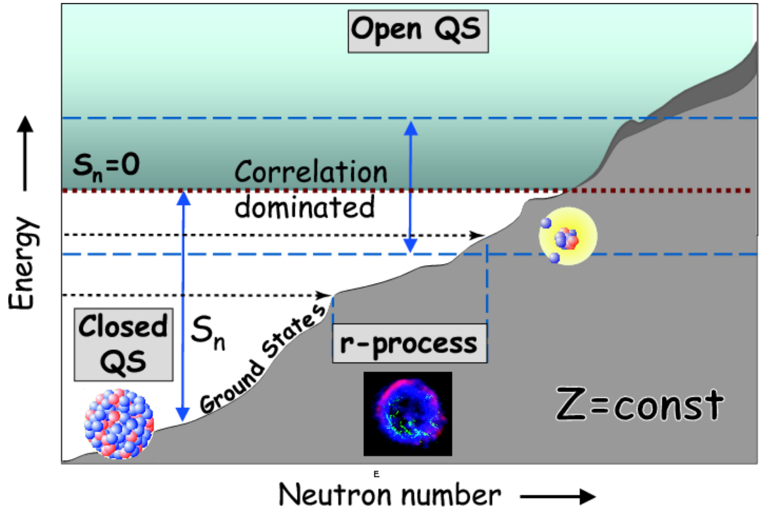
Therefore, 3NF is an important in calculations of both energy and radius

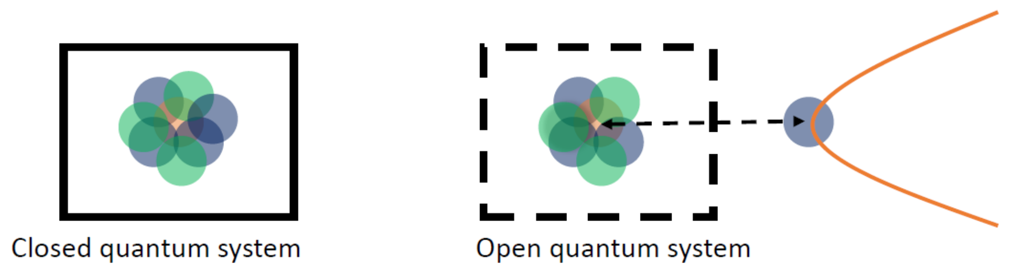
Element production

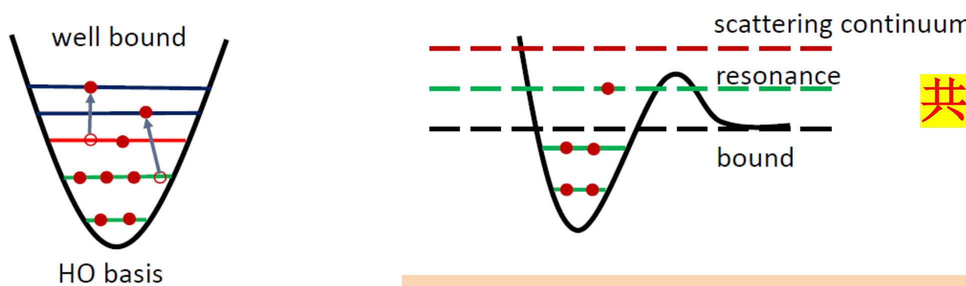


II. The continuum effect

弱束缚和不束缚的原子核属于“开放的量子体系”Open Quantum Systems







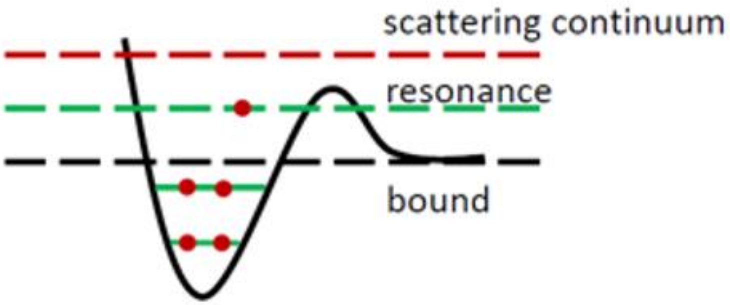
共振+连续

Time-dependent problem

Gamow-Berggren ensemble provides a good basis for many-body calculations of nuclei as OQS

T. Berggren, Nucl. Phys. A109 (1968) 265

$\psi(\boldsymbol{r},t)$=$e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})$ **(Stationary or quasi-stationary)**



$[-\frac{\hbar^2}{2m}\nabla^2+\mathrm{V}]\varphi_E(\boldsymbol{r})=\mathrm{E}\varphi_E(\boldsymbol{r})$

But E can be complex, and

the inner product: $\int\varphi_E(\boldsymbol{r})\varphi_E(\boldsymbol{r})=1$ (not the complex conjugate)

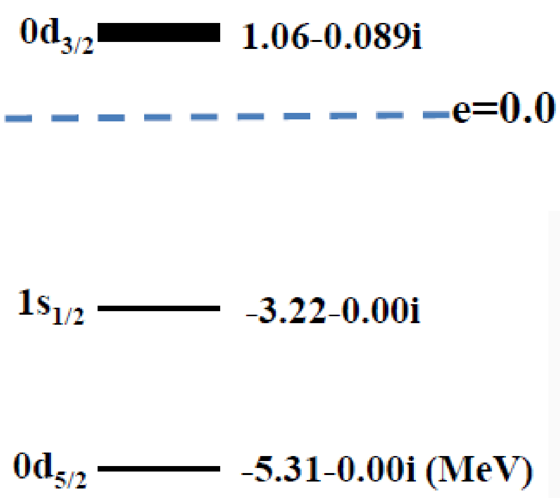
E is a negative real number: a bound state

E is a positive real number: a continuum (scattering) state

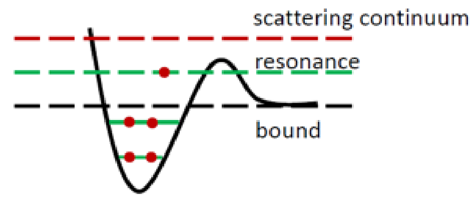
$E=\frac{\hbar^2k^2}{2m}=E_R-i\frac{\Gamma}{2}$: a resonant state

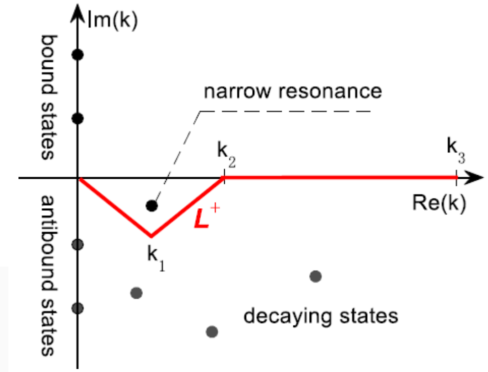
$\psi(\boldsymbol{r},t)$=$e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})$=$e^{-iE_Rt/\hbar}\varphi_E(\boldsymbol{r})e^{-\Gamma\hbar t/2}$ $\quad T_{1/2}=\hbar\ln 2/\Gamma$

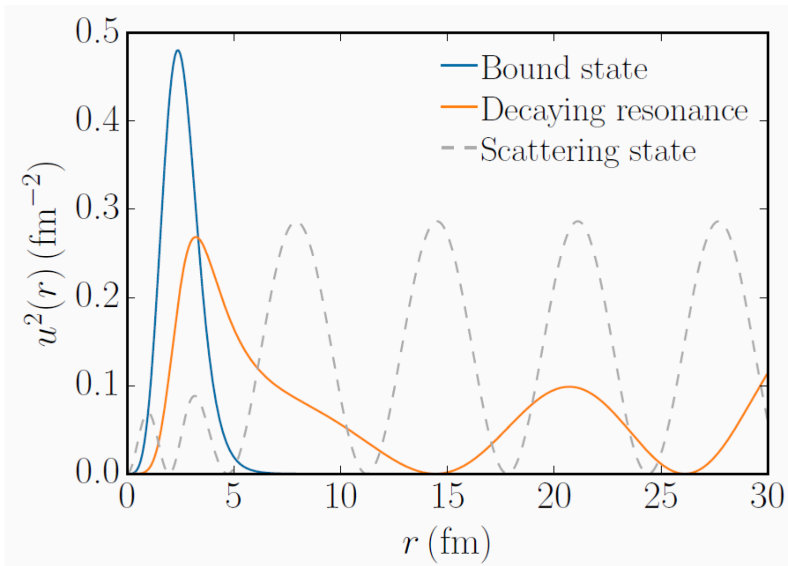
Complex-momentum space: **bound, resonance and continuum**



WS SPE's







$$C^+(k)\,C^-(k) = \frac{1}{2\pi}$$

$$\int_0^\infty dr\, u^2_{\ell,\eta}(r) = 1$$

Non-integrable

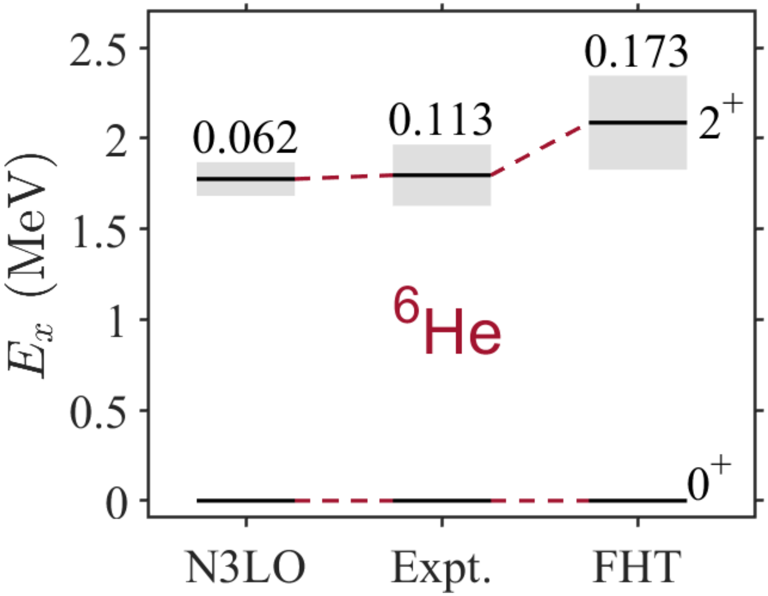
complex scaling:

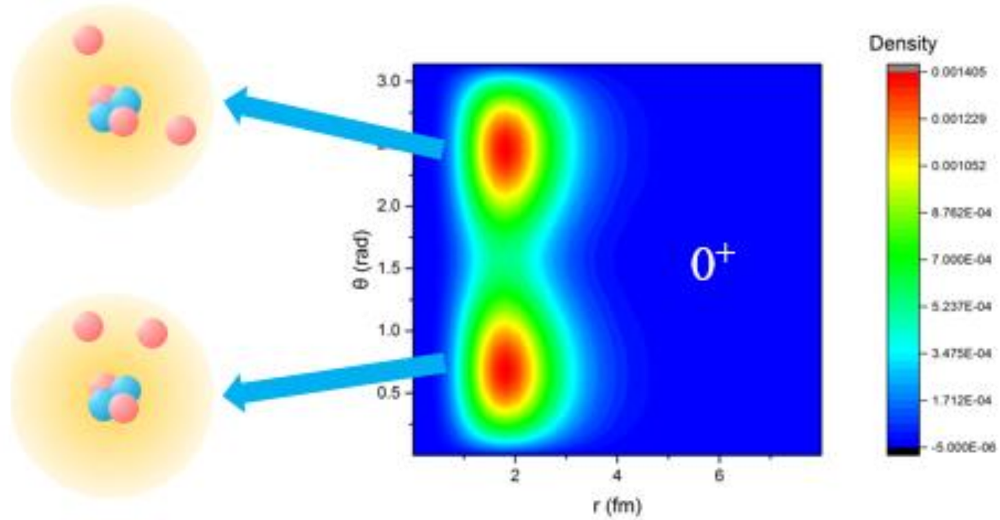
$$U(\theta)\boldsymbol{r}_i U^{-1}(\theta) = \boldsymbol{r}_i\, e^{i\theta}, \qquad U(\theta)\boldsymbol{k}_i U^{-1}(\theta) = \boldsymbol{k}_i\, e^{-i\theta} \qquad U(\theta)\, U^{-1}(\theta) = 1$$

J. Aguilar and J. M. Combes, Comm. Math. Phys. 22, 269 (1971);

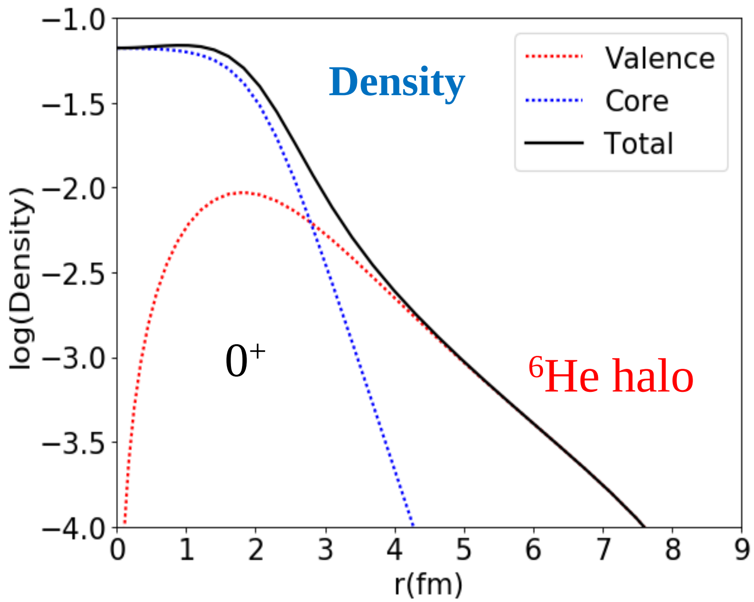
E. Balslev and J. M. Combes, Comm. Math. Phys. 22, 280 (1971)

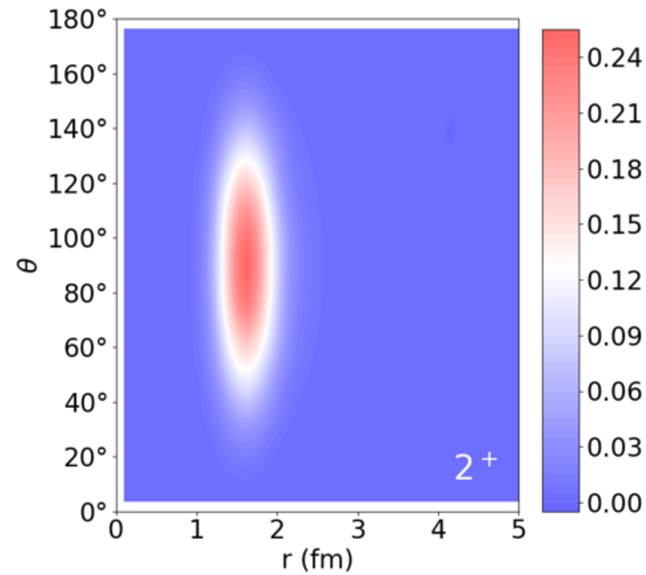
^{6}He halo and 2^{+} resonance calculated by GSM with ^{4}He core





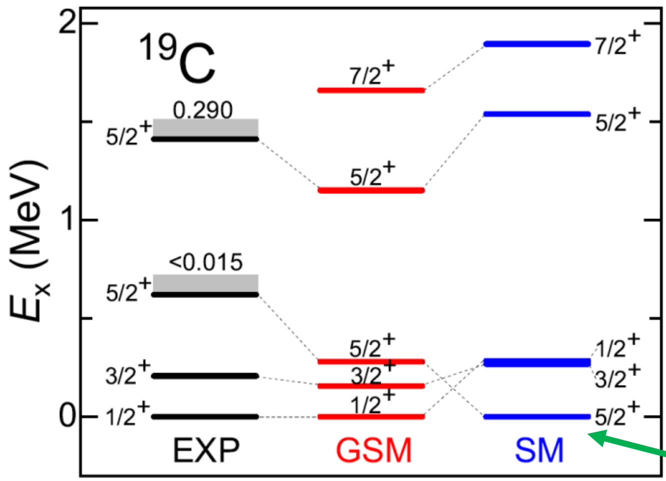
$$\rho(r,\theta) = \langle \Psi | \delta(r_1 - r)\delta(r_2 - r)\delta(\theta_{12} - \theta) | \Psi \rangle$$

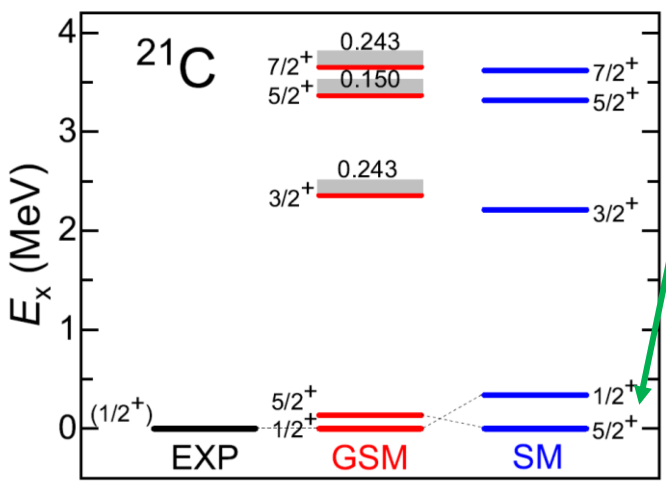




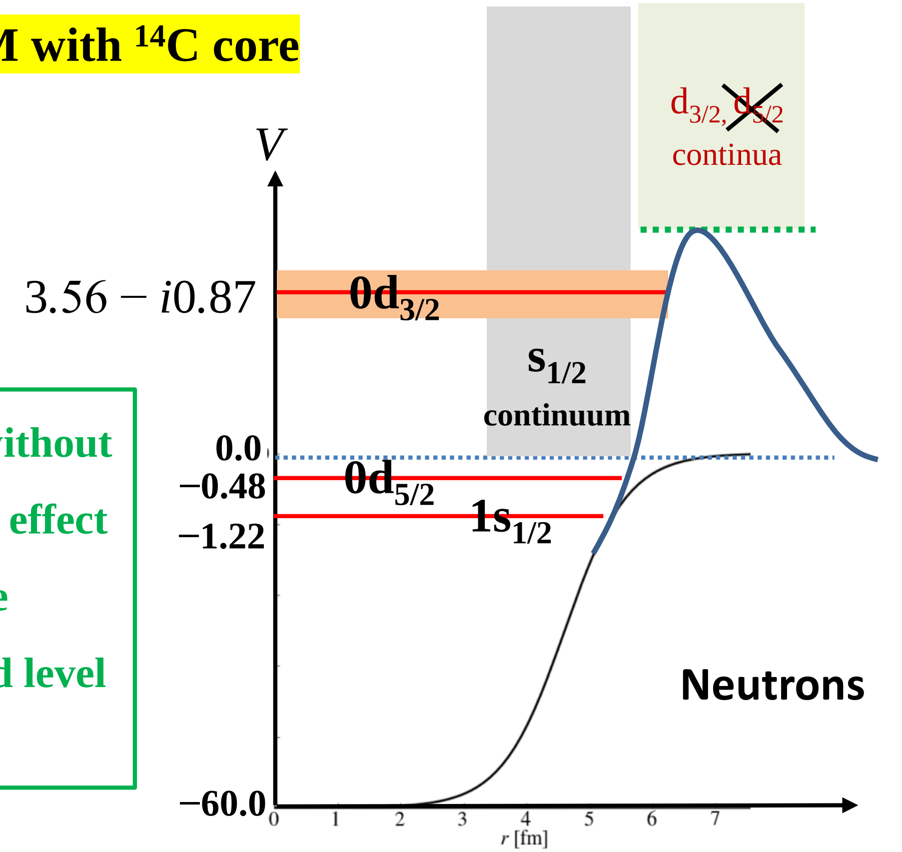
The continuum coupling is important in descriptions of carbon spectra, giving correct orders of levels

CD Bonn GSM with ^{14}C core





Calculations without the continuum effect cannot give the correct g.s. and level order.



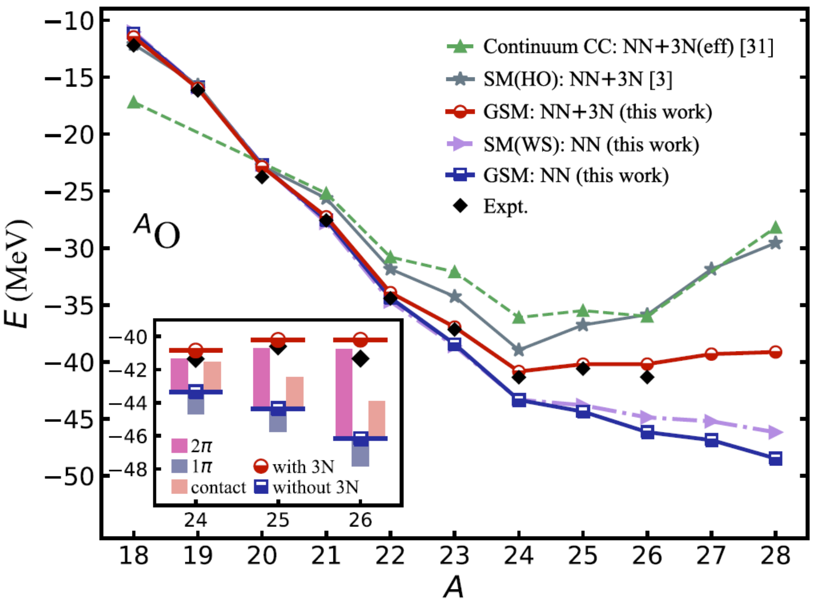
GSM space:

$0d_{5/2}$, $1s_{1/2}$+s-continuum, $0d_{3/2}$ (resonance)+$d_{3/2}$-continuum

Y. F. Geng, J. G. Li, Y. Z. Ma, B. S. Hu, Q. Wu, Z. H. Sun, S. Zhang, and **FRX**, PRC 106, 024304 (2022)

III. Combined effects of 3NF and the coupling to the continuum

GSM with ^{16}O core: N^{3}LO(NN) + N^{2}LO(NNN)



Y.Z. Ma, **FRX** *et al.*, PLB 802, 135257 (2020)

N^{3}LO(NN): Entem and Machleidt, PRC **66**, 014002 (2002)

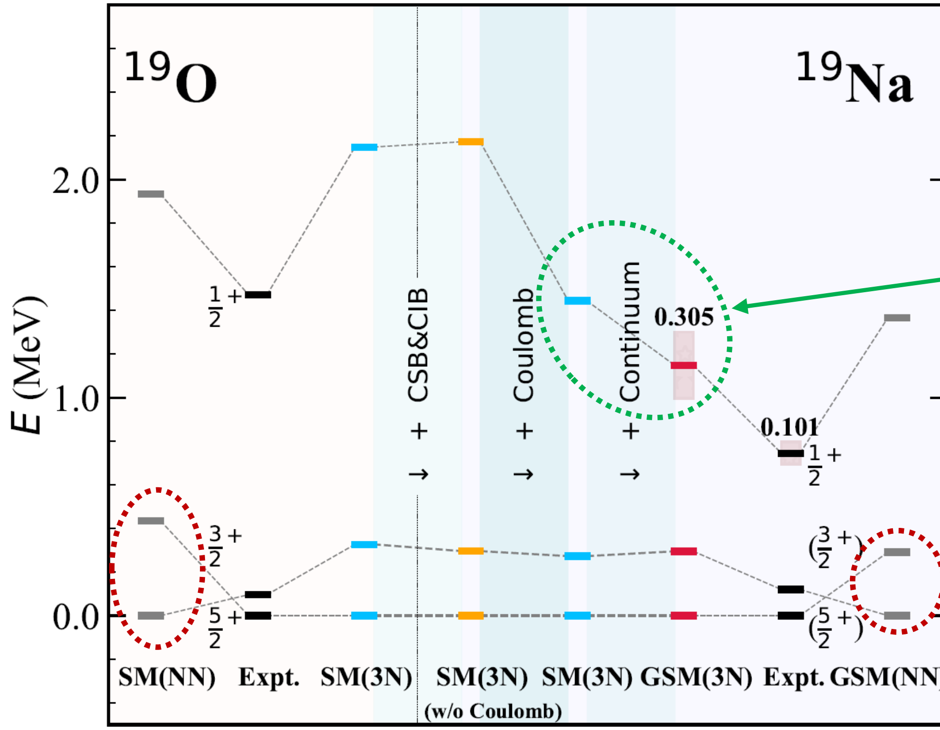
N^{2}LO(NNN): c_D=-1, c_E=-0.34, P. Navrátil *et al.*, PRL 99, 042501 (2007)

[3] Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL 105 (2010) 032501

[31] Hagen, Hjorth-Jensen, Jansen, Machleidt, Papenbrock, PRL108 (2012) 242501

Thomas-Ehrman shift (TES) in mirror pair

Both continuum effect and 3NF are important to make the observed TES



Continuum effect

If no 3NF, calculation cannot give the correct order of the levels.

CIB: charge independence breaking, a violation of rotation invariance in isospin space.

T=1 *NN* interaction: Tz=+1 (pp), 0 (np) and -1 (nn)

The main reasons: $m_p \neq m_n$, π^0, $\pi^\pm$ **mass splitting**

CIB is more significant than CSB

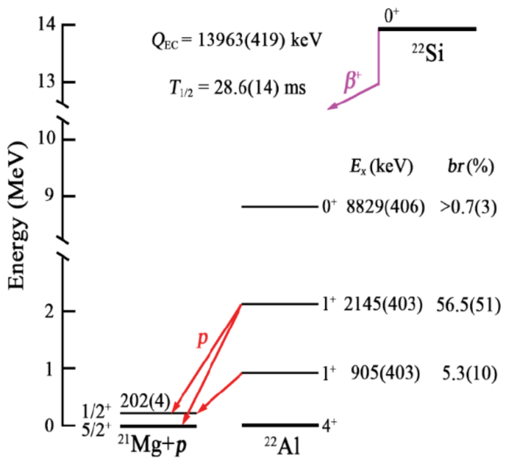
CSB: charge symmetry breaking, a violation of rotation invariance by 180°

Only for *pp* and *nn*

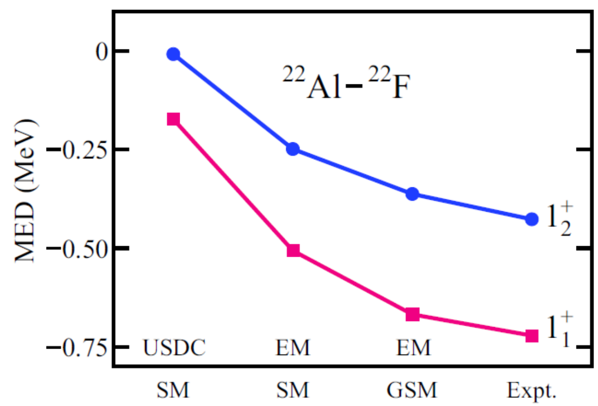
S. Zhang, Y.Z. Ma, J.G. Li, B.S. Hu, Q. Yuan, Z.H. Cheng, F.R. Xu, PLB 827, 136958 (2022)

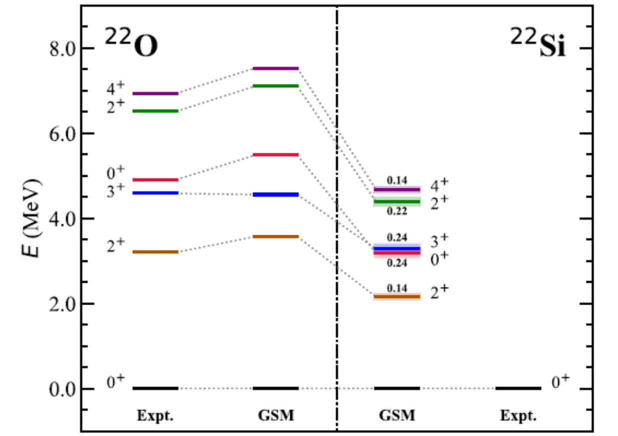
GT transitions at driplines

J. Lee *et al.*,PRL125，192503 (2020)



EM1.8/2.0





GT transition matrix elements $|M_{GT}|$

		SM		GSM	Ref. [54]	
		USDC	EM	EM	Expt.	Cal.
$^{22}Si \rightarrow {}^{22}Al$	1_1^+	0.236	0.343	0.257	0.176(16)	0.242
	1_2^+	0.721	1.042	1.012	0.750(41)	0.863
$^{22}O \rightarrow {}^{22}F$	1_1^+	0.198	0.569	0.497	0.310(32)	0.428
	1_2^+	0.719	1.092	1.068	0.775(77)	0.848

[54] J. Lee *et al.*, PRL 125, 192503 (2020)

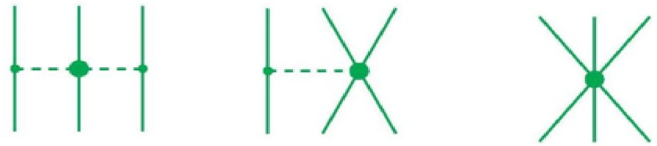
Z.C. Xu, S. Zhang, J.G. Li, S.L.Jin, Q. Yuan, Z.H. Cheng, N. Michel, and FRX, PRC 108, **L**031301 (2023)

IV. Conclusions

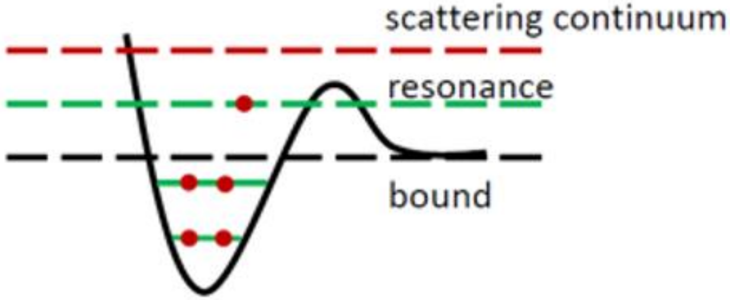
***Ab initio* nuclear many-body calculations show:**

Both 3NF and continuum effect plays important roles in describing nuclei around driplines

Chiral 2NF + 3NF



Complex-energy resonance with continuum effect



Thank you for your attention

2025 ANPhA Symposium

(Taipei, 27-29 Nov. 2025)

J.G. Li, Q. Yuan, Z.C. Xu, Y.Z. Ma, S. Zhang, R.Z. Hu, Y.F. Geng, N. Michel, L. Coraggio, T. Fukui, L. DeAngelis, N. Itaco, A. Gargano