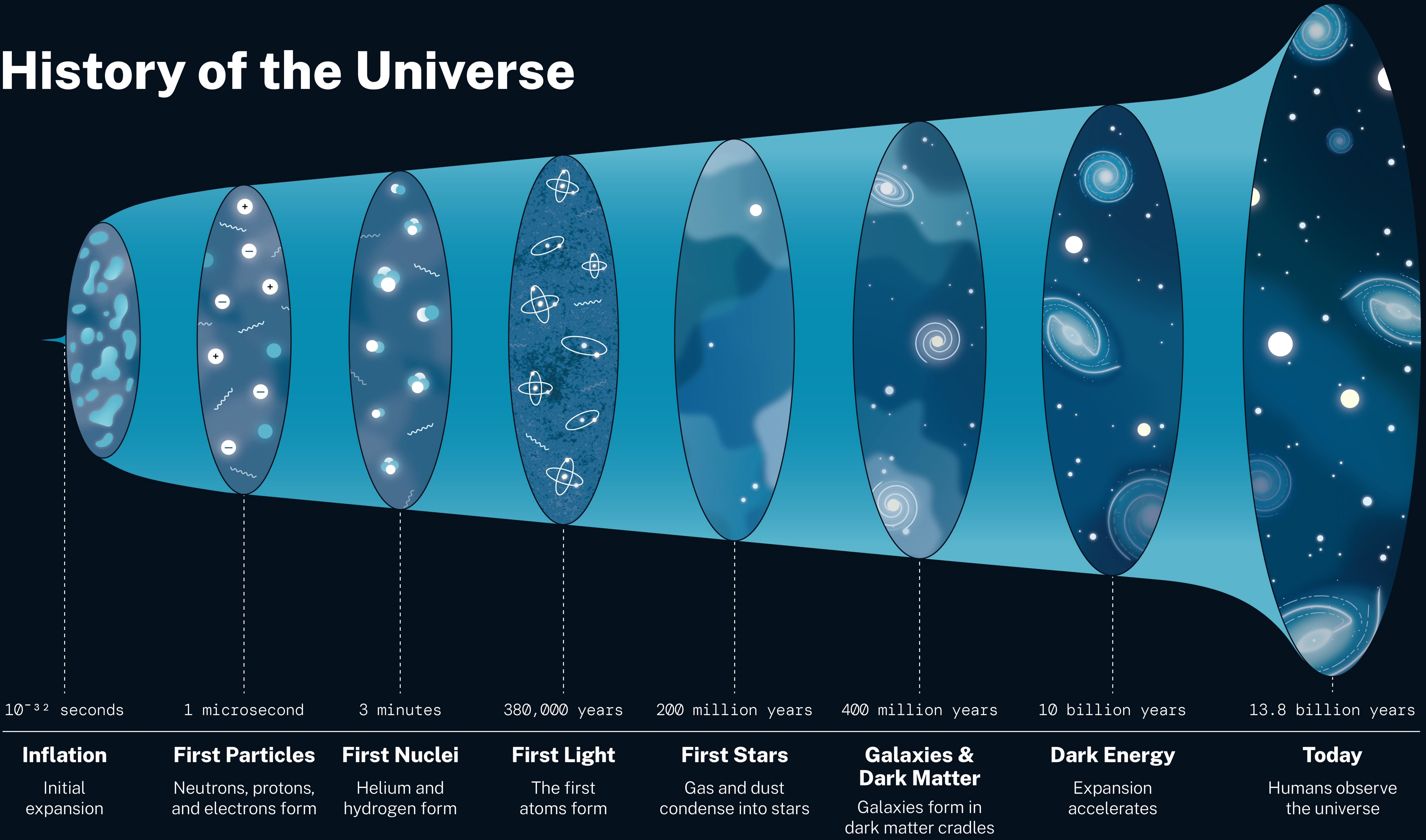
Unveiling the Strongest Force in Nature through EIC and Lattice QCD

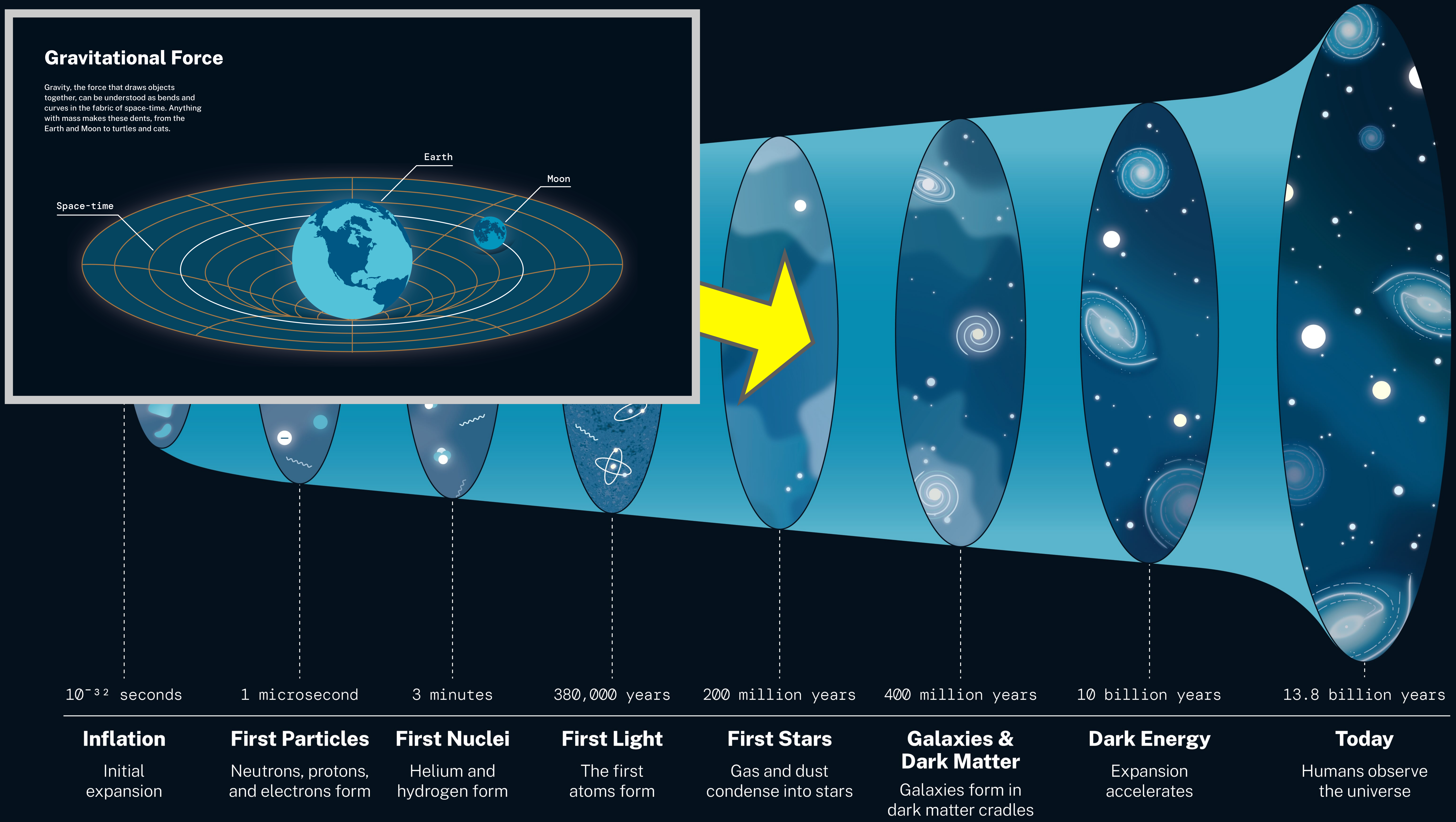
June 2025,
Taipei, Taiwan

Swagato Mukherjee

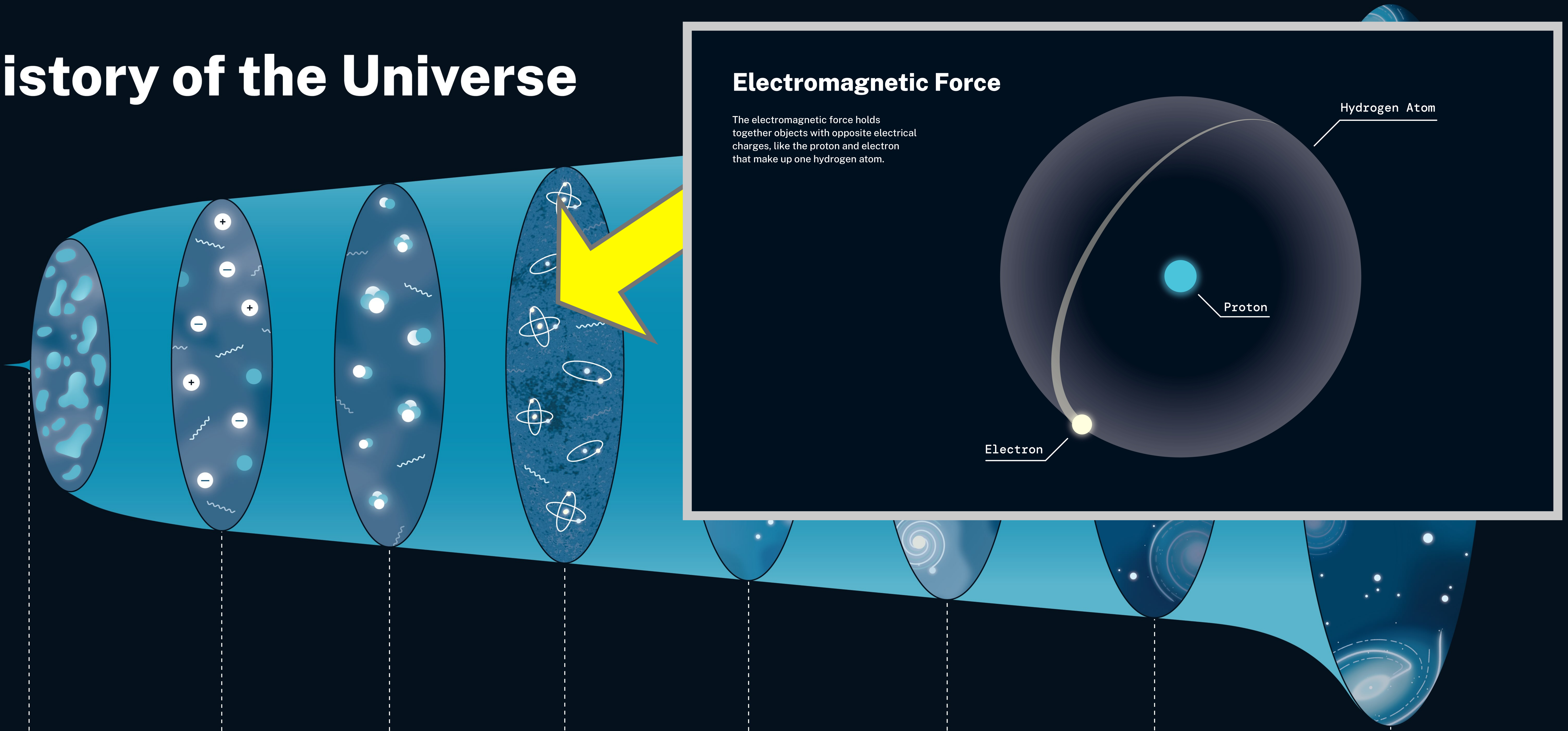
Brookhaven National Laboratory

History of the Universe



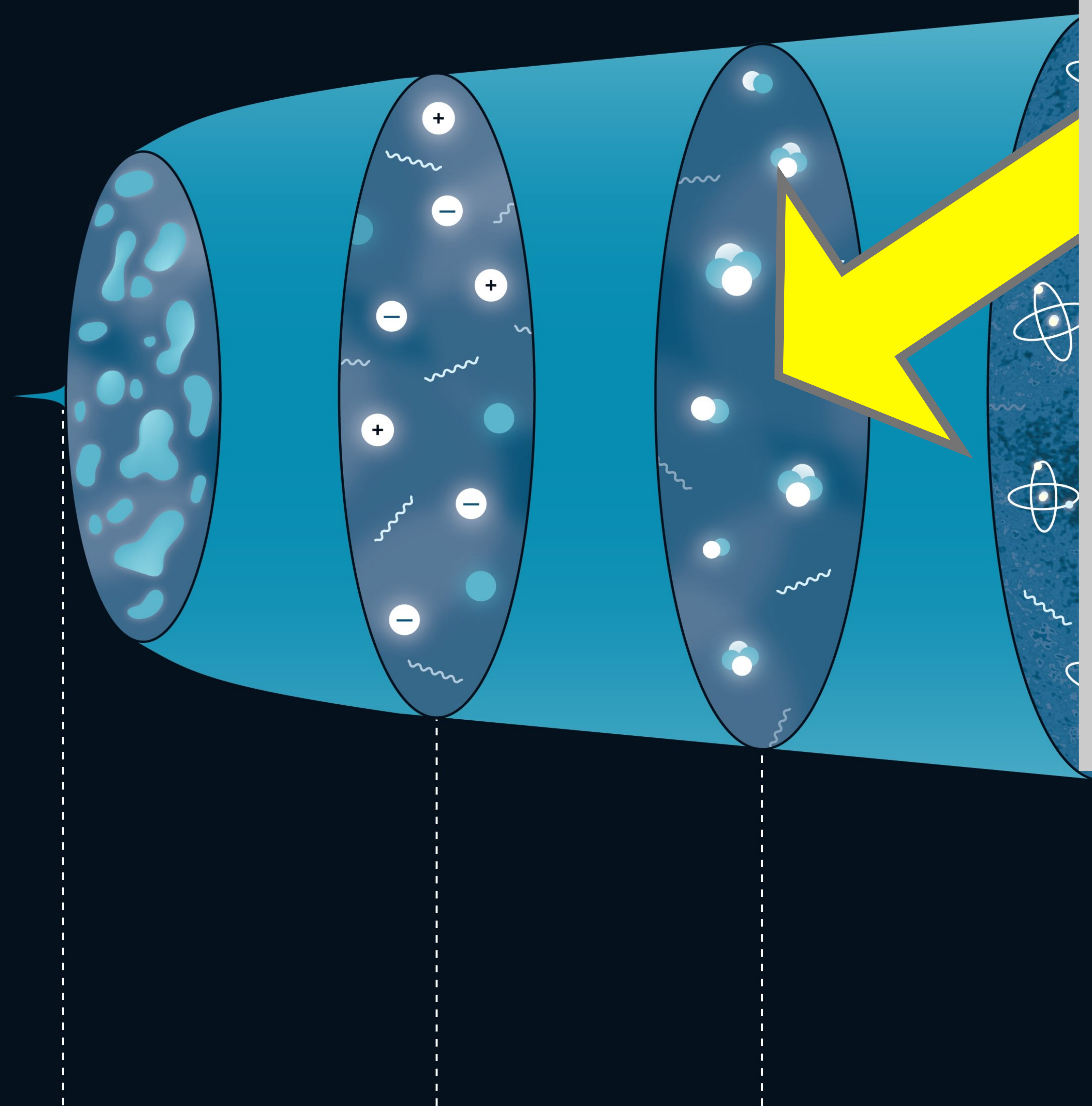

Gravitational Force
Gravity, the force that draws objects together, can be understood as bends and curves in the fabric of space-time. Anything with mass makes these dents, from the Earth and Moon to turtles and cats.
Earth
Moon
Space-time
10⁻³² seconds
1 microsecond
3 minutes
380,000 years
200 million years
400 million years
10 billion years
13.8 billion years
Inflation
Initial expansion
First Particles
Neutrons, protons, and electrons form
First Nuclei
Helium and hydrogen form
First Light
The first atoms form
First Stars
Gas and dust condense into stars
Galaxies & Dark Matter
Galaxies form in dark matter cradles
Dark Energy
Expansion accelerates
Today
Humans observe the universe

History of the Universe



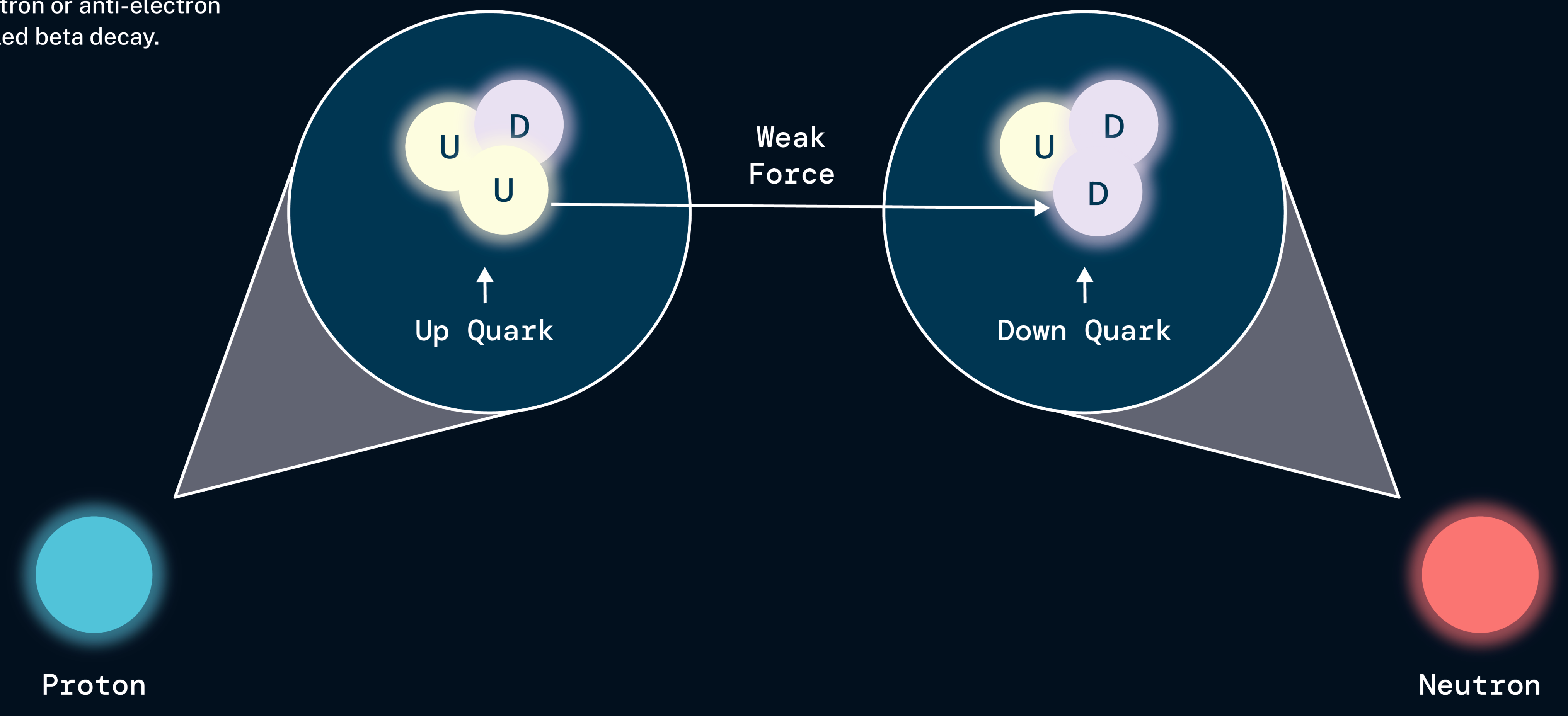
10⁻³² seconds	1 microsecond	3 minutes	380,000 years	200 million years	400 million years	10 billion years	13.8 billion years
Inflation	**First Particles**	**First Nuclei**	**First Light**	**First Stars**	**Galaxies & Dark Matter**	**Dark Energy**	**Today**
Initial expansion	Neutrons, protons, and electrons form	Helium and hydrogen form	The first atoms form	Gas and dust condense into stars	Galaxies form in dark matter cradles	Expansion accelerates	Humans observe the universe

History of the Univers



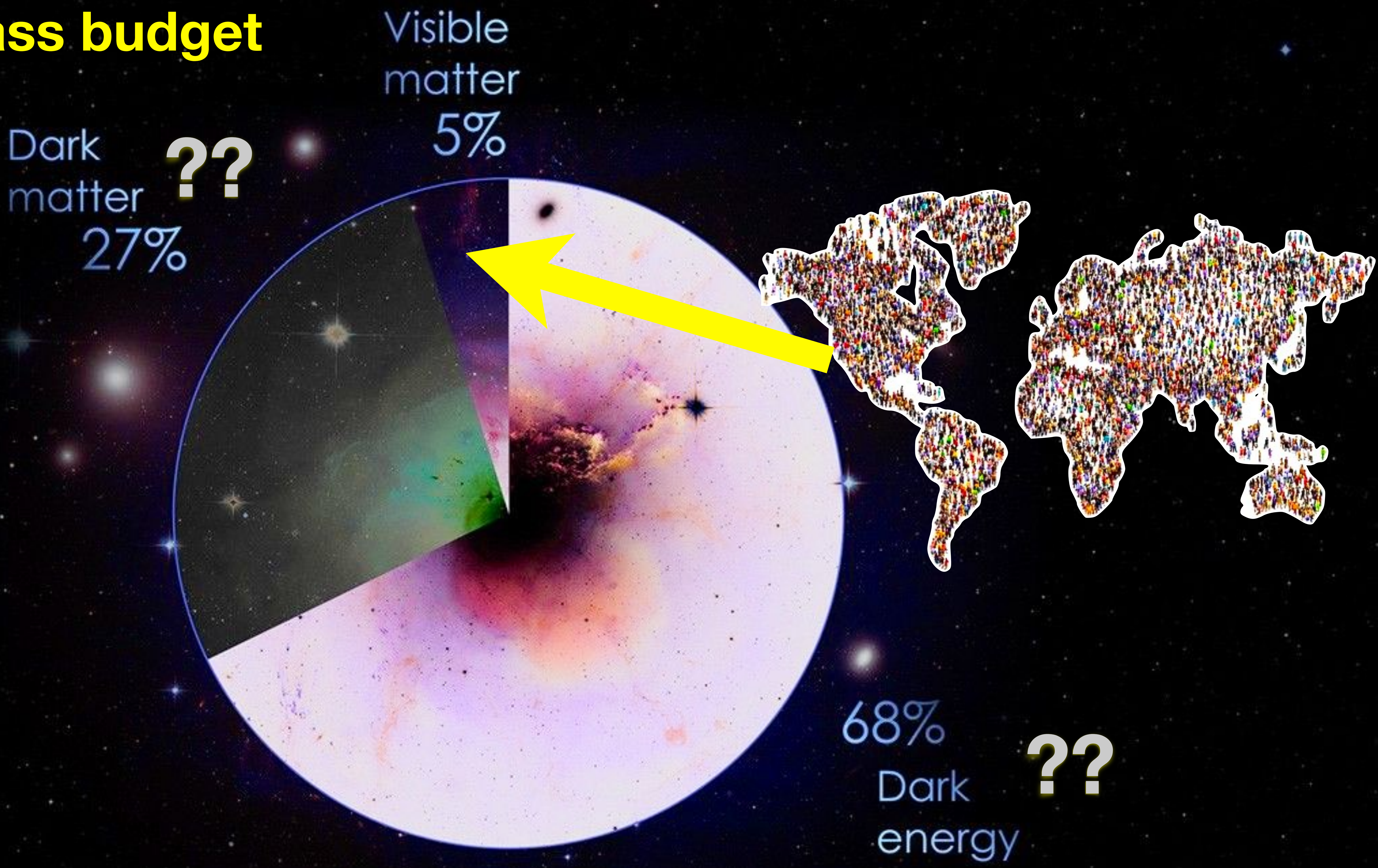
Weak Force

The weak force can change a quark from one type to another: from up to down. That change could turn a proton into a neutron or vise versa. This process would also produce a neutrino and an electron or anti-electron through a process called beta decay.

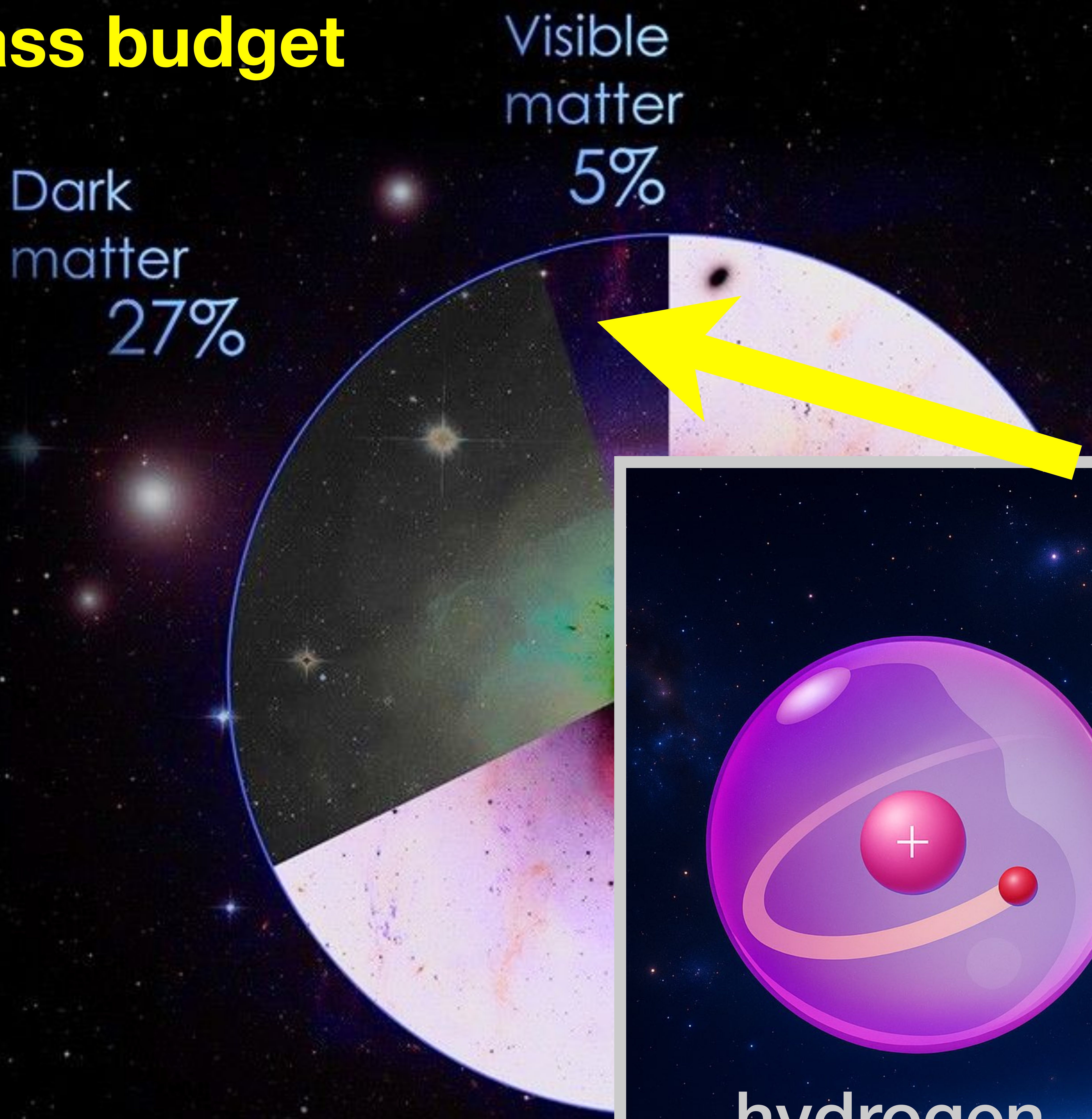


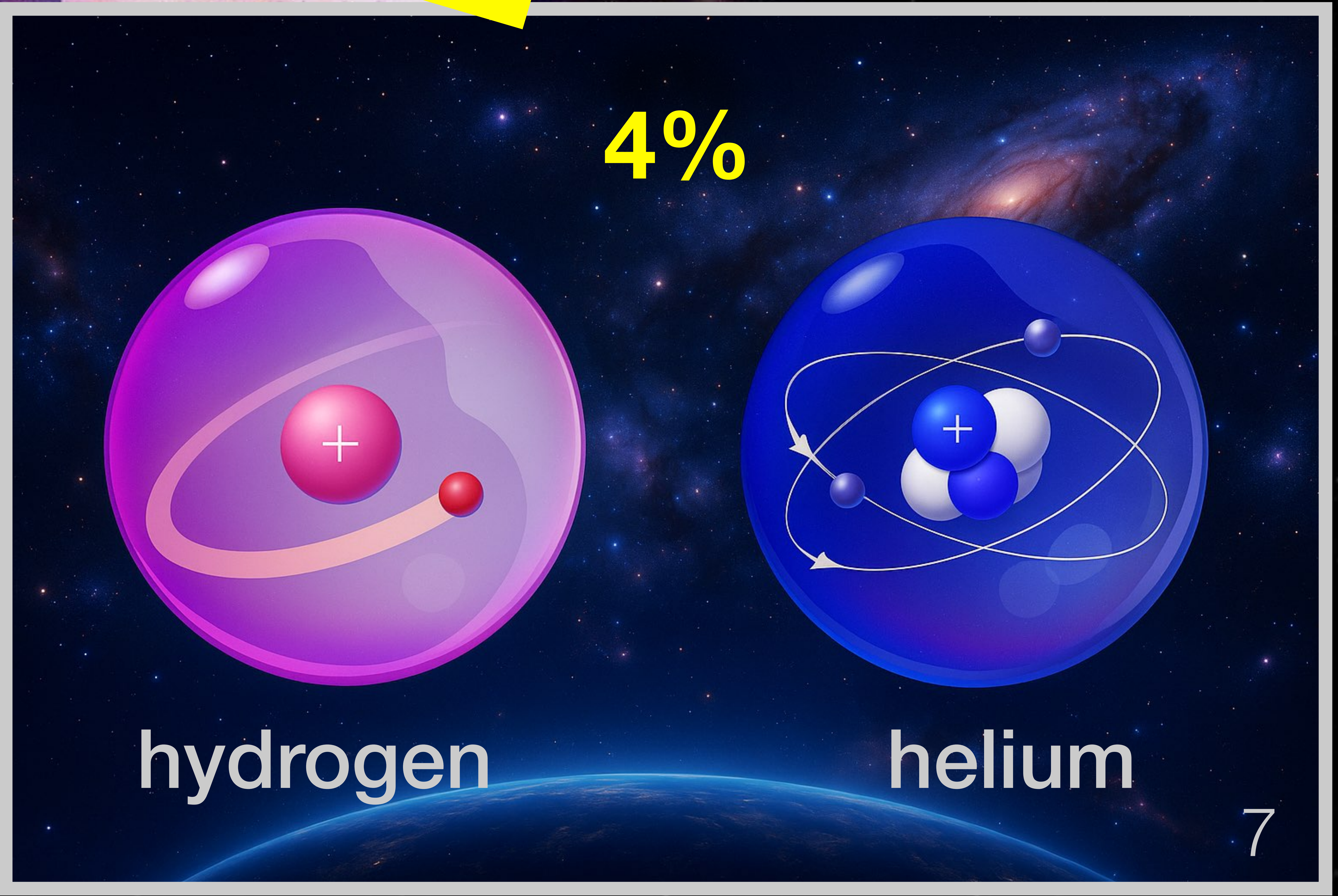
10^{-32} seconds	1 microsecond	3 minutes	380,000 years	200 million years	400 million years	10 billion years	13.8 billion years
Inflation	**First Particles**	**First Nuclei**	**First Light**	**First Stars**	**Galaxies & Dark Matter**	**Dark Energy**	**Today**
Initial expansion	Neutrons, protons, and electrons form	Helium and hydrogen form	The first atoms form	Gas and dust condense into stars	Galaxies form in dark matter cradles	Expansion accelerates	Humans observe the universe

Universe: mass budget

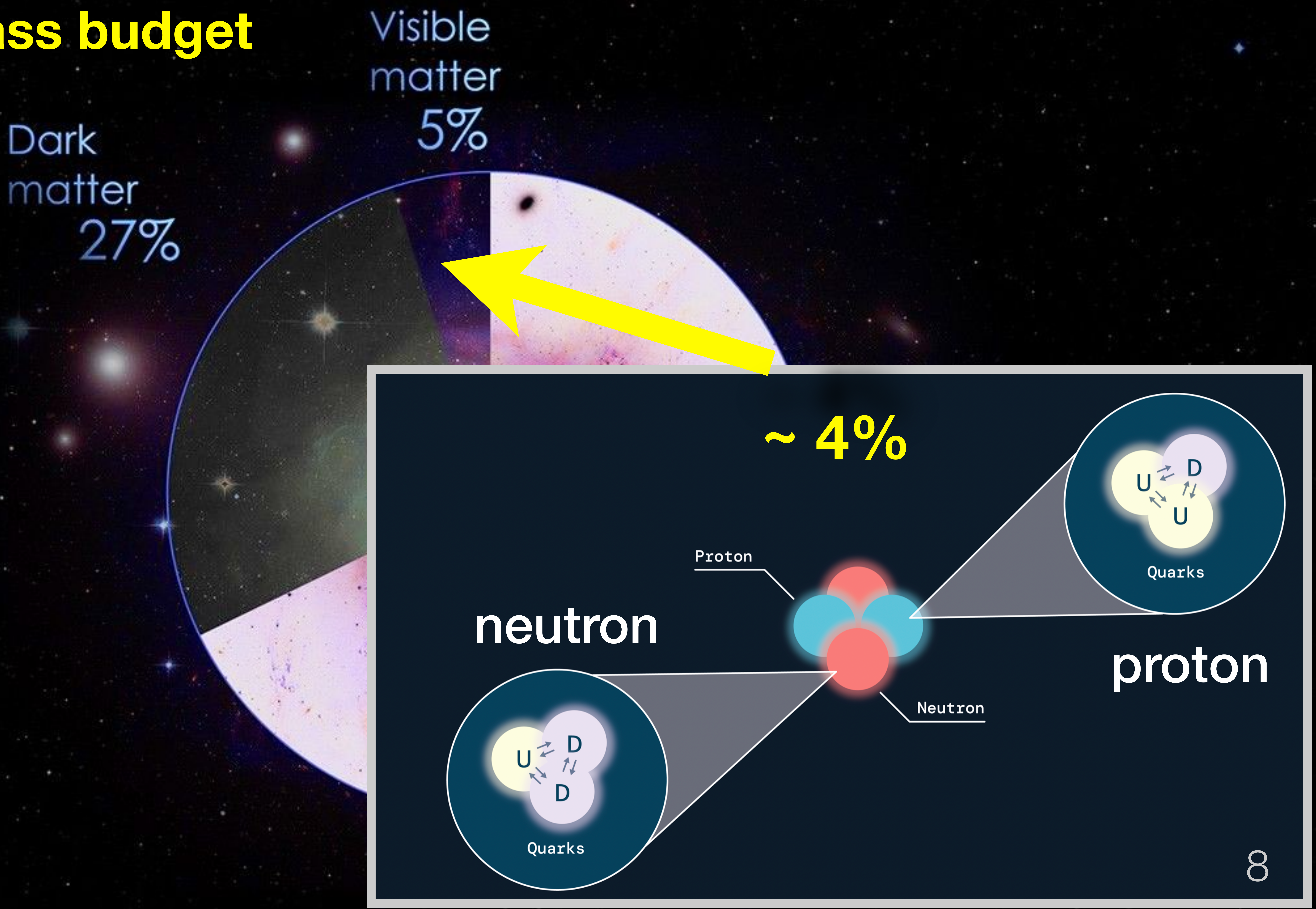


Universe: mass budget

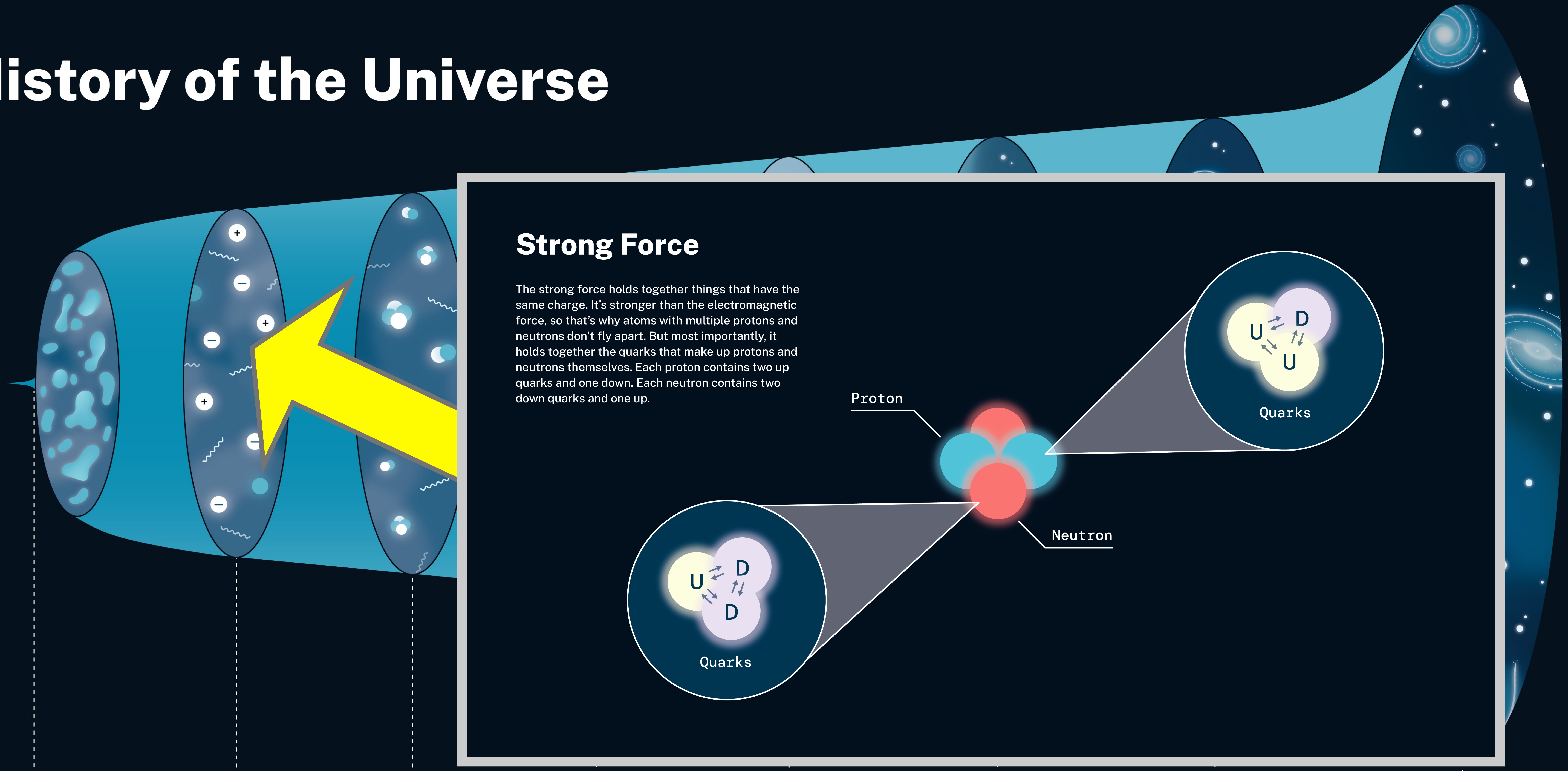




Universe: mass budget

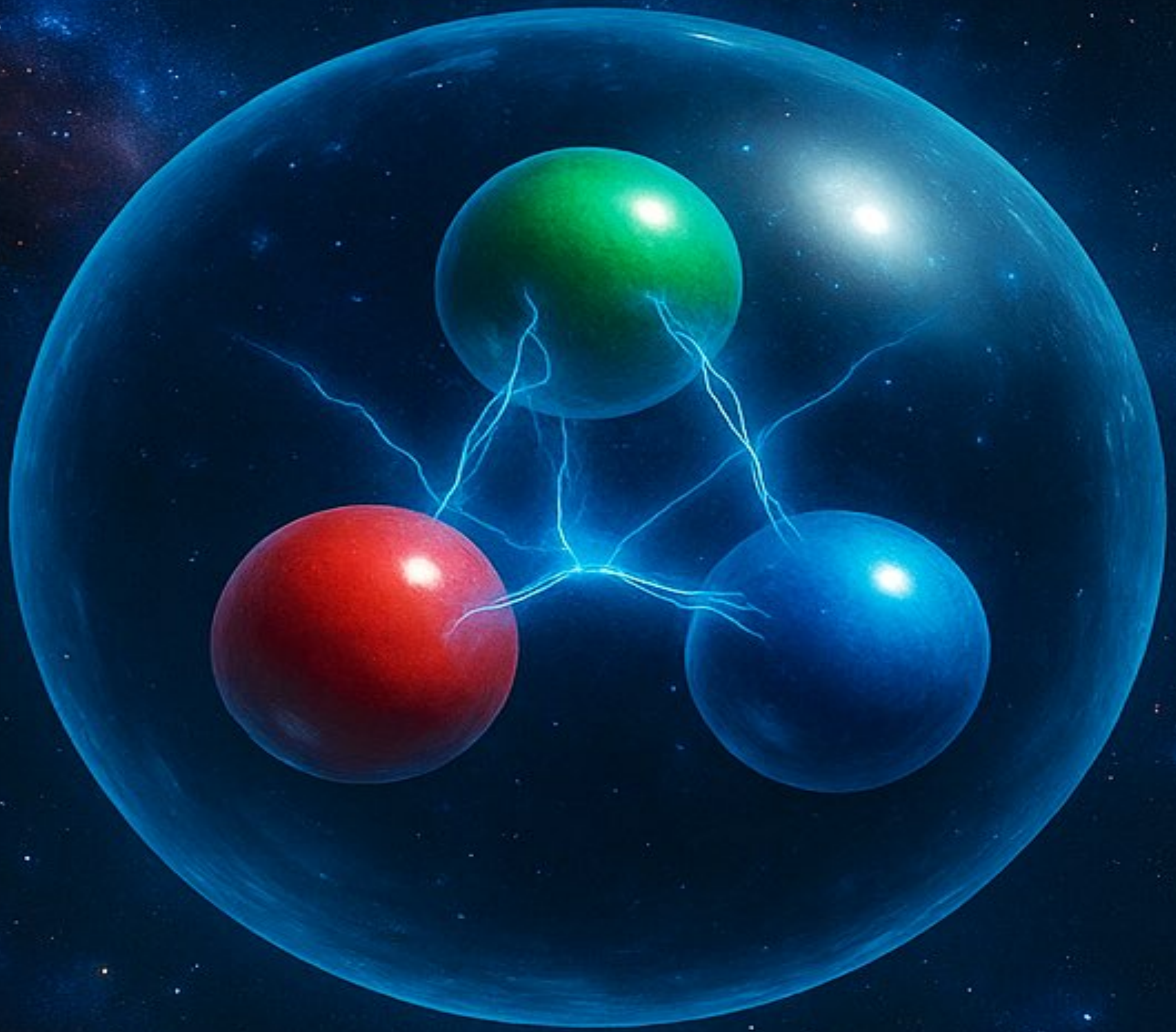


History of the Universe



10⁻³² seconds	1 microsecond	3 minutes	380,000 years	200 million years	400 million years	10 billion years	13.8 billion years
Inflation	**First Particles**	**First Nuclei**	**First Light**	**First Stars**	**Galaxies & Dark Matter**	**Dark Energy**	**Today**
Initial expansion	Neutrons, protons, and electrons form	Helium and hydrogen form	The first atoms form	Gas and dust condense into stars	Galaxies form in dark matter cradles	Expansion accelerates	Humans observe the universe

strong force between quarks inside proton

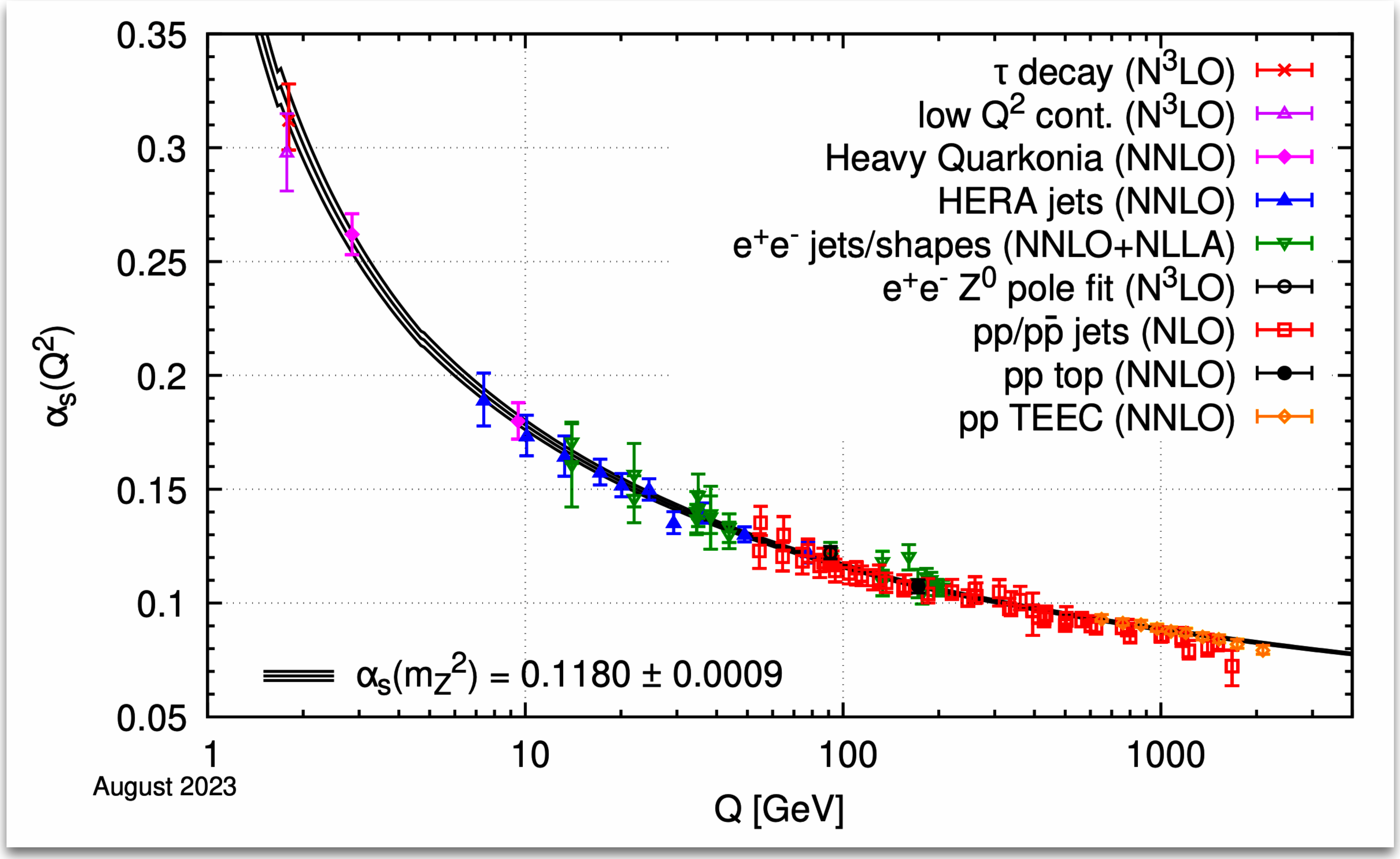


$\approx$

$10\ \times$

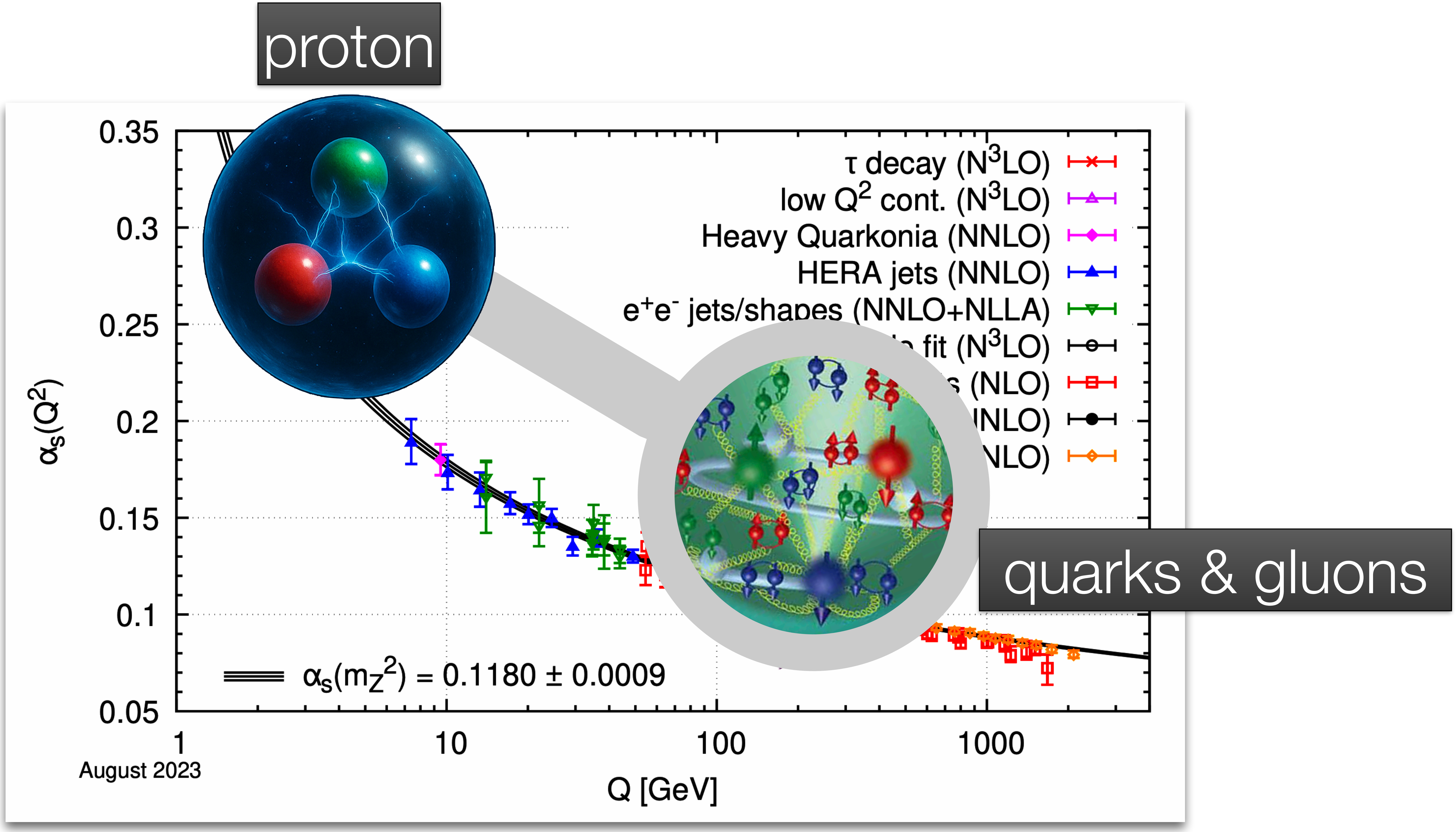
gravity

strength of strong force



coupling strength

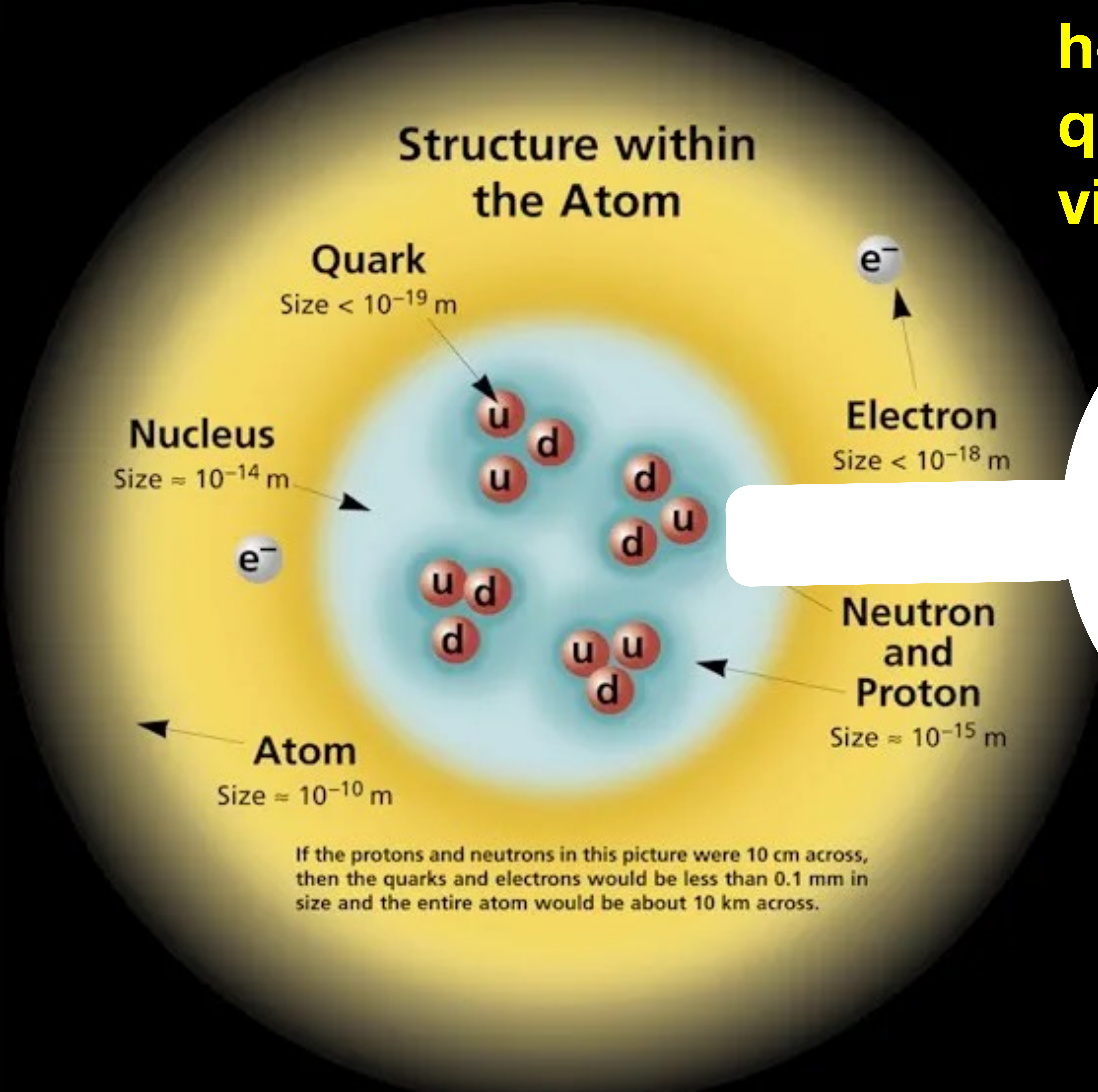
energy

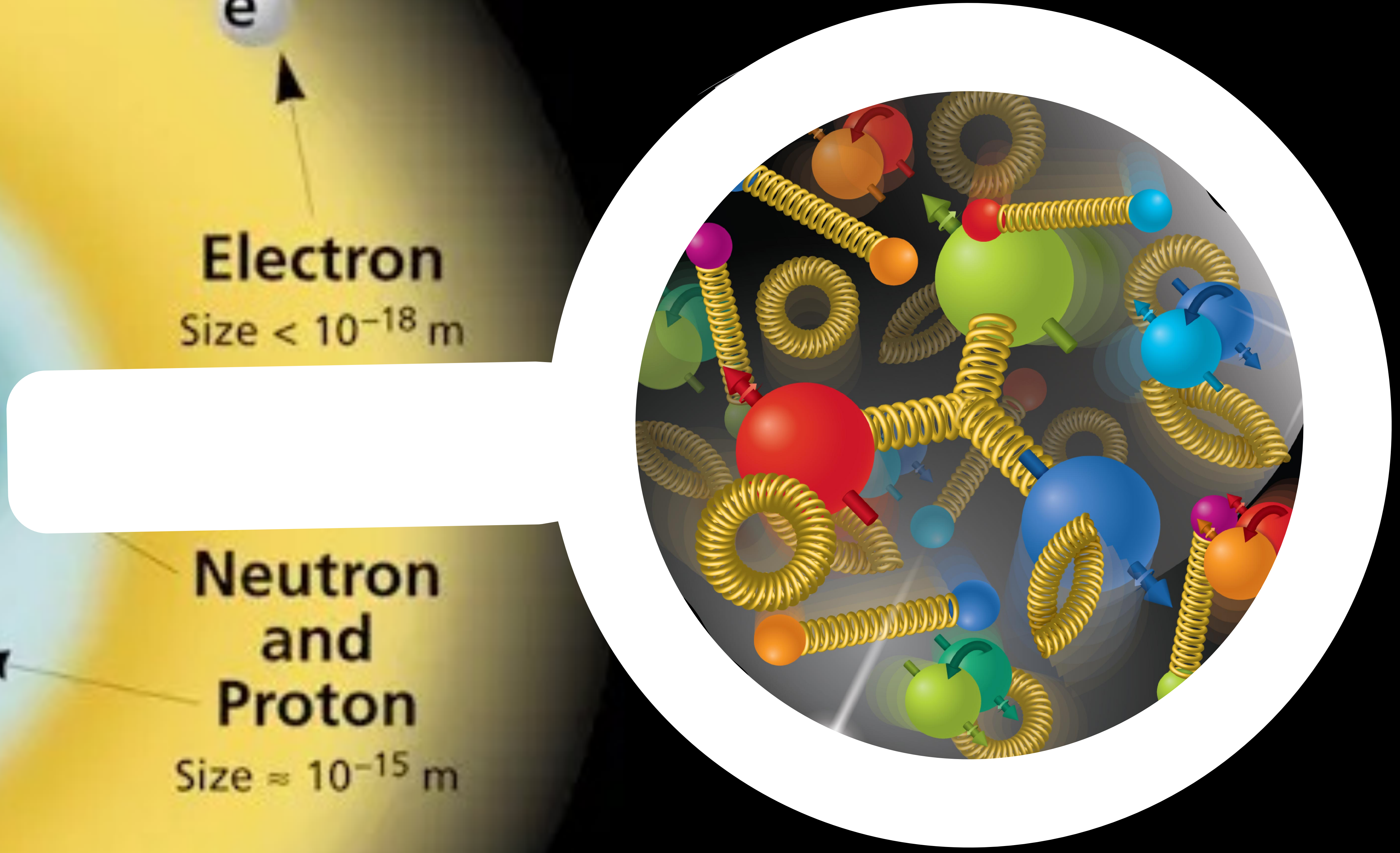


coupling strength

energy

how proton emerges from quarks & gluons interacting via the strong force ?



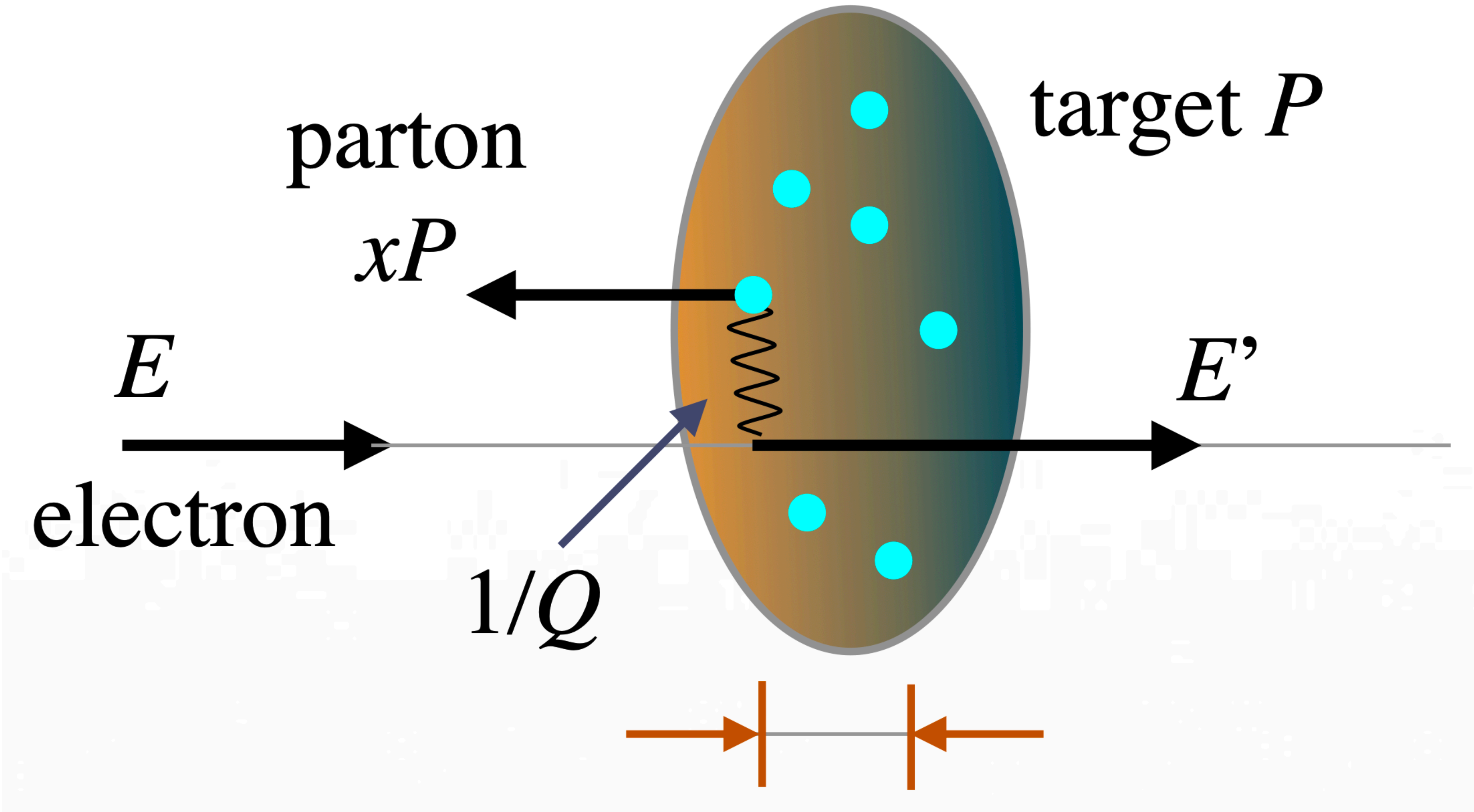


Electron Ion Collider (EIC) ...

... shining light on proton

proton in the eyes of the light — partons

an effective description observed from an infinite-momentum frame

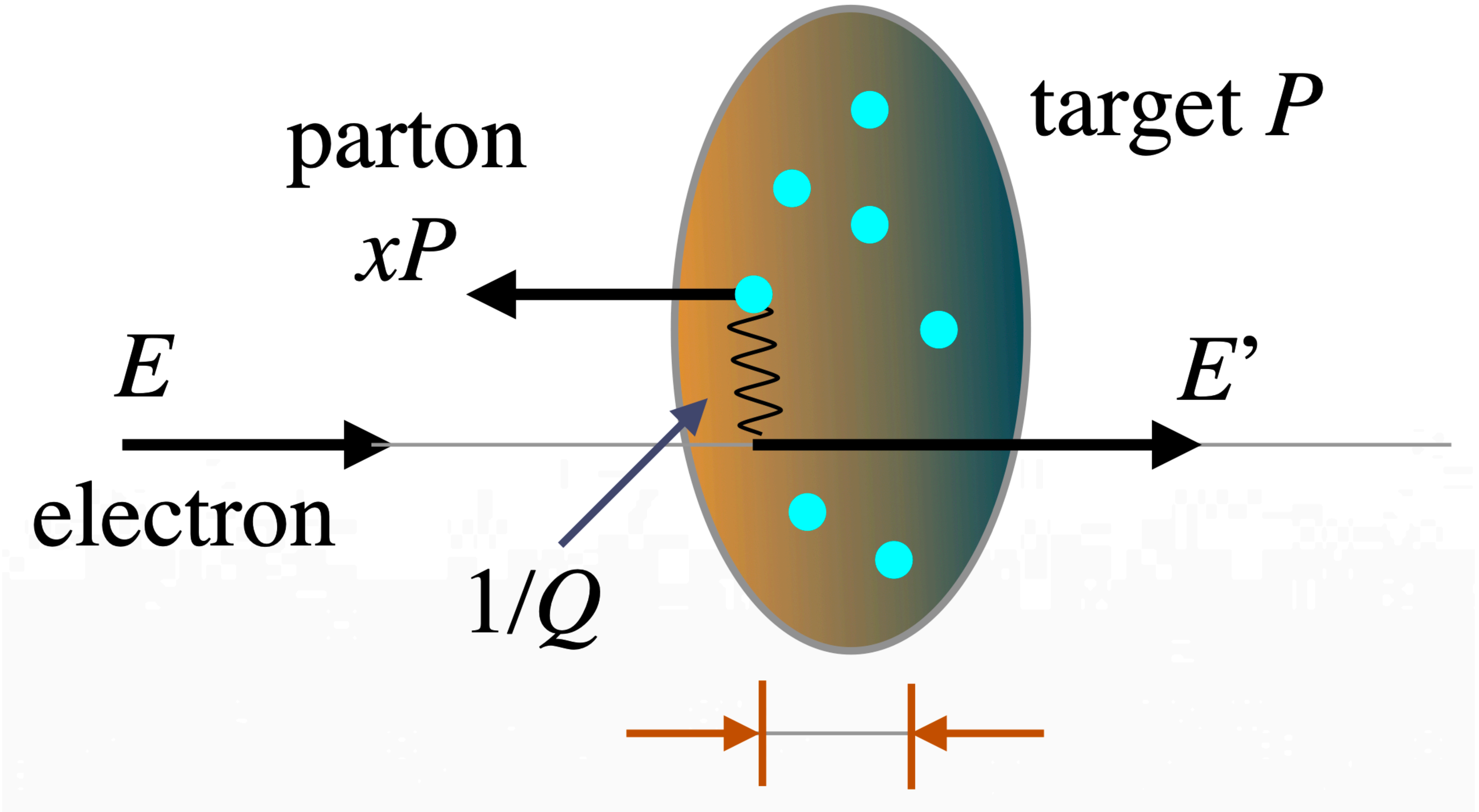


in c.m. frame electron sees a Lorentz-contracted proton; parton's virtual life Lortentz-dialted

$Q^2 \to \infty$, electron crosses proton in $t \to 0$: sees partons 'frozen' on approx. mass-shell

proton in the eyes of the light — partons

an effective description observed from an infinite-momentum frame



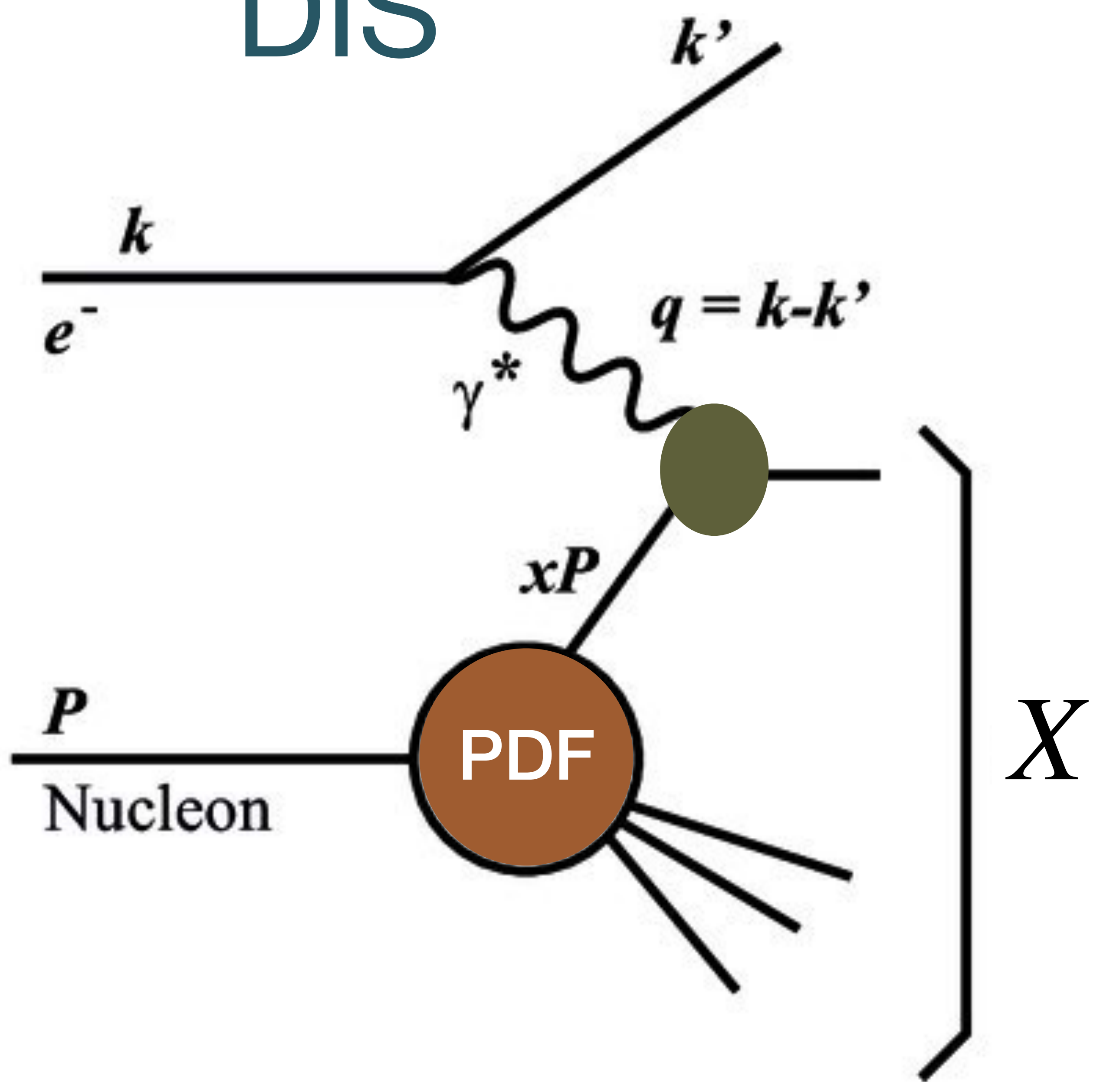
x : fraction of the longitudinal momentum of proton carried by a parton

partons: effective d.o.f—collinear momenta-modes of proton how to discover partonic structures of proton ?

QCD factorization

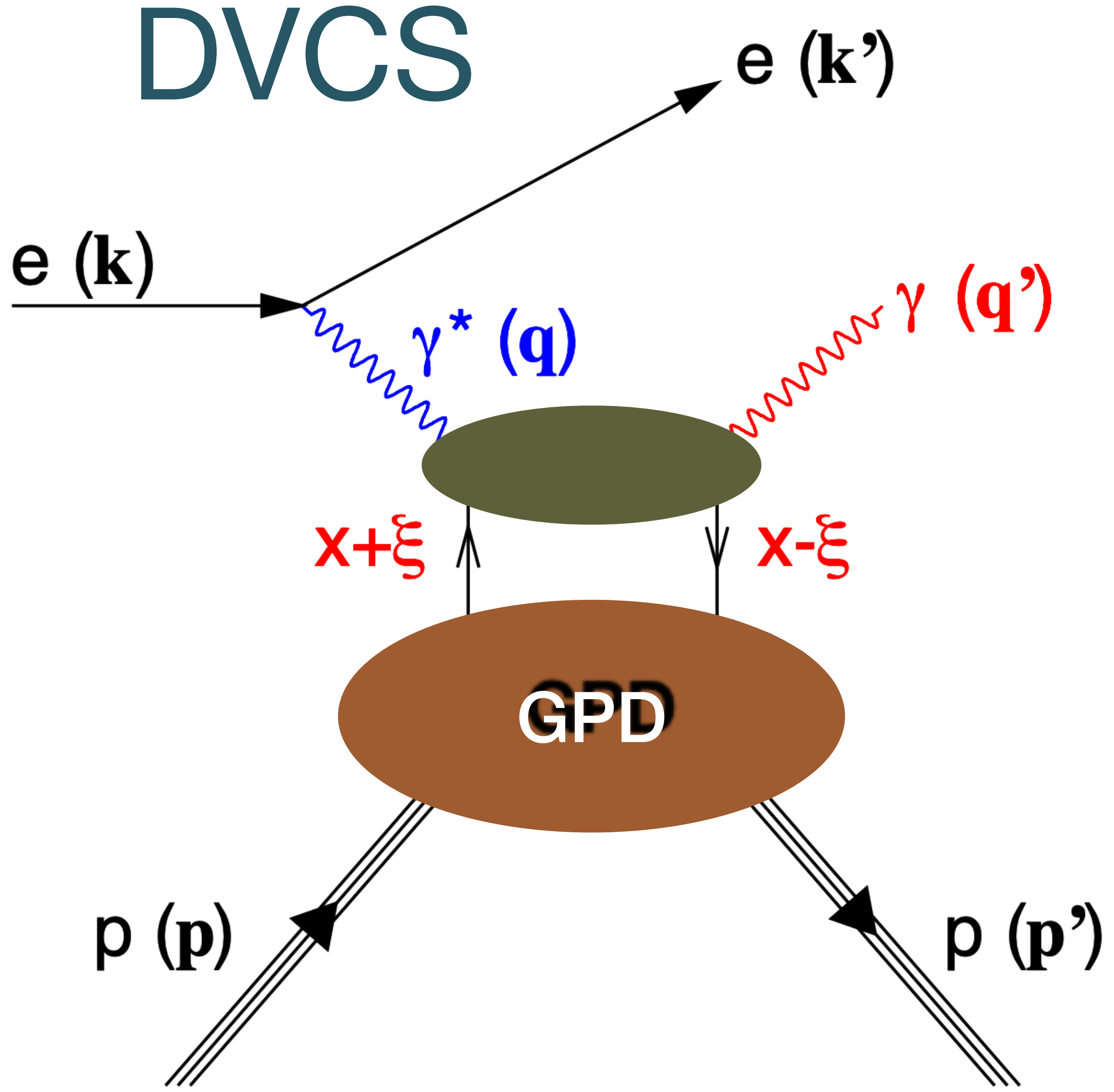
$e + P \to e + X$

DIS



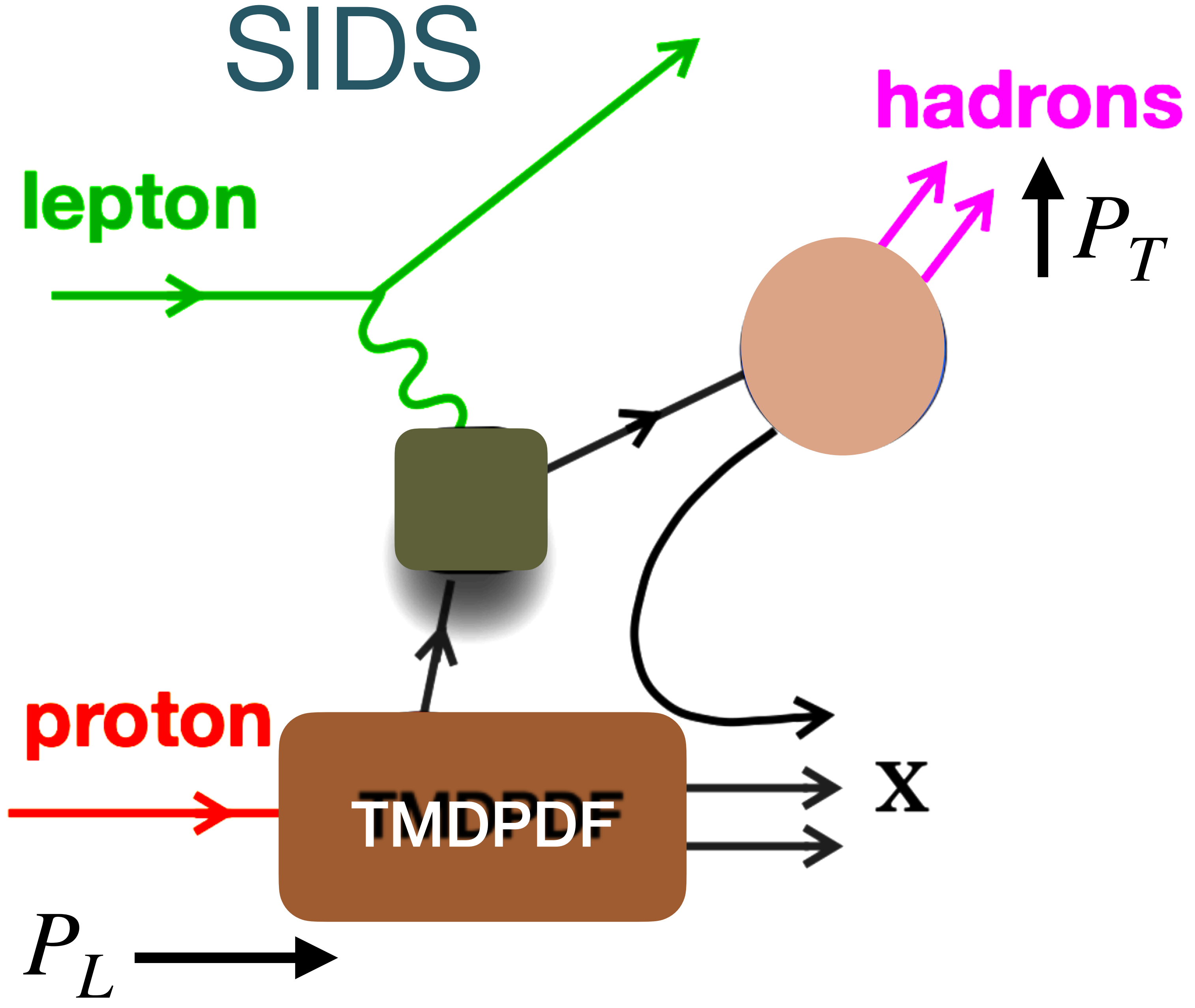
$e + P \to e + P' + \gamma$

DVCS



$e + P \to e + H + X$

SIDS



cross-section $\sim \boxed{c(y, x, \mu)} \otimes \boxed{f(x, \mu)}$ nonperturbative QCD

perturbative QCD

analytically calculable

encodes proton's partonic structures

knowing proton

cross-section ~ $c(y, x, \mu) \otimes f(x, \mu)$

measure at EIC

perturbative QCD

nonperturbative proton structure

extract by fitting experimental data
using perturbative QCD inputs

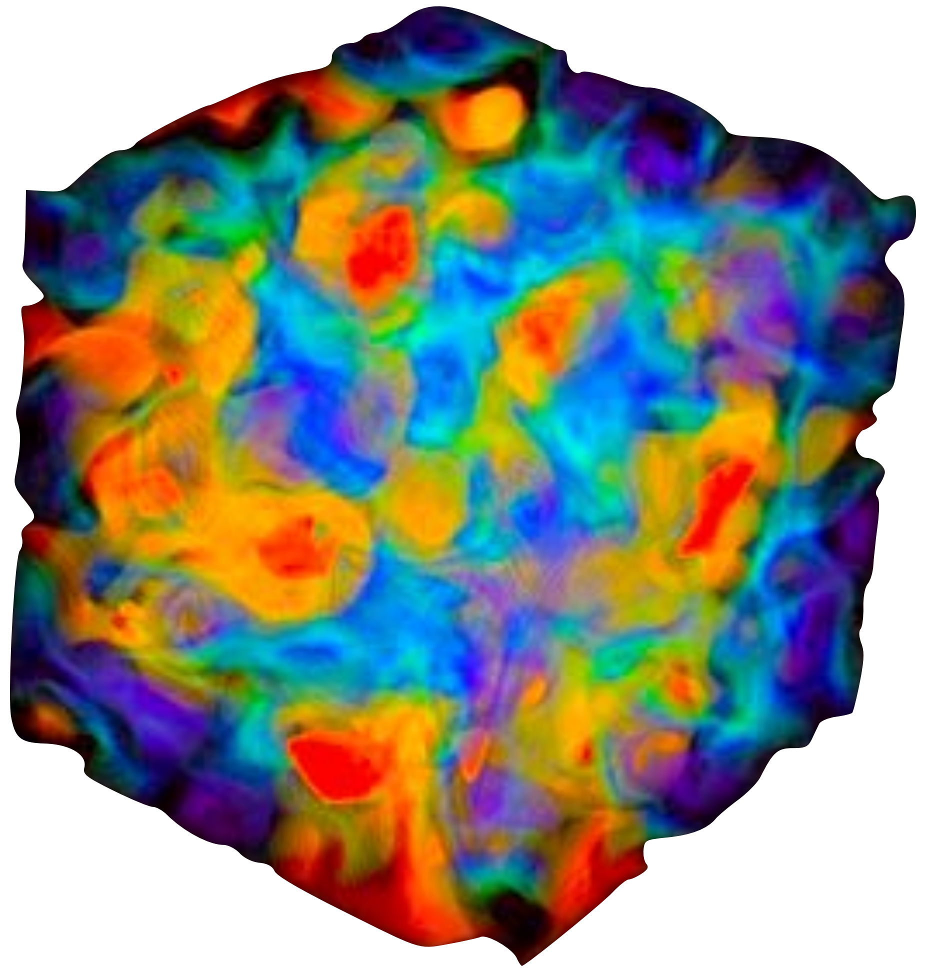
dream: understanding proton — from QCD to cross-section

cross-section ← $c(y, x, \mu) \otimes f(x, \mu)$

measure at EIC

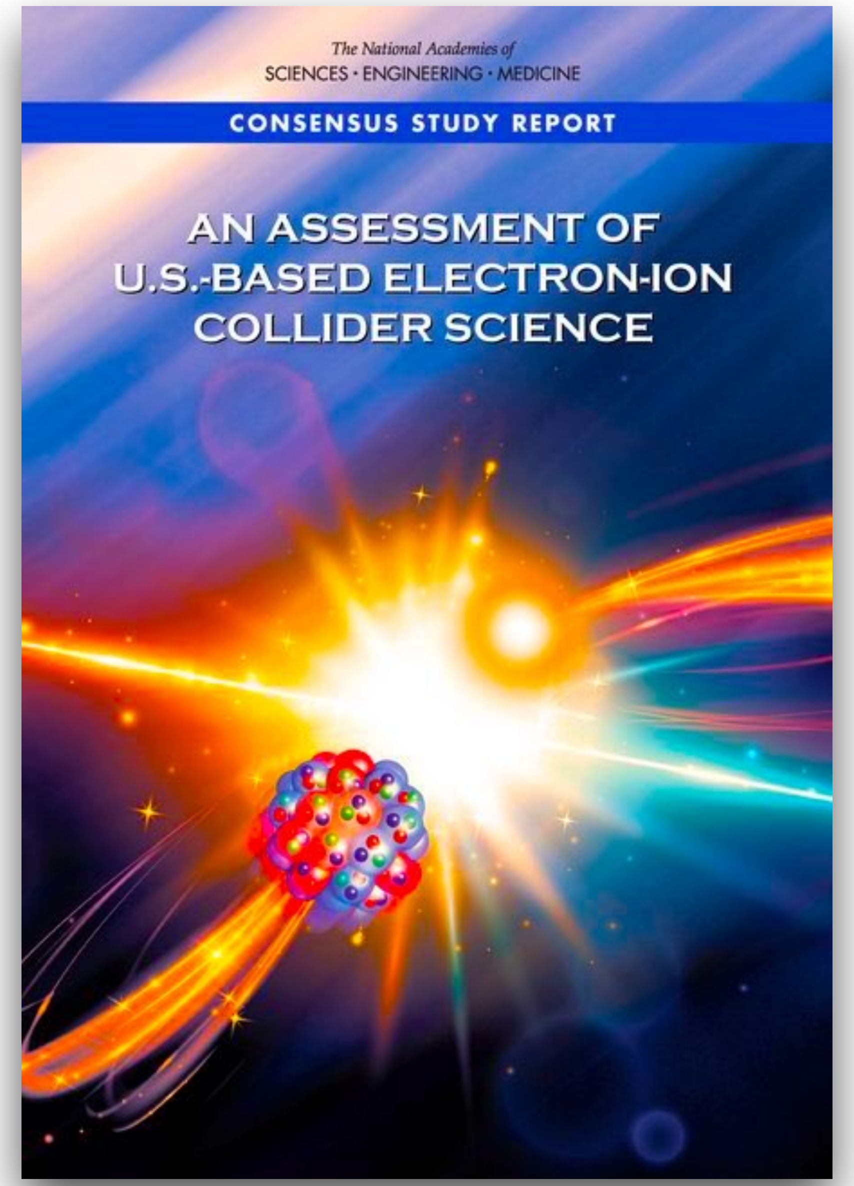
perturbative QCD

nonperturbative QCD



science at EIC needs help

Summary of the National Academy of Science report

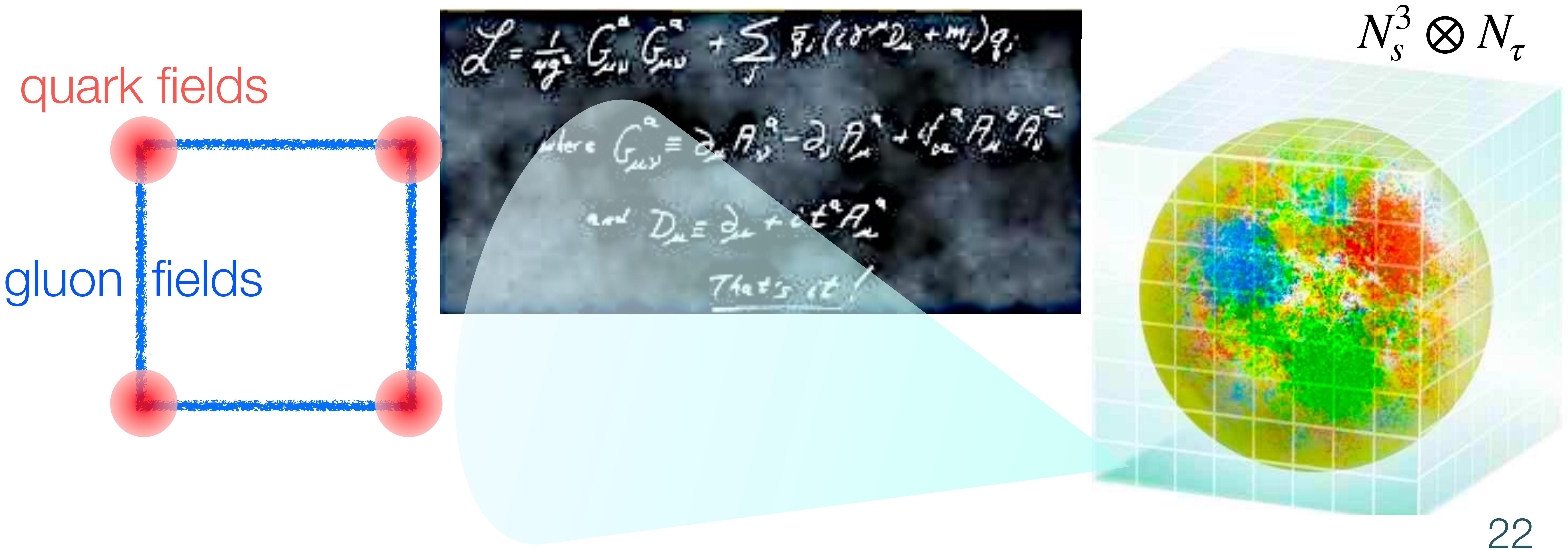


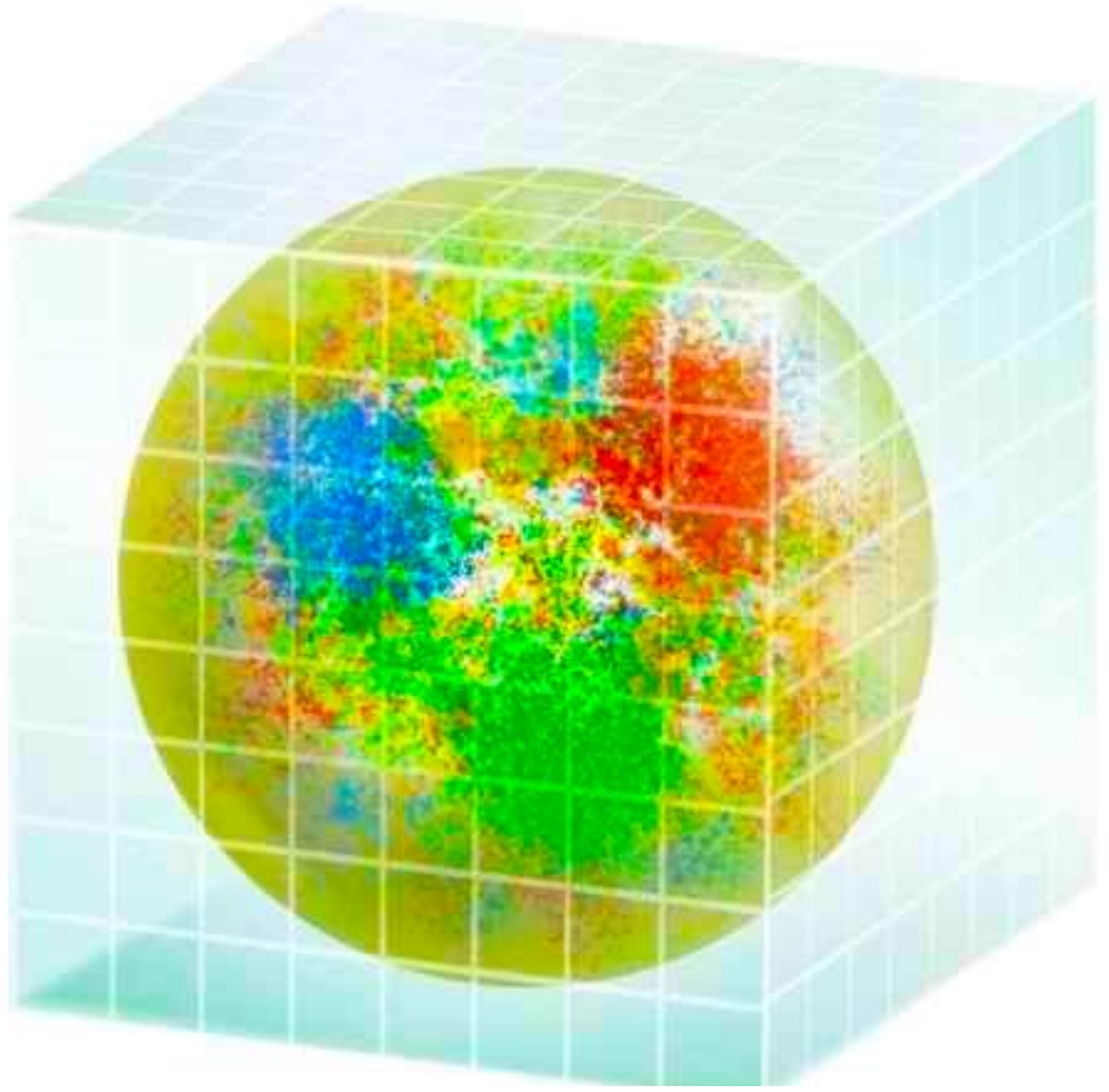
“The scientific challenges that would unfold with EIC require a robust theory program, not simply to design and interpret experiments, but also to develop the broad implications in an understanding of the quantum world, both through analytic theory as well as through lattice QCD simulations on large-scale computers.”

lattice quantum chromodynamics (QCD)

nonperturbative regularization of field theory (QCD)

discretized space and Euclidean (imaginary) time





QCD path integral $\sim \int \mathscr{D}[U]\mathscr{D}[\psi]\mathscr{D}[\bar{\psi}]e^{-S_{QCD}[U,\psi,\bar{\psi}]}$

$$N_s^3 \otimes N_\tau \otimes N_{color} \otimes N_{spin} \otimes N_{flavor}$$

> 10 billion degrees of freedom

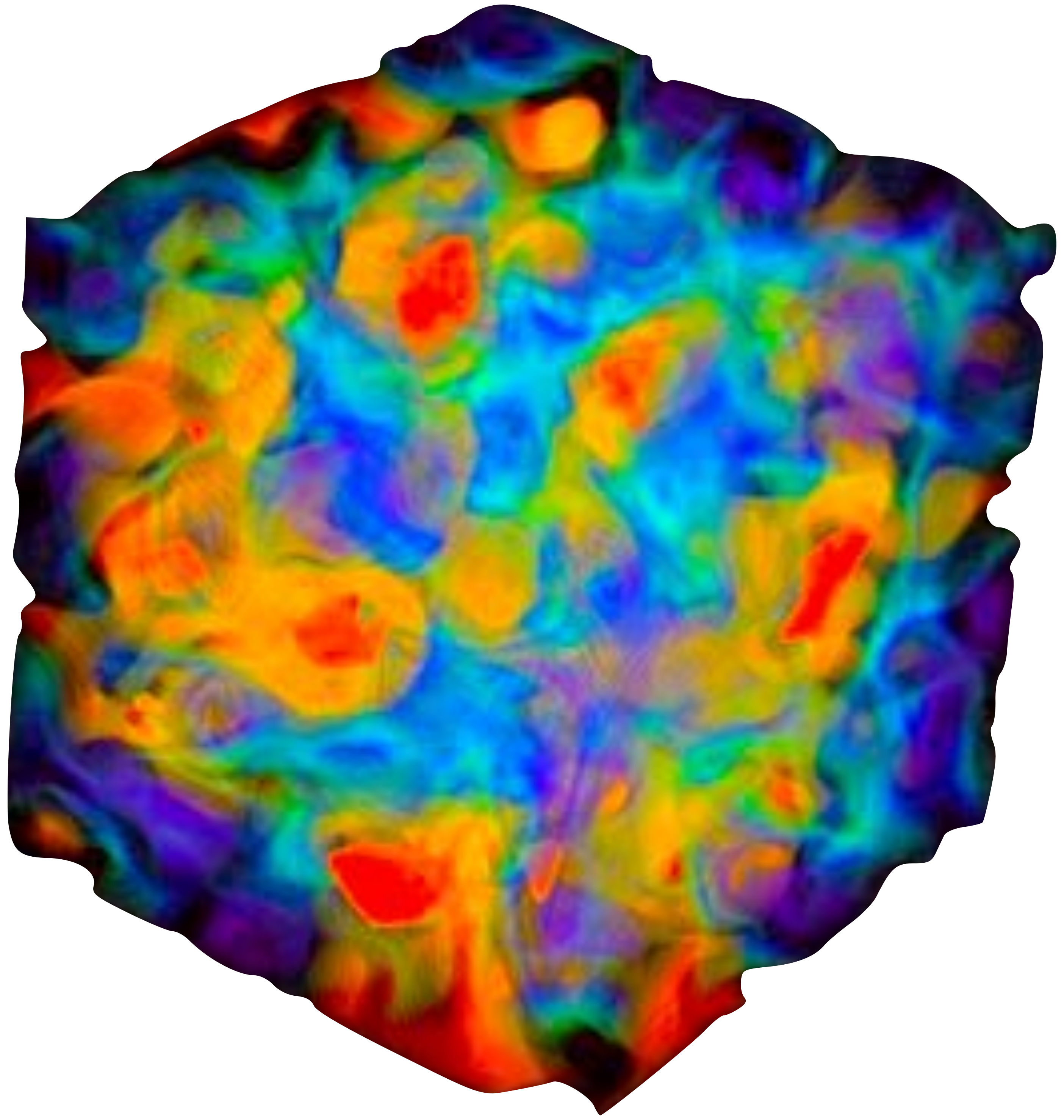
solve via numerical Monte-Carlo using computers

exascale supercomputers

1 sec

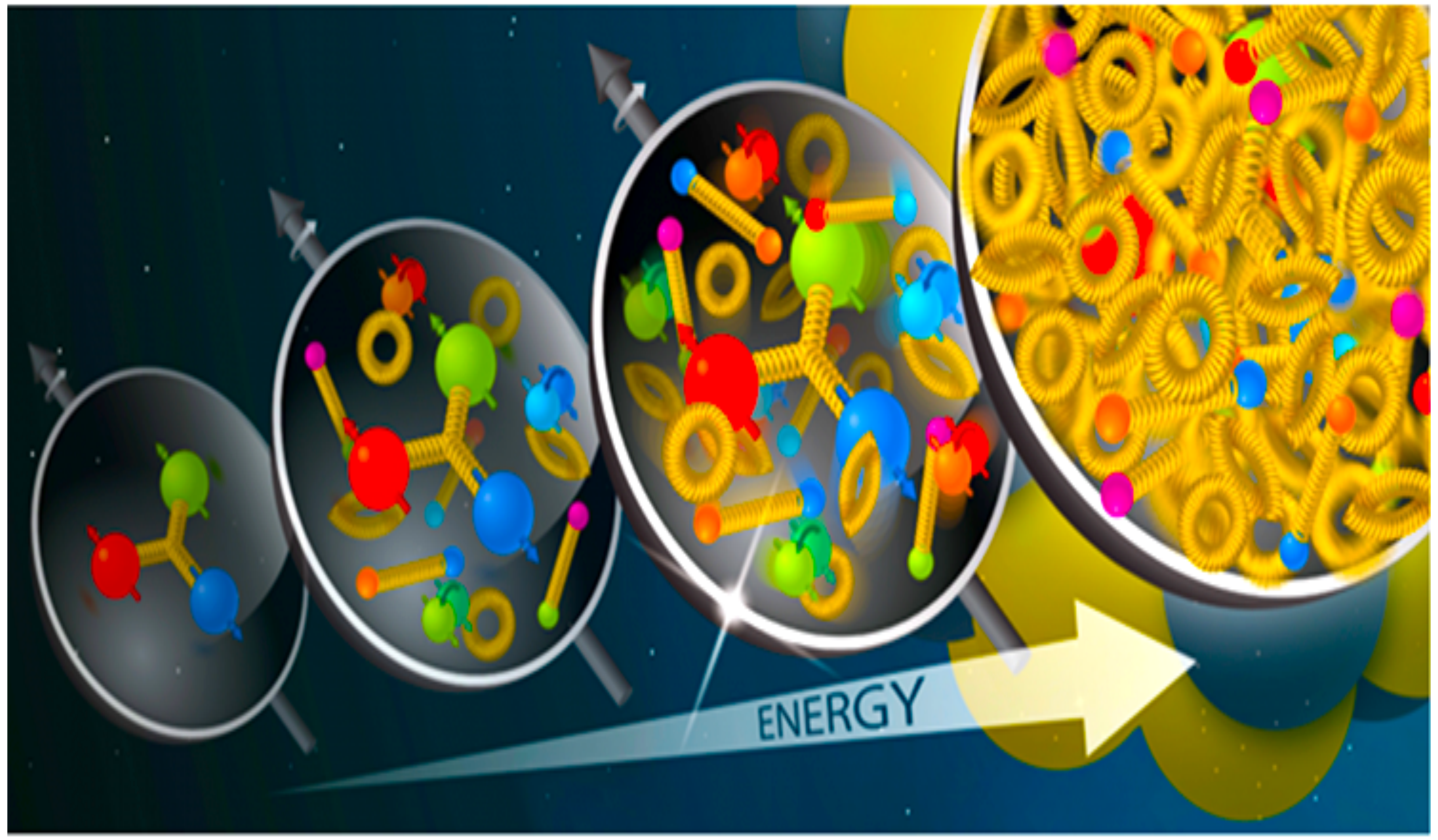
each person, one calculation/sec, approx. 4 years

how to see a parton on the lattice ?



nonperturbative field theory

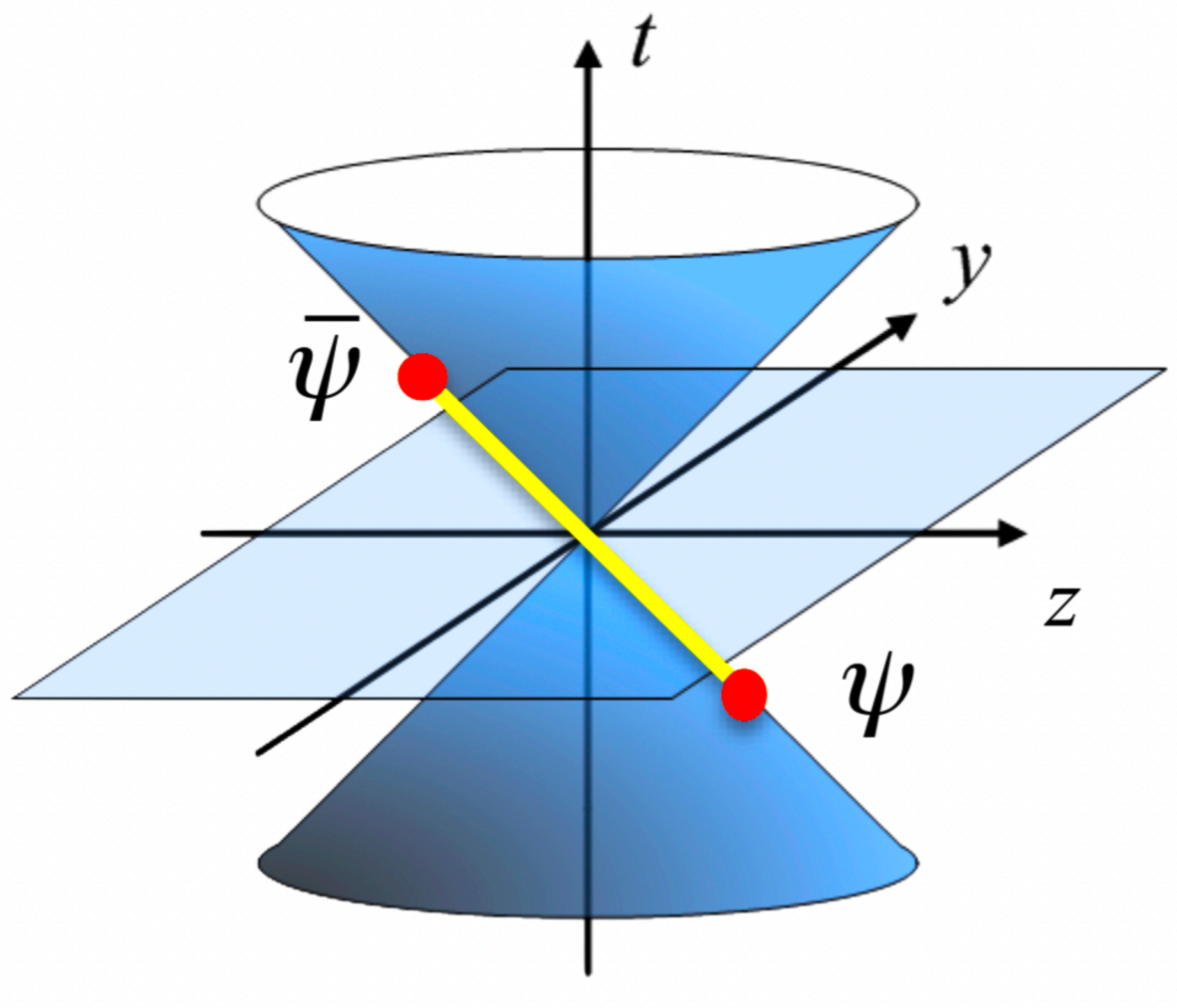
????



perturbative effective d.o.f

partonic structure of hadron

regularize QCD after taking the lightcone, $P_z \to \infty$ / $z^2 \to 0$, limit

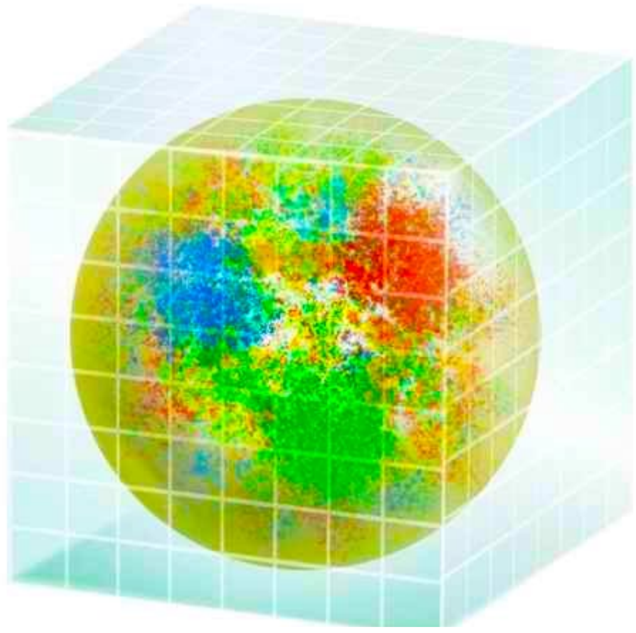


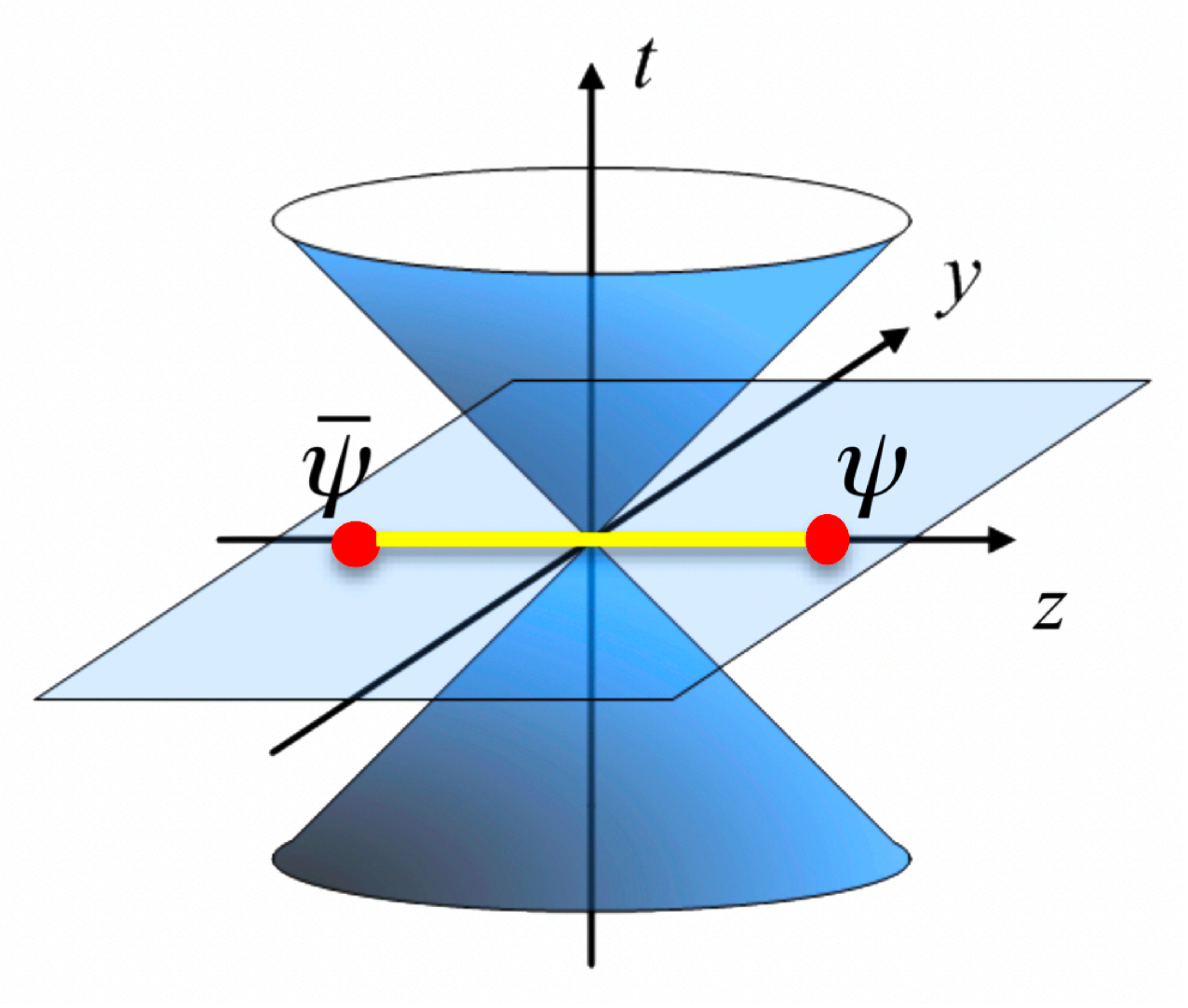
$$f(x,\mu) \sim \left\langle H(P_z) \,|\, \boxed{\hat{O}(z^-,\mu)} \,|\, H(P_z) \right\rangle$$

timelike separated bilocal operator

partonic structures from lattice QCD

hadron at rest



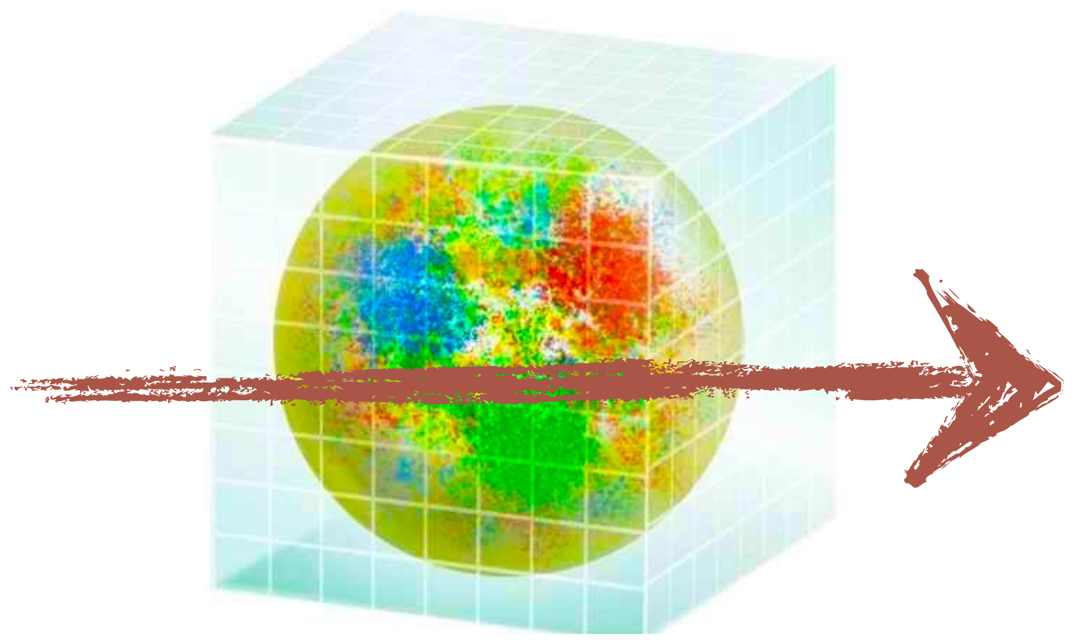


$$M(z^2, \mu) \sim \left\langle H(0) \,|\, \hat{O}(z, \mu) \,|\, H(0) \right\rangle$$

spacelike separated bilocal operator

renormalize: scale μ

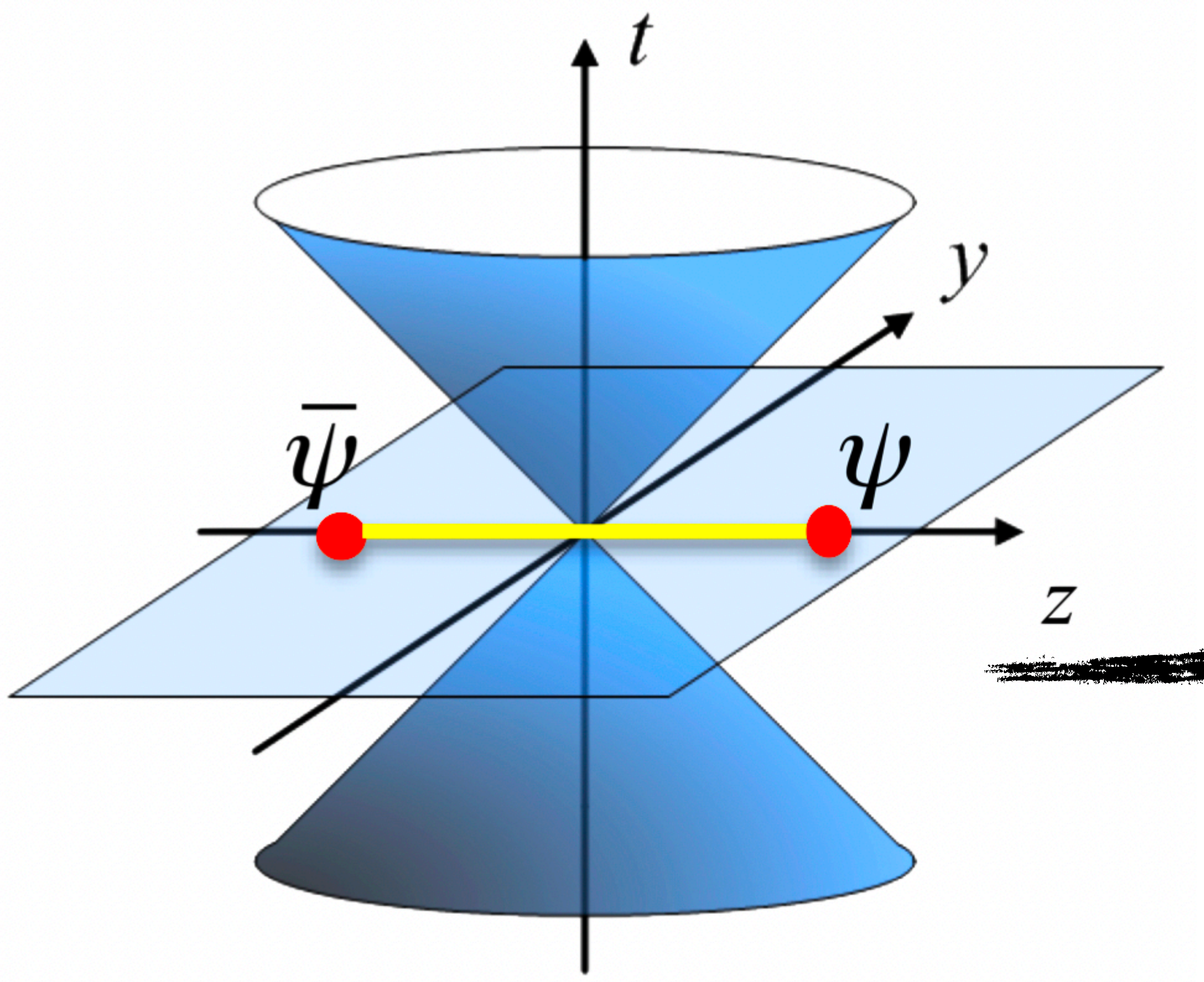
fast-moving hadron

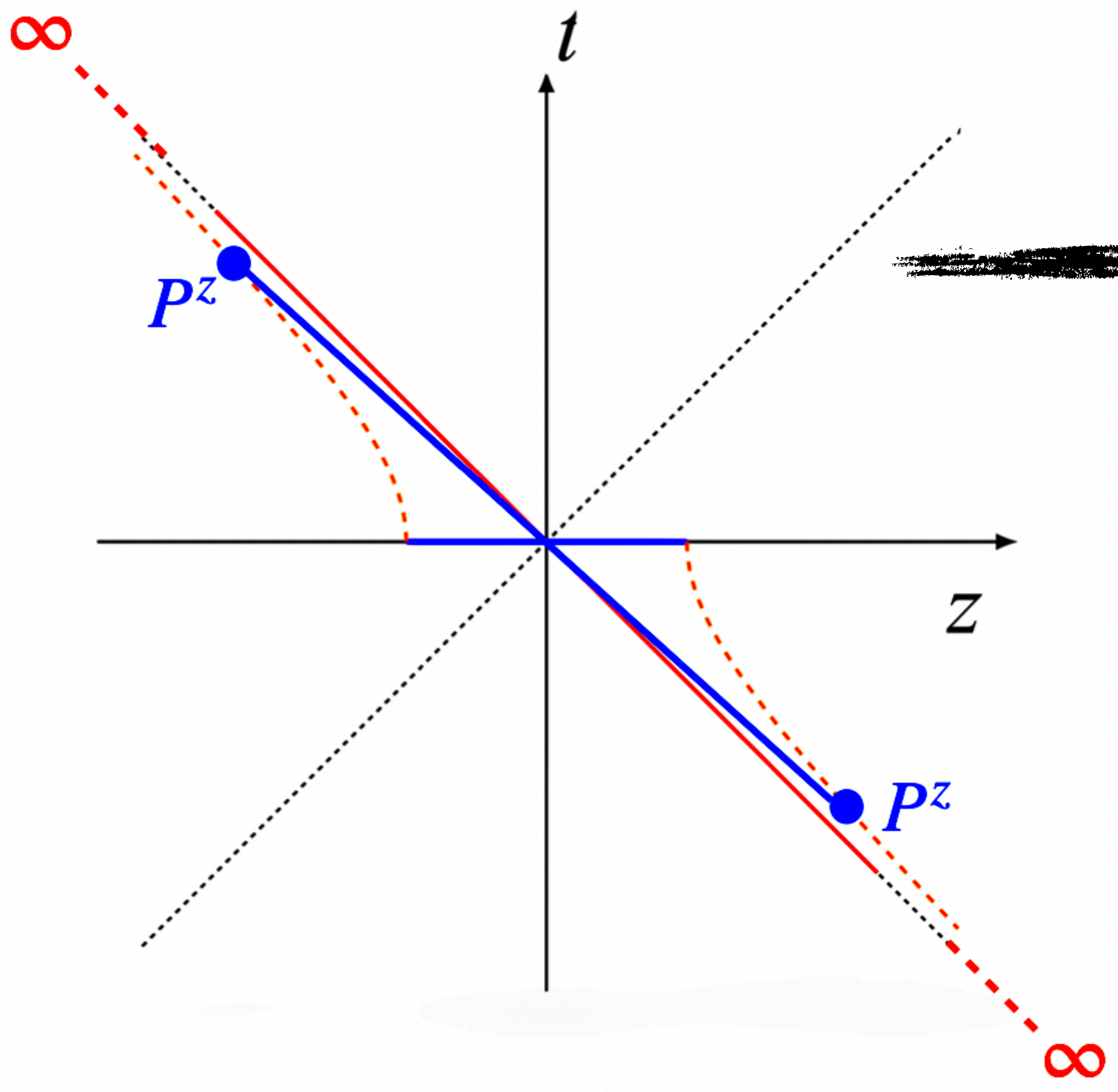


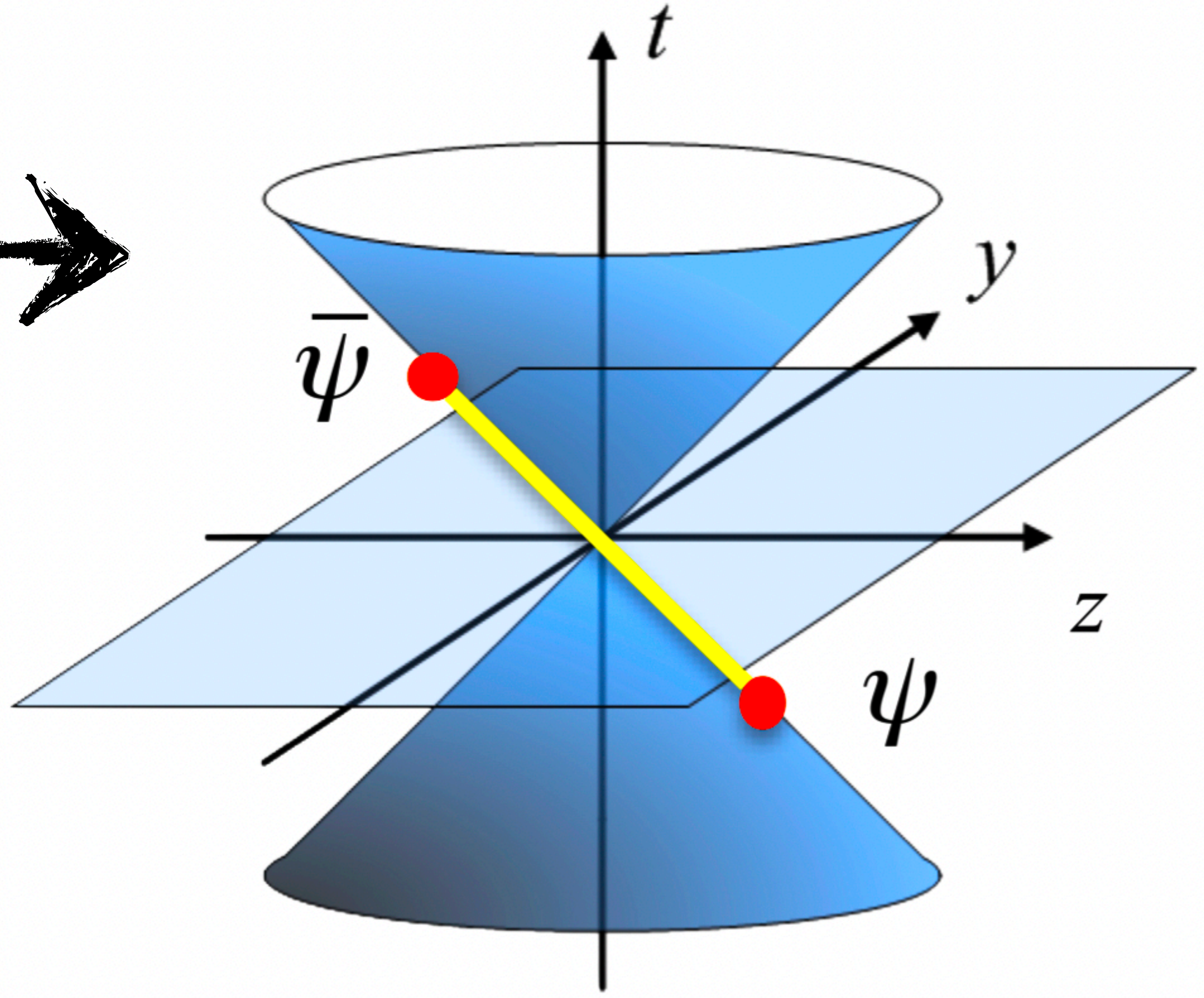
$P_z \approx E$

$$\left\langle H(0) \,|\, \hat{O}(z,\mu) \,|\, H(0) \right\rangle$$

$$\left\langle H(P_z) \,|\, \boxed{\hat{O}(z^-,\mu)} \,|\, H(P_z) \right\rangle$$







factorization of $M(y, \mu, P_z) \sim$ perturbative $\otimes$ non-perturbative

$\tilde{c}(y, x, \mu, P_z) \otimes \tilde{f}(x, \mu)$

momentum space

$\tilde{c}(y, x, \mu, z^2) \otimes \tilde{f}(x, \mu)$

position space

nonperturbative objects on the lightcone, $f(x, \mu)$, and from lattice QCD, $\tilde{f}(x, \mu)$, shares same infrared singularities, i.e. governed by same evolution equations factorization: perturbative $\otimes$ non-perturbative

$$M(y,\mu,P_z) \sim \boxed{\tilde{\sigma}(y,x,\mu,P_z)} \otimes \boxed{\tilde{f}(x,\mu)} \qquad M(y,\mu,z^2) \sim \boxed{\tilde{\sigma}(y,x,\mu,z^2)} \otimes \boxed{\tilde{f}(x,\mu)}$$

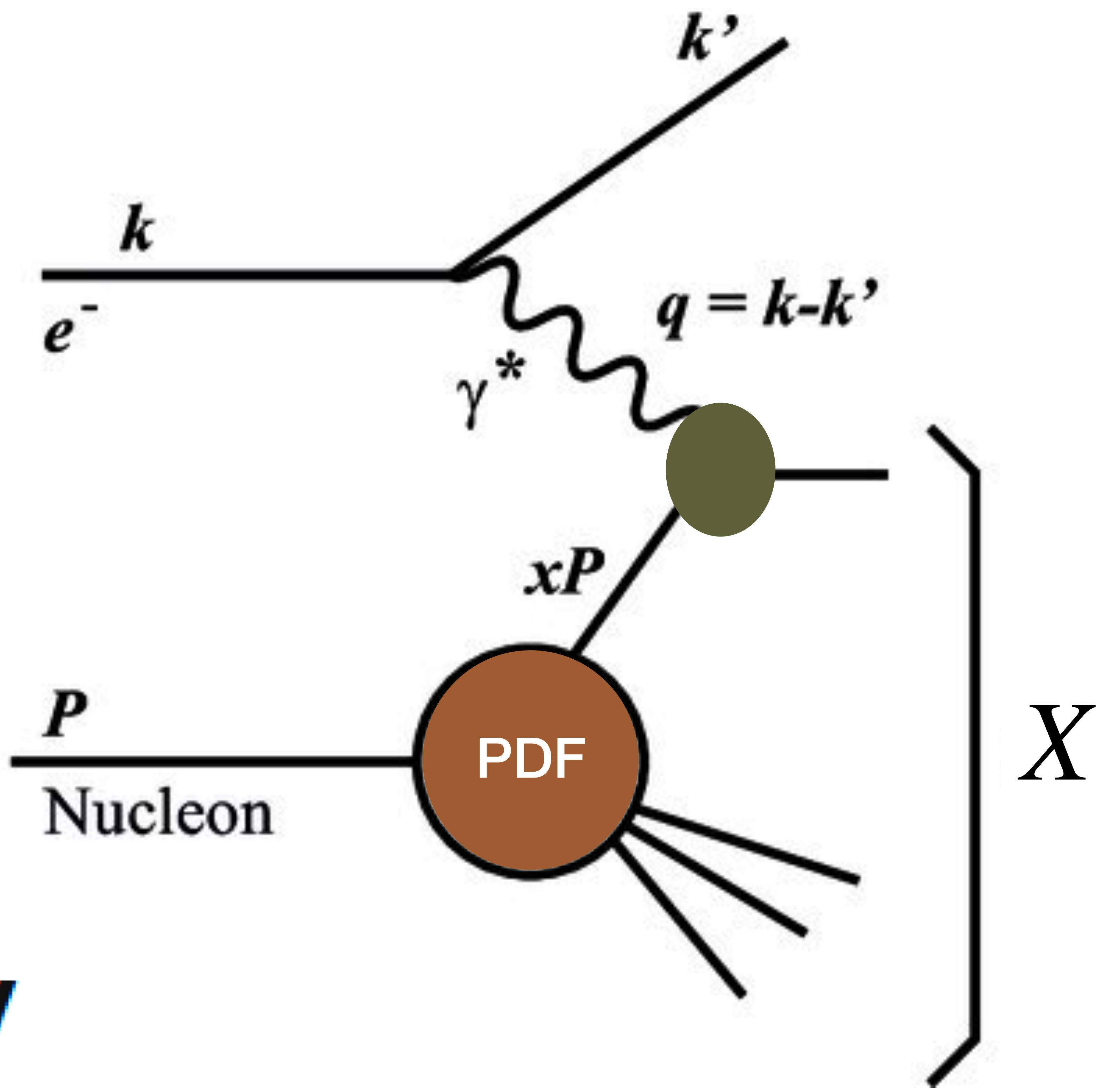
regularize QCD on a lattice, then $P_z \to \infty$ / $z^2 \to 0$; opposite order of limits from light-cone quantization

difference is UV physics, can be taken care of through perturbative matching

partonic origin of proton mass

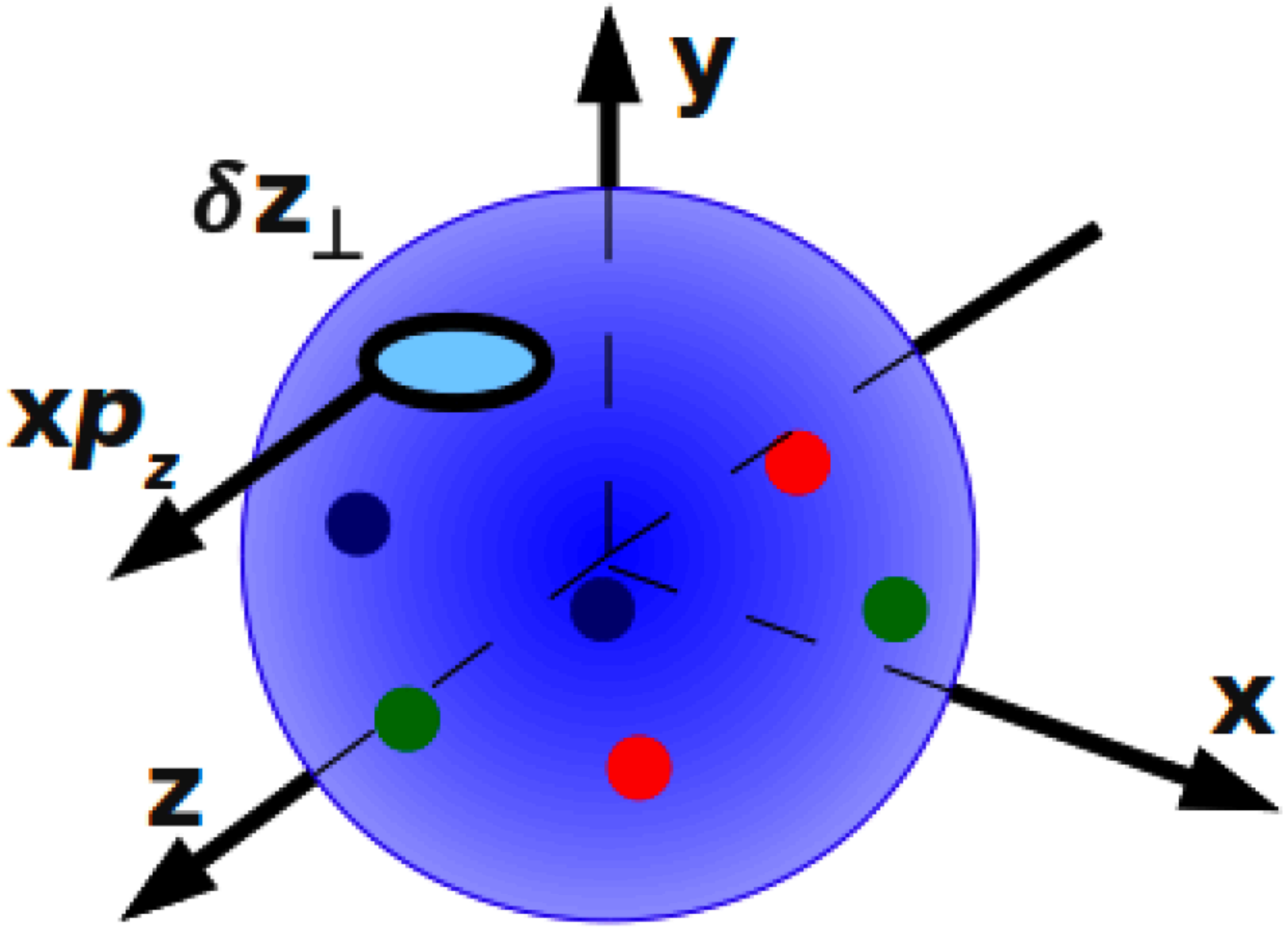
deep inelastic scattering: $e + P \longrightarrow e + X$

cross-section ~ perturbative $\otimes$ PDF

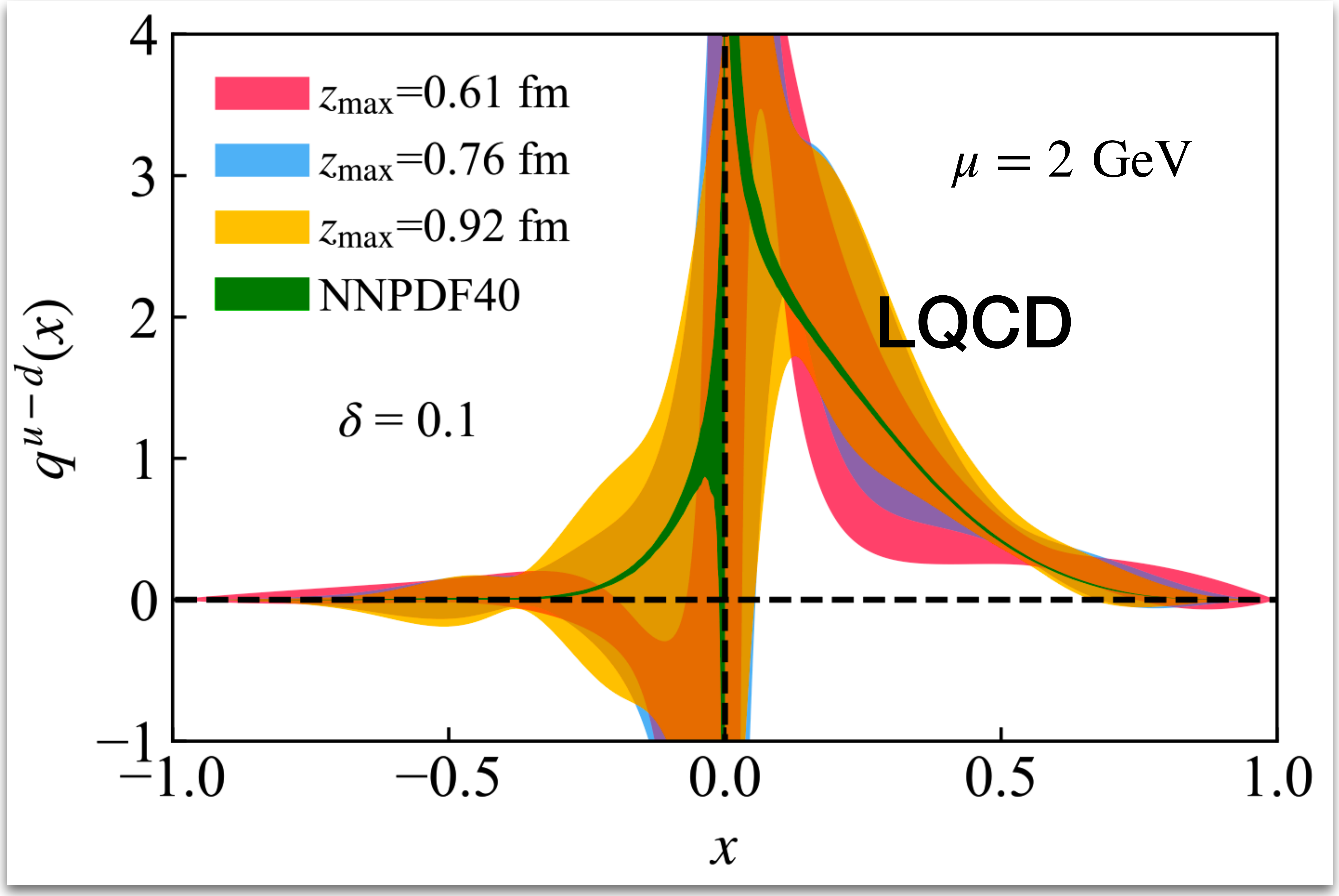


parton distribution function (PDF)

distribution, at scale μ, of longitudinal momentum fractions of a parton inside hadron moving with infinite momentum



isovector quark PDF of unpolarized proton at NNLO

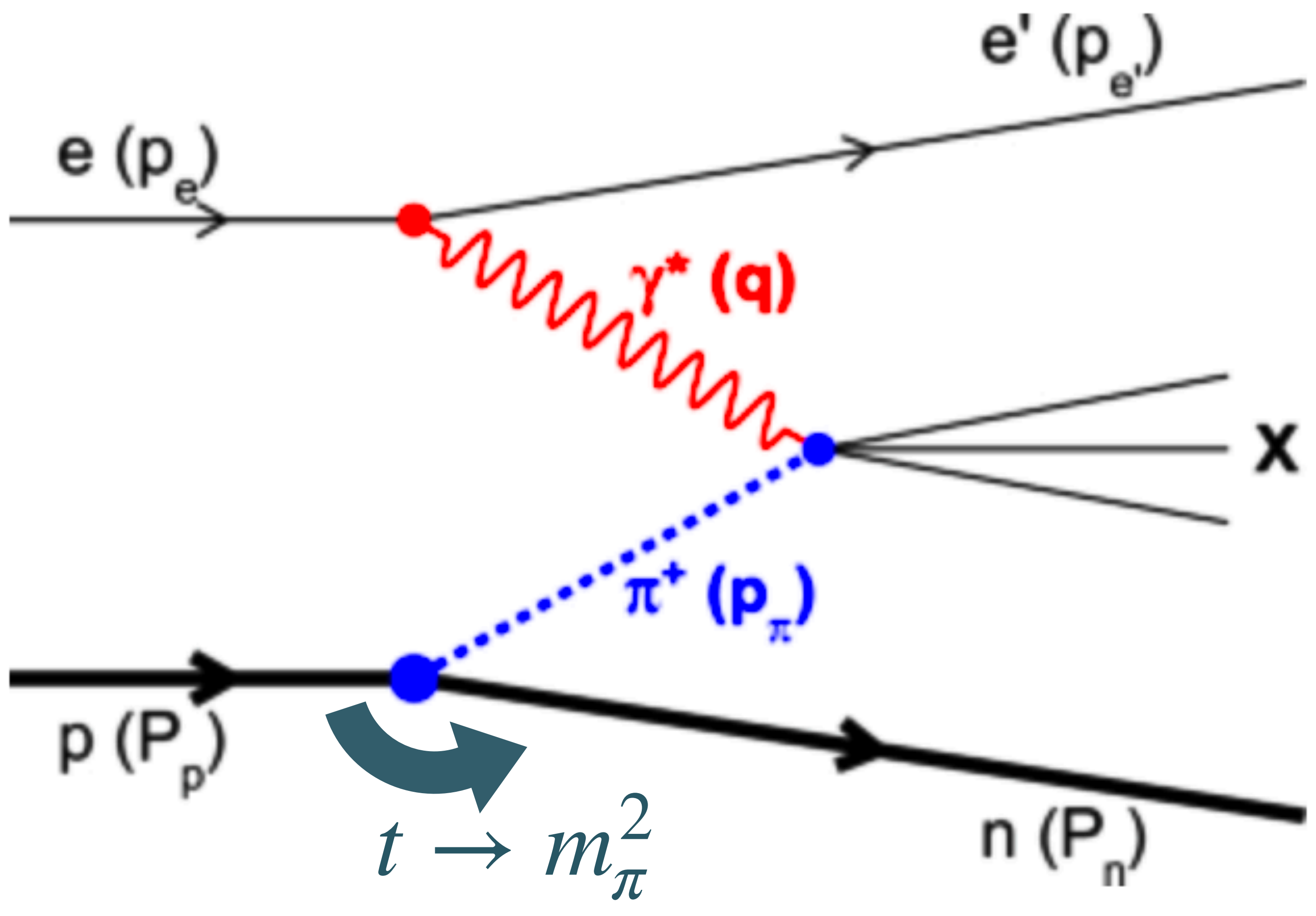


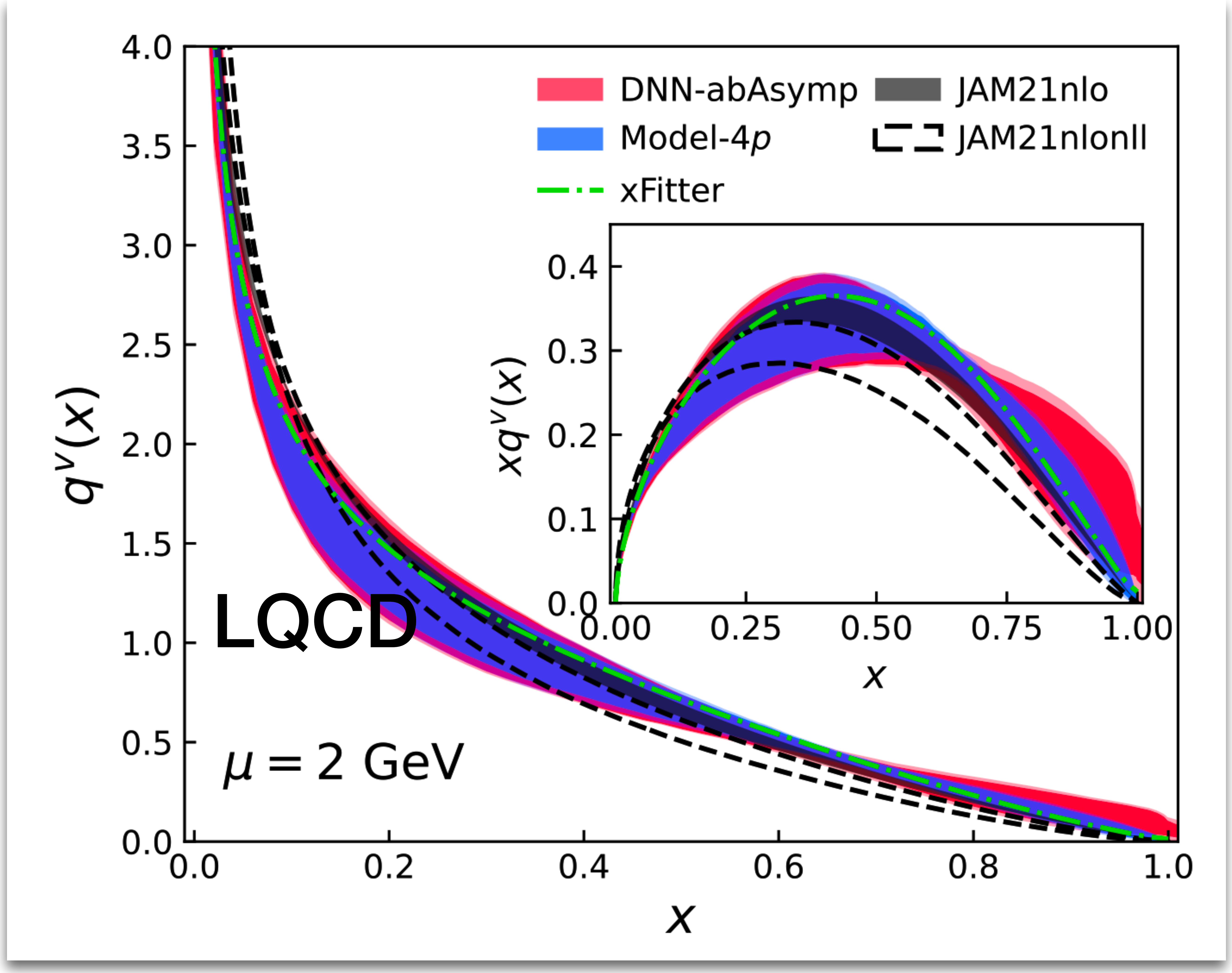
physical quark masses

A. Hanlon et al., Phys. Rev. D107, 7, 074509 (2023)

valance quark PDF of pion at NNLO

Sullivan process @ EIC





physical quark masses,
continuum-extrapolated

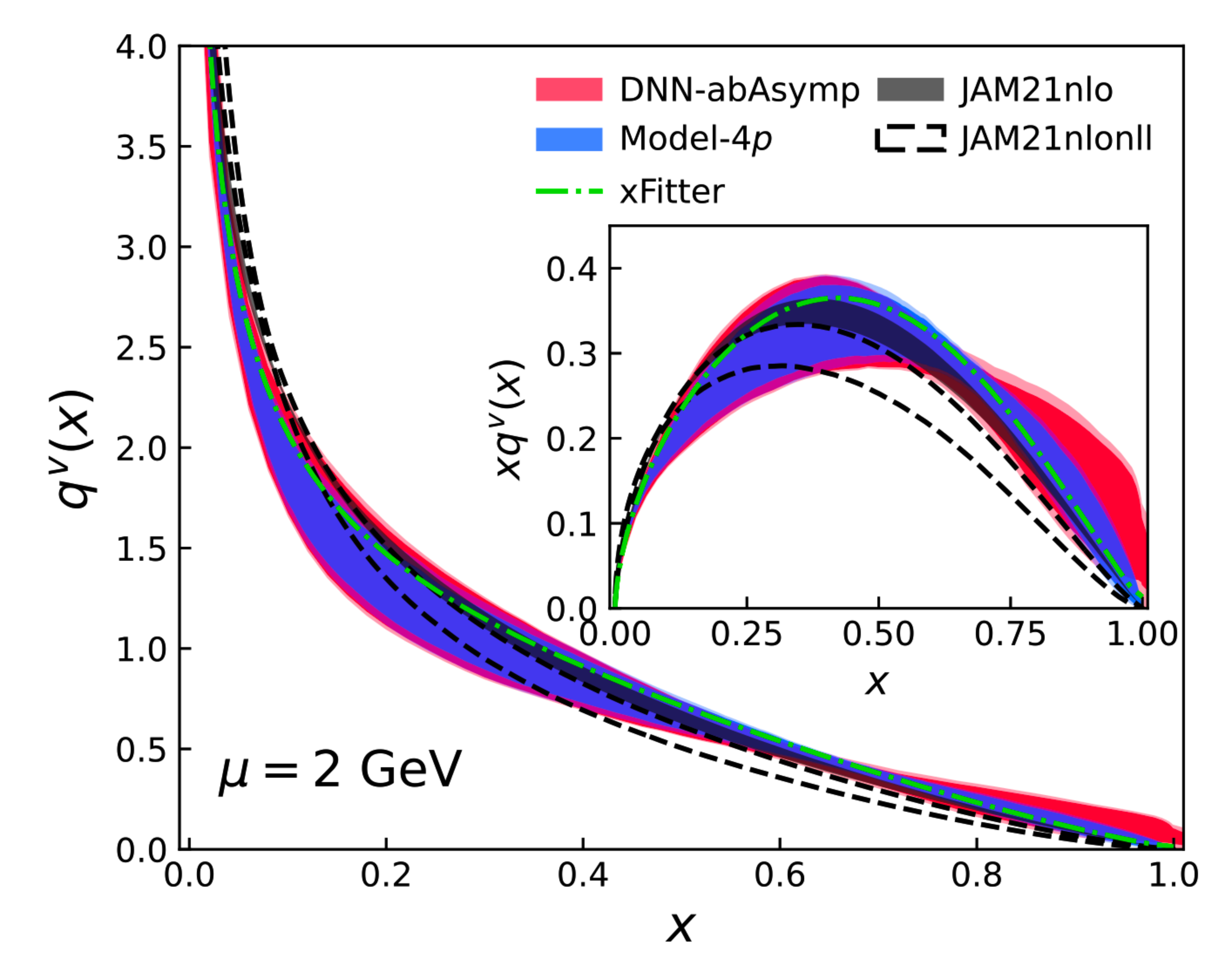
Y. Zhao et al., Phys. Rev. Lett. 128, 14, 142003 (2022)

X. Gao et al., Phys. Rev. D06, 11, 114510 (2022)

quark energy contributions to hadron masses

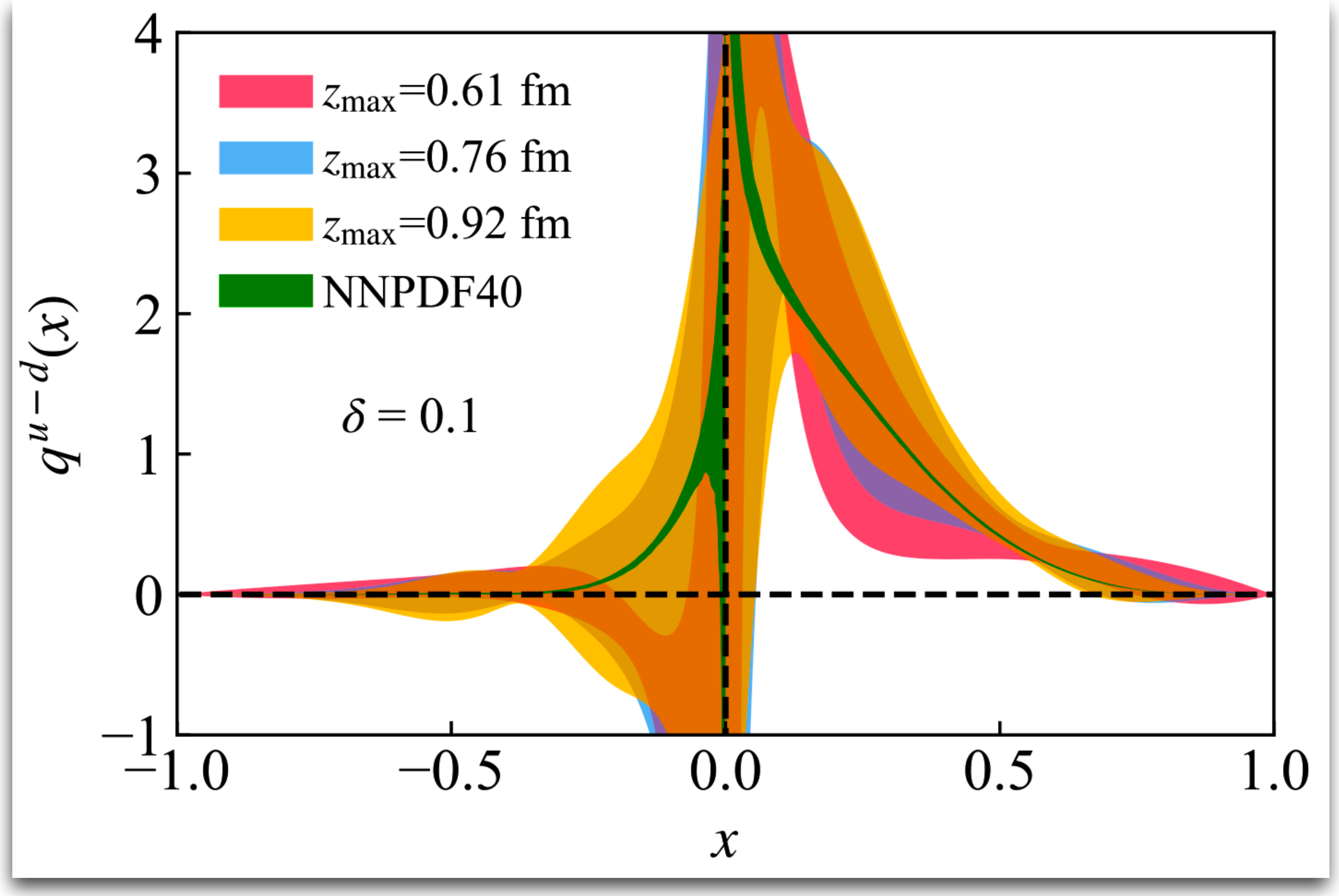
$$E_q(\mu) = \frac{3}{4} m_H \int_0^1 x f(x,\mu)\, dx$$

pion: $E_q(2\text{ GeV})/m_\pi \approx 30\,\%$



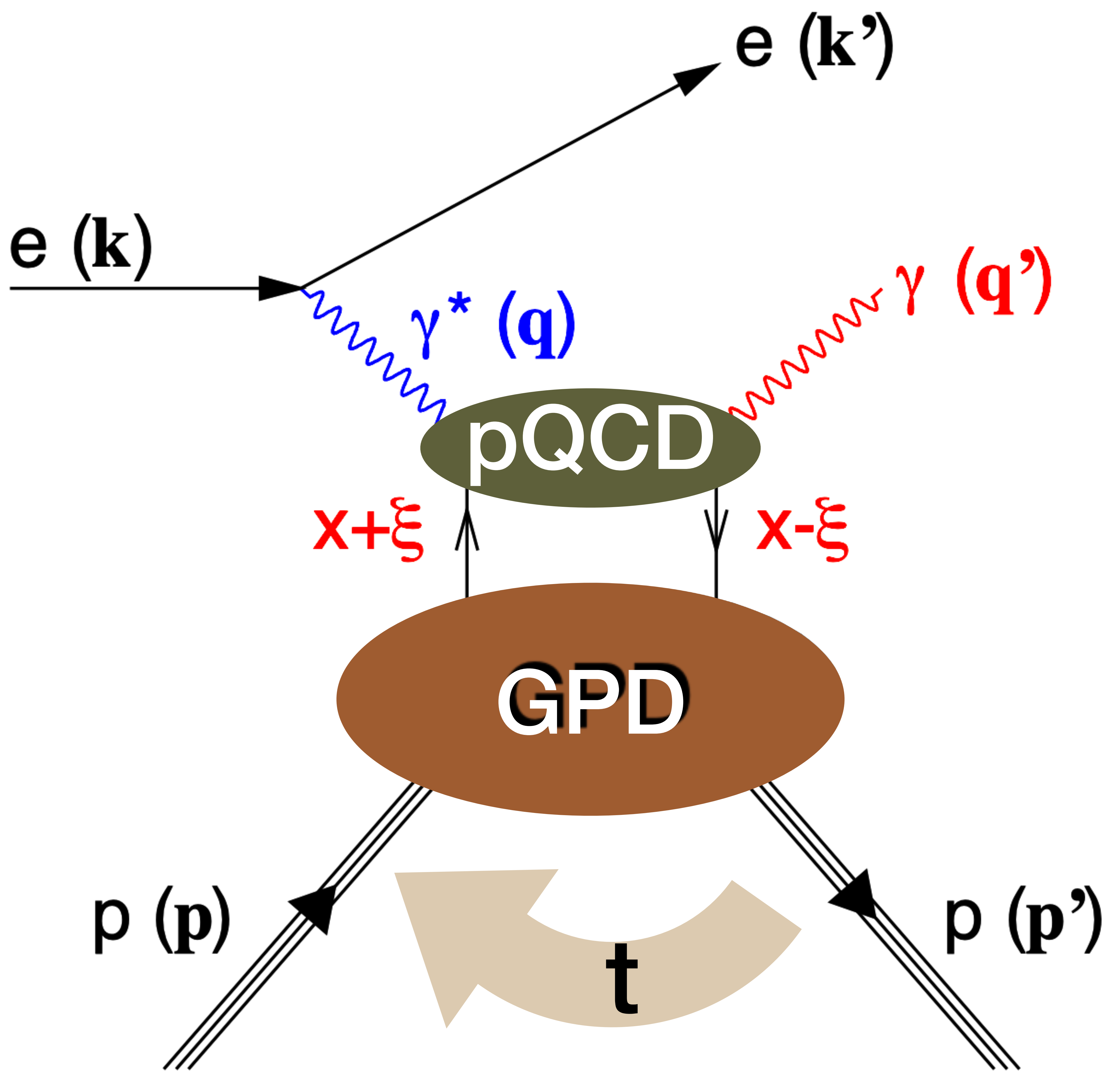
vs.

proton: $E_q(2\text{ GeV})/m_p \approx 40\,\%$

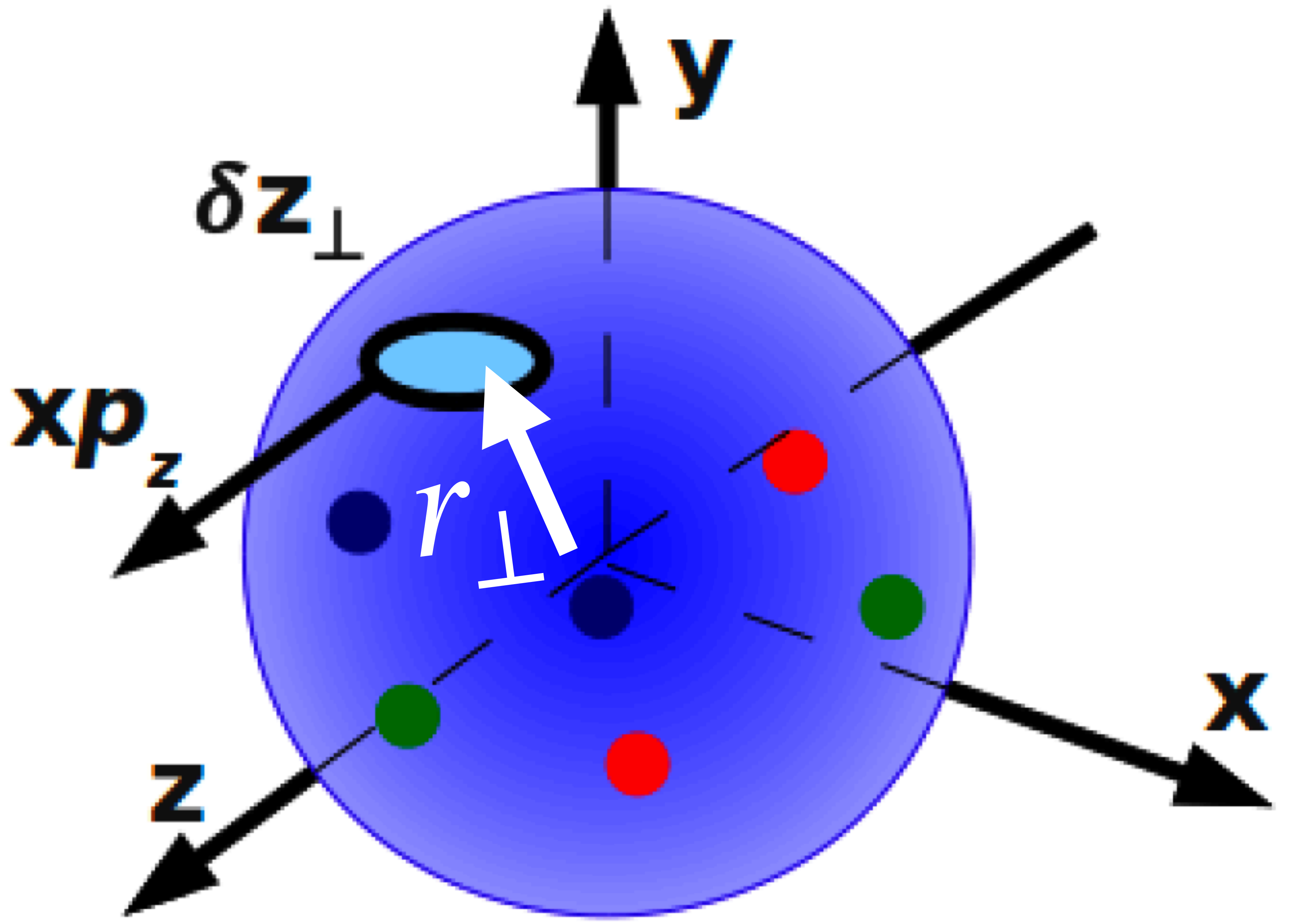


partonic origin of proton spin

deeply virtual compton scattering: $e + P \rightarrow e + P' + \gamma$



$r_\perp$ is Fourier conjugate of t



generalized parton distributions (GPD)

distribution of the longitudinal momentum fractions of partons in the transverse plane the hadron

from proton GPD to proton spin

contributions of quarks' total angular momentum to proton spin:

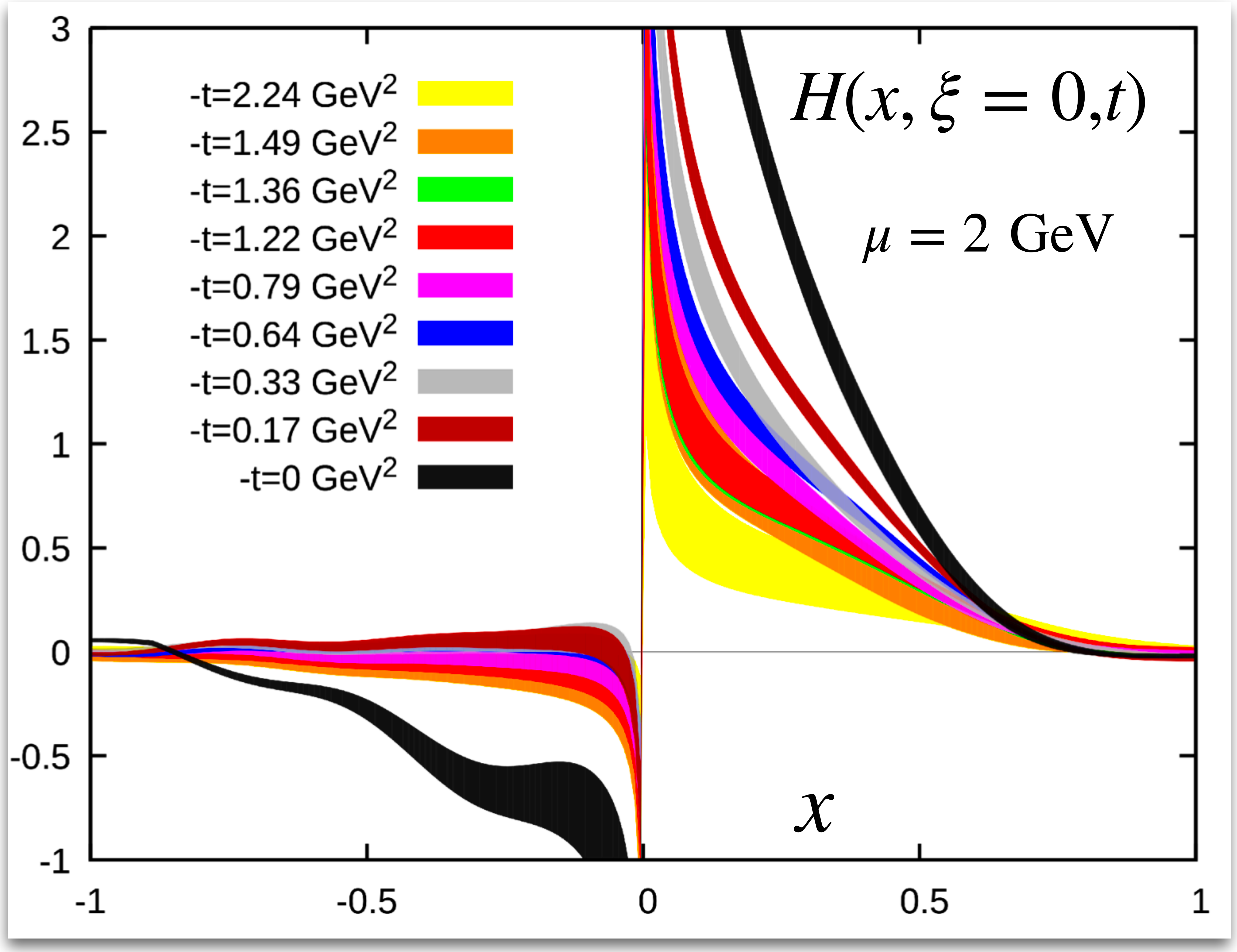
$$J^q = \frac{1}{2}\left[A_{20}(0) + B_{20}(0)\right]$$

$$A_{2,0}(t) = \int_{-1}^{1} xH^q(x, \xi = 0,t)dx \qquad B_{2,0}(t) = \int_{-1}^{1} xE^q(x, \xi = 0,t)dx$$

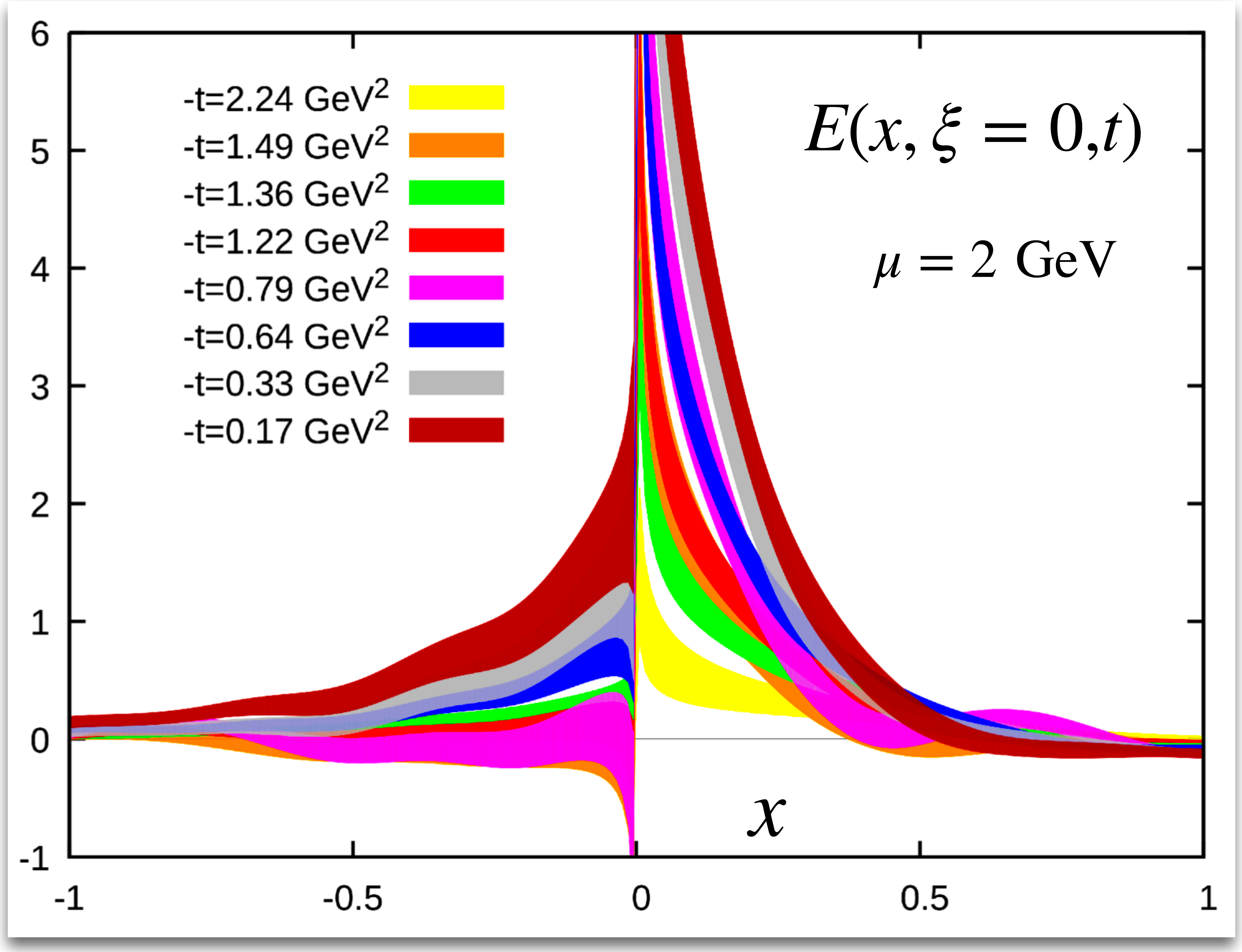
and it's distribution in the transverse plane $J^q(r_\perp)$

GPD: unpolarized quarks inside …

unpolarized proton



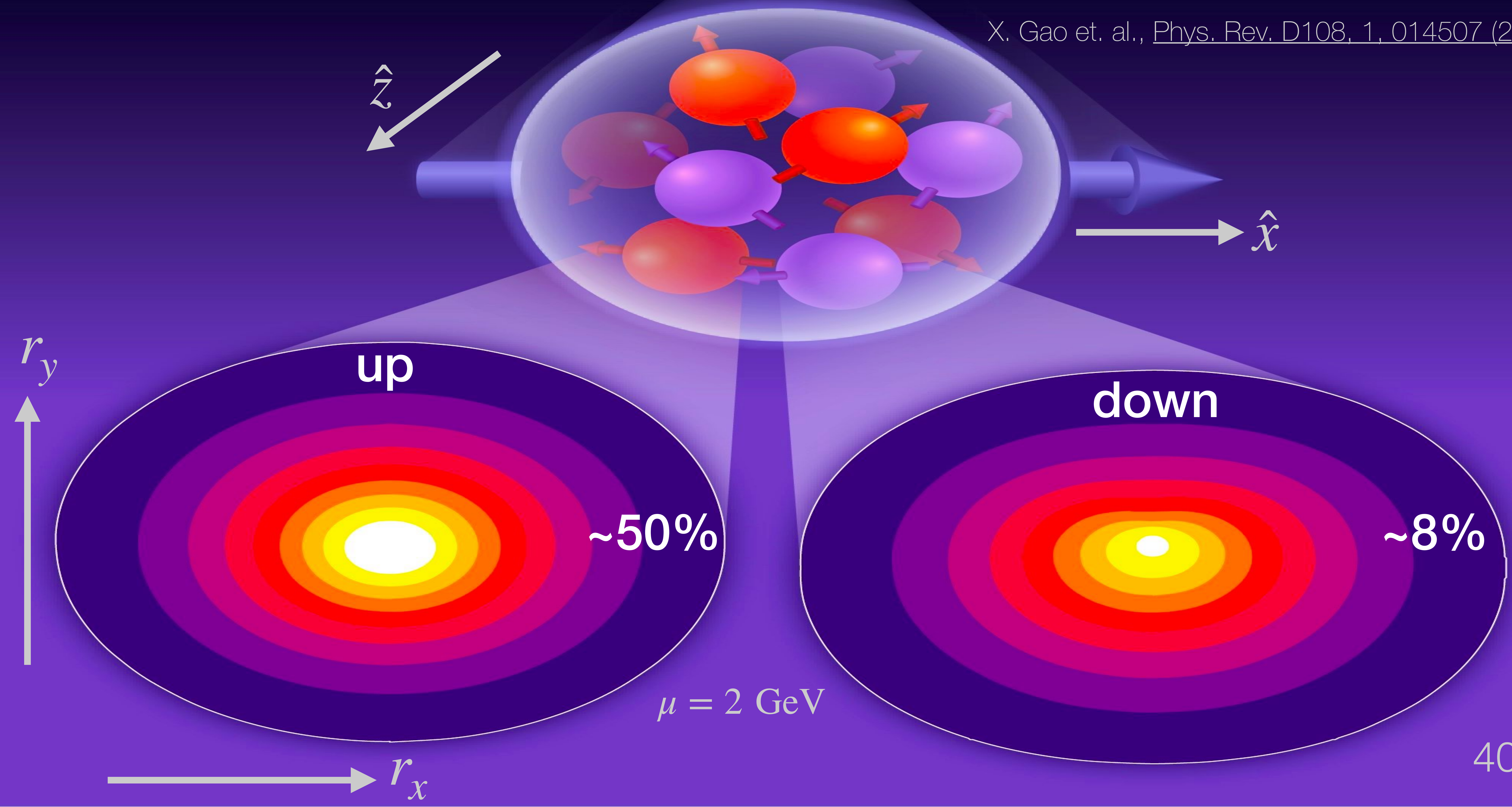
longitudinally-polarized proton



S. Bhattacharya et al., Phys. Rev. D 106, 1, 114512 (2022)

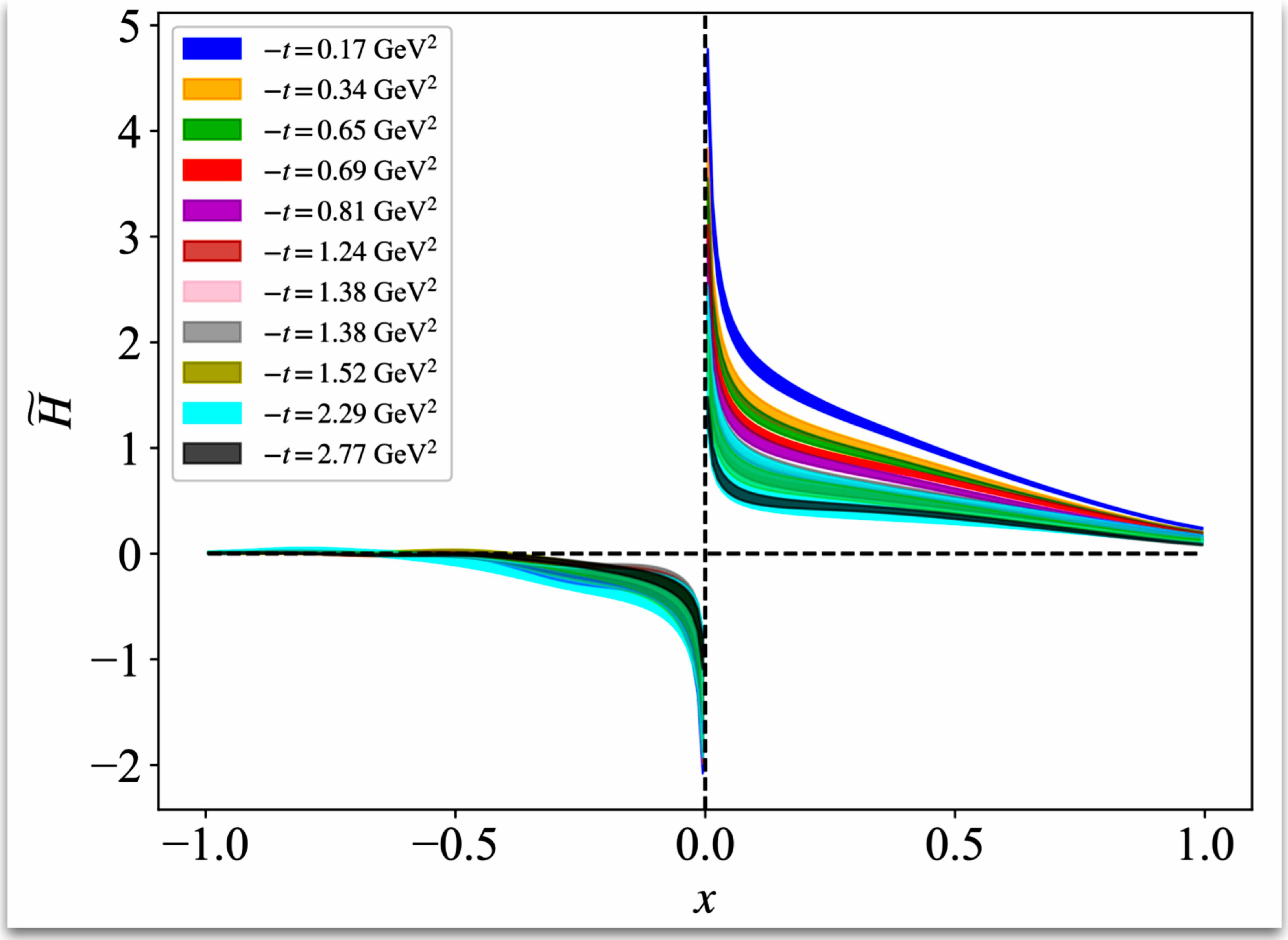
spatial distributions of quarks' total angular momenta

X. Gao et. al., Phys. Rev. D108, 1, 014507 (2023)



GPD: longitudinally-polarized quarks inside …

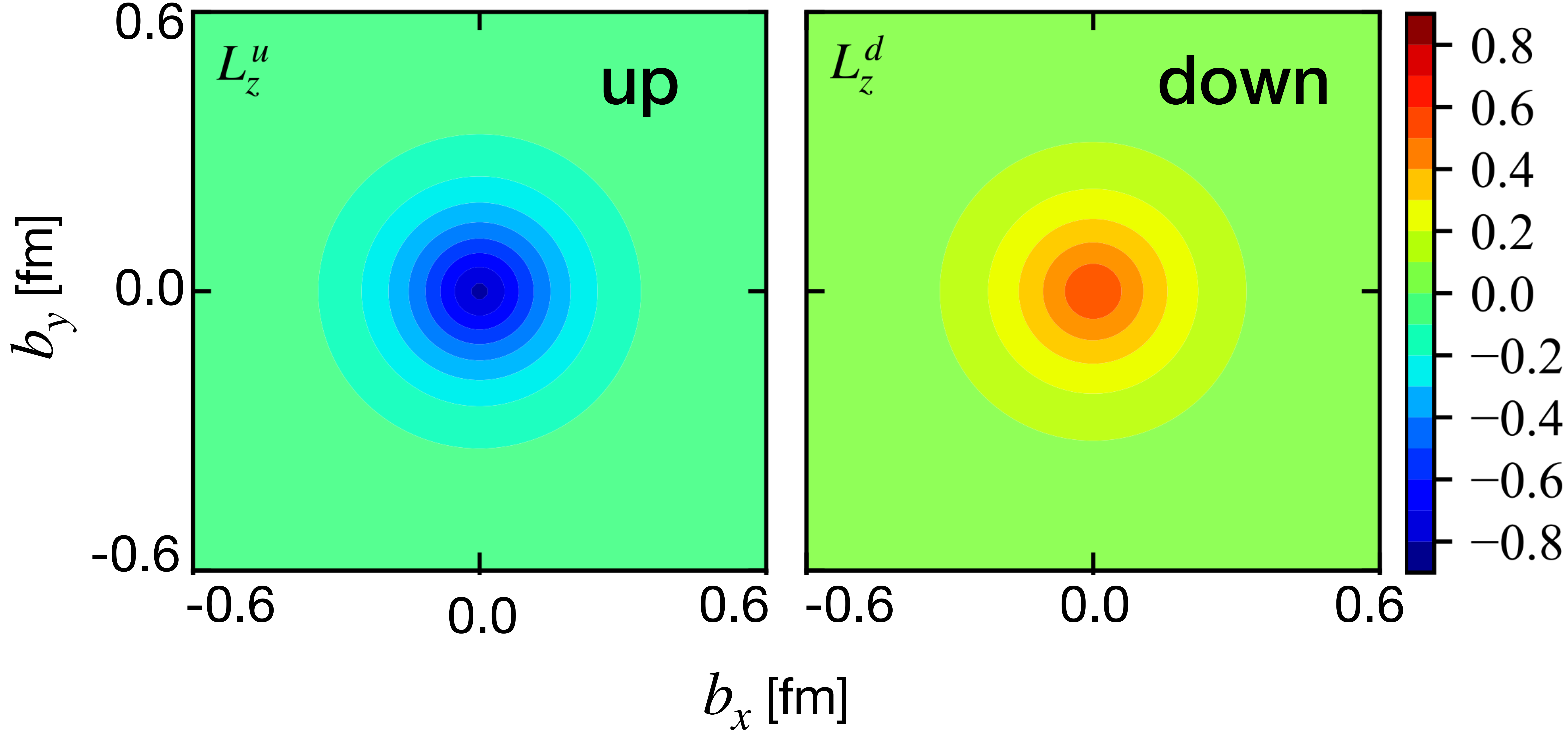
longitudinally-polarized proton



J. Miller et al., Phys. Rev. D 109, 3, 034508 (2024)

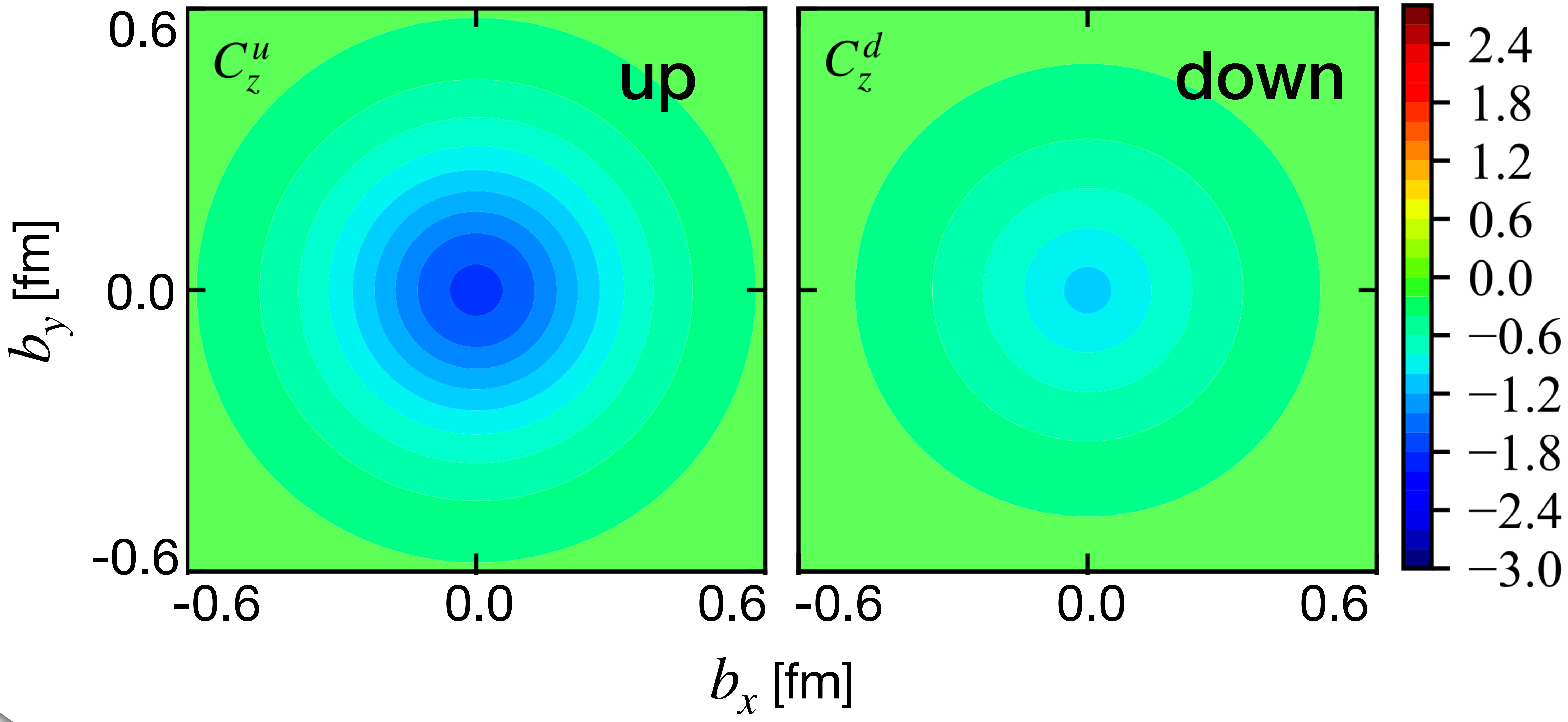
spatial distribution of quarks' orbital angular momenta

X. Gao et. al., JHEP 01, 146 (2025)



spatial distribution of quarks' spin-orbit correlation

X. Gao et. al., JHEP 01, 146 (2025)

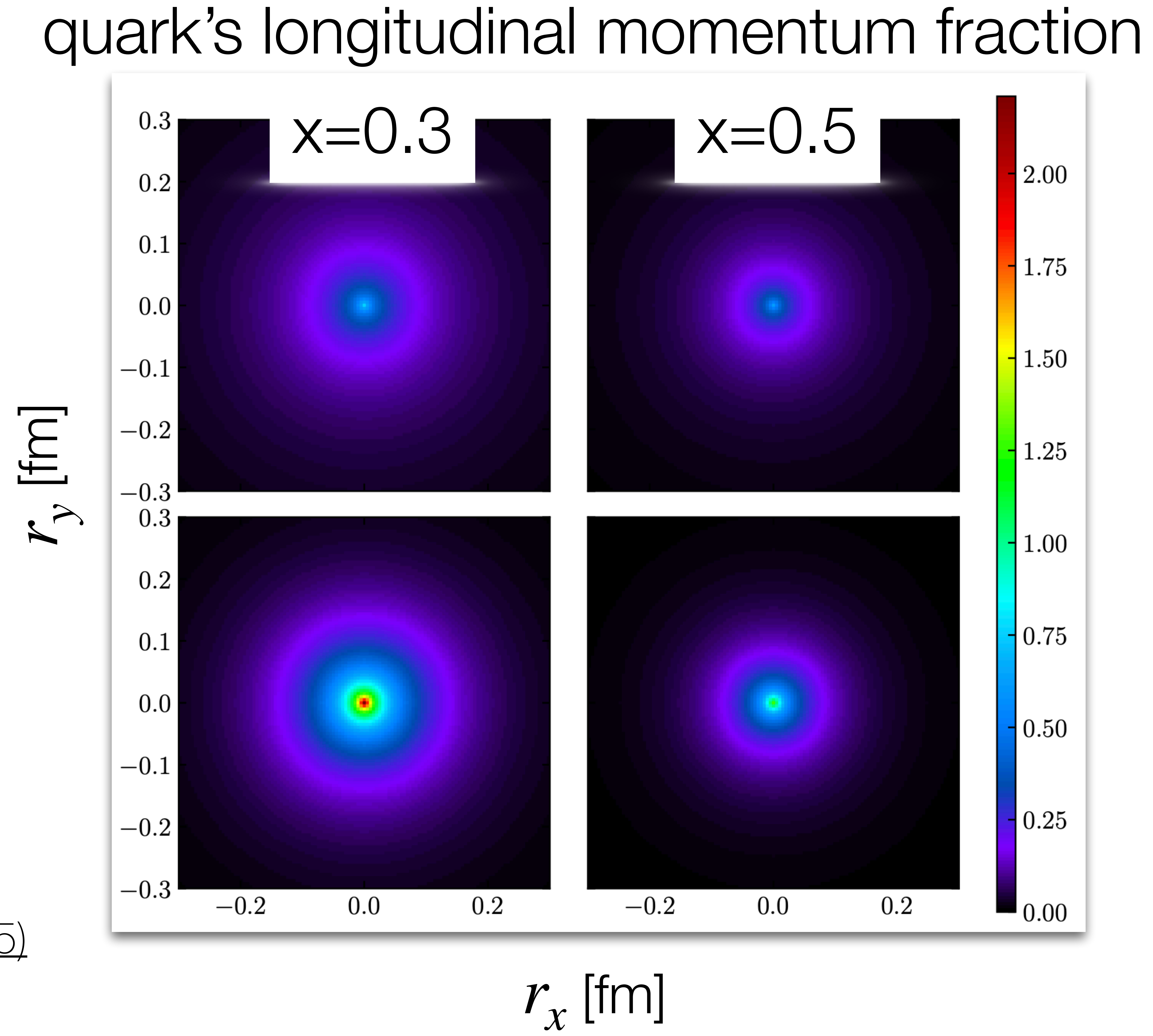


2+1 dimensional image

pion

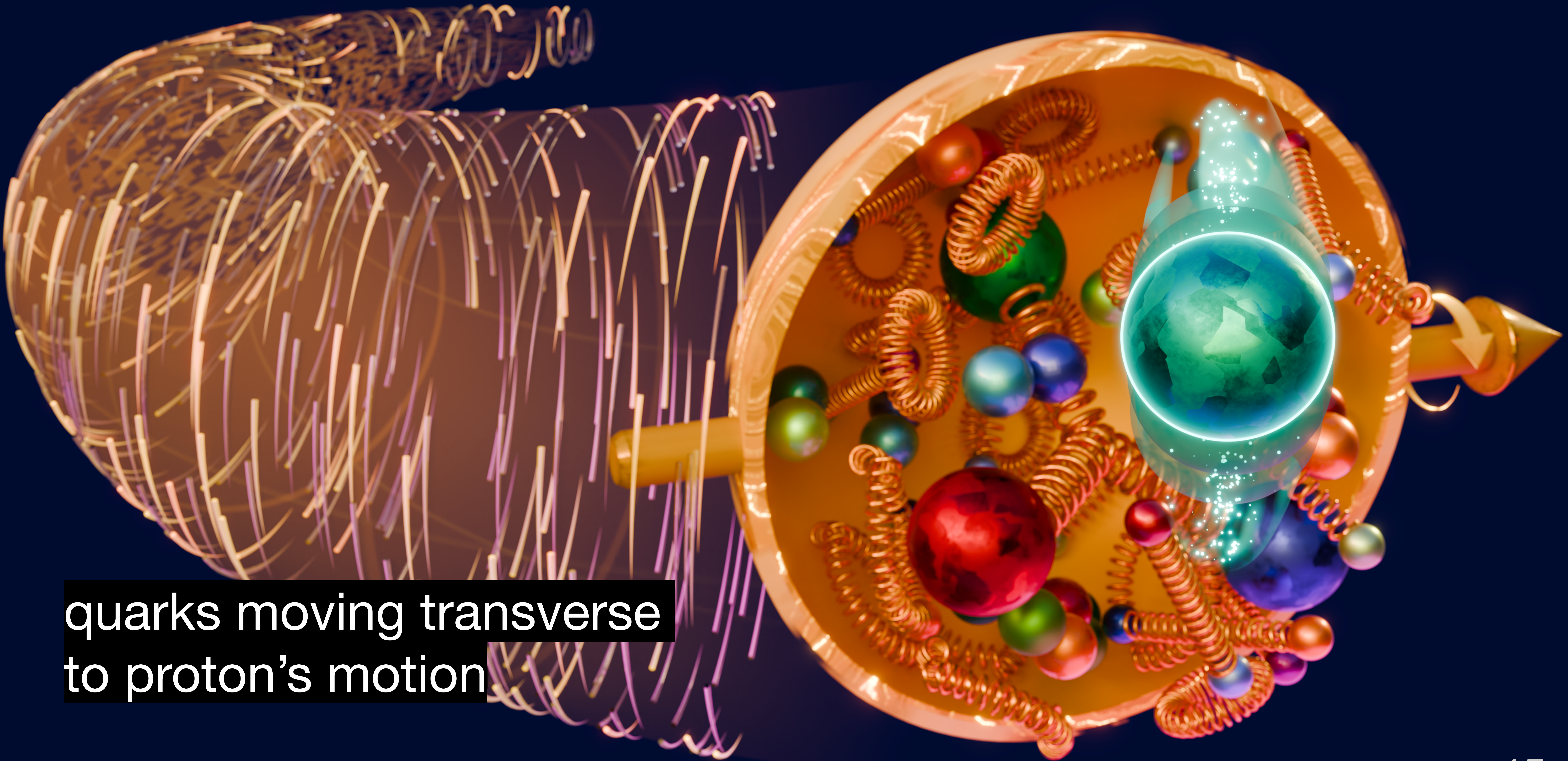
vs.

proton

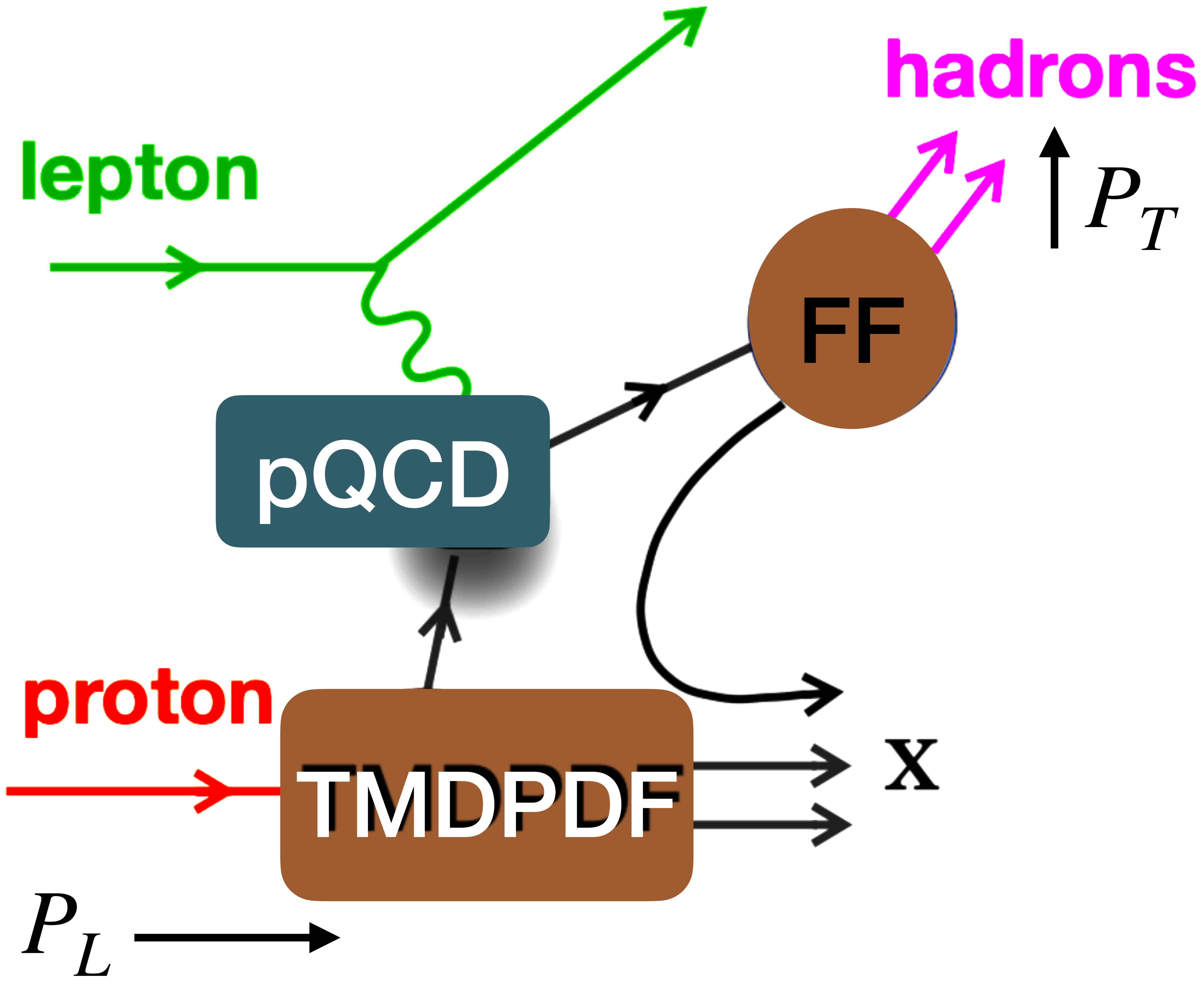


Q. Shi et. al., JHEP 02, 056 (2025)

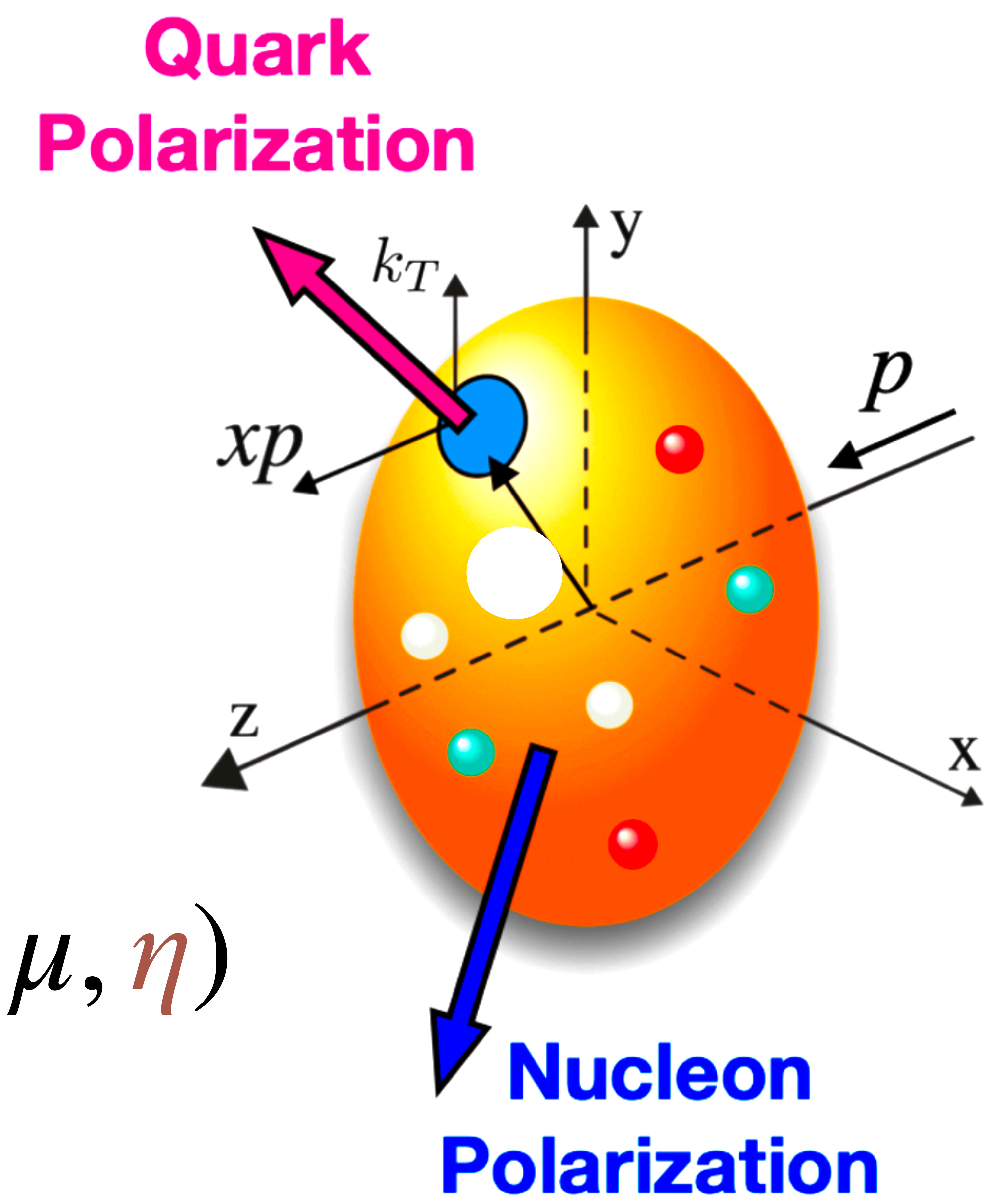
partonic image of proton: 3-dimensional momentum space



semi-inclusive deep inelastic scattering: $e + P \to e + H + X$



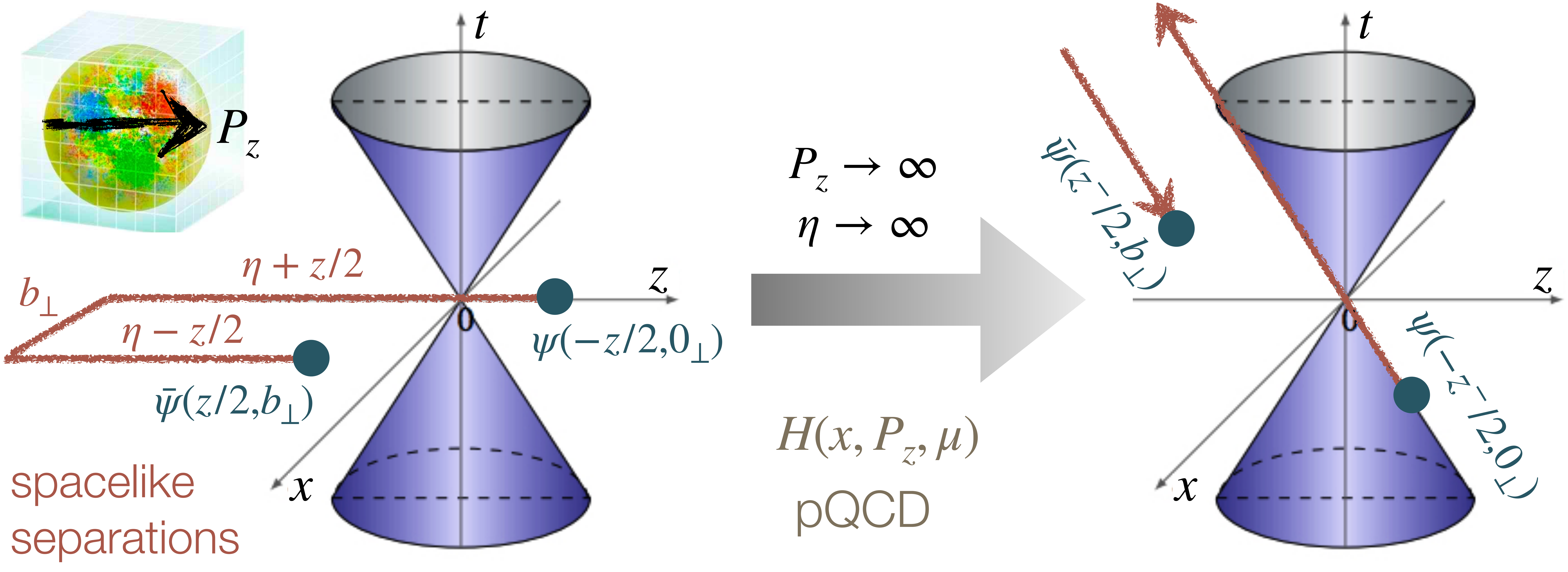
$$f(x, k_T, \mu, \eta)$$



transverse momentum-dependent PDF (TMDPDF)

b_T is Fourier conjugate of k_T

TMD distributions from lattice QCD



$\tilde{\phi}(z, b_\perp, \eta, P_z)$

quasi-TMD beam function

lighcone-TMD beam function

TMD factorization of LQCD beam function

LQCD

CS kernel

TMDPDF

$$\sqrt{S_I(b_T,\mu)} \cdot \tilde{f}(x, b_T, P_z, \mu) = H(x, P_z, \mu) \cdot \exp\left[\frac{1}{2} \ln \frac{(2xP_z)^2}{\zeta} \gamma^{\overline{\mathrm{MS}}}(b_T, \mu)\right] \cdot f(x, b_T, \zeta, \mu)$$

intrinsic soft factor

pQCD kernel

nonperturbative Collins-Soper kernel

b_T

$b_T \sim \Lambda_{\mathrm{QCD}}^{-1}$

nonperturbative

$\gamma^{\overline{\mathrm{MS}}}(b_\perp, \mu)$

$b_T \ll \Lambda_{\mathrm{QCD}}^{-1}$

perturbative

JLAB

EIC

RHIC

LHC

$\sqrt{\zeta}$

nonperturbative Collins-Soper kernel from LQCD

LQCD

CS kernel

TMDPDF

$$\sqrt{S_I(b_T,\mu)} \cdot \tilde{f}(x,b_T,P_z,\mu) = H(x,P_z,\mu) \cdot \exp\left[\frac{1}{2}\ln\frac{(2xP_z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}(b_T,\mu)\right] \cdot f(x,b_T,\zeta,\mu)$$

intrinsic soft factor

pQCD kernel

universal CS kernel

$$\gamma^{\overline{\mathrm{MS}}}(b_T,\mu) = \frac{1}{\ln(P_2/P_1)}\ln\left[\frac{\tilde{f}(x,b_T,P_2,\mu)}{\tilde{f}(x,b_T,P_1,\mu)}\right] + \delta\gamma^{\overline{\mathrm{MS}}}(b_T,\mu,P_1,P_2)$$

LQCD

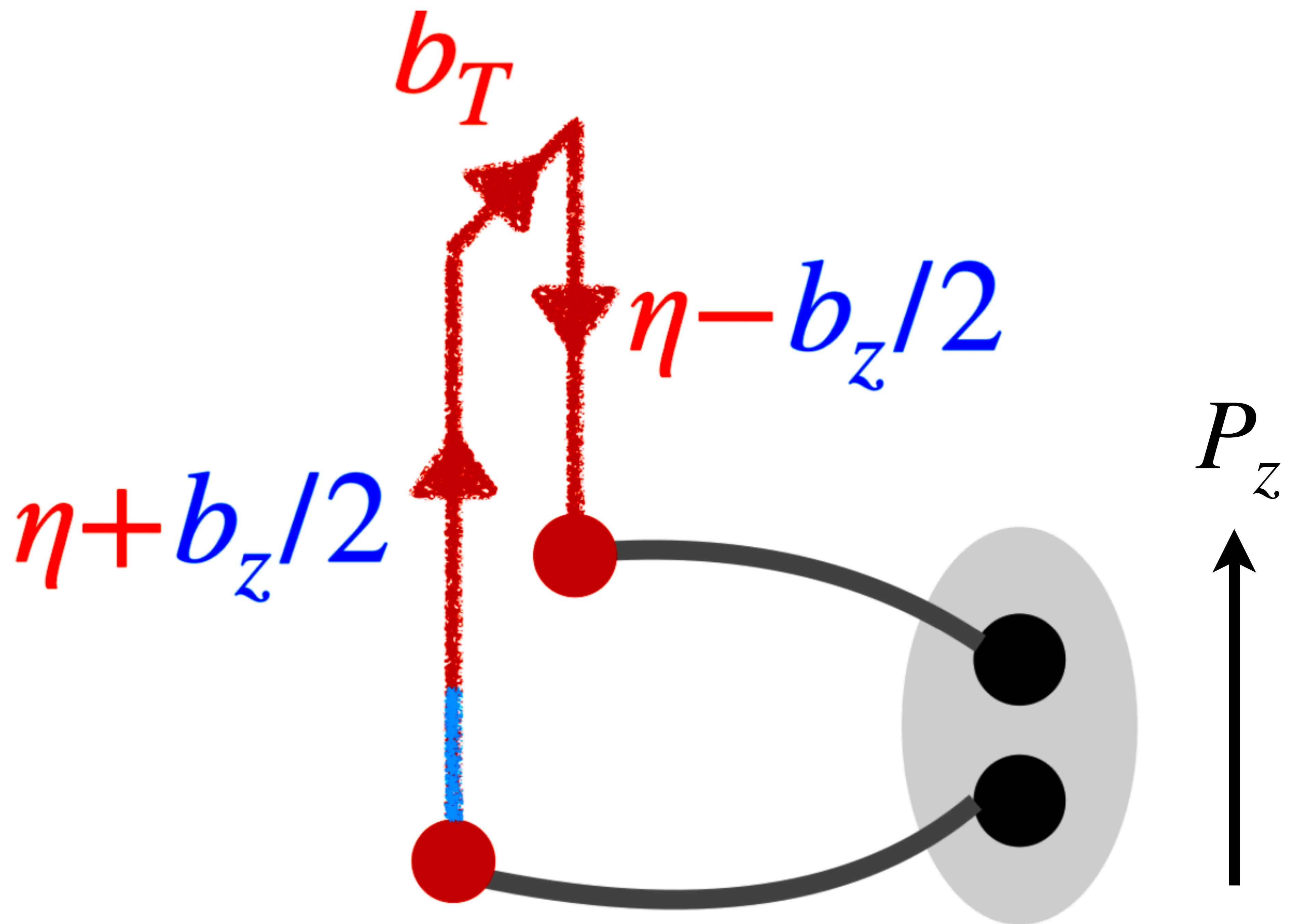
pQCD kernel

lattice QCD calculations of CS kernel

simplest choice for the quasi-TMD beam function $\tilde{\phi}(b_z, b_\perp, \eta, P_z)$

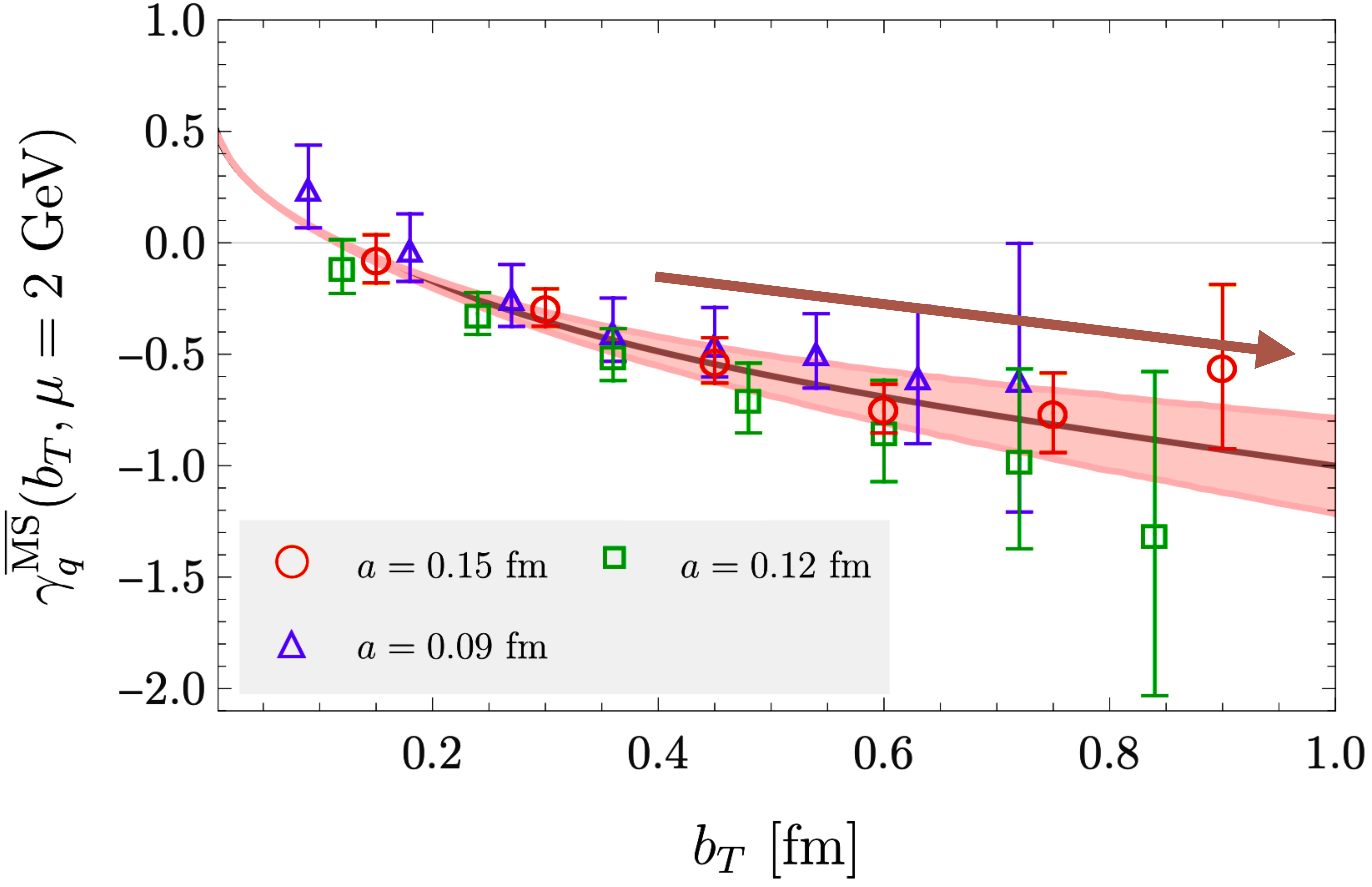
pion TMD wave function (TMDWF)

$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp)\Gamma W_{\sqsupset}(\frac{\mathbf{b}}{2}, -\frac{\mathbf{b}}{2}, \eta)\psi(-\frac{b_z}{2}, 0) | \pi^+, P_z \rangle$$



the challenge

rapidly growing errors with increasing $b_\perp$

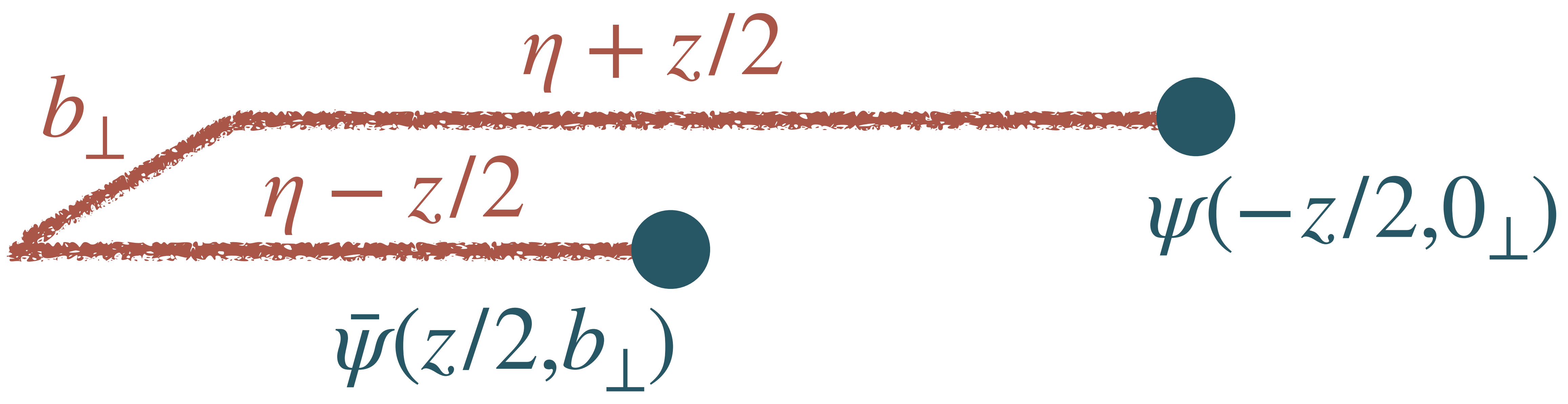


Avkhadiev et al., Phys. Rev. Lett. 23, 231901 (2024)

understanding the challenge

multiplicative renormalization factor of the Wilson line:

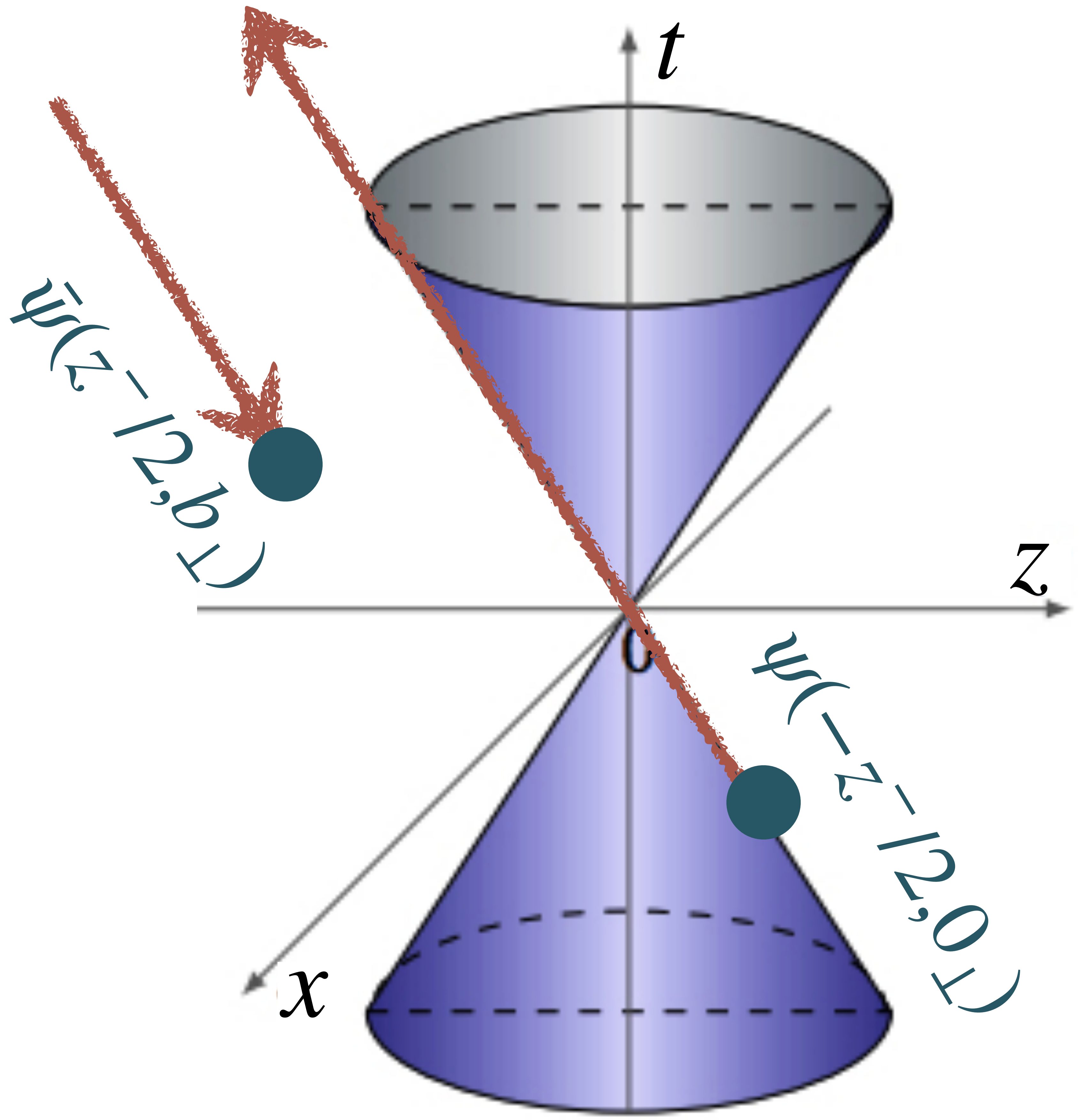
$$\sim e^{-\delta m(\eta + b_\perp)}$$



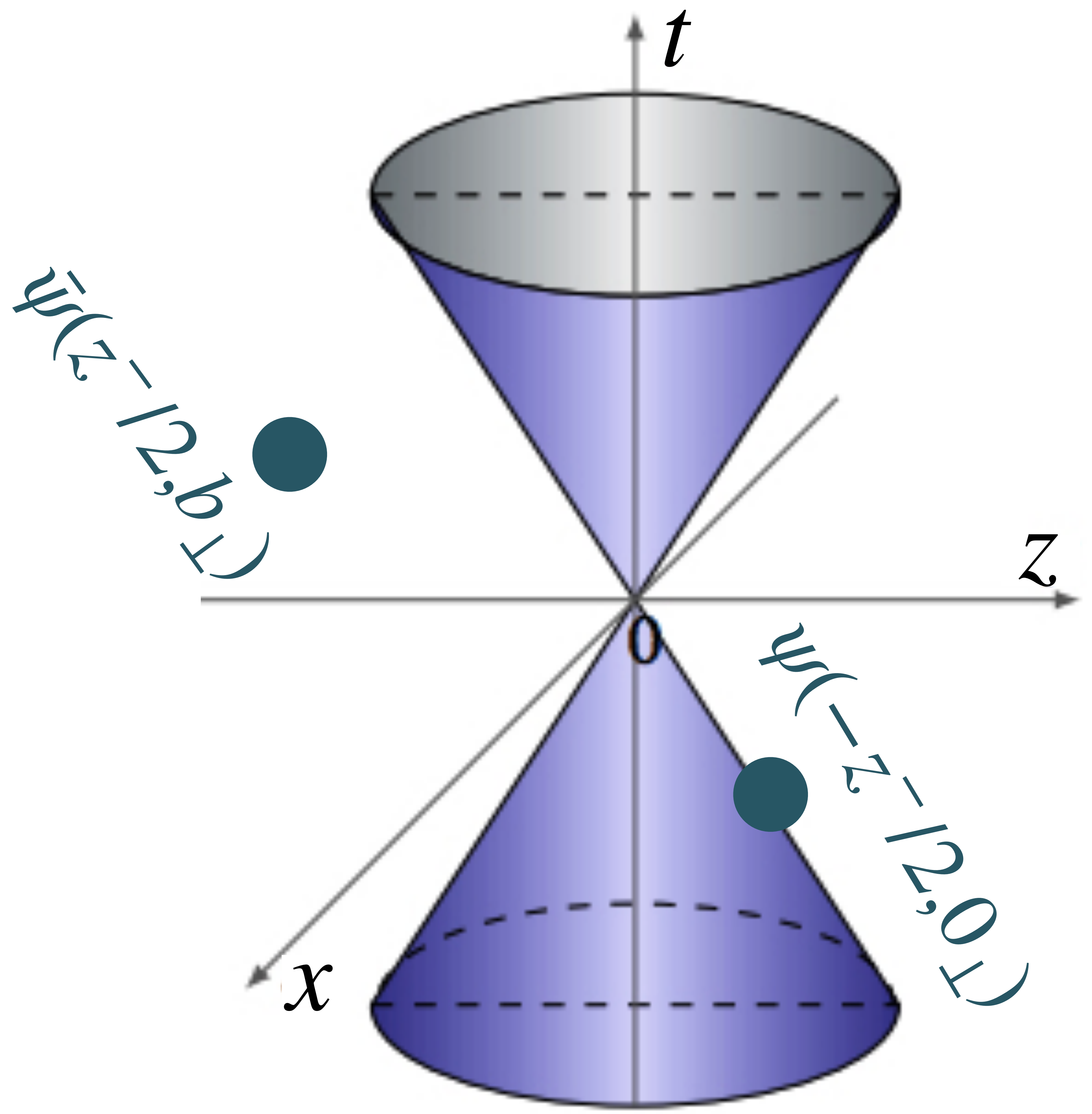
exponential decrease of signal for large η and increasing $b_\perp$

overcoming the challenge

physical lightcone gauge $A^+ = 0$



$\equiv$



how can we access $A^+ = 0$ in lattice QCD calculations ?

find a gauge that becomes equivalent to $A^+ = 0$ in the limit $P_z \to \infty$

Coulomb gauge

$$\vec{\nabla} \cdot \vec{A} = 0$$

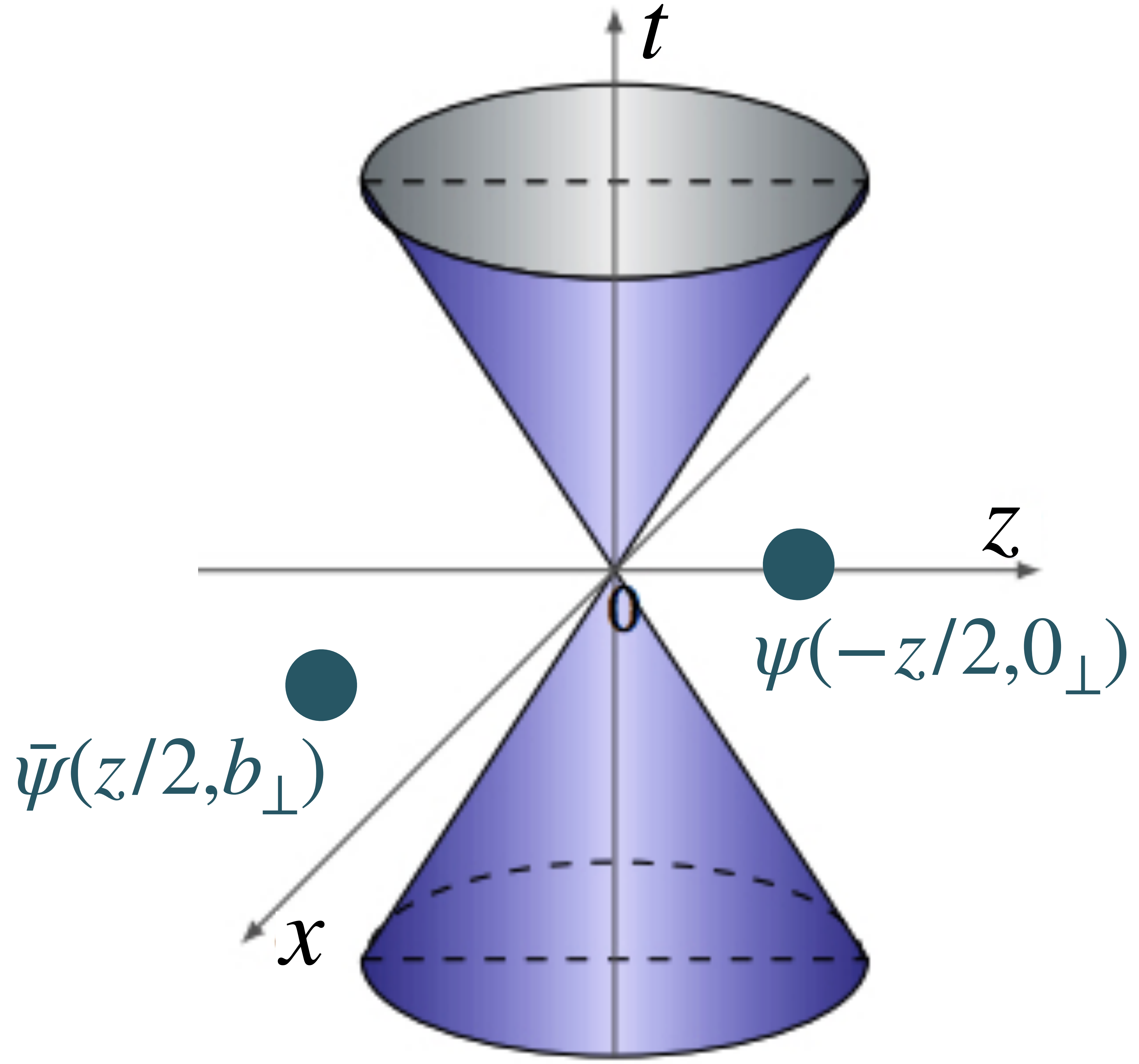
$P_z \to \infty$

lightcone gauge

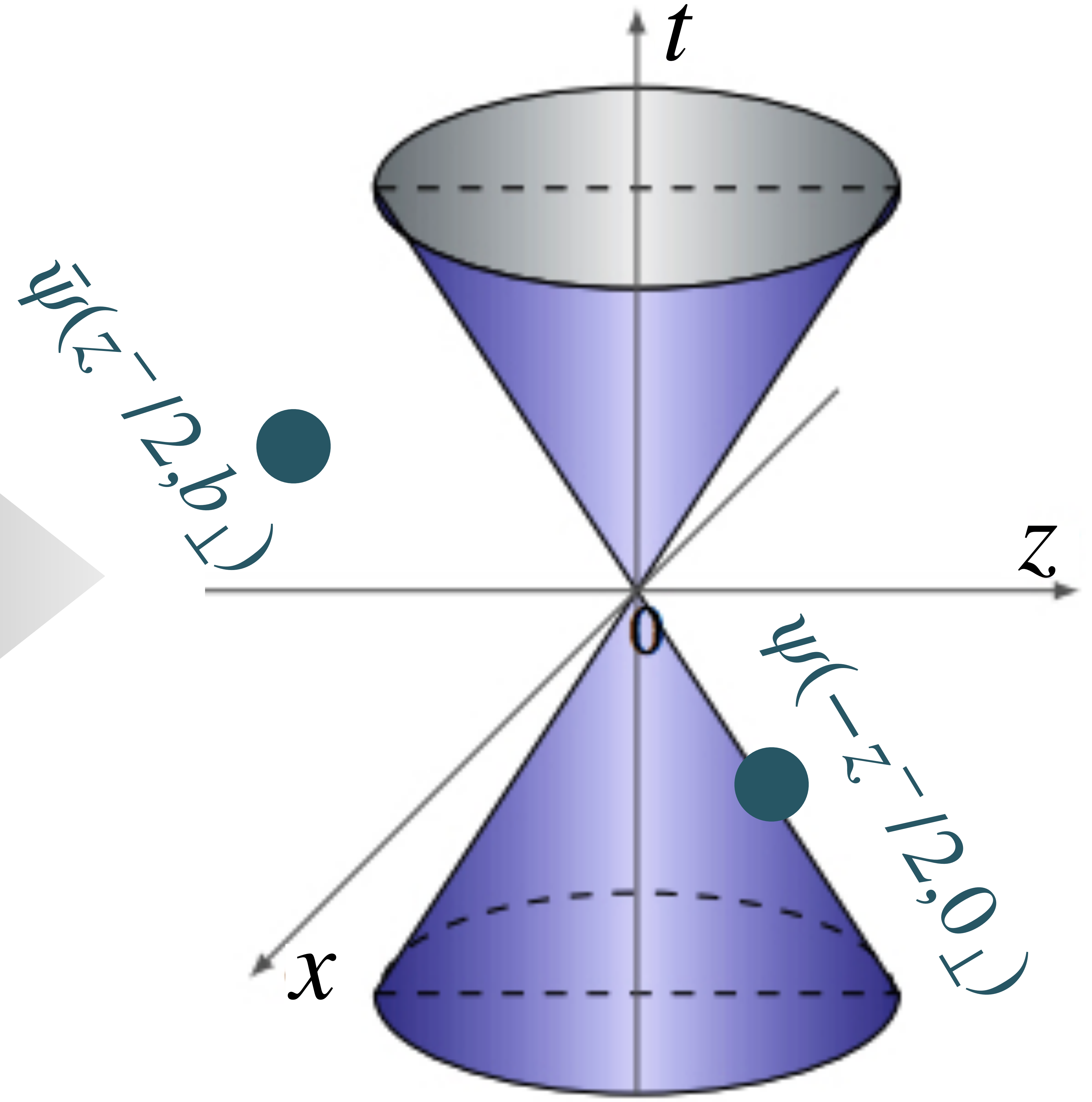
$$A^+ = 0$$

quasi-TMD beam function in Coulomb gauge (CG)

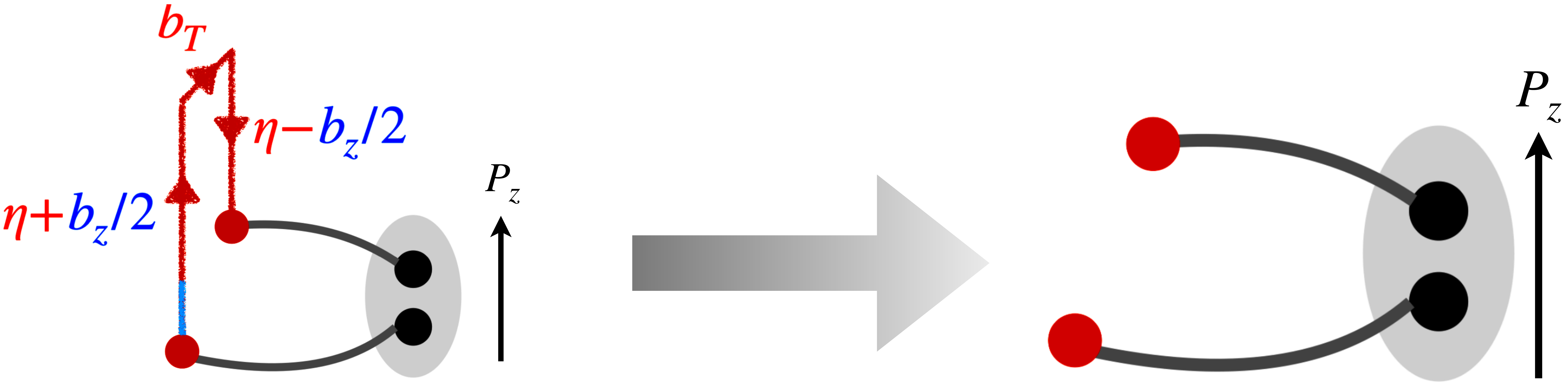
$$\bar{\psi}(z/2, b_\perp)\Gamma\psi(-z/2, 0_\perp)\Big|_{\nabla\cdot A=0}$$



$P_z \to \infty$



CG quasi-TMD beam function

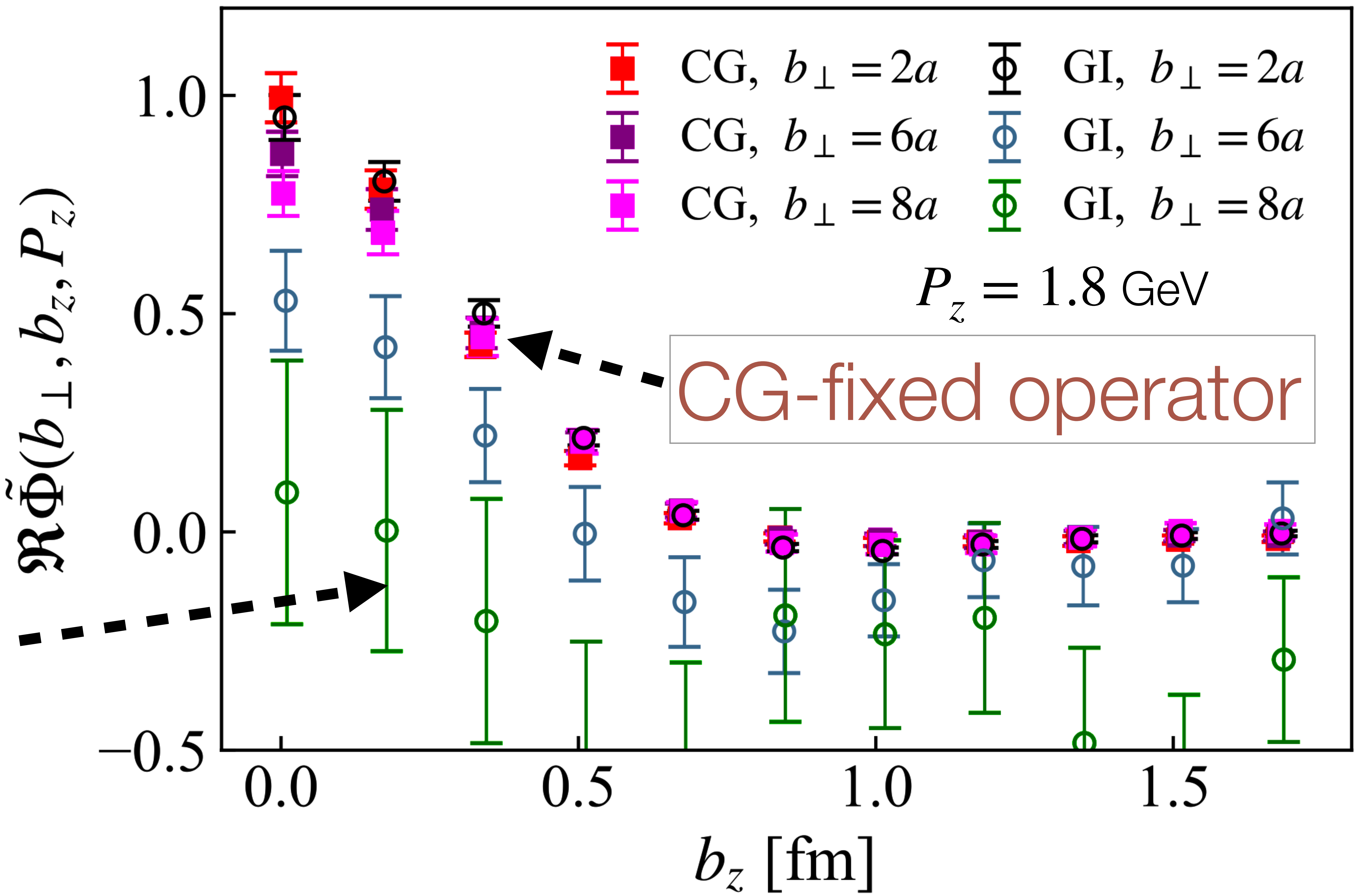


+ re-computation of pQCD matching function $\delta\gamma^{\overline{\mathrm{MS}}}(x, \mu, P_1, P_2)$

next-to-leading-log (NLL) accuracy

Y. Zhao, Phys. Rev. Lett. 133, 241904 (2024)

renormalized quasi-TMD beam functions

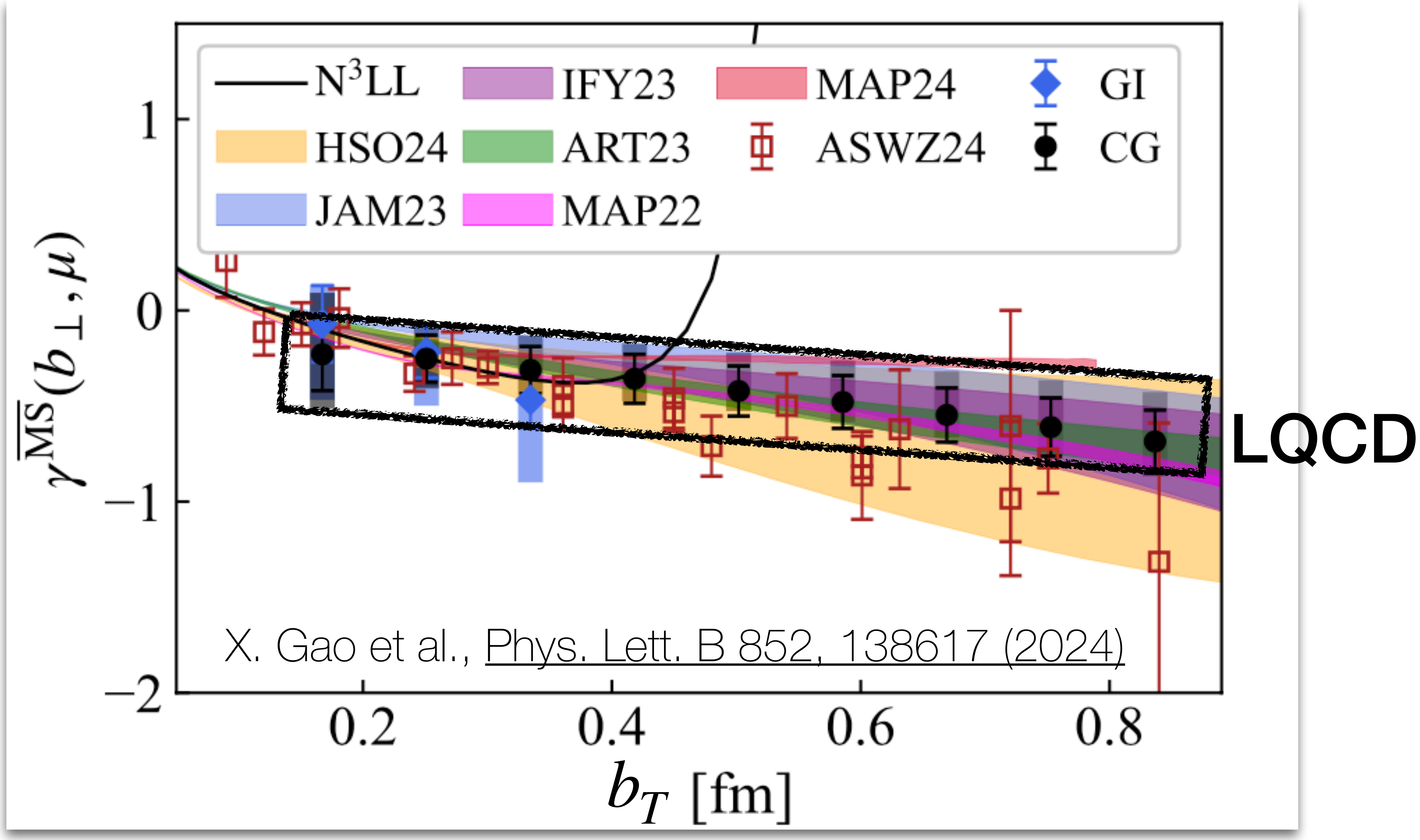


gauge invariant (GI) operator

unitary chiral (Domain Wall) fermions, physical pion mass, lattice spacing a=0.085 fm

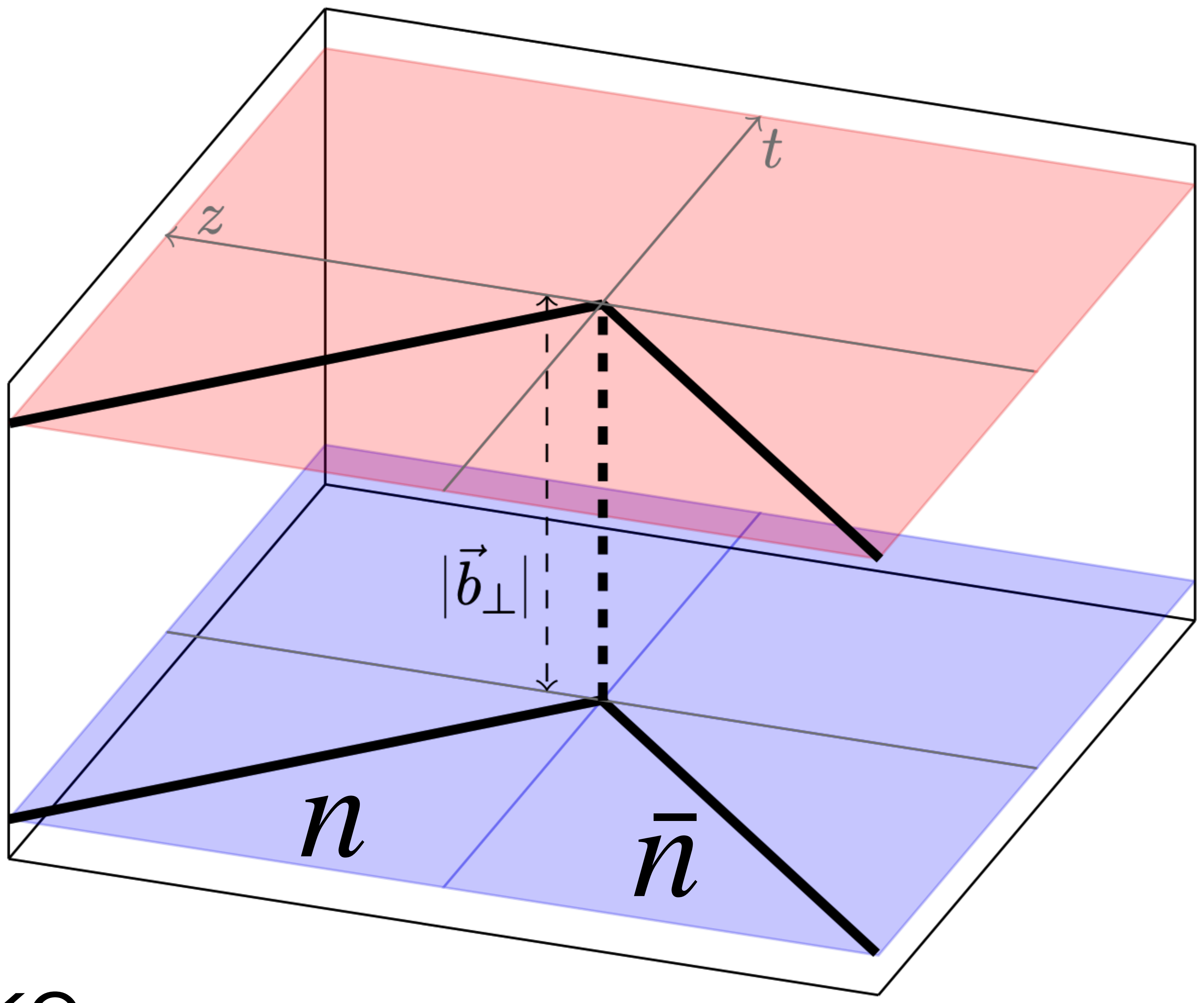
Bollweg et al.: Phys. Lett. B 852, 138617 (2024)

nonperturbative Collins-Soper kernel from LQCD



unitary chiral quarks, physical mass

intrinsic soft factor

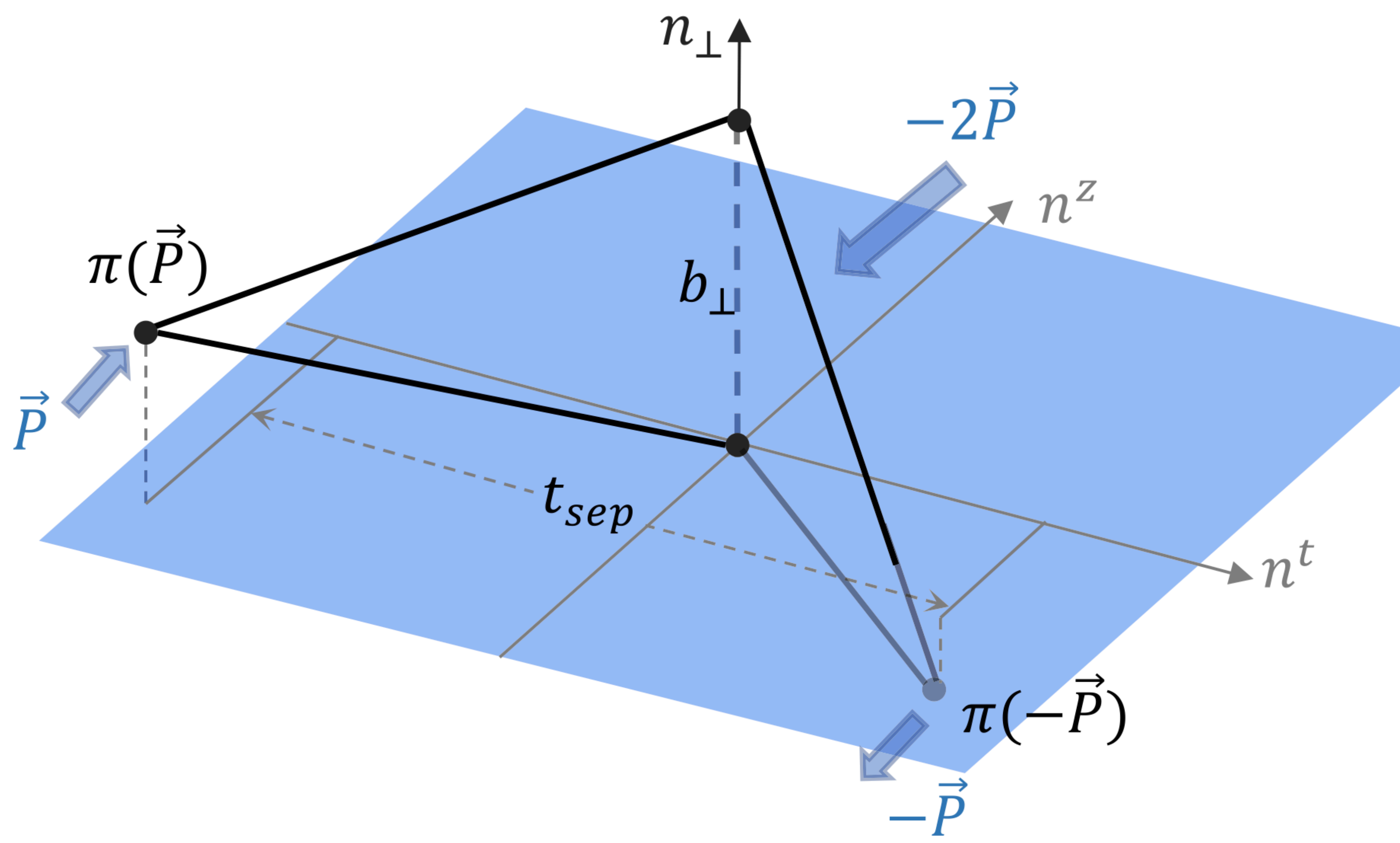


operator involves two lightcone directions

impossible to obtain by computing a space-like operator within a hadron boosted in a direction

need an alternative indirect approach

form factor of two transversely-separated currents within boosted pions from LQCD



$$F(b_T, P_z) \sim \langle P_z | [\bar{q}(b_T)\gamma_T q(b_T)][\bar{q}(0)\gamma_T q(0)] | -P_z \rangle$$

factorizes into pion TMD wave function

$$F(b_T, P_z) \sim H_F(x_1, x_2, P_z, \mu) \otimes \phi^\dagger(x_1, b_T, \mu, \zeta_1, \bar{\zeta}_1) \otimes \phi(x_2, b_T, \mu, \zeta_2, \bar{\zeta}_2)$$

pQCD

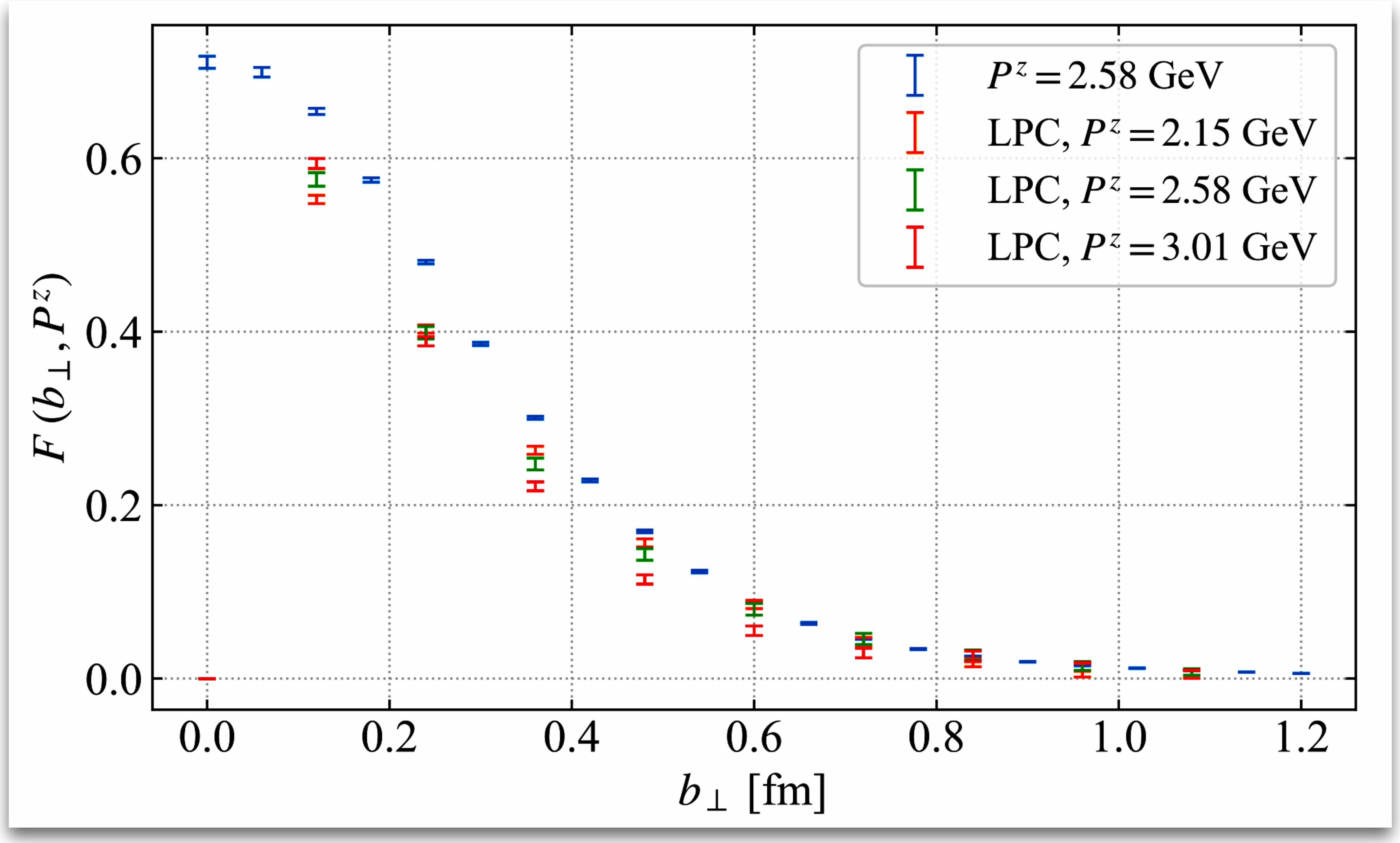
pion TMD wave function from LQCD

$$\sqrt{S_I(b_T, \mu)} \cdot \tilde{\phi}(x, b_T, P_z, \mu) = H_\phi(x, \bar{x}, P_z, \mu) \cdot \phi(x, b_T, \mu, \zeta, \bar{\zeta})$$

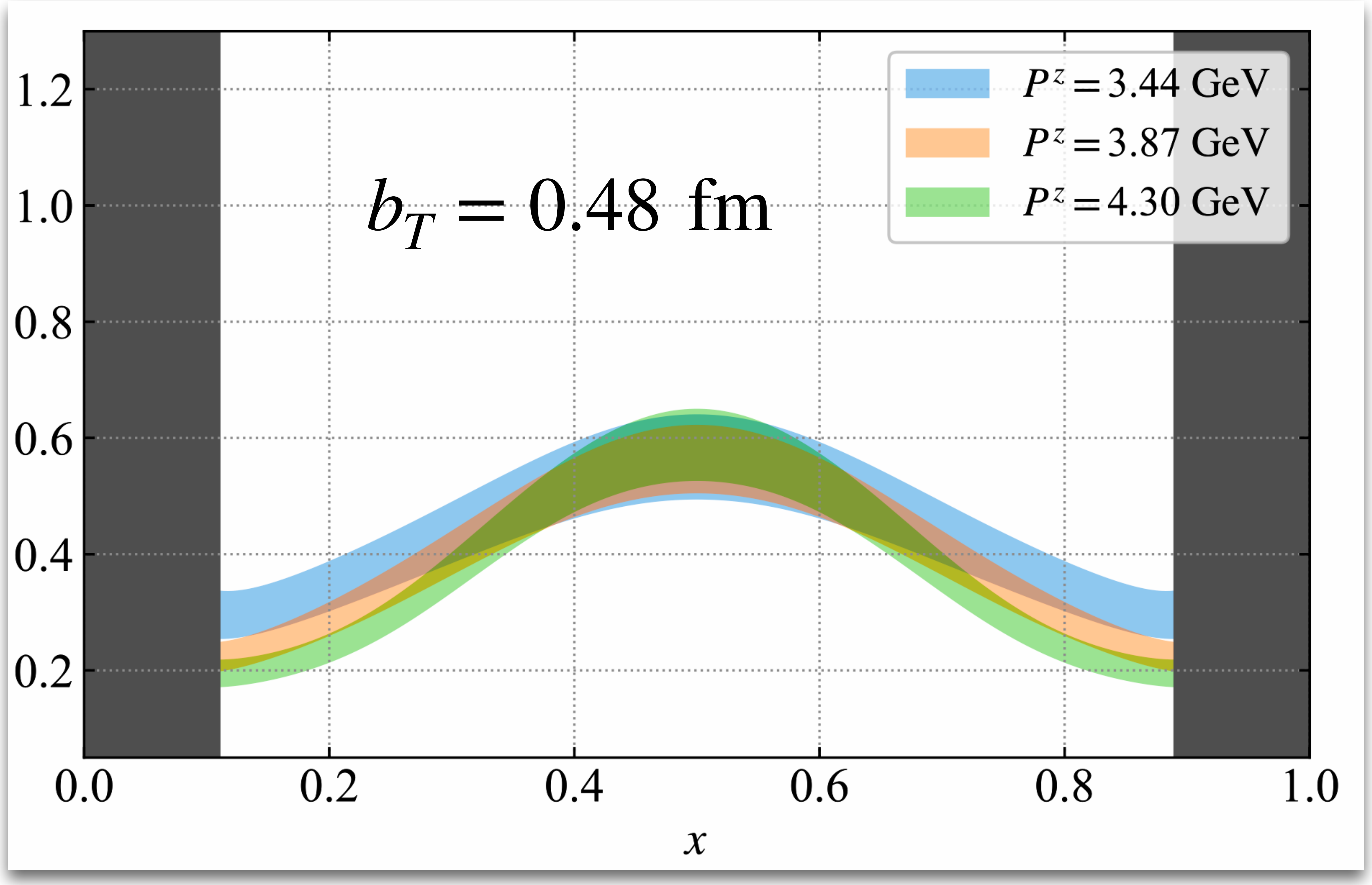
intrinsic soft factor

pQCD

form factor

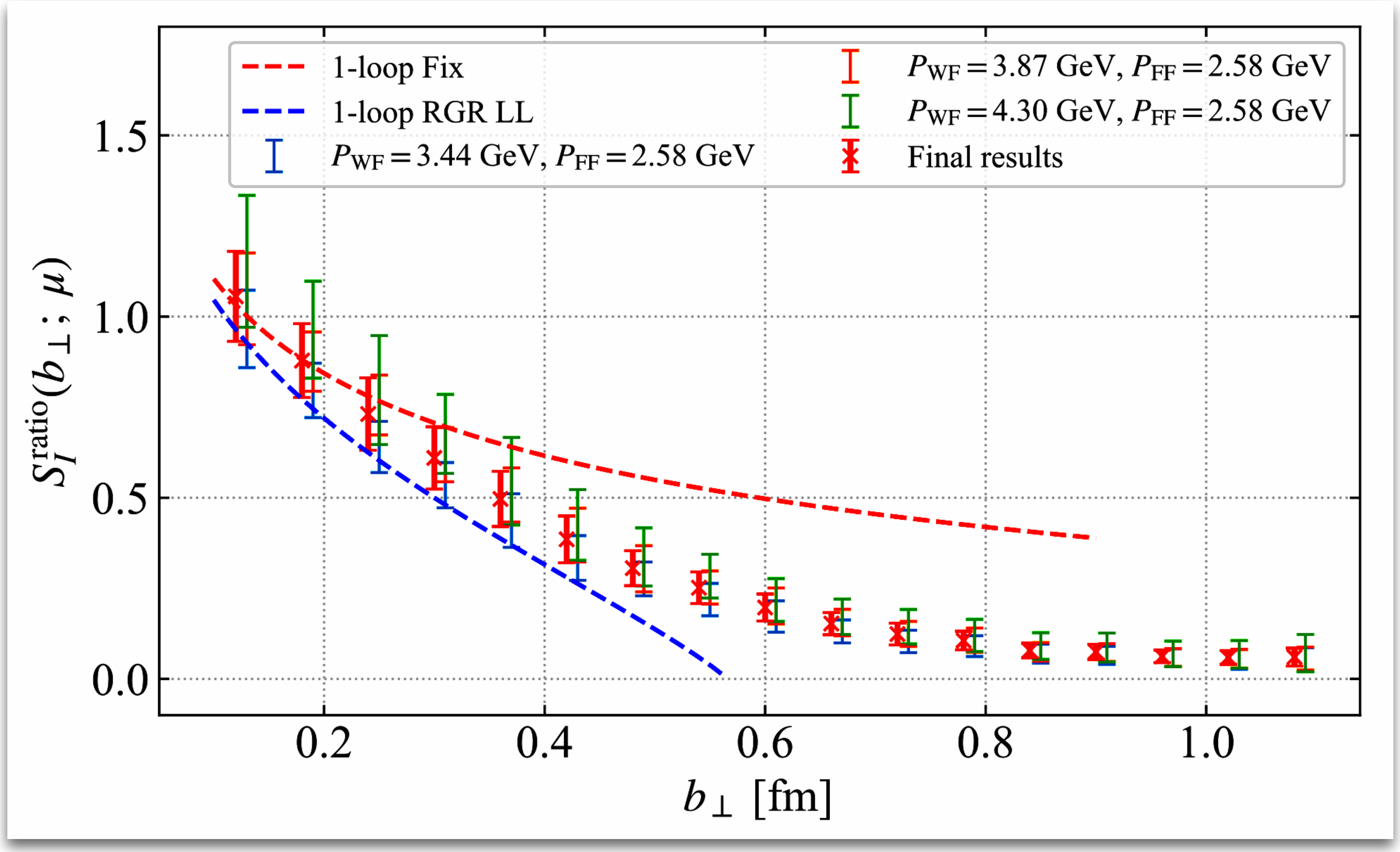


pion TMD wave function



J. C. He et al., arXiv:2504.04625

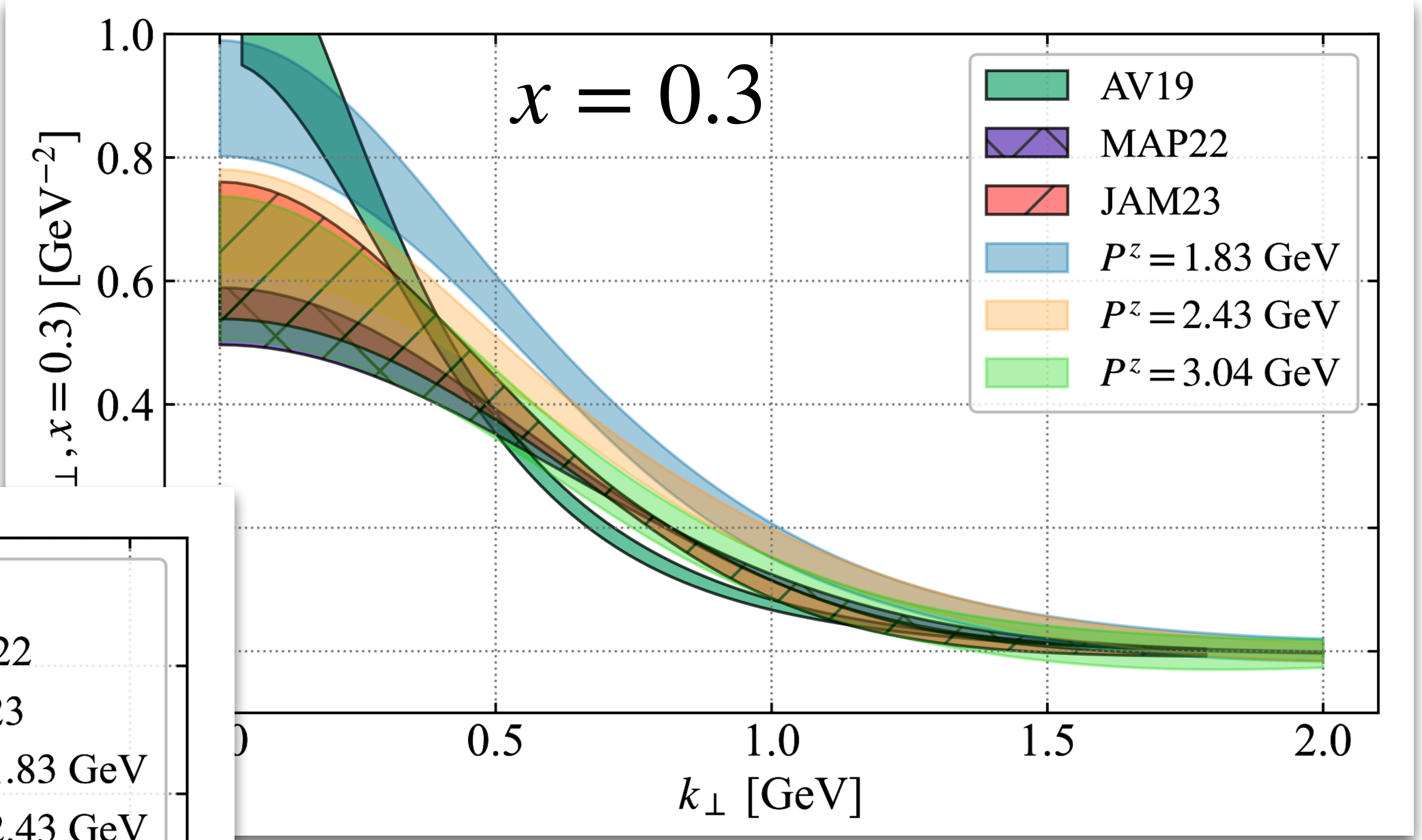
intrinsic soft factor

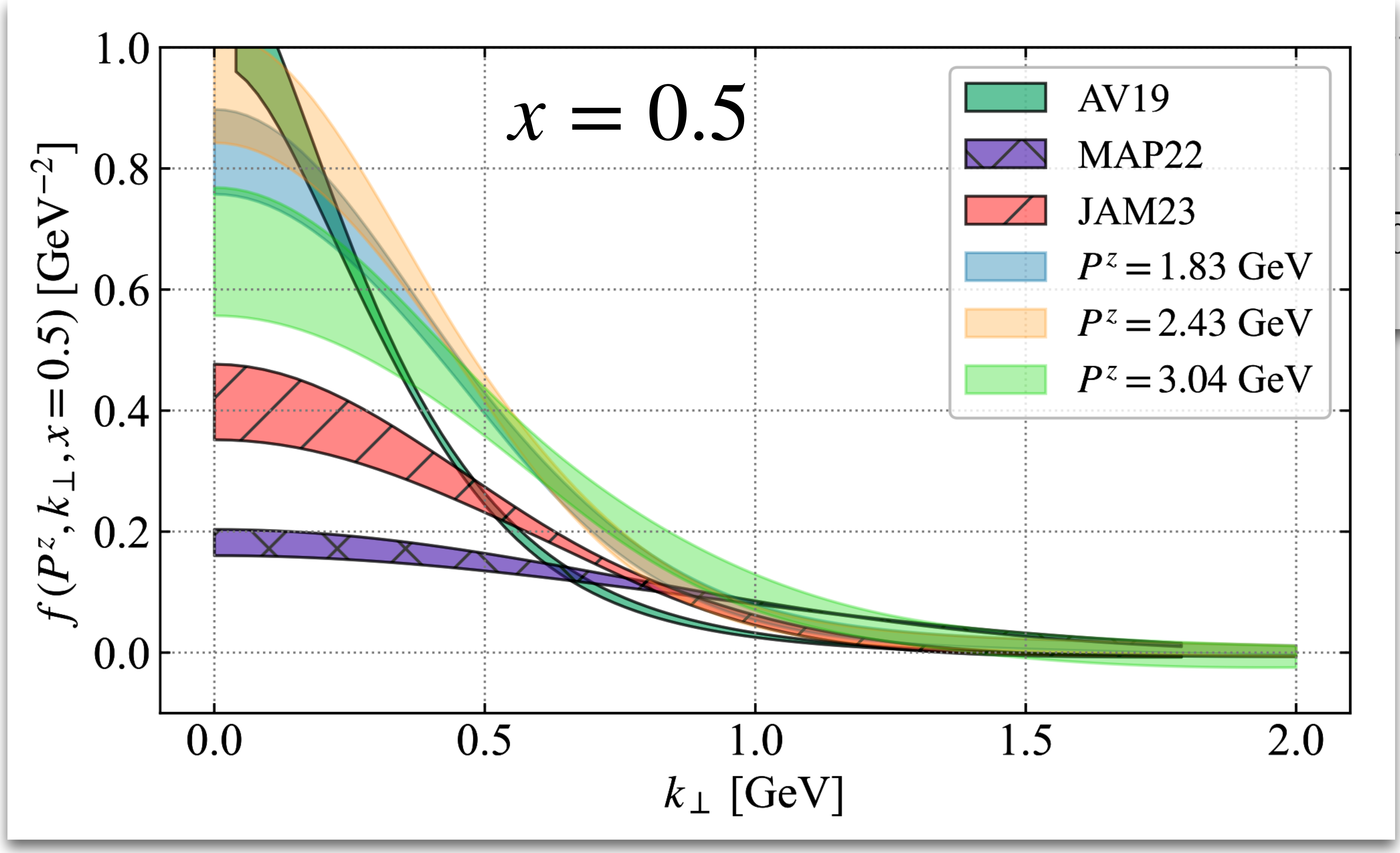


scheme dependent

J. C. He et al., arXiv:2504.04625

pion TMDPDF from LQCD





J. C. He et al., arXiv:2504.04625

TMD factorization of LQCD beam function

LQCD

CS kernel

TMDPDF

$$\sqrt{S_I(b_T,\mu)} \cdot \tilde{f}(x,b_T,P_z,\mu) = H(x,P_z,\mu) \cdot \exp\left[\frac{1}{2}\ln\frac{(2xP_z)^2}{\zeta}\gamma^{\overline{\text{MS}}}(b_T,\mu)\right] \cdot f(x,b_T,\zeta,\mu)$$

intrinsic soft factor

pQCD kernel

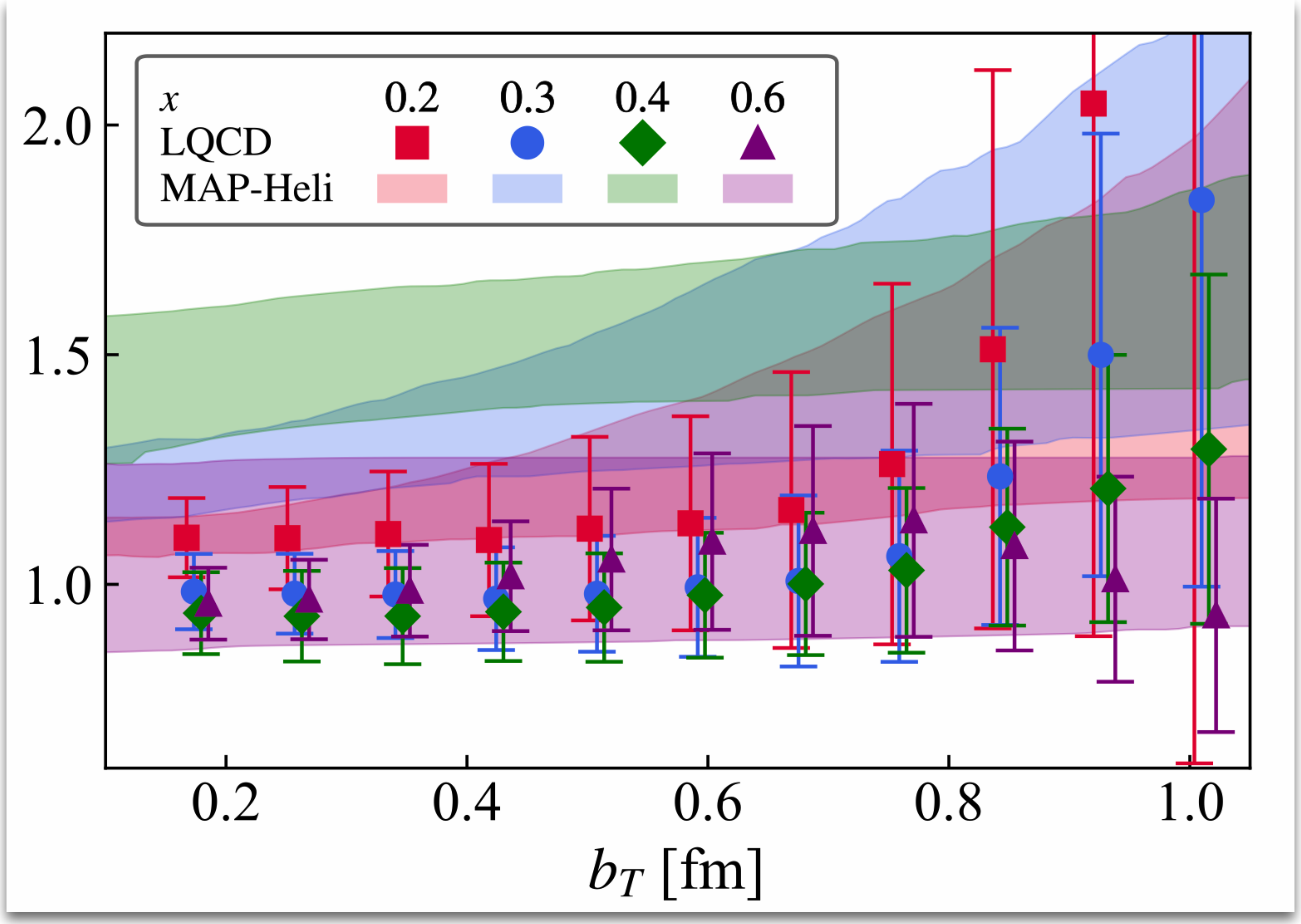
scale-independent ratios of TMDPDF

$$\frac{f_a(x,b_T,\zeta,\mu)}{f_b(x,b_T,\zeta,\mu)} = \frac{\tilde{f}_a(x,b_T,P_z,\mu)}{\tilde{f}_b(x,b_T,P_z,\mu)}$$

TMDPDF of proton: helicity to unpolarized TMDPDF

$$\frac{g_{1L}^{\Delta u_+ - \Delta d_+}(x, b_T, \zeta, \mu)}{g_A \cdot f_1^{u_v - d_v}(x, b_T, \zeta, \mu)}$$

unitary chiral quarks, physical mass

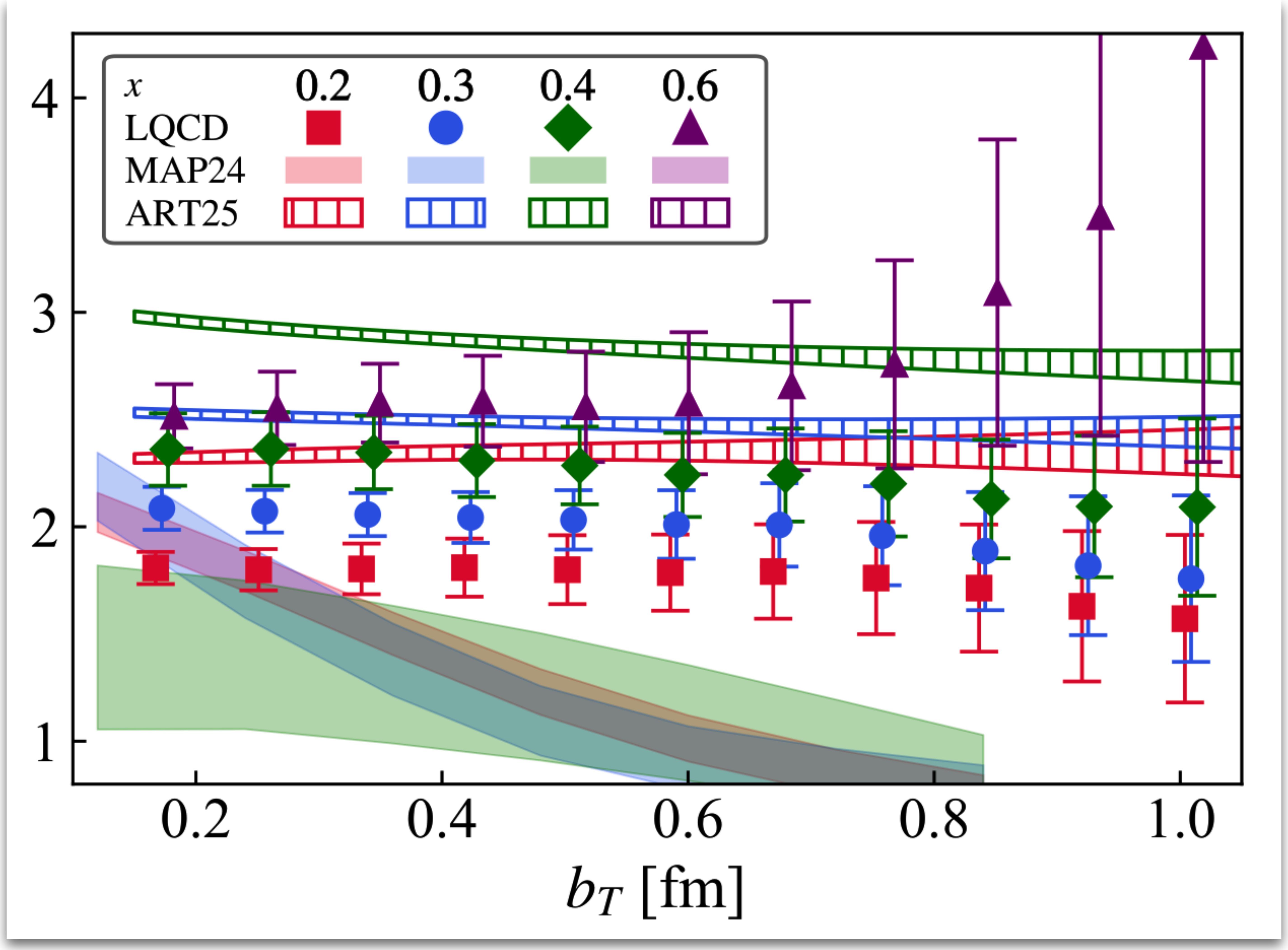


X. Gao et al., arXiv:2505.18430

TMDPDF of proton: up to down unpolarized TMDPDF

$$\frac{f_1^{u_v}(x, b_T, \zeta, \mu)}{f_1^{d_v}(x, b_T, \zeta, \mu)}$$

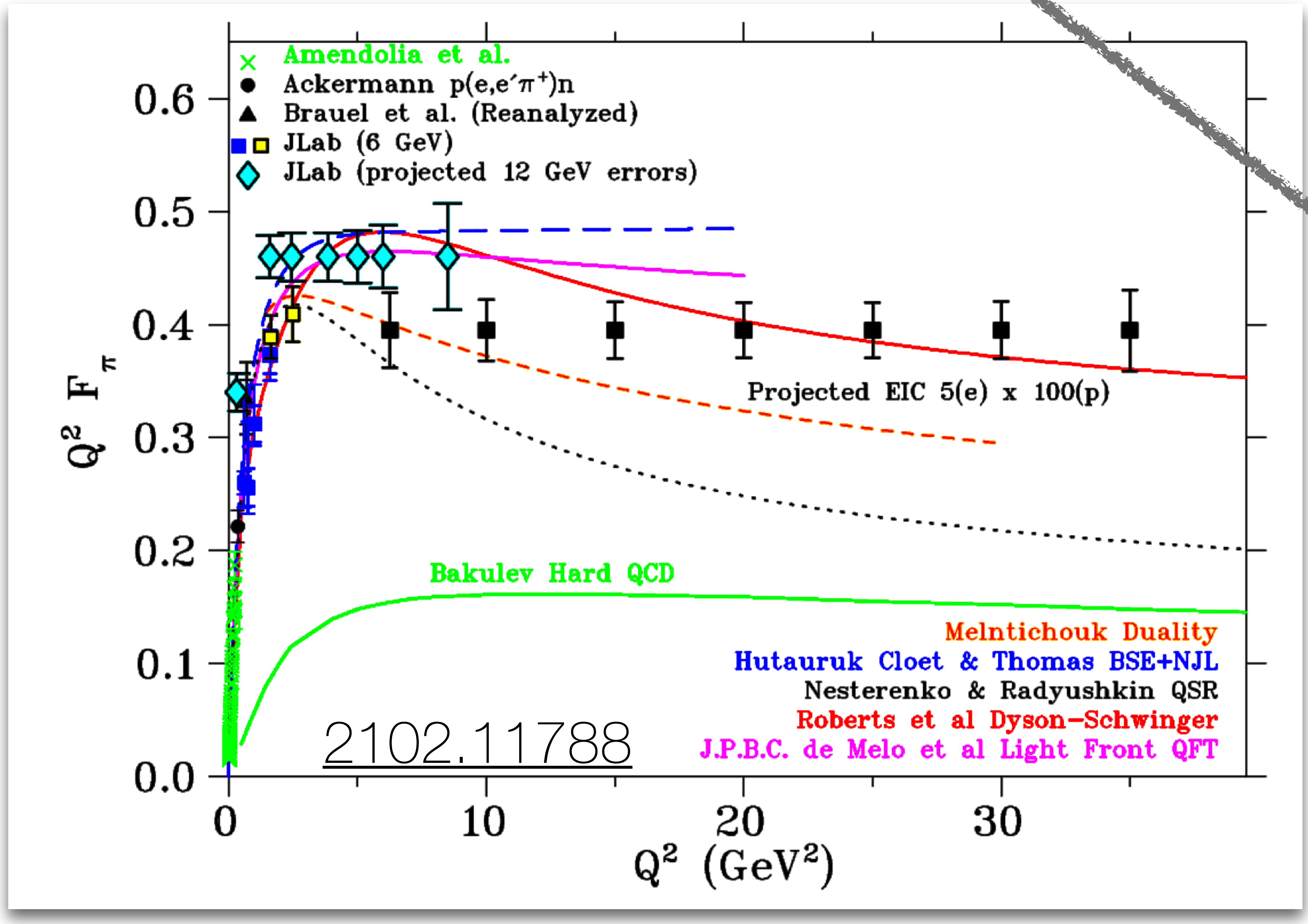
unitary chiral quarks,
physical mass



X. Gao et al., arXiv:2505.18430

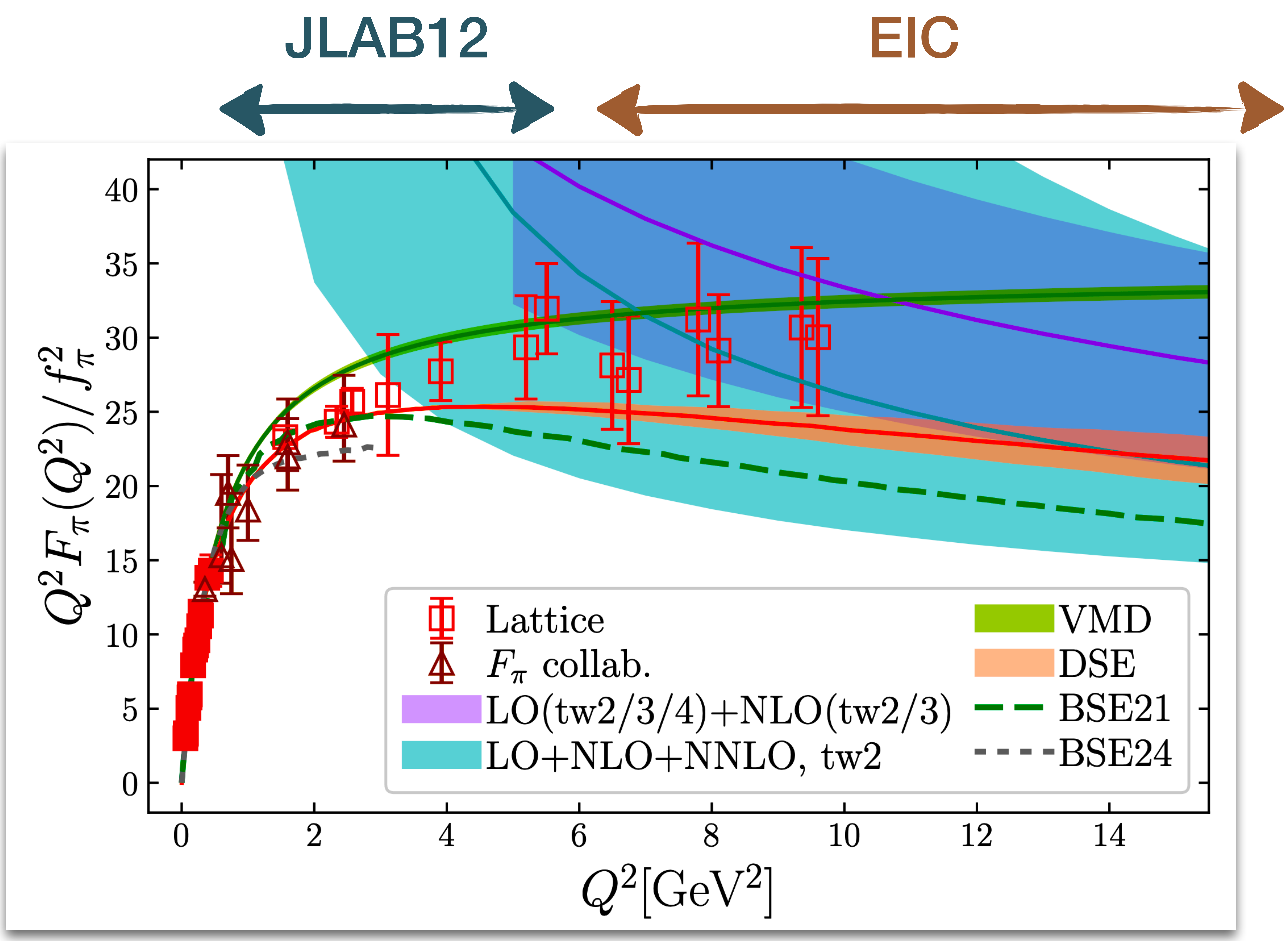
from QCD to cross-section: e.g. pion/kaon EM form factor

EIC $F(Q^2 \gg \Lambda_{QCD}) \sim$ NNLO pQCD $\otimes$ meson DA LQCD



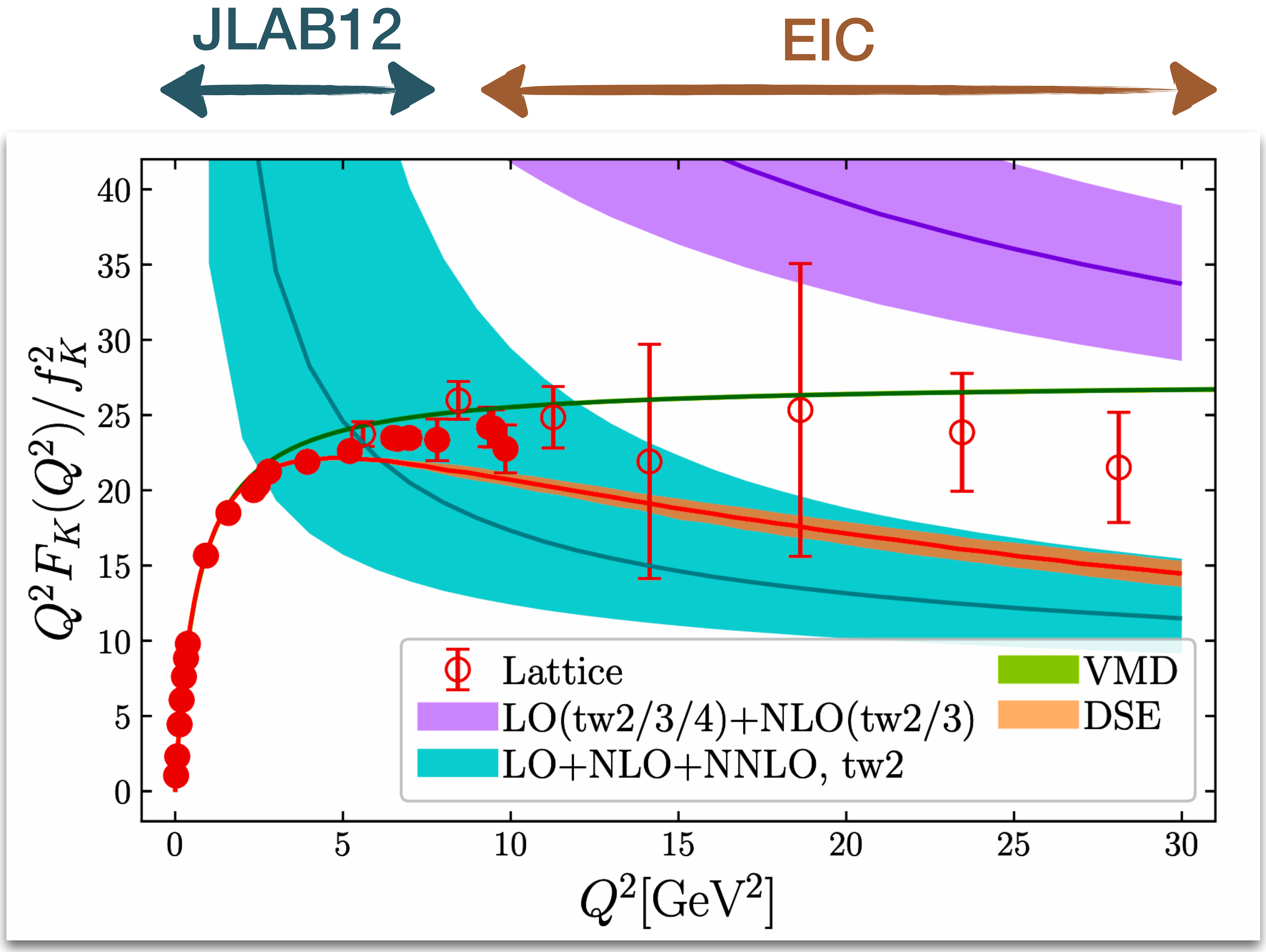
also can predict from LQCD !!!

LQCD predictions for pion EM form factor



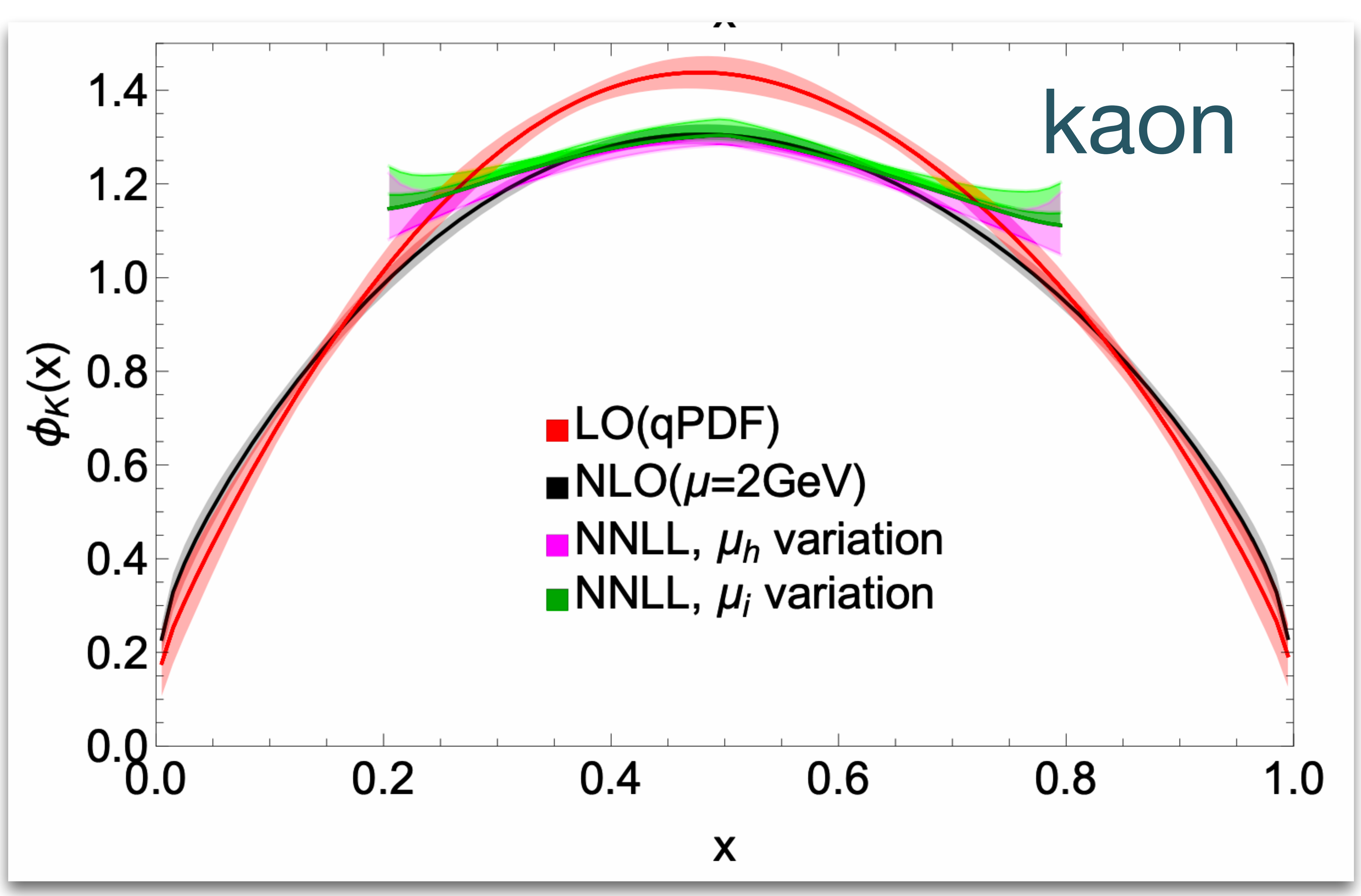
X. Gao, Q. Shi et al., Phys.Rev.Lett. 133, 181902 (2024)

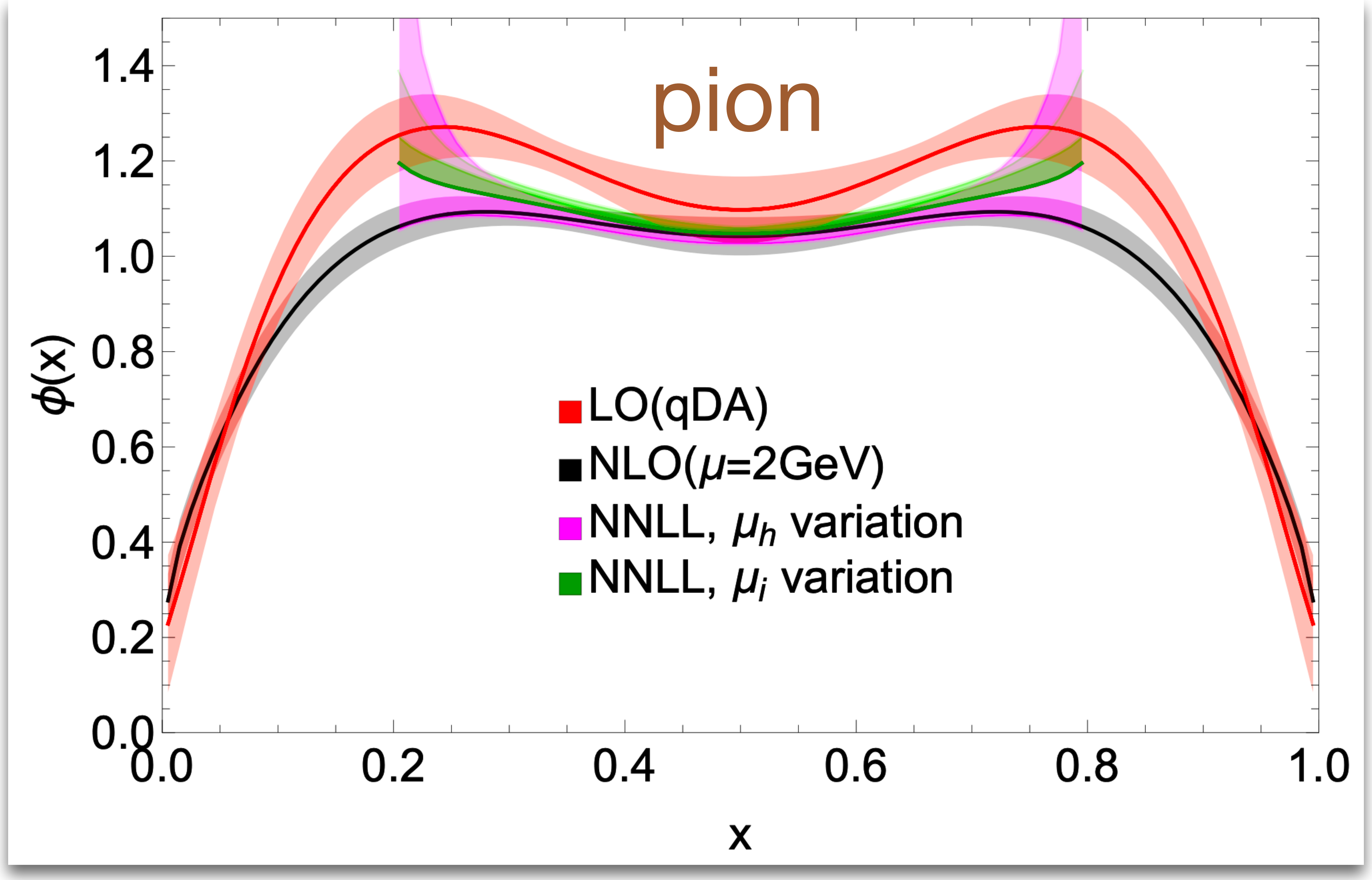
LQCD predictions for kaon EM form factor



X. Gao, Q. Shi et al., Phys.Rev.Lett. 133, 181902 (2024)

meson distribution amplitudes from LQCD



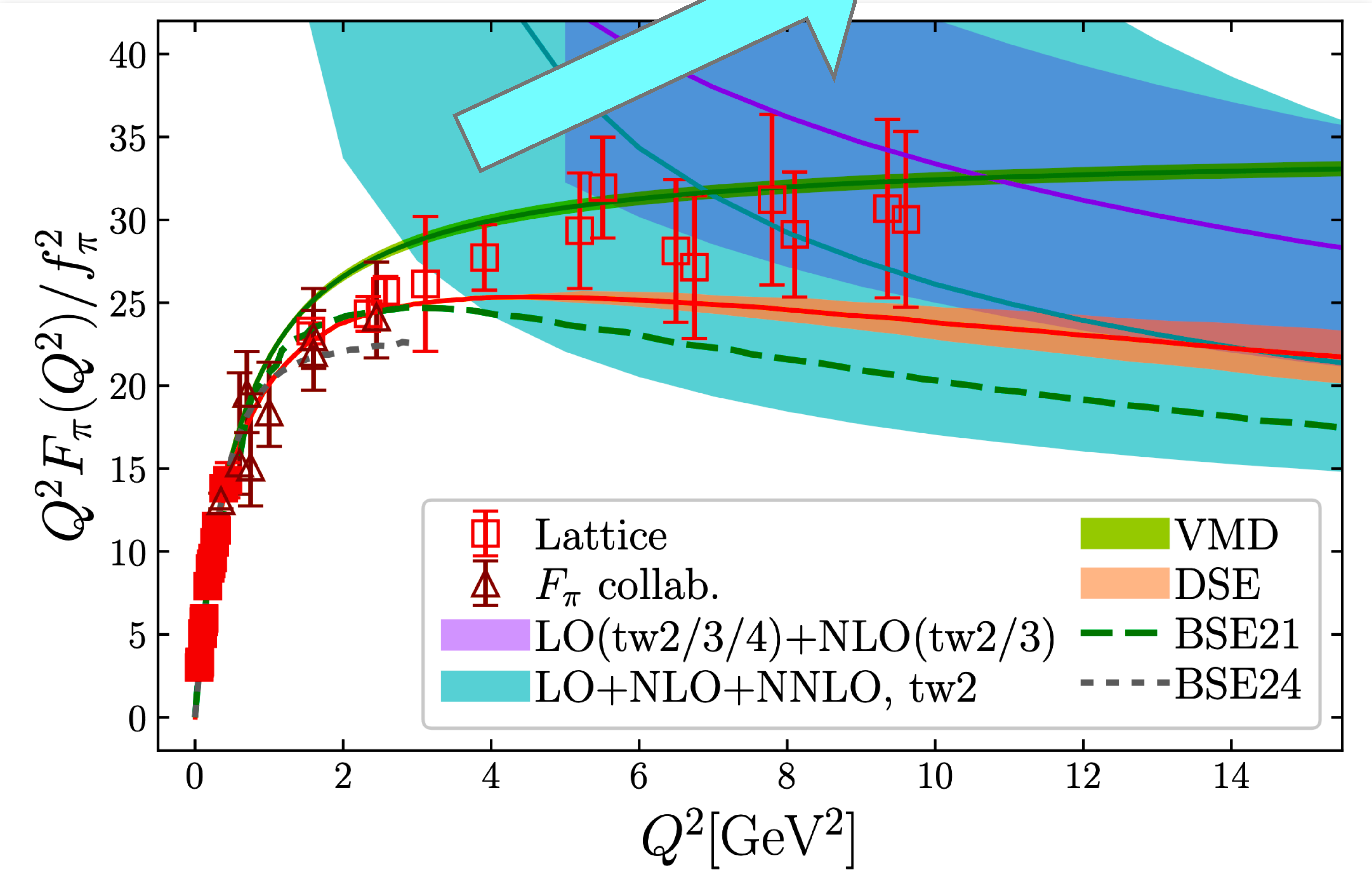


R. Zhang et al., Phys. Rev. D 110, 114502 (2024)

R. Zhang et al., JHEP 07, 211 (2024)

from QCD to cross-section …

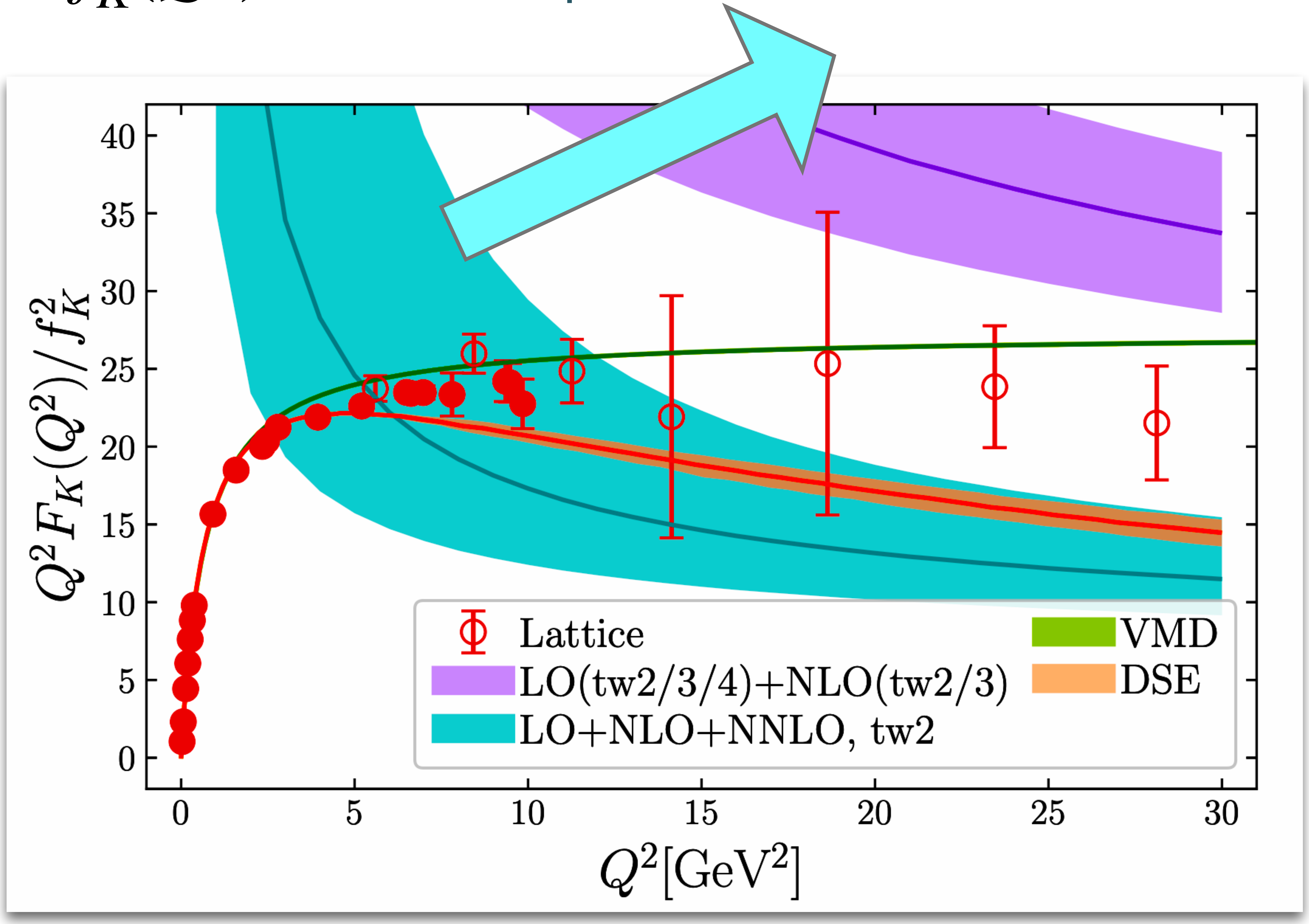
$f_\pi(Q^2)$ ~ NNLO pQCD $\otimes$ LQCD pion DA



X. Gao, Q. Shi et al., Phys.Rev.Lett. 133, 181902 (2024)

from QCD to cross-section ...

$f_K(Q^2)$ ~ NNLO pQCD $\otimes$ LQCD kaon DA



X. Gao, Q. Shi et al., Phys.Rev.Lett. 133, 181902 (2024)

from QCD to EIC ...