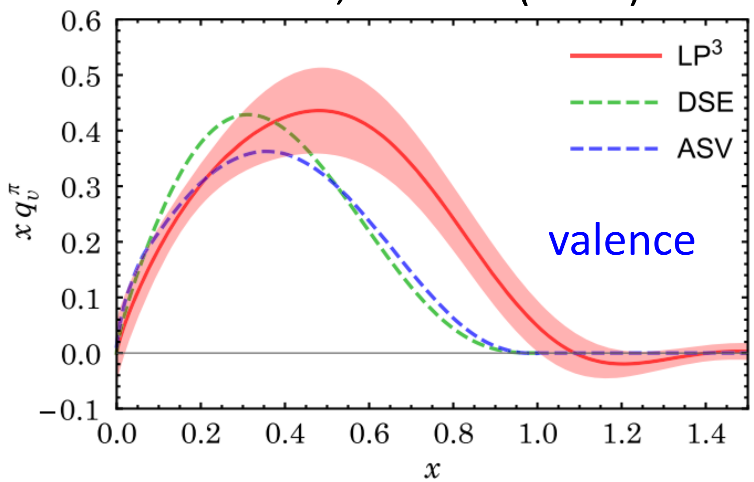
Works in the past week (2025/3/6)

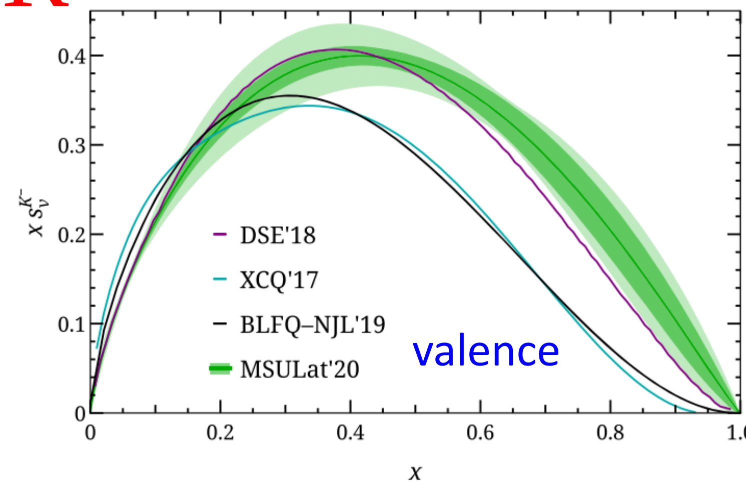
- Monday: prepare for the NSTC review report.
- Tuesday: Chi2 expression; seminar talk at NYCU.
- Wednesday: 驗收 in Tainan; NSTC review report.
- Thursday: CEM/NRQCD calculations of difference of sigma(pi+)-sigma(pi-).

LQCD: Valence & Gluon

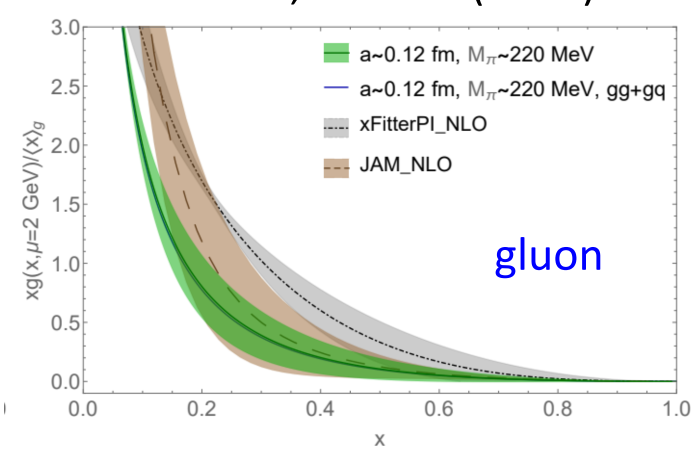
π PRD 100, 034505 (2019)



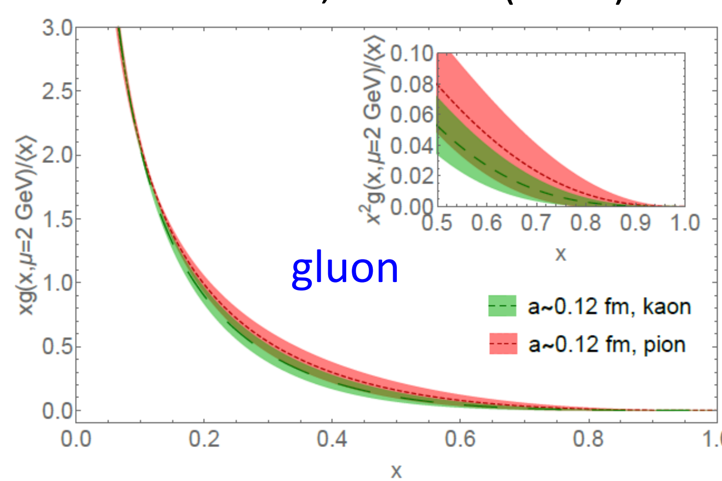
K PRD 103, 014516 (2021)



PLB 823, 136778 (2021)



PRD 106, 094510 (2022)



LQCD: Pion Momentum Fractions

[ETM Collaboration, PRL 127, 252001 (2021)]

TABLE I. Compilation of results and comparison to literature. All values are at 2 GeV in the $\overline{\text{MS}}$ scheme.

	This work	RQCD [20]	JAM [45]	xFitter [46]
$\langle x\rangle_l^R)$	$0.601(28)(_{-21})$	⋯	⋯	⋯
$\langle x\rangle_s^R$	$0.059(13)(_{-10})$	⋯	⋯	⋯
$\langle x\rangle_c^R$	$0.019(05)(_{-10})$	⋯	⋯	⋯
$\langle x\rangle_g^R$	$0.52(11)(^{+02})$	⋯	0.42(4)	0.25(13)
$\Sigma_f \langle x\rangle_f^R$	$0.68(05)(_{-03})$	0.220(207)	0.58(9)	0.75(18)
$\langle x\rangle_{u+d-2s}^R$	0.48(01)	0.344(28)	⋯	⋯
$\langle x\rangle_{u+d+s-3c}^R$	0.60(03)	⋯	⋯	⋯

The gluon momentum fraction from LQCD is larger than those of JAM and xFitter.

Pion/Kaon PDFs: Mellin moments

ETM Collaboration, PRD 104, 054504 (2021)

$$\langle x^2\rangle^u_{\pi^+} = 0.110(7)(12),$$

$$\langle x^2\rangle^u_{K^+} = 0.096(2)(2),$$

$$\langle x^2\rangle^s_{K^+} = 0.139(2)(1),$$

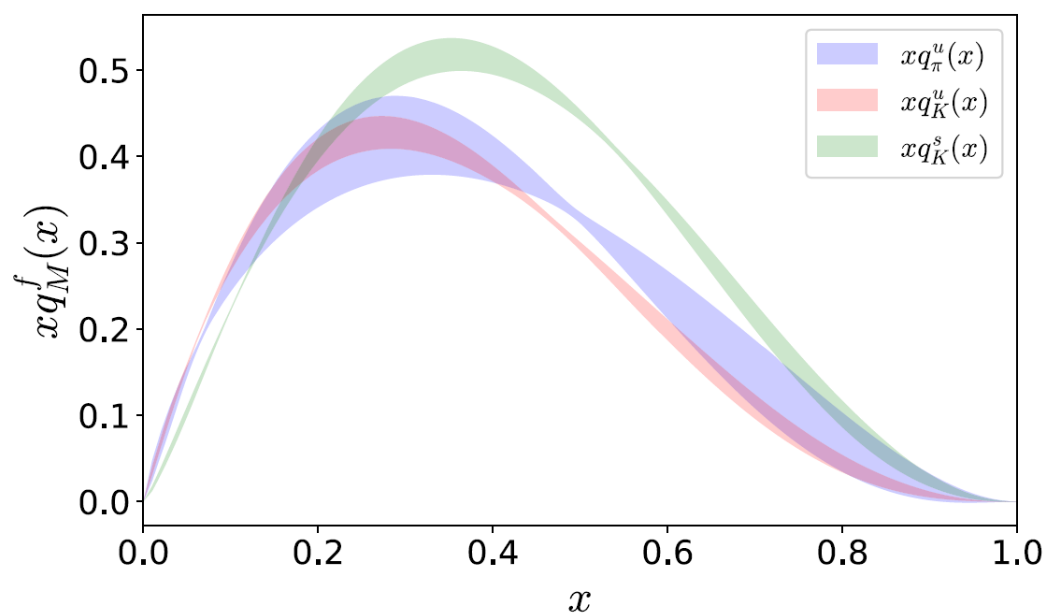
$$\langle x^3\rangle^u_{\pi^+} = 0.024(18)(2),$$

$$\langle x^3\rangle^u_{K^+} = 0.033(6)(1),$$

$$\langle x^3\rangle^s_{K^+} = 0.073(5)(2),$$

$$\langle x^n\rangle = N\int_0^1 dx x^\alpha (1-x)^\beta (1+\gamma x).$$

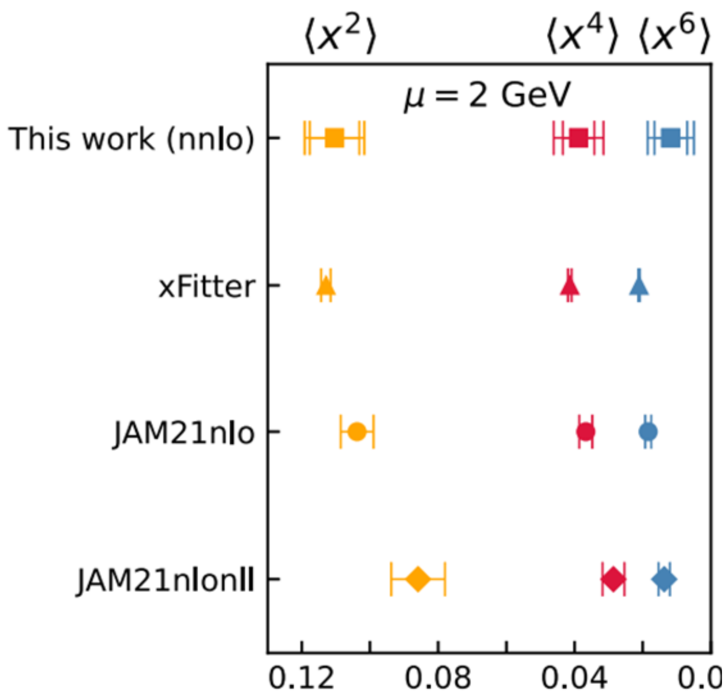
$$\langle x^n\rangle = \frac{\left(\prod_{i=1}^n (i+\alpha)\right)(n+2+\alpha+\beta+(i+1+\alpha)\gamma)}{\left(\prod_{i=1}^n (i+2+\alpha+\beta)\right)(2+\alpha+\beta+(1+\alpha)\gamma)}, \quad n>0.$$

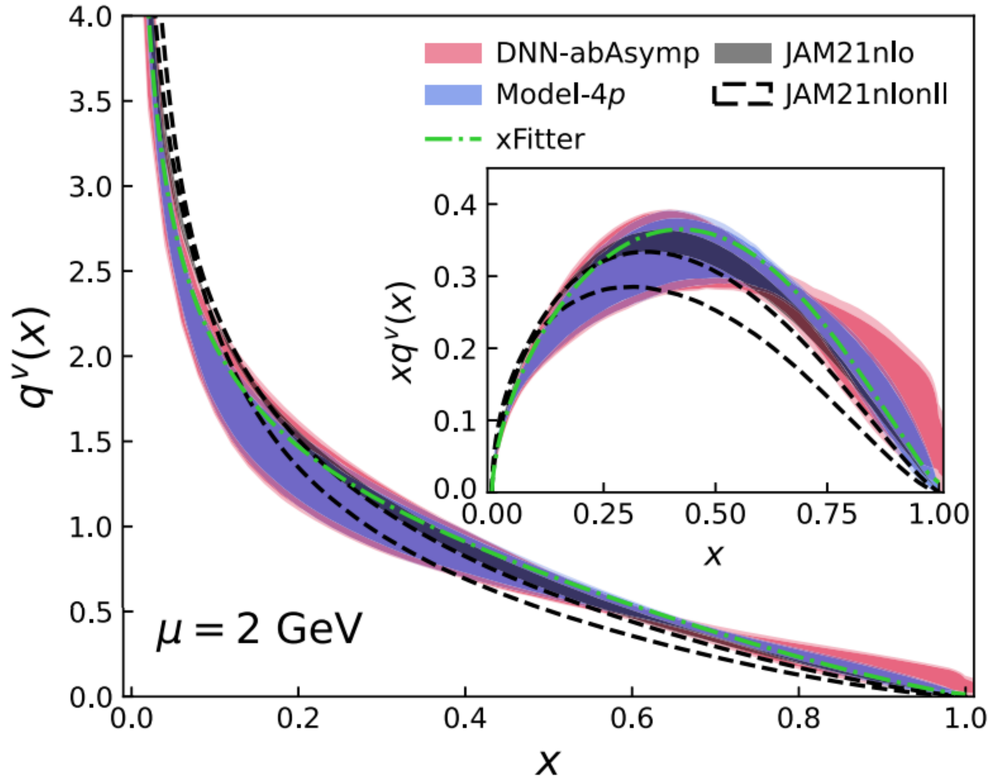


Reconstructed by the lattice data up to <x^3>.

Pion PDFs: Mellin Moments

[Gao et al., [PRD 106, 114510 (2022)]]





Previous work: PRD **102**, 094513 (2020)

Pion PDFs: Mellin Moments

[Gao et al., PRD 106, 114510 (2022)]

$$q(x;\alpha,\beta) = \mathcal{N} x^\alpha (1-x)^\beta,$$
$$q(x;\alpha,\beta,s,t) = \mathcal{N}' x^\alpha (1-x)^\beta (1 + s\sqrt{x} + tx), \qquad (23)$$

denoted by Model-$2p$ and Model-$4p$, in which $\mathcal{N}$ and $\mathcal{N}'$ are normalization factors so that $\int_0^1 q^v(x)dx = 1$. The moments of the PDF therefore can be determined from the model parameters, e.g.,

$$\langle x^n \rangle_{(\alpha,\ldots)} = \int_0^1 x^n q^v(x;\alpha,\ldots)dx. \qquad (24)$$

We then reexpress Eq. (20) using the model parameters and minimize

$$\chi^2 = \sum_{P_z > P_z^0}^{P_z^{\max}} \sum_{z_{\min}}^{z_{\max}} \frac{(\mathcal{M}(z,P_z,P_z^0) - \mathcal{M}_{\text{model}}(z,P_z,P_z^0;\alpha,\ldots))^2}{\sigma^2(z,P_z,P_z^0)}, \qquad (25)$$

$$\mathcal{M}(z,P_z,P_z^0=0) = \sum_{n=0} \frac{(-izP_z)^n}{n!} \langle x^n \rangle_{\text{LO}}(z), \qquad (\text{B1})$$

where we define

$$\langle x^n \rangle_{\text{LO}}(z) = c_n(z^2\mu^2)\langle x^n \rangle(\mu), \qquad c_n(z^2\mu^2) = \frac{C_n(z^2\mu^2)}{C_0(z^2\mu^2)}. \qquad (\text{B2})$$

$$\mathcal{M}(z,P_z,P_z^0) = \frac{\sum_n c_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \langle x^n \rangle + r(aP_z)^2}{\sum_n c_n(\mu^2 z^2) \frac{(-izP_z^0)^n}{n!} \langle x^n \rangle + r(aP_z^0)^2}. \qquad (20)$$

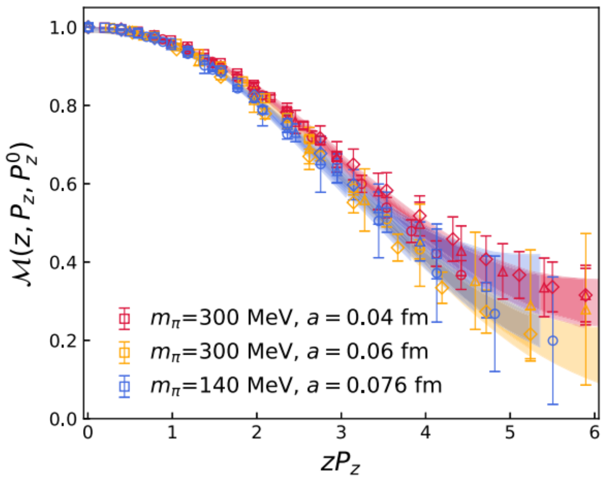


FIG. 11. Ratio scheme renormalized matrix elements $\mathcal{M}(z,P_z,P_z^0)$ of $P_z > P_z^0 = 0.25$ GeV are shown for the three ensembles, with the bands being the fit results of the Model-$4p$ [cf. Eq. (23)] using $z \in [2a, 0.61 \text{ fm}]$.

Pion/Kaon PDFs: Momentum Fractions
ETM Collaboration, arXiv:2404.08529

TABLE II. Compilation of results for the pion and for the kaon in the continuum limit. All quantities are presented at the scale 2 GeV in the $\overline{\text{MS}}$ scheme.

	π	K
$\langle x \rangle_{l,\mathrm{R}}$	0.448(34)	0.260(09)
$\langle x \rangle_{s,\mathrm{R}}$	0.043(15)	0.333(11)
$\langle x \rangle_{c,\mathrm{R}}$	0.019(17)	0.024(17)
$\langle x \rangle_{g,\mathrm{R}}$	0.388(49)	0.408(61)
$\langle x \rangle_{q,\mathrm{R}}$	0.532(56)	0.618(32)
$\langle x \rangle_{u+d-2s,\mathrm{R}}$	0.382(17)	−0.409(16)
$\langle x \rangle_{u+d+s-3c,\mathrm{R}}$	0.445(48)	0.487(39)