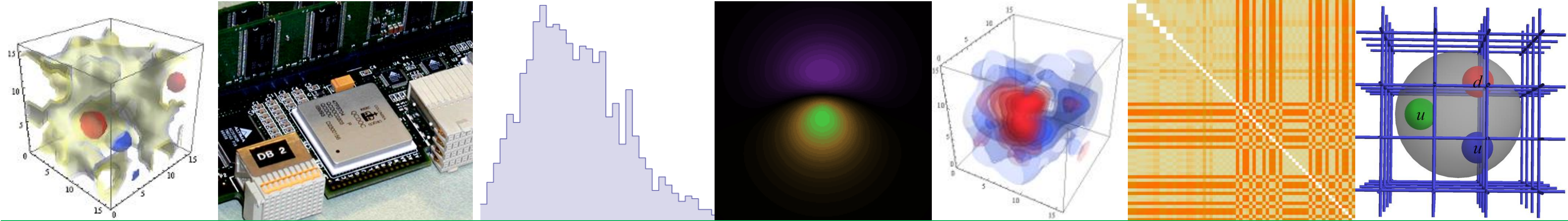


Advancing Nucleon and Pion Parton Distributions with Lattice QCD: Insights and Impact on Global Analyses

HUEY-WEN LIN

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

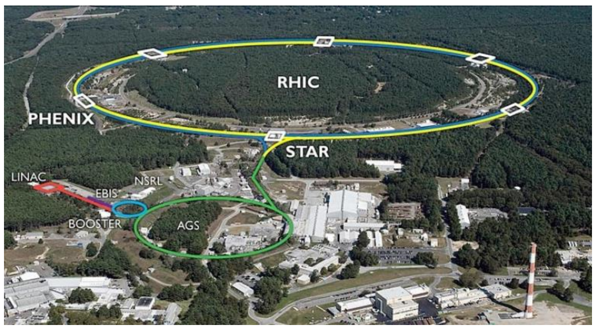
Outline

- Lattice QCD and Parton Distribution Functions
- Selected Nucleon and Meson *x*-Dependent Parton Distributions
- Impact of Lattice-QCD PDFs on Global Fits

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, EIcC, LHeC, …)



Electron Ion Collider:
The Next QCD Frontier

Imaging of the proton

How are the *sea* *quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?*

EIC White Paper, 1212.1701; [The Present and Future of QCD](2303.02579) (2303.02579)

Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

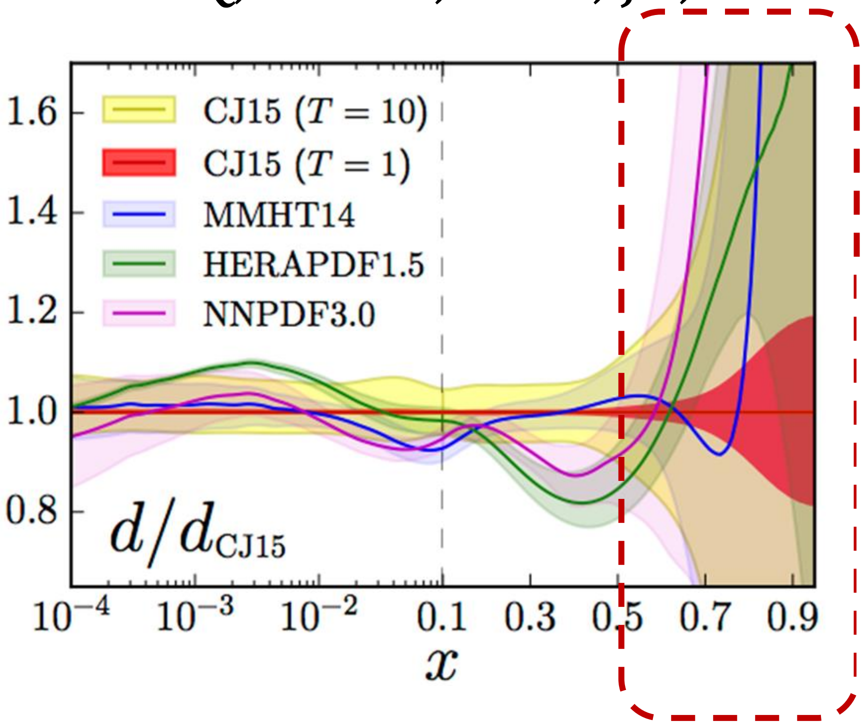
- Assumptions imposed

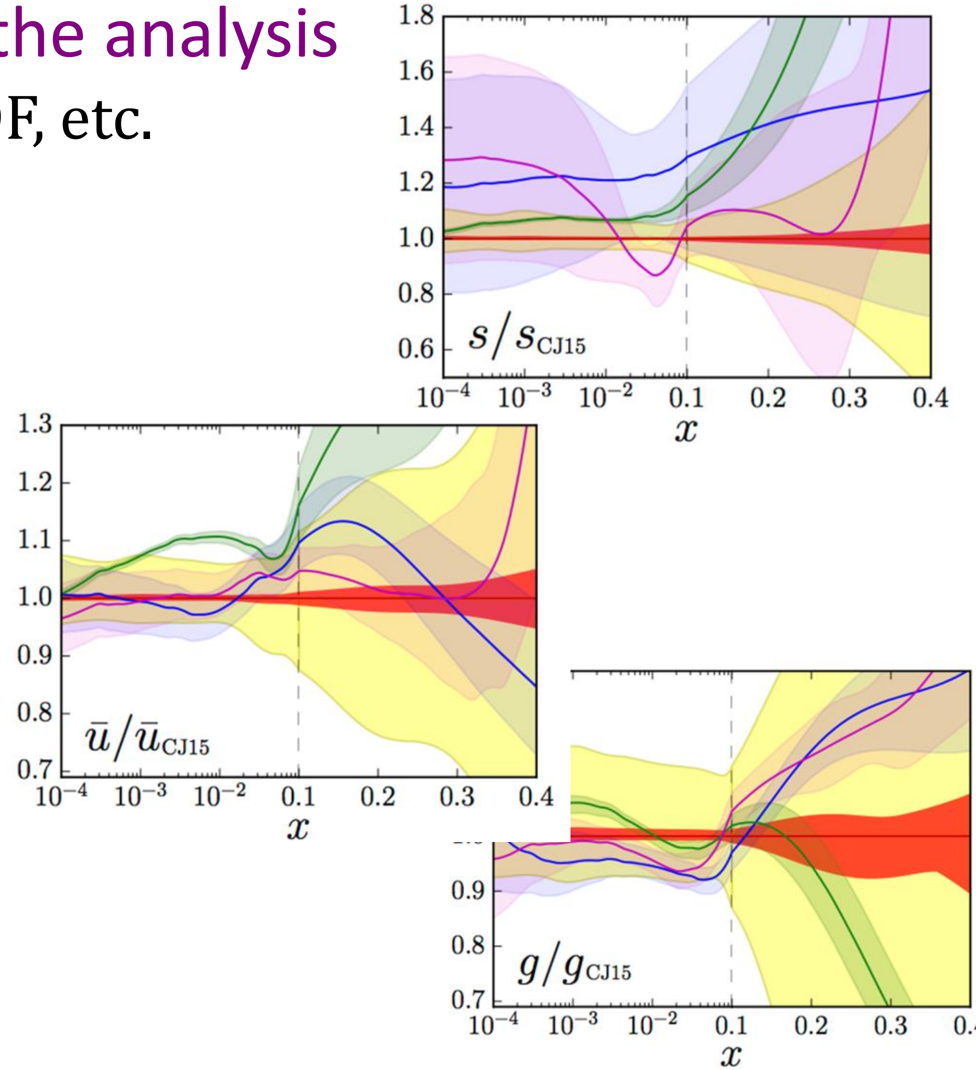
SU(3) flavor symmetry, charge symmetry, strange and sea distributions

$$s = \bar{s} = \kappa\left(\bar{u} + \bar{d}\right)$$

Global Analysis

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.





CTEQ-JLAB
https://www.jlab.org/theory/cj/

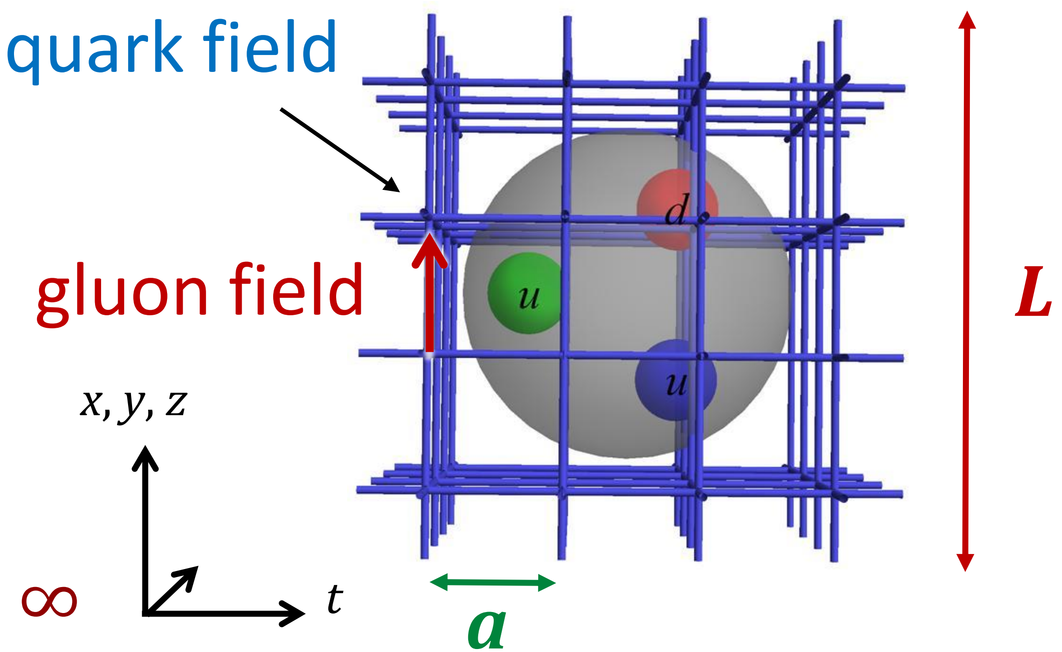
Lattice QCD 101

- Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}A \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, e^{iS(\bar{\psi},\psi,A)} O(\bar{\psi}, \psi, A)$$

  in **Euclidean** space

  - Quark mass parameter (described by m_π)
  - Impose a UV cutoff
    discretize spacetime
  - Impose an infrared cutoff
    finite volume



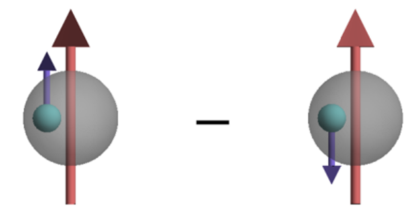
- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}}, \boldsymbol{a} \to \boldsymbol{0}, \boldsymbol{L} \to \infty$$

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	0.10 — 1.1
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	-0.14 — 0.91
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	-0.97 — 0.47
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	N/A
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
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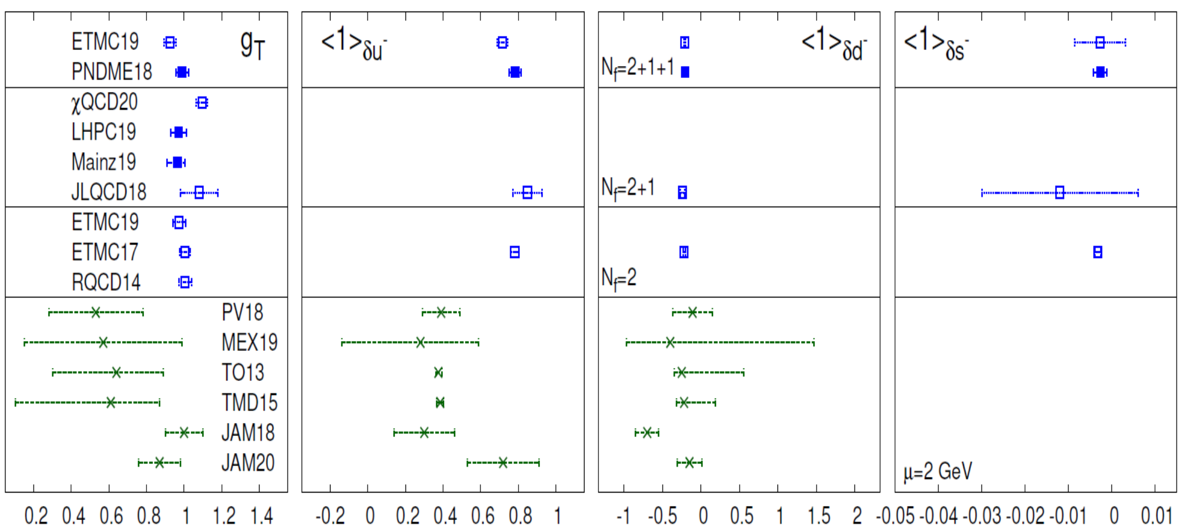
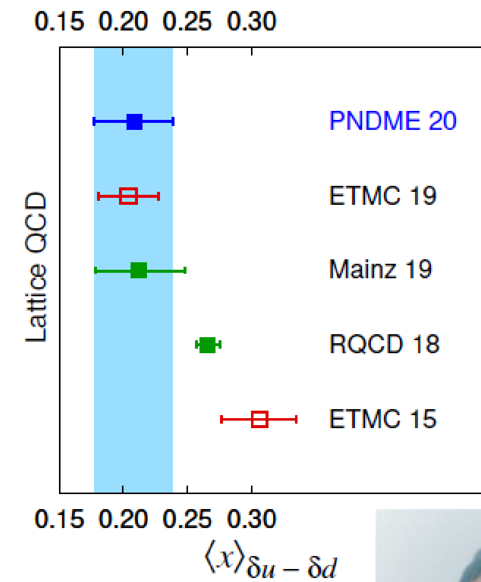
- Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$

LatticePDF Reports, 1711.07916, 2006.08636



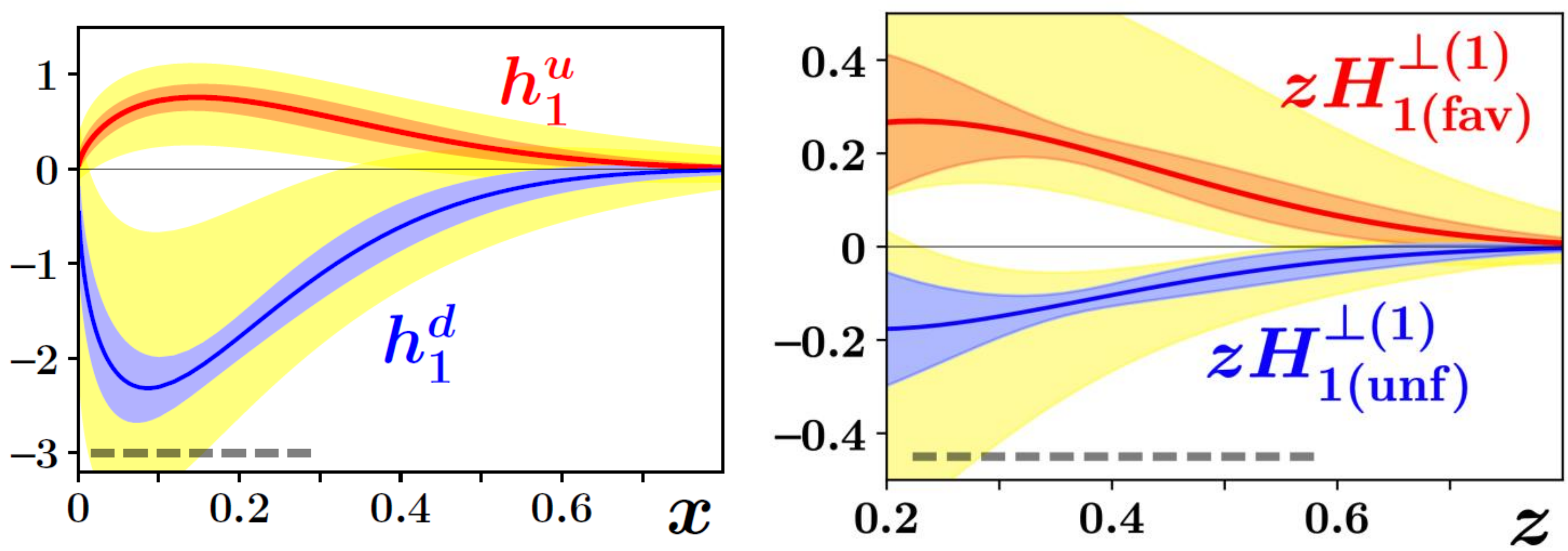


S. Mondal et al 2005.13779

From Charges to PDFs

§ Improved **transversity** distribution with LQCD tensor charge, $g_T = \int_{-1}^{1} dx\,(\,h_1^u(x) - h_1^d(x)\,)$

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets

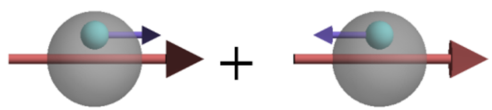


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

PDFs on the Lattice

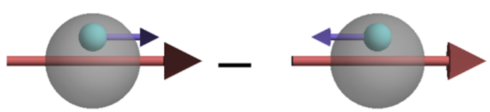
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

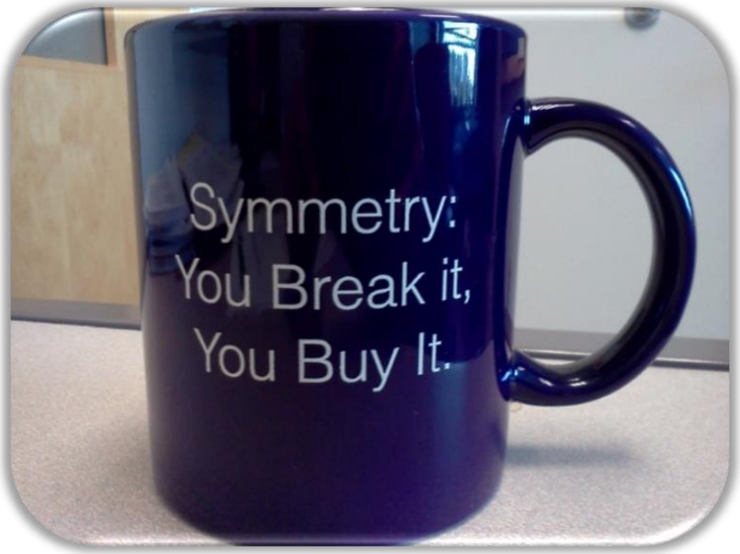
spin-dependent transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

§ Limited to the lowest few moments

- For higher moments, all ops mix with lower-dimension ops
- Novel proposals to overcome this problem

W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501

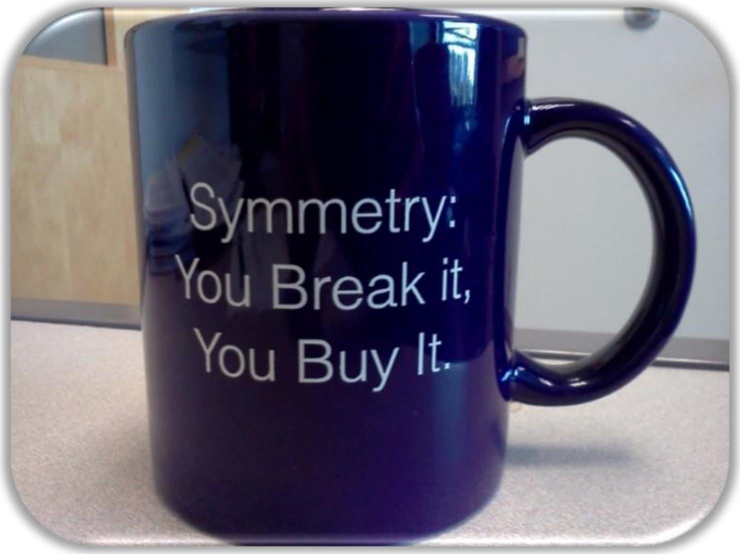
Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505

A. Shindler, arXiv:2311.18704

THE HOPE COLLABORATION

Sheng Pin Chang (Kaon moments) @ Tue. 4:00 PM

Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

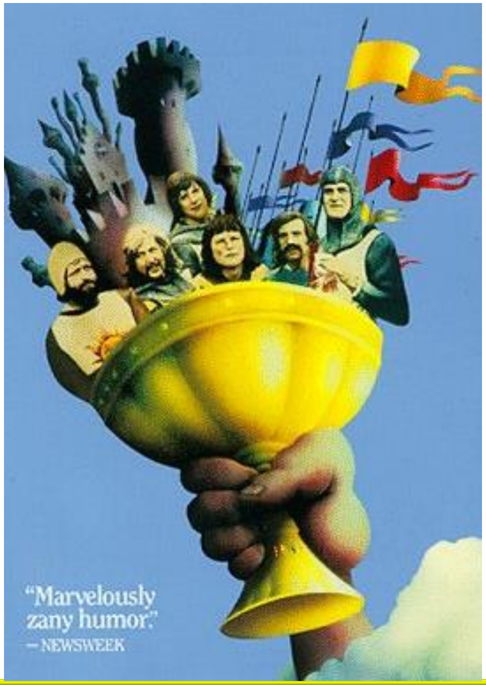
§ Longstanding obstacle!

- Holy grail of structure calculations

§ Applies to many structure quantities:

- Parton distribution functions (PDF)
- Generalized parton distributions (GPD)
- Transverse-momentum distributions (TMD)

⋮



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

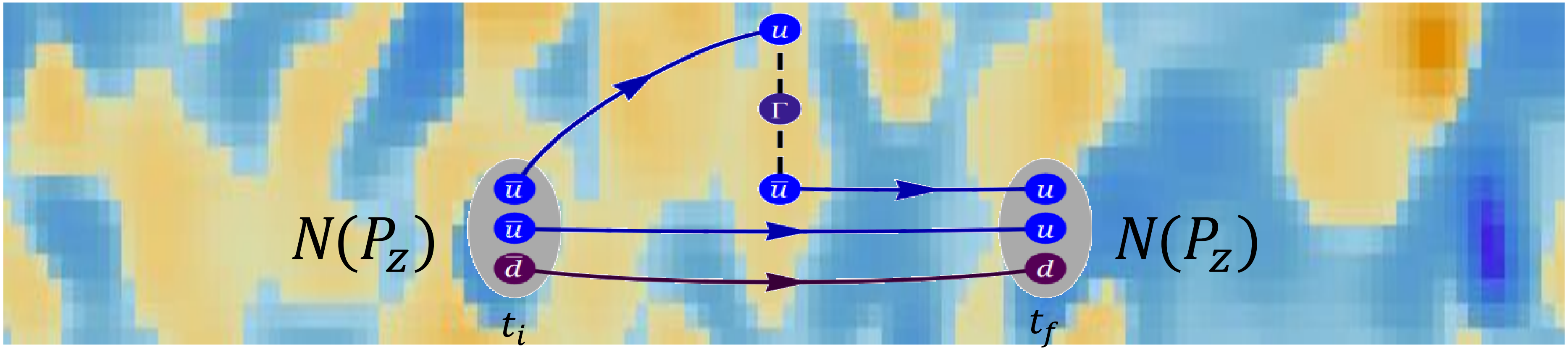
Outline

- Lattice QCD and Parton Distribution Functions
- **Selected Nucleon and Meson *x*-Dependent Parton Distributions**
- Impact of Lattice-QCD PDFs on Global Fits

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF

(X. Ji, 2013; See 2004.03543 for review)



§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0)\right| P\right\rangle$$

§ Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{\boldsymbol{P_z^2}}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x\boldsymbol{P_z})^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{((1-x)\boldsymbol{P_z})^2}\right)$$

X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF

(X. Ji, 2013; See 2004.03543 for review)

Additional source of systematics: $\boldsymbol{P_z}$

Smaller P_z gives better signal but larger systematics (like how heavier pion mass gives better precision)

New parameters in x-dependent methods to pay attention to

§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

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X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today $= \sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

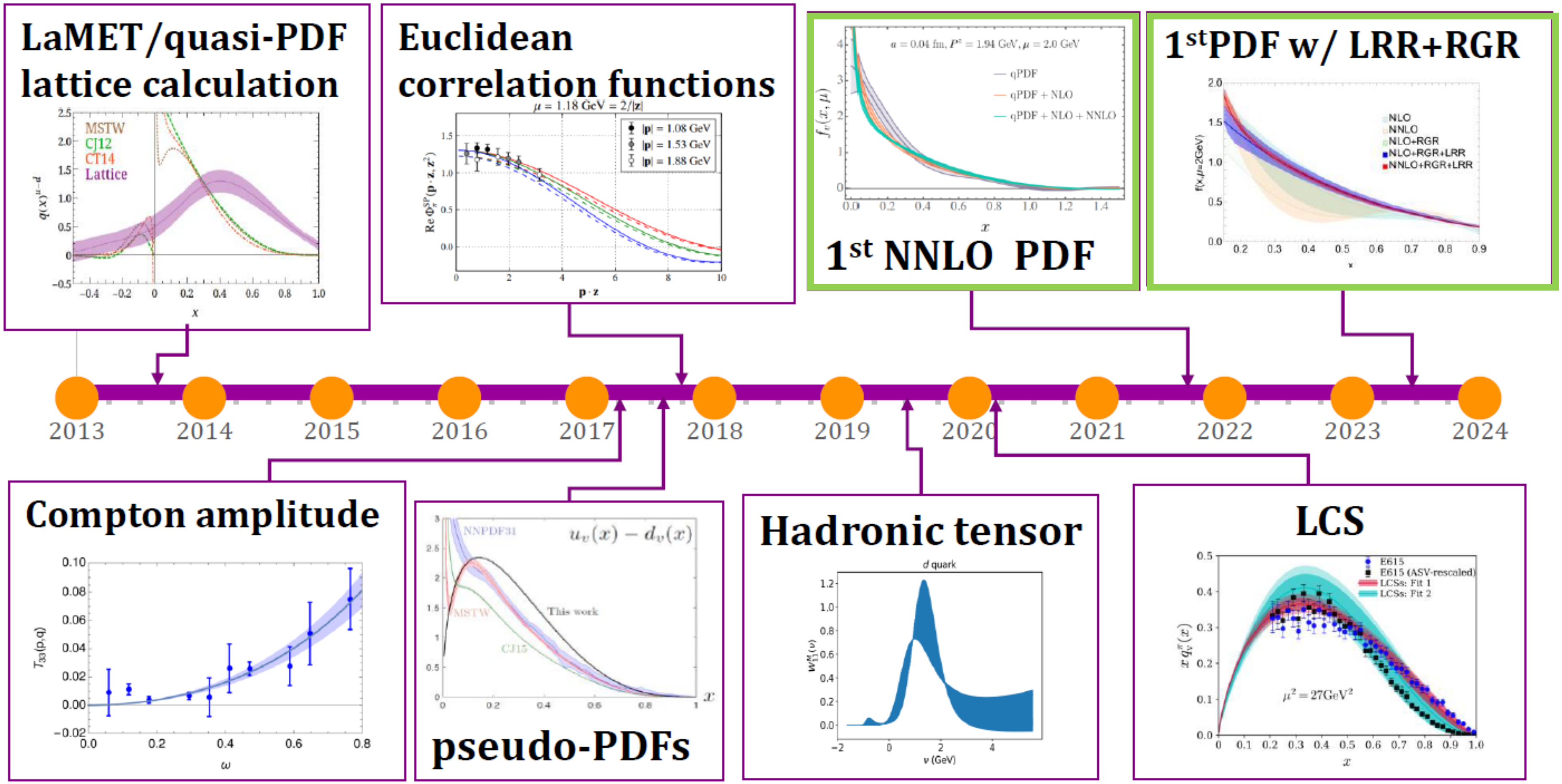
- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Compton amplitude method (A.J. Chambers et al., 1703.01153)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)

⋮

Lattice Parton Calculations

§ Rapid developments!

HL, Prog.Part.Nucl.Phys. 144 (2025)

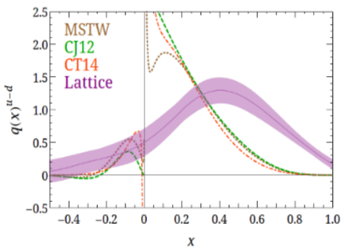


CSSM/QCDSF; Ross Young @ Tue. 3:00pm

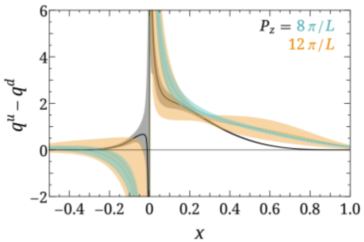
Lattice Parton Calculations

§ Physics-quantity milestones

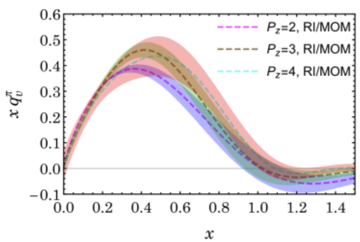
First unpol. lattice PDF



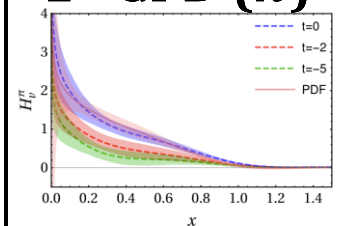
First PDFs at $M_\pi^{\rm phys}$



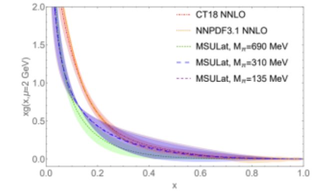
Pion v-PDF



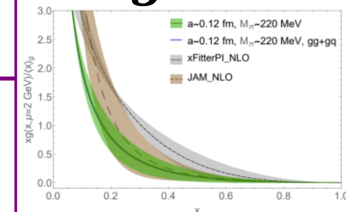
1st GPD (π)



N g-PDF

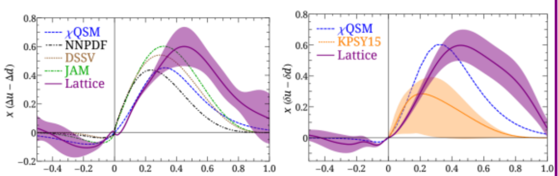


π g-PDF

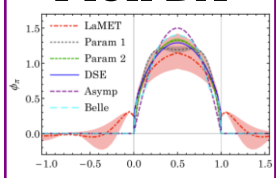


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

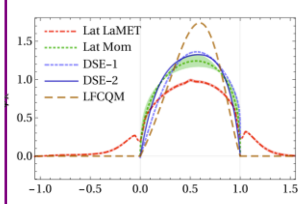
Pol. PDFs and mass corrections



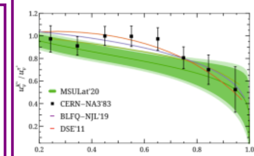
Pion DA



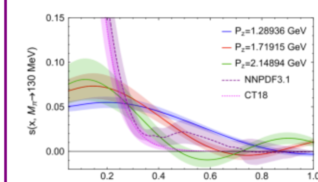
Kaon DA



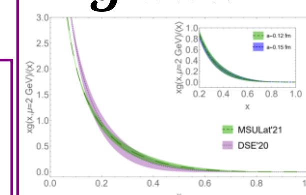
K PDF



s,c PDF



Kaon g-PDF

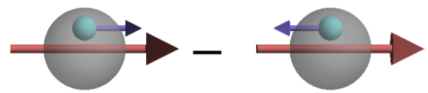


HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

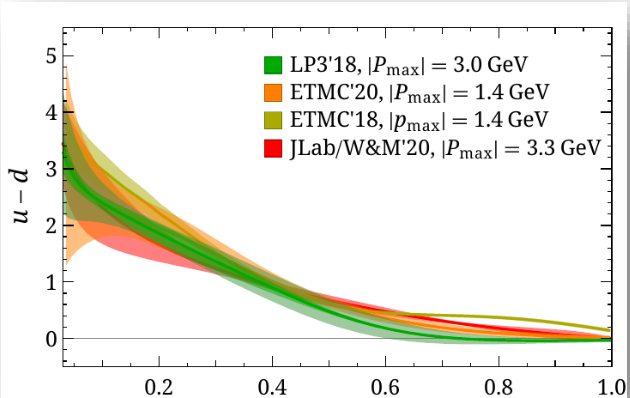
Lattice Example Results

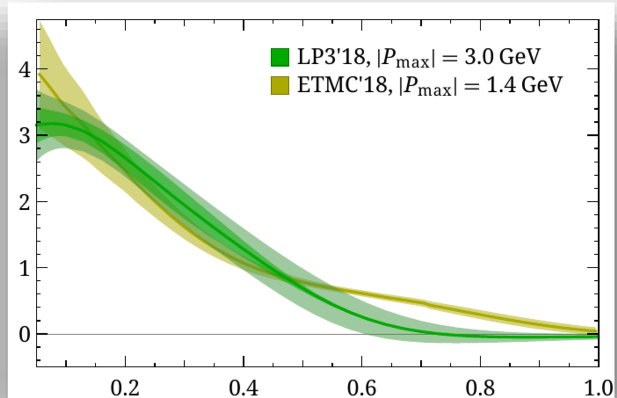
§ Summary of PDF results at physical pion mass

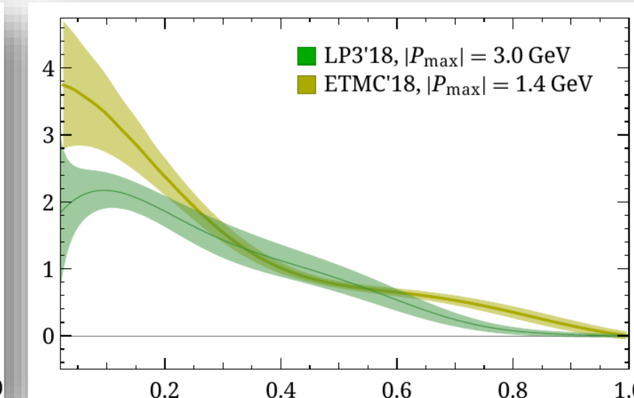
unpolarized	longitudinally polarized	transversely polarized

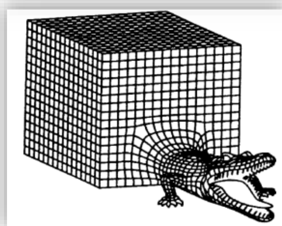


$u(x) - d(x)$ $\Delta u(x) - \Delta d(x)$ $\delta u(x) - \delta d(x)$









Finite volume,
Discretization,
…

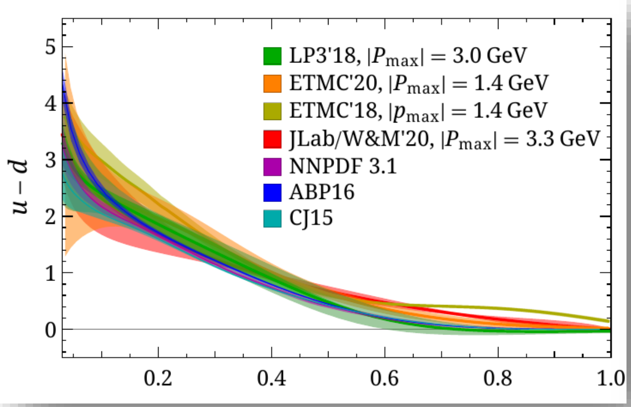
2006.08636 (PDFLattice2019)

Lattice Example Results

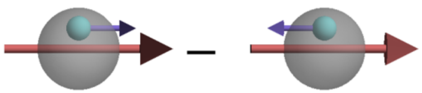
§ Summary of PDF results at physical pion mass

unpolarized

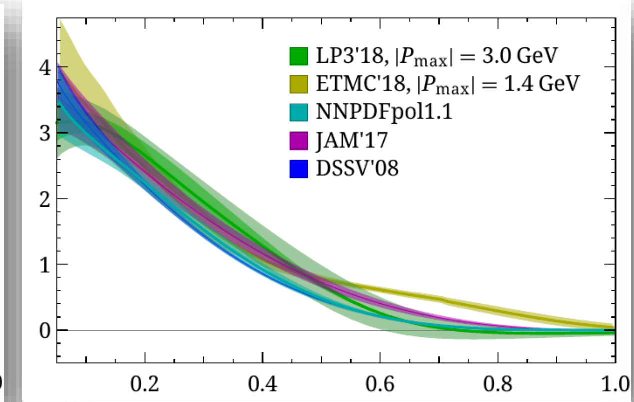
$u(x) - d(x)$



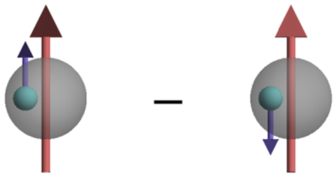
longitudinally polarized



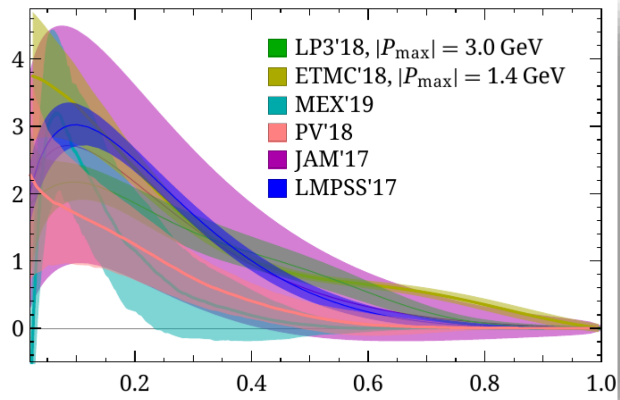
$\Delta u(x) - \Delta d(x)$

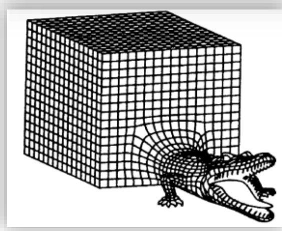


transversely polarized



$\delta u(x) - \delta d(x)$





Finite volume,
Discretization,
...

2006.08636 (PDFLattice2019)

Isovector PDFs Update

§ Nucleon isovector PDF calculated directly at **physical pion mass**

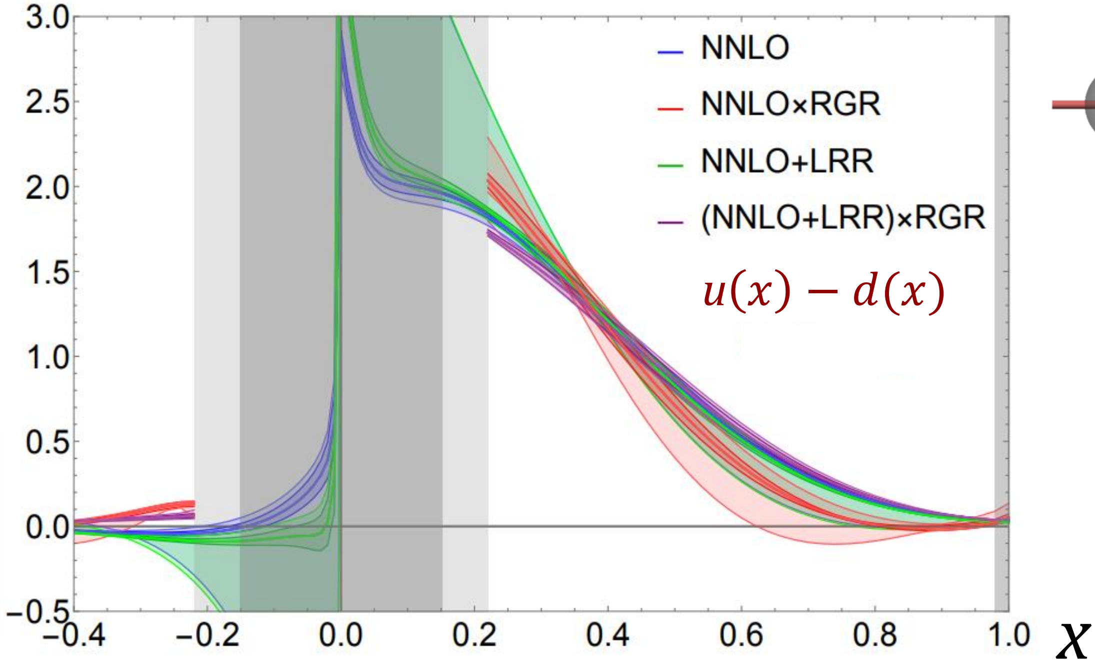
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

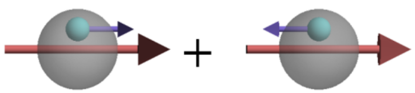
R. Zhang, et. al.
PLB 844, 138081 (2023)

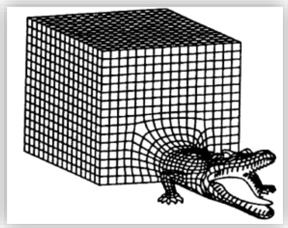
- $N_f = 2+1+1$ clover/HISQ, $a \approx 0.09$ fm, $P_z \approx 2$ GeV

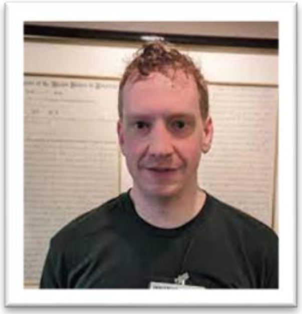
J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…









P: Jack Holligan

Isovector PDFs Update

§ Nucleon isovector PDF calculated directly at **physical pion mass**

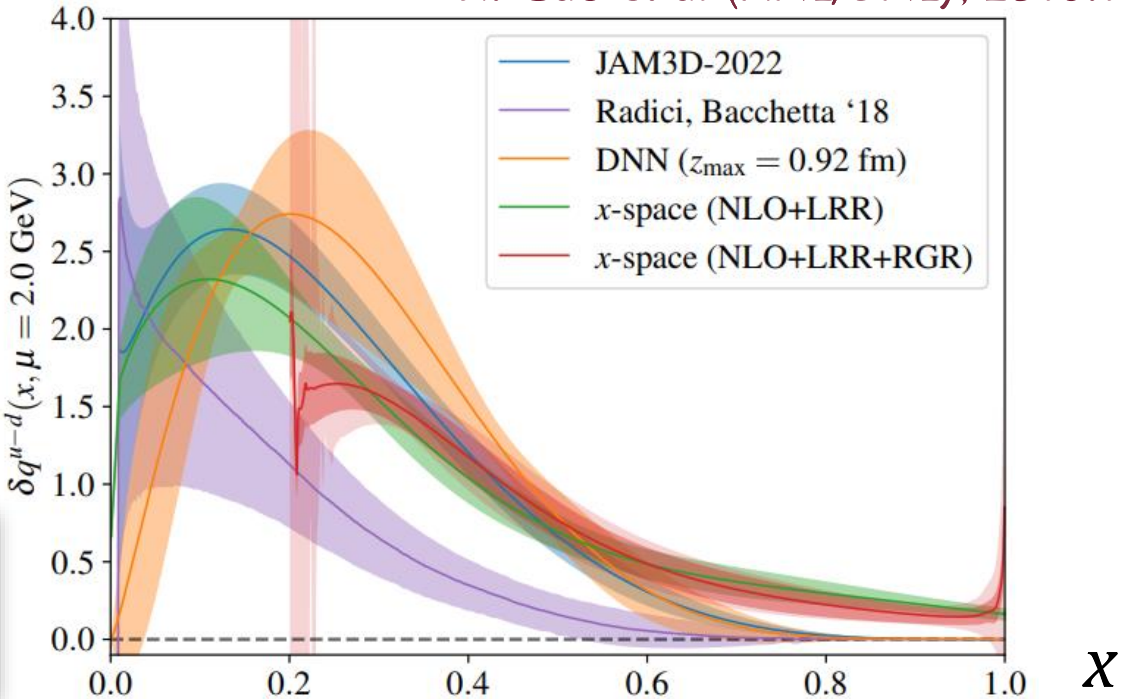
- NNLO matching & treat leading-renormalon effects 
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al.
PLB 844, 138081 (2023)

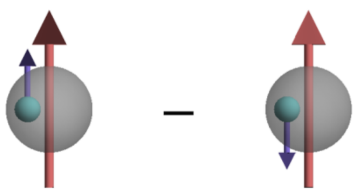
- $N_f = 2+1$ clover/HISQ, $a \approx 0.076$ fm, $P_z \approx 1.5$ GeV

X. Gao et al (ANL/BNL), 2310.19047 [hep-lat]

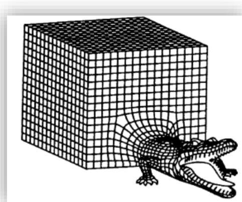
Wanted PDFs, GPDs, etc…



Transversity



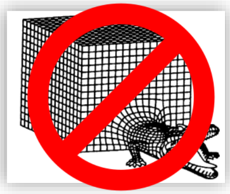
$\delta u(x) - \delta d(x)$



Continuum PDF

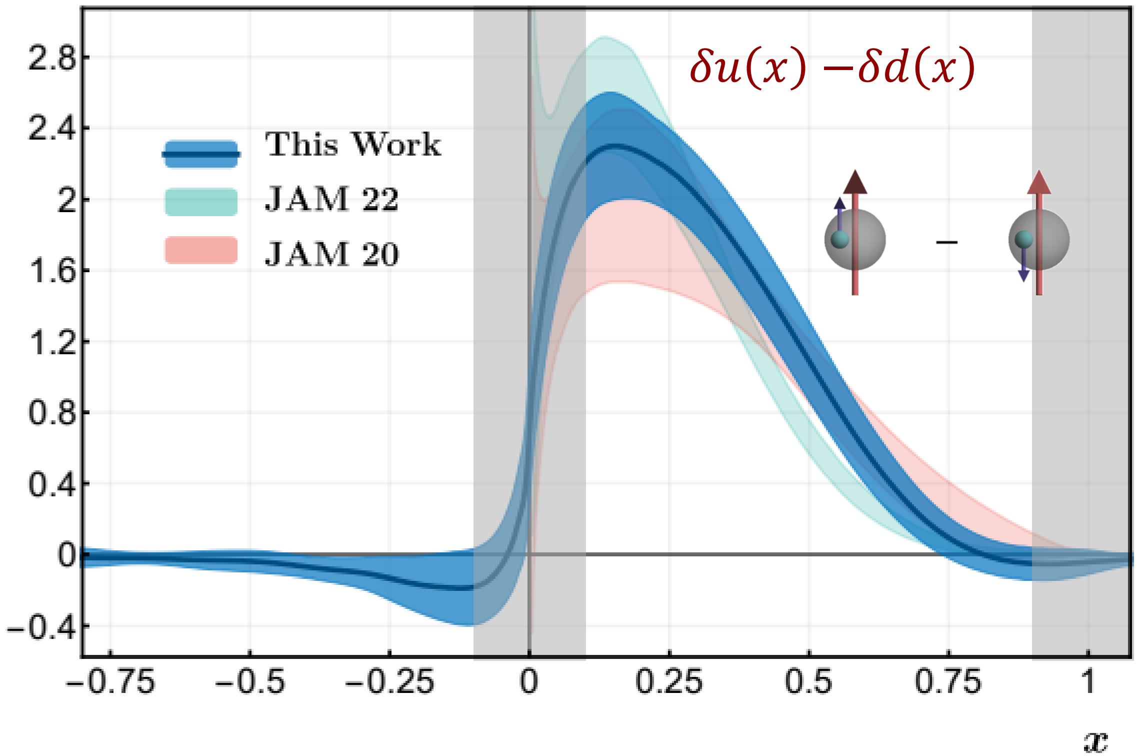
§ Nucleon PDFs using quasi-PDFs in the continuum limit

- Lattice details: clover/2+1 clover (LPC)
  - $a \approx \{0.49, 0.64, 0.85, 0.98\}$ fm,
  - $M_\pi \in [222, 354]$-MeV pion,
  - $M_\pi L \in [3.9, 8.1]$
  - $P_z \in [1.8, 2.8]$



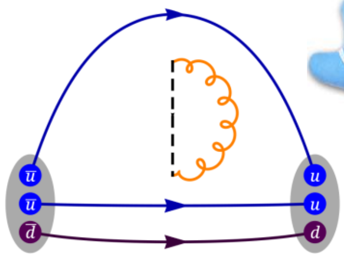
F. Yao et al (LPC), 2208.08008

Wanted PDFs, GPDs, etc…



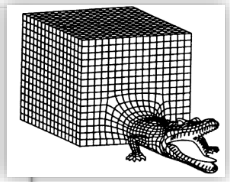
Nucleon Gluon PDF (2020)

§ Gluon PDF using pseudo-PDF

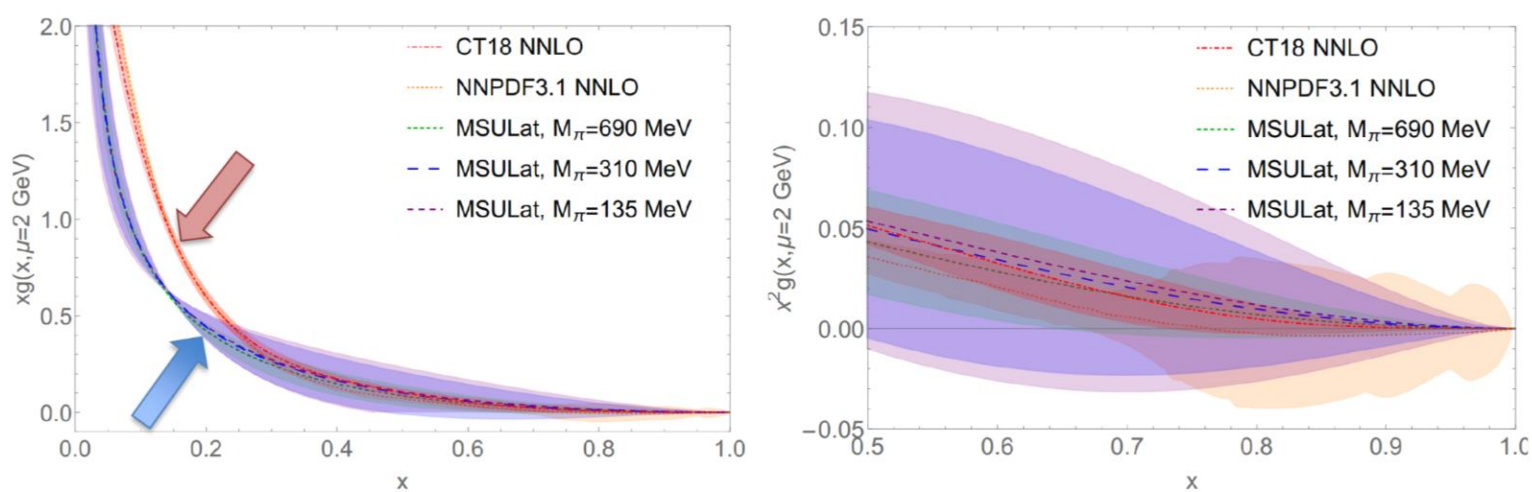


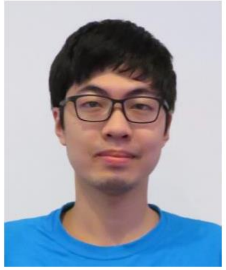
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

Z. Fan. et al (MSULat), 2007.16113



The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\mathrm{MS}}$ scheme.

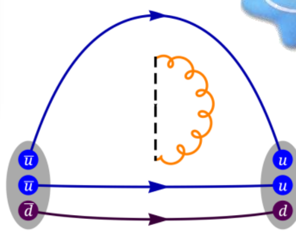




Slide by Zhouyou Fan@DNP2020

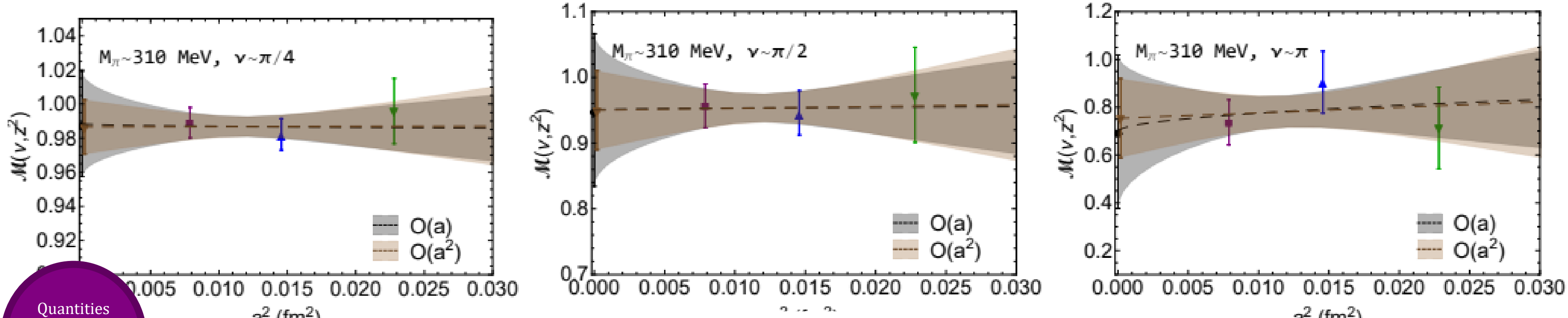
FG: Zhouyou Fan

§ Continuum Gluon PDF w/ pseudo-PDF

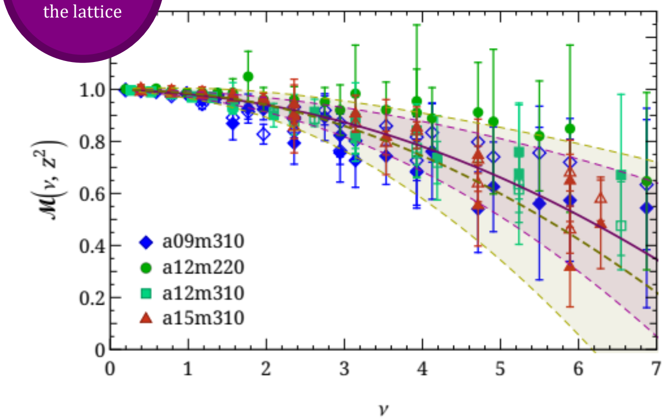


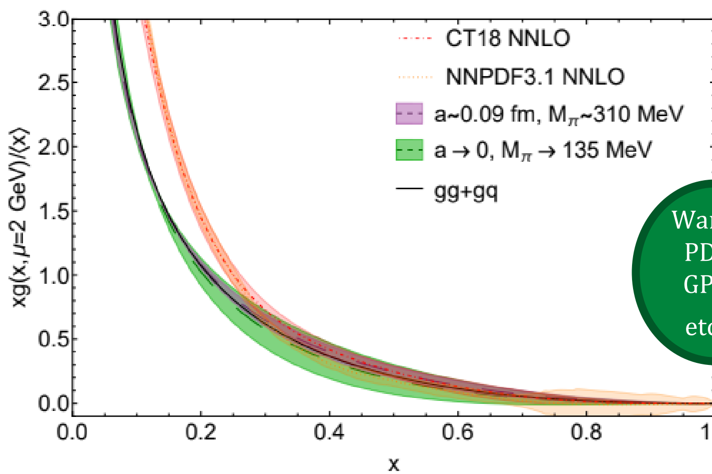
- 2+1+1 HISQ {0.09, 0.12, 0.15} fm
  [220, 310, 700]-MeV pion, 10^5–10^6 statistics

Z. Fan et al (MSULat), 2210.09985



Quantities that can be calculated on the lattice



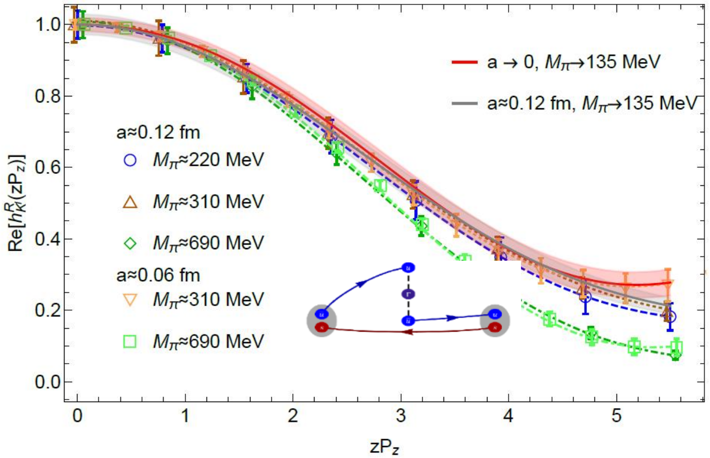


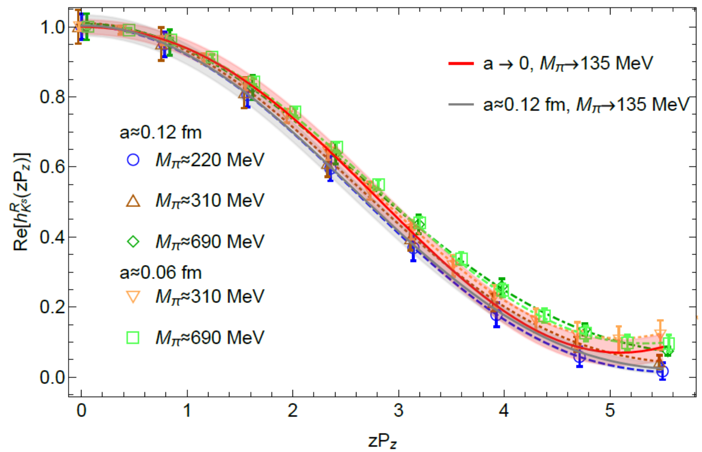
Wanted PDFs, GPDs, etc…

G: Bill Good

Meson Valence-quark PDFs

§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

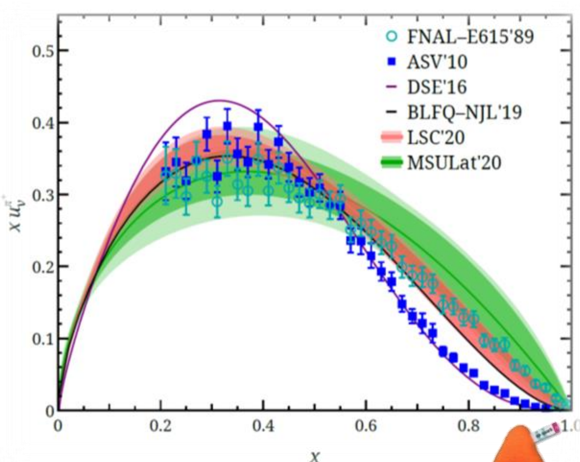


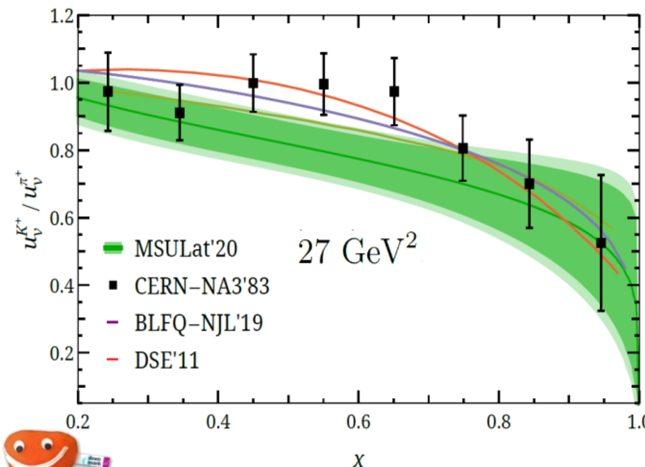


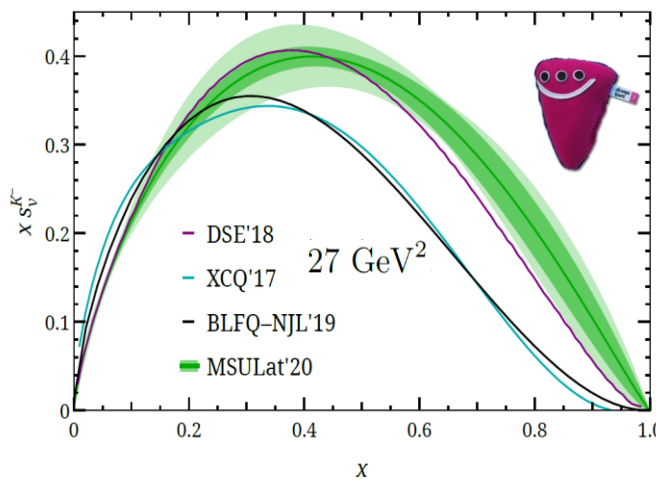
Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

MSULat, 2003.14128







Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

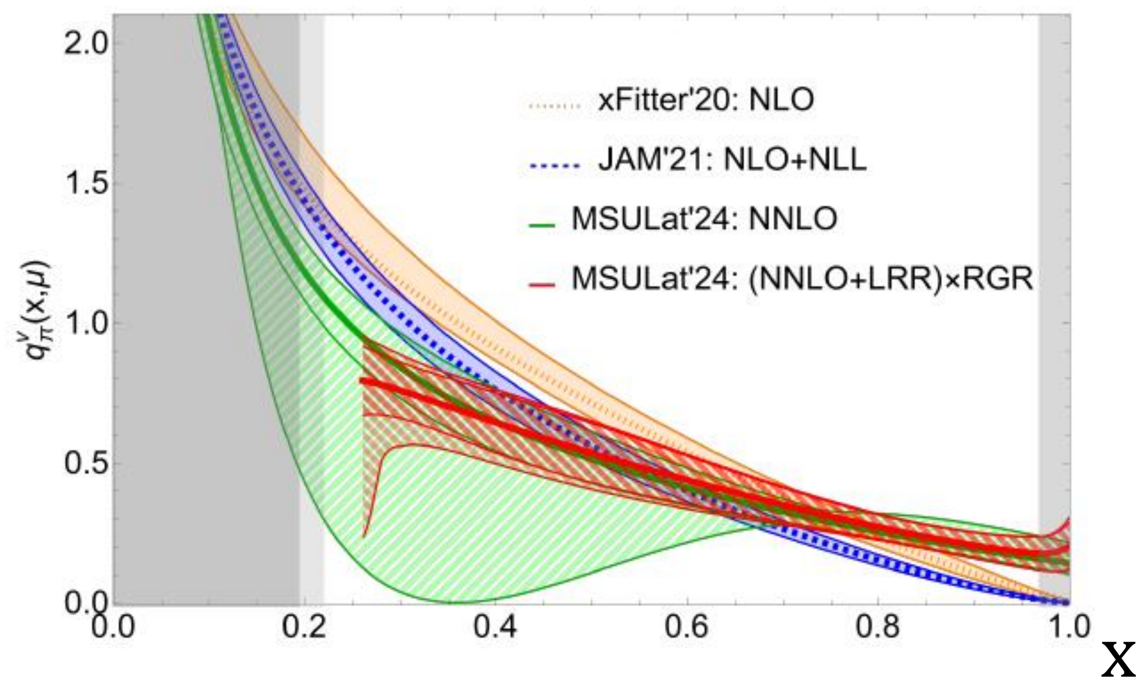
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

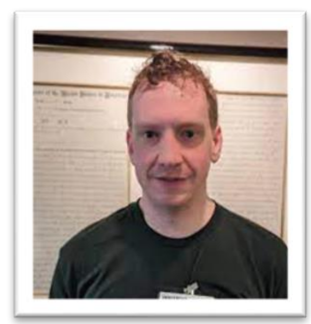
R. Zhang, et. al.
PLB 844, 138081 (2023)

- N_f=2+1+1 clover/HISQ, a~0.09 fm

J. Holligan, HL (MSULat), [10.1088/1361-6471/ad3162](https://doi.org/10.1088/1361-6471/ad3162)

Wanted PDFs, GPDs, etc…

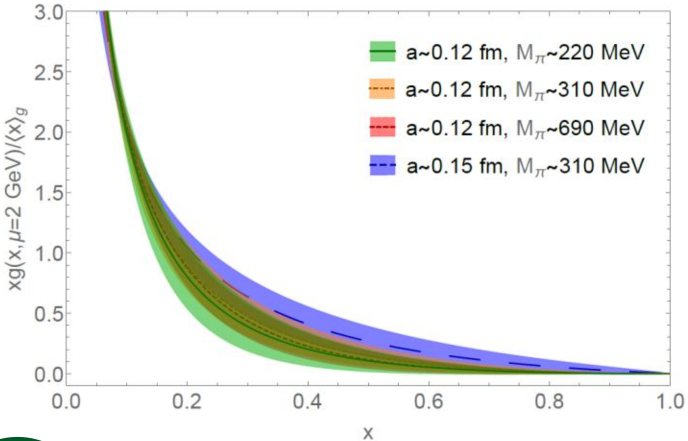


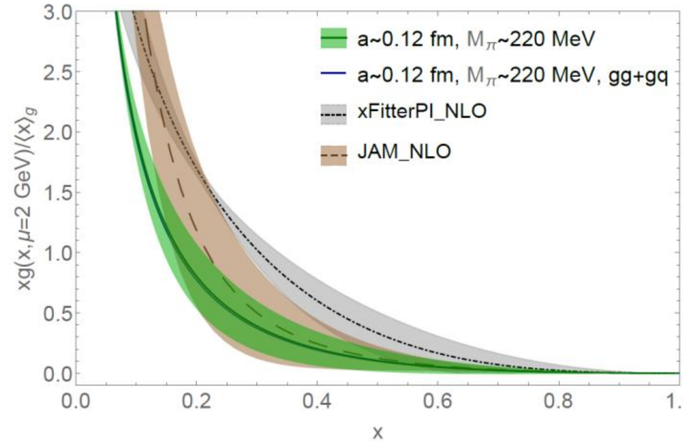


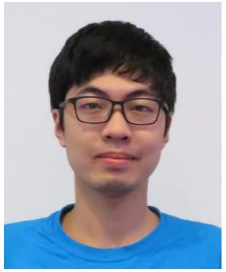
P: Jack Holligan

Meson Gluon PDFs

§ First pion and kaon gluon PDFs $g(x)/\langle x\rangle$ using pseudo-PDF



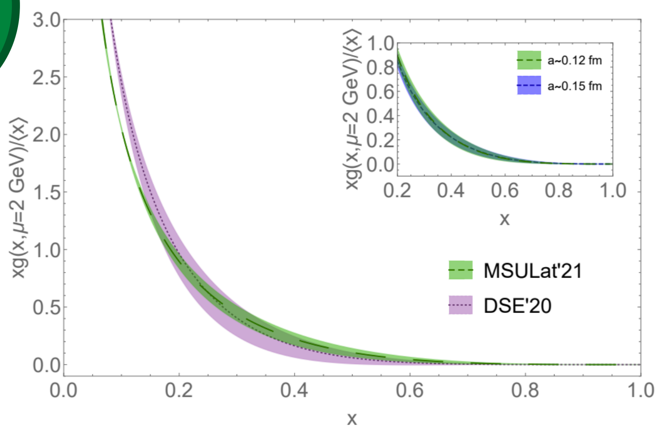




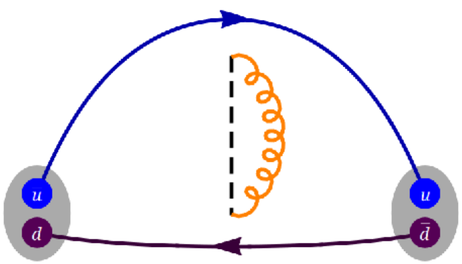
G: Zhouyou Fan

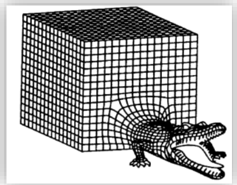
Wanted PDFs, GPDs, etc…

2104.06372, Fan et al. (MSULat); 2112.03124, Salas-Chavira et al. (MSULat)



G: Alejandro Salas-Chavira





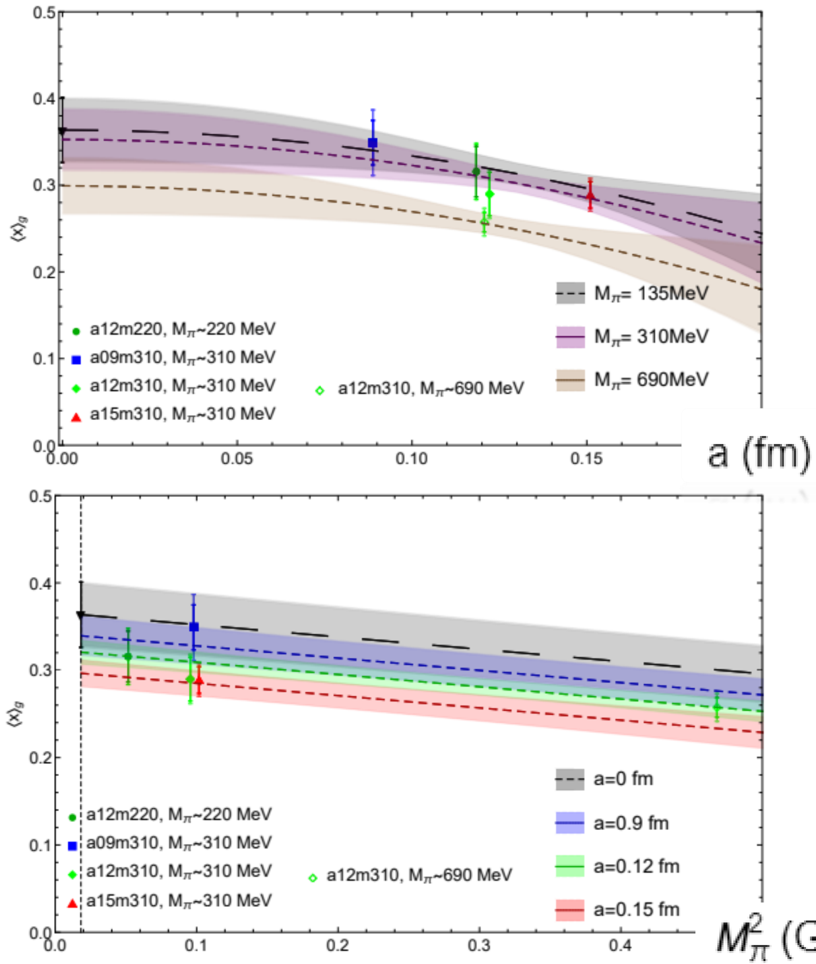
finite-volume, discretization, heavy quark mass, …

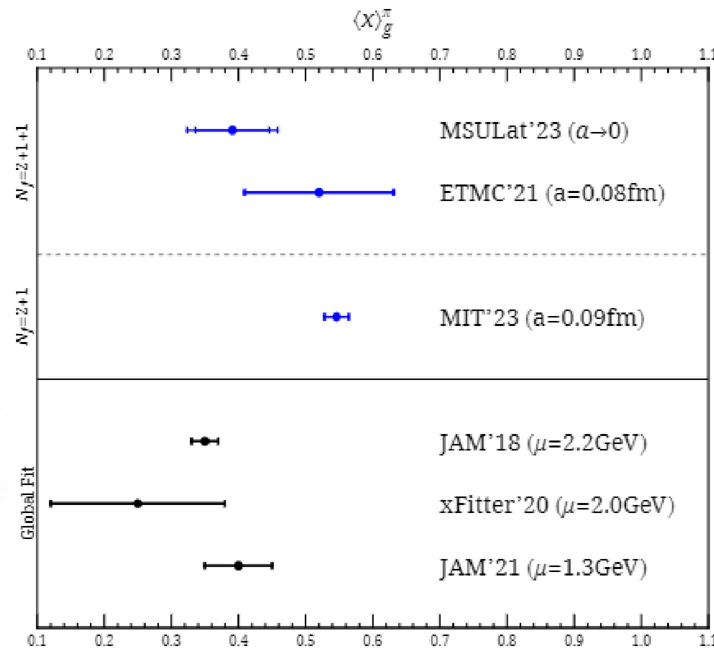
§ What does lattice QCD say about $g(x)$?

Pion Gluon PDF Update

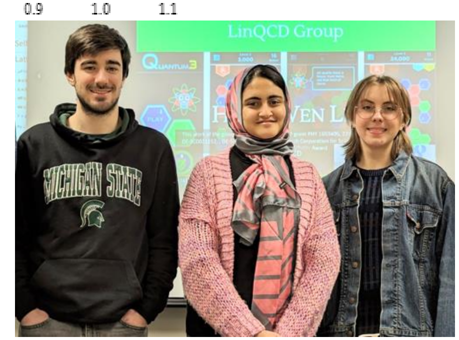
§ Study discretization systematic in $\langle x\rangle_{\{\pi,g\}}$

- Lattice details: clover/HISQ, HISQ, a ~{0,15, 0.12, 0.09} fm





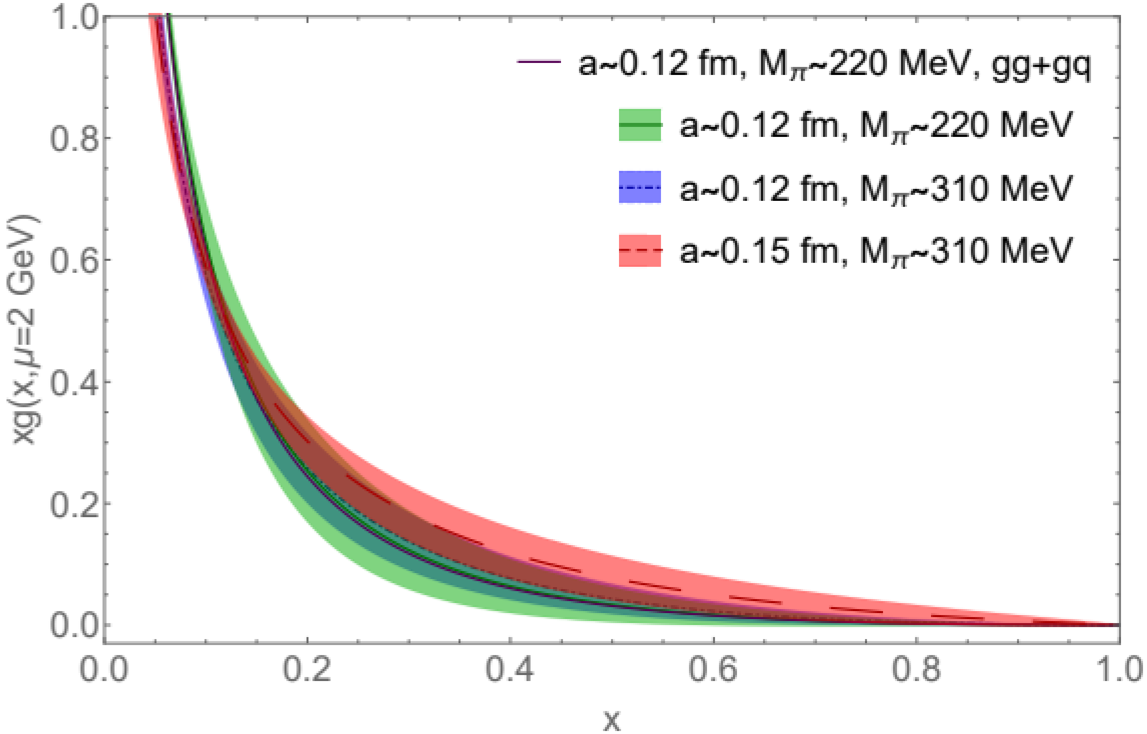
2310.12034, Good et al. (MSULat)



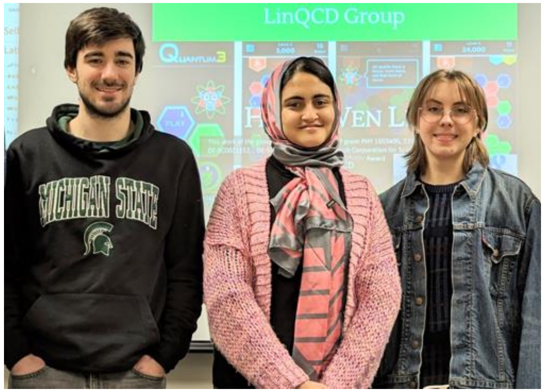
Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

- Update previous calculated $g(x)/\langle x\rangle$ in 2021

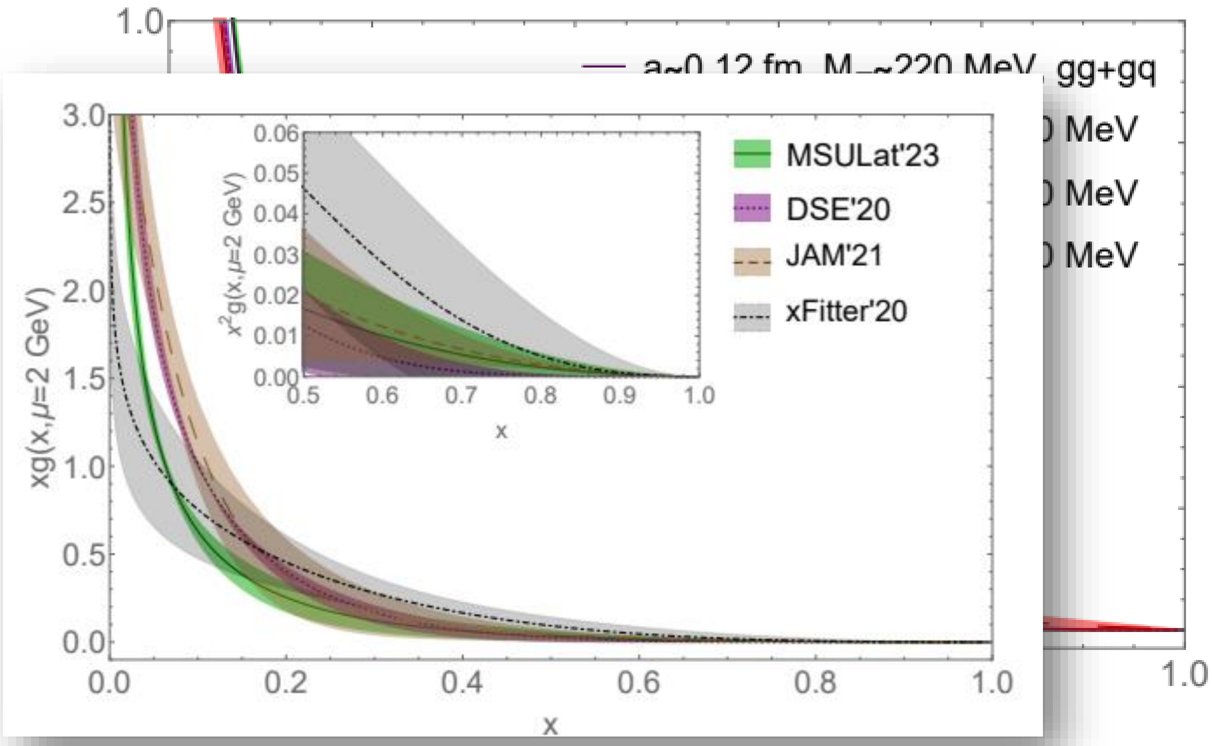


2310.12034, Good et al. (MSULat)

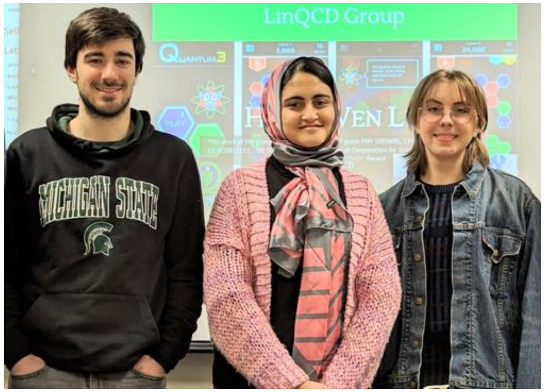


§ Back to Pion gluon PDF $g(x)$

- Update previous calculated $g(x)/\langle x\rangle$ in 2021



2310.12034, Good et al. (MSULat)

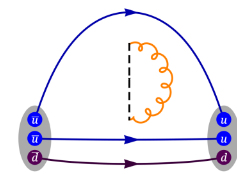


First LaMET Gluon PDF

§ Gluon PDF w/ quasi-PDF (no parameterization)

- 2+1+1 clover/HISQ 0.12 fm, 690-MeV pion, 10^6 statistics

  Good et al (MSULat), 2505.13321



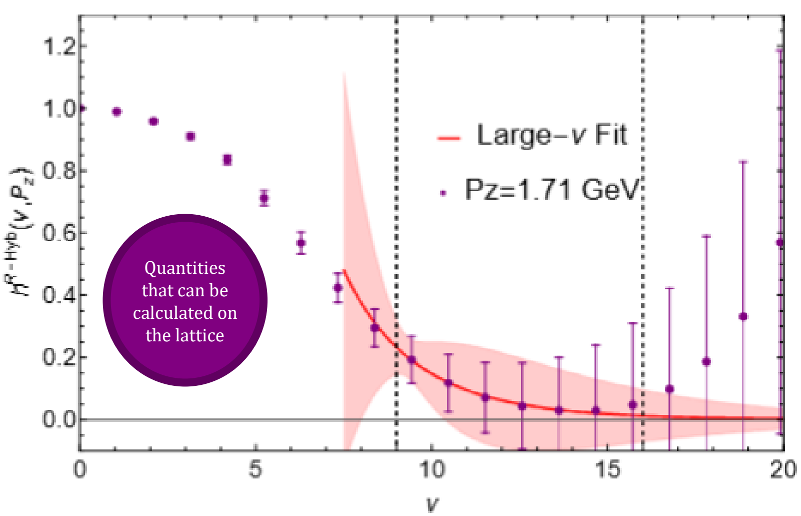
- PQCD calculated Wilson coefficient and matching kernel

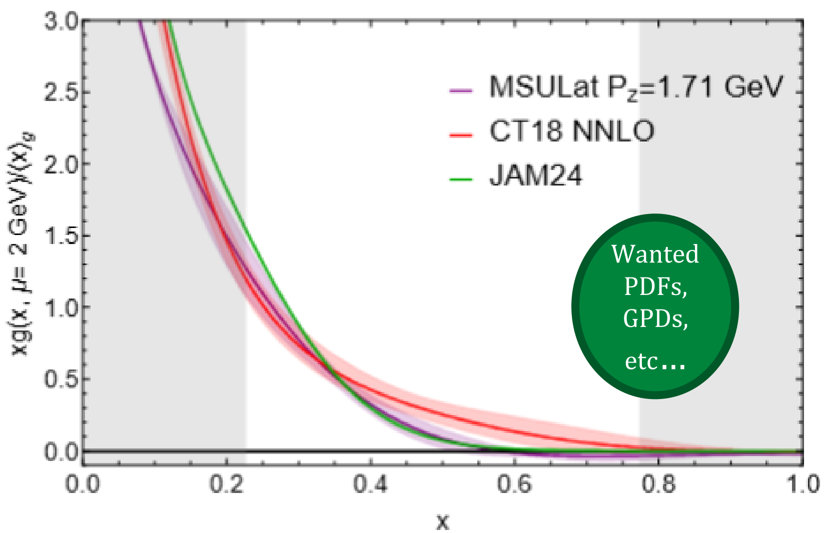
$$C_0 = 1 + \frac{\alpha_s C_A}{2\pi}\left(\frac{5}{6}\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{3}{2}\right)$$

$$C^{\text{hyb.}}\left(\xi, \frac{\mu}{p^z}\right) = C^{\text{ratio}}\left(\xi, \frac{\mu}{p^z}\right) + \frac{\alpha_s C_A}{2\pi}\frac{5}{6}\left[-\frac{1}{|1-\xi|} + \frac{2\text{Si}((1-\xi)z_s p^z)}{\pi(1-\xi)}\right]_+$$

Formula by Fei Yao (BNL)

Plots by Bill Good (MSU)





Hadron Tomography

§ Lots of progress on tomography by many collaborations

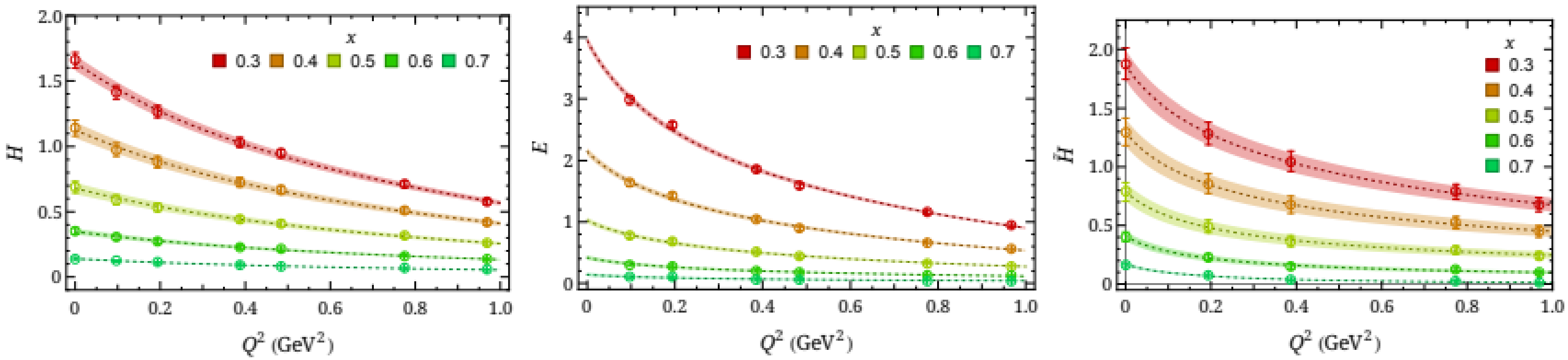
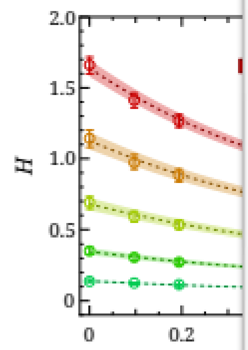


Figure 4.1: Nucleon isovector H (left), E (middle) and $\tilde{H}$ (right) GPDs at $\xi = 0$ with z-expansion to Q^2 at selected x values.

HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

Hadron Tomography

§ Lots of … borations



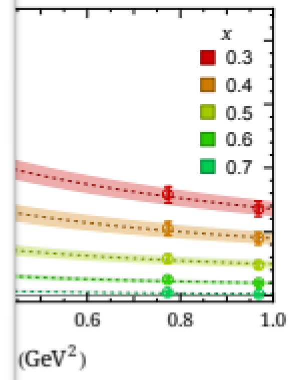
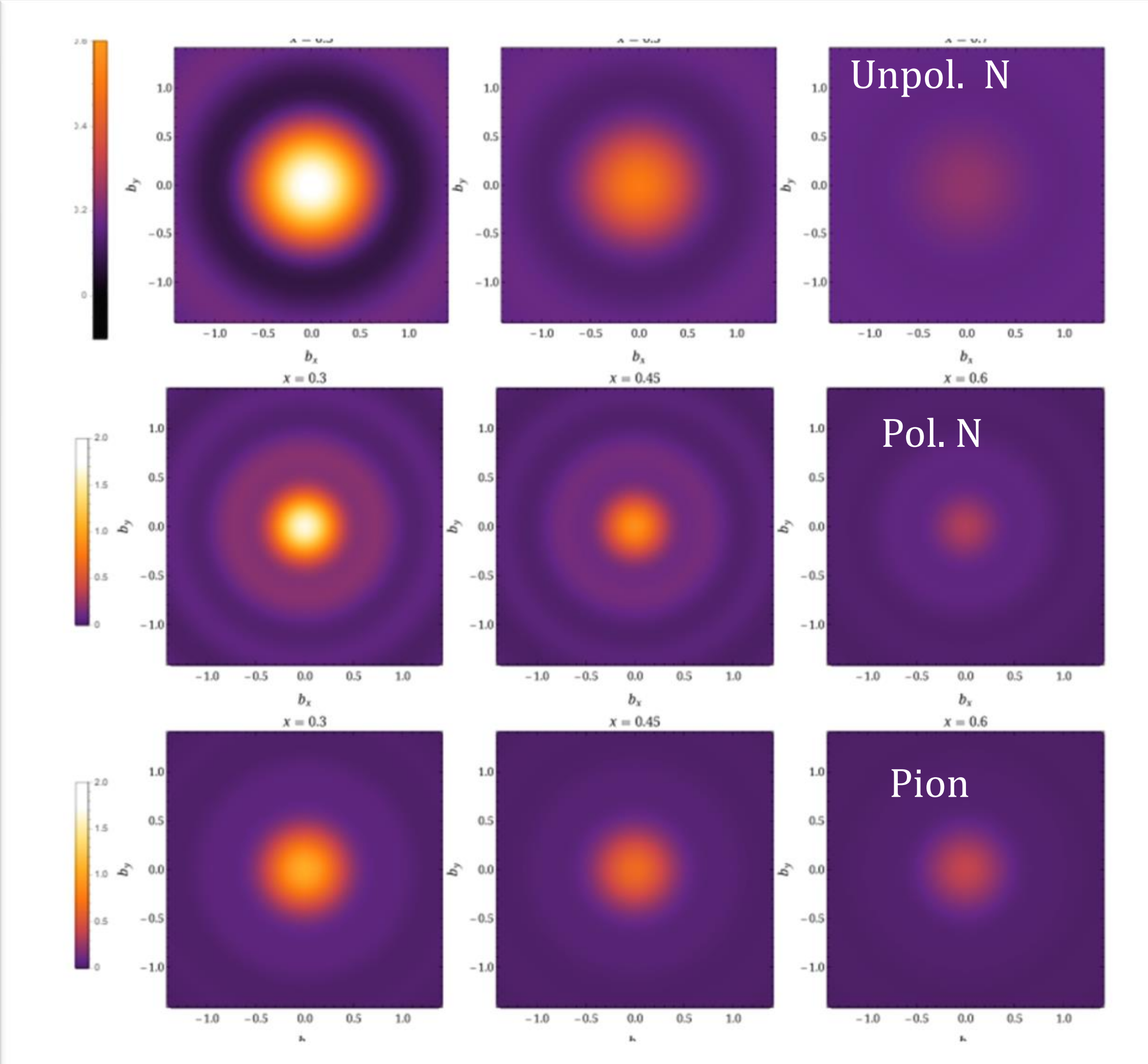


Figure 4.1: … selected x values.



HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

GPD Systematic Update

§ Nucleon isovector GPDs & pion valence-quark GPDs

- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al.
PLB 844, 138081 (2023)

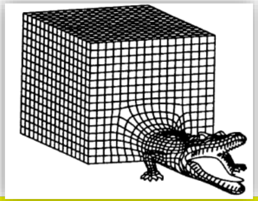
- N_f= 2+1+1 clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV

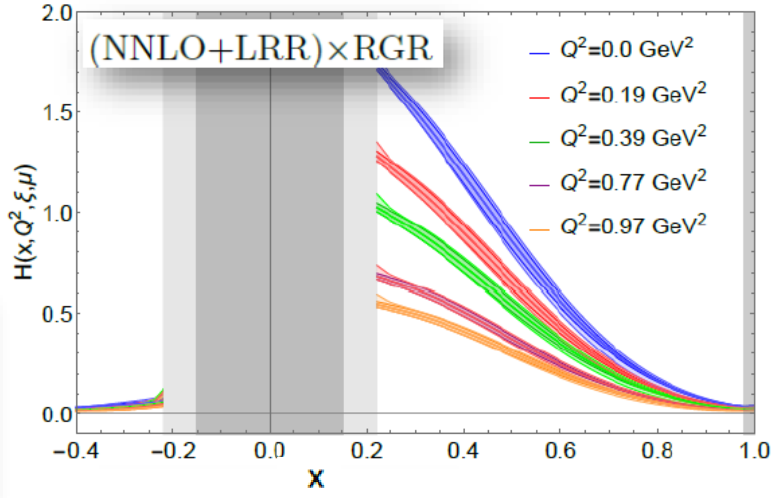
  J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

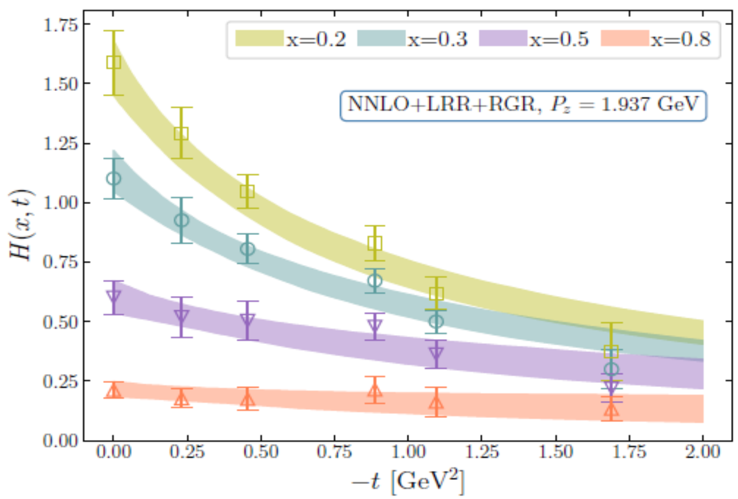
- N_f= 2+1 clover/HISQ, $a \approx 0.09$ fm, 300-MeV pion, $P_z \approx 2$ GeV

  H. Ding et al (ANL/BNL/Wuhan), 2407.03516 [hep-lat]

Wanted PDFs, GPDs, etc…







Impact of Lattice-QCD PDFs on Global Fits

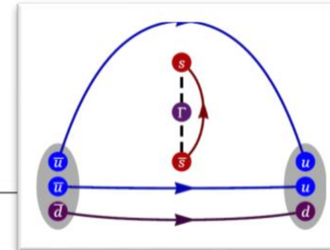
First Lattice Strange PDF

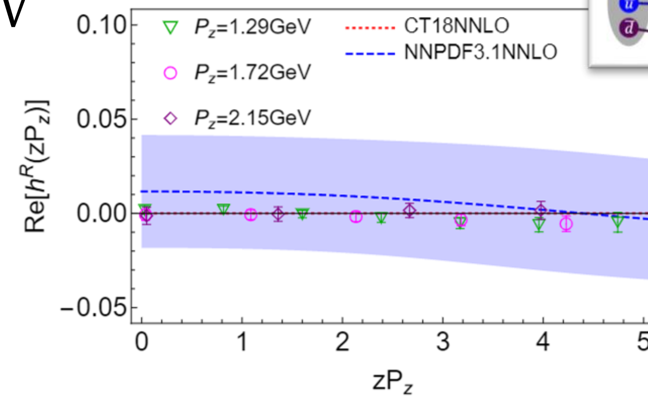
§ Results by MSULat/quasi-PDF method

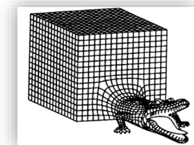
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

R. Zhang et al (MSULat), 2005.01124

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x)) \cos(xzP_z)$$







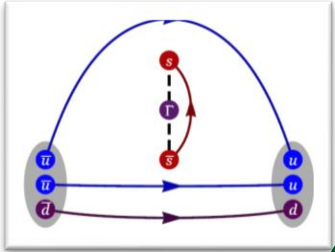
Quantities that can be calculated on the lattice

FG: Rui Zhang

Lattice Strangeness Asymmetry Impact

§ Results by MSULat/quasi-PDF method

- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV, $P_z \approx 1.7$ GeV



Wanted PDFs, GPDs, etc…

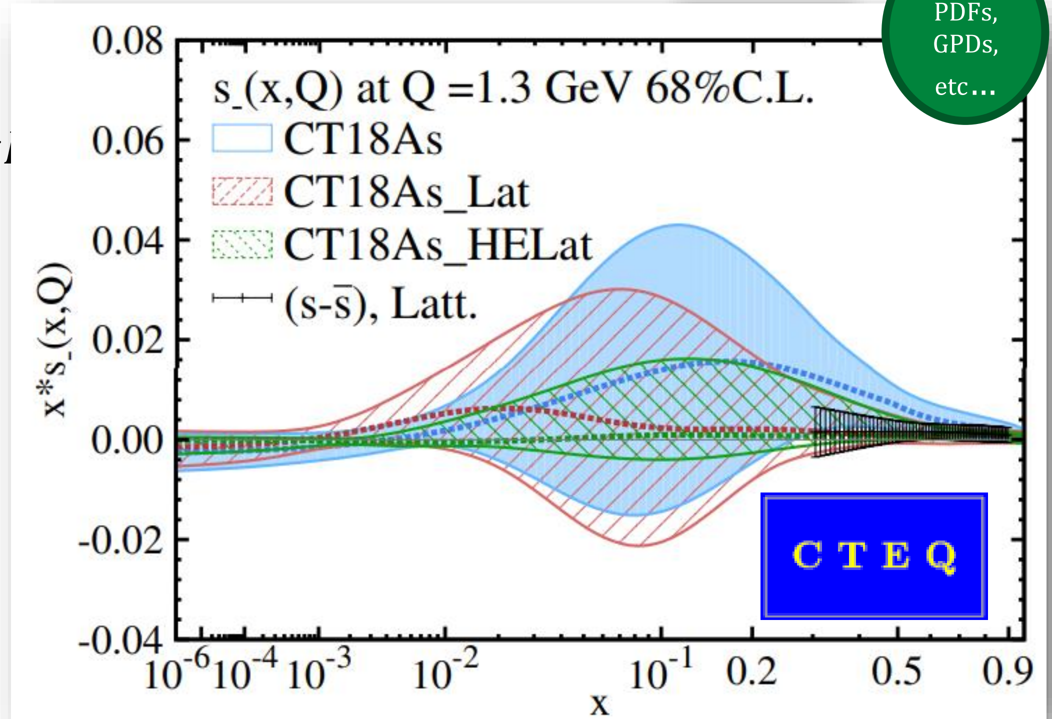
R. Zhang et al (MSULat), 2005.01124

$$\mathrm{Re}[h(z)] \propto \int dx\,\left(s(x) - \bar{s}(x)\right)\cos(xz\ldots$$

§ From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2 P_z^2}\right)$$

T. Hou, HL, M. Yan, C. Yuan, 2211.11064



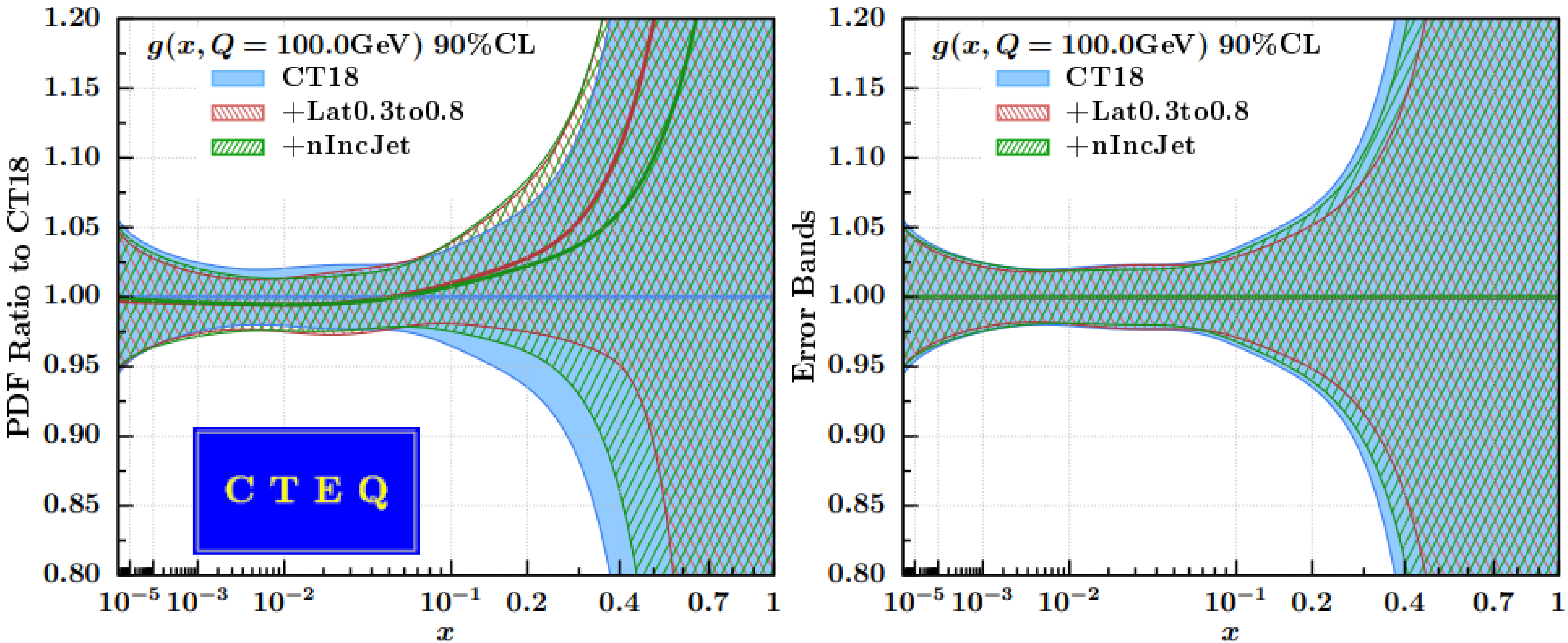
§ The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

Nucleon Gluon PDF Impact

§ Impact study with CTEQ-TEA analysis

- Take lattice inputs in the region where no strong experimental data constraints, $x \in [0.3, 0.8]$

Plots by Alim Ablat (Xinjiang U.); 2502.10630

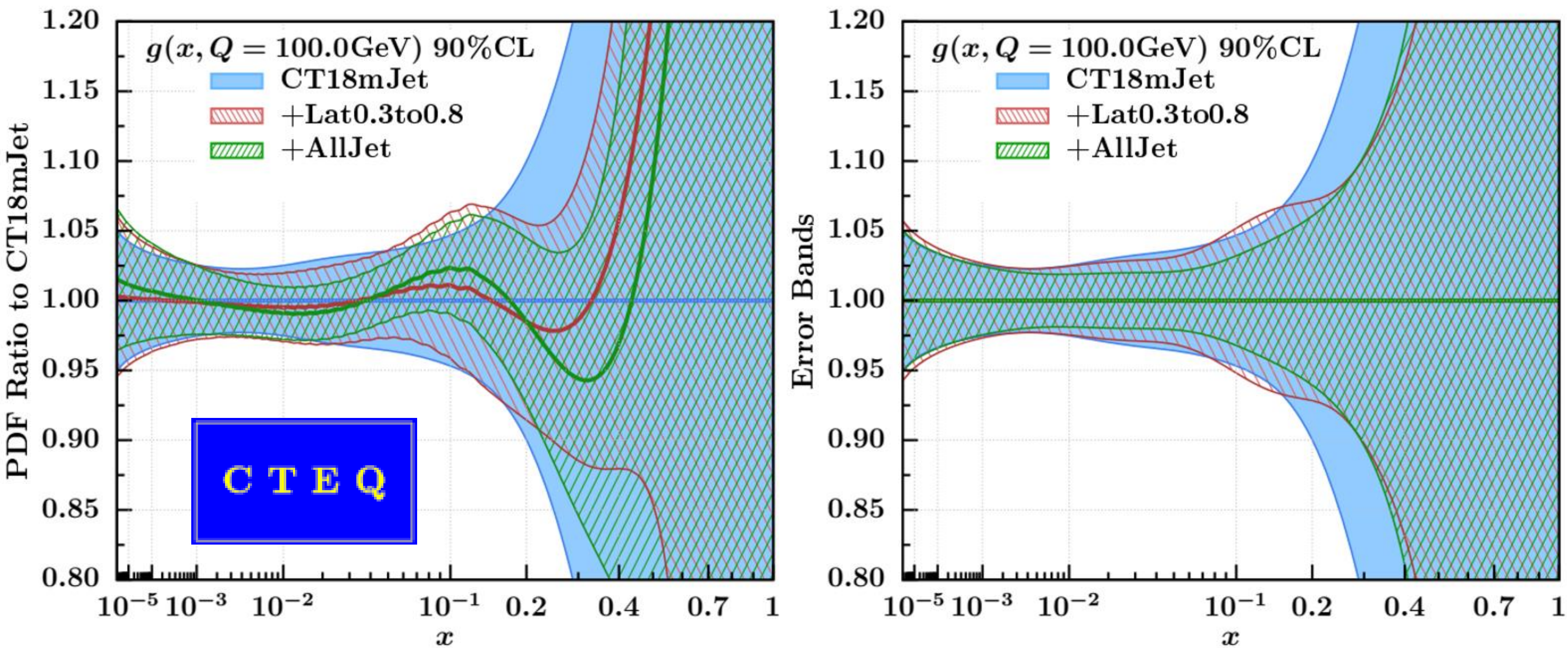


Nucleon Gluon PDF Impact

§ Impact study with CTEQ-TEA analysis

- Take lattice inputs in the region where no strong experimental data constraints, $x \in [0.3, 0.8]$

Plots by Alim Ablat (Xinjiang U.); 2502.10630

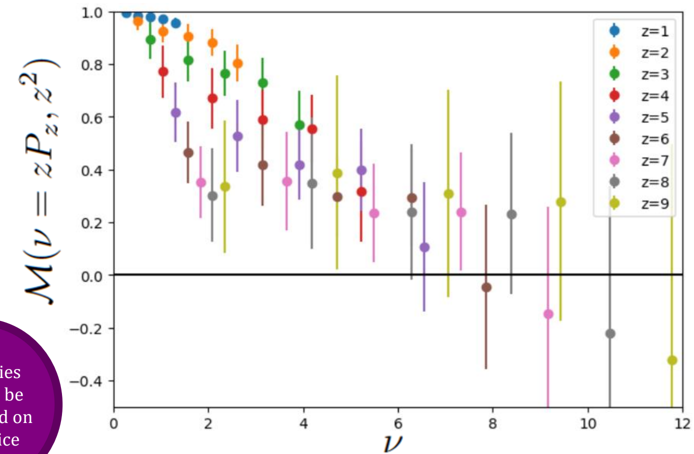


Pion Gluon PDF Impact

§ Preliminary study with JAM analysis

Plots by Bill Good (MSU)

- Treat lattice reduced pseudo-Ioffe time distribution (RpITD) as "cross-section" inputs in the JAM global fit



W. Good et al, 2409.02750 [hep-lat]

Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

Quantities that can be calculated on the lattice

$$\boxed{\mathcal{M}(\nu, z^2)} = \int_0^1 dx \boxed{\frac{xg(x)}{\langle x_g \rangle}} \left[\boxed{R_{gg}(x\nu, z^2\mu^2) + R_r(x\nu, z^2\mu^2)} \boxed{\frac{\langle x_S \rangle}{\langle x_g \rangle}} \right] + \int_0^1 dx \boxed{\frac{x f_S(x)}{\langle x_g \rangle}} \boxed{R_{gq}(x\nu, z^2\mu^2)} + \boxed{\mathcal{O}(z^2 m^2, z^2 \Lambda_{\rm QCD})}$$

Lattice data

Non-perturbative PDFs and moments

Perturbative matching kernels

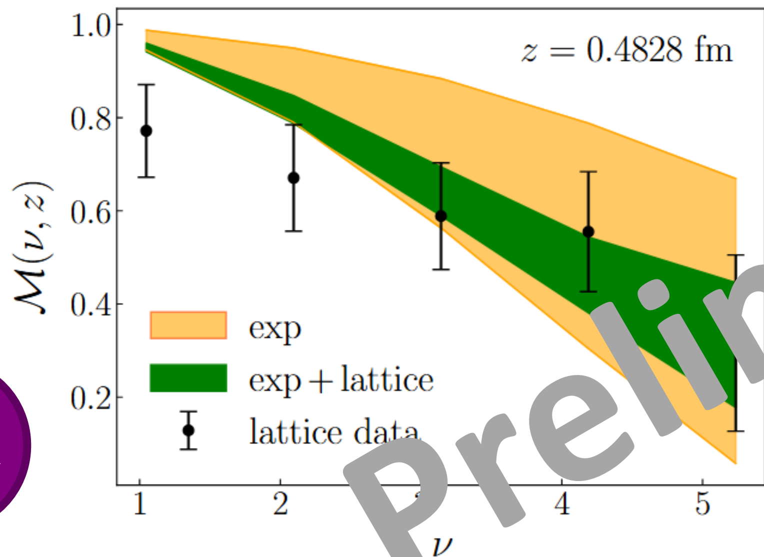
Residual systematic error

A. V. Radyushkin, PRD96, 034025 (2017); Balitsky, et al., PLB808, 135621 (2020)

Pion Gluon PDF Impact

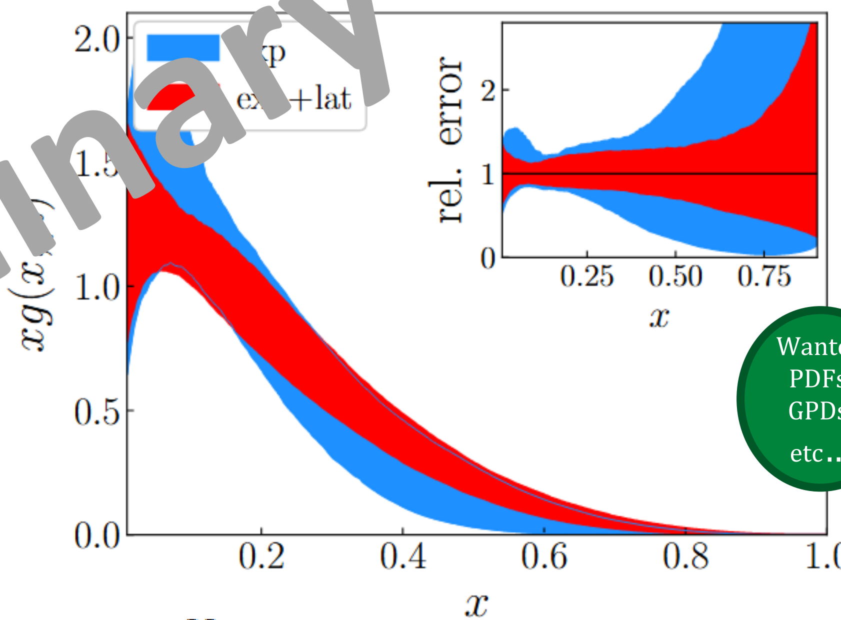
§ Preliminary study with JAM analysis

Plots by Bill Good (MSU)



Preliminary

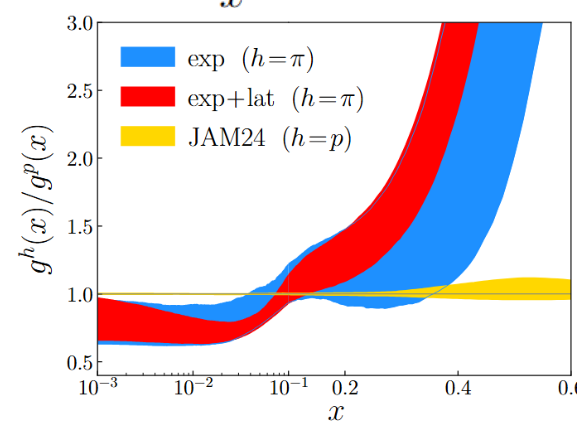
Quantities that can be calculated on the lattice



Wanted PDFs, GPDs, etc…

x

also in Collaboration with Wally and Nobuo



Lattice Progress & Challenges

§ Beyond the standard twist-2 collinear PDFs

- Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
- Transverse-momentum– dependent distributions (TMDs)
  - Collins-Soper kernel, soft function and wavefunctions
- Twist-3 PDFs and GPDs

  For more details and references, refer to 2202.07193

Swagato Mukherjee,
Wayne Morris
@ Wed. Morning

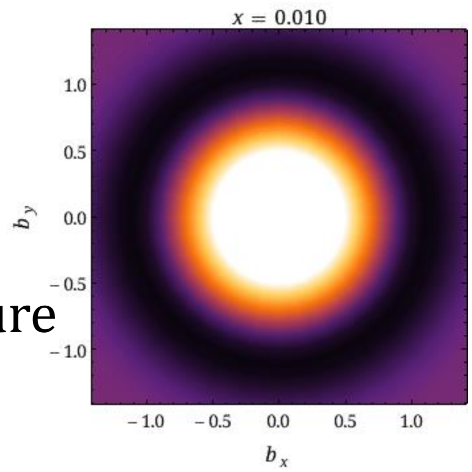
§ Challenges ahead for precision PDFs

- Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- Access small-x physics; some methods have inverse problem in PDF extraction, more computational resources, etc.

Summary and Outlook

§ Exciting era using LQCD to study *x*-dependent parton distributions

- Bjorken-*x* dependence of parton distributions now widely studied
- More study of systematics planned for the near future

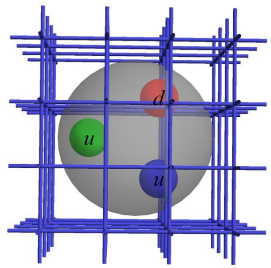


§ Lattice strange and gluon PDFs can have impacts

- Treat lattice matrix elements as expt inputs in the future
- Computational resources are needed for precision calculations (community support!)

§ Precision and progress are limited by resources

Theory Input → **Global Analysis of PDFs/GPDs** ← **Exp't Input** +



EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
This work is partially sponsored by grants NSF PHY 1653405 & 1653405, DOE DE-SC0024053 & RCSA Cottrell Scholar Award

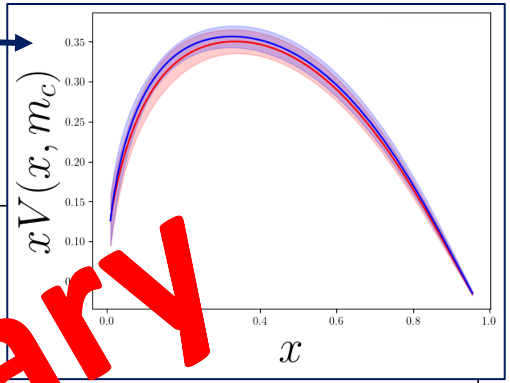
Students Wanted

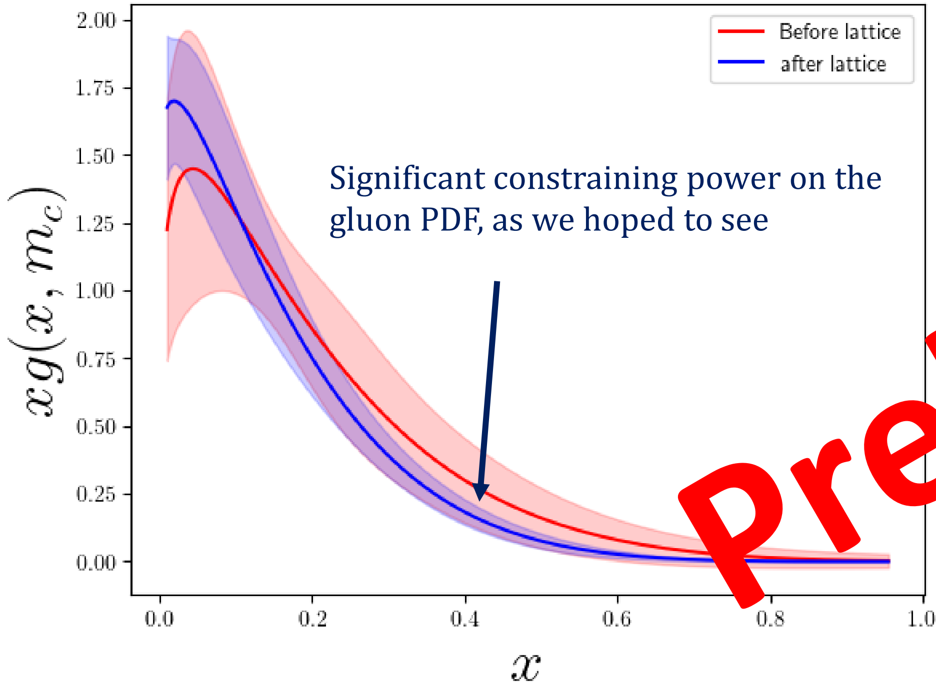
LGT4HEP website: https://lgt4hep.github.io/

Backup Slides

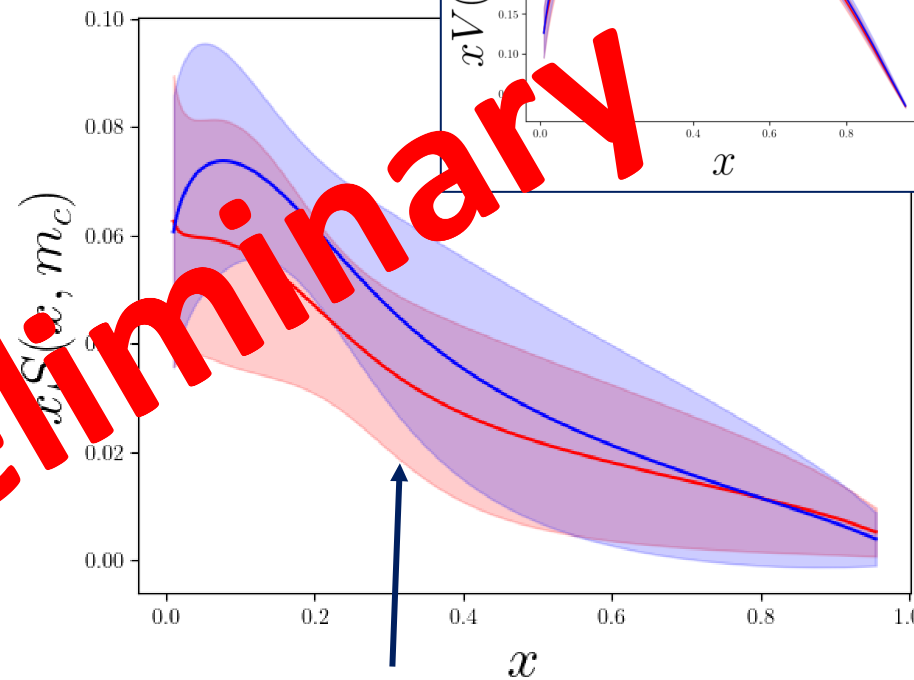
More Pion PDF Updates

Not much effect on the valence quarks (as expected)





Significant constraining power on the gluon PDF, as we hoped to see



Overall compatibility with "before" lattice PDFs

Some shape change in the sea quark PDF, need to look at relative errors

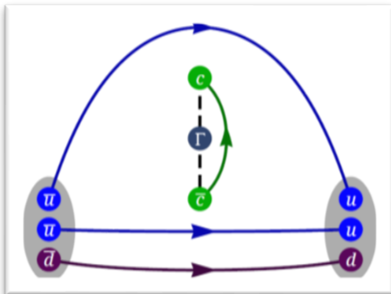
First Lattice Charm PDF

§ Large uncertainties in global PDFs

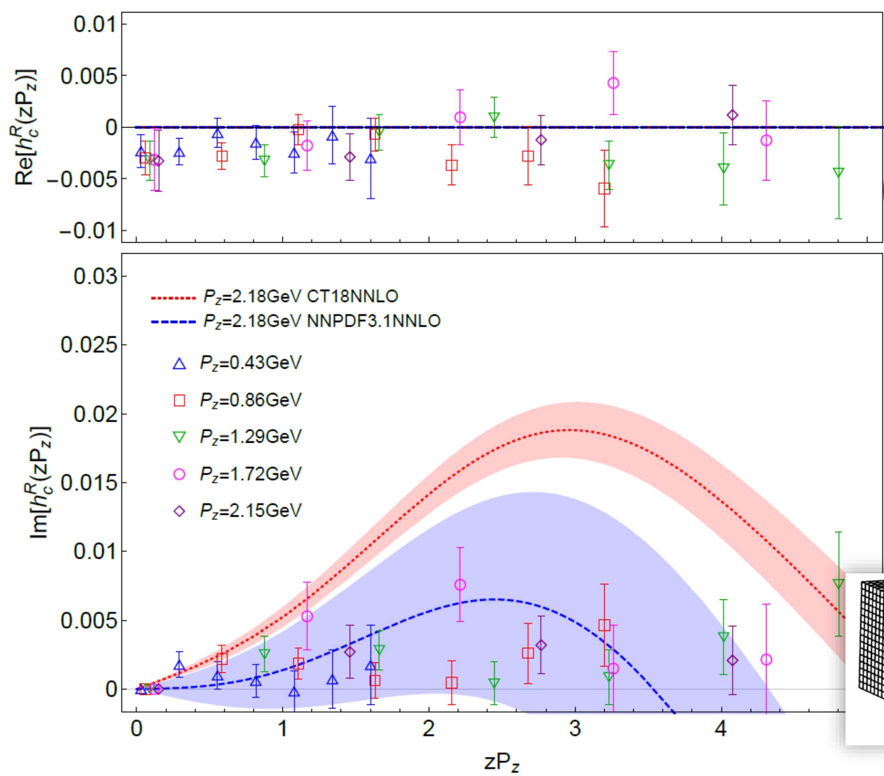
§ Results by MSULat/quasi-PDF method

- Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

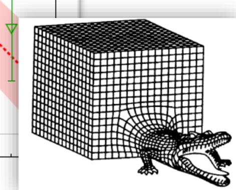
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Quantities that can be calculated on the lattice



First Lattice Charm PDF

- Large uncertainties in global PDFs
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  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

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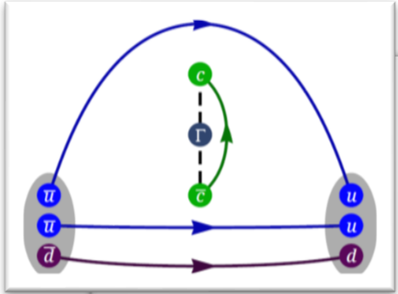

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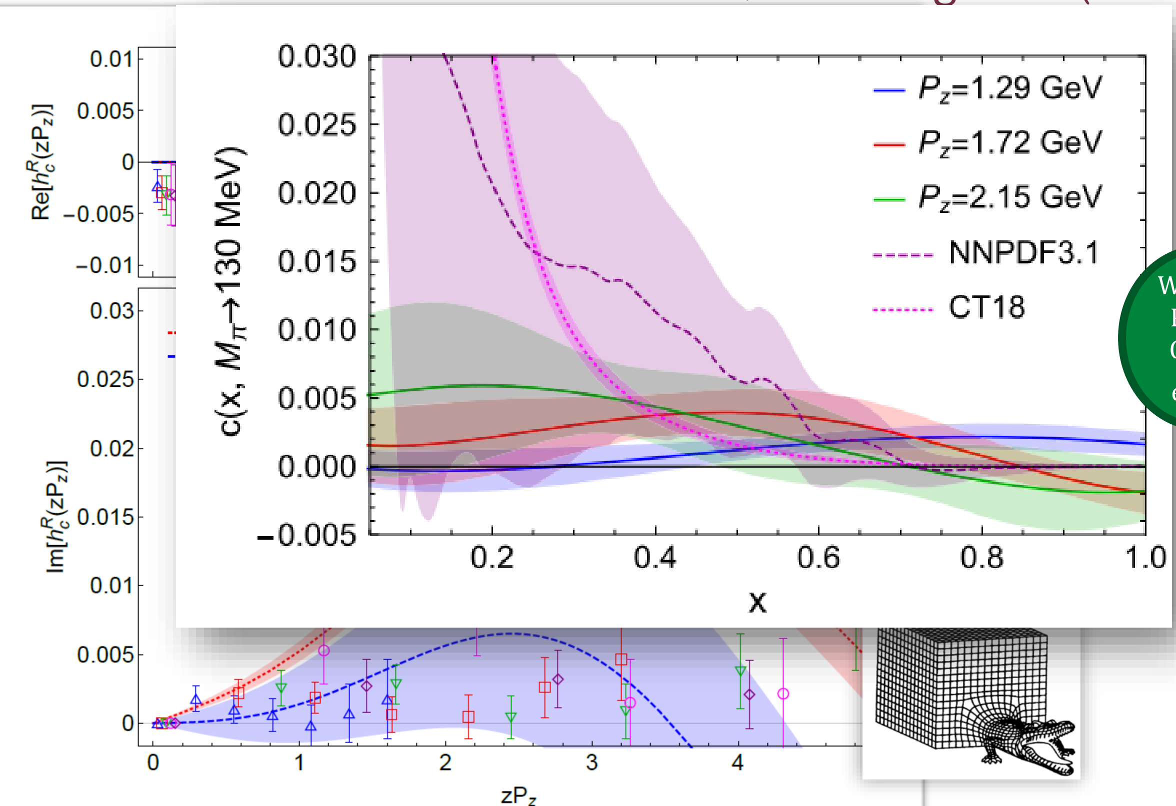
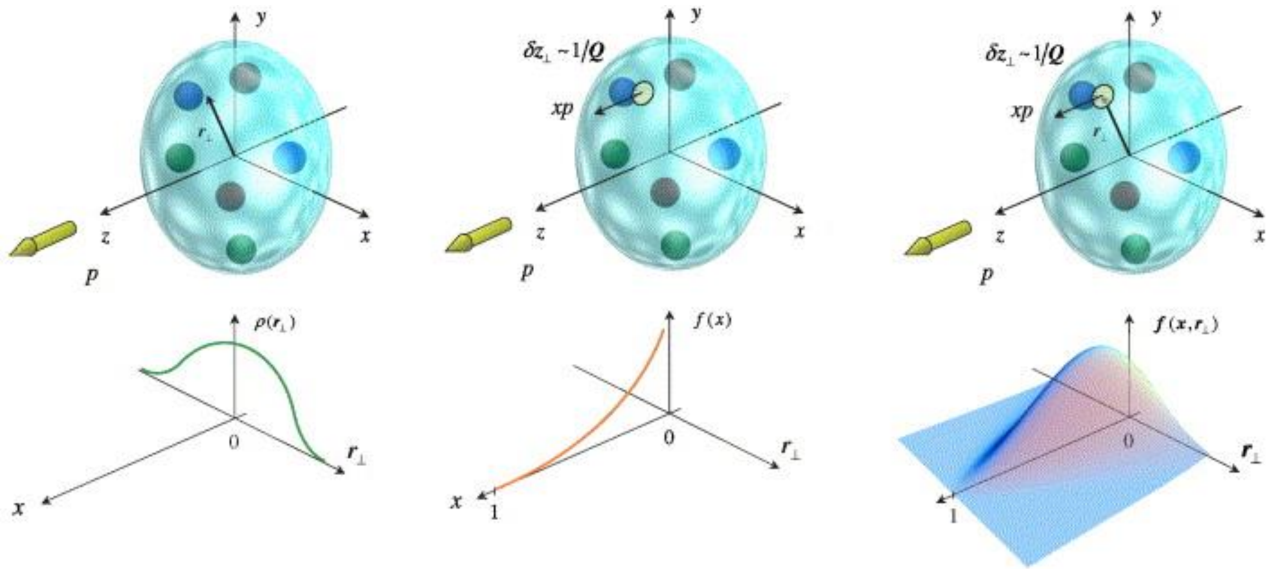


Wanted PDFs, GPDs, etc…

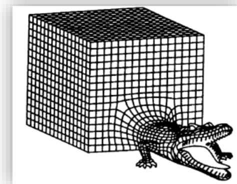
Bjorken-x Dependent GPDs



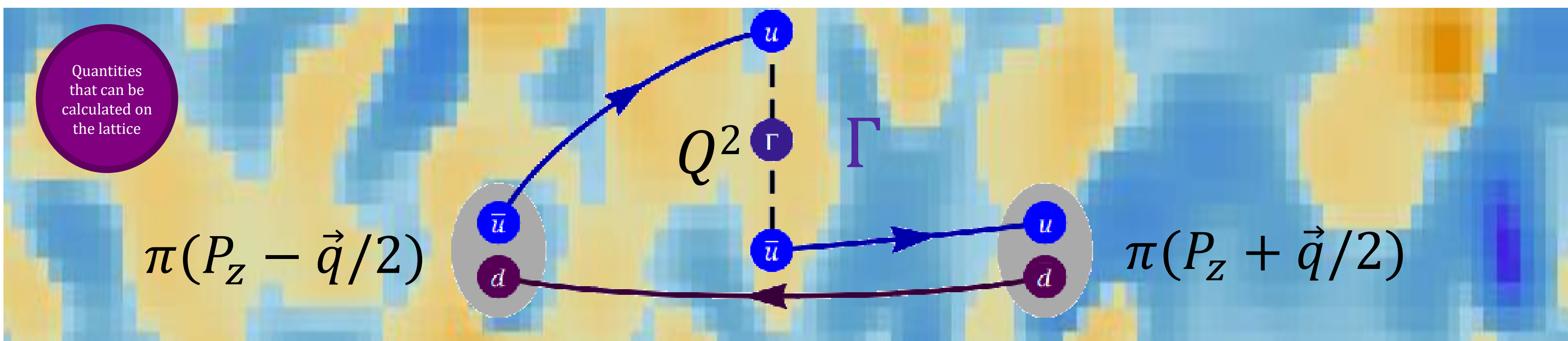
Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

Generalized Parton Distributions

Single-ensemble result



finite-volume,
discretization,
heavy quark mass,
…



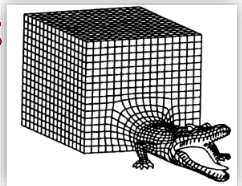
First Lattice GPDs

§ First glimpse into pion GPD using Quasi-PDF/LaMET

- Lattice details: clover/HISQ, **0.12fm**, **310-MeV** pion mass

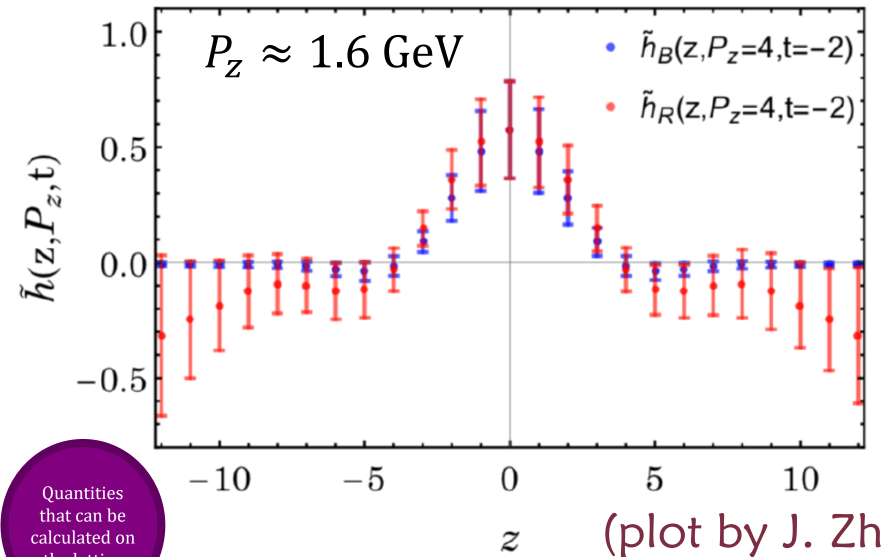
$P_z \approx 1.3, 1.6$ GeV

MILC, Phys. Rev. D, 82 (2010), 074501;
Phys. Rev. D, 87 (2013), 0545056

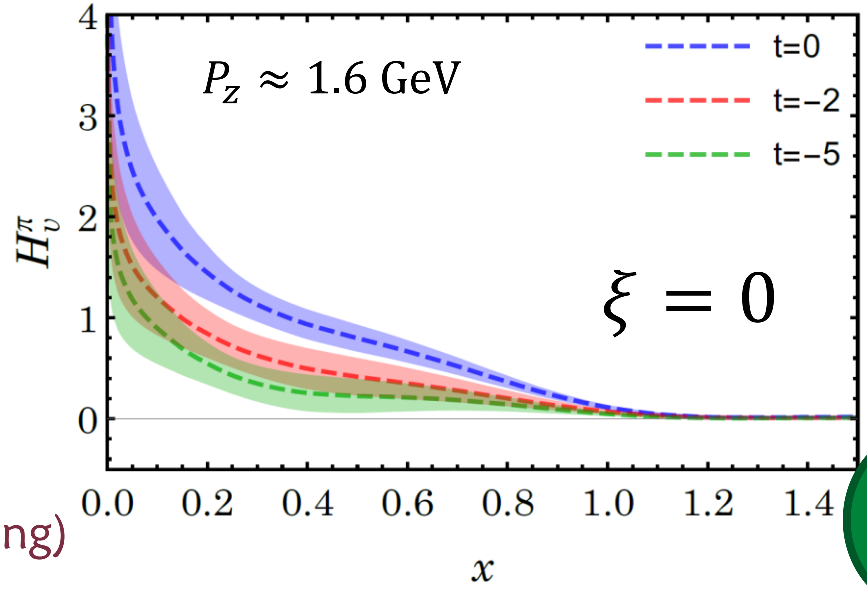


J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right)\gamma^+ \Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right)\right| \pi(P-\Delta/2)\right\rangle$$



(plot by J. Zhang)



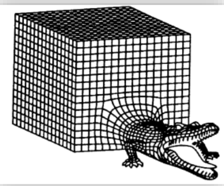
Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

Valence-Quark Pion GPD

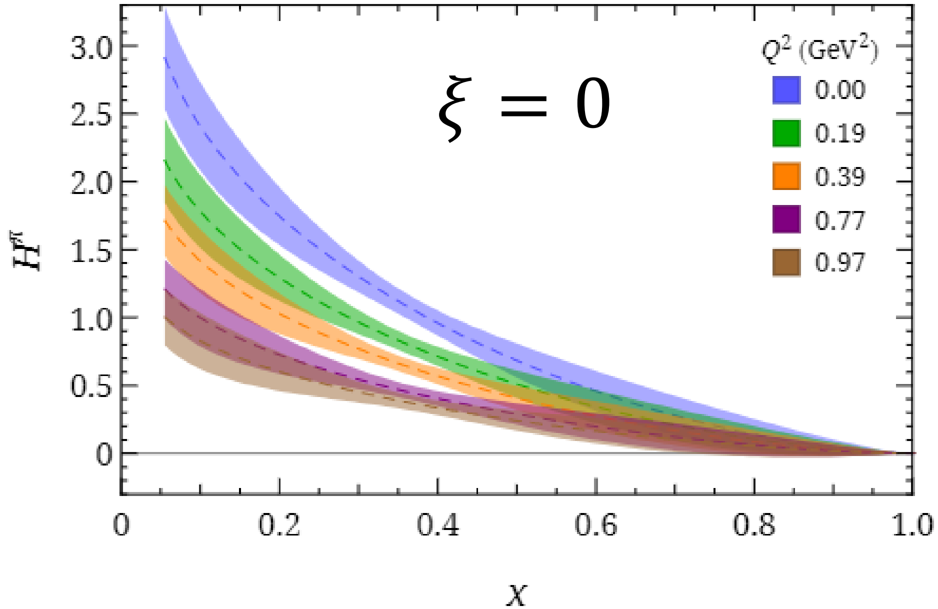
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results

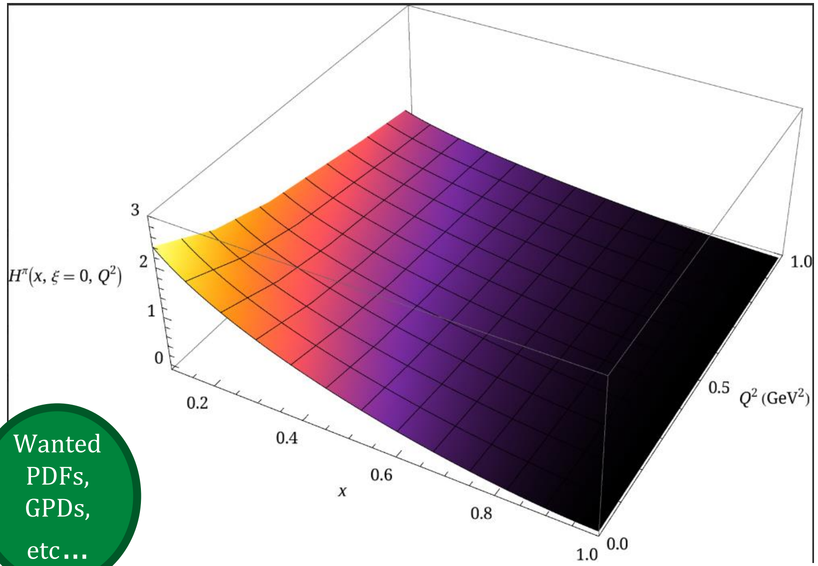


finite-volume,
discretization,

HL (MSULat), Phys. Lett. B 846 (2023) 138181



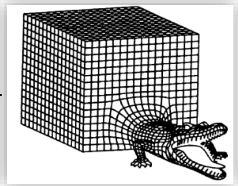
Wanted PDFs, GPDs, etc…



Valence-Quark Pion GPD

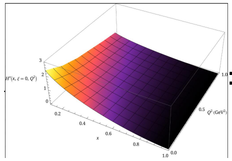
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

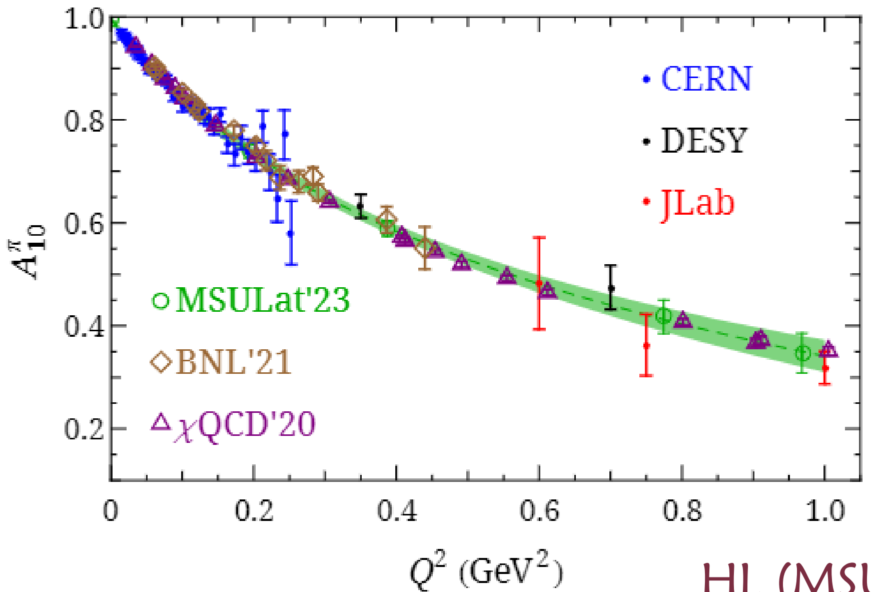
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV



finite-volume,
discretization,

- $\xi = 0$ valence-quark Pion GPD results

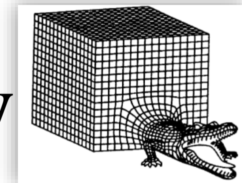
$$\int_{-1}^{+1} dx\, x^{n-1}$$  $$= A^{\pi}_{ni}(t)$$



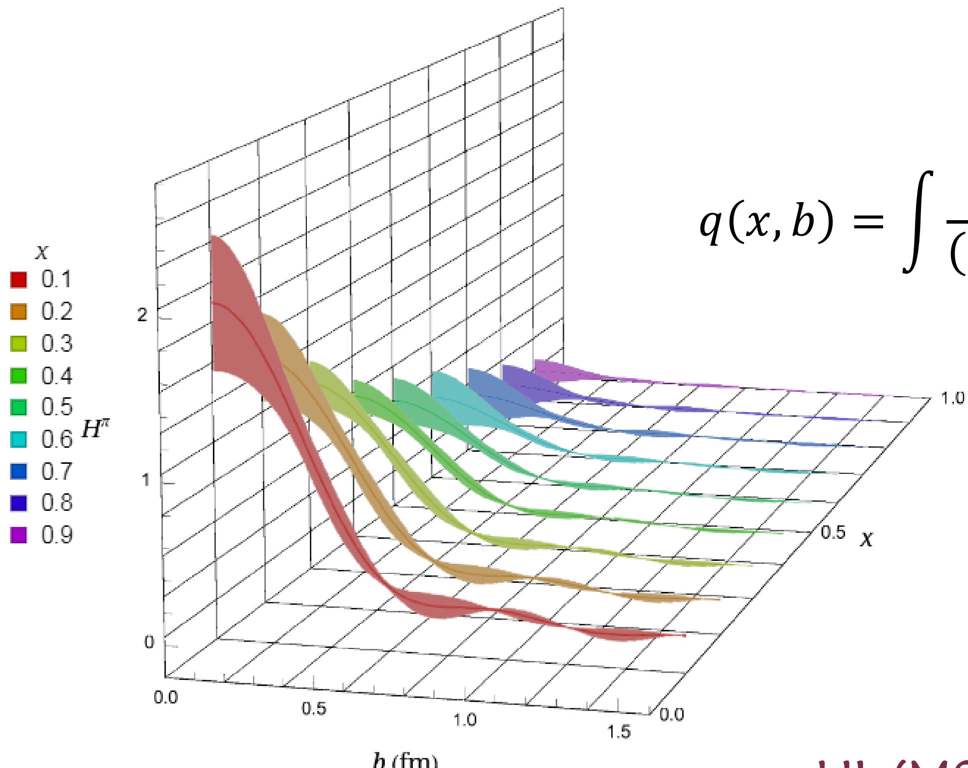
HL (MSULat), Phys. Lett. B 846 (2023) 138181

§ Nucleon GPD using quasi-PDFs at physical pion mass

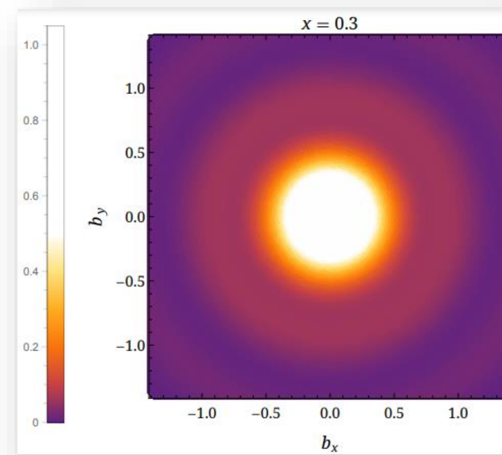
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results



finite-volume, discretization,



$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^{\,2}) e^{i\vec{q}\cdot\vec{b}}$$



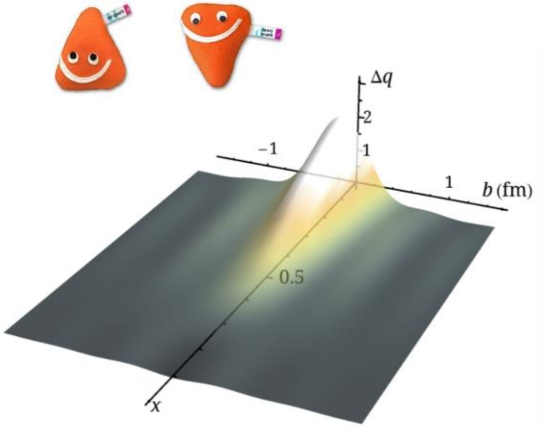
HL (MSULat), Phys. Lett. B 846 (2023) 138181

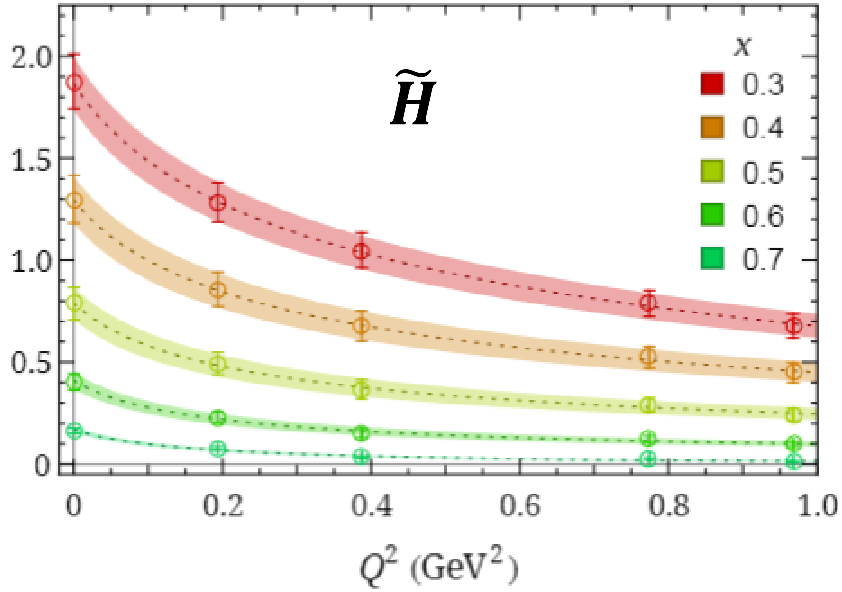
Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

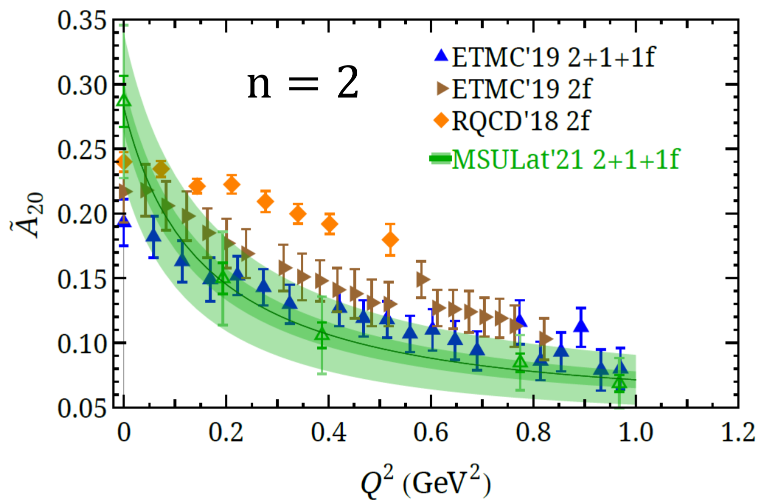
- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821





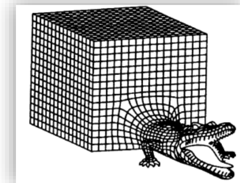
- Take the integral to form moments



Nucleon Tomography

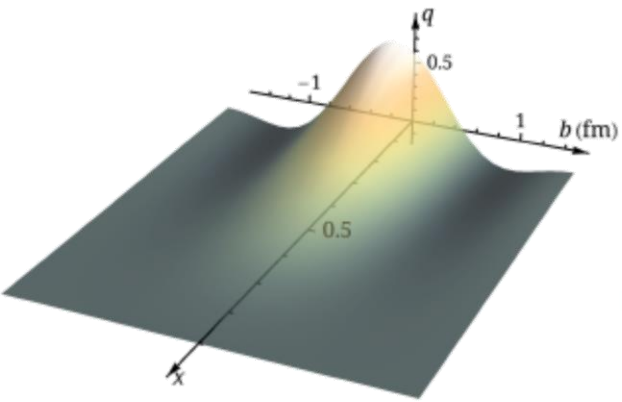
§ Nucleon GPD using quasi-PDFs at physical pion mass

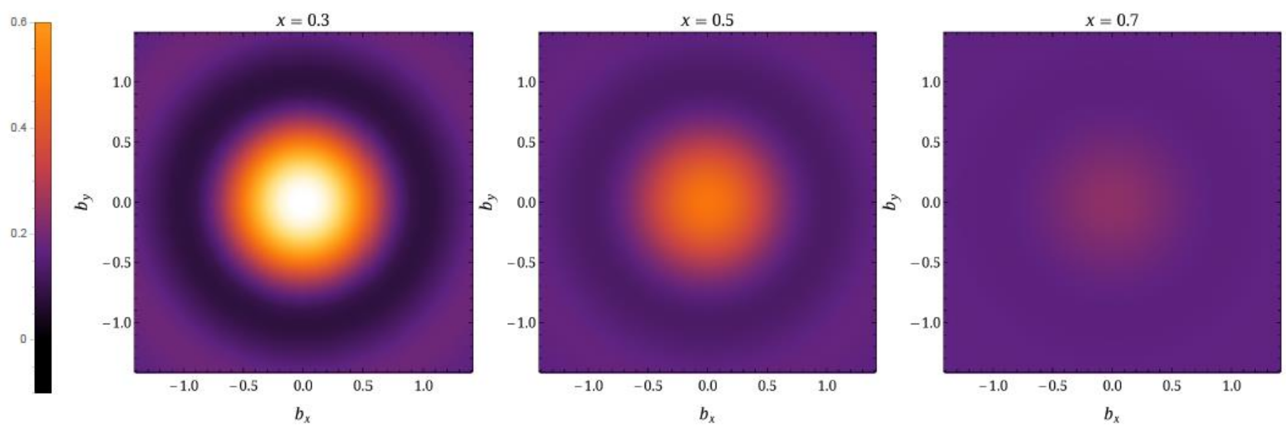
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



finite-volume, discretization,

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x,\xi=0,t=-\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$





HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Also see work done by ANL/BNL/ETMC, [2209.05373](2209.05373) ,[2310.13114](2310.13114)

GPD Systematic Update

§ Nucleon isovector GPDs calculated directly at physical pion mass

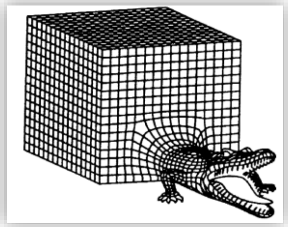
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

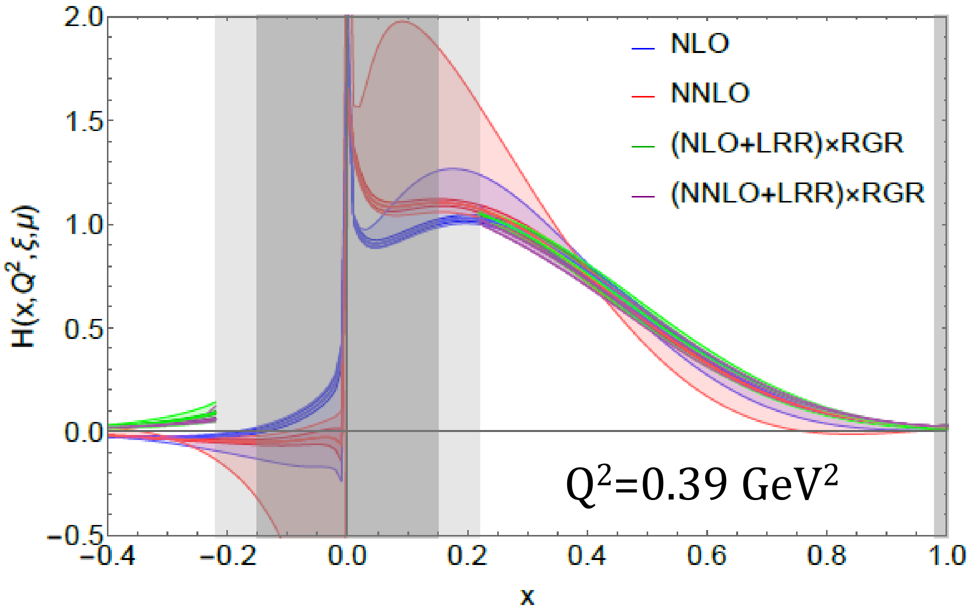
R. Zhang, et. al.
PLB 844, 138081 (2023)

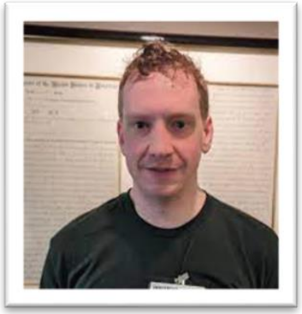
- N_f= 2+1+1 clover/HISQ, $a \approx$ 0.09 fm, 135-MeV pion, $P_z \approx$ 2 GeV

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…







P: Jack Holligan