Dynamical interpretation of the Standard-Model flavor structure

Hsiang-nan Li

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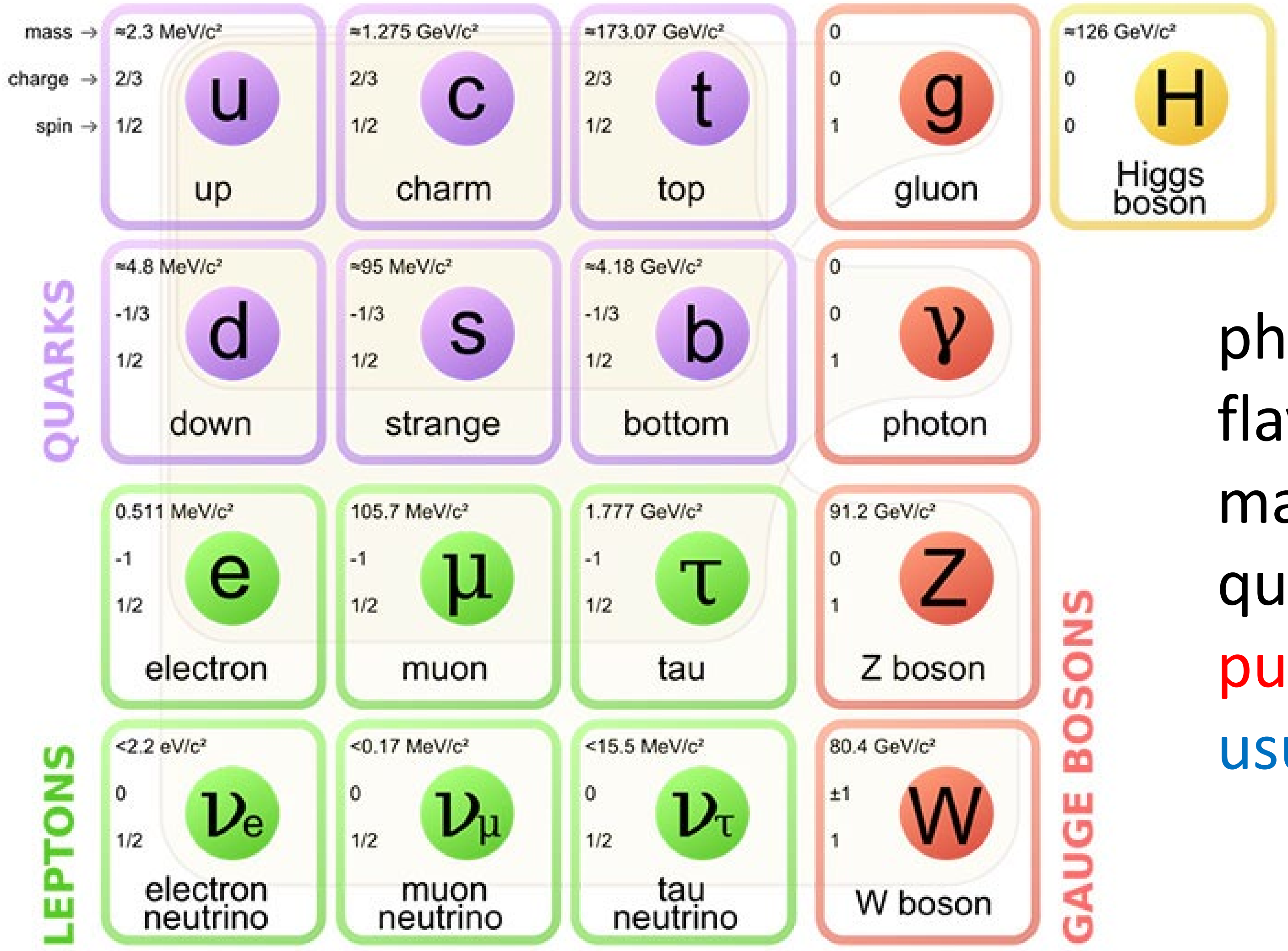
2306.03463, 2404.16626, 2407.07813

Standard Model



physicists are curious about
flavor structure:
mass hierarchy (order 10E11),
quark, lepton mixing patterns,…
puzzles for decades
usually explained by new physics

Speculation

- Physical observables, being analytical, must respect dispersion relations
- Dispersion relation connects various dynamics at different scales; heavy meson lifetimes link EW and strong interactions; Higgs decays into b quark pairs link Yukawa coupling and strong interactions,…
- Numerous observables imply numerous links --- nontrivial constraints
- Perhaps SM parameters may not be completely free?
- SM flavor structure governed by dispersive constraints?
- If yes, SM flavor structure can be understood dynamically
- Our speculation: only the three gauge couplings are fundamental

Mixing patterns

Why are quark and lepton mixings so different?

A simple example to demonstrate our formalism

Li, 2306.03463

Issues about fermion mixing

- Neutrino mass ordering

$$\Delta m^2_{21} \equiv m^2_2 - m^2_1 = (7.55^{+0.20}_{-0.16}) \times 10^{-5}\ \text{eV}^2 \qquad \Delta m^2_{32} \equiv m^2_3 - m^2_2 = (2.424 \pm 0.03) \times 10^{-3}\ \text{eV}^2$$

  but normal ordering or inverted ordering?

- Why small mixing in quark sector, but large mixing in lepton sector?

  CKM: $\theta_{12} = 13.04° \pm 0.05°$, $\theta_{13} = 0.201° \pm 0.011°$, $\theta_{23} = 2.38° \pm 0.06°$

  Pontecorvo–Maki–Nakagawa–Sakata: $\theta_{12} = 33.41°^{+0.75°}_{-0.72°}$ $\theta_{13} = 8.54°^{+0.11°}_{-0.12°}$

- Why lepton mixing has maximal angle $\theta_{23} \approx 45°$?

Dispersion relation

- $\mu^- e^+$-$\mu^+ e^-$ mixing amplitude $\Pi(s) \equiv M(s) - i\Gamma(s)/2$

$$M(s) = \frac{1}{2\pi}\int^{R} ds' \frac{\Gamma(s')}{s - s'} + \frac{1}{2\pi i}\int_{C_R} ds' \frac{\Pi(s')}{s' - s}$$

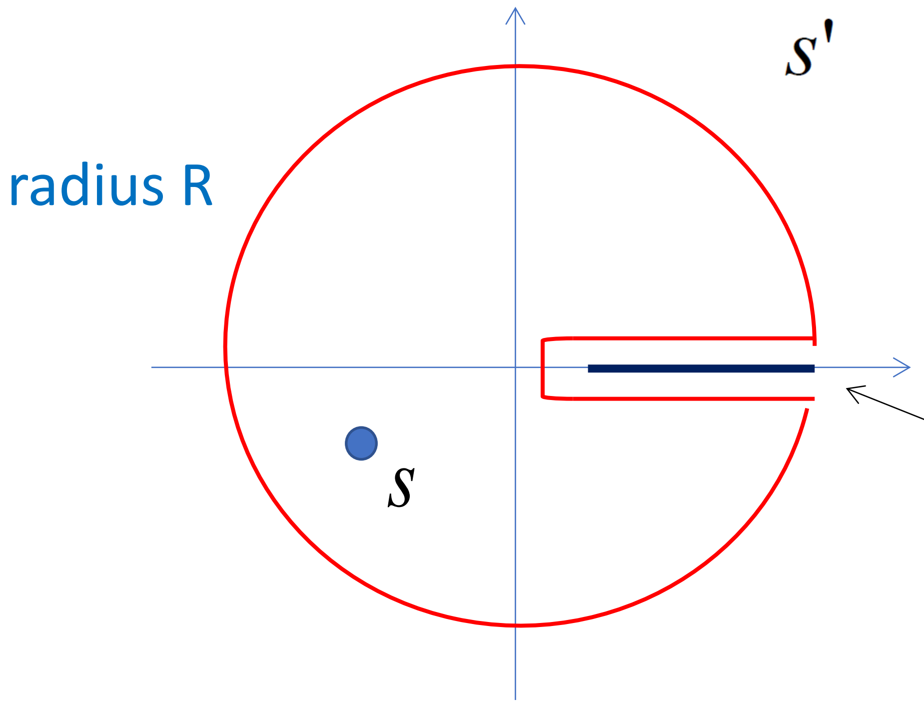
so far, it's identity for physical observable based on analyticity

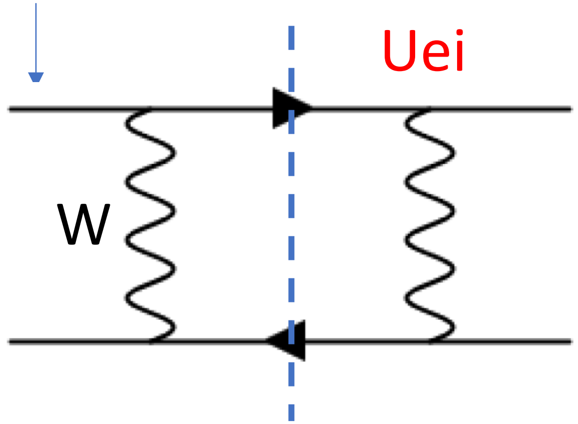
real part imaginary part

invariant mass squared s

PMNS matrix element

Uei





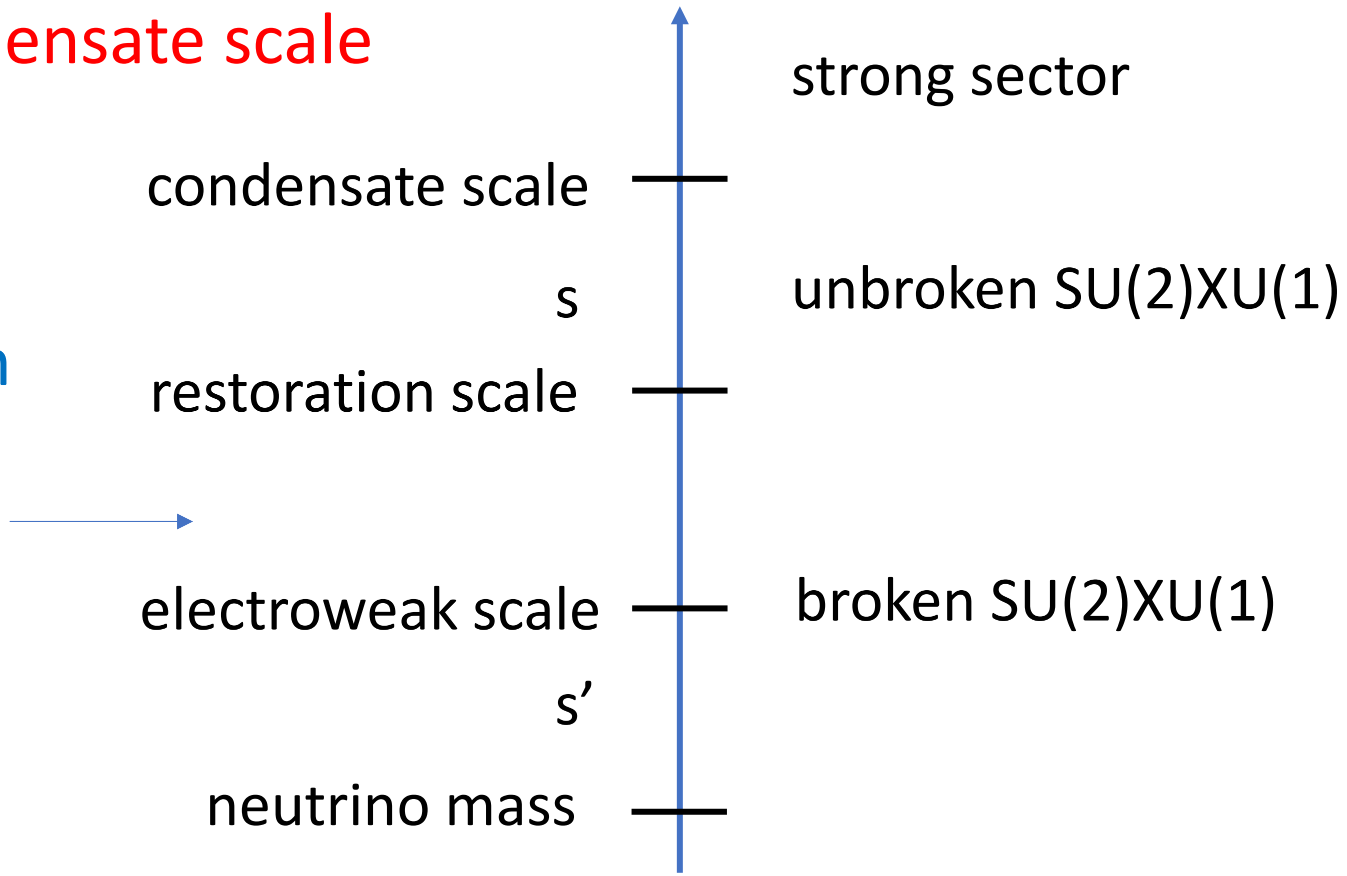
branch cut caused by real intermediate states

Crazy idea

- Dispersion relation reminds QCD sum rules, which link perturbative and nonperturbative dynamics
- Nonperturbative properties (hadron mass, decay constant) can be constrained by perturbative dynamics
- No condensate, no bound state
- Related to chiral symmetry restoration and breaking
- Neutrino properties related to electroweak symmetry restoration and breaking?
- Then EW symmetric phase, corresponding to perturbative phase in QCD, provides inputs, which constrain EW broken phase

What if EW symmetry restored at high energy?

- Composite Higgs model, Kaplan and Georgi, Phys. Lett. B136, 183 (1984):
- Electroweak group is broken at a scale much smaller than the condensate scale
- This sequential breaking can be realized in SM4 with fourth generation fermions



Mixing in symmetric phase

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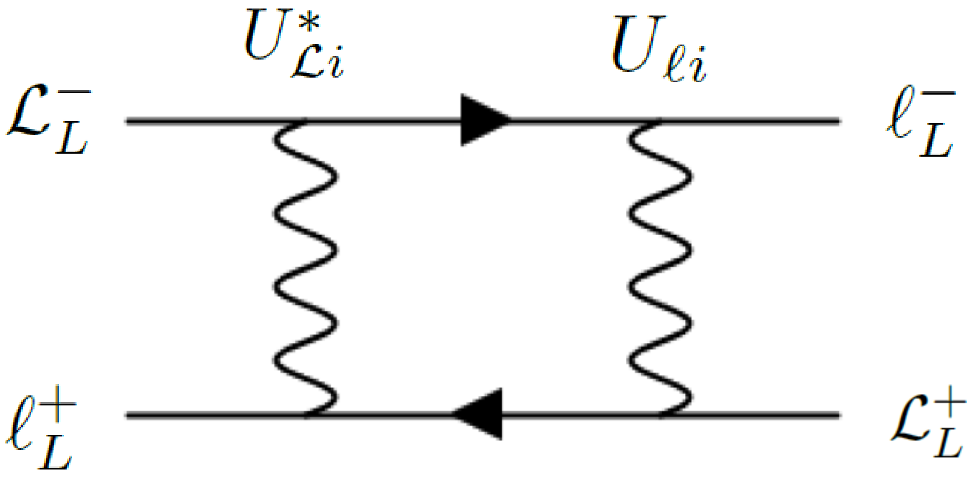
- Internal particles massless
- All intermediate channels give same contribution
- Sum over all channels vanishes due to unitarity $\sum_i U^*_{\mathcal{L}i} U_{\ell i} = 0$
- Mixing phenomenon disappears!



restoration scale

$$M(s) = \frac{1}{2\pi} \int^{\Lambda} ds' \frac{\Gamma(s')}{s - s'} \approx 0$$

$s > \Lambda$

EW symmetry broken at low energy;

constrains fermion masses and mixing angles

Box diagram in broken phase

- s' can be low, so $\Gamma(s')$ depends on PMNS matrix elements and intermediate neutrino masses in broken phase.
- Box-diagram contribution from channel with two real neutrinos

Cheng 1982
Buras et al 1984

$$\Gamma_1\,\Gamma(s) \propto \sum_{i,j=1}^{3} \lambda_i \lambda_j \Gamma_{ij}(s), \qquad \lambda_i = U_{\mathcal{L}i} U_{\ell i}$$

$$\Gamma_{ij}(s) = \frac{1}{s^2} \frac{\sqrt{s^2 - 2s(m_i^2 + m_j^2) + (m_i^2 - m_j^2)^2}}{(m_W^2 - m_i^2)(m_W^2 - m_j^2)} \times \left\{ \left(m_W^4 + \frac{m_i^2 m_j^2}{4} \right) [2s^2 - 4s(m_i^2 + m_j^2) + 2(m_i^2 - m_j^2)^2] + 3m_W^2 s (m_i^2 + m_j^2)(m_i^2 + m_j^2 - s) \right\}$$

Constraints

- How to diminish dispersive integral $\int^{\Lambda} ds' \frac{\Gamma(s')}{s-s'}$?
- Asymptotic expansion

to have finite integral

$$\sum_{i,j} \lambda_i \lambda_j \Gamma_{ij}^{(m)} \approx 0, \quad m = 1, 0, -1$$

$$\Gamma_{ij}(s') \approx \Gamma_{ij}^{(1)} s' + \Gamma_{ij}^{(0)} + \frac{\Gamma_{ij}^{(-1)}}{s'} + \cdots$$

$$\Gamma_{ij}^{(1)} = \frac{4m_W^4 - 6m_W^2(m_i^2+m_j^2) + 4m_i^2 m_j^2}{2(m_W^2-m_i^2)(m_W^2-m_j^2)}, \quad \Rightarrow \Lambda^2/s$$

$$\Gamma_{ij}^{(0)} = -\frac{3(m_i^2+m_j^2)\left[4m_W^4 - 4m_W^2(m_i^2+m_j^2) + m_i^2 m_j^2\right]}{2(m_W^2-m_i^2)(m_W^2-m_j^2)} \quad \Rightarrow (m_i^2+m_j^2)\Lambda/s$$

$$\Gamma_{ij}^{(-1)} = \frac{3(m_i^4+m_j^4)\left[4m_W^4 - 2m_W^2(m_i^2+m_j^2) + m_i^2 m_j^2\right]}{2(m_W^2-m_i^2)(m_W^2-m_j^2)}. \quad \Rightarrow (m_i^4+m_j^4)\ln\Lambda/s$$

to diminish integral

$$\int ds' \frac{\Gamma_{12}(s')}{s-s'} \approx \frac{1}{s}\sum_{i,j}\lambda_i\lambda_j g_{ij} \qquad g_{ij} \equiv \int_{t_{ij}}^{\infty} ds' \left[\Gamma_{ij}(s') - \Gamma_{ij}^{(1)} s' - \Gamma_{ij}^{(0)} - \frac{\Gamma_{ij}^{(-1)}}{s'}\right]$$

$$\sum_{i,j} \lambda_i \lambda_j g_{ij} \approx 0$$

These four conditions constrain neutrino masses and mixing angles!

Test quark mixing first---constrain quark masses and CKM matrix elements

for D meson mixing $\lambda_i \equiv V_{ci}^* V_{ui} \quad i, j = d, s, b$

Minimization

- Use unitarity to eliminate λ_b and rewrite constraints

$$r^2 R_{dd}^{(m)} + 2r R_{ds}^{(m)} + 1 \approx 0, , \quad m = 1, 0, -1, i$$

← refer to finite integral g_{ij}

$$R_{dd}^{(m)} = \frac{\Gamma_{dd}^{(m)} - 2\Gamma_{db}^{(m)} + \Gamma_{bb}^{(m)}}{\Gamma_{ss}^{(m)} - 2\Gamma_{sb}^{(m)} + \Gamma_{bb}^{(m)}}, \quad R_{ds}^{(m)} = \frac{\Gamma_{ds}^{(m)} - \Gamma_{db}^{(m)} - \Gamma_{sb}^{(m)} + \Gamma_{bb}^{(m)}}{\Gamma_{ss}^{(m)} - 2\Gamma_{sb}^{(m)} + \Gamma_{bb}^{(m)}} \quad m = 1, 0, -1$$

- Expression for $m = i$ similar, but with g_{ij}
- Ratio of CKM elements

$$r = \frac{\lambda_d}{\lambda_s} = \frac{V_{cd}^* V_{ud}}{V_{cs}^* V_{us}} \equiv u + iv,$$

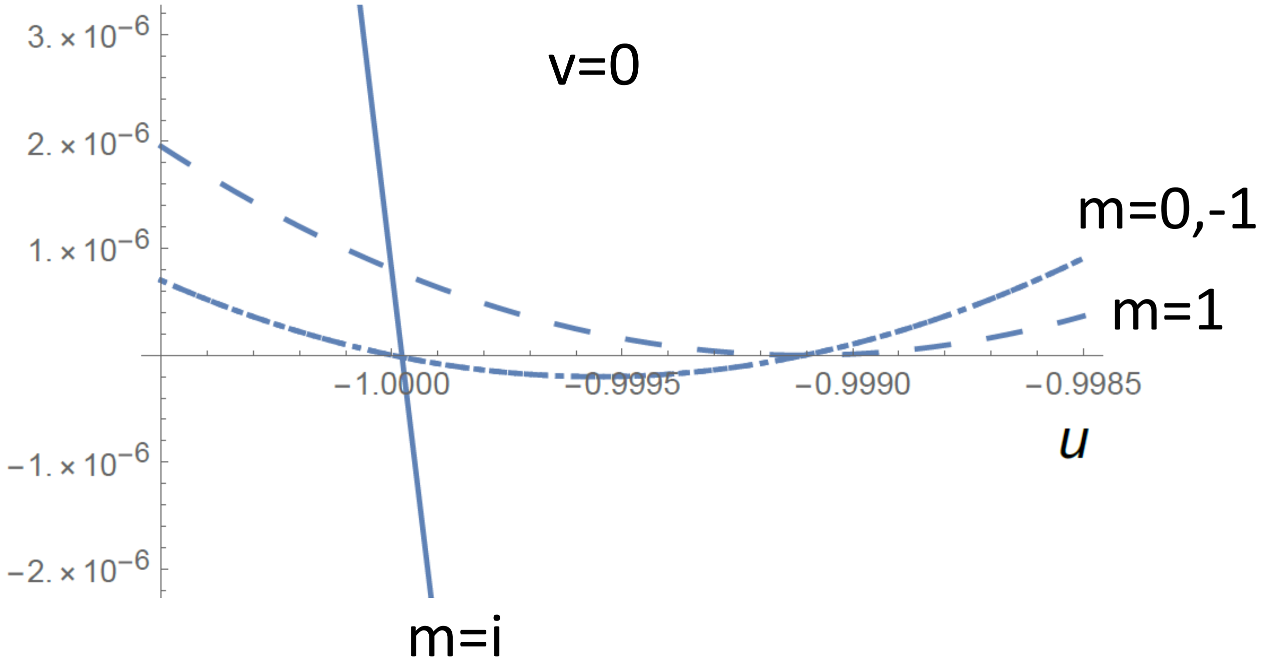
- Tune u and v to minimize the sum (real parts of constraints)

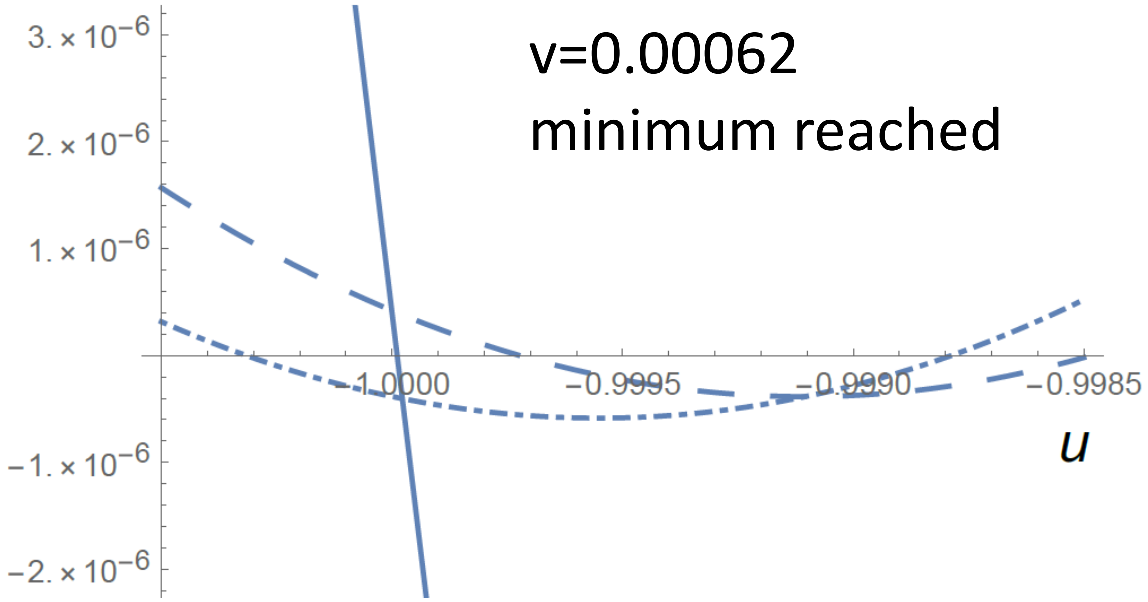
$$\sum_{m=1,-1,i} \left[(u^2 - v^2) R_{dd}^{(m)} + 2u R_{ds}^{(m)} + 1 \right]^2$$

then imaginary parts also small

Results

$m_d = 0.005$ GeV $\quad m_s = 0.12$ GeV $\quad m_b = 4.0$ GeV $\quad m_W = 80.377$ GeV





PDG

$$r = \frac{V_{cd}^* V_{ud}}{V_{cs}^* V_{us}} = -1.0 + (6.2^{+1.2}_{-1.0}) \times 10^{-4} i$$

variation of m_s by 0.01 GeV

$$u = -1.00029 \pm 0.00002, \quad v = 0.00064 \pm 0.00002$$

they agree well; CP phase must exist

Global fits

experimental discrimination of NO, IO difficult

	Ref. [181] w/o SK-ATM		Ref. [181] w SK-ATM		Ref. [182] w SK-ATM		Ref. [183] w SK-ATM	
NO	Best Fit Ordering		Best Fit Ordering		Best Fit Ordering		Best Fit Ordering	
Param	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.03^{+0.12}_{-0.11}$	$2.70 \to 3.41$	$3.03^{+0.12}_{-0.12}$	$2.70 \to 3.41$	$3.03^{+0.13}_{-0.13}$	$2.63 \to 3.45$	$3.18^{+0.16}_{-0.16}$	$2.71 \to 3.69$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.40^{+0.80}_{-0.82}$	$30.85 \to 35.97$	$34.3^{+1.0}_{-1.0}$	$31.4 \to 37.4$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.72^{+0.18}_{-0.23}$	$4.06 \to 6.20$	$4.51^{+0.19}_{-0.16}$	$4.08 \to 6.03$	$4.55^{+0.18}_{-0.15}$	$4.16 \to 5.99$	$5.74^{+0.14}_{-0.14}$	$4.34 \to 6.10$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$42.4^{+1.0}_{-0.9}$	$40.2 \to 50.7$	$49.3^{+0.8}_{-0.8}$	$41.2 \to 51.3$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.203^{+0.056}_{-0.059}$	$2.029 \to 2.391$	$2.225^{+0.056}_{-0.059}$	$2.052 \to 2.398$	$2.23^{+0.07}_{-0.06}$	$2.04 \to 2.44$	$2.200^{+0.069}_{-0.062}$	$2.00 \to 2.405$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \to 8.89$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.59^{+0.13}_{-0.12}$	$8.21 \to 8.99$	$8.53^{+0.13}_{-0.12}$	$8.13 \to 8.92$
$\delta_{CP}/^\circ$	197^{+42}_{-25}	$108 \to 404$	232^{+36}_{-26}	$144 \to 350$	223^{+32}_{-23}	$139 \to 355$	194^{+24}_{-22}	$128 \to 359$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.36^{+0.16}_{-0.15}$	$6.93 \to 7.93$	$7.50^{+0.22}_{-0.20}$	$6.94 \to 8.14$
$\frac{\Delta m^2_{32}}{10^{-3}\ \text{eV}^2}$	$2.437^{+0.028}_{-0.027}$	$2.354 \to 2.523$	$2.433^{+0.026}_{-0.027}$	$2.353 \to 2.516$	$2.448^{+0.023}_{-0.031}$	$2.367 \to 2.521$	$2.47^{+0.02}_{-0.03}$	$2.40 \to 2.46$
IO	$\Delta\chi^2 = 2.3$		$\Delta\chi^2 = 6.4$		$\Delta\chi^2 = 6.5$		$\Delta\chi^2 = 6.4$	
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.03^{+0.12}_{-0.11}$	$2.70 \to 3.41$	$3.03^{+0.12}_{-0.11}$	$2.70 \to 3.41$	$3.03^{+0.13}_{-0.13}$	$2.63 \to 3.45$	$3.18^{+0.16}_{-0.16}$	$2.71 \to 3.69$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.40^{+0.80}_{-0.82}$	$30.85 \to 35.97$	$34.3^{+1.0}_{-1.0}$	$31.4 \to 37.4$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.78^{+0.16}_{-0.21}$	$4.12 \to 6.23$	$5.69^{+0.16}_{-0.21}$	$4.12 \to 6.13$	$5.69^{+0.13}_{-0.21}$	$4.17 \to 6.06$	$5.78^{-0.10}_{+0.17}$	$4.33 \to 6.08$
$\theta_{23}/^\circ$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$	$49.0^{+0.7}_{-1.4}$	$40.2 \to 51.1$	$49.5^{+0.6}_{-1.0}$	$41.2 \to 51.2$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.219^{+0.060}_{-0.057}$	$2.047 \to 2.396$	$2.223^{+0.058}_{-0.058}$	$2.048 \to 2.416$	$2.23^{+0.06}_{-0.06}$	$2.03 \to 2.45$	$2.225^{+0.064}_{-0.070}$	$2.02 \to 2.42$
$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.11}$	$8.23 \to 8.90$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$	$8.59^{+0.13}_{-0.12}$	$8.19 \to 9.00$	$8.58^{+0.12}_{-0.14}$	$8.17 \to 8.96$
$\delta_{CP}/^\circ$	286^{+27}_{-32}	$192 \to 360$	276^{+22}_{-29}	$194 \to 344$	274^{+25}_{-27}	$193 \to 342$	284^{+26}_{-28}	$200 \to 353$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.36^{+0.16}_{-0.15}$	$6.93 \to 7.93$	$7.50^{+0.22}_{-0.20}$	$6.94 \to 8.14$
$\frac{\Delta m^2_{32}}{10^{-3}\ \text{eV}^2}$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \to -2.408$	$-2.486^{+0.028}_{-0.025}$	$-2.570 \to -2.406$	$-2.492^{+0.025}_{-0.030}$	$-2.578 \to -2.413$	$-2.52\pm^{+0.03}_{-0.02}$	$-2.60 \to -2.44$

PDG 2024

Chau-Keung parametrization

Pontecorvo–Maki–Nakagawa–Sakata matrix

U =

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}.$$

Neutrino mass orderings

- Apply to lepton $\mu^- e^+$-$\mu^+ e^-$ mixing with intermediate neutrino channels
- Normal ordering (NO) $m_1^2 = 10^{-6}\ \mathrm{eV}^2$ (as long as it is small enough)

$\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = (7.55^{+0.20}_{-0.16}) \times 10^{-5}\ \mathrm{eV}^2$ $\Delta m_{32}^2 \equiv m_3^2 - m_2^2 = (2.424 \pm 0.03) \times 10^{-3}\ \mathrm{eV}^2$

de Salas et al, 2018

- Predict

$$r = \frac{U_{\mu 1}^* U_{e1}}{U_{\mu 2}^* U_{e2}} \approx -1.0 - 0.02i$$

global fit

$$r = -(0.738^{+0.050}_{-0.048}) - (0.179^{+0.136}_{-0.125})i$$

- Be reminded that it is LO analysis with 3 generations
- Inverted ordering (IO) $r \approx -1.0 - O(10^{-5})i$ $r = -(1.03^{+0.05}_{-0.16}) - (0.356^{+0.015}_{-0.048})i$

dramatically different

- NO and observed PMNS matrix satisfy constraint at order of magnitude

Mixing patterns

- Insert u=-1 into m=1 constraint to get analytical expression of v

$$v \approx \frac{(m_W^2 - m_b^2)(m_s^2 - m_d^2)}{(m_W^2 - m_s^2)(m_b^2 - m_d^2)} \approx \frac{m_s^2}{m_b^2}$$

- v proportional to sine of mixing angle
- Larger mixing angles in lepton sector due to smaller mass hierarchy

$$\boxed{\frac{m_2^2}{m_3^2} \approx 3.1 \times 10^{-2} \gg \frac{m_s^2}{m_b^2} \approx 9.0 \times 10^{-4}}$$

Mixing of generations 1-3

- Heavy lepton could be μ or τ , same intermediate neutrinos
- $\tau^- e^+$-$\tau^+ e^-$ and $\mu^- e^+$-$\mu^+ e^-$ satisfy same constraints?
- Magnitude of PMNS matrix elements

NuFIT, 2023

$$|U| = \begin{bmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{bmatrix} = \begin{bmatrix} 0.803 \sim 0.845 & 0.514 \sim 0.578 & 0.142 \sim 0.155 \\ 0.233 \sim 0.505 & 0.460 \sim 0.693 & 0.630 \sim 0.779 \\ 0.262 \sim 0.525 & 0.473 \sim 0.702 & 0.610 \sim 0.762 \end{bmatrix}$$

These two rows are indeed similar, $\mu - \tau$ reflection symmetry

Maximal mixing angle θ_{23}

- Recall v has two solutions with opposite signs, so one for $\mu^- e^+$-$\mu^+ e^-$ another for $\tau^- e^+$-$\tau^+ e^-$?
- Check data

	$U^*_{\tau 1} U_{e1}/(U^*_{\tau 2} U_{e2})$	$r = U^*_{\mu 1} U_{e1}/(U^*_{\mu 2} U_{e2})$	
	$-(1.231^{+0.078}_{-0.186}) + (0.204^{+0.085}_{-0.138})i$	$r = -(0.738^{+0.050}_{-0.048}) - (0.179^{+0.136}_{-0.125})i$	de Salas et al, 2018
	$-(1.139^{+0.139}_{-0.207}) + (0.266^{+0.050}_{-0.124})i$	$r = -(0.801^{+0.219}_{-0.097}) - (0.265^{+0.090}_{-0.145})i$	Capozzi et al, 2018

- Implication: $\theta_{23} \approx 45^\circ$

roughly equal

$$\frac{U^*_{\mu 1} U_{e1}}{U^*_{\mu 2} U_{e2}} = -\frac{c_{12}}{s_{12}} \frac{c_{12}s_{12}(c_{23}^2 - s_{13}^2 s_{23}^2) + c_{23}s_{13}s_{23}c_\delta(c_{12}^2 - s_{12}^2) - c_{23}s_{13}s_{23}s_\delta i}{(c_{12}c_{23} - s_{12}s_{13}s_{23})^2 + 2c_{12}c_{23}s_{12}s_{13}s_{23}(1 - c_\delta)}$$

$$\frac{U^*_{\tau 1} U_{e1}}{U^*_{\tau 2} U_{e2}} = -\frac{c_{12}}{s_{12}} \frac{c_{12}s_{12}(s_{23}^2 - c_{23}^2 s_{13}^2) - c_{23}s_{13}s_{23}c_\delta(c_{12}^2 - s_{12}^2) + c_{23}s_{13}s_{23}s_\delta i}{(c_{12}s_{23} + c_{23}s_{12}s_{13})^2 - 2c_{12}c_{23}s_{12}s_{13}s_{23}(1 - c_\delta)}$$

roughly equal

$$(c_{12}^2 + s_{12}^2 s_{13}^2)(c_{23}^2 - s_{23}^2) \approx 0$$

Impact from 4^{th} generation

How to improve constraints on neutrino masses and mixing angles?

Have predicted $m_L \approx 270\ \mathrm{GeV}$, $m_4 \approx 170\ \mathrm{GeV}$

Li, 2407.07813

Motivation

- If SM flavor structure constrained dynamically, scalar sector of SM may not be free
- Bold conjecture: SM contains only three fundamental (gauge) parameters, and other parameters, governing interplay among various generations of fermions, are fixed by SM dynamics itself
- To maintain simplicity and beauty, natural extension of SM is to introduce sequential fourth generation of fermions, since associated parameters in scalar sector are not free
- Solutions for PMNS matrix with 3 generations deviate from data
- Check whether consistency can be improved in SM4
- QED corrections also estimated

Solutions

- Formalism basically the same, solve for two PMNS ratios in SM4

$$r_1 \equiv \frac{U^*_{\mathcal{L}1}U_{e1}}{U^*_{\mathcal{L}2}U_{e2}} \equiv u_1 + iv_1, \qquad r_3 \equiv \frac{U^*_{\mathcal{L}3}U_{e3}}{U^*_{\mathcal{L}2}U_{e2}} \equiv u_3 + iv_3$$

- Suppress detail, solutions $r_1 \approx -0.83 - 0.04i, \quad -0.98 + 0.04i$

  assigned to μe τe

- Data

$$\frac{U^*_{\mu 1}U_{e1}}{U^*_{\mu 2}U_{e2}} = \begin{cases} -(0.676^{+0.045}_{-0.005}) - (0.072^{+0.118}_{-0.113})i \\ -(0.786^{+0.116}_{-0.046}) - (0.178^{+0.094}_{-0.092})i \end{cases}$$

de Sala et al, 2021

Capozzi et al, 2021

$$\frac{U^*_{\tau 1}U_{e1}}{U^*_{\tau 2}U_{e2}} = \begin{cases} -(1.289^{+0.008}_{-0.068}) + (0.079^{+0.115}_{-0.125})i \\ -(1.260^{+0.106}_{-0.104}) + (0.284^{+0.076}_{-0.134})i \end{cases}$$

de Sala et al, 2021

Capozzi et al, 2021

- Consistent within experimental uncertainties
- In terms of mixing angles, we predict $\theta_{12} \approx 34^\circ$, $\underline{\theta_{23} \approx 47^\circ}$, $\theta_{13} \approx 5^\circ$? and $\delta \approx 200^\circ$

data $\theta_{12} = (34.3\pm1.0)^\circ$, $\theta_{13} = (8.53^{+0.13}_{-0.12})^\circ$ and $\theta_{23} = (49.26\pm0.79)^\circ$, and the CP phase $\delta = (194^{+24}_{-22})^\circ$

$\theta_{12} = (33.40^{+0.80}_{-0.82})^\circ$, $\theta_{13} = (8.59^{+0.13}_{-0.12})^\circ$, $\theta_{23} = (42.4^{+1.0}_{-0.9})^\circ$ and $\delta = (223^{+32}_{-23})^\circ$

Summary

- Only electroweak symmetry restoration is assumed
- Quark and lepton mixing patterns due to different mass hierarchies
- $\mu - \tau$ reflection symmetry due to same intermediate states in mixing
- Normal ordering favored, θ_{23} tends to be in the 2^{nd} octant
- Consistency with the neutrino mixing angles improved in SM4
- Consistency further improved by QED corrections
- Predicted small CP violation in lepton sector
- Scalar sector, governing interplay among generations, fixed by dynamics
- Our explanation sheds light on model building for new physics