Momentum, structure, and forces in the nucleon from lattice QCD

James Zanotti
The University of Adelaide

QCDSF Collaboration

Workshop on parton distribution functions in the EIC era
June 16 - 18, 2025
Institute of Physics, Academia Sinica, Taipei, Taiwan

QCDSF Collaboration

- M. Batelaan (Adelaide, PhD 2023 -> W&M)
- K. U. Can (Adelaide)
- **J. Crawford (Adelaide, PhD 2025?)***
- **A. Hannaford-Gunn (Adelaide, PhD 2023)***
- R. Horsley (Edinburgh)
- **T. Howson (Adelaide, PhD 2024)***
- J. McKee (Adelaide, PhD)
- J. Perks (Adelaide, Masters)

- D. Pleiter (KTH)
- P. Rakow (Liverpool)
- G. Schierholz (DESY)
- **R. Smail (Adelaide, PhD 2024)***
- H. Stüben (Hamburg)
- I. van Schalkwyk (Adelaide, PhD)
- T. Schar (Adelaide, Masters 2025)
- R. Young (Adelaide)

(* this talk)

Topics

- Precision isovector axial, tensor, scalar charges [PRD108 (2023)]
- Quark and gluon momentum fractions, $\langle x \rangle_q$, $\langle x \rangle_g$ [PLB714 (2012) + in preparation]
    - Renormalisation and mixing
- Off-forward Compton amplitude [PRD105 (2022), PRD110 (2024)]
    - Reconstruction of generalised parton distribution functions
- Transverse forces [PRL134 (2025)]

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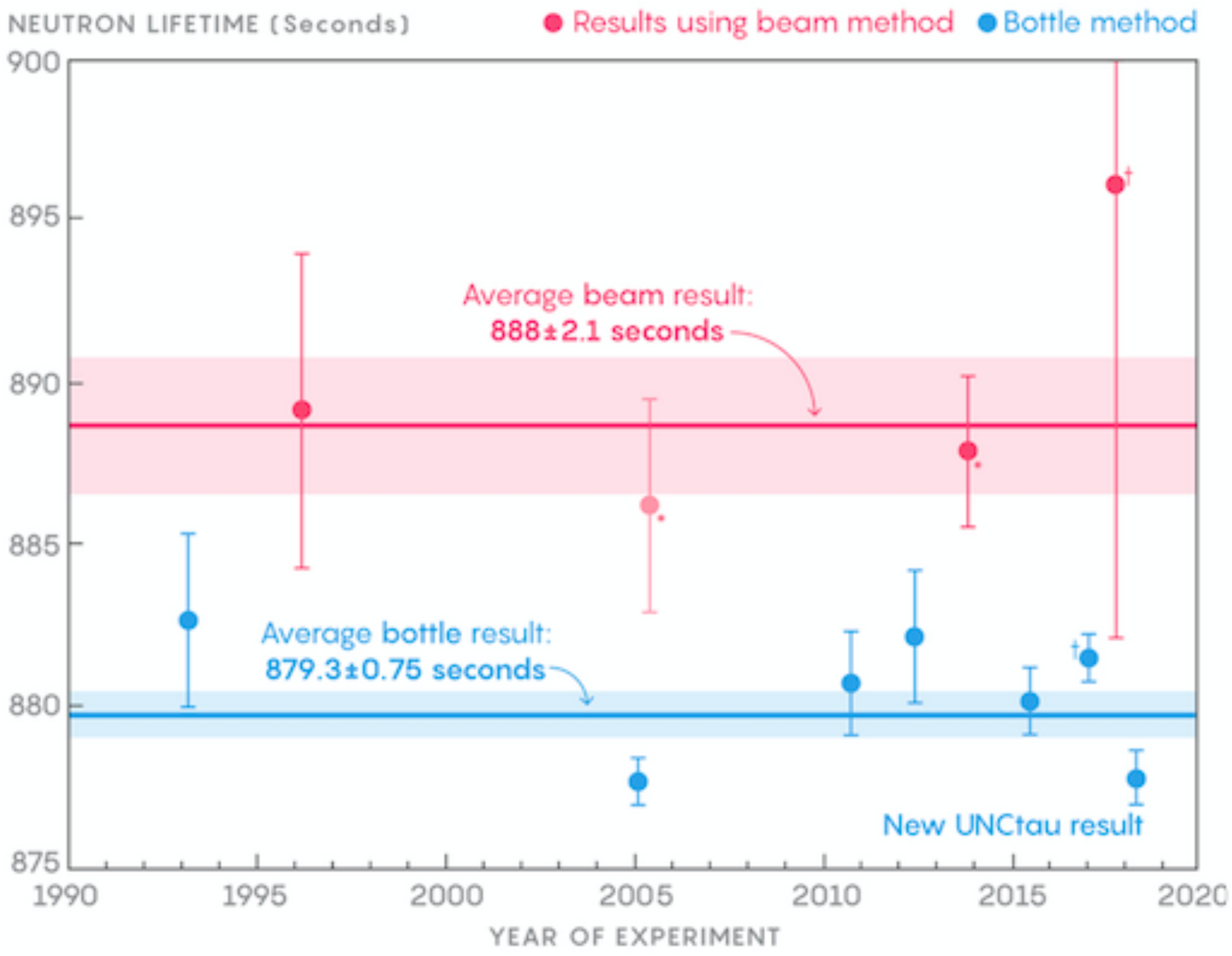
Precision isovector axial, tensor, scalar charges

[PRD108 (2023)]

Motivation

- Current $\tau^n_{\text{bottle}} - \tau^n_{\text{beam}} \sim 4\sigma$
- Unconsidered systematic error in the experiments? or evidence of new physics?
- Bottle counts how many neutrons left
- Beam counts final state protons only
- *Evidence of some unknown decay in bottle?*



Motivation

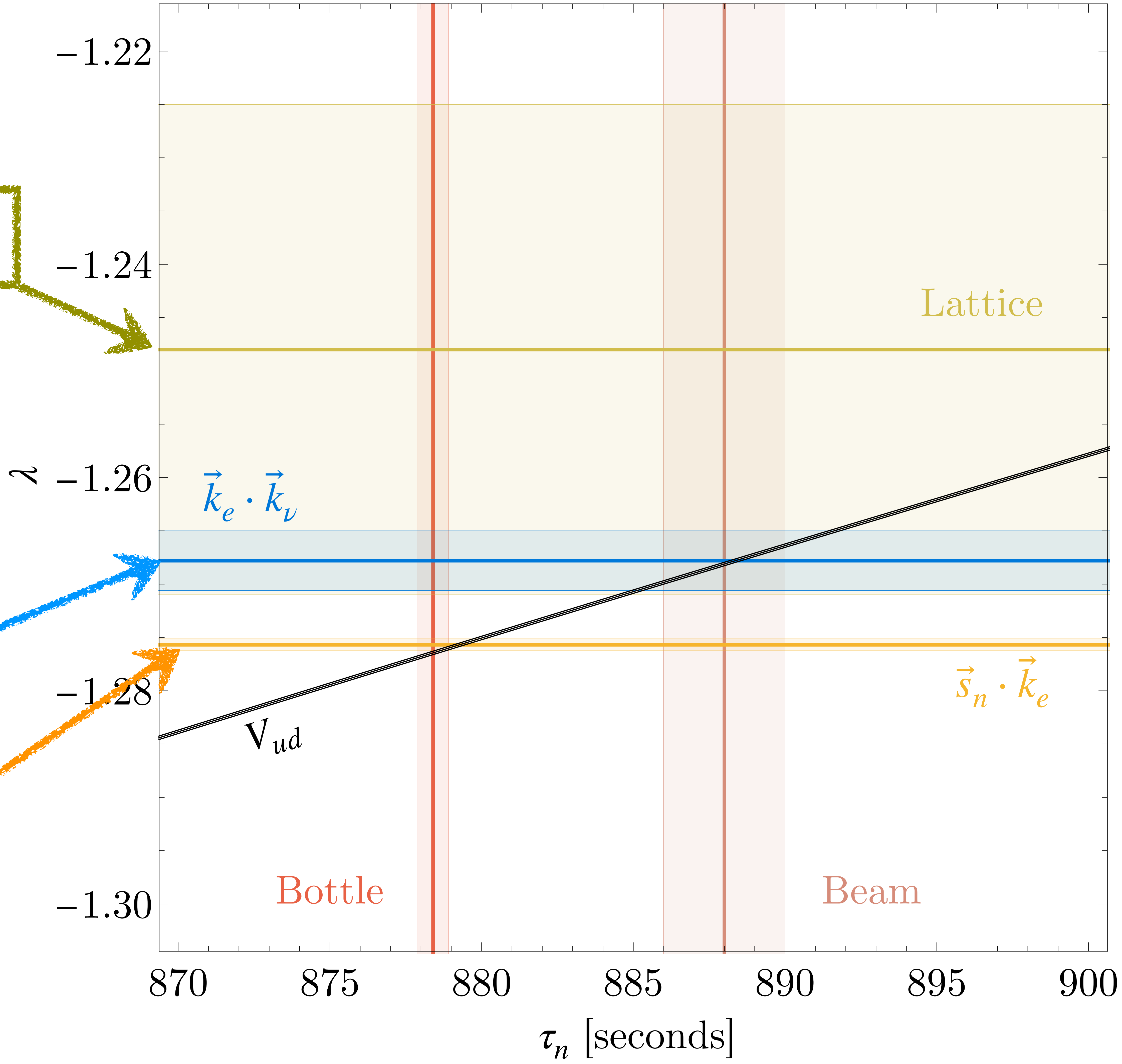
[QCDSF, PRD108 (2023)]

- Nucleon isovector charges ($g_A^{u-d}, g_T^{u-d}, g_S^{u-d}$) can have an impact on searches for New Physics
  - Neutron lifetime puzzle
  - Neutron β-decay
  - CP-violation and neutron EDM
- Importance of lattice input to these reflected in appearing in FLAG in 2021

FLAG '21

electron — anti-neutrino momenta correlation

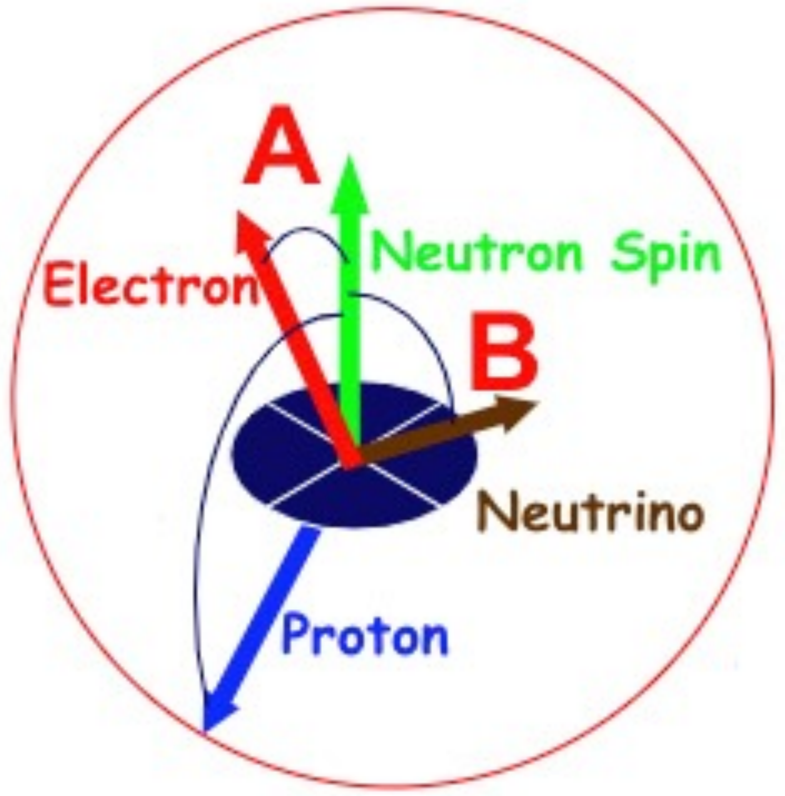
electron momentum — neutron polarisation correlation



Motivation

➤ For a beam of polarised neutrons the differential decay rate is described by:

$$dW \propto \frac{1}{\tau_n} F(E_n) \left[1 + a \frac{\vec{k}_e \cdot \vec{k}_\nu}{E_e E_\nu} + b \frac{m_e}{E_e} + A \frac{\vec{s}_n \cdot \vec{k}_e}{E_e} + B \frac{\vec{s}_n \cdot \vec{k}_\nu}{E_\nu} \right]$$



Fierz interference term

➤ SM: $b = 0$

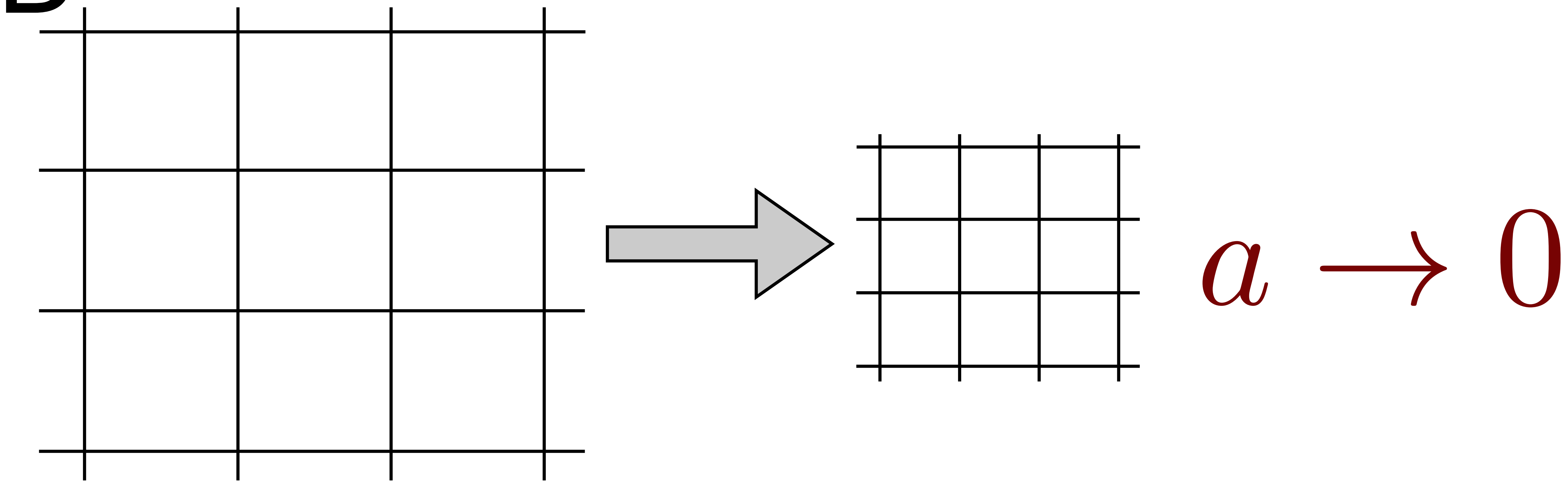
➤ Added to account for the possible BSM *scalar* and *tensor* interactions

SM	**BSM**
$\langle p \| V/A \| n \rangle$	$\langle p \| T/S \| n \rangle$
$g_V \approx 1, g_A = 1.2756(13)$	$g_S \approx ?, g_T \approx ?$

Systematics of Lattice QCD

Extrapolations:

- Continuum
  - Unavoidable
  - Improved actions (errors O(a2))
  - Finer lattice spacings



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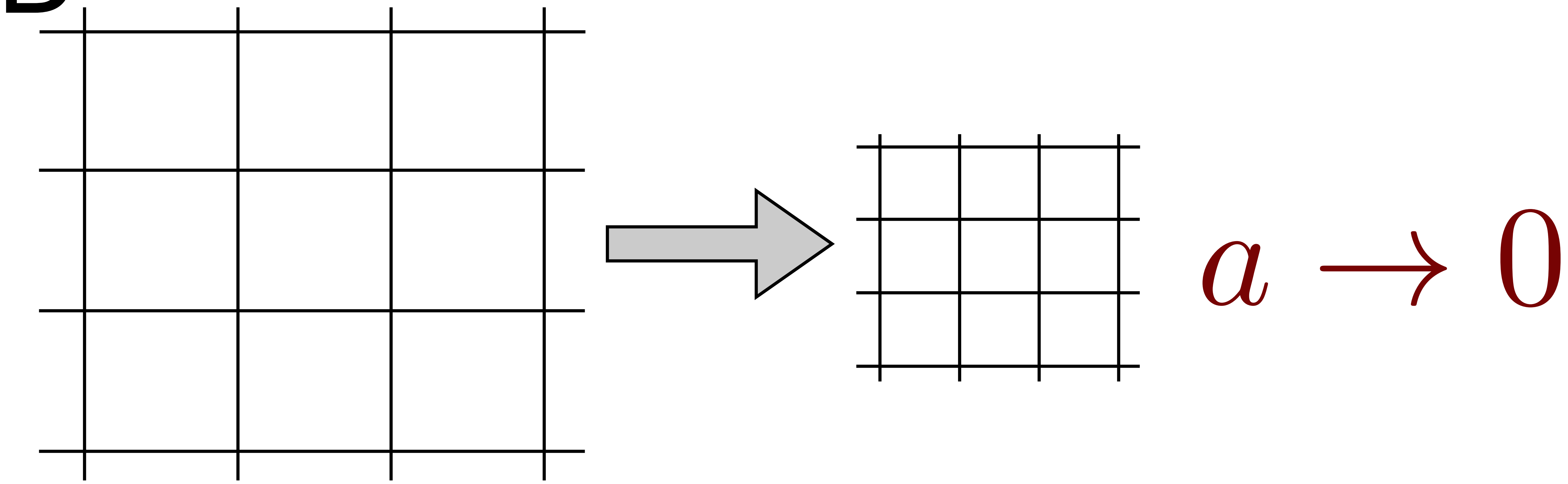
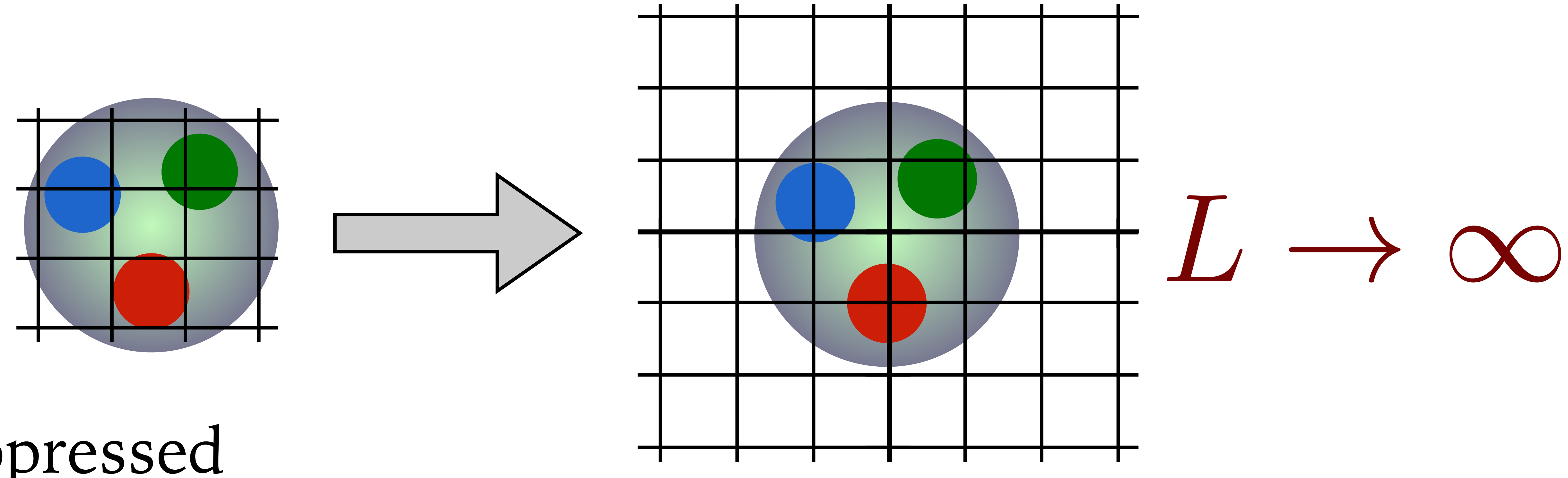


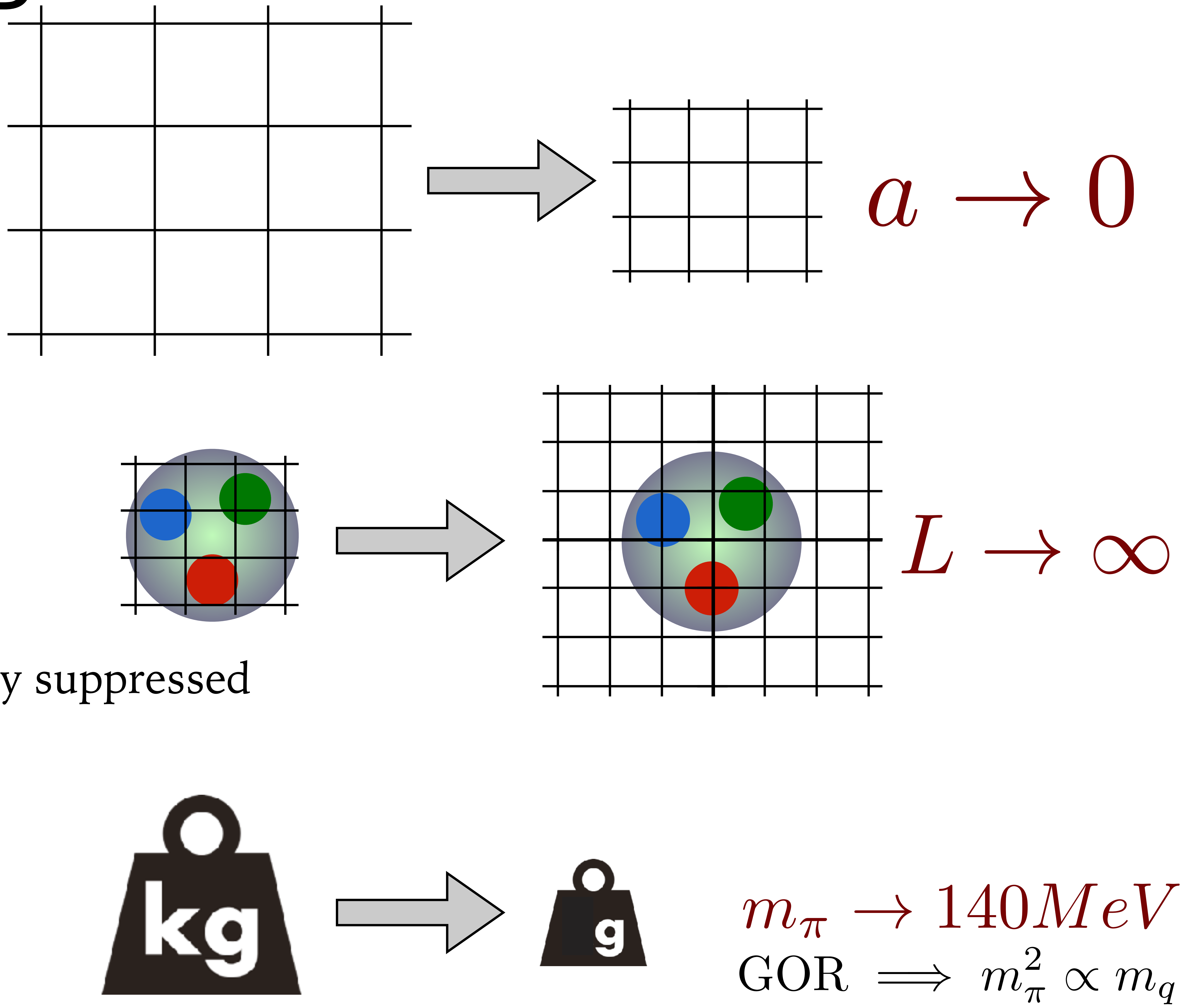
- Finite volume
  - Large volumes so effects are exponentially suppressed



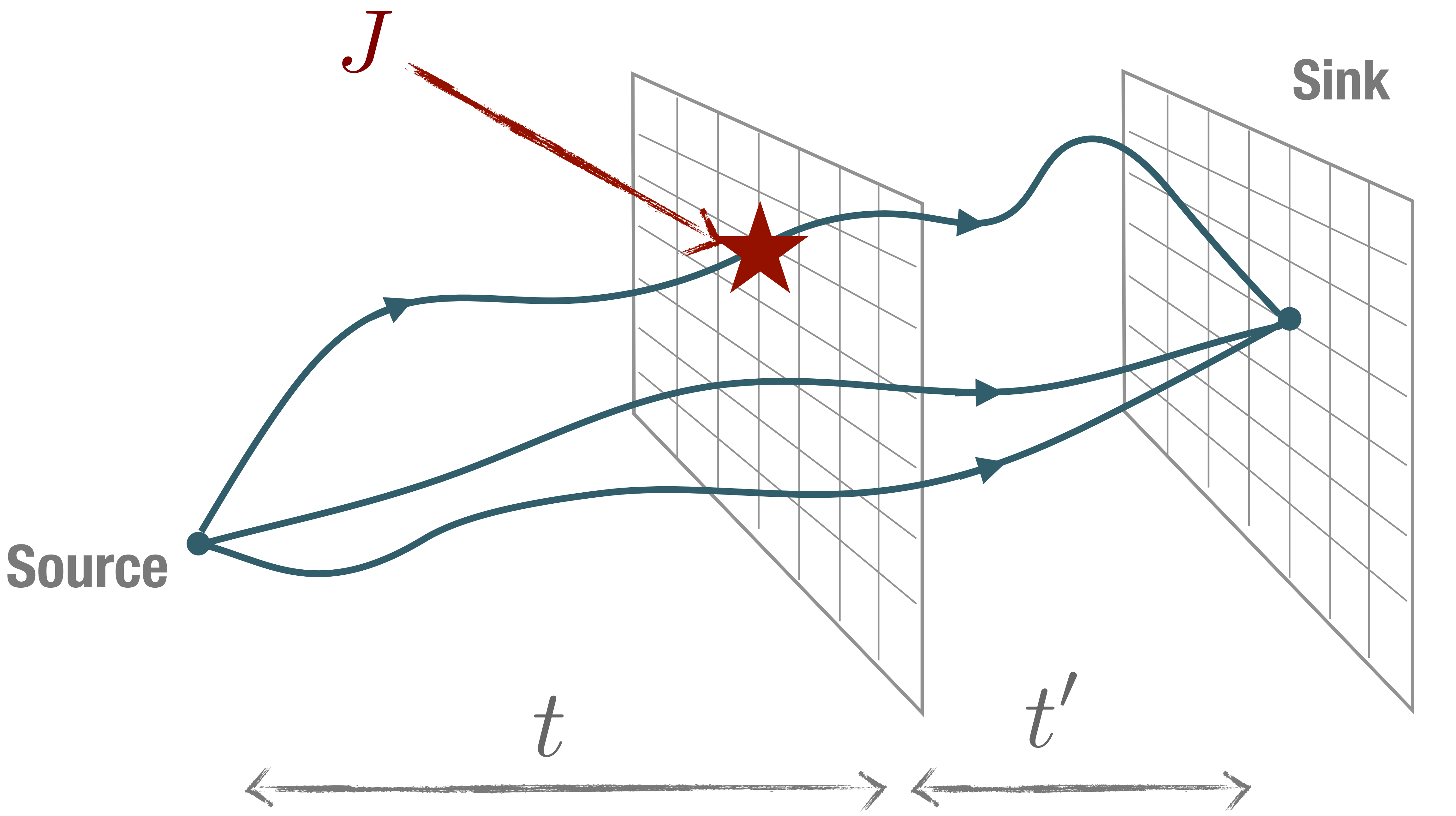
Systematics of Lattice QCD

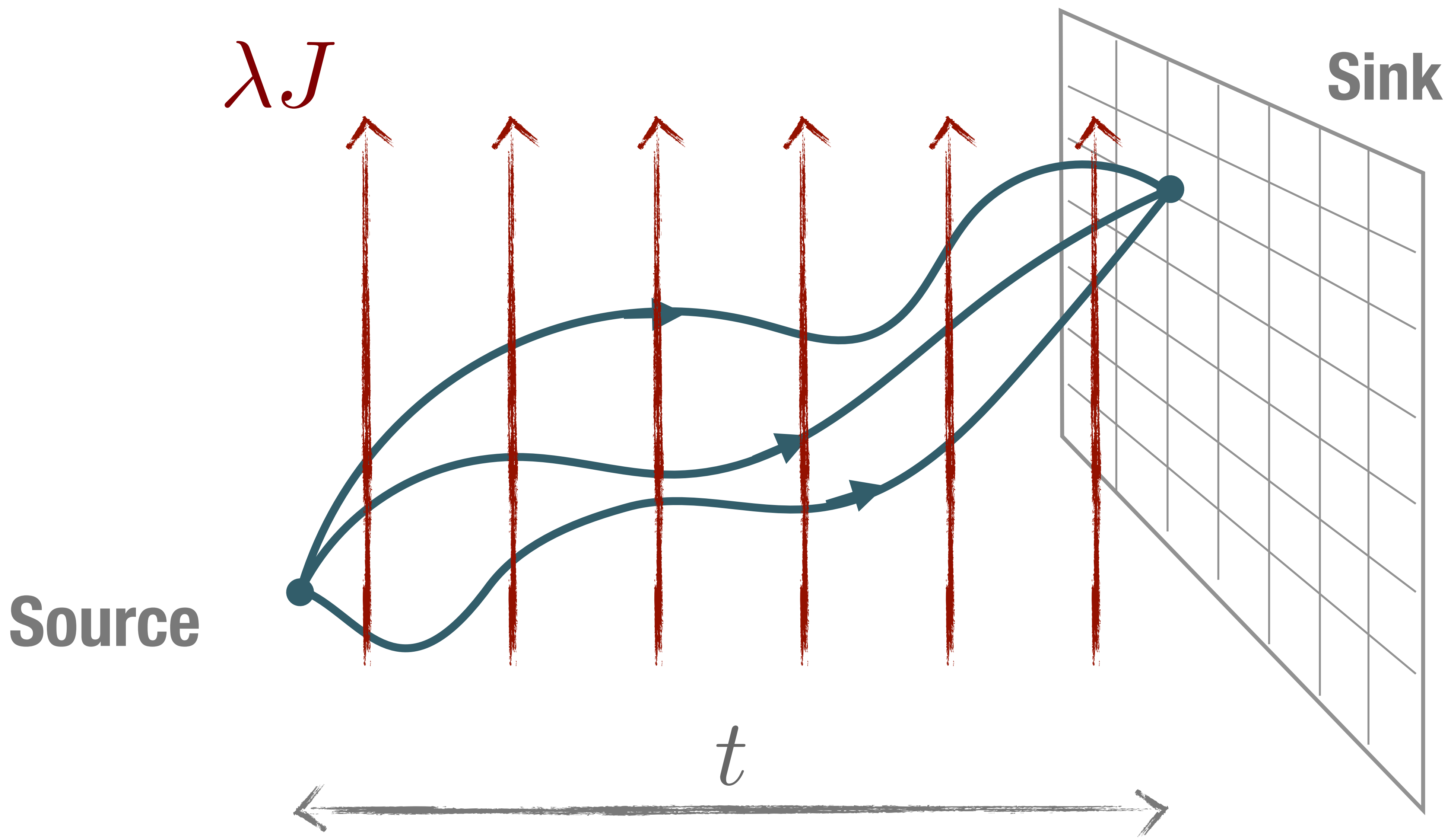
Extrapolations:

- Continuum
  - Unavoidable
  - Improved actions (errors O(a2))
  - Finer lattice spacings
- Finite volume
  - Large volumes so effects are exponentially suppressed
- Chiral
  - Simulate at physical quark masses
  - Chiral perturbation theory
  - Flavour-breaking expansion



Matrix elements on the lattice





3-pt functions

$$t, t' \gg \frac{1}{\Delta E}$$ ← energy gap to lowest excitation

$$\frac{\langle C_3(t,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|J|N\rangle$$

Feynman–Hellmann

$$t \gg \frac{1}{\Delta E}$$

$$\left.\frac{\partial E}{\partial \lambda}\right|_{\lambda\to 0} \propto \langle N|J|N\rangle$$

Feynman-Hellmann Theorem

Suppose we want: $\langle H | \mathcal{O} | H \rangle$

Modify action with external field:

real parameter

$$S \to S + \lambda \int d^4x \, \mathcal{O}(x)$$

local operator, e.g. $\bar{q}(x)\gamma_3 q(x)$

Measure hadron energy while changing λ

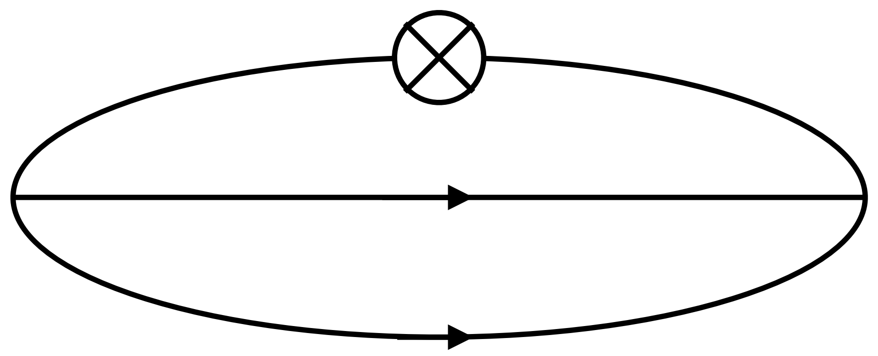
$$G(\lambda; \vec{p}; t) = \int dx \, e^{-i\vec{p}\cdot\vec{x}} \langle \chi'(x)\chi(0) \rangle \overset{\text{large t}}{\propto} e^{-E_H(\lambda,\vec{p})t}$$

Calculation of matrix elements ≡ hadron spectroscopy $\left.\frac{\partial E_H(\lambda, \vec{p})}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\vec{p})} \langle H(\vec{p}) | \mathcal{O}(0) | H(\vec{p}) \rangle$

Feynman-Hellmann Theorem

- Can modify fermion action in 2 places:

  - quark propagators *Connected*



g_A, $\Delta\Sigma$ [PRD90 (2014)]

NPR [PLB740 (2015)]

G_E, G_M [PRD96 (2017)]

$F_{1,2}(\omega, Q^2)$ [PRL118 (2017), PRD102 (2020), PRD107 (2023)]

GPDs [PRD105 (2022), PRD110 (2024)]

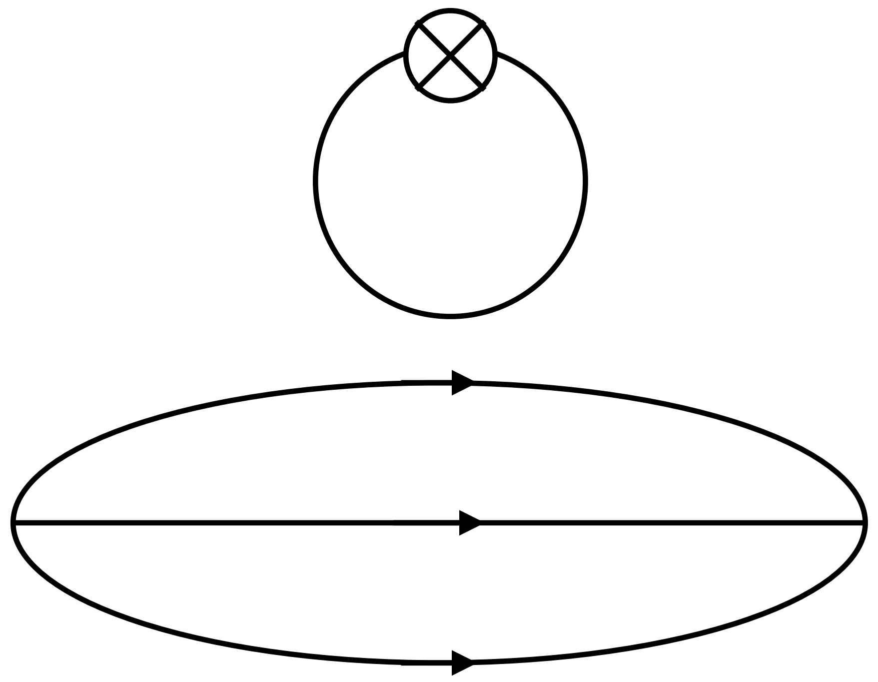
$\Sigma \to n$ [PRD108 (2023)]

g_A, g_T, g_S [PRD108 (2023)]

$S_1(Q^2)$ [PRD111 (2025)]

$F_3(\omega = 0, Q^2)$ [PRD111 (2025)]

  - fermion determinant *Disconnected*



(Requires new gauge configurations)

$\langle x \rangle_g$ [PLB714 (2012)]

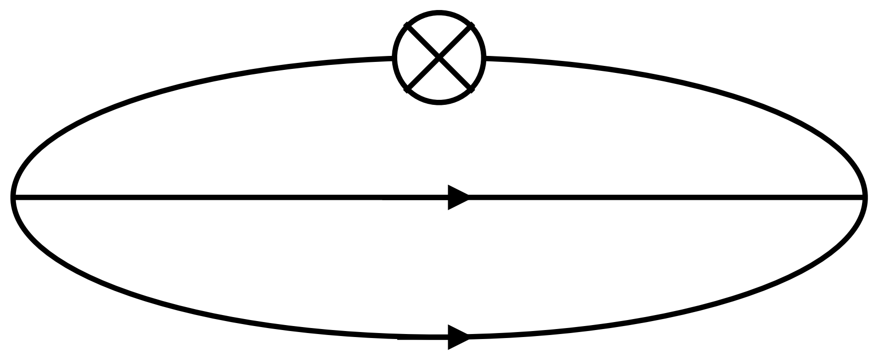
NPR [PLB740 (2015)]

Δs [PRD92 (2015)]

Feynman-Hellmann Theorem

- Can modify fermion action in 2 places:

◉ quark propagators *Connected*



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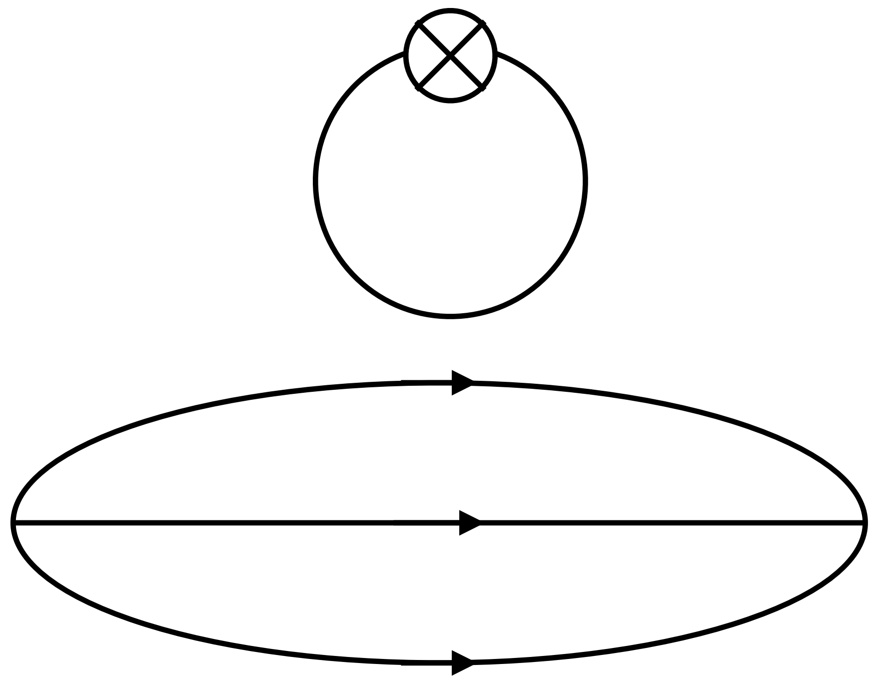


(Requires new gauge configurations)

$\langle x \rangle_g$ [PLB714 (2012)]

NPR [PLB740 (2015)]

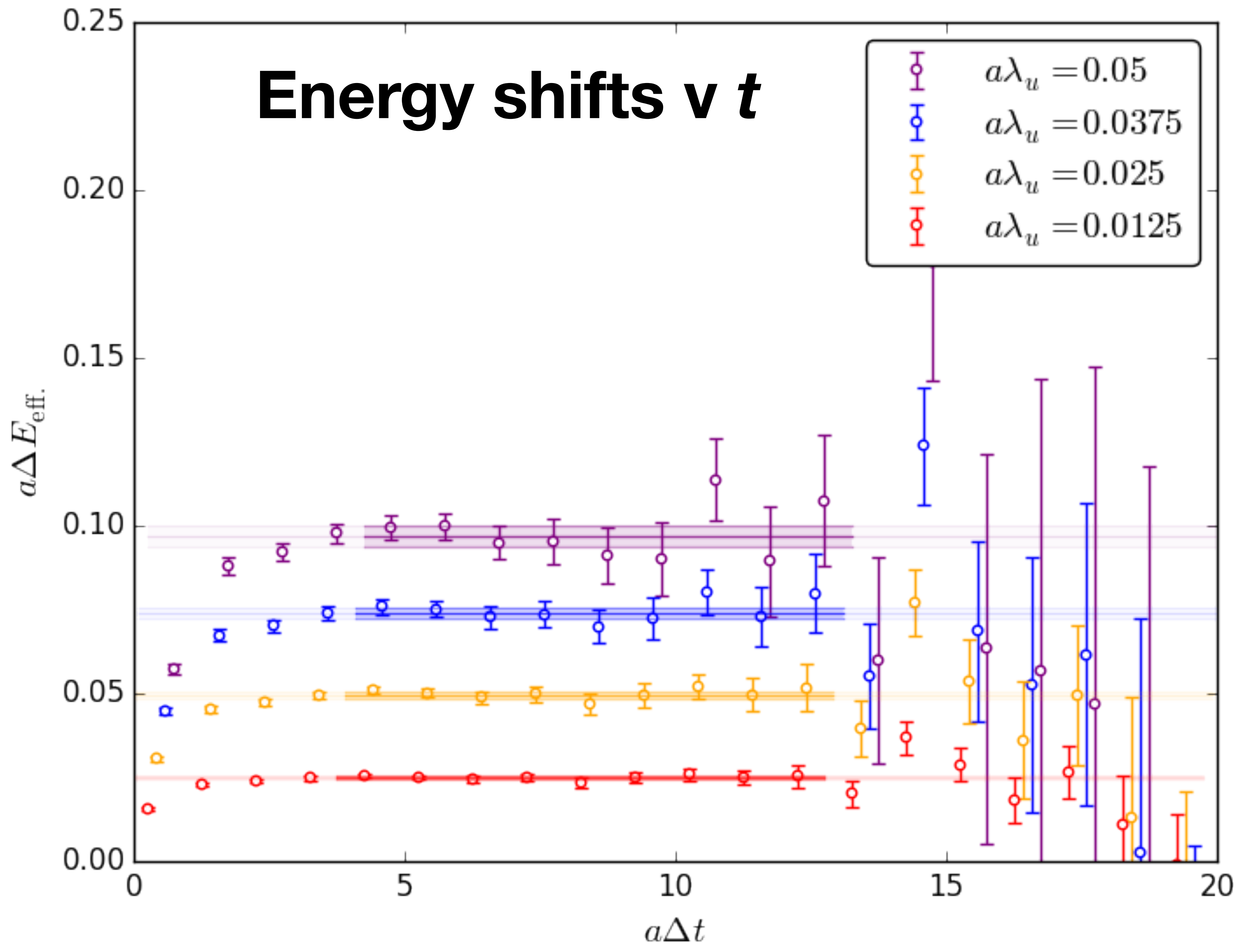
Δs [PRD92 (2015)]

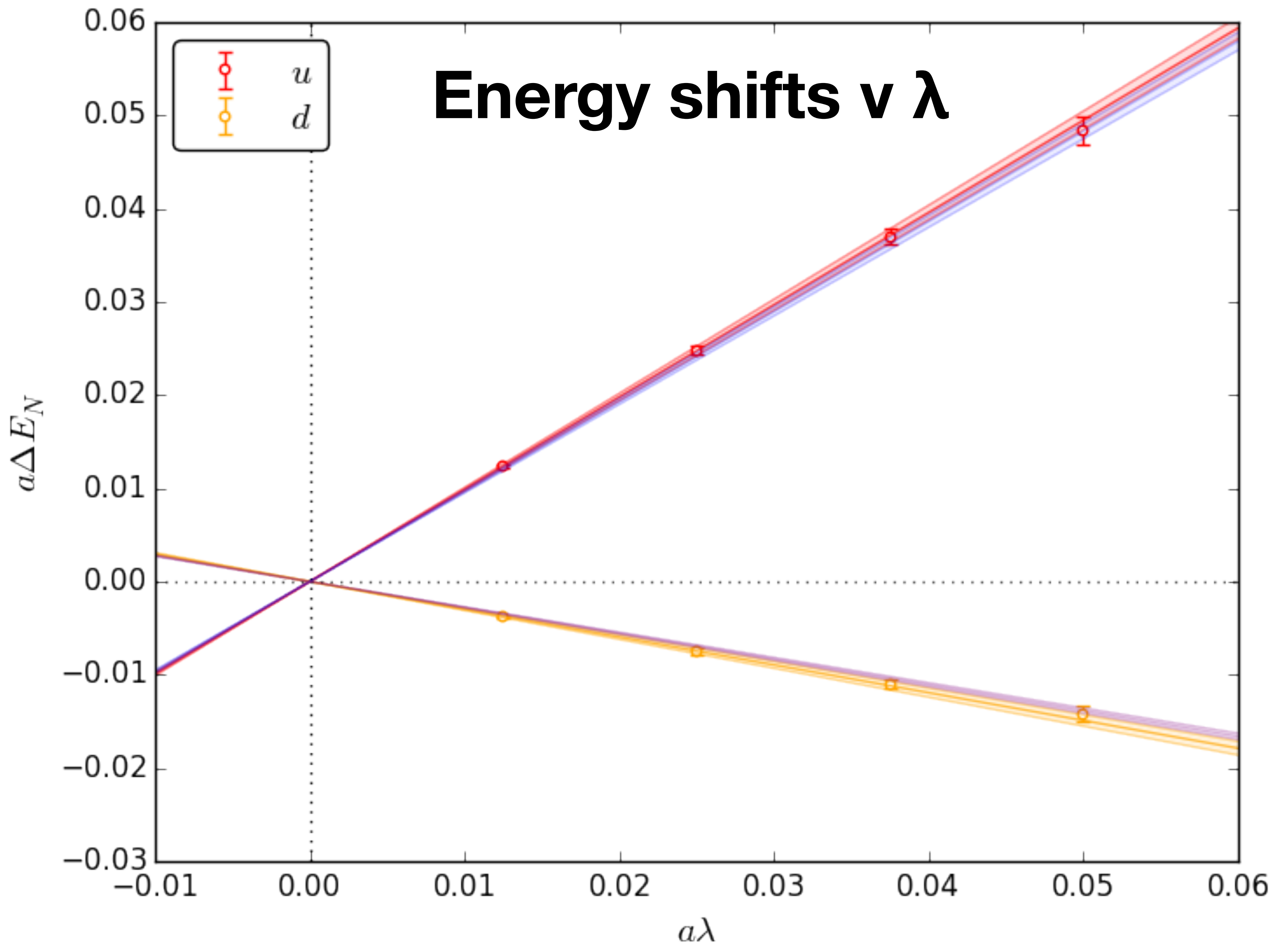
Demonstration: Axial charges

(Connected only, [PRD90 (2014)])

- Want $\langle N_s(\vec{p})|\bar{q}(0)\gamma_\mu\gamma_5 q(0)|N_s(\vec{p})\rangle = 2is_\mu\Delta q \qquad q \in (u,d)$

- Employ $\mathcal{L} \to \mathcal{L} + \lambda\bar{q}(-i\gamma_3\gamma_5)q \implies \left.\frac{\partial E_N(\lambda)}{\partial\lambda}\right|_{\lambda=0}^{\Gamma_\pm} = \pm\Delta q_{\text{conn.}}$

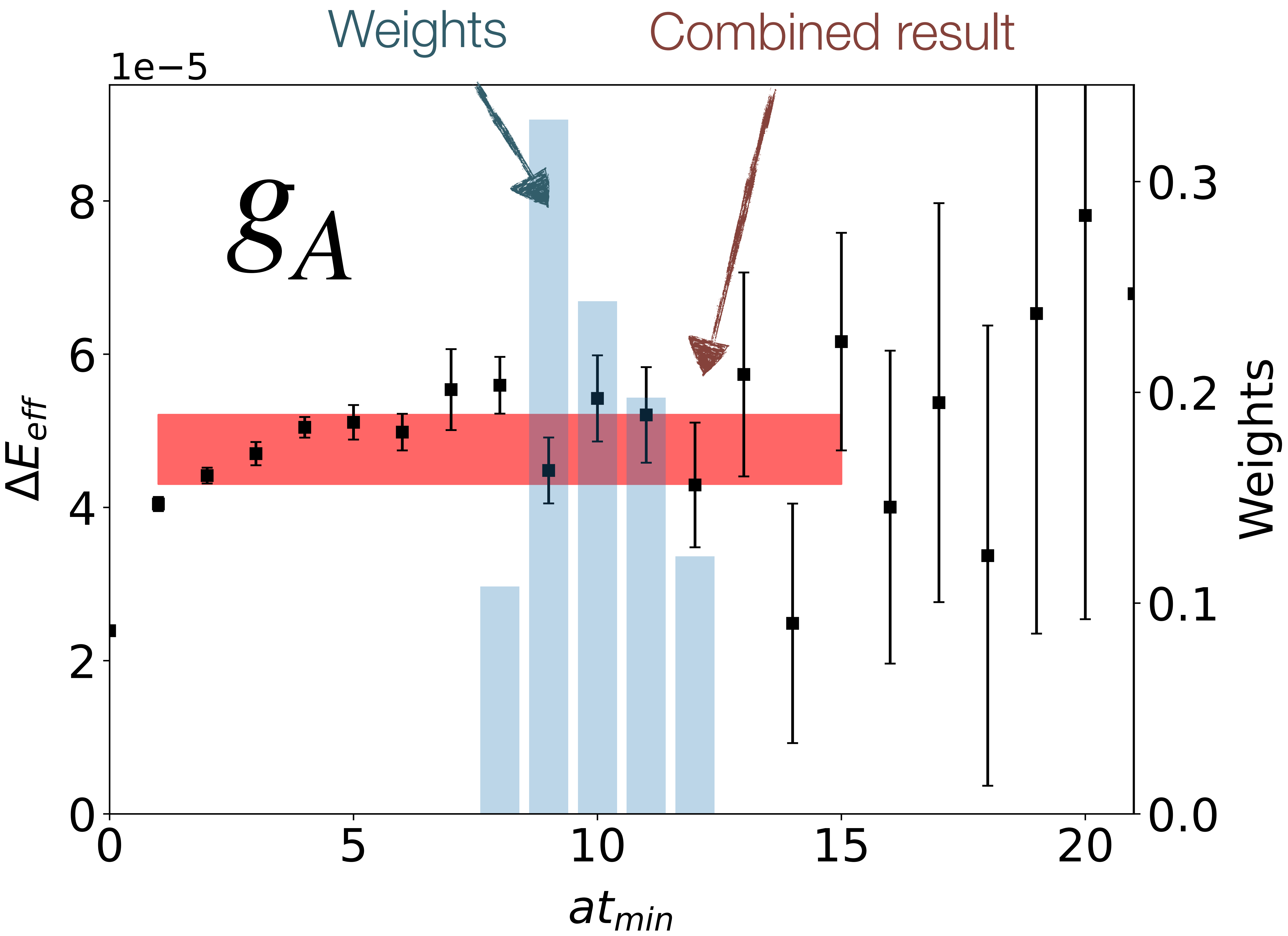


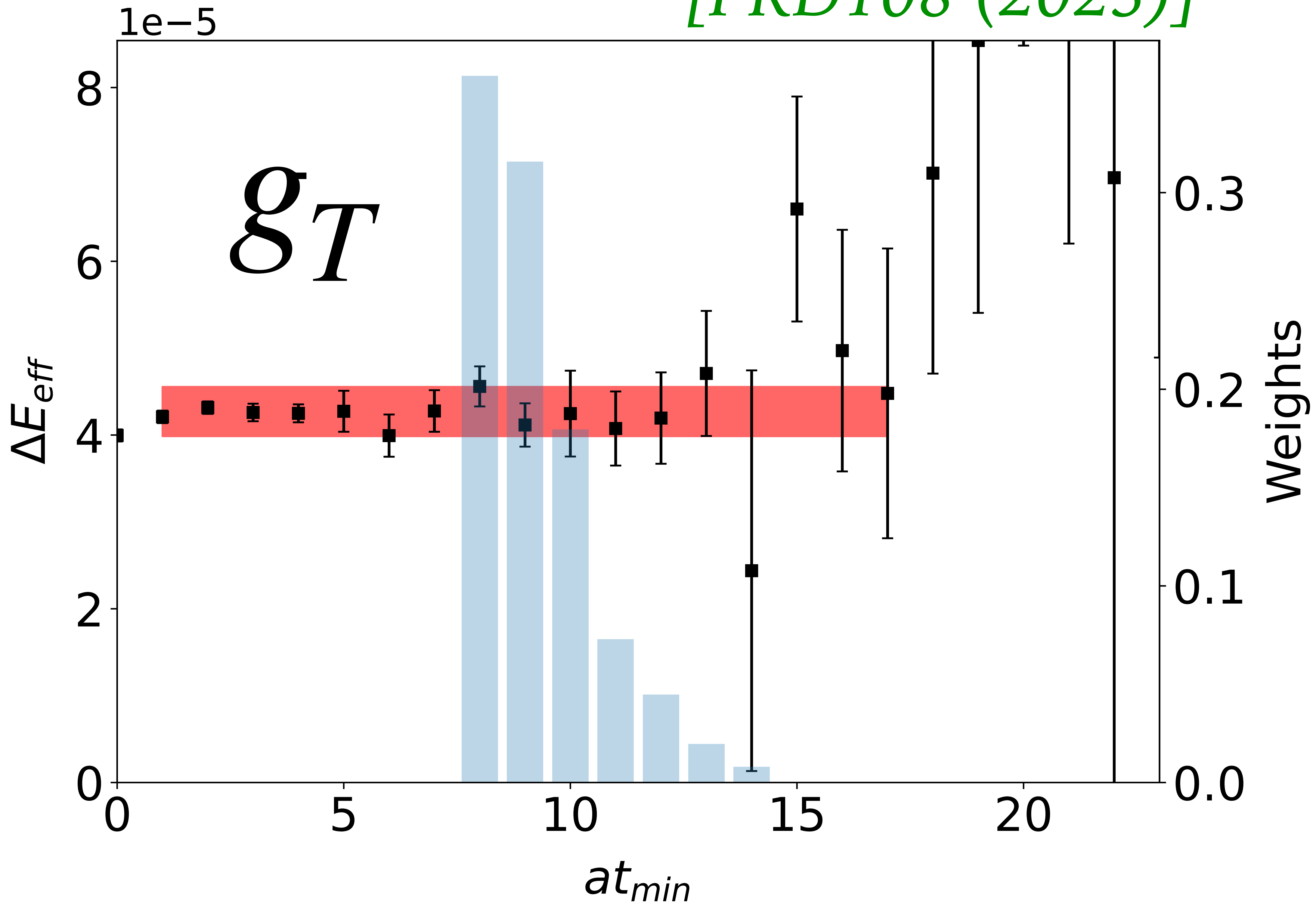


$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$

Energy shifts: weighted average

[PRD108 (2023)]





Minimum time used in fit~0.5-0.55fm

$t = 10, 9, 8, 7, 6$ for $a = 0.052,\ 0.058, 0.068, 0.074, 0.082$ fm

$m_\pi \approx 265\,\text{MeV},\ \text{a} = 0.068\,\text{fm},\ \text{V} = 48^3 \times 96,\ \lambda = 5 \times 10^{-4}$

see also: Beane *et al*. NPLQCD/QCDSF, PRD(2021), Rinaldi *et al*., PRD(2019)

(Non-normalised) weights:

$$\tilde{w}_f = \frac{p_f}{\sigma_f^2}$$

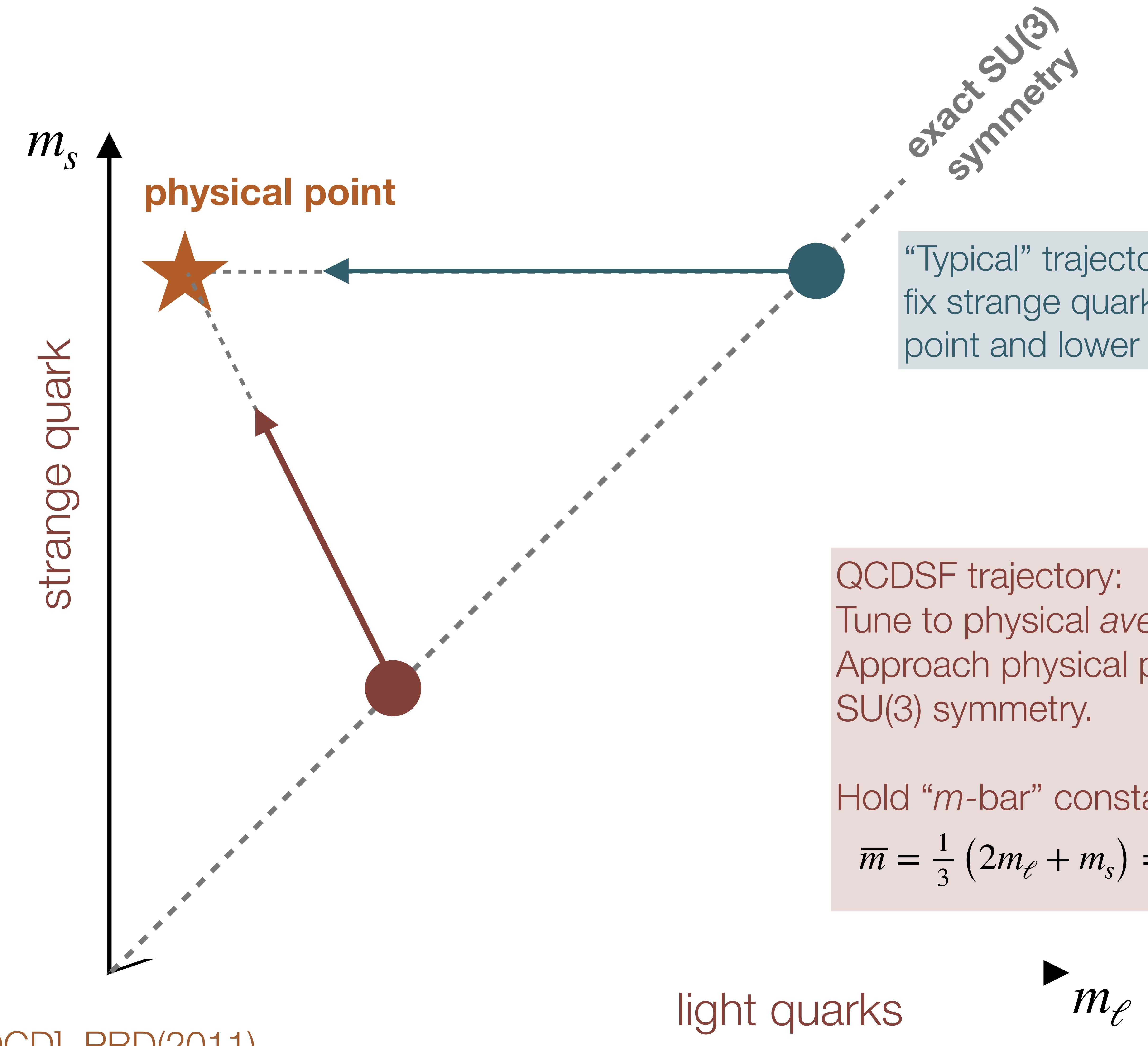
fit *p*-value

result uncertainty

Comparison to 3-point functions

Excellent agreement between Feynman-Hellmann and standard 3-point function methods

Quark mass trajectory



"Typical" trajectory:
fix strange quark mass to physical point and lower light quark mass

QCDSF trajectory:
Tune to physical *average* quark mass.
Approach physical point by breaking SU(3) symmetry.

Hold "*m*-bar" constant:

$$\overline{m} = \frac{1}{3}\left(2m_\ell + m_s\right) = \frac{1}{3}\left(2m_\ell^{\text{phys}} + m_s^{\text{phys}}\right)$$

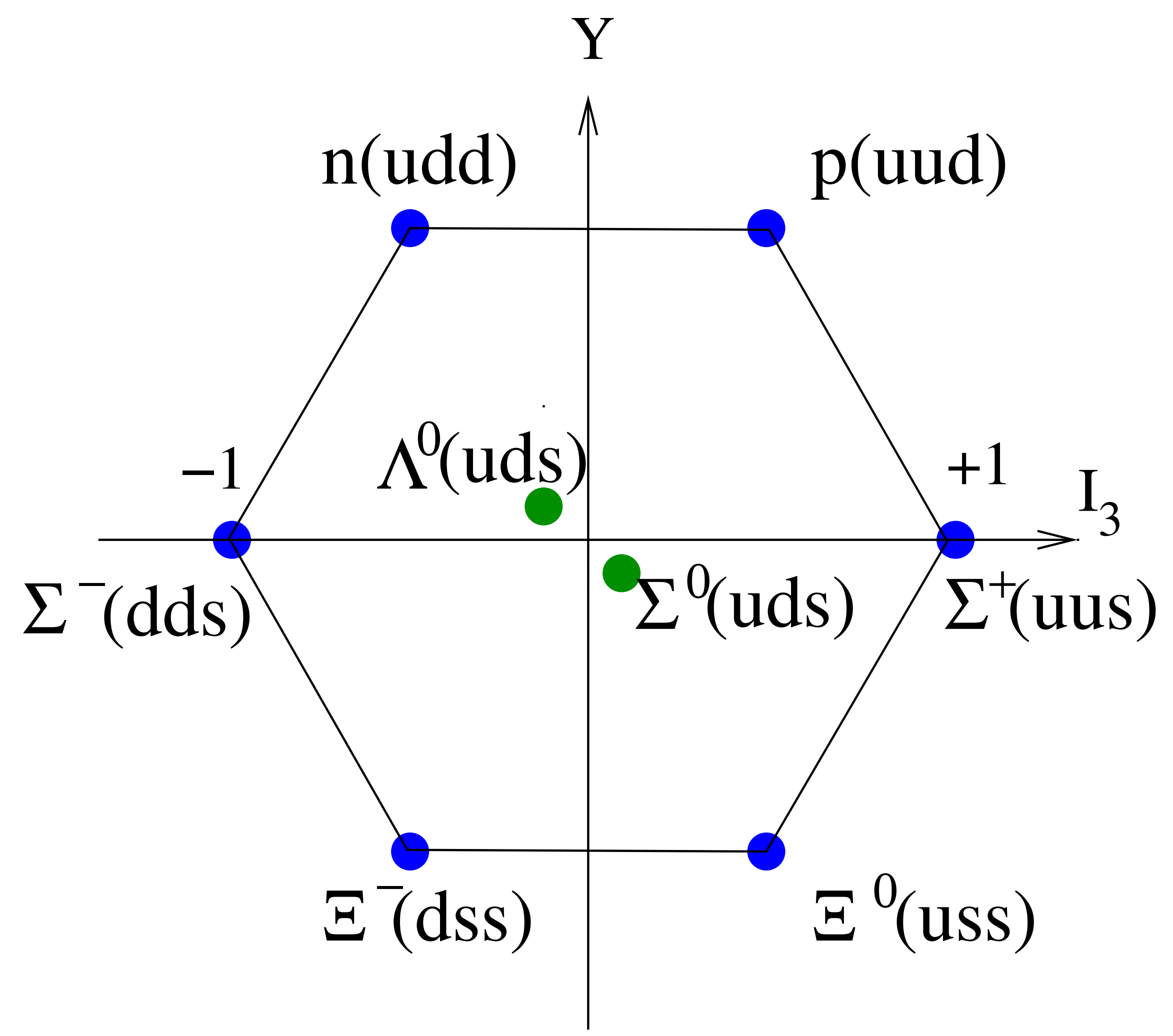
Bietenholz *et al*. [QCDSF-UKQCD], PRD(2011)

Flavour-breaking expansion

Bickerton, Horsley *et al.* [QCDSF], PRD(2019)

Consider general flavour matrix elements of octet baryons:

$$\langle B' | J^F | B \rangle = A_{B'FB}$$



In exact SU(3) limit, just 2 independent constants:

- *F*- and *D*-type couplings

At linear order in SU(3) breaking: 5 slope parameters (3 D's & 2 F's)

- # of parameters (polynomials/operators) reduced by restricting to $\bar{m}$ = constant line

$$F_1 \equiv \frac{1}{\sqrt{3}}(A_{\bar{N}\eta N} - A_{\bar{\Xi}\eta\Xi}) = 2f - \frac{2}{\sqrt{3}} s_2 \delta m_l,$$

$$F_2 \equiv (A_{\bar{N}\pi N} + A_{\bar{\Xi}\pi\Xi}) = 2f + 4 s_1 \delta m_l,$$

$$F_3 \equiv A_{\bar{\Sigma}\pi\Sigma} = 2f + (-2s_1 + \sqrt{3} s_2)\delta m_l,$$

$$F_4 \equiv \frac{1}{\sqrt{2}}(A_{\bar{\Sigma}K\Xi} - A_{\bar{N}K\Sigma}) = 2f - 2 s_1 \delta m_l,$$

$$F_5 \equiv \frac{1}{\sqrt{3}}(A_{\bar{\Lambda}K\Xi} - A_{\bar{N}K\Lambda}) = 2f + \frac{2}{\sqrt{3}}(\sqrt{3} s_1 - s_2)\delta m_l.$$

All matrix elements identical in the SU(3) symmetric limit

Index	Baryon (B)	Meson (F)	Current (J^F)
1	n	K^0	$\bar{d}\gamma s$
2	p	K^+	$\bar{u}\gamma s$
3	Σ^-	π^-	$\bar{d}\gamma u$
4	Σ^0	π^0	$\frac{1}{\sqrt{2}}(\bar{u}\gamma u - \bar{d}\gamma d)$
5	Λ^0	η	$\frac{1}{\sqrt{6}}(\bar{u}\gamma u + \bar{d}\gamma d - 2\bar{s}\gamma s)$
6	Σ^+	π^+	$\bar{u}\gamma d$
7	Ξ^-	K^-	$\bar{s}\gamma u$
8	Ξ^0	$\bar{K}^0$	$\bar{s}\gamma d$
0		η'	$\frac{1}{\sqrt{6}}(\bar{u}\gamma u + \bar{d}\gamma d + \bar{s}\gamma s)$

Fan plots

a=0.068fm

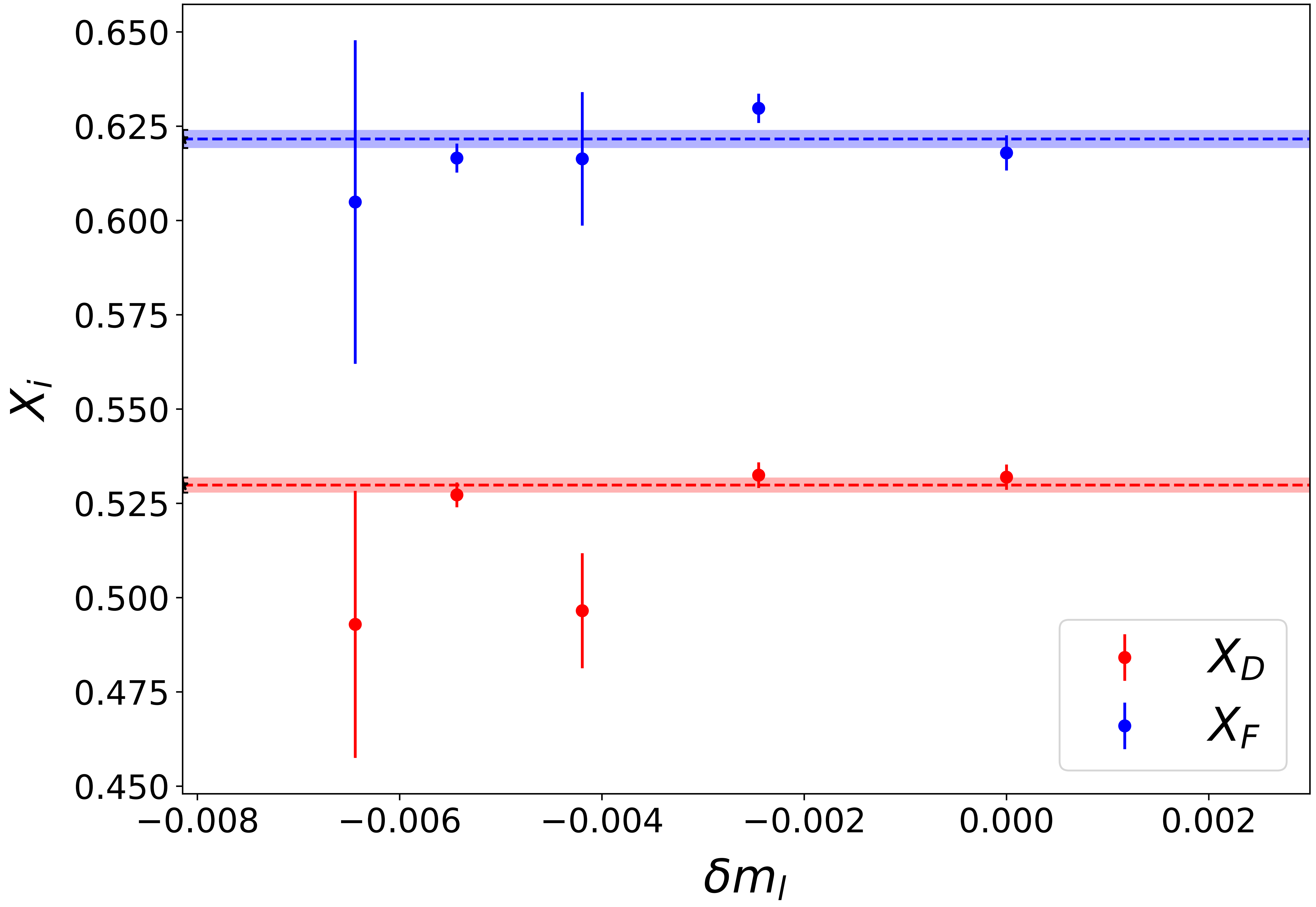
Can form a "singlet" combination

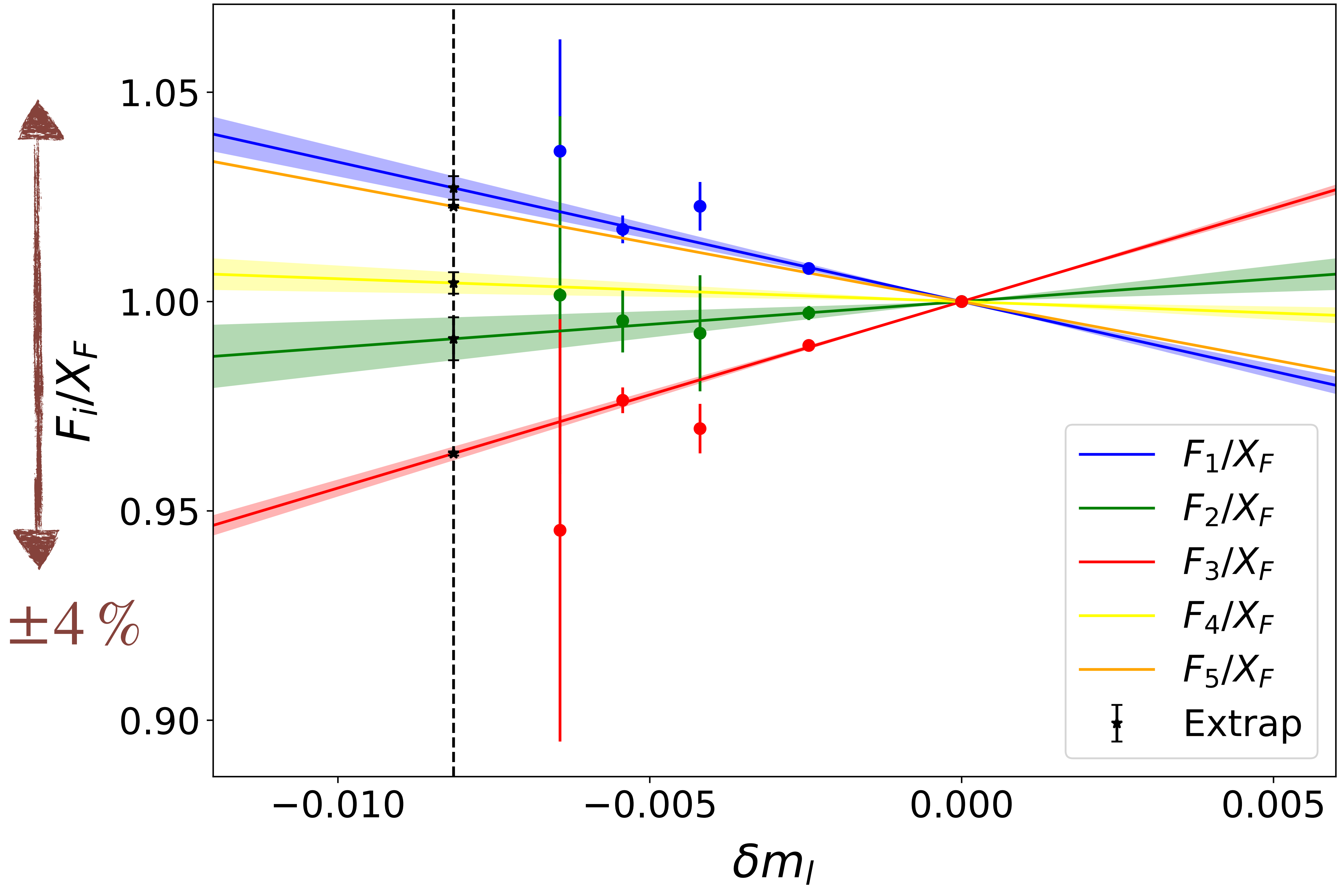
$$X_F = \frac{1}{6}(3F_1 + F_2 + 2F_3) = 2f + \mathcal{O}(\delta m_\ell^2)$$

General result: Singlet quantities only vary at 2nd-order in SU(3) breaking.

$$\begin{aligned}
F_1 \equiv \frac{1}{\sqrt{3}}(A_{\bar{N}\eta N} - A_{\bar{\Xi}\eta\Xi}) &= 2f - \frac{2}{\sqrt{3}}s_2\delta m_l, \\
F_2 \equiv (A_{\bar{N}\pi N} + A_{\bar{\Xi}\pi\Xi}) &= 2f + 4s_1\delta m_l, \\
F_3 \equiv A_{\bar{\Sigma}\pi\Sigma} &= 2f + (-2s_1 + \sqrt{3}s_2)\delta m_l, \\
F_4 \equiv \frac{1}{\sqrt{2}}(A_{\bar{\Sigma}K\Xi} - A_{\bar{N}K\Sigma}) &= 2f - 2s_1\delta m_l, \\
F_5 \equiv \frac{1}{\sqrt{3}}(A_{\bar{\Lambda}K\Xi} - A_{\bar{N}K\Lambda}) &= 2f + \frac{2}{\sqrt{3}}(\sqrt{3}s_1 - s_2)\delta m_l.
\end{aligned}$$

F fan

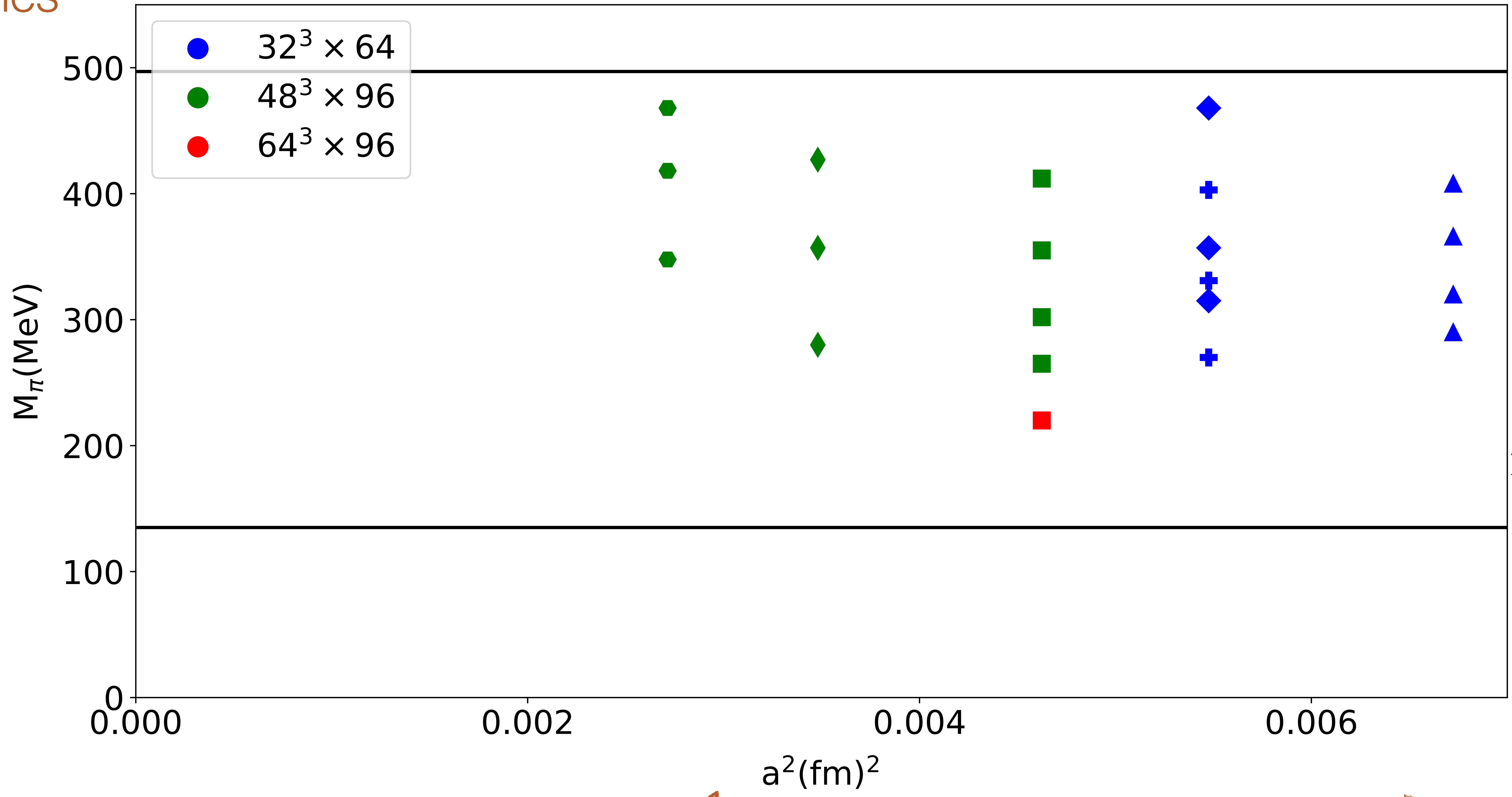




Simulation details

2+1 flavour, NP-improved Wilson fermions

3 volumes



pion masses

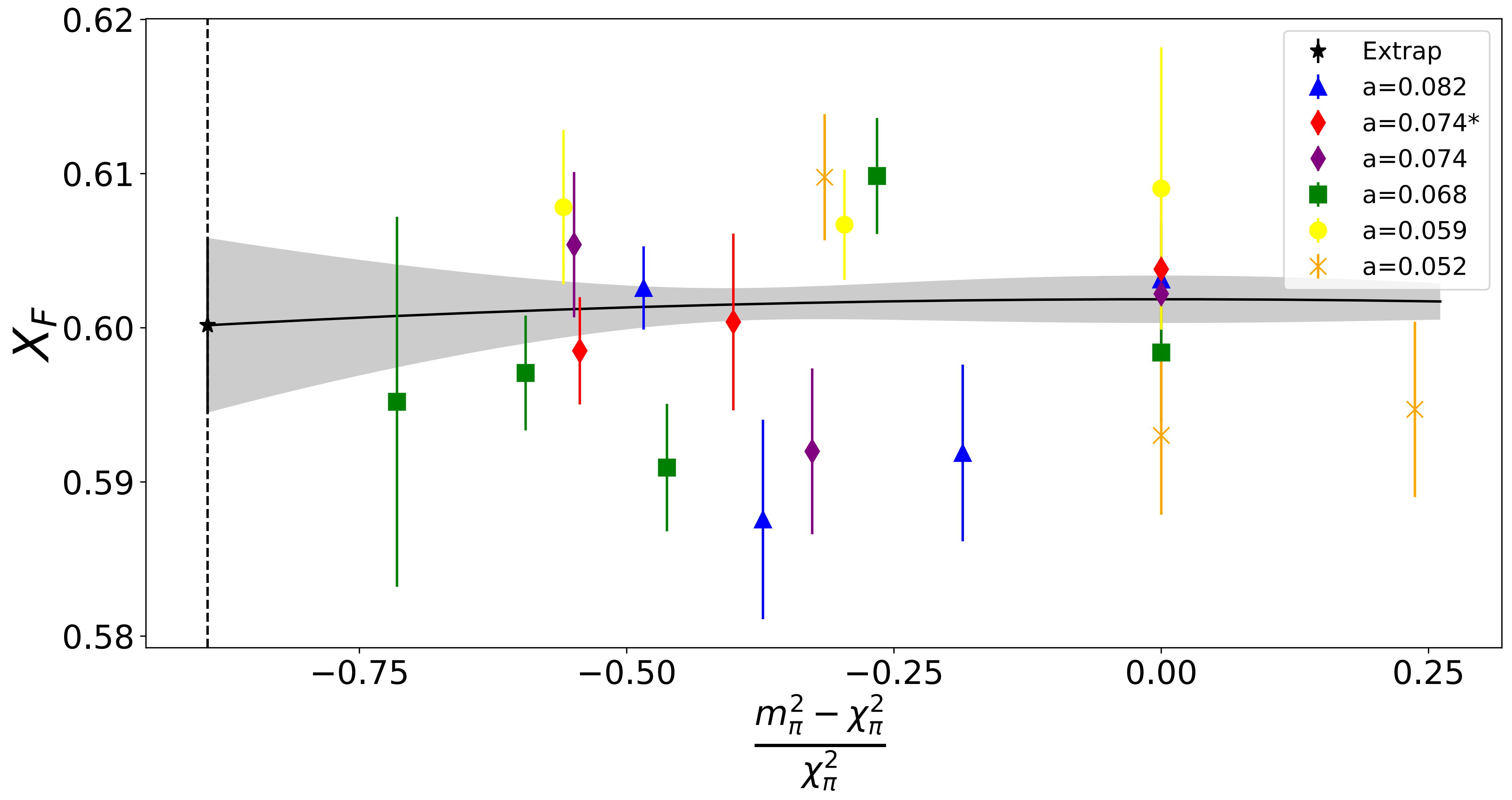
$220 \leftrightarrow 500\,\mathrm{MeV}$

5 lattice spacings $0.052 \leftrightarrow 0.082\,\mathrm{fm}$

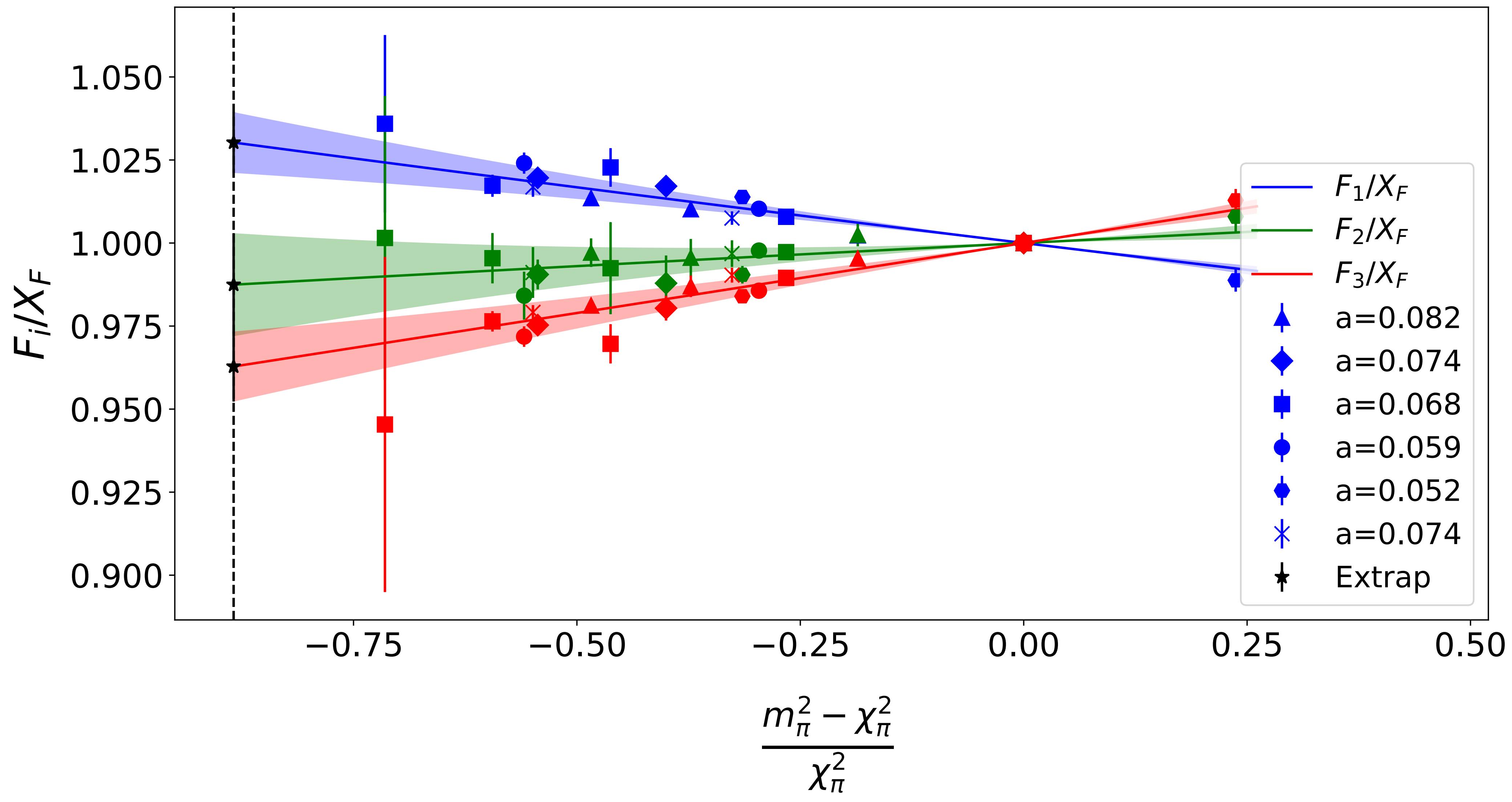
Global fits

Singlet X_F

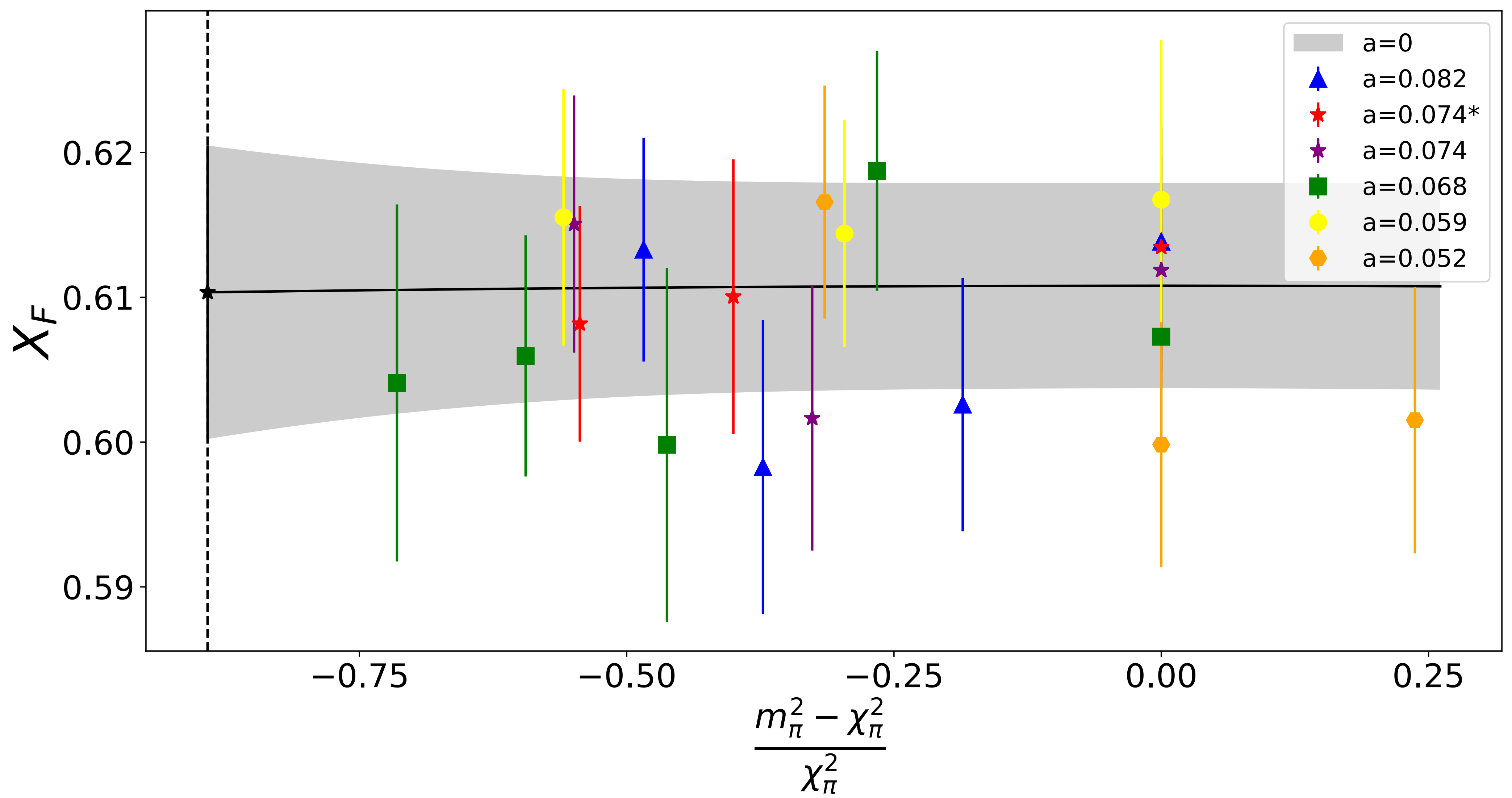
Quark mass only

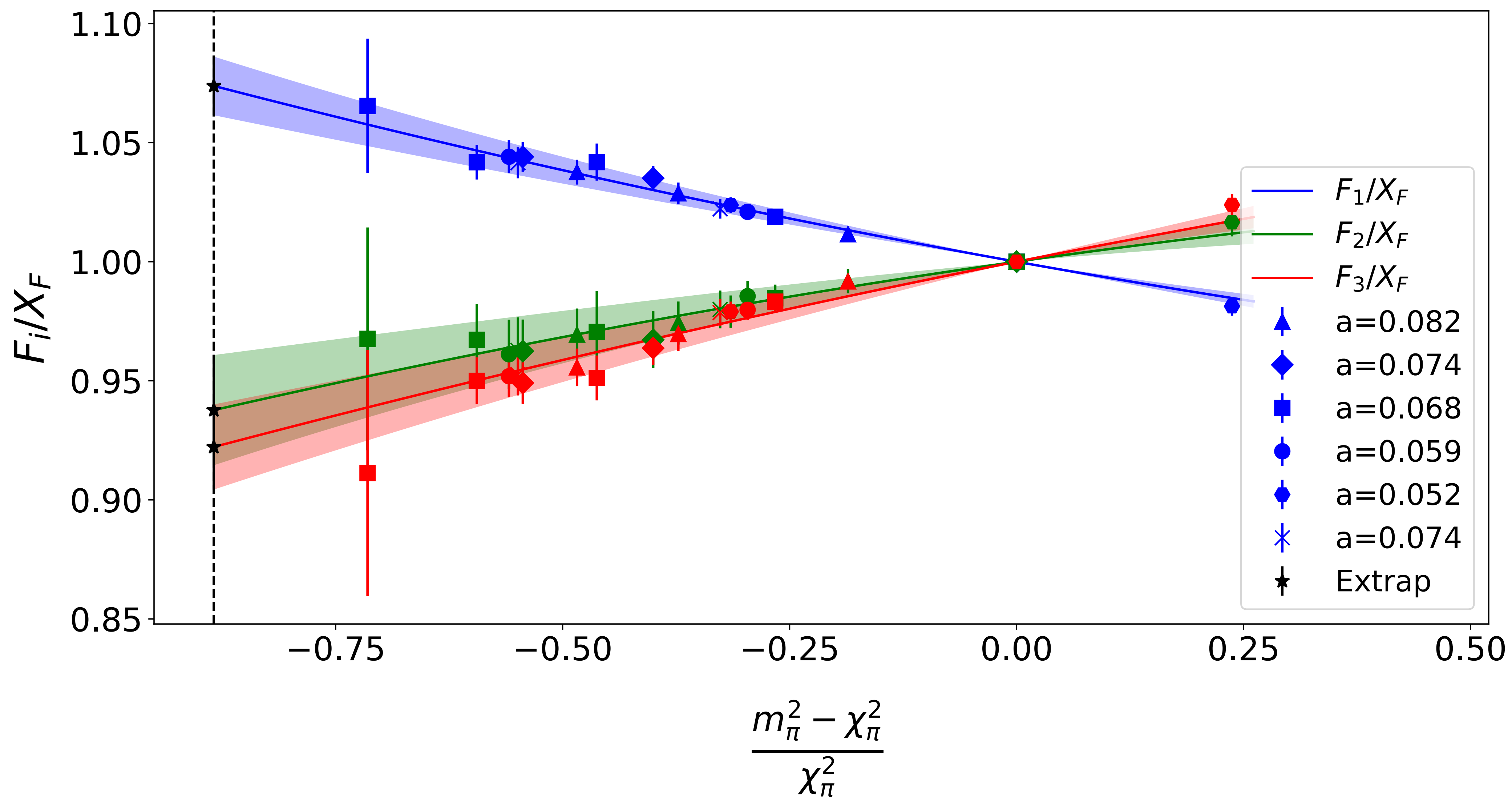


F slope parameters



With O(a) and FV

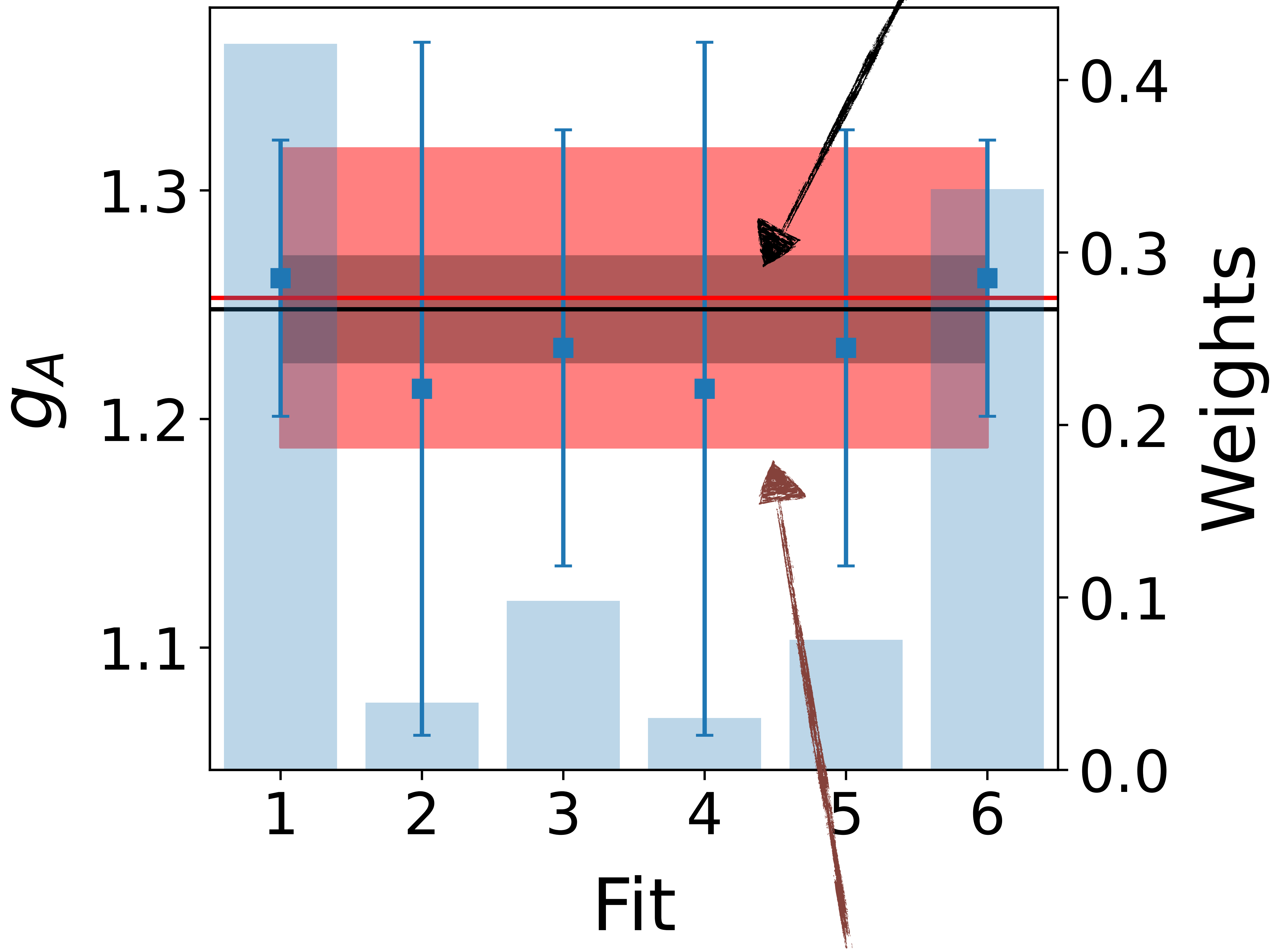




Results - g_A (isovector)

weighted average among models (as above)

FLAG result, ~2.2%

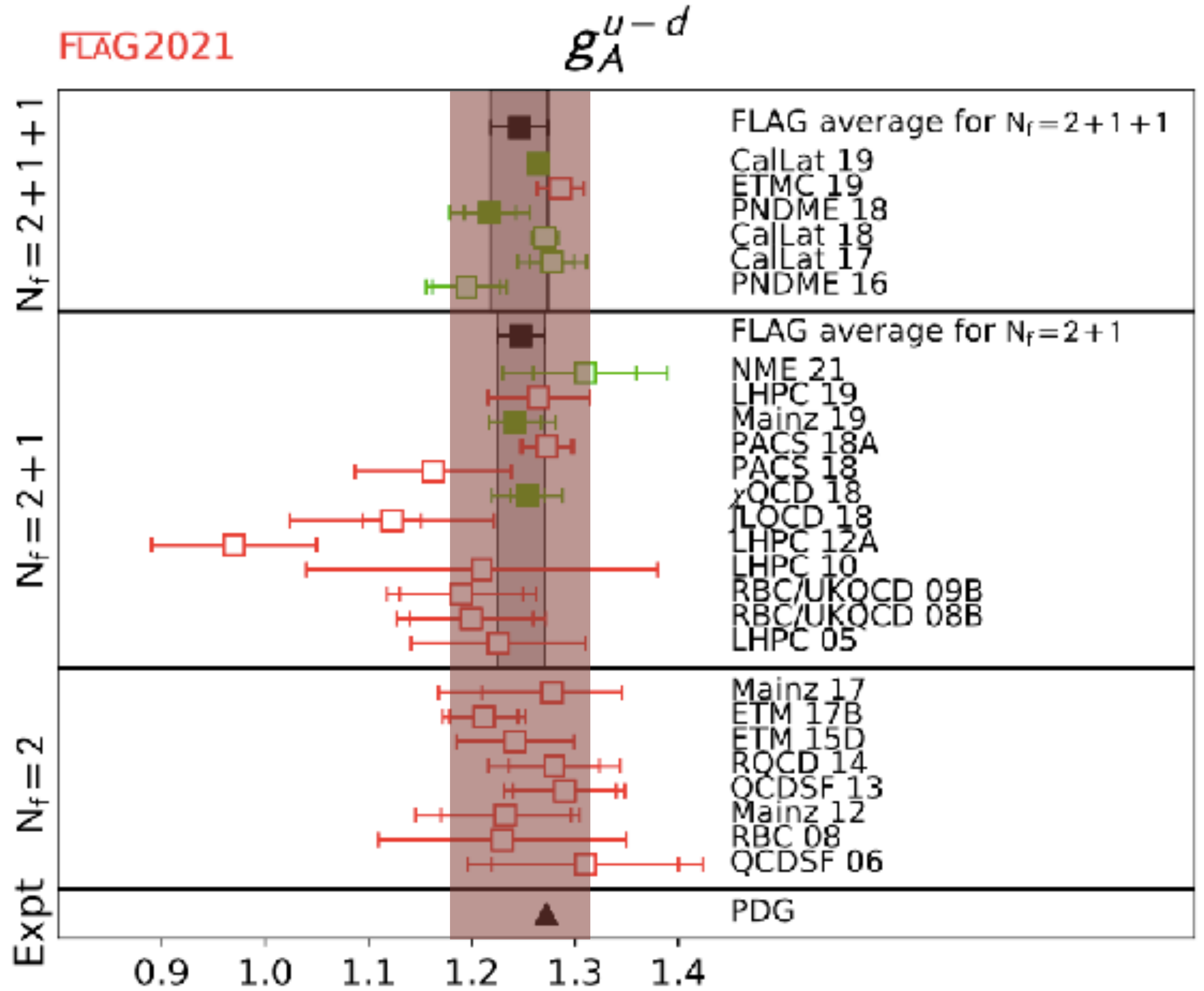


Our result, (stat+sys)~5.5%

$$g_A^{u-d} = 1.253(63)_{\text{stat}}(41)_a(03)_{\text{FV}}$$

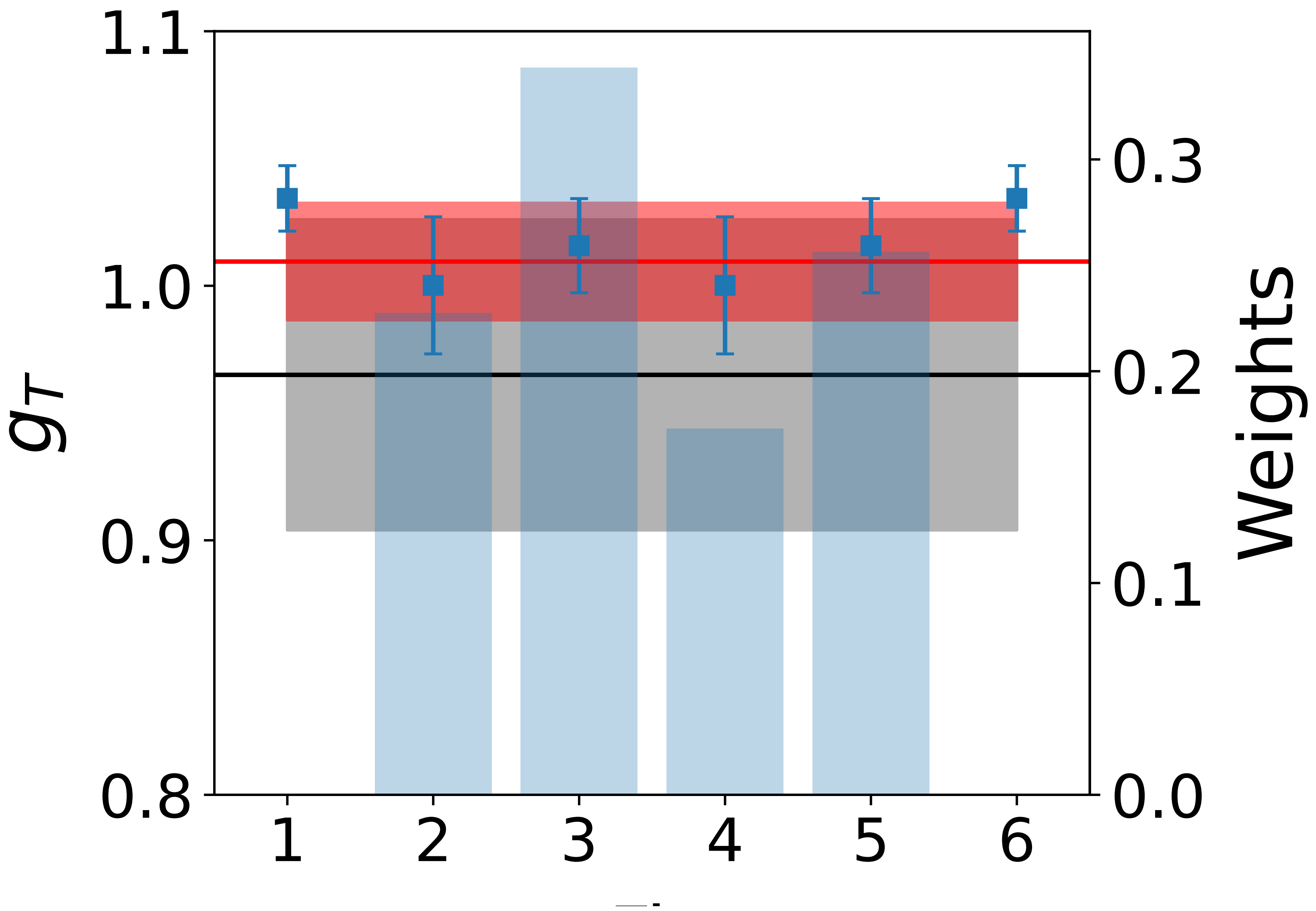
Different model parameterisations

1. δm_l^2
2. $a,\ \delta m_l^2$
3. $a^2,\ \delta m_l^2$
4. $a,\ \delta m_l^2,\ m_\pi L$
5. $a^2,\ \delta m_l^2,\ m_\pi L$
6. $\delta m_l^2,\ m_\pi L$



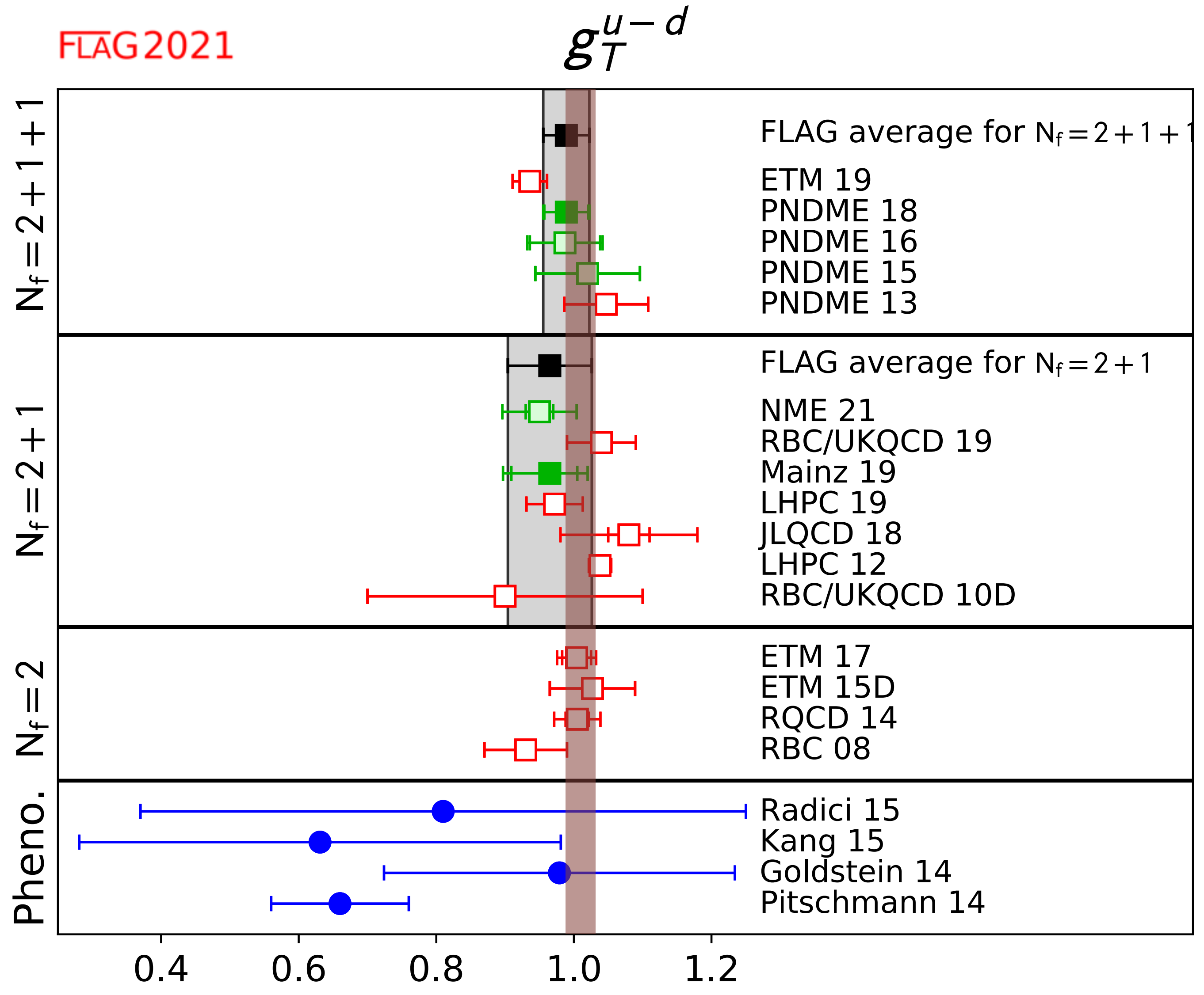
Results - isovector charges $N_f = 2 + 1$

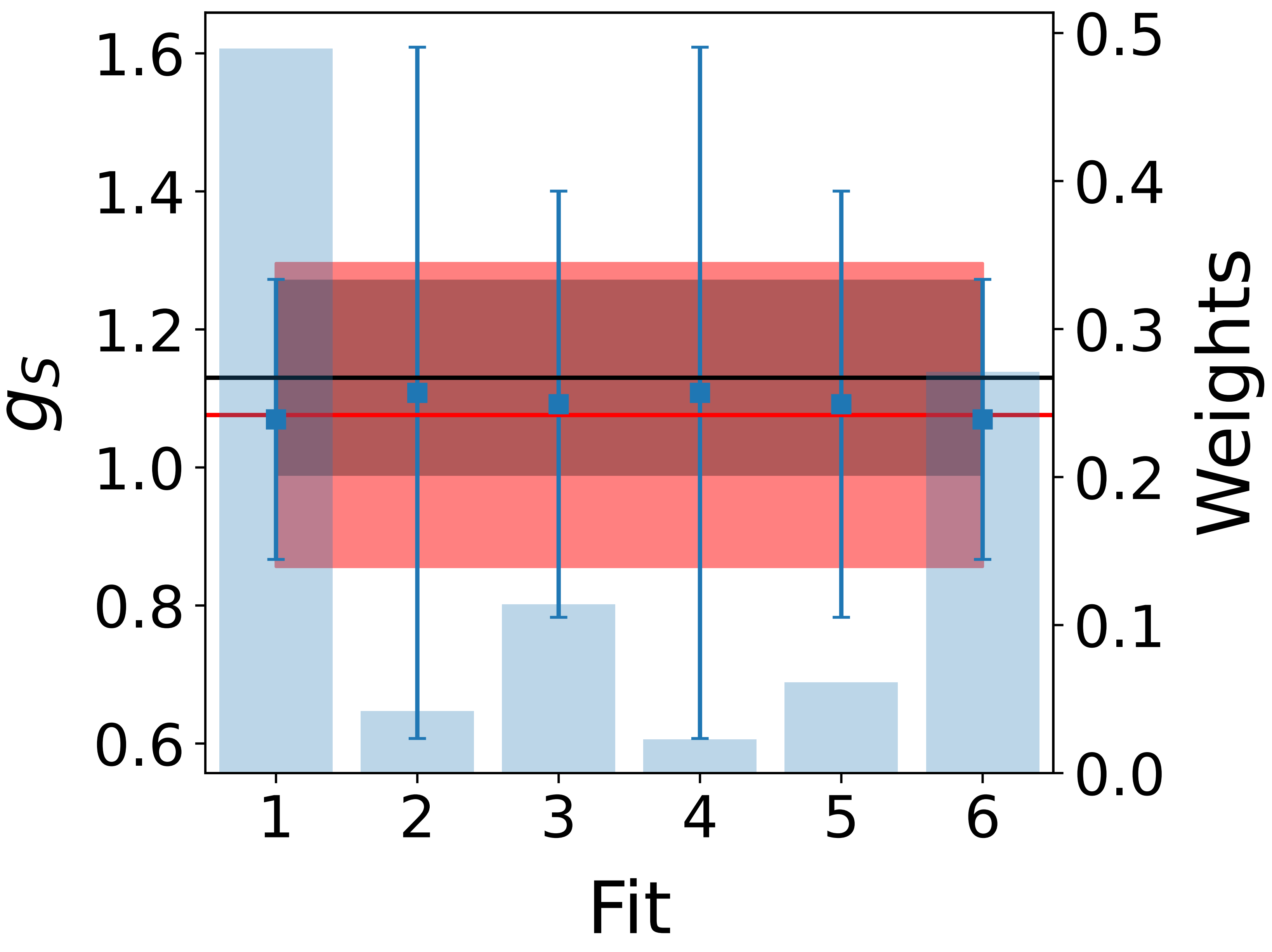
$\overline{\text{MS}}, \mu = 2\,\text{GeV}$



$$g_T^{u-d} = 1.010(21)_{\text{stat}}(12)_a(01)_{\text{FV}}$$

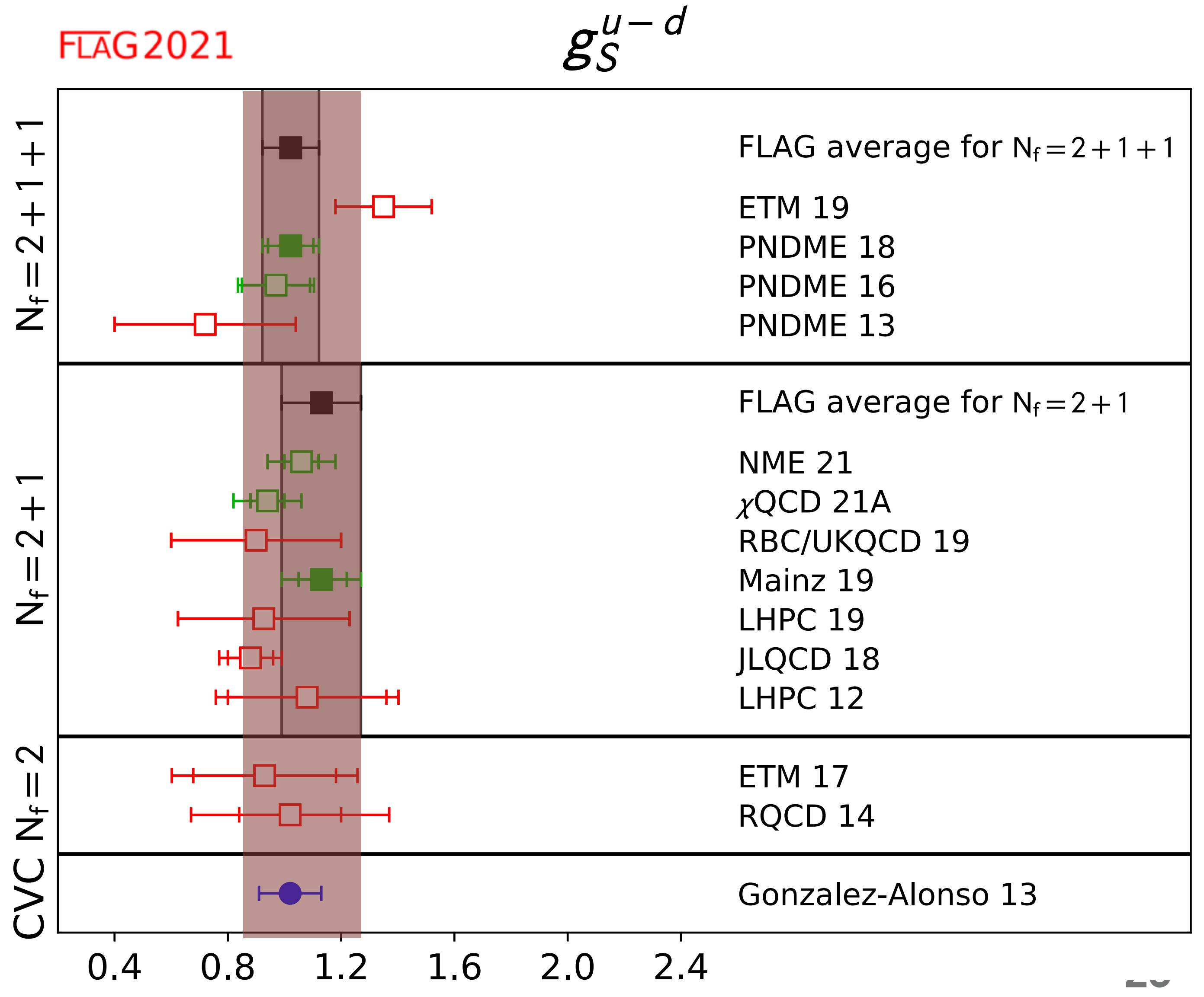
FLAG 2+1: ~6%
FLAG 2+1+1: ~3%
Our result: ~2%





$$g_S^{u-d} = 1.08(21)_{\text{stat}}(03)_a(01)_{\text{FV}}$$

FLAG 2+1: ~12%
Our result: ~19%



Quark and gluon momentum fractions, $\langle x \rangle_q$, $\langle x \rangle_g$

[PLB714 (2012) + in preparation]

Motivation

- Long-standing question re: nucleon momentum:

  How is the nucleon's momentum distributed amongst its constituents?

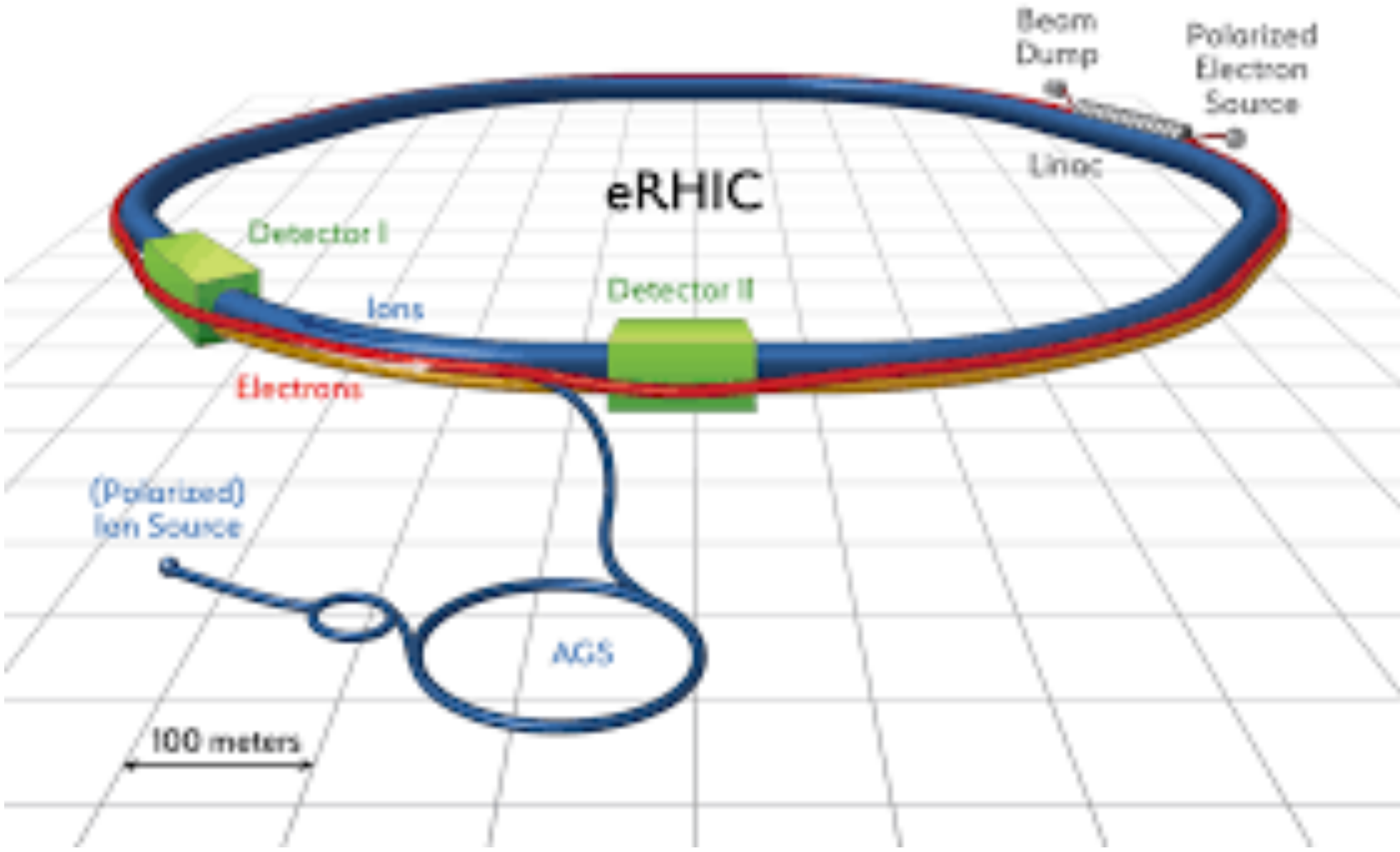
- Addressed experimentally @ JLab (now), EIC (future)
- Must satisfy the momentum rule

$$\sum_q \langle x \rangle_q + \langle x \rangle_g = 1$$

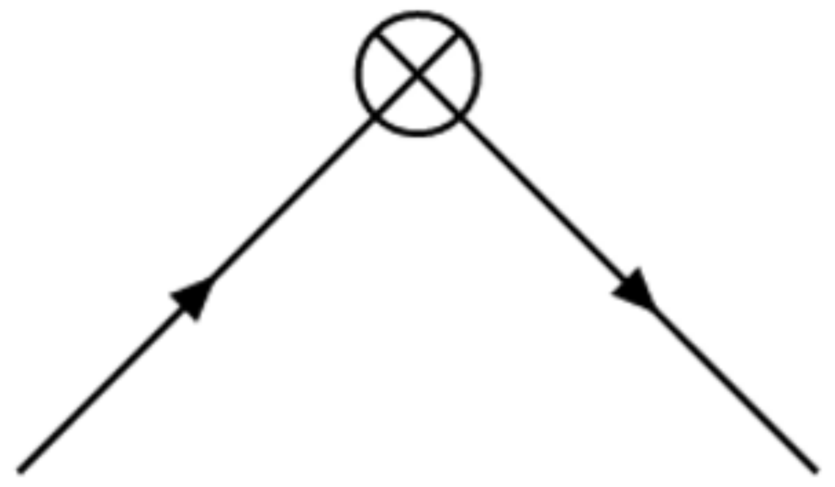
where

$\langle x \rangle_f$ = fraction of nucleon momentum carried by parton $f=q,g$

- Experimentally : $\langle x \rangle_g \sim \frac{1}{2}$
- Received much interest from Lattice QCD, but with challenges,
  - e.g. statistical noise in $\langle x \rangle_g$ due disconnected nature

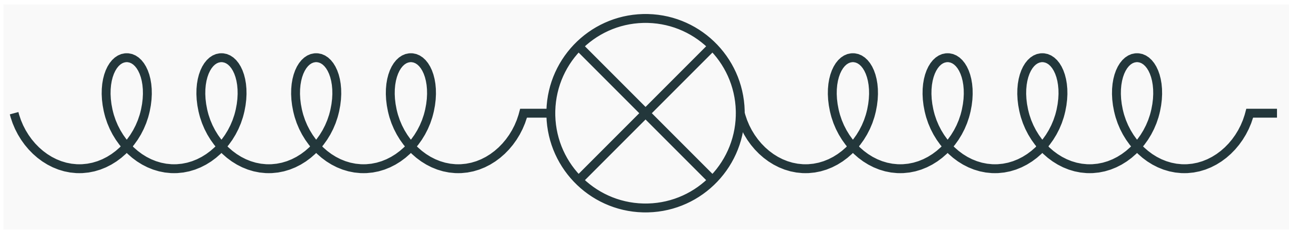


Motivation



Renormalisation: Mixing between $\langle x \rangle_q$ and $\langle x \rangle_g$

$$\text{i.e.} \quad \sum_q \langle x \rangle_q^R + \langle x \rangle_g^R \; = \; 1 \; = \; Z_q \sum_q \langle x \rangle_q^{lat} + Z_g \langle x \rangle_g^{lat}$$

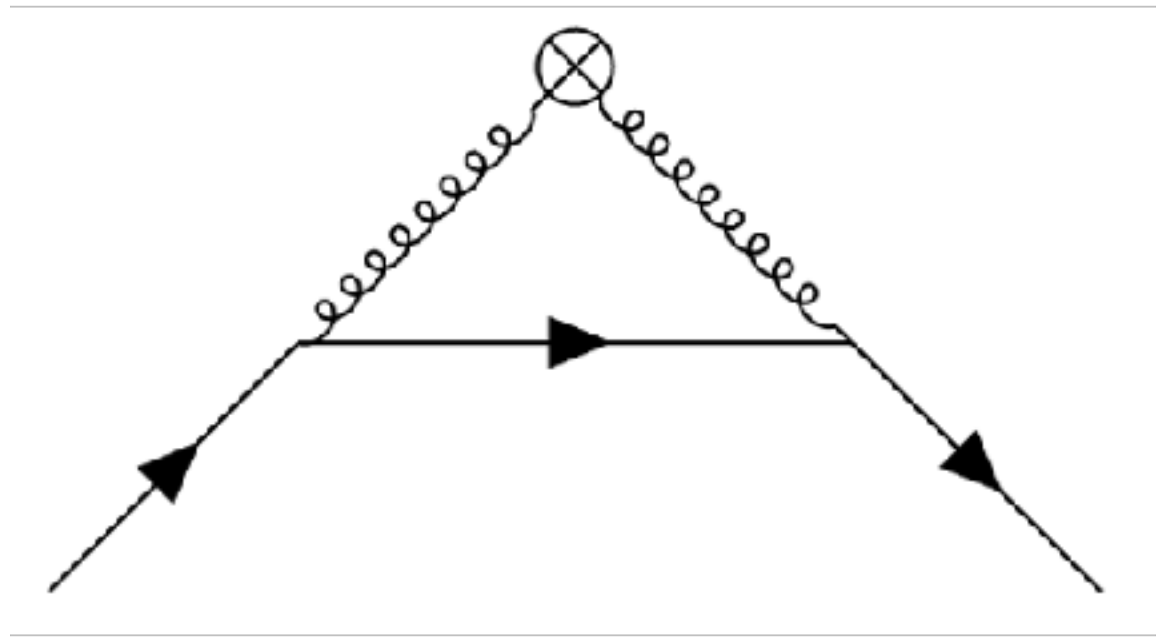


does not necessarily mean

$$\langle x \rangle_q^R = Z_q \langle x \rangle_q^{lat} \quad \text{or} \quad \langle x \rangle_g^R = Z_g \langle x \rangle_g^{lat}$$

e.g.

$$\begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_q \end{pmatrix}^R = \begin{pmatrix} Z_{gg} & Z_{gq} \\ Z_{qg} & Z_{qq} \end{pmatrix} \begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_q \end{pmatrix}^{lat}$$



$$\Rightarrow \quad Z_g = Z_{gg} + Z_{qg} \qquad Z_q = Z_{gq} + Z_{qq}$$

Recent progress in NP determination of Z_{gg}

Mixing due to Z_{qg}, Z_{qg} often ignored or computed perturbatively

Determining $\langle x \rangle_{q,g}$

Require matrix elements

$$\langle N(\vec{p})\,|\,\mathcal{O}_f^{(b)}\,|\,N(\vec{p})\rangle = 2(m_N^2 + \frac{4}{3}\vec{p}^2)\langle x\rangle_f$$

$$\mathcal{O}^{(b)} = \mathcal{O}_{44} - \frac{1}{3}\mathcal{O}_{ii}$$

which can be computed at $\vec{p} = 0$ (for $\mathcal{O}^{(b)}$)

$$\mathcal{O}_{\mu\nu}^{(g)} = -\,\mathrm{Tr}_c F_{\mu\alpha}F_{\nu\alpha}\ , \quad \mathcal{O}_g^{(b)} = \frac{2}{3}\mathrm{Tr}_c(-\mathcal{E}^2 + \mathcal{B}^2)$$

$$\mathcal{O}_{\mu\nu}^{(q)} = \bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q\ , \quad \mathcal{O}_q^{(b)} = \bar{q}\gamma_4 \overleftrightarrow{D}_4 q - \frac{1}{3}\bar{q}\gamma_i \overleftrightarrow{D}_i q$$

Typically obtained via 3-point functions

$$\mathcal{O}(\tau) = \int d^3x\,\mathcal{O}(\tau,\vec{x})$$

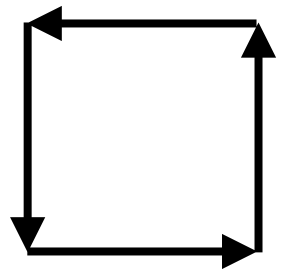
This work: Feynman-Hellmann theorem *[following QCDSF(2012)]*

Compute 2-point functions in the presence of a modification to the action $S \to S(\lambda) = S + \lambda \sum_z \mathcal{O}(z)$

Matrix elements determined from energy shifts $\left.\frac{\partial E_\lambda}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E}\left\langle N \left| : \frac{\partial S_\lambda}{\partial \lambda} : \right| N \right\rangle\Big|_{\lambda=0}$

The modified action

Wilson gluonic action: $S_g = \frac{1}{3}\beta \sum_{x\,\mu<\nu} \mathrm{Re}\,\mathrm{Tr}_c\left[1 - U^{\square}_{\mu\nu}\right] = \sum_{\tau} \frac{1}{2}\mathrm{Tr}_c\left[\mathscr{E}^2(\tau) + \mathscr{B}^2(\tau)\right]$

$U^{\square}_{\mu\nu} =$ 

Modify with gluon operator $\mathcal{O}_g^{(b)}$:

$$S_g(\lambda_g) = \frac{1}{3}\beta(1+\lambda_g) \sum_{x,i} \mathrm{Re}\,\mathrm{Tr}_c(1 - U^{\square}_{i4}(x)) + \frac{1}{3}\beta(1-\lambda_g) \sum_{x,i<j} \mathrm{Re}\,\mathrm{Tr}_c(1 - U^{\square}_{ij}(x))$$

→ *anisotropic action*

Similary modify Wilson/Clover action with $\mathcal{O}_q^{(b)}$:

$$S_q^W(\lambda) = \sum_x \bar{q}(x)q(x) - \kappa\Big[\sum_x \bar{q}(x)\big(1-(1+\lambda_q)\gamma_4\big)\,U_4(x)\,q(x+\hat{4}) + \sum_x \bar{q}(x+\hat{4})\big(1+(1+\lambda_q)\gamma_4\big)\,U_4^{\dagger}(x)\,q(x) +$$
$$\sum_{x,i} \bar{q}(x)\big(1-(1-\tfrac{1}{3}\lambda_q)\gamma_i\big)\,U_i(x)\,q(x+\hat{\imath}) + \sum_{x,i} \bar{q}(x+\hat{\imath})\big(1+(1-\tfrac{1}{3}\lambda_q)\gamma_i\big)\,U_i^{\dagger}(x)\,q(x)\Big]$$

→ *modified hopping term*

Simulation details

Quenched QCD → *no disconnected quarks and* $Z_{qg} = 0$

Volume: $24^3 \times 48$

Wilson glue, $\beta = 6.0 \implies a = 0.1\text{fm}$

5 values of λ_g

N_s	N_t	β	λ_g	β_{input}	ξ_{input}	N_{cfg}
24	48	6.0	-0.0666	5.9867	0.9354	1000
24	48	6.0	-0.0333	5.9967	0.9672	1000
24	48	6.0	0	6.0	1	1000
24	48	6.0	$+0.0333$	5.9967	1.0340	1000
24	48	6.0	$+0.0666$	5.9867	1.0689	1000

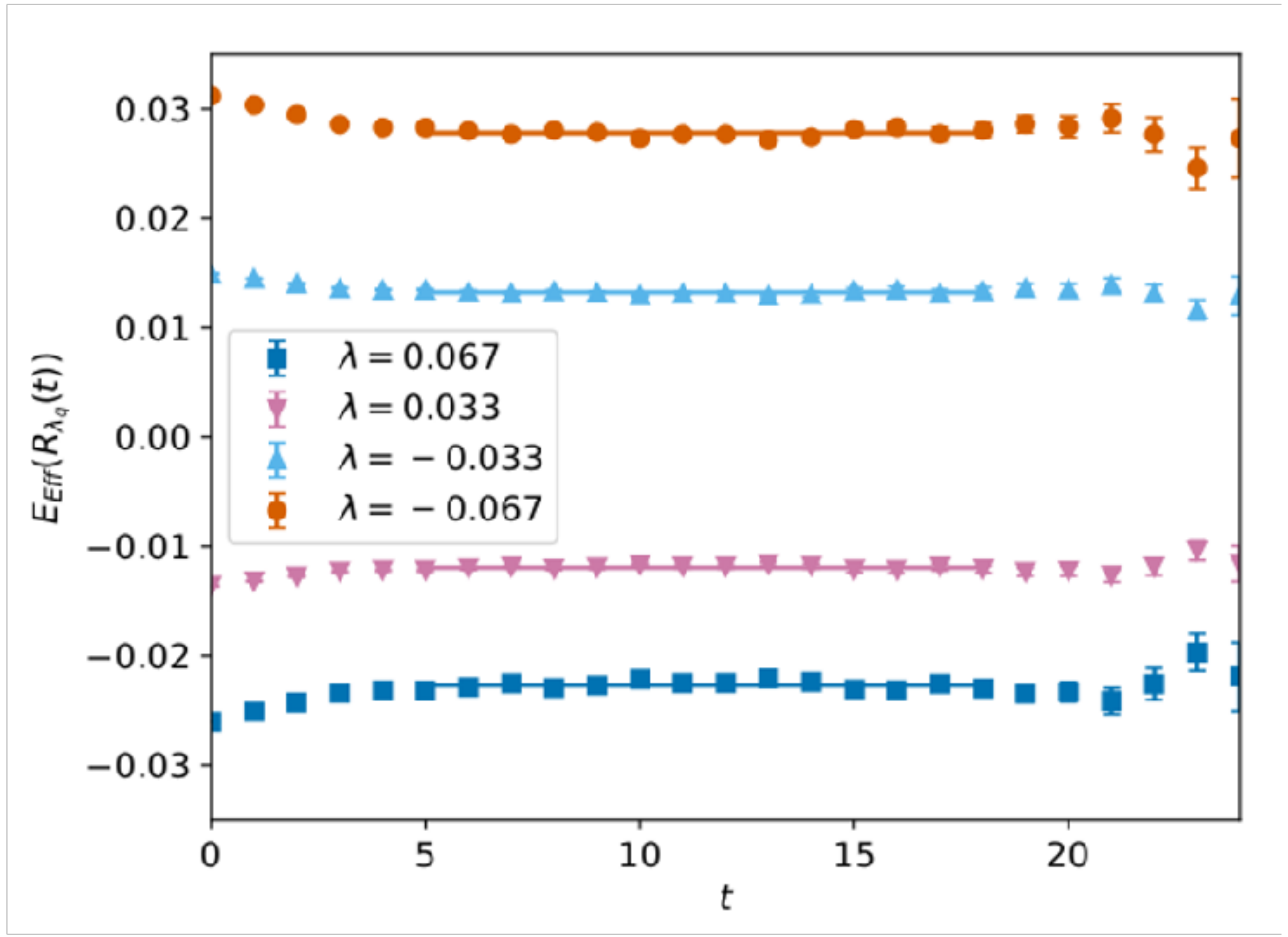
NP-clover action for valence quarks

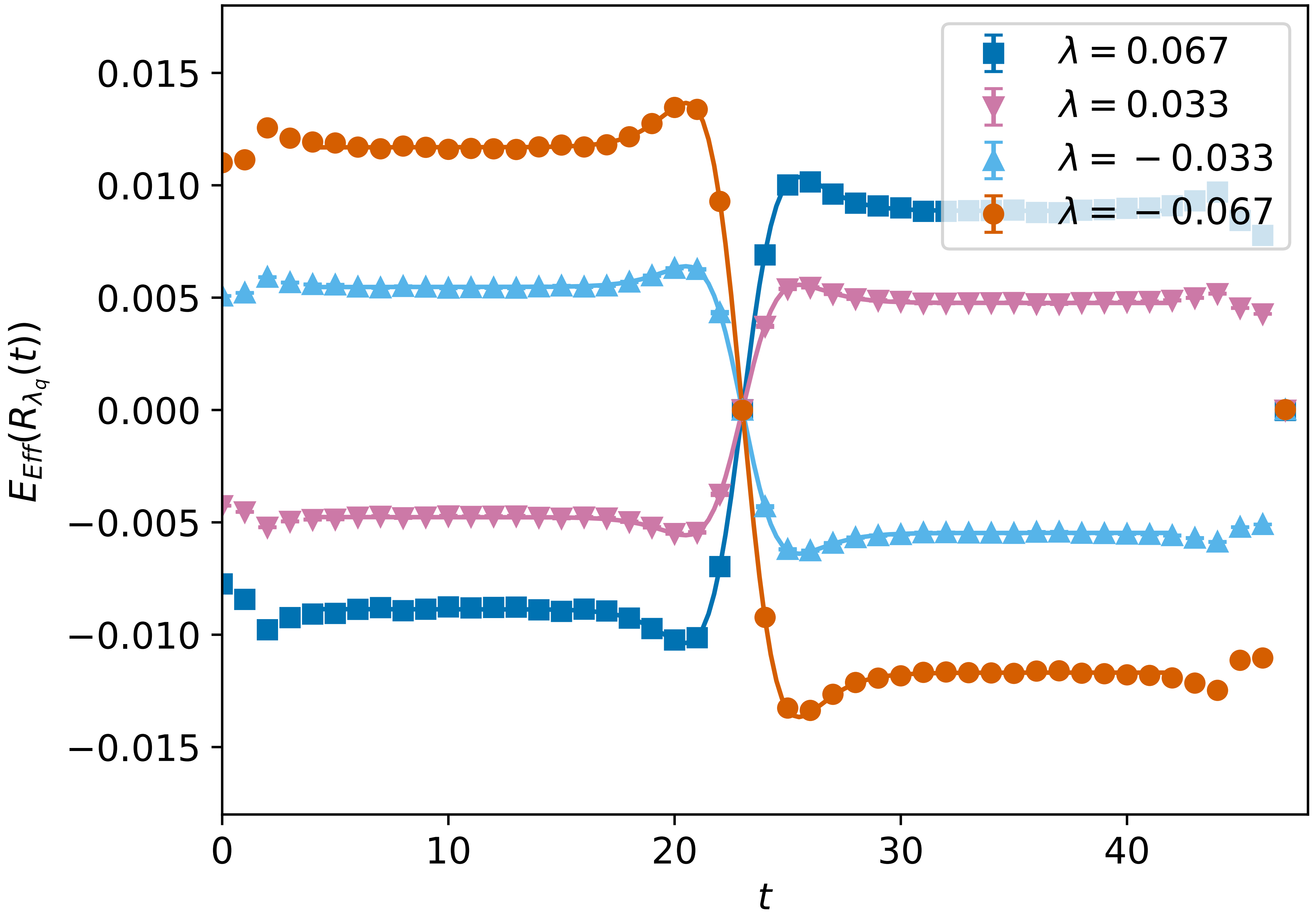
$\kappa = 0.1320, 0.1333, 0.1342$ → $m_\pi \approx 1080, 820, 600\,\text{MeV}$

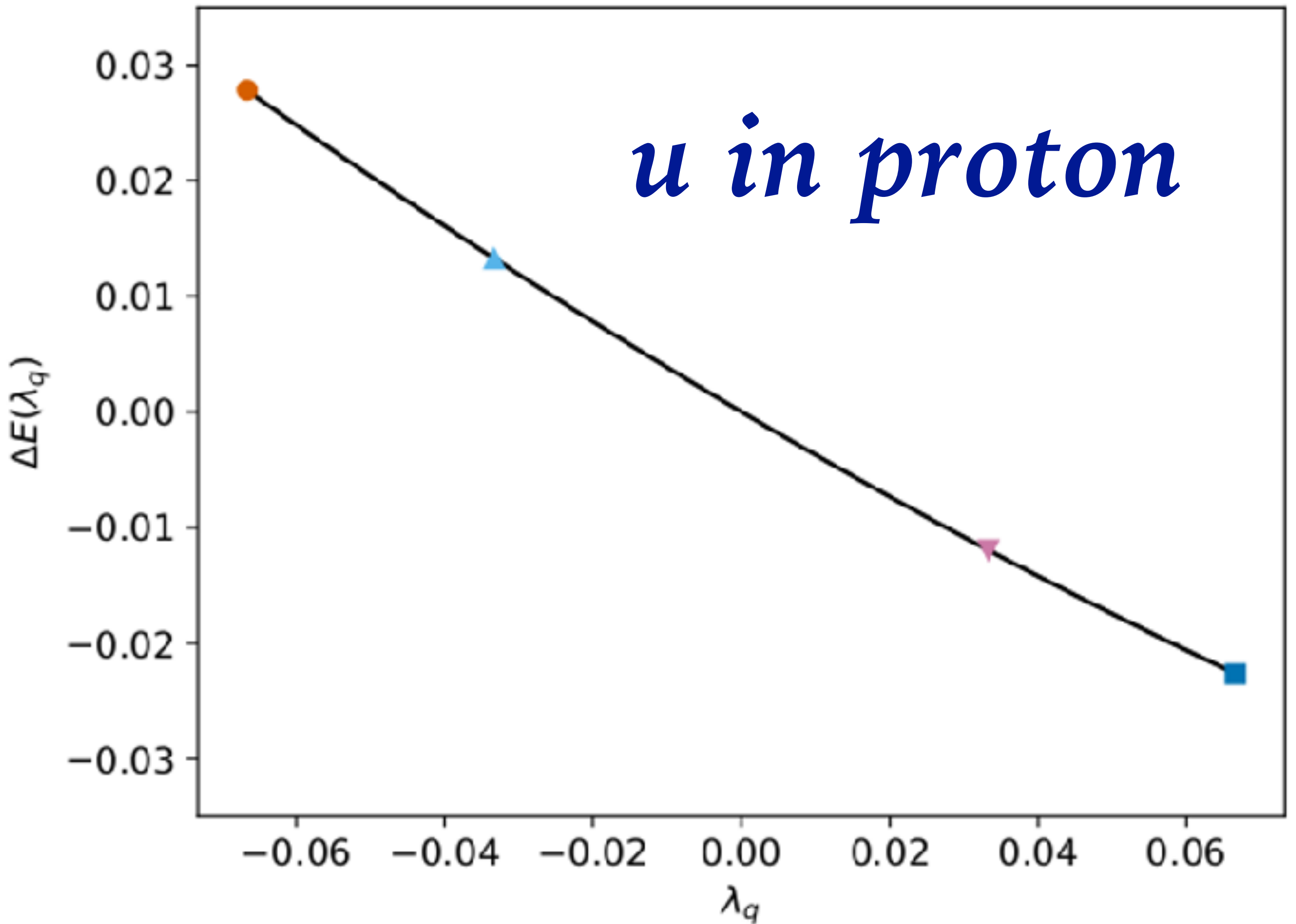
5 values of $\lambda_q = -0.0666, \ -0.0333, 0, \ +0.0333, \ +0.0666$

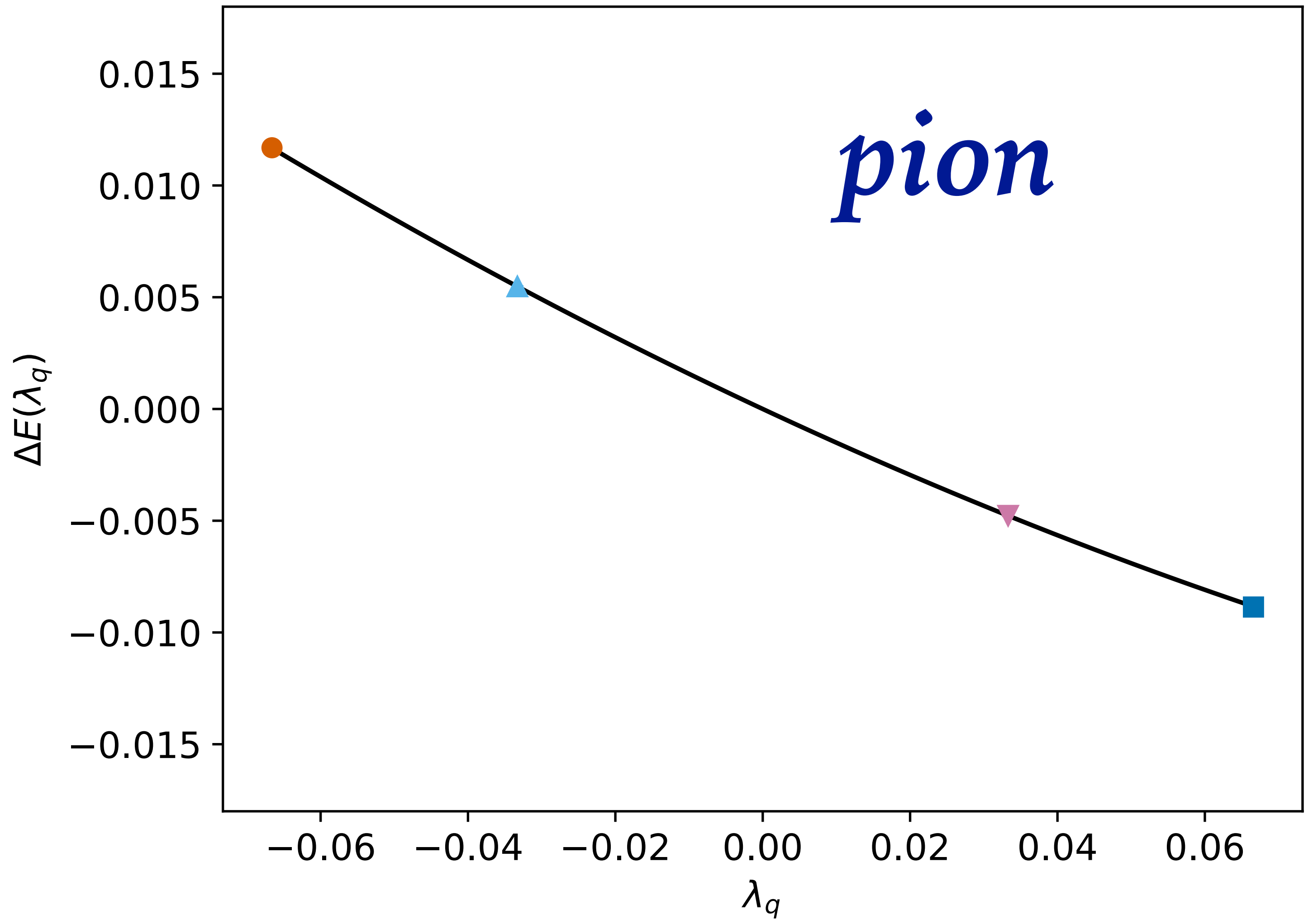
Energy shifts: Quark operator

$m_\pi \approx 1065\,\text{MeV}$



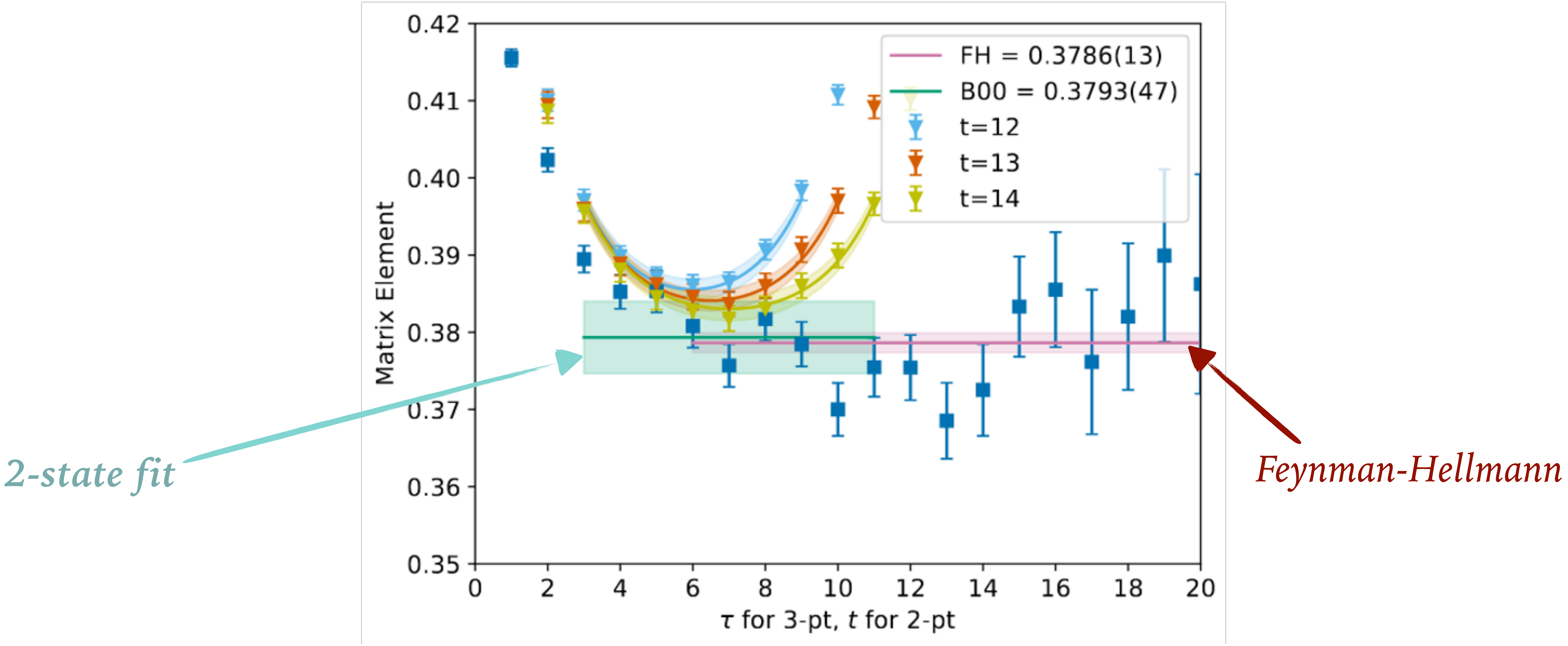






Quark operator - comparison to 3-point functions

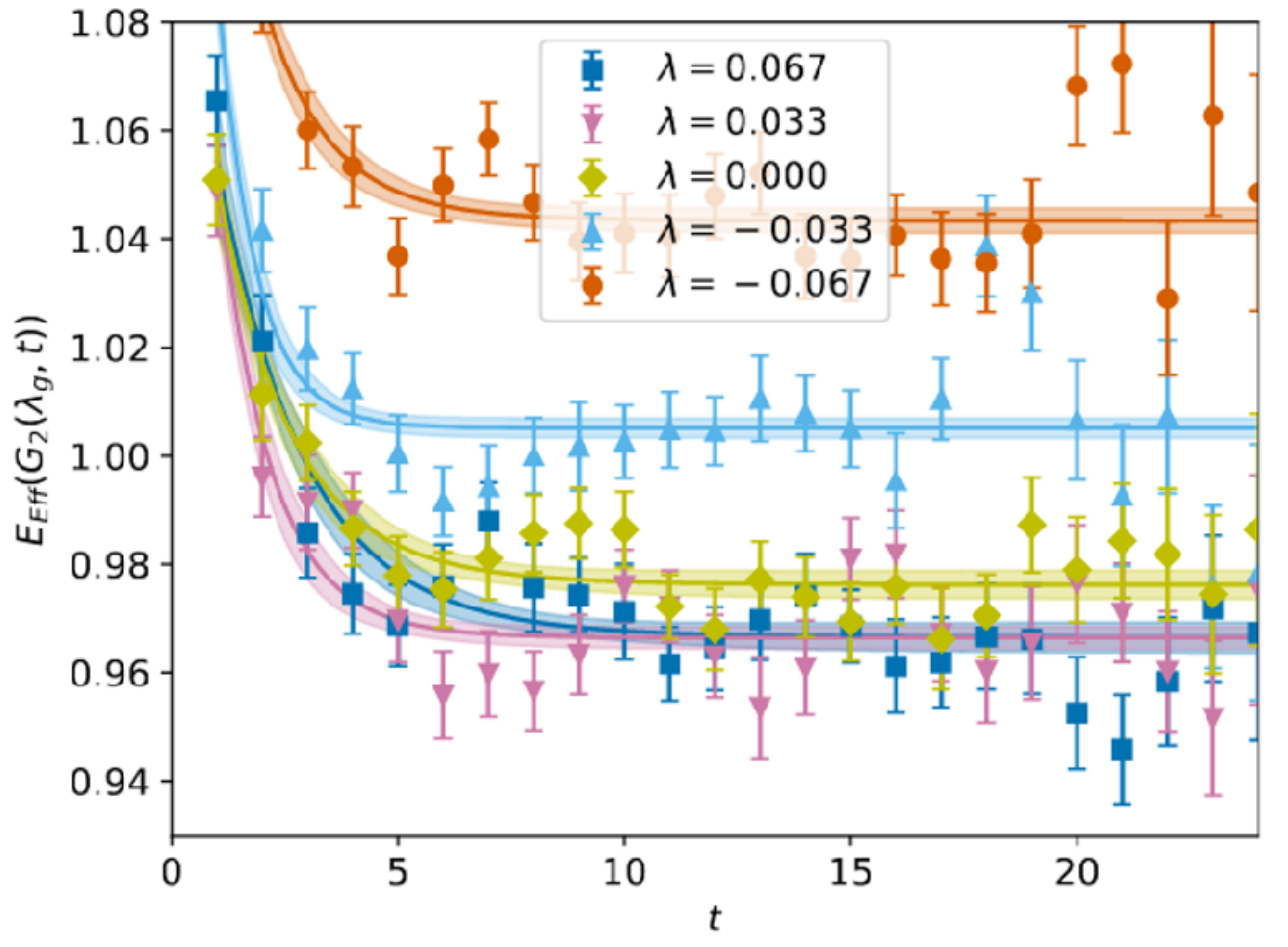
$m_\pi \approx 1065\,\text{MeV}$

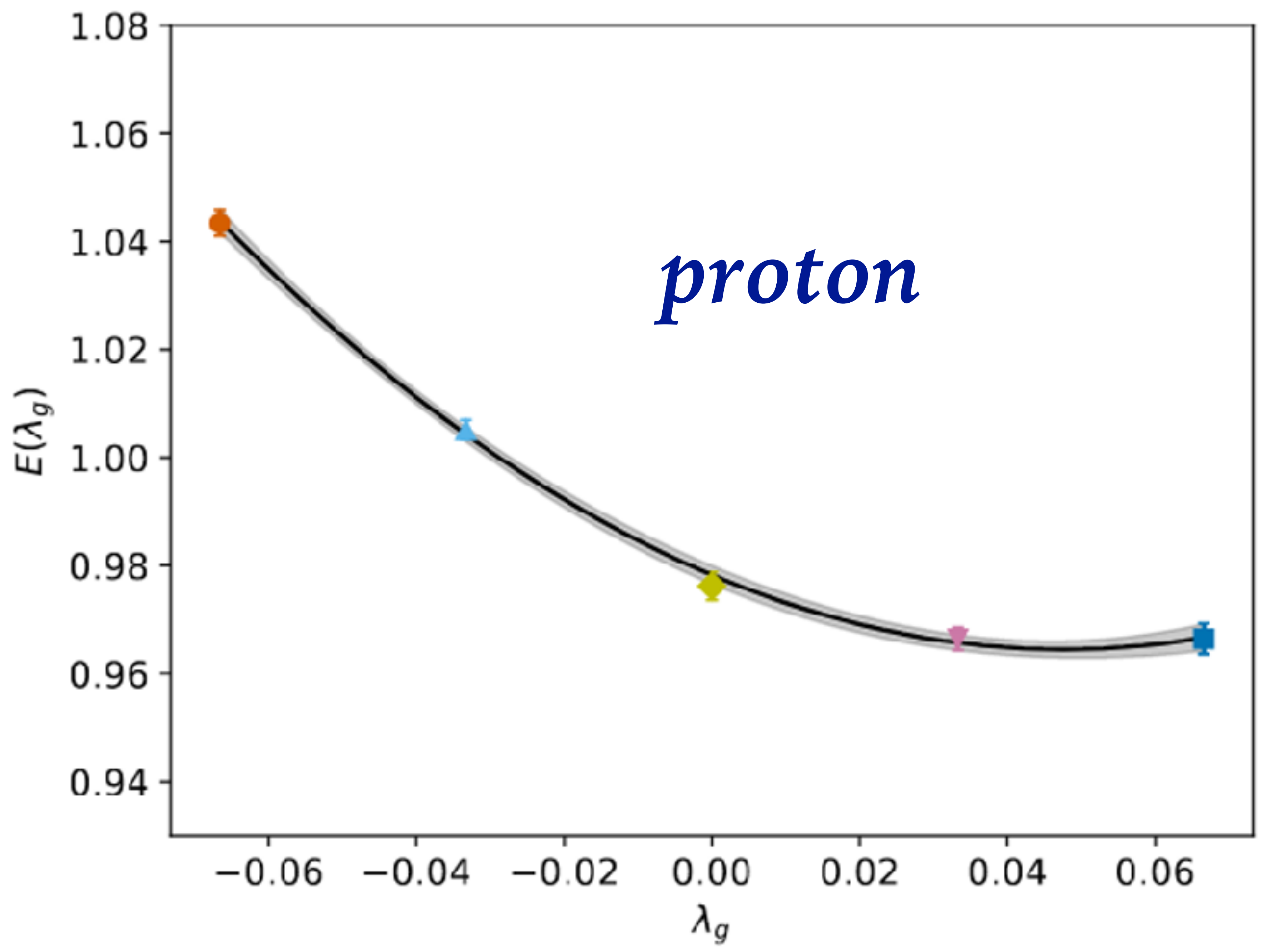


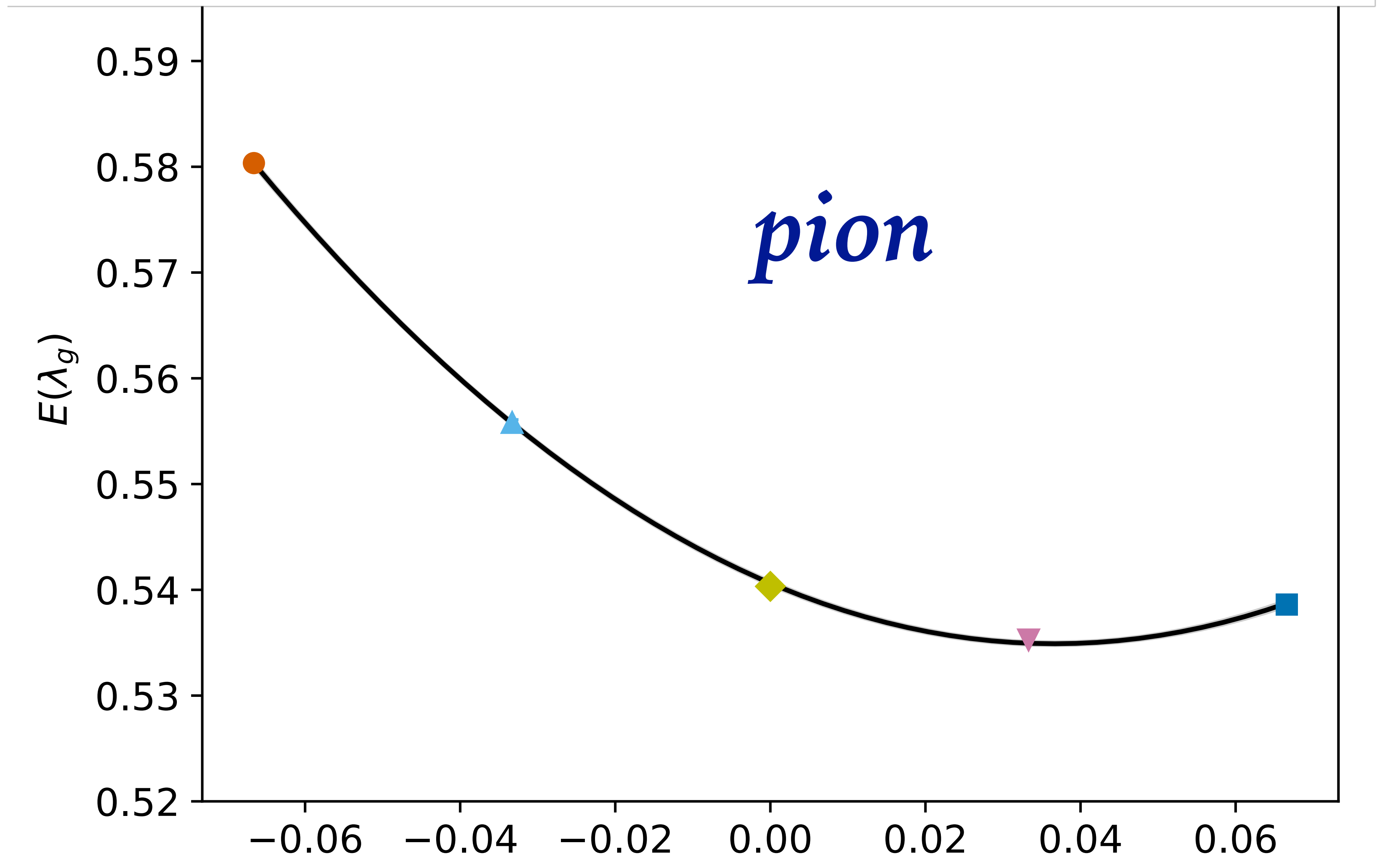
Excellent agreement between Feynman-Hellmann and standard 3-point function methods

Energies: Gluon operator

$m_\pi \approx 1065\,\text{MeV}$



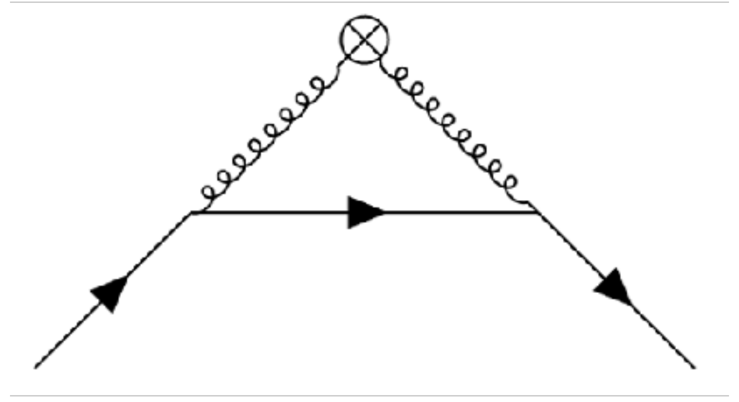




Data points are entirely uncorrelated, from separate ensembles.

Good agreement with quadratic fit, no significant cubic term.

Renormalisation



Recall quark-glue mixing under renormalisation
$$\begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_q \end{pmatrix}^R = \begin{pmatrix} Z_{gg} & Z_{gq} \\ Z_{qg} & Z_{qq} \end{pmatrix} \begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_q \end{pmatrix}^{lat}$$

But $Z_{qg} = 0$ in quenched QCD

➡ for $n_f = 0$ with $m_u = m_d$

$$\begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_u \\ \langle x \rangle_d \end{pmatrix}^R = \begin{pmatrix} Z_{gg} & Z_{gq} & Z_{gq} \\ 0 & Z_{qq} & 0 \\ 0 & 0 & Z_{qq} \end{pmatrix} \begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_u^{con} \\ \langle x \rangle_d^{con} \end{pmatrix}^{lat}$$

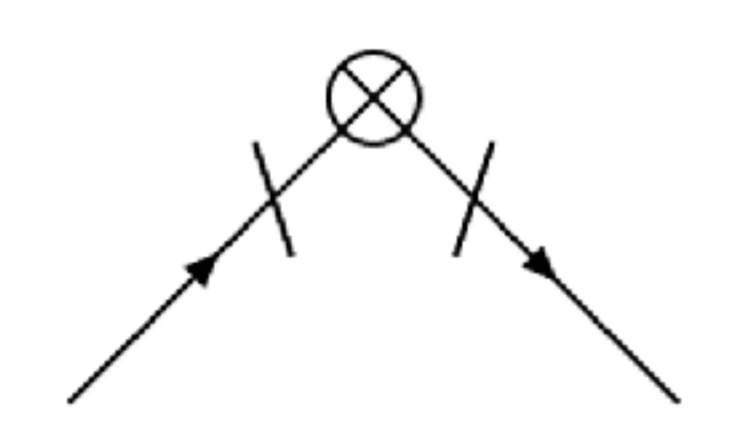
and

$$\Big(\langle x \rangle_g + \langle x \rangle_u + \langle x \rangle_d\Big)^R = Z_g \langle x \rangle_g^{lat} + Z_q \Big(\langle x \rangle_u + \langle x \rangle_u\Big)^{lat} = 1$$

with Z_g, Z_q depending only on coupling g and

$$Z_g = Z_{gg} \qquad \text{and} \qquad Z_q = Z_{gq}^{\overline{\text{MS}}} + Z_{qq}^{\overline{\text{MS}}}$$

We will employ RI'-MOM, e.g. $\frac{1}{12}\text{Tr}\Big(\Gamma^R [\Gamma^{\text{Tree}}]^{-1}\Big) = 1, \quad \Gamma^R = Z_{\mathcal{O}} Z_\psi^{-1} \Gamma^{lat}$ and $\Gamma^{lat} =$



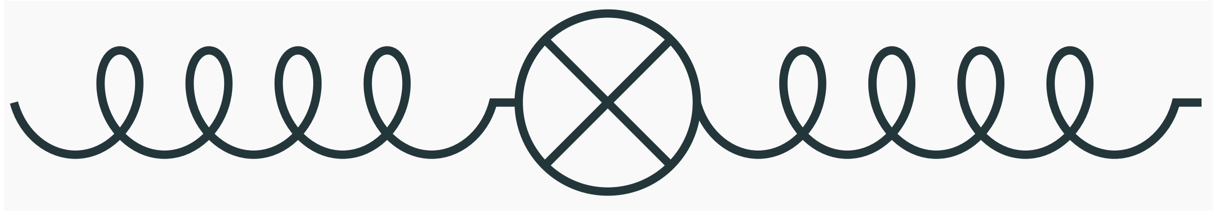
[NPB445(1994), NPB544(1999)]

Renormalisation - FH

Similar to: QCDSF(2015)
[PLB740 (2015)]

Extract 3-point functions from perturbed quark/gluon propagators

Generate propagators on same modified gauge fields as above

Gluon: $\left.\frac{\partial D_{\lambda_g}(p)}{\partial \lambda_g}\right|_{\lambda_g=0} = -\langle A(p)O(0)A(-p)\rangle^{lat} =$ 

with $\langle A(p)O_g(0)A(-p)\rangle^R = Z_A Z_{\mathcal{O}_g}\langle A(p)O_g(0)A(-p)\rangle^{lat}$ $\quad D(p)^R = Z_A D(p)^{lat}$

To avoid mixing with non-physical operators in the EMT [Collins&Scalise(1994),Shanahan&Detmold(2019)]

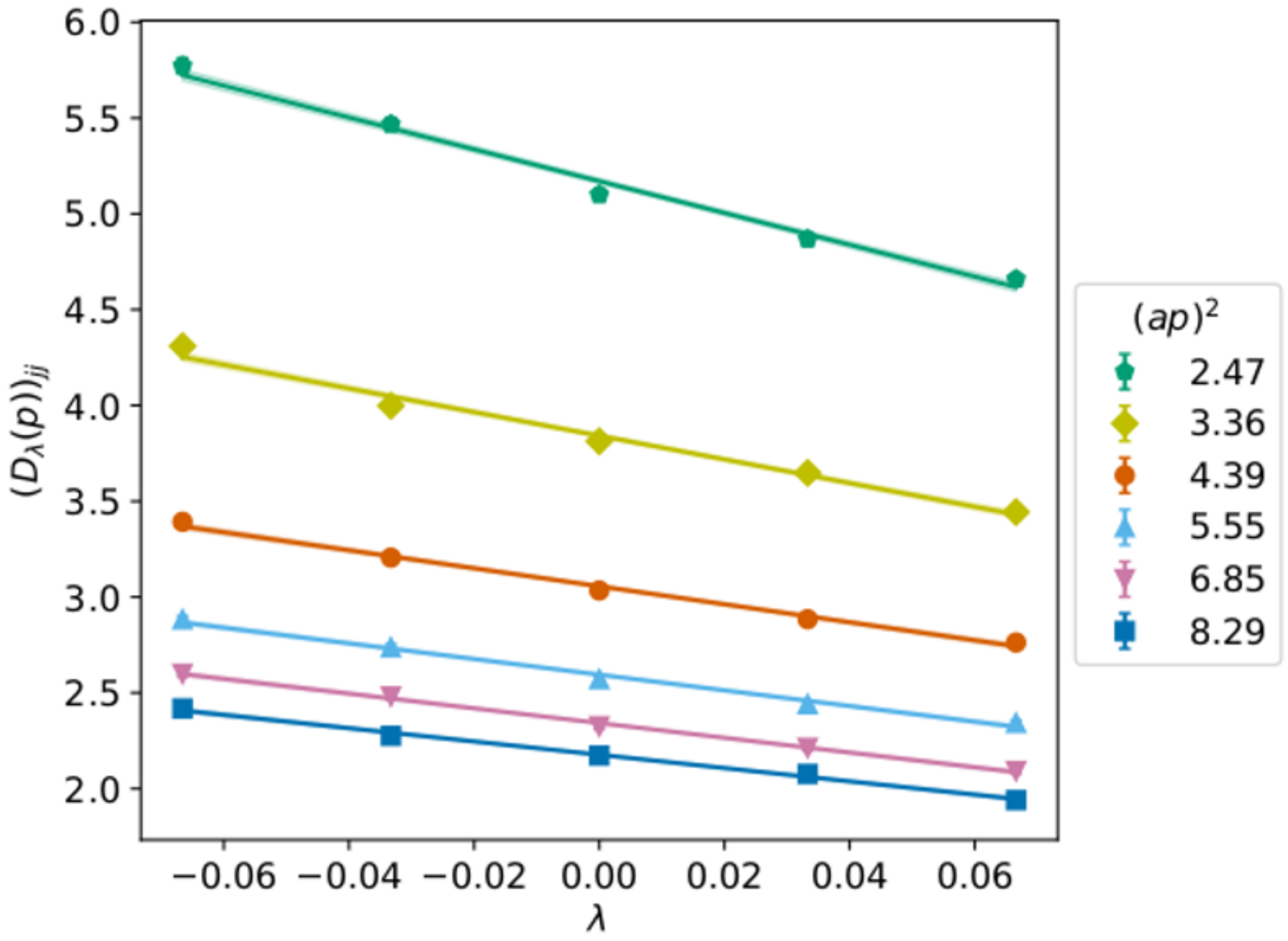
➡ take combination $\left\langle A_\rho(p)\bar{T}^g_{44}A_\tau(-p)\right\rangle - \left\langle A_\rho(p')\bar{T}^g_{44}A_\tau(-p')\right\rangle = 2p_4^2$

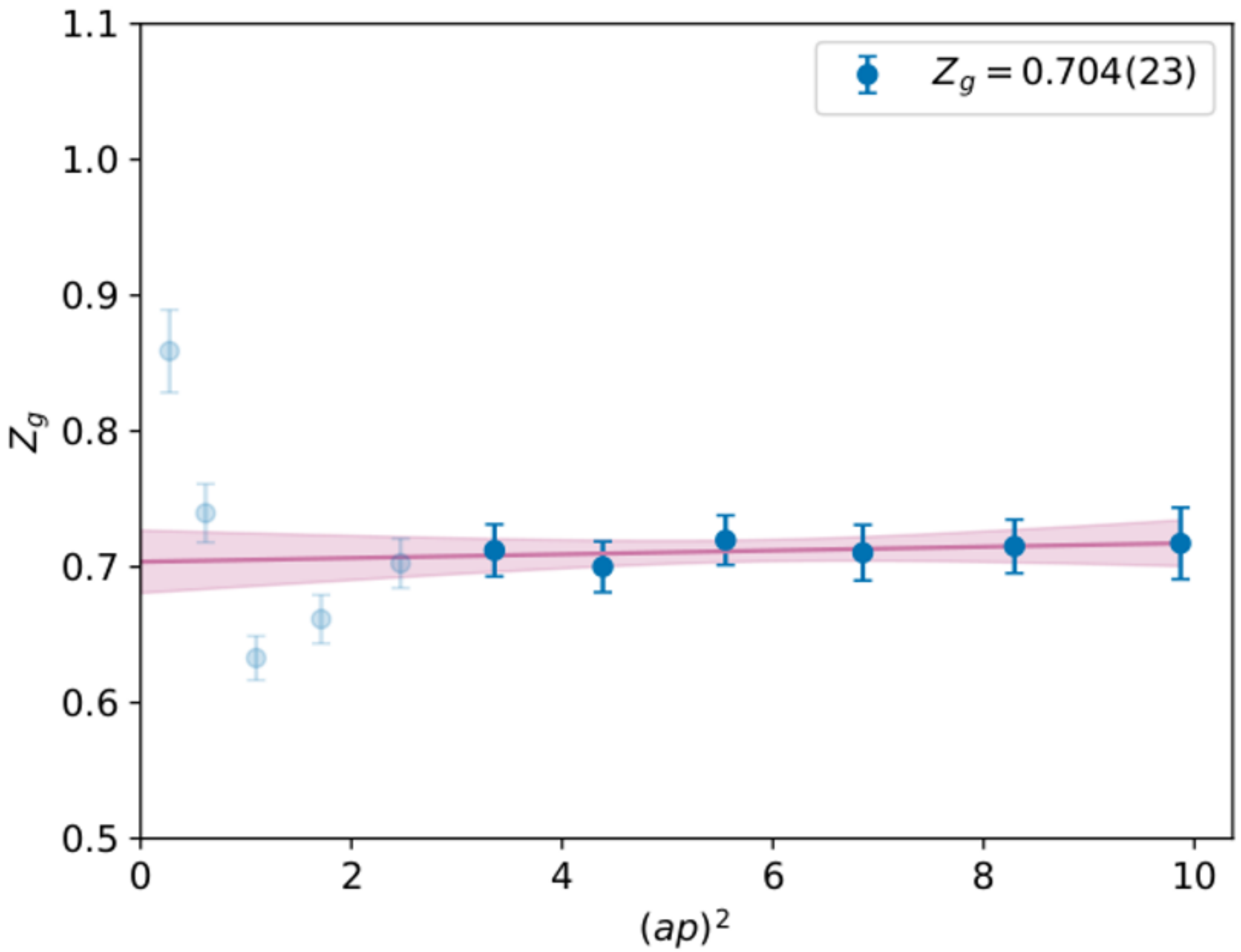
when $\rho \neq 4, p_4 \neq 0, p'_4 = 0, p_\rho = p'_\rho = 0$ and $p^2 = p'^2$

$$\bar{T}^g_{44} = \frac{3}{4}\mathcal{O}^{(b)}_g$$

➡ $$Z_g(\mu) = 2p_4^2 p^2\, D_0^{lat}(p)\left[\left.\frac{\partial (D^{latt}_{\lambda_g}(p))_{jj}}{\partial \lambda_g}\right|_{\lambda_g=0} - \left.\frac{\partial (D^{latt}_{\lambda_g}(p'))_{jj}}{\partial \lambda_g}\right|_{\lambda_g=0}\right]^{-1}\Bigg|_{\substack{p_j=p'_j=0 \\ p^2=p'^2=\mu^2}}$$

Renormalisation - glue





Data points at different λ_g are entirely uncorrelated, from separate ensembles.

Appear linear in λ_g, quadratic terms small, no significant cubic term.

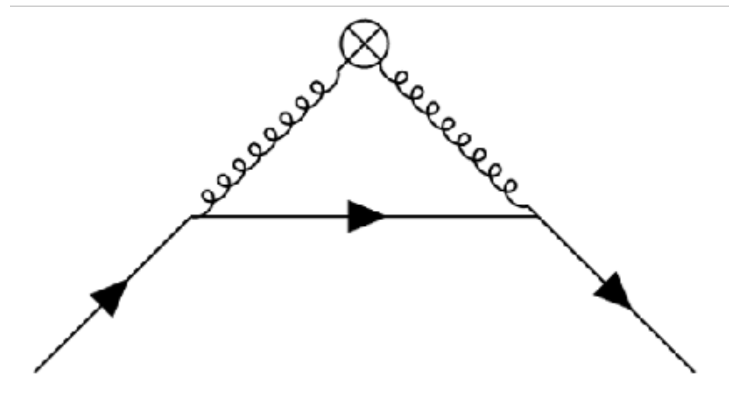
Good signal for Z_g

Renormalisation - quark

Need to account for quark-glue mixing

Z_{qq} can be obtained via usual RI'-MOM (e.g. QCDSF(2005))

To account for mixing, generate quark propagators on same modified gauge fields

$$\left.\frac{\partial S_{\lambda_g}(p)}{\partial \lambda_g}\right|_{\lambda_g=0} = -\langle \bar{q}(p) O(0) q(p) \rangle =$$ 

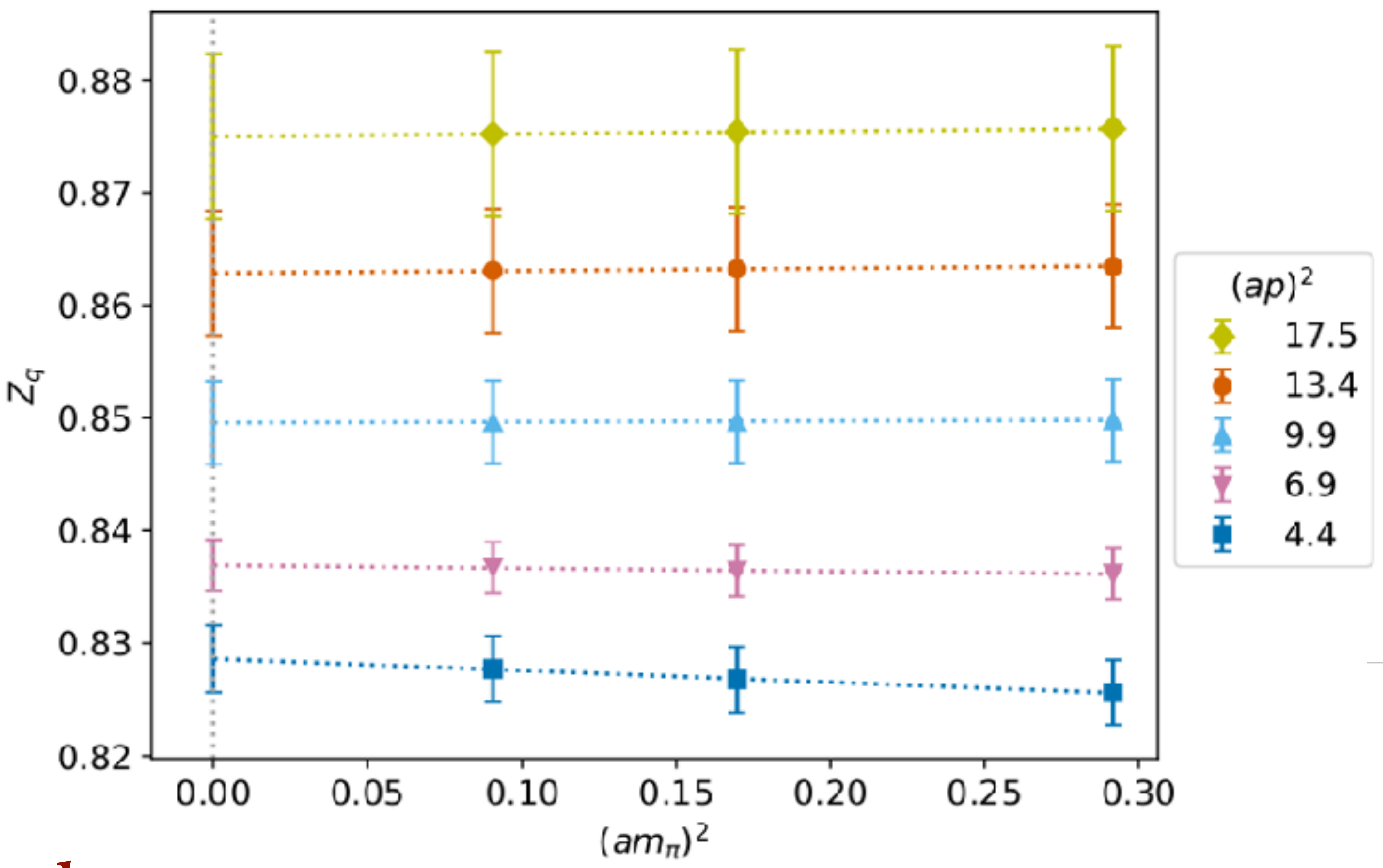
and invoke $\mathcal{O}_q^R + \mathcal{O}_g^R = Z_q \mathcal{O}_q^{lat} + Z_g \mathcal{O}_g^{lat}$ with $Z_q = Z_{qq} + Z_{gq}$ and $Z_g = Z_{qg} + Z_{gg}$ $\quad n_f = 0 : Z_{qg} = 0$

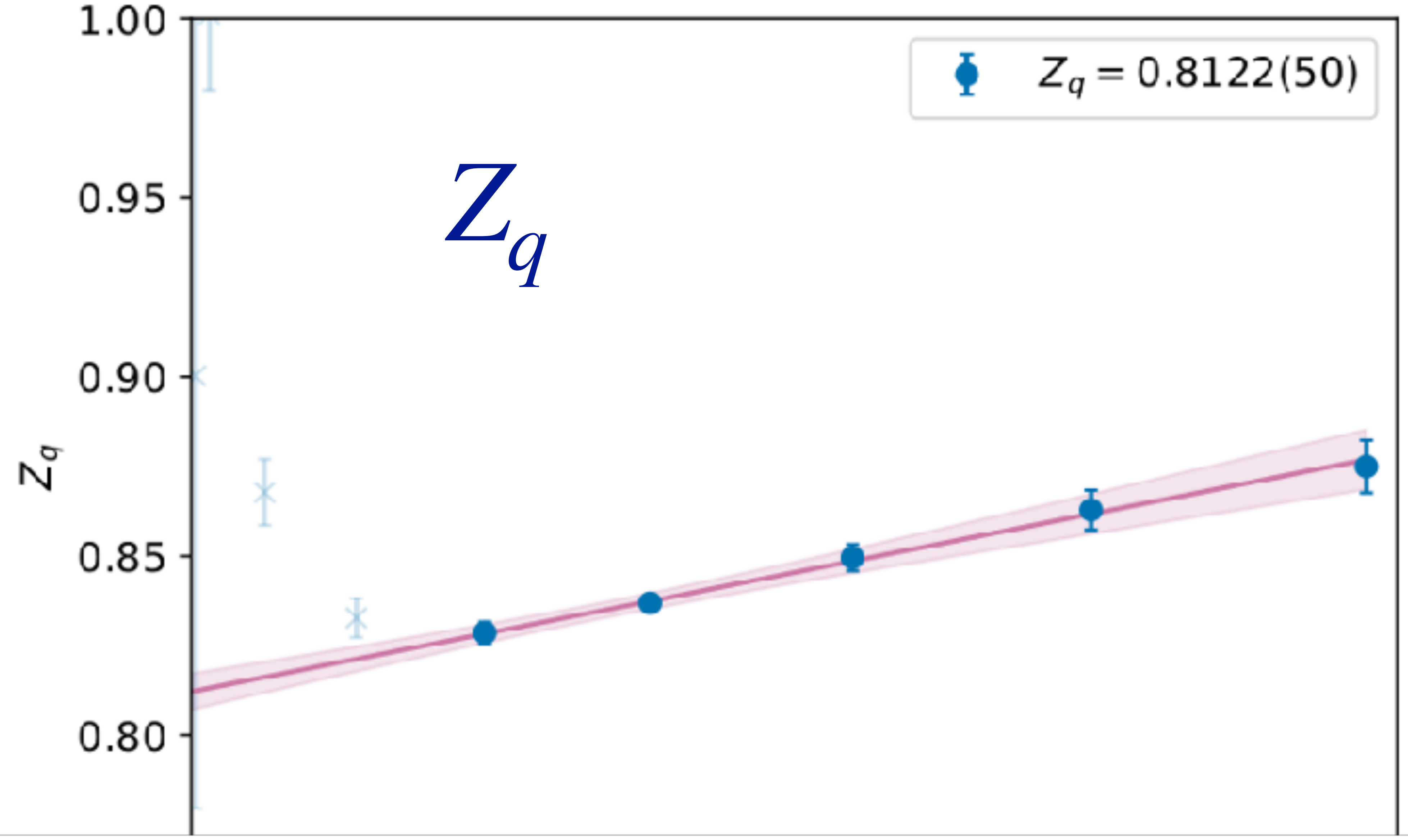
$$Z_q^{-1}(\mu) = \frac{1}{12}\mathrm{Tr}\left\{ \Gamma_{qq}^{lat}(p) \left[Z_\psi(p) \Gamma_{qq}^{\mathrm{Born}}(p) - Z_g(p) \left([S_0^{lat}(p)]^{-1} \left.\frac{\partial S_{\lambda_g}(p)}{\partial \lambda_g}\right|_{\lambda_g=0} [S_0^{lat}(p)]^{-1} \right) \right]^{-1} \right\} \Bigg|_{p^2=\mu^2}$$

use standard quark 3-point methods

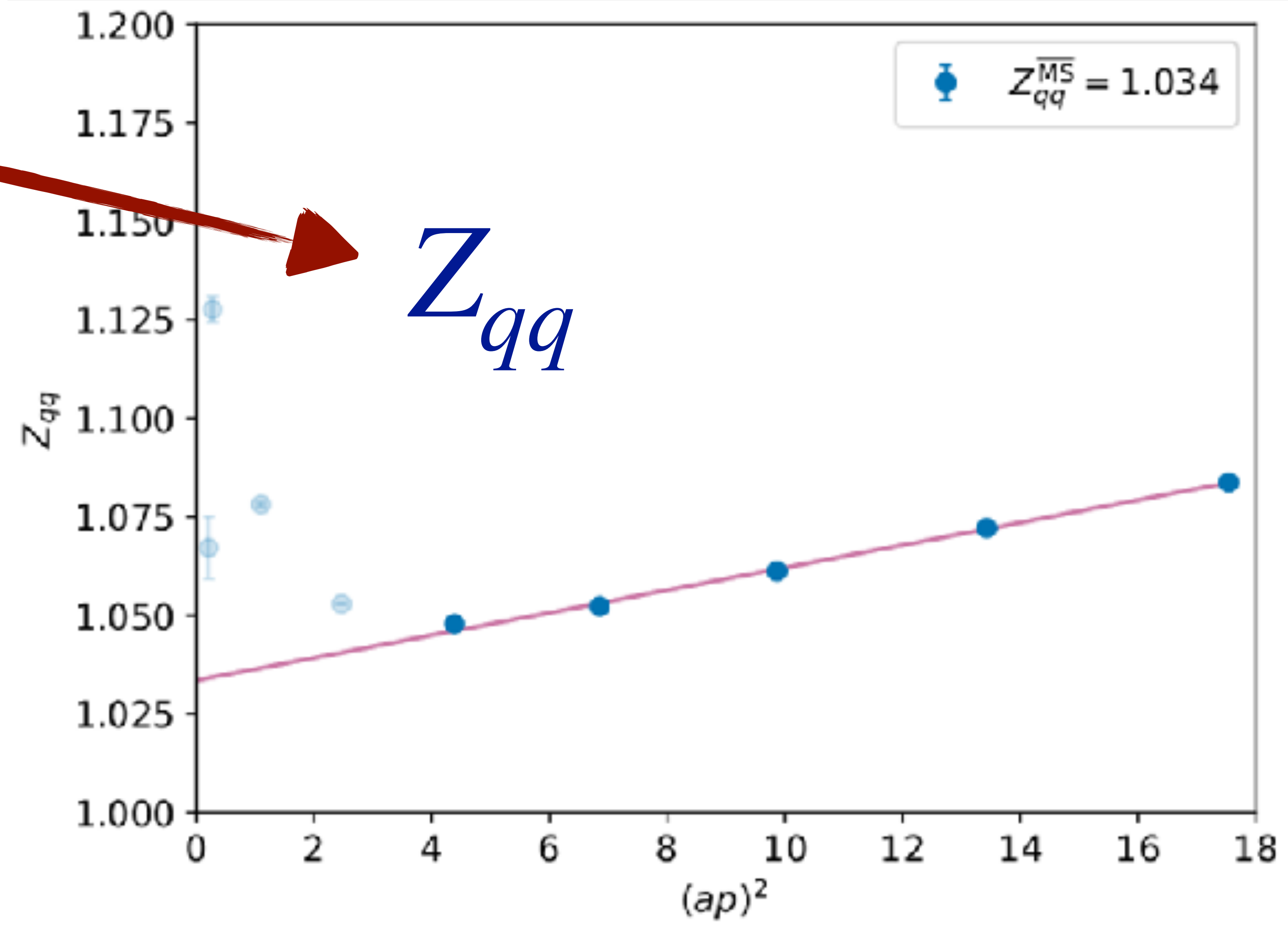
then isolate mixing term $Z_{gq} = Z_q - Z_{qq}$

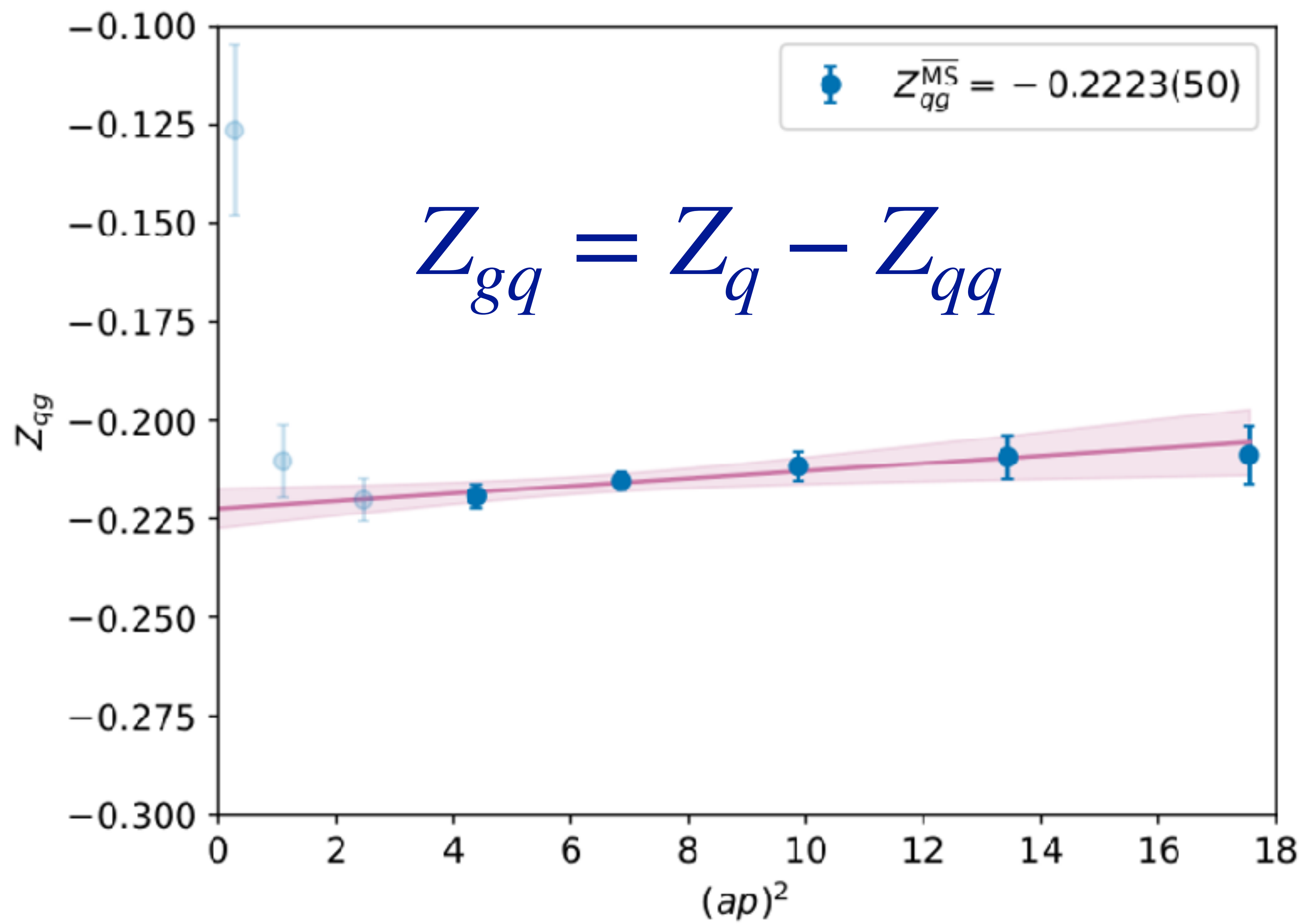
Renormalisation - quark





use standard RI'-MOM with quark 3-point methods





Momentum sum rule

$$\begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_u \\ \langle x \rangle_d \end{pmatrix}^R = \begin{pmatrix} Z_{gg} & Z_{gq} & Z_{gq} \\ 0 & Z_{qq} & 0 \\ 0 & 0 & Z_{qq} \end{pmatrix} \begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_u^{con} \\ \langle x \rangle_d^{con} \end{pmatrix}^{lat}$$

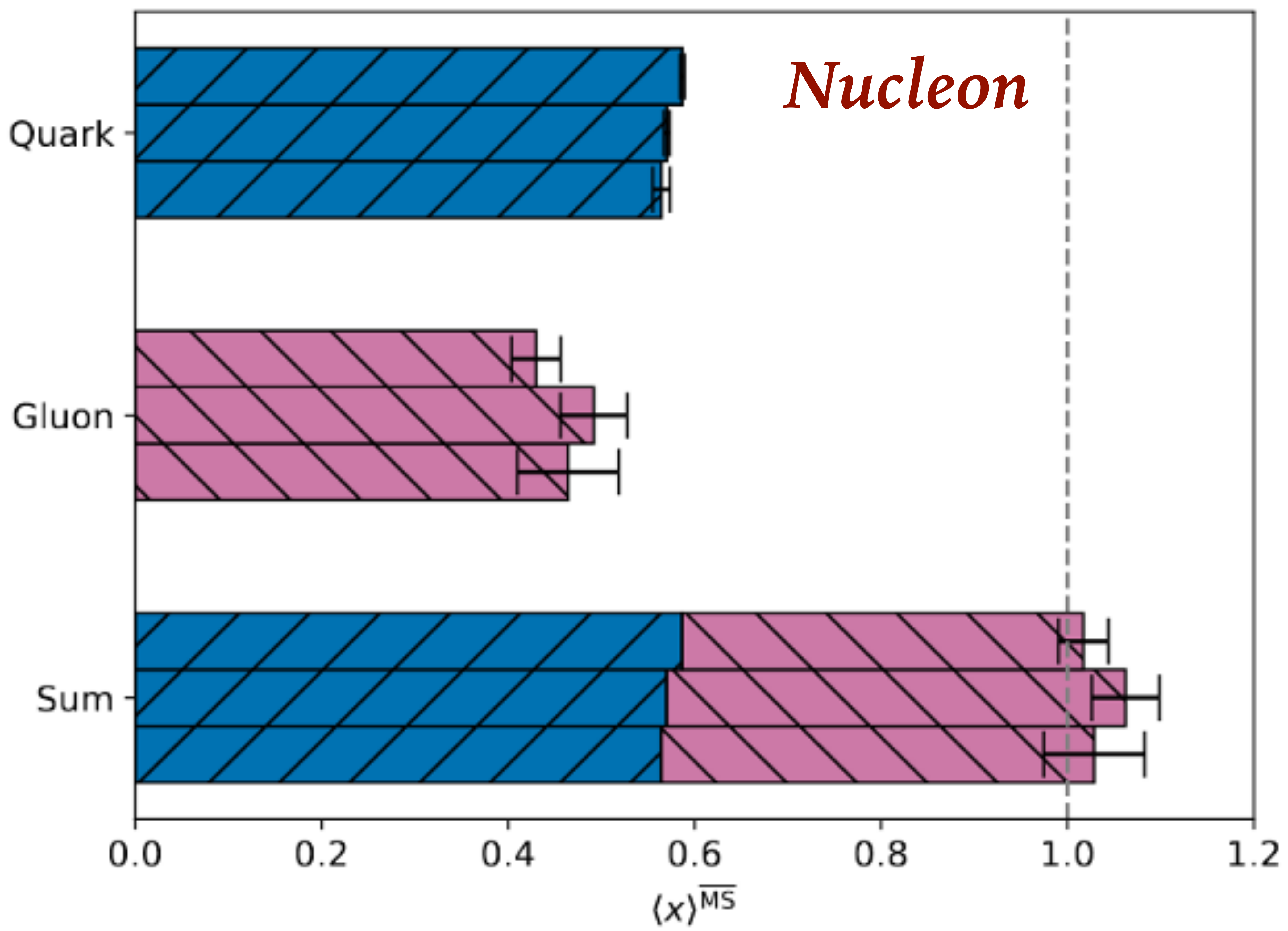
$$Z^{\overline{\mathrm{MS}}} = \begin{pmatrix} 0.704(23) & -0.2223(50) & -0.2223(50) \\ 0 & 1.034(1) & 0 \\ 0 & 0 & 1.034(1) \end{pmatrix}$$

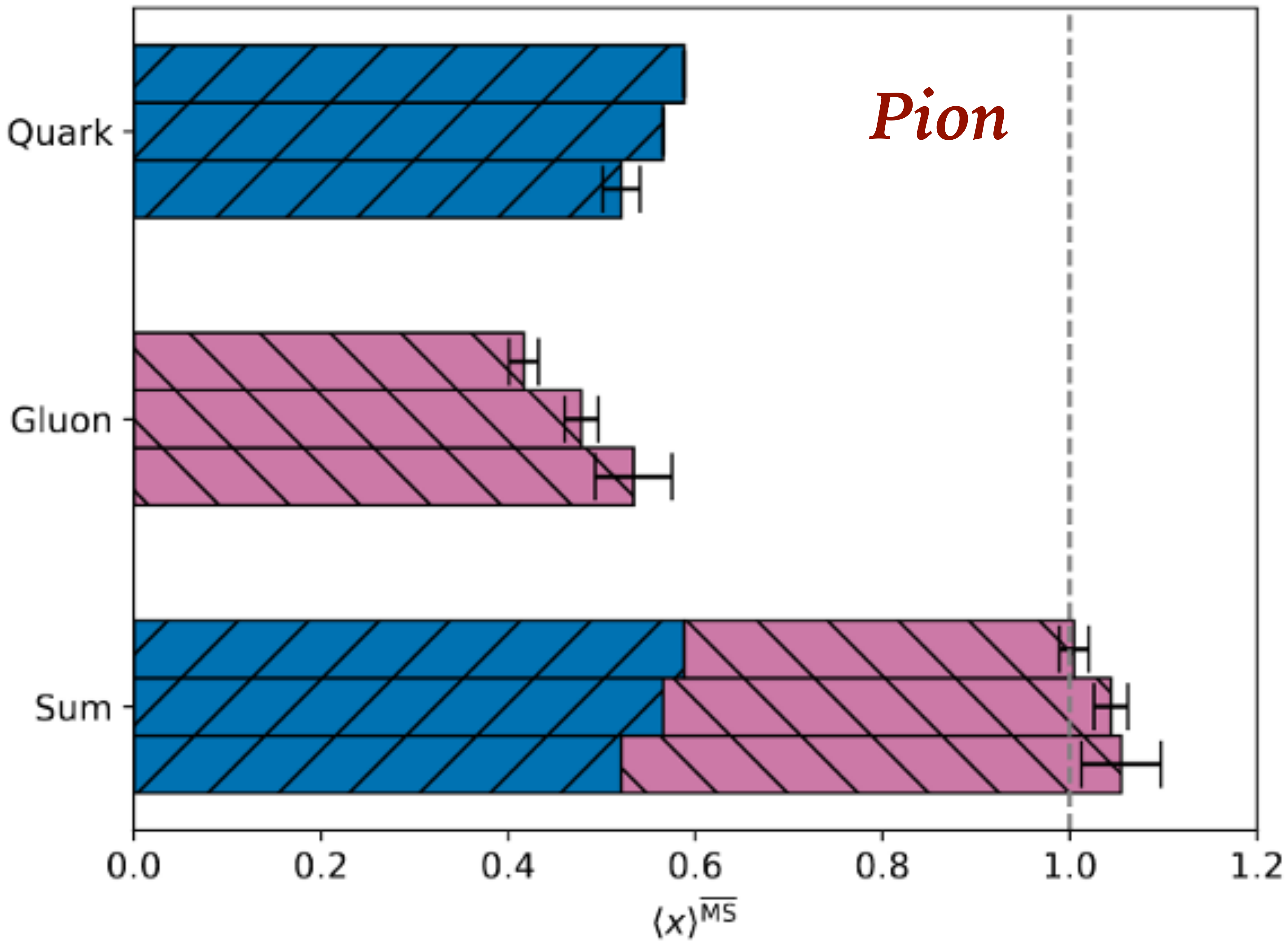
Nucleon

am_π	$\langle x \rangle_q^{\overline{\mathrm{MS}}}$	$\langle x \rangle_g^{\overline{\mathrm{MS}}}$	$\langle x \rangle_q^{\overline{\mathrm{MS}}} + \langle x \rangle_g^{\overline{\mathrm{MS}}}$
0.540	0.5869(23)	0.430(26)	1.018(27)
0.412	0.5703(31)	0.492(36)	1.063(36)
0.300	0.5645(92)	0.464(54)	1.029(54)

Pion

am_π	$\langle x \rangle_q^{\overline{\mathrm{MS}}}$	$\langle x \rangle_g^{\overline{\mathrm{MS}}}$	$\langle x \rangle_q^{\overline{\mathrm{MS}}} + \langle x \rangle_g^{\overline{\mathrm{MS}}}$
0.540	0.58803(58)	0.417(16)	1.005(16)
0.412	0.56569(80)	0.478(18)	1.045(18)
0.300	0.521(20)	0.534(41)	1.056(42)





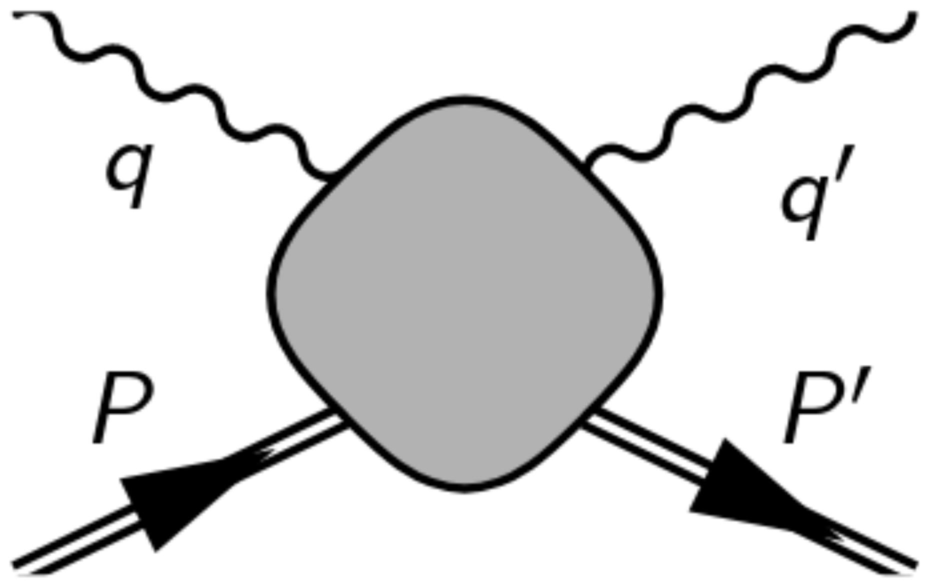
Off-forward Compton amplitude

[PRD105 (2022), PRD110 (2024)]

Off-forward Compton

[PRD105 (2022), PRD110 (2024)]

$$T^{\mu\nu} = \int d^4z e^{\frac{i}{2}(\mathbf{q}+\mathbf{q}')\cdot\mathbf{z}} \langle N(p') \,|\, TJ^\mu(z)J^\nu(0) \,|\, N(p)\rangle$$



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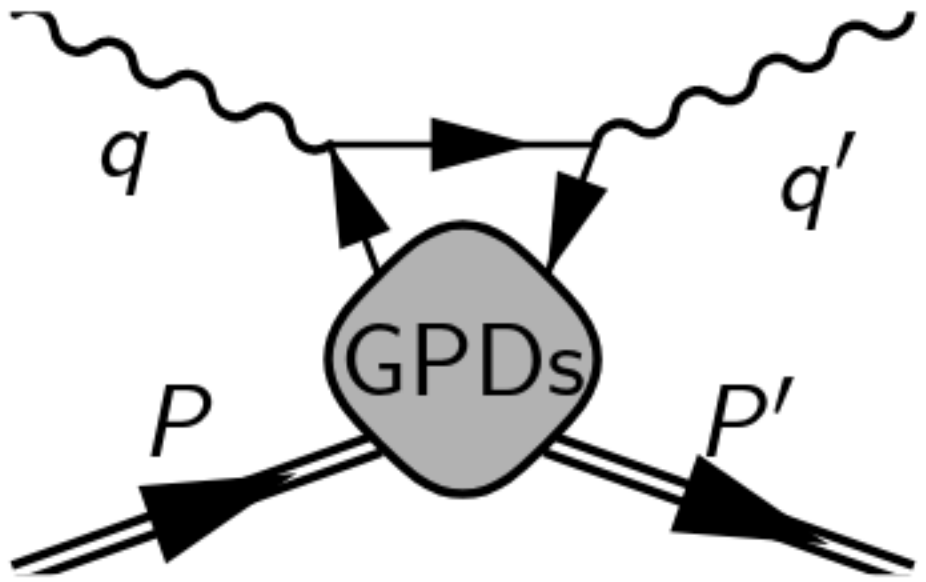


$+ \frac{\Lambda^2_{QCD}}{Q^2} + S_1$

power corrections

subtraction "constant"

$S_1(t, Q^2)$

$$T^{\mu\nu}(P, q, q') = \sum_{i=1}^{18} \mathscr{A}_i(\overline{\omega}, \theta, t, \overline{Q}^2) L_i^{\mu\nu}$$

18 tensor structures

$$\overline{P} = \frac{1}{2}(P + P'), \quad \overline{q} = \frac{1}{2}(q + q'), \quad \Delta = P' - P$$

$$t = \Delta^2, \quad \overline{Q}^2 = -\overline{q}^2,$$

$$\overline{\omega} = \frac{2\overline{P}\cdot\overline{q}}{\overline{Q}^2}, \quad \theta = -\frac{\Delta\cdot\overline{q}}{\overline{Q}^2}$$

Off-forward Compton

$$h^\mu = \bar{u}'\gamma^\mu u, \quad e^\mu = \bar{u}'\frac{i\sigma^{\mu\alpha}\Delta_\alpha}{2m_N}u$$

$$\tilde{h}^\mu = \bar{u}'\gamma^\mu\gamma_5 u, \quad \tilde{e}^\mu = \frac{\Delta^\mu}{2m_N}\bar{u}'\gamma_5 u$$

$$\begin{aligned}\bar{T}_{\mu\nu} = &\frac{1}{2\bar{P}\cdot\bar{q}}\Big[-\Big(h\cdot\bar{q}\mathcal{H}_1 + e\cdot\bar{q}\mathcal{E}_1\Big)g_{\mu\nu} + \frac{1}{\bar{P}\cdot\bar{q}}\Big(h\cdot\bar{q}\mathcal{H}_2 + e\cdot\bar{q}\mathcal{E}_2\Big)\bar{P}_\mu\bar{P}_\nu + \mathcal{H}_3 h_{\{\mu}\bar{P}_{\nu\}}\Big]\\ &+ \frac{i}{2\bar{P}\cdot\bar{q}}\epsilon_{\mu\nu\rho\kappa}\bar{q}^\rho\Big(\tilde{h}^\kappa\tilde{\mathcal{H}}_1 + \tilde{e}^\kappa\tilde{\mathcal{E}}_1\Big) + \frac{i}{2(\bar{P}\cdot\bar{q})^2}\epsilon_{\mu\nu\rho\kappa}\bar{q}^\rho\Big[(\bar{P}\cdot\bar{q}\tilde{h}^\kappa - \tilde{h}\cdot\bar{q}\bar{P}^\kappa)\tilde{\mathcal{H}}_2 + (\bar{P}\cdot\bar{q}\tilde{e}^\kappa - \tilde{e}\cdot\bar{q}\bar{P}^\kappa)\tilde{\mathcal{E}}_2\Big]\\ &+ \Big(\bar{P}_\mu q'_\nu + \bar{P}_\nu q_\mu\Big)\Big(h\cdot\bar{q}\mathcal{K}_1 + e\cdot\bar{q}\mathcal{K}_2\Big) + \Big(\bar{P}_\mu q'_\nu - \bar{P}_\nu q_\mu\Big)\Big(h\cdot\bar{q}\mathcal{K}_3 + e\cdot\bar{q}\mathcal{K}_4\Big) + q_\mu q'_\nu(h\cdot\bar{q} - e\cdot\bar{q})\mathcal{K}_5\\ &+ h_{[\mu}\bar{P}_{\nu]}\mathcal{K}_6 + \Big(h_\mu q'_\nu + h_\nu q_\mu\Big)\mathcal{K}_7 + \Big(h_\mu q'_\nu - h_\nu q_\mu\Big)\mathcal{K}_8 + \bar{P}_{\{\mu}\bar{u}(P')i\sigma_{\nu\}\alpha}u(P)\bar{q}^\alpha\mathcal{K}_9,\end{aligned}$$

$\mathcal{K}$ vanish at leading twist

Diehl, EPJC(2001)
Belitsky, Müller, Kirchner, NPB(2002)
Belitsky, Müller, Ji, NPB(2014)

simple mapping to forward limit

$$\mathcal{H}_1 \xrightarrow{t\to 0} \mathcal{F}_1, \quad \mathcal{H}_2 + \mathcal{H}_3 \xrightarrow{t\to 0} \mathcal{F}_2,$$

$$\tilde{\mathcal{H}}_1 \xrightarrow{t\to 0} \tilde{g}_1, \quad \tilde{\mathcal{H}}_2 \xrightarrow{t\to 0} \tilde{g}_2,$$

- Kinematics chosen carefully
- With current chosen $\hat{e}_k \propto \vec{\Delta} = \vec{q}_1 - \vec{q}_2$ $T_{\mu\nu}$ reduces to

$$T_{kk} = -\frac{1}{2\bar{P}\cdot\bar{q}}\left(h\cdot\bar{q}\,\mathcal{H}_1 + e\cdot\bar{q}\mathcal{E}_1\right)$$

$\frac{L}{2\pi}\mathbf{q}_1$, $\frac{L}{2\pi}\mathbf{q}_2$	$\frac{L}{2\pi}\boldsymbol{\Delta}$	$\frac{L}{2\pi}\bar{\mathbf{q}}$	t [GeV^2]	$\bar{Q}^2$ [GeV^2]	N_{meas}
$(5,3,0)$	—	—	0	4.86	1605
$(4,3,3)$ $(3,4,3)$	$(1,-1,0)$	$(\frac{7}{2},\frac{7}{2},3)$	-0.29	4.79	1031
$(5,3,1)$ $(5,3,-1)$	$(0,0,2)$	$(5,3,0)$	-0.57	4.86	1072
$(4,2,4)$ $(2,4,4)$	$(2,-2,0)$	$(3,3,4)$	-1.14	4.86	1031

Off-forward Compton

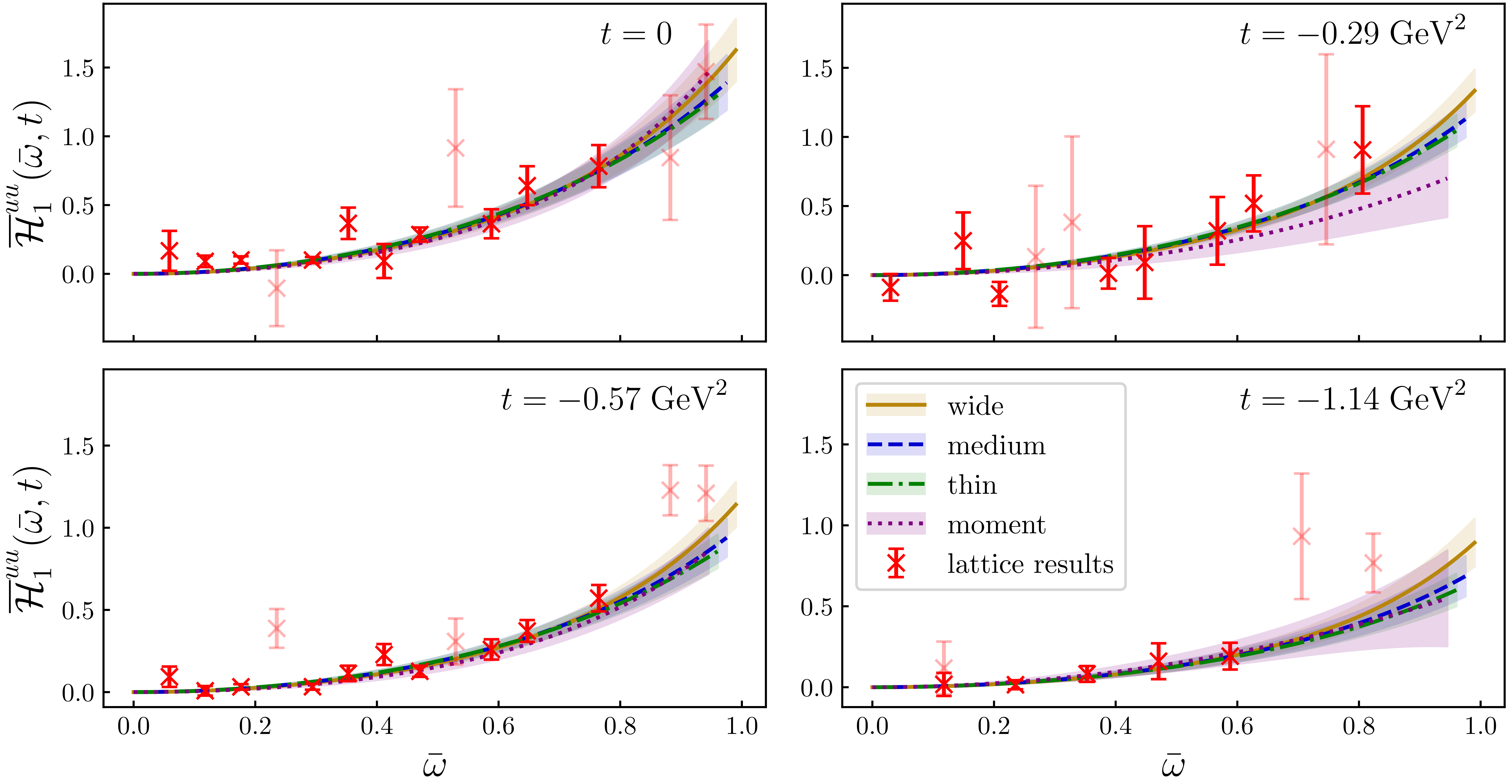
$$T_{kk} = -\frac{1}{2\overline{P}\cdot\bar{q}}\left(h\cdot\bar{q}\,\mathcal{H}_1 + e\cdot\bar{q}\,\mathcal{E}_1\right)$$

- 2 x spin projectors

➡ isolate $\mathcal{H}_1$, $\mathcal{E}_1$

- Expand subtracted $\overline{\mathcal{H}}$, $\overline{\mathcal{E}}$ in moments, e.g.

$$\overline{\mathcal{H}}_1(\bar{\omega}, t, \overline{Q}^2) = 2\sum_n \bar{\omega}^n M_n(t, \overline{Q}^2)$$



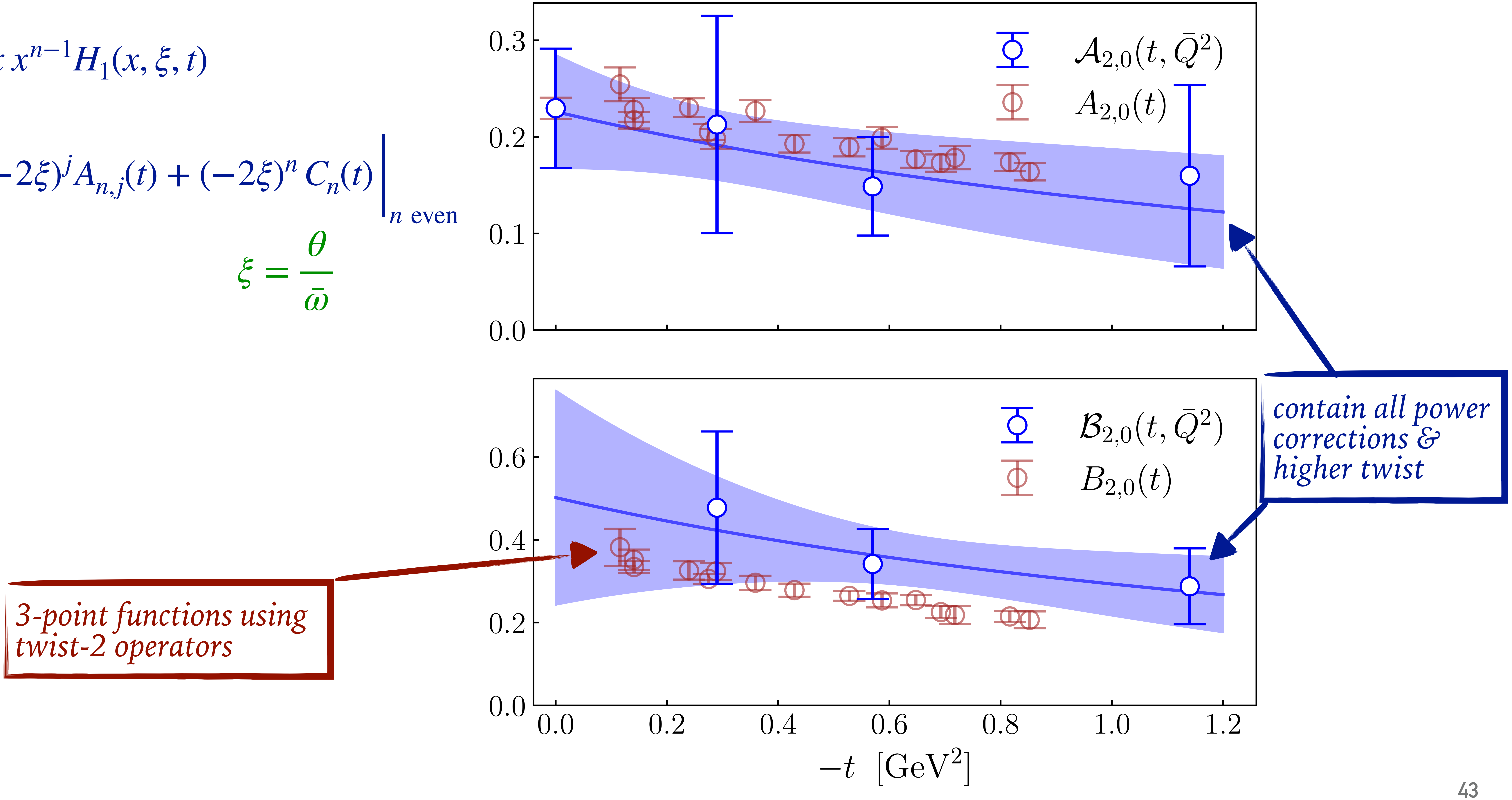
Off-forward Compton

Moments match onto Mellin moments of GPDs

$$M_n(t,\theta,\overline{Q}^2) \xrightarrow{\overline{Q}^2\to\infty} \int_{-1}^{1} dx\, x^{n-1} H_1(x,\xi,t)$$

$$= \sum_{j=0,2,4,\ldots}^{n-1} (-2\xi)^j A_{n,j}(t) + (-2\xi)^n\, C_n(t)\Big|_{n\ \mathrm{even}}$$

$$\xi = \frac{\theta}{\bar{\omega}}$$



Off-forward Compton

GPD reconstruction

- Employ model-dependent ansatz

$$H(x,t) = Cx^{-\alpha(t)}(1-x)^{\beta}$$

$$\alpha(t) = \alpha_0 + \alpha' t$$

(dispersion relation)

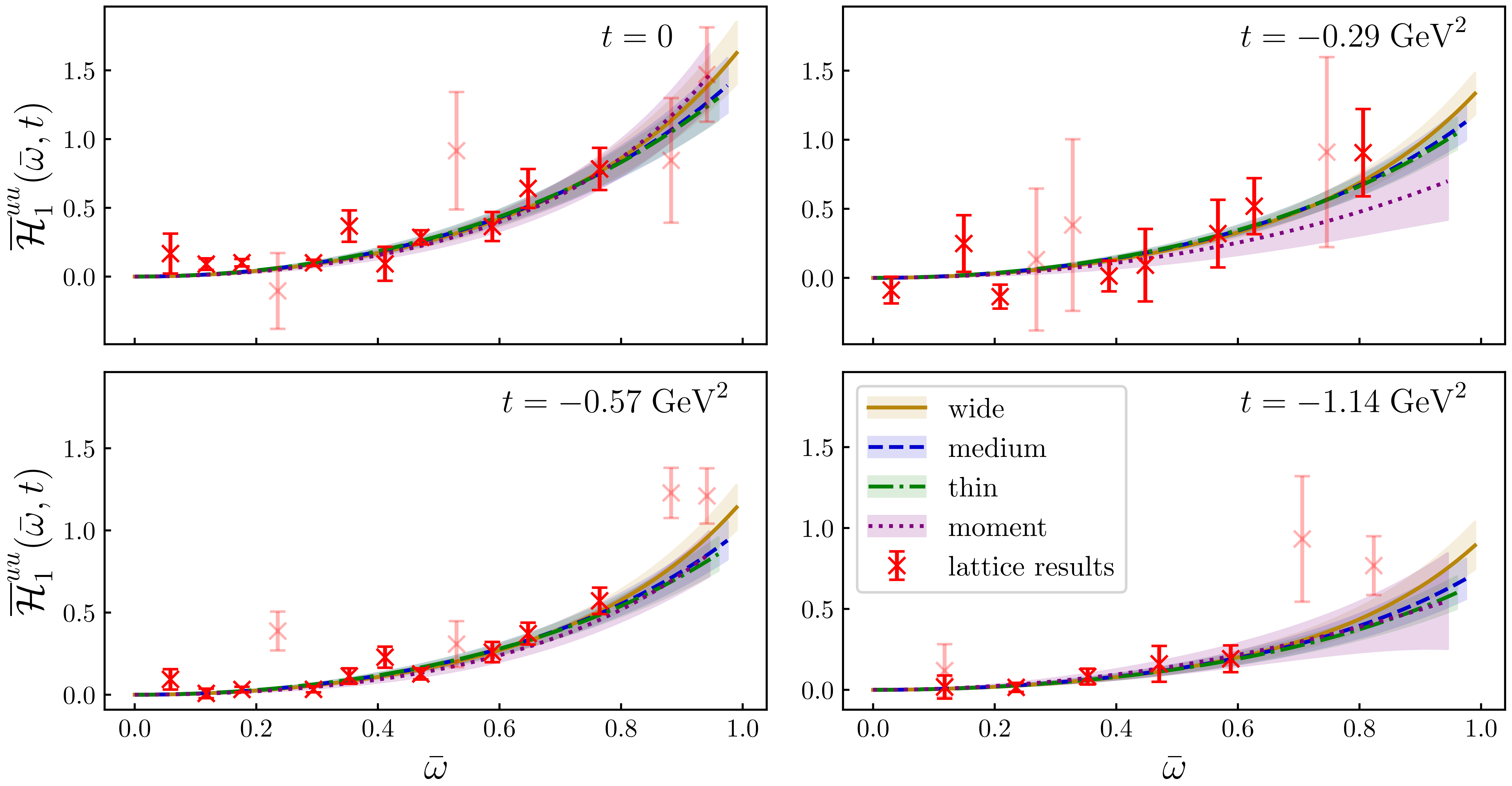
$$\overline{\mathcal{H}}_1(\bar{\omega},t) = 2C\sum_{n=1}^{\infty}\bar{\omega}^{2n}\frac{\Gamma(2n-\alpha(t))\Gamma(\beta+1)}{\Gamma(1+2n-\alpha(t)+\beta)}$$

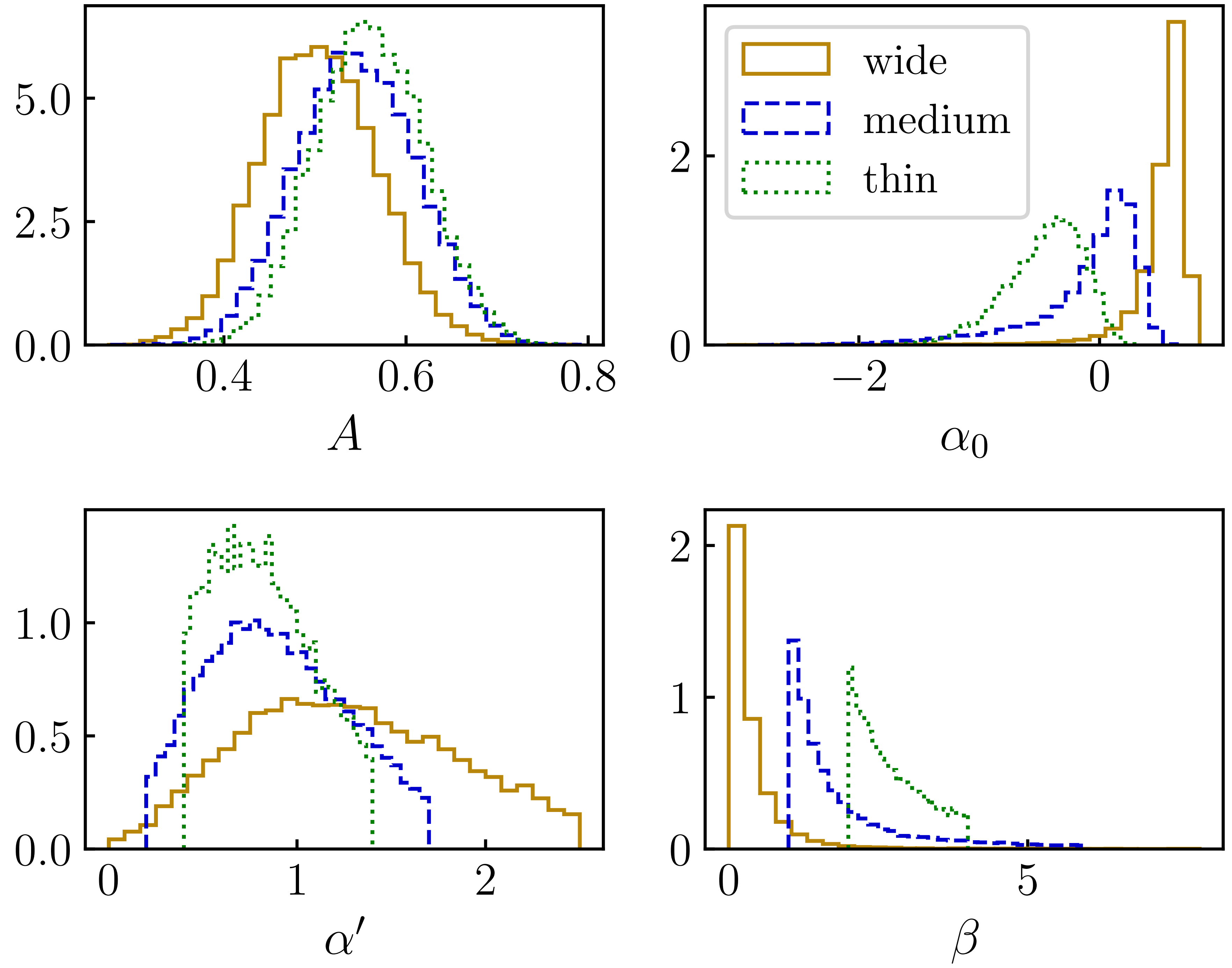
- Perform Bayesian fit with 3 priors of differing widths
- Drawing on positivity constraint of leading-twist GPDs

enforce

$$|\mathscr{A}_{2n,0}(t)| \leq \mathscr{A}_{2n,0}(0)$$

$$|\mathscr{B}_{2n,0}(t)| \leq \frac{2m_N}{\sqrt{-t}}\mathscr{A}_{2n,0}(0)$$

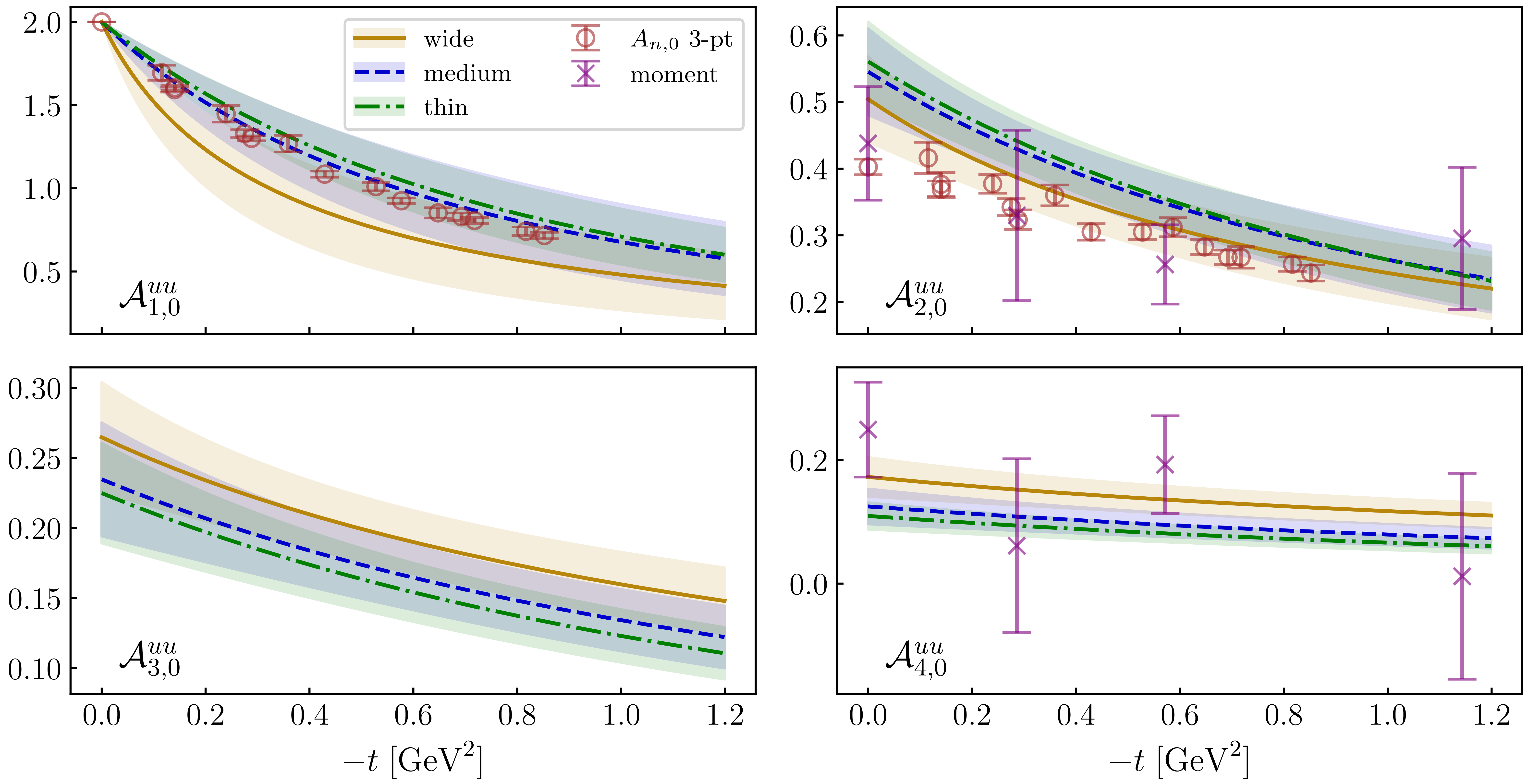




Off-forward Compton

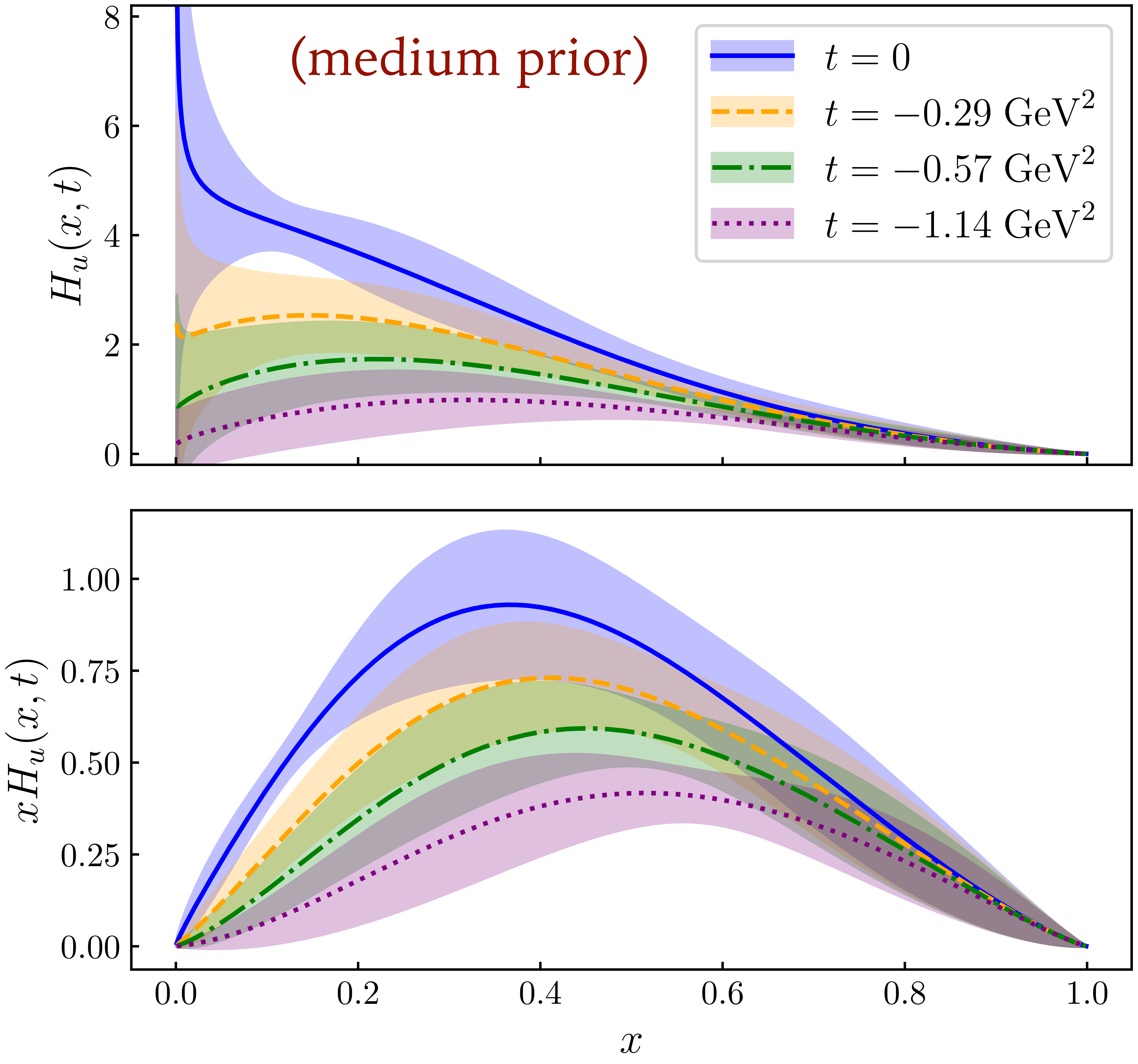
GPD reconstruction

- Enforce quark counting (leading-twist)
- Truncate series at n=50 moments
- Compare with model-independent moment fit and 3-point results



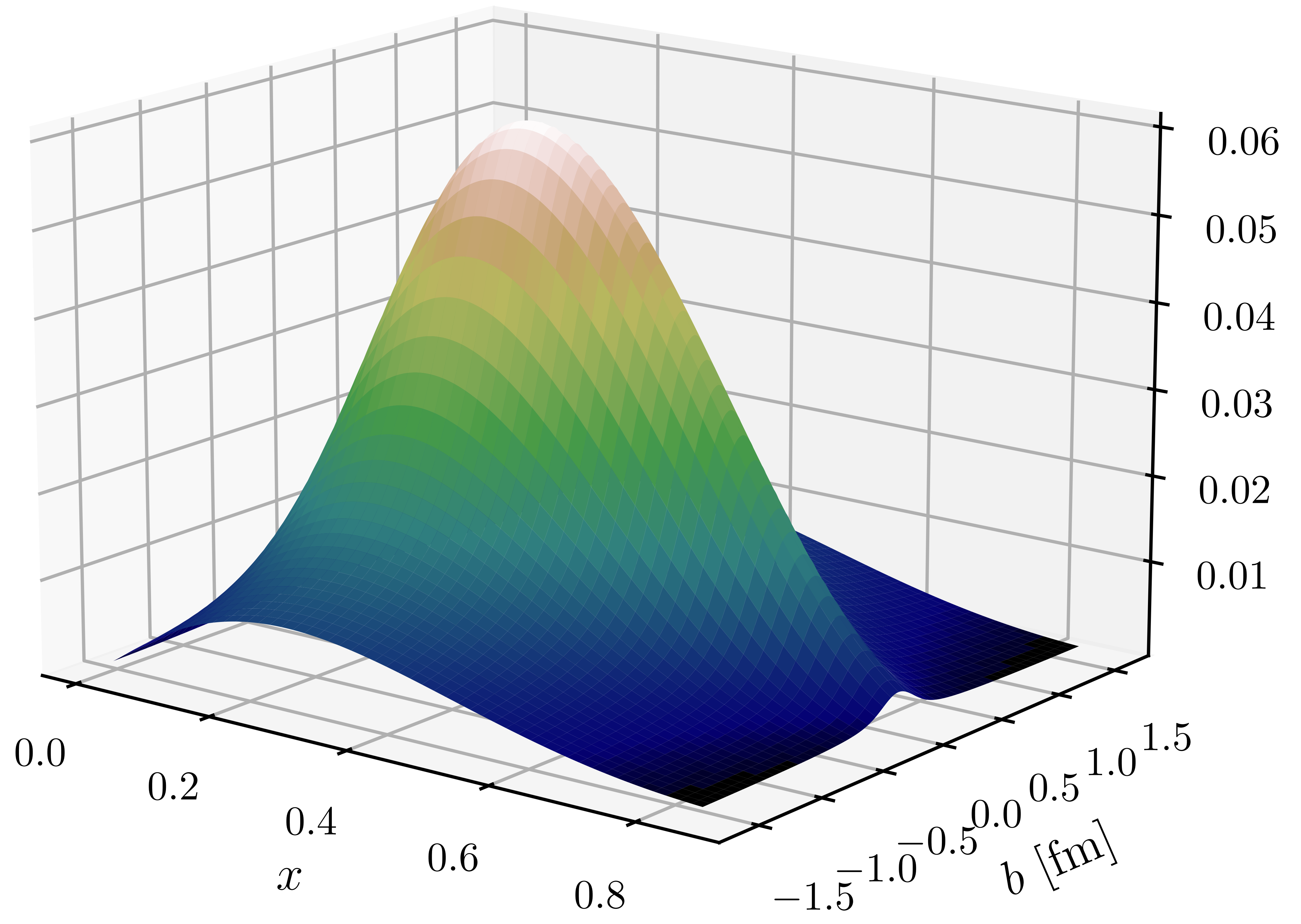
Off-forward Compton GPD reconstruction

$$H(x,t) = Cx^{-\alpha(t)}(1-x)^{\beta}$$



- Fourier transform $\vec{\Delta} \to \vec{b}$

➡ Impact parameter GPD



Transverse force distributions in the nucleon

Physical Review Letters 134 (2025) — Editor's suggestion

*"The study reveals a spin-independent force that reflects the confinement of quarks, with local forces reaching up to 3 billion electron volts per femtometre — about half a million Newtons, or the **weight of roughly 10 elephants**. A spin-dependent force has also been mapped, which offers new insights into how the dynamics of quarks are influenced by the spin of the proton in which they live."*

Protons' Internal Forces Are As Strong As The Weight Of 5 Schoolbuses
Raw News Health, 26 Feb 2025
Protons sit in the nuclei of all atoms, but they are not fundamental particles: They are made of three quarks...

Protons' Internal Forces Are As Strong As The Weight Of 5 Schoolbuses
Iflscience, 26 Feb 2025
Space and Physicsphysics PUBLISHED31 minutes ago This is why you need very powerful particle accelerators to smash them! Dr.

Force As Strong As 10 Compressed Elephants Rests Inside A Proton, Suggests Study
Wonderful Engineering, 24 Feb 2025
Protons, the building blocks of all matter, hold some of the deepest mysteries in modern physics.

Illuminating the Inner Workings of the Proton
Space Daily, 24 Feb 2025
The international collaboration, which includes researchers from the University of Adelaide, is focused on uncovering the...

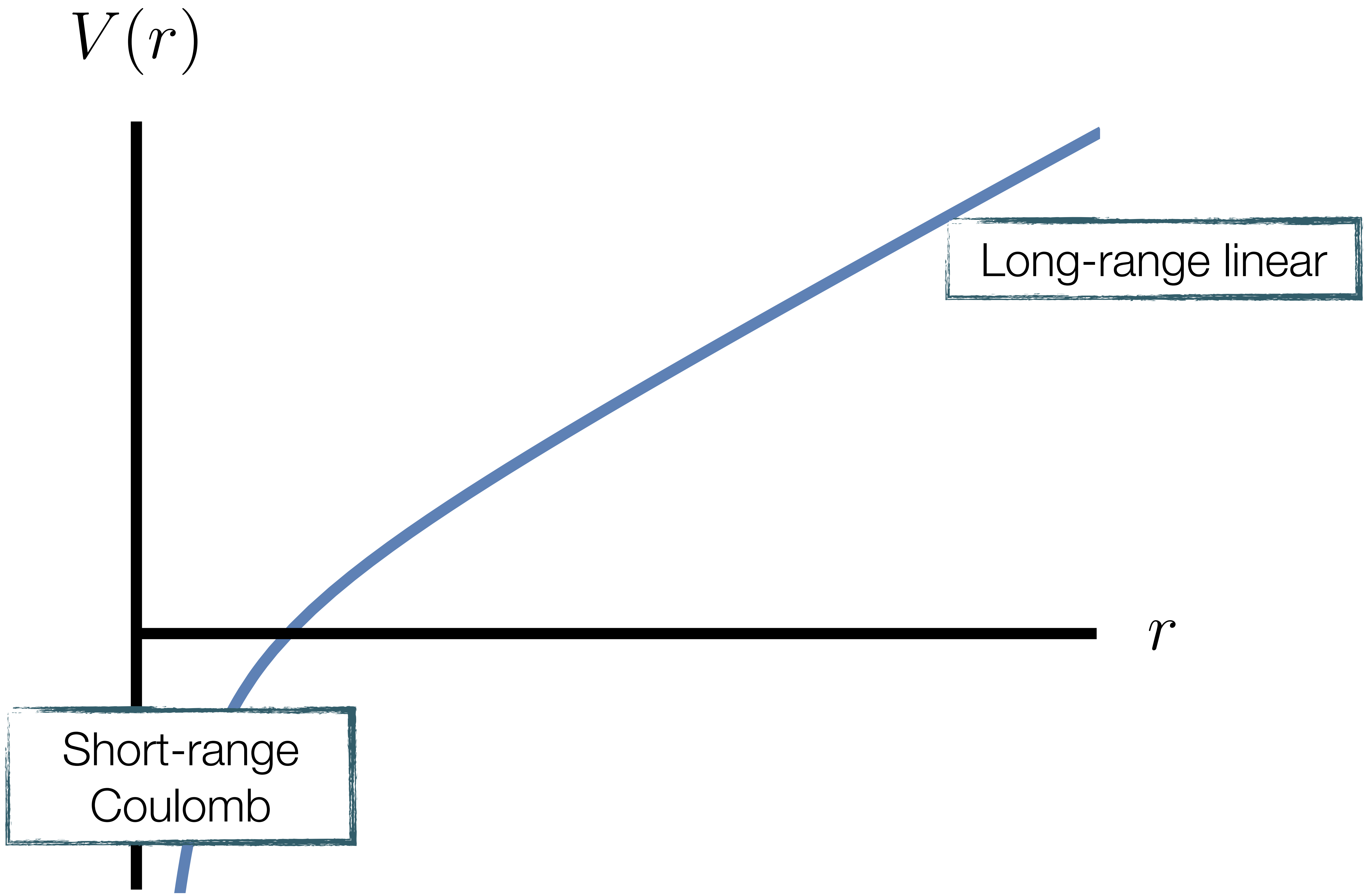
Mapping the forces inside protons
COSMOS magazine, 24 Feb 2025
You might be surprised to hear that there's anything "inside" a proton. But scientists have mapped the forces between the...

Illuminating the Inner Workings of the Proton
Sky Nightly , 24 Feb 2025
Illuminating the Inner Workings of the Proton by Simon Mansfield Sydney, Australia (SPX) Feb 21, 2025 A team of scientists has...

Force as strong as 10 compressed elephants rests inside a proton, suggests study
Interesting Engineering, 23 Feb 2025
To uncover the forces within a proton, the study authors used a computational technique called lattice quantum chromodynamics (La...

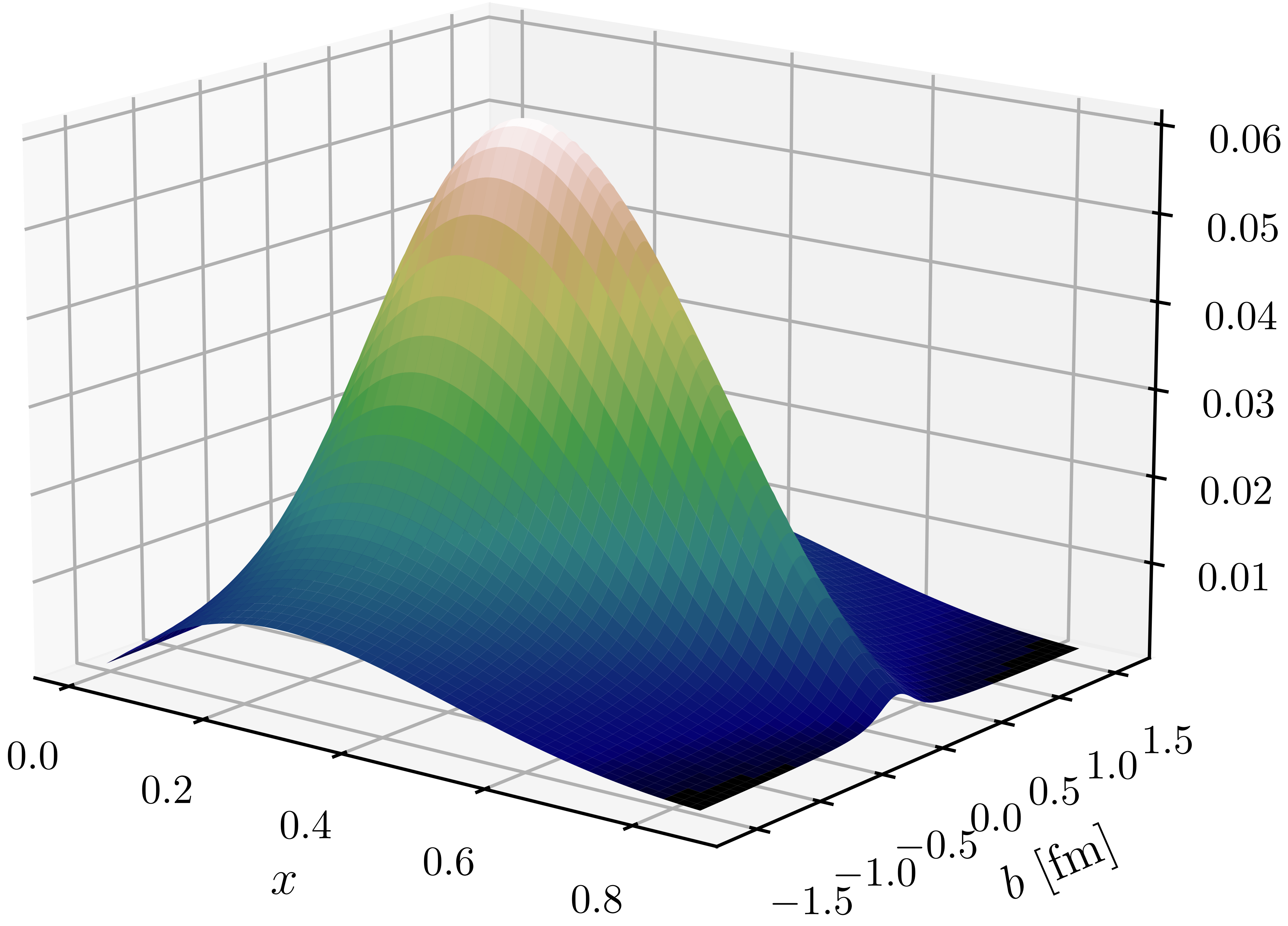
Illuminating the proton's inner workings
Science Daily, 21 Feb 2025
Scientists have now mapped the forces acting inside a proton, showing in unprecedented detail how quarks -- the tiny particles...

Confinement: static quark potential



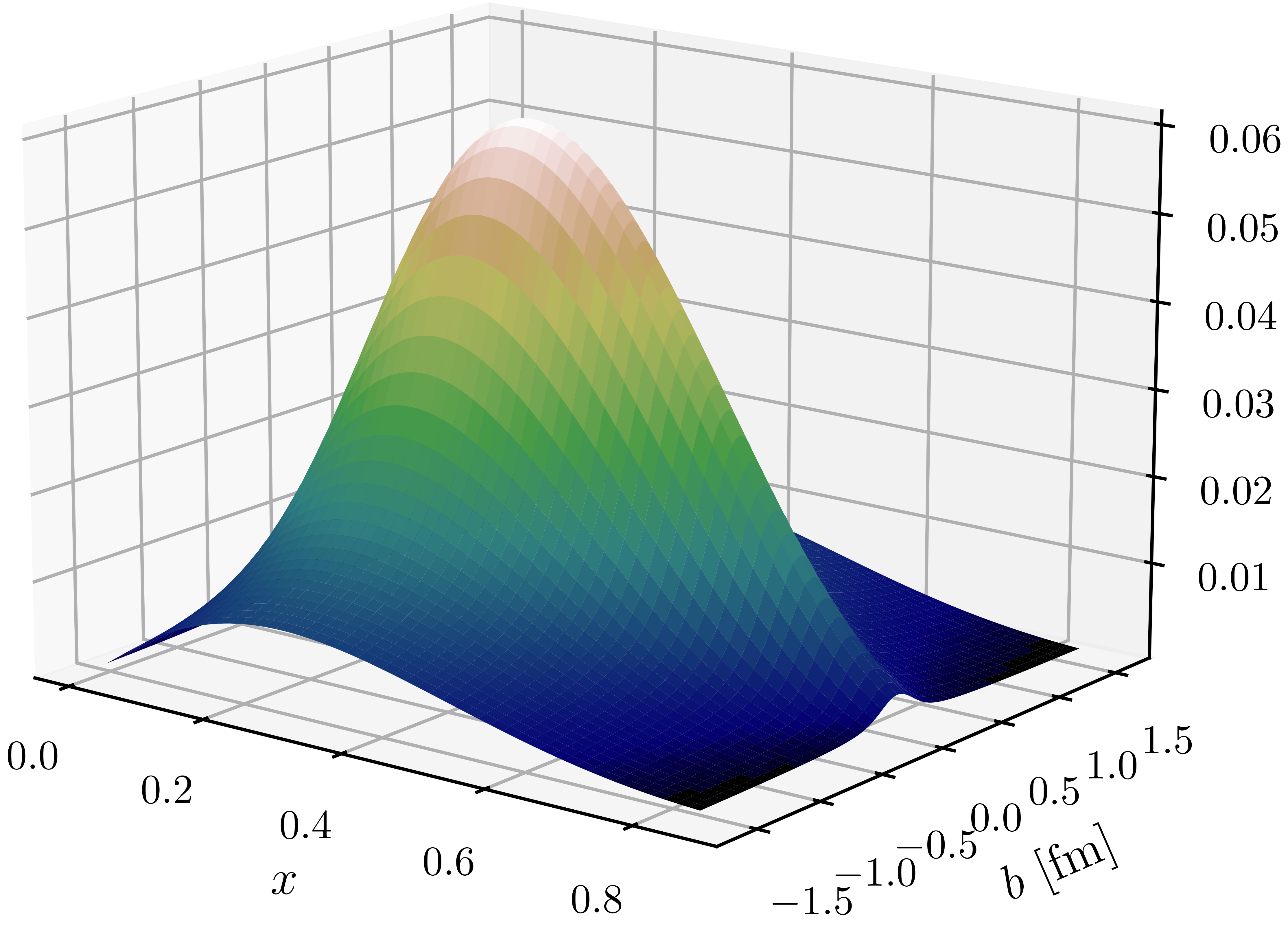
Charting the energy dependence between static quark-antiquark pair (in pure Yang-Mills)

Imaging nucleon structure



Recall: Generalised parton distributions
Describe the (longitudinal) **momentum**
and (transverse) **position** of quarks

Imaging nucleon structure



Recall: Generalised parton distributions
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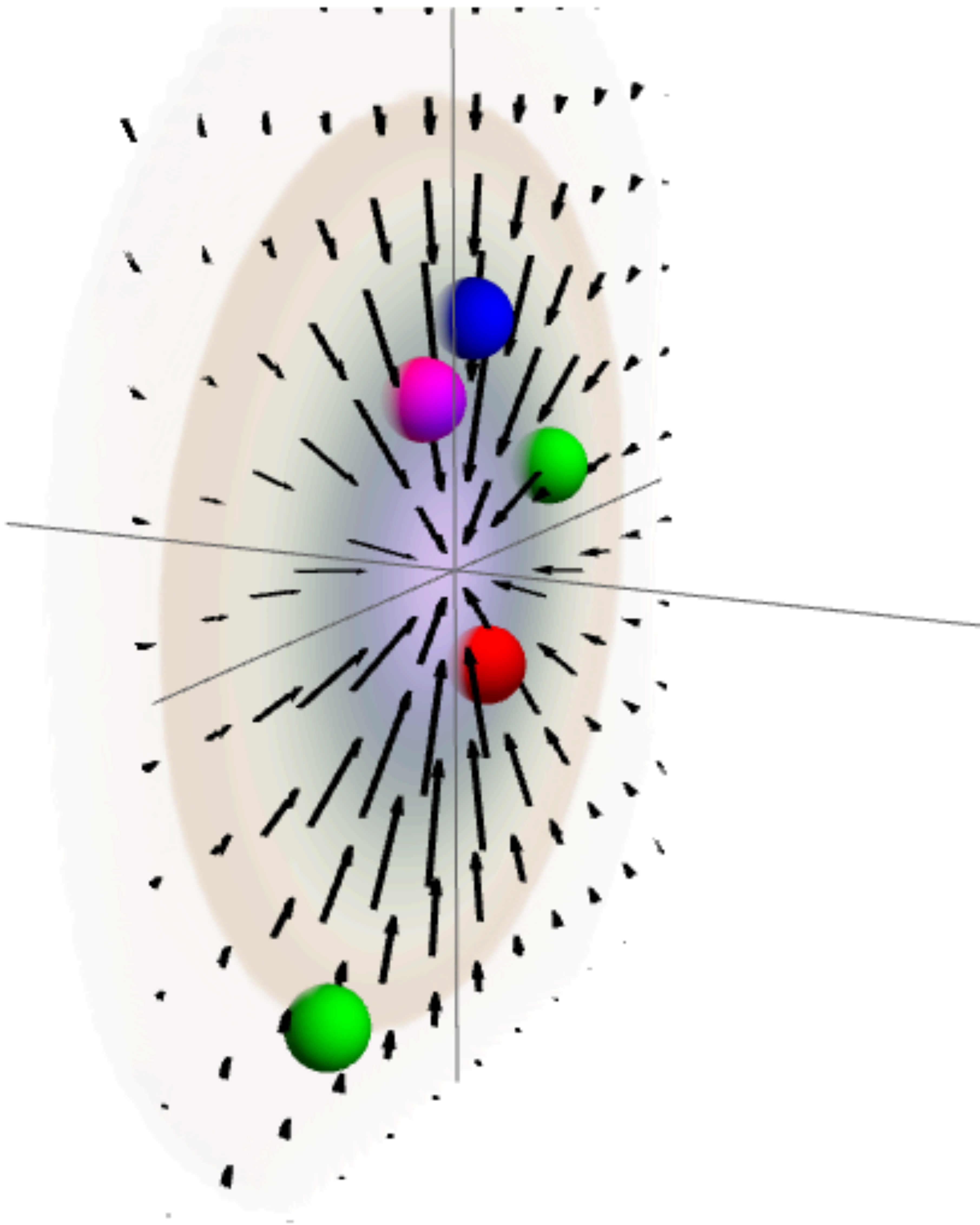
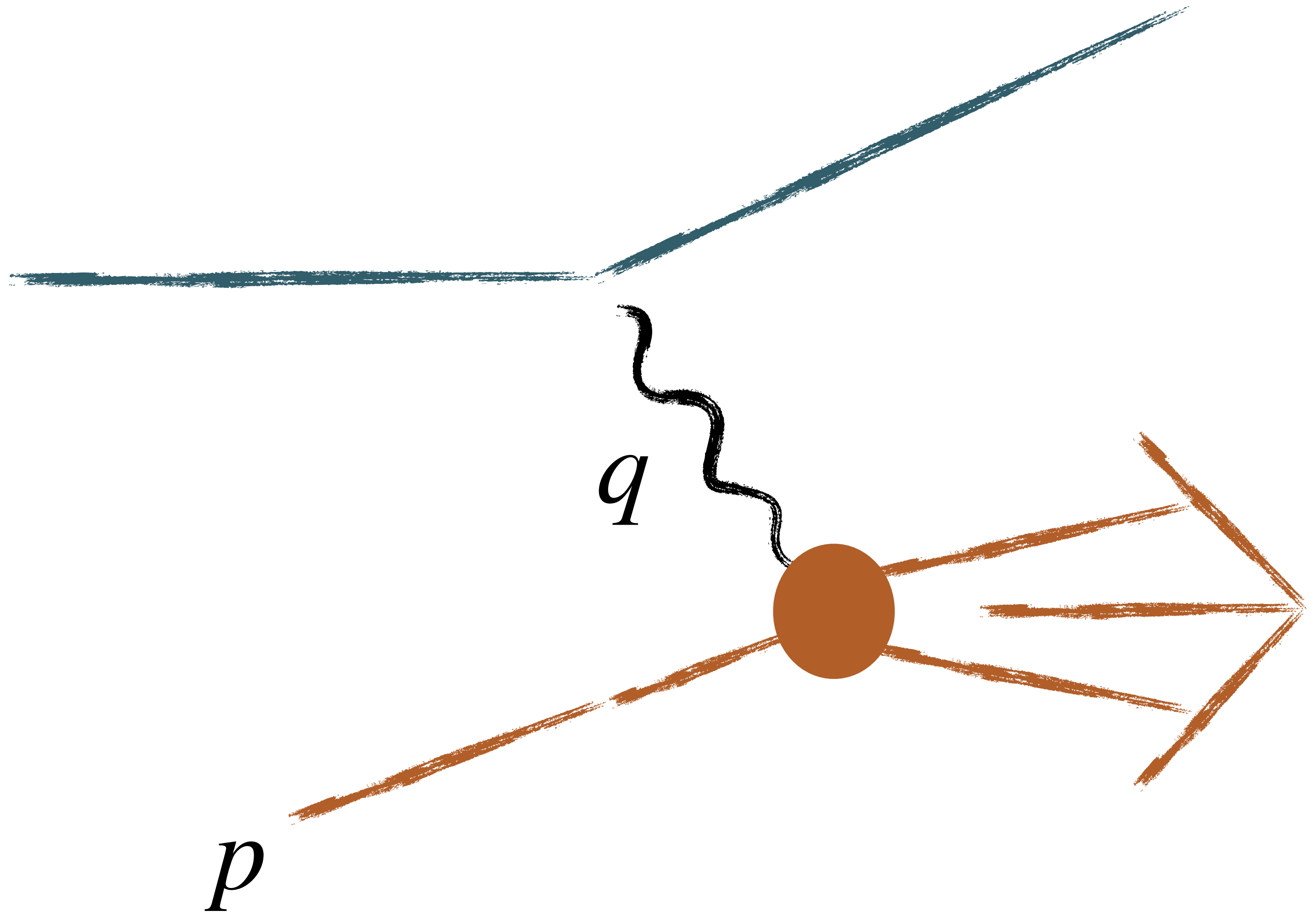


Can we go further to describe the
forces acting on these quarks?

Inelastic scattering

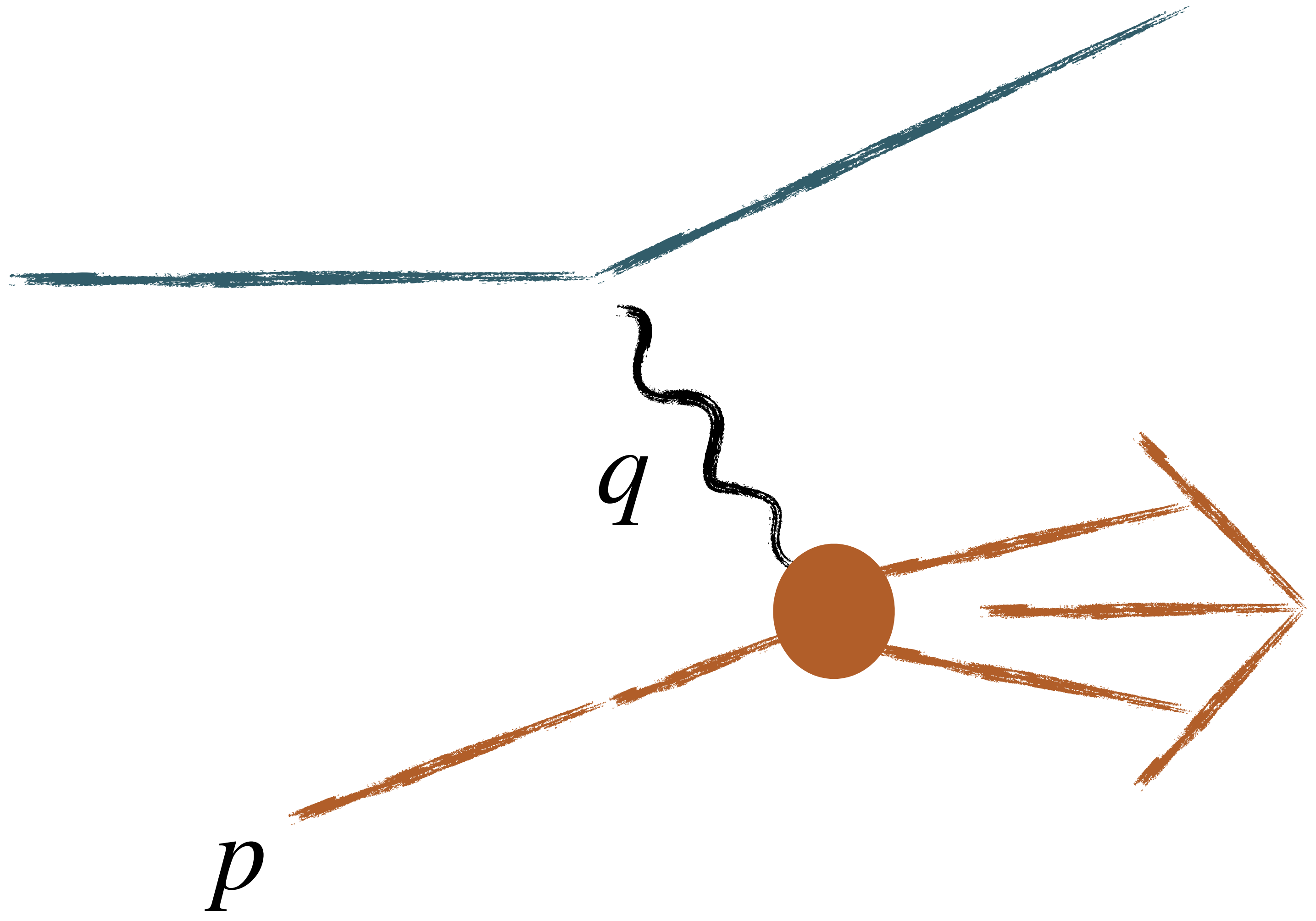


Hadron tensor

$$W_{\mu\nu} = F_1 \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{F_2}{p \cdot q} \left(p_\mu - \frac{p \cdot q \, q_\mu}{q^2} \right) \left(p_\nu - \frac{p \cdot q \, q_\nu}{q^2} \right)$$

$$+ \frac{i g_1}{p \cdot q} \, \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + \frac{i g_2}{(p \cdot q)^2} \, \epsilon_{\mu\nu\lambda\sigma} q^\lambda \left(p \cdot q \, s^\sigma - s \cdot q \, p^\sigma \right) \ ,$$

Inelastic scattering



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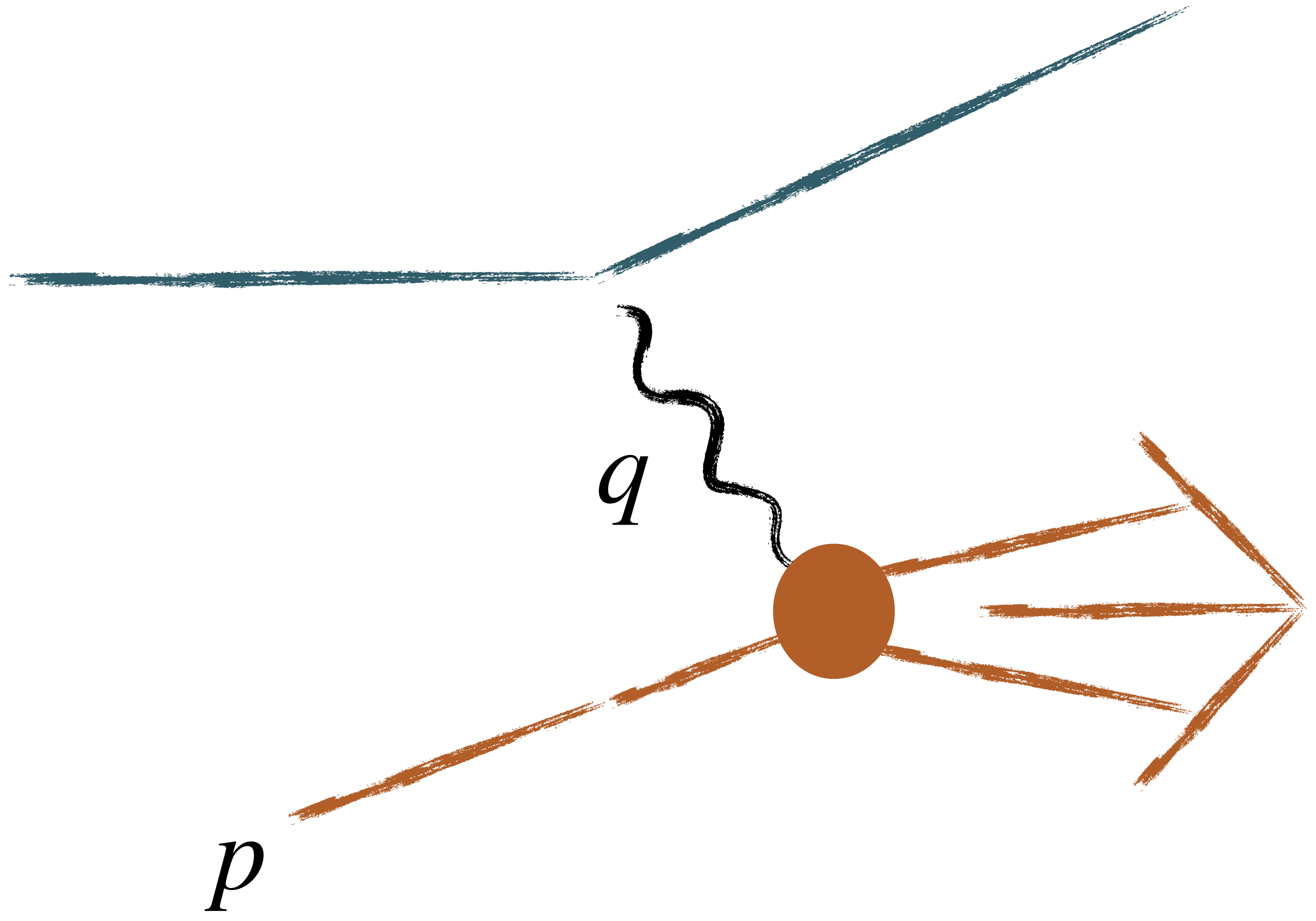
$$+ \frac{ig_1}{p\cdot q}\,\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{(p\cdot q)^2}\,\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left(p\cdot q\, s^\sigma - s\cdot q\, p^\sigma\right)\ ,$$

Scaling functions

In the deep inelastic region, large Q^2, these functions map onto the parton distributions

Inelastic scattering



Hadron tensor

$$W_{\mu\nu} = F_1\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{F_2}{p\cdot q}\left(p_\mu - \frac{p\cdot q\, q_\mu}{q^2}\right)\left(p_\nu - \frac{p\cdot q\, q_\nu}{q^2}\right)$$
$$+ \frac{ig_1}{p\cdot q}\,\epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + \frac{ig_2}{(p\cdot q)^2}\,\epsilon_{\mu\nu\lambda\sigma} q^\lambda \left(p\cdot q\, s^\sigma - s\cdot q\, p^\sigma\right)\ ,$$

Scaling functions

In the deep inelastic region, large Q^2, these functions map onto the parton distributions

g_2**: No simple partonic interpretation!**

Colour-Lorentz force

Burkardt, PRD, 2013

While no simple parton interpretation, moment of the g_2 structure function can be expressed in terms of a local matrix element

$$\int dx\, x^2 \bar{g}_2(x) = \frac{d_2}{3} \equiv \frac{1}{6} \sum_q e_q^2 d_2^q$$

where

$$d_2^q = \frac{1}{2MP^+P^+S^x} \langle P, S | \overline{\psi_q}(0) \gamma^+ g G^{+y}(0) \psi_q(0) | P, S \rangle$$

Quark current density coupled to colour-Lorentz force

$$G^{+y} = \frac{1}{\sqrt{2}} \left(G^{0y} + G^{zy} \right) = -\frac{1}{\sqrt{2}} \left[\vec{E}_c + \vec{v} \times \vec{B}_c \right]^y = -\frac{1}{\sqrt{2}} F^y!$$

Transverse densities

Electromagnetic current

$$\langle p', s' | \overline{\psi} \gamma^\mu \psi | p, s \rangle = \overline{u}(p', s') \left[\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2(t) \right] u(p, s)$$

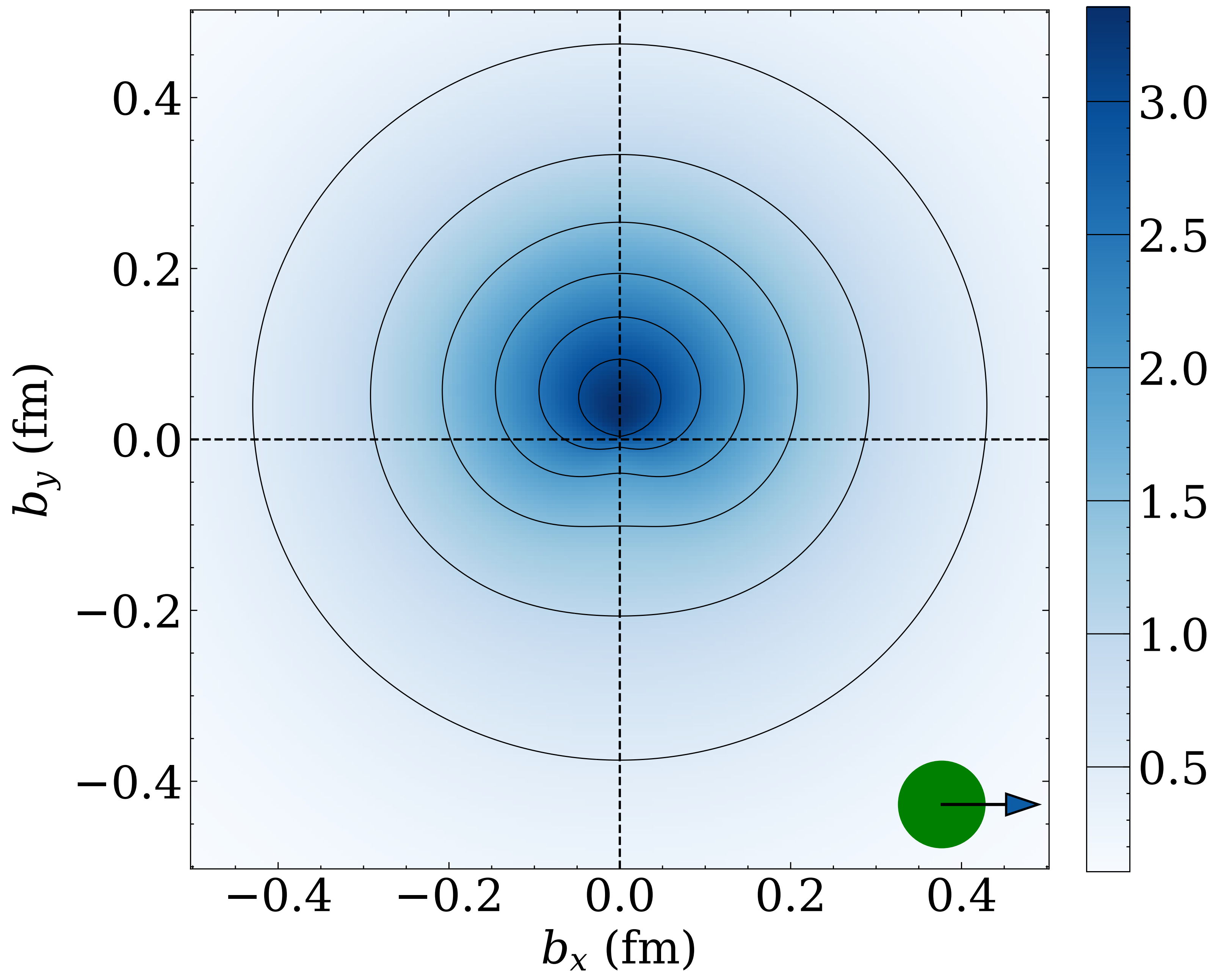
Dirac *Pauli*

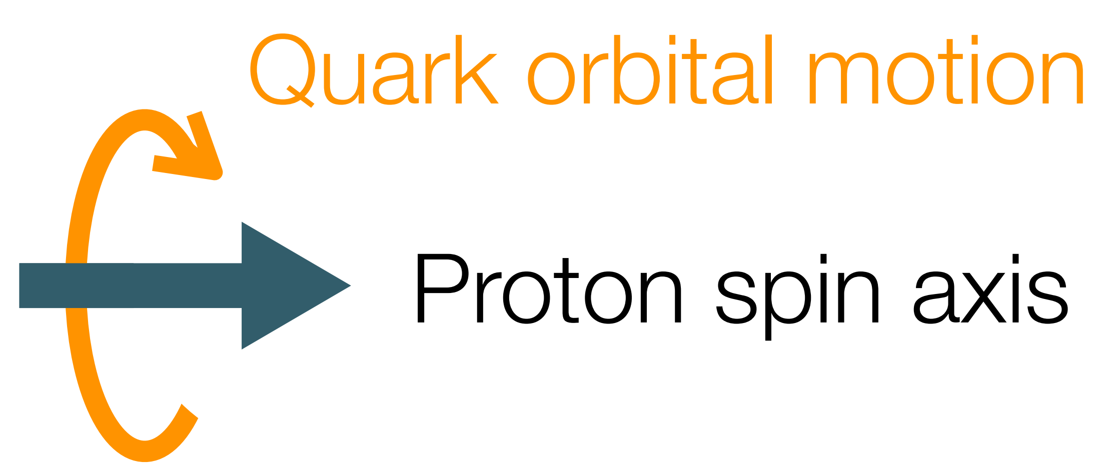
2D Fourier transforms

$$\tilde{F}_1(b^2),\ \tilde{F}_2(b^2)$$

Quark densities

$$q(\mathbf{b}) = \frac{1}{2} \left[\tilde{F}_1(b^2) + \frac{b^j \epsilon^{ji} S^i}{m_N} \frac{d}{db^2} \tilde{F}_2(b^2) \right]$$





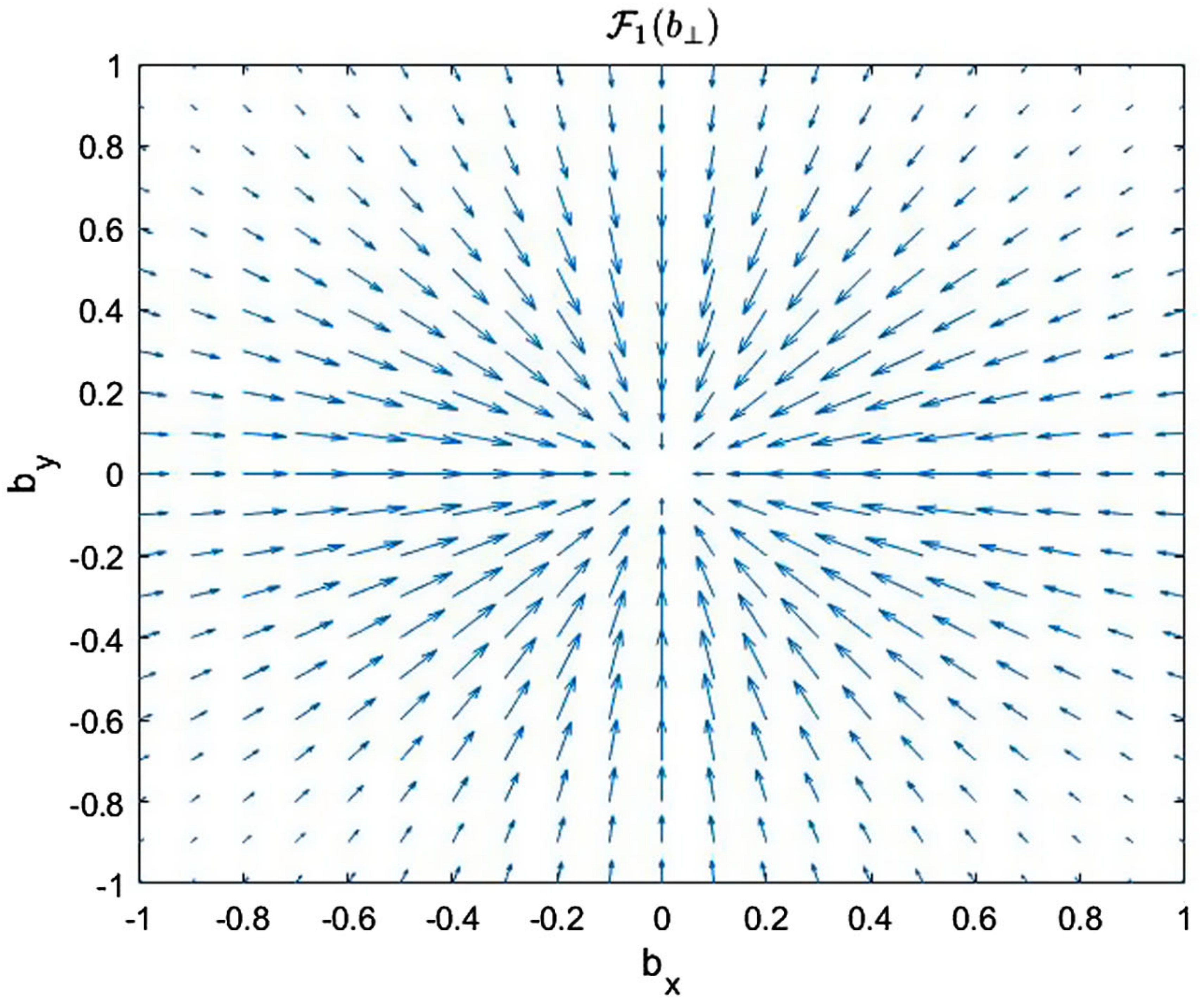
Twist-3 off-forward matrix elements

Aslan, Burkardt, Schlegel, PRD, 2019

$$\langle p', s'|\overline{\psi}\gamma^+ igG^{+i}\psi|p, s\rangle = \overline{u}(p', s')\Big[P^+\Delta^i\gamma^+\Phi_1(t) + MP^+ i\sigma^{+i}\Phi_2(t) + \frac{1}{M}P^+\Delta^i i\sigma^{+\Delta}\Phi_3(t)\Big]u(p, s),$$

where $P^\mu = (p' + p)^\mu/2$, $\Delta^\mu = (p' - p)^\mu$, $t = -\Delta^2$ and $\sigma^{\mu\Delta} = \sigma^{\mu\nu}\Delta_\nu$.

Off-forward matrix elements → Form factors → 2D Fourier transform → Force distributions in the transverse plane

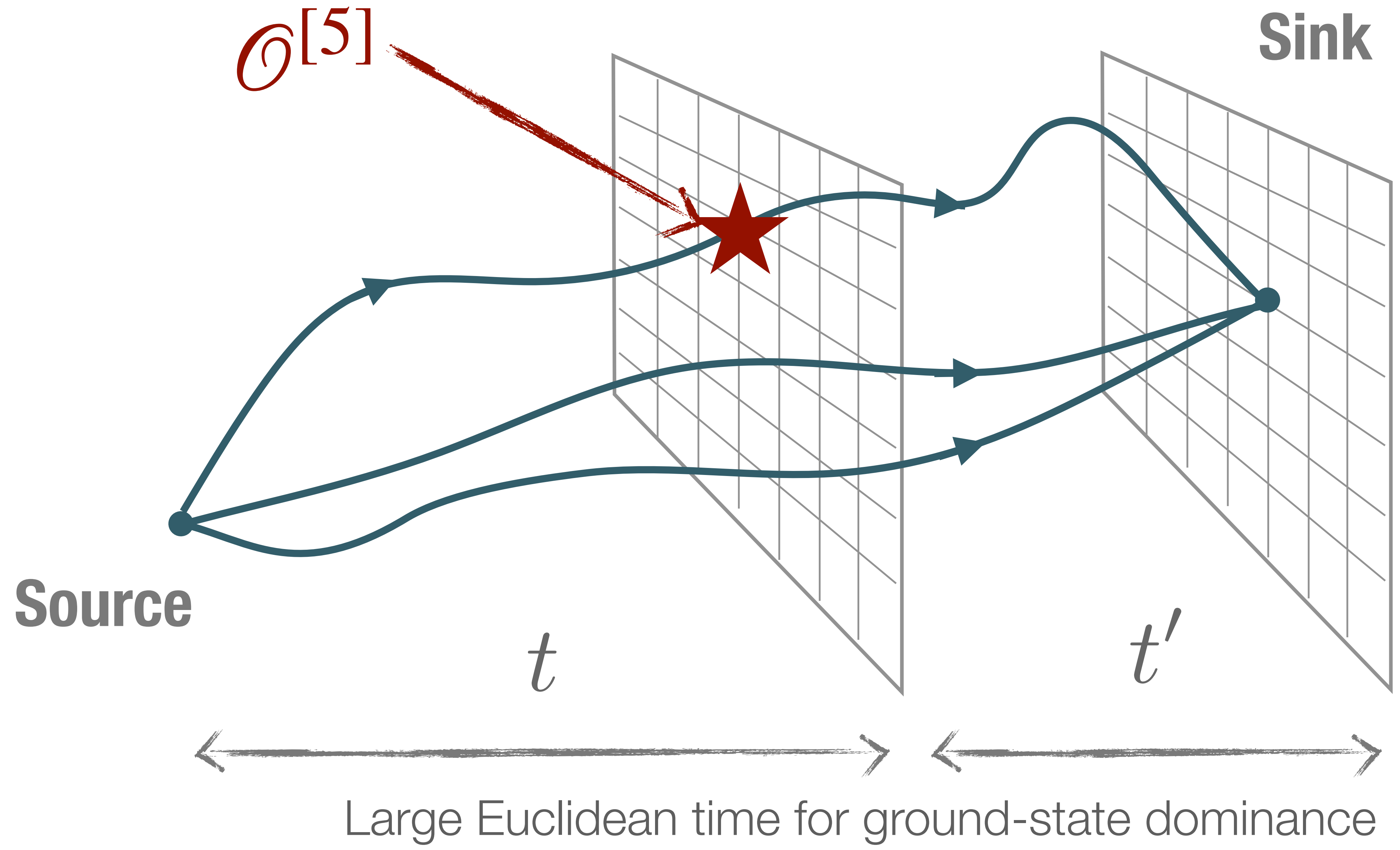


Recall: 3-point functions

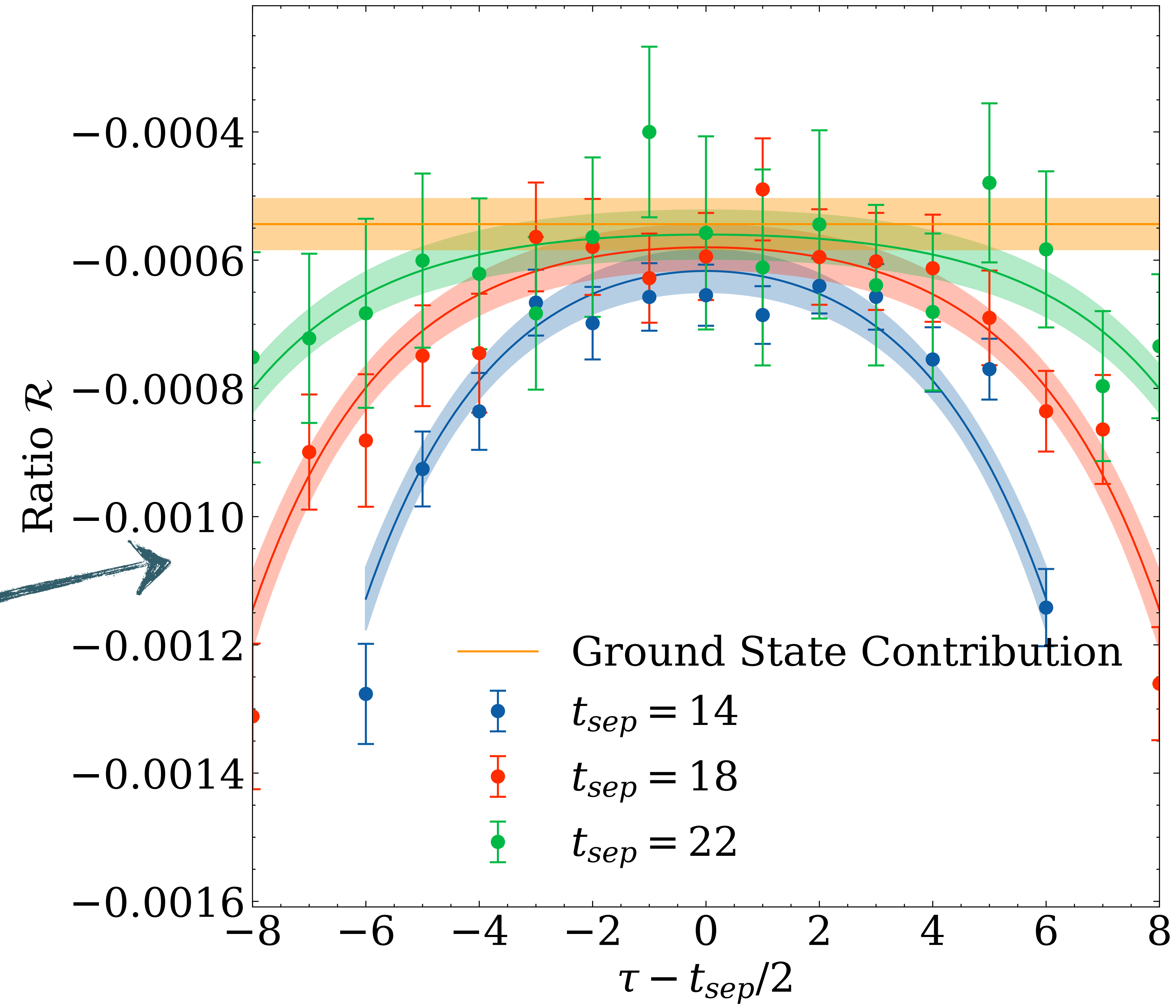
Compute 3-point correlators on the lattice

$$\mathcal{O}^{[5]}_{[i\{j]4\}} = -\frac{g}{6}\overline{\psi}\left(\tilde{G}_{ij}\gamma_4 + \tilde{G}_{i4}\gamma_j\right)\psi - \text{traces}$$

$$\frac{\langle C_3(t,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|J|N\rangle$$



3 source–sink separations to establish ground-state dominance



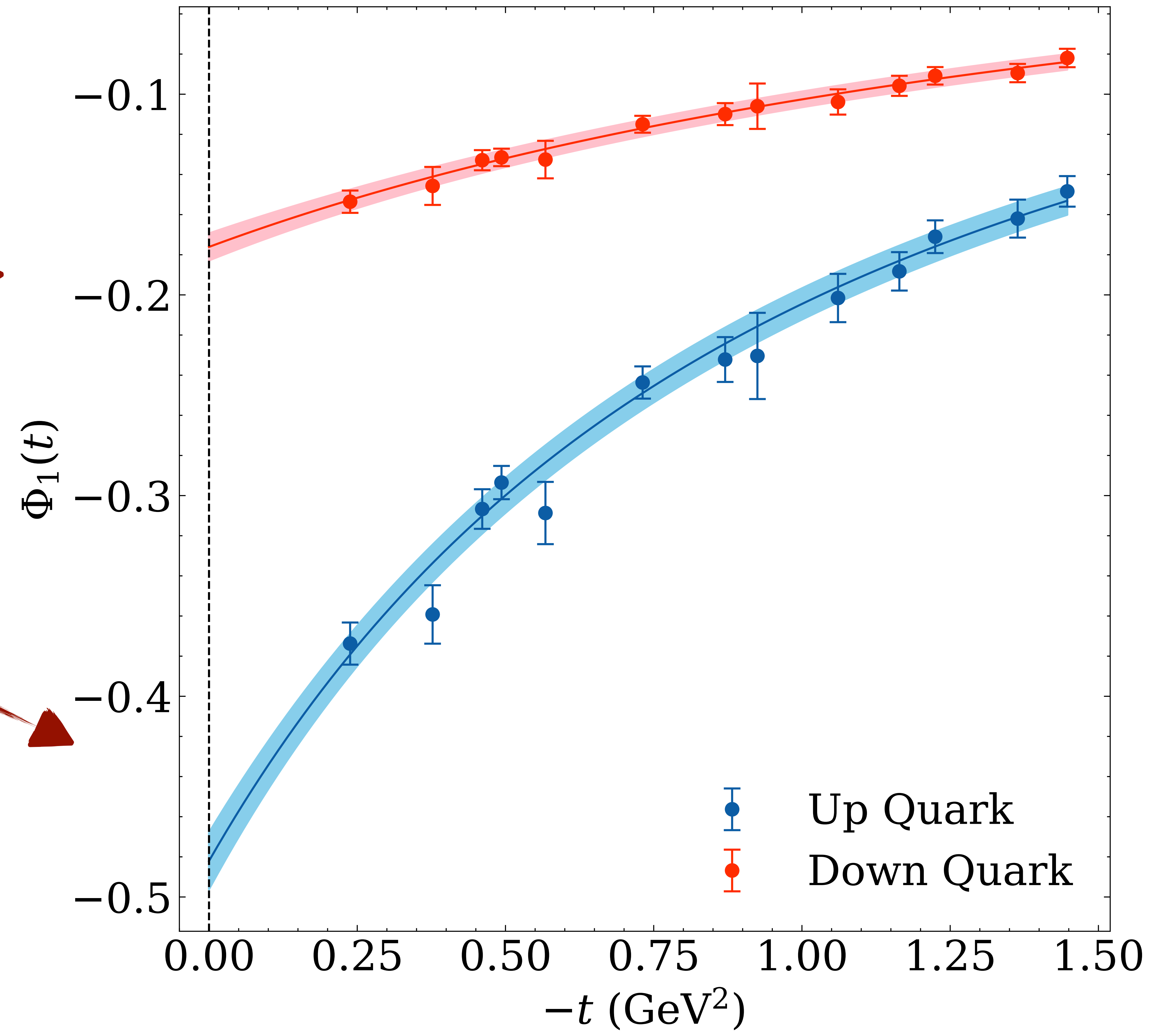
Φ_1 form factor

- Φ_1: isotropic force distribution
- Dipole fits to lattice results

Negative form factors
$\Longrightarrow$ *attractive forces*

$m_\pi \sim 410\,\mathrm{GeV}$

$\beta = 5.95\ (a \sim 0.052\,\mathrm{fm})$



$$\langle p', s' | \overline{\psi} \gamma^+ i g G^{+i} \psi | p, s \rangle = \overline{u}(p', s') \Big[P^+ \Delta^i \gamma^+ \Phi_1(t) + M P^+ i \sigma^{+i} \Phi_2(t) + \frac{1}{M} P^+ \Delta^i i \sigma^{+\Delta} \Phi_3(t) \Big] u(p, s),$$

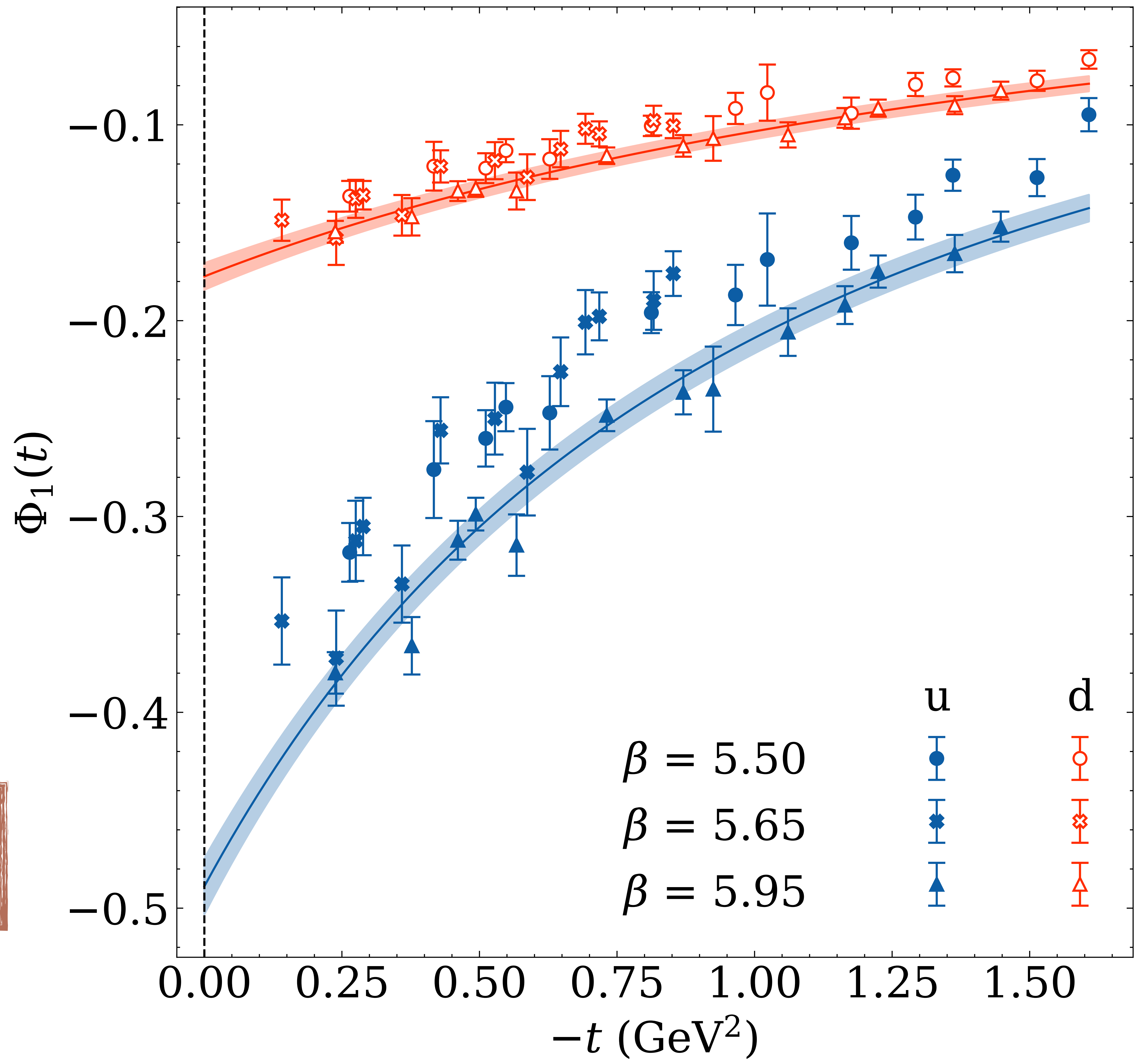
Discretisation effects

Extract form factors at 3 lattice spacings,

$$a \sim 0.74, 0.68, 0.52\,\mathrm{fm}$$

$$m_\pi \sim 410\,\mathrm{GeV}$$

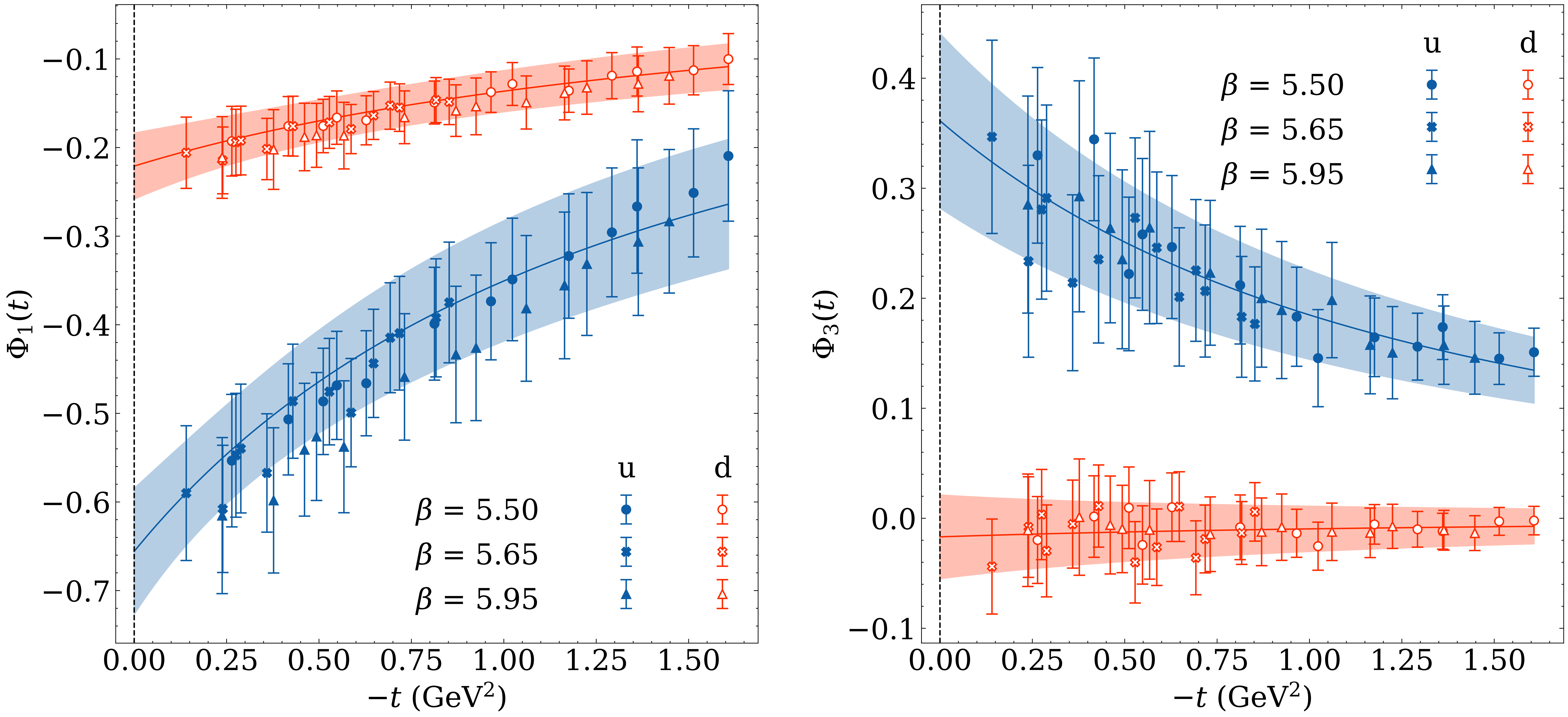
Some tension between different lattices; mostly in overall normalisation



Global fit

Model a dependence in magnitude and slope

$$\Phi_i(t,a) = \frac{\Phi_i(0) + b_i a}{\left(1 + t\left(\frac{1}{\Lambda_i^2} + c_i a\right)\right)^2},$$



Error bars here include estimate for $a \to 0$

Force densities

2D Fourier transform to impact parameter space

Quark densities

$$q(\mathbf{b}) = \frac{1}{2}\left[\tilde{F}_1(b^2) + \frac{b^j\epsilon^{ji}S^i}{m_N}\frac{d}{db^2}\tilde{F}_2(b^2)\right]$$

Force densities

$$\mathscr{F}^i_{ss'}(\mathbf{b}) = -2\sqrt{2}P^+\frac{d}{db^2}\tilde{\Phi}_1(b^2) + \sqrt{2}m_N\epsilon^{ij}S^j\tilde{\Phi}_2(b^2) - \frac{2\sqrt{2}\epsilon^{jk}S^k}{m_N}\left[\delta^{ij}\frac{d}{db^2} + 2b^ib^j\frac{d^2}{(db^2)^2}\right]\tilde{\Phi}_3(b^2)$$

Force densities

2D Fourier transform to impact parameter space

Unpolarised

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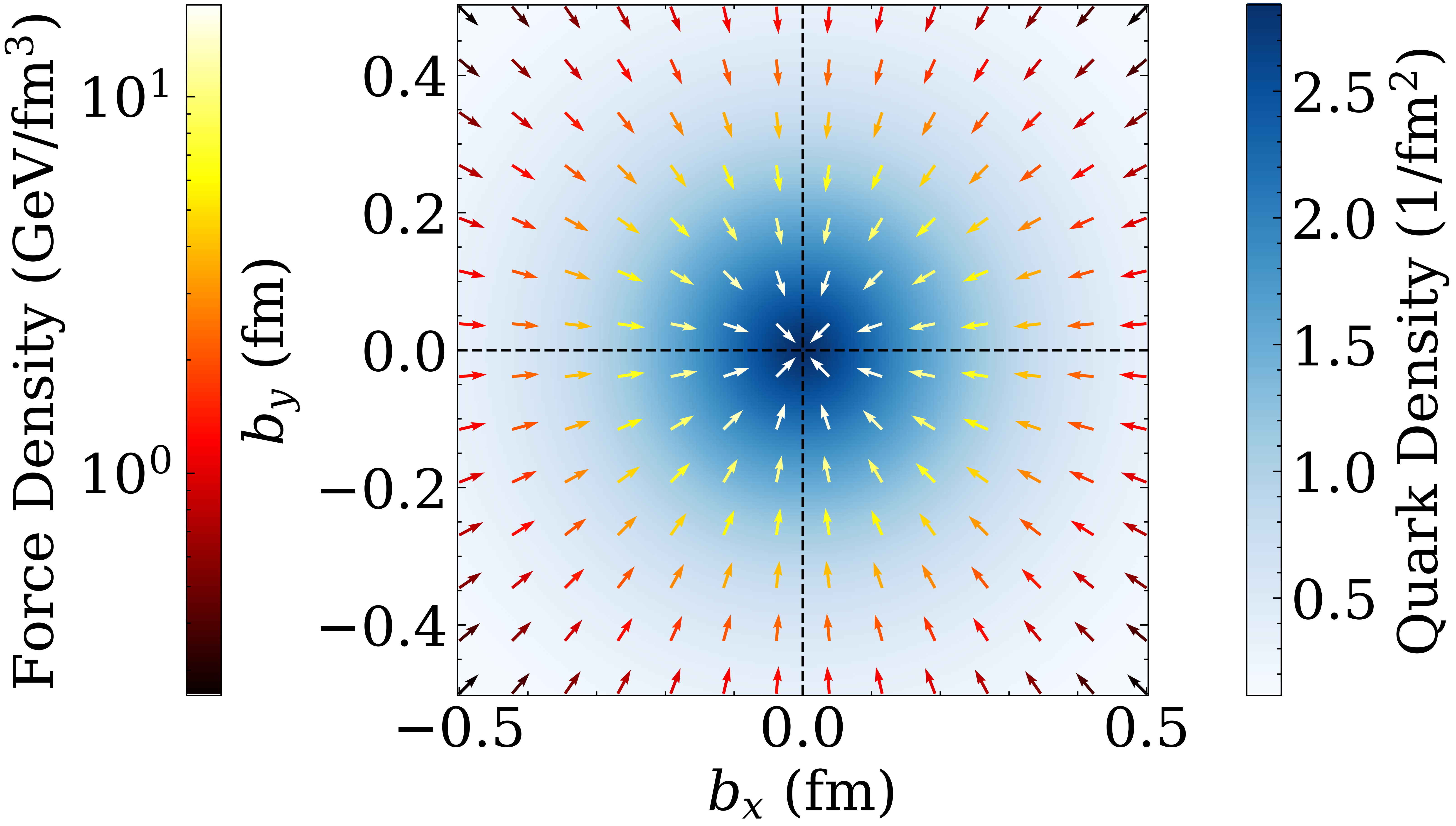

Spherically symmetric

Force densities

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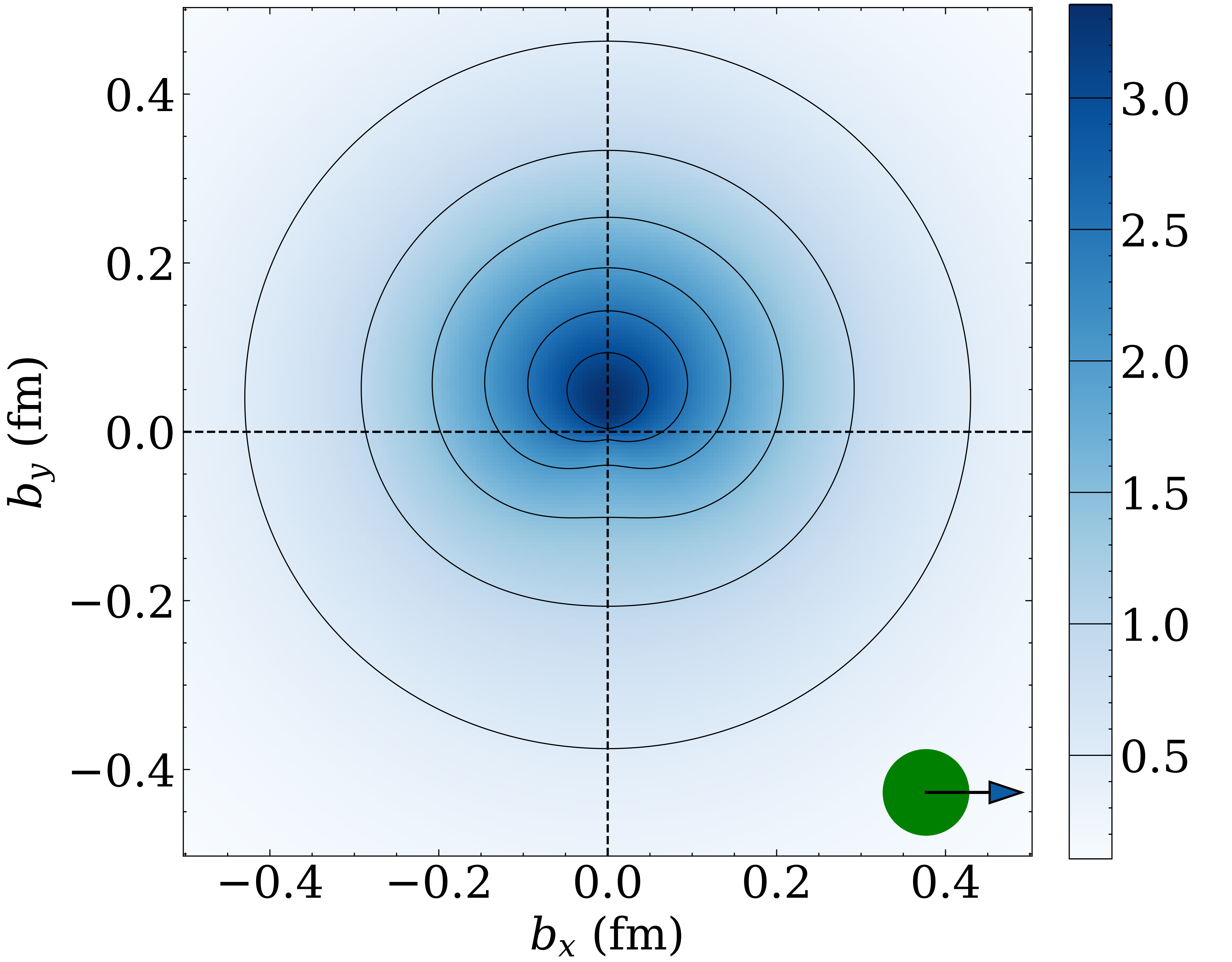
Force densities

2D Fourier transform to impact parameter space

Transversely polarised
densities sensitive to
impact parameter

→ **distorted distributions**



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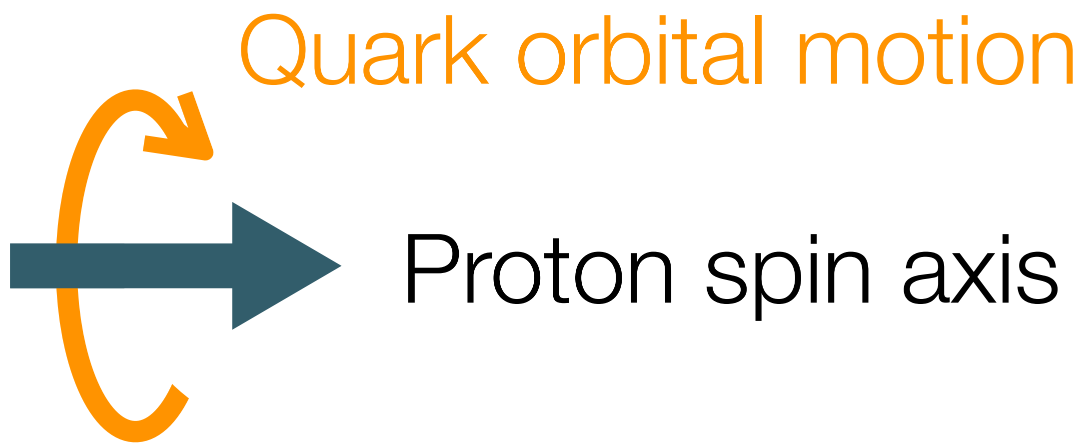

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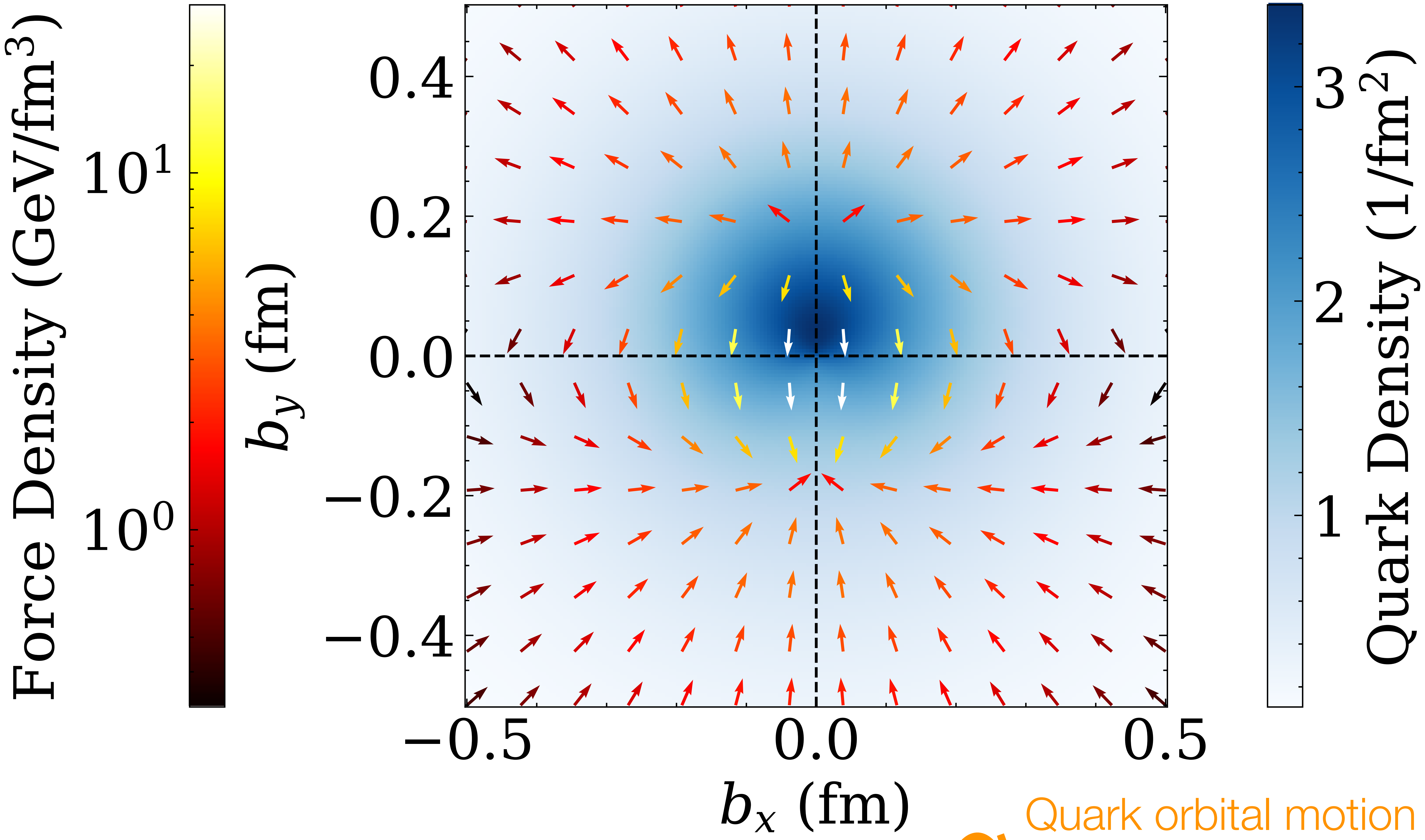
Force densities

2D Fourier transform to impact parameter space

Transversely polarised densities sensitive to impact parameter

➡ **distorted distributions**



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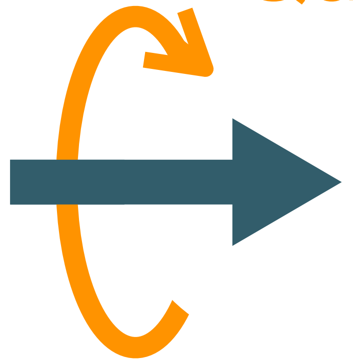

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Local forces

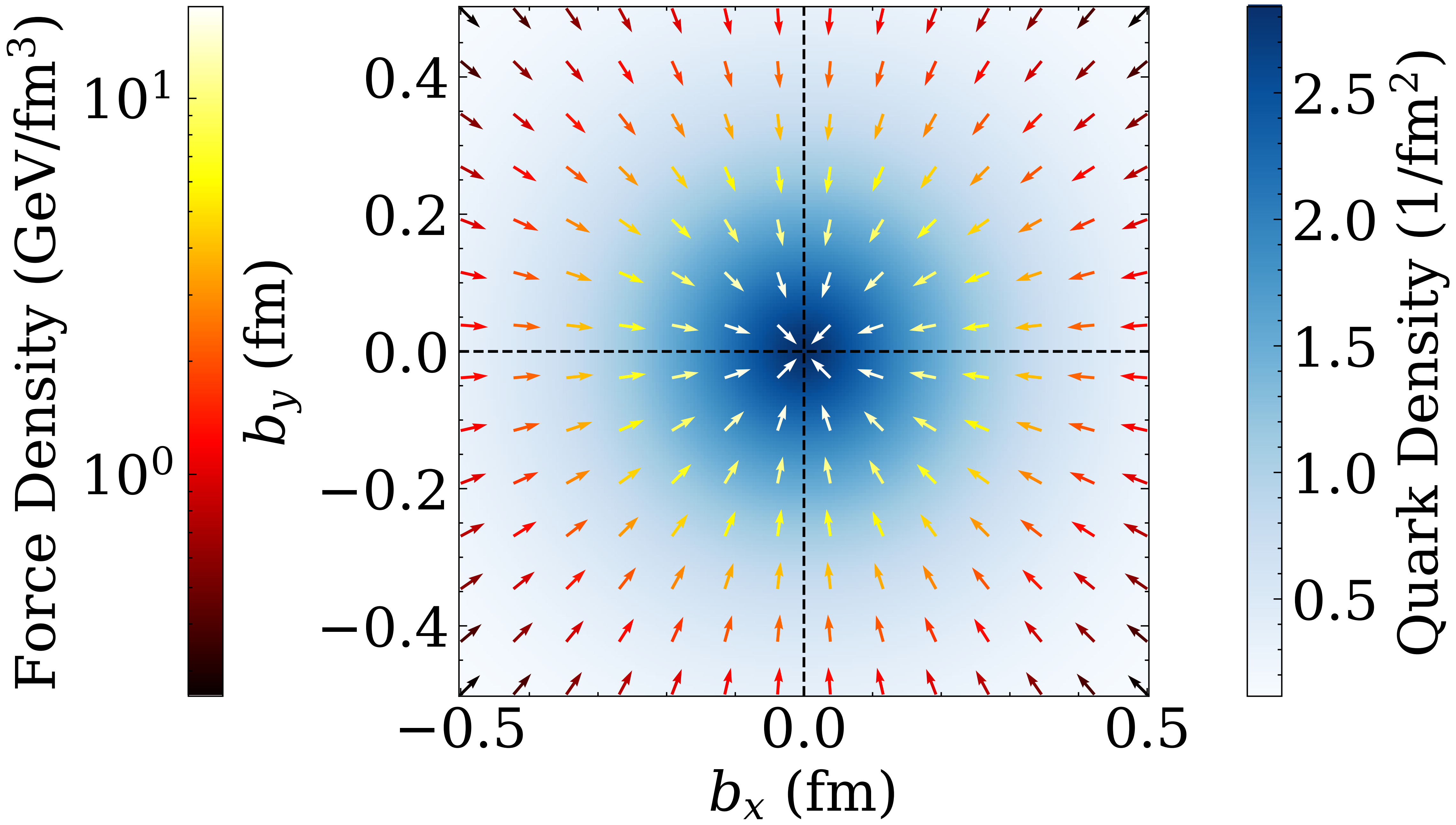
Dividing out the quark densities

Force densities = “quark density” x “force”

$$\mathscr{F} \sim \langle \rho F \rangle$$

Motivates modelling the local forces as:

$$F \sim \frac{\langle \rho F \rangle}{\langle \rho \rangle}$$

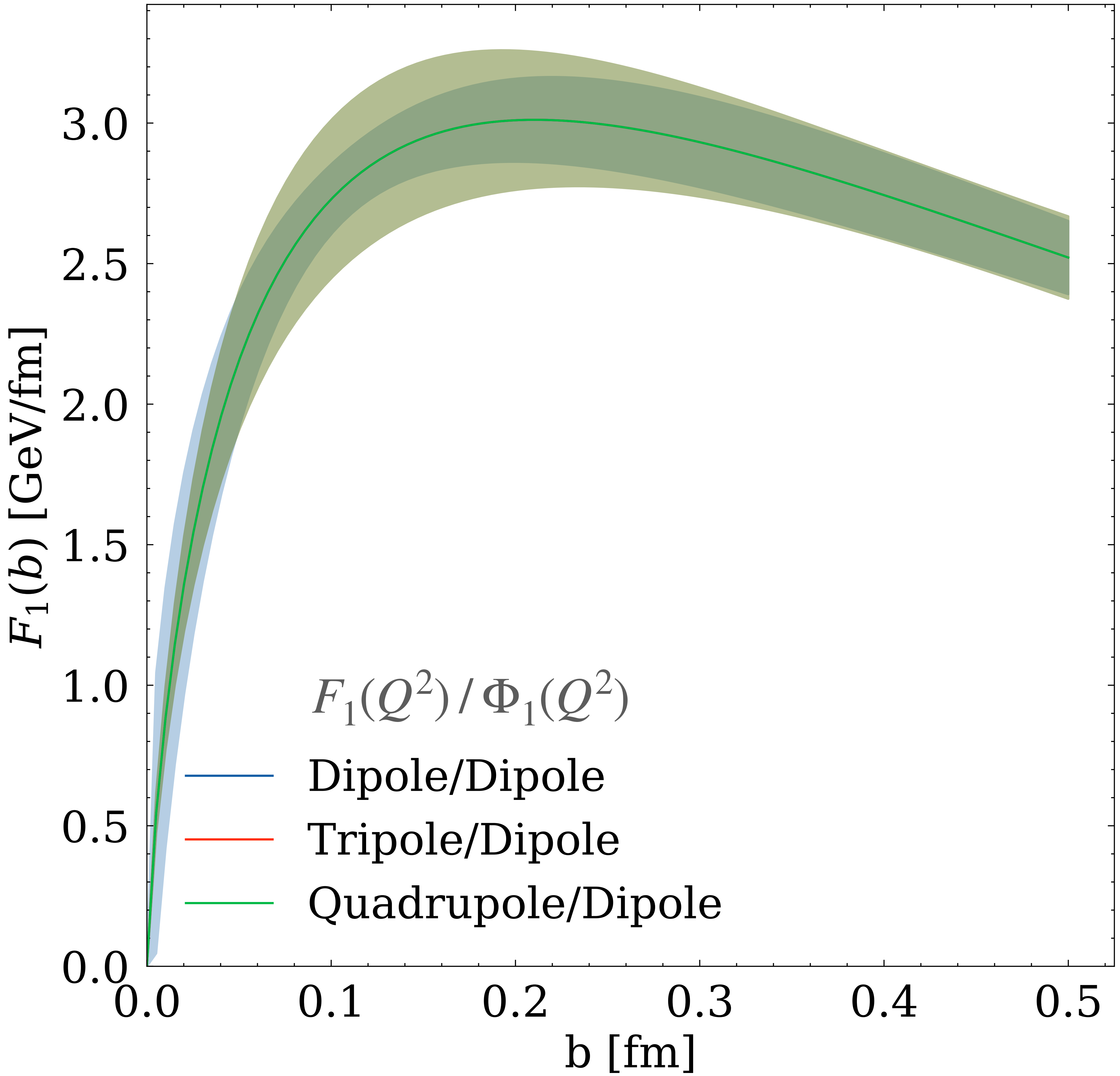


Local forces

spin independent Φ_1

$$\sim \frac{\langle \overline{\psi} \gamma^+ F^{+b} \psi \rangle}{\langle \overline{\psi} \gamma^+ \psi \rangle}$$

Strong forces at intermediate distances



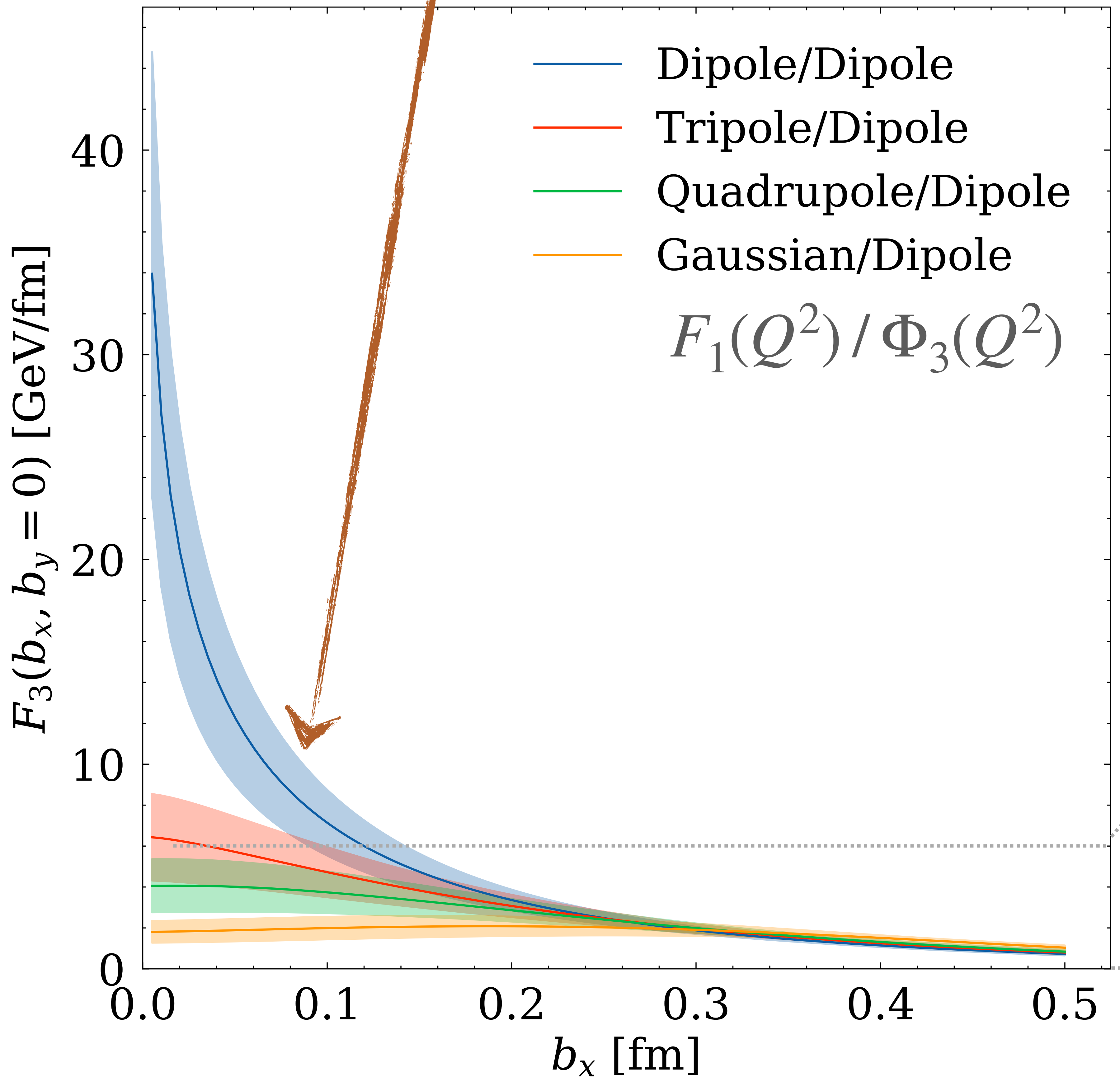
Does this resemble the static quark potential anyone?

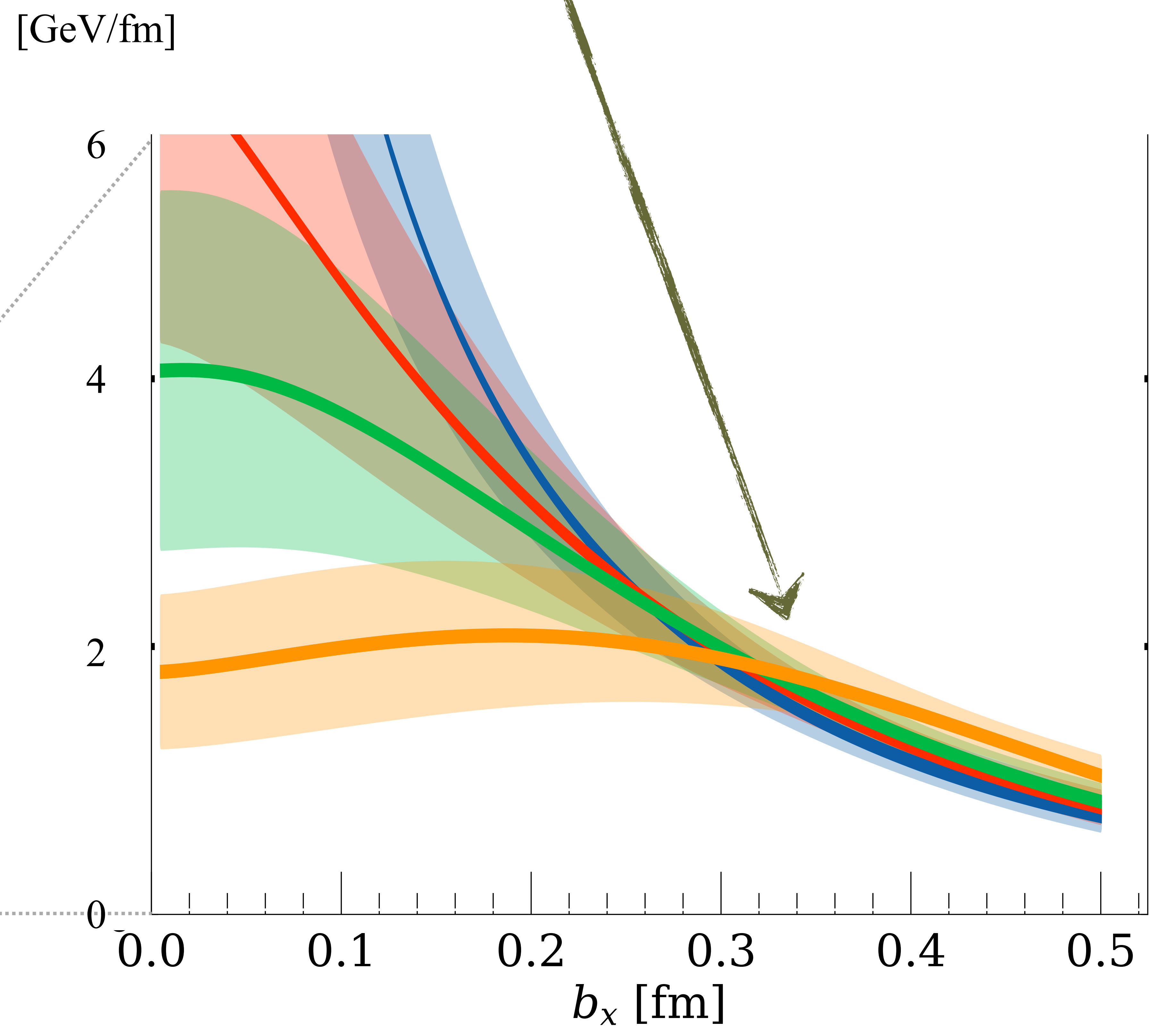
Local forces spin dependent Φ_3

$$\sim \frac{\langle \bar{\psi}\gamma^+ F^{+j}\psi\rangle}{\langle \bar{\psi}\gamma^+\psi\rangle}$$

Short distance behaviour quite sensitive to model: need good FFs at large Q^2

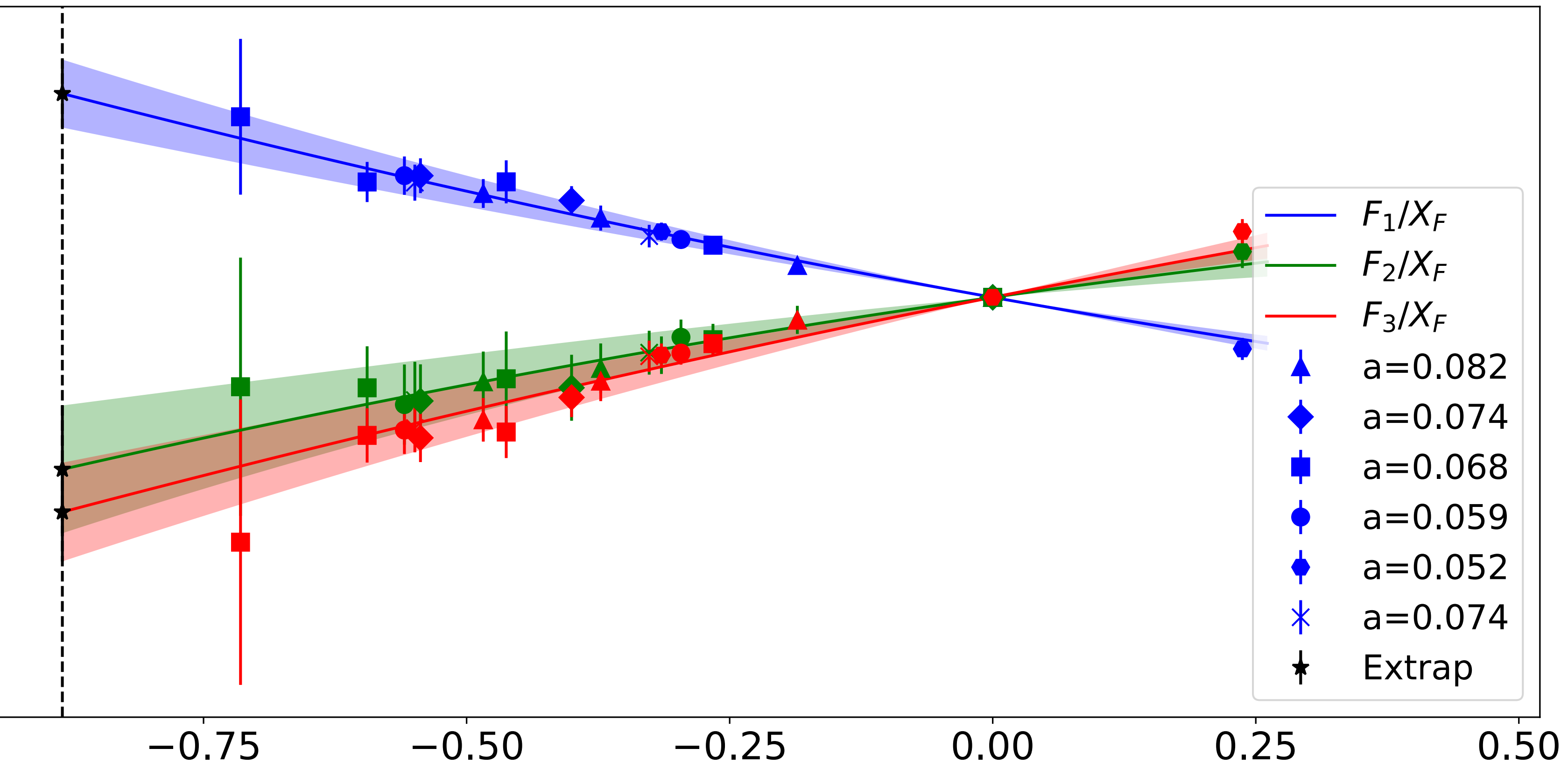
Asymptotic trend? … need better FFs at smaller Q^2

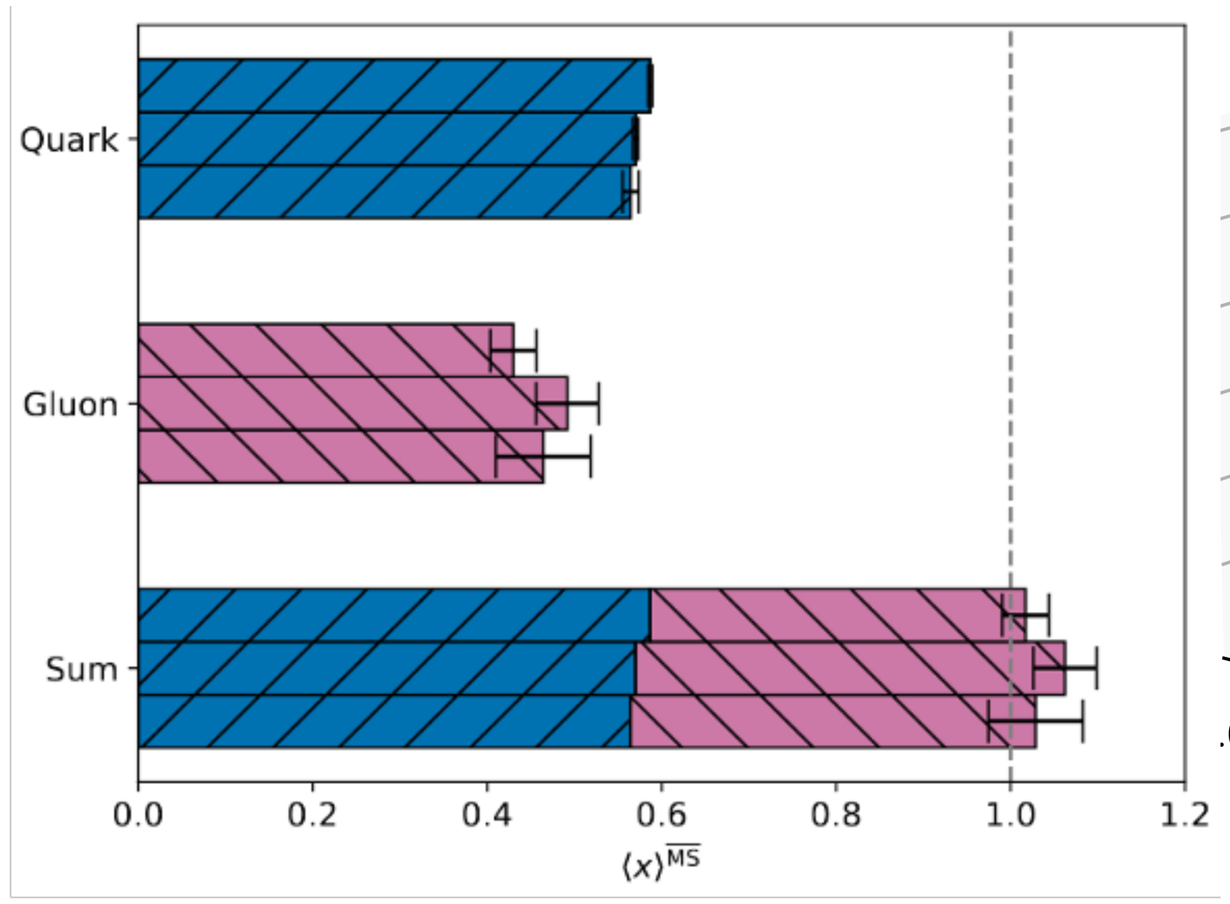


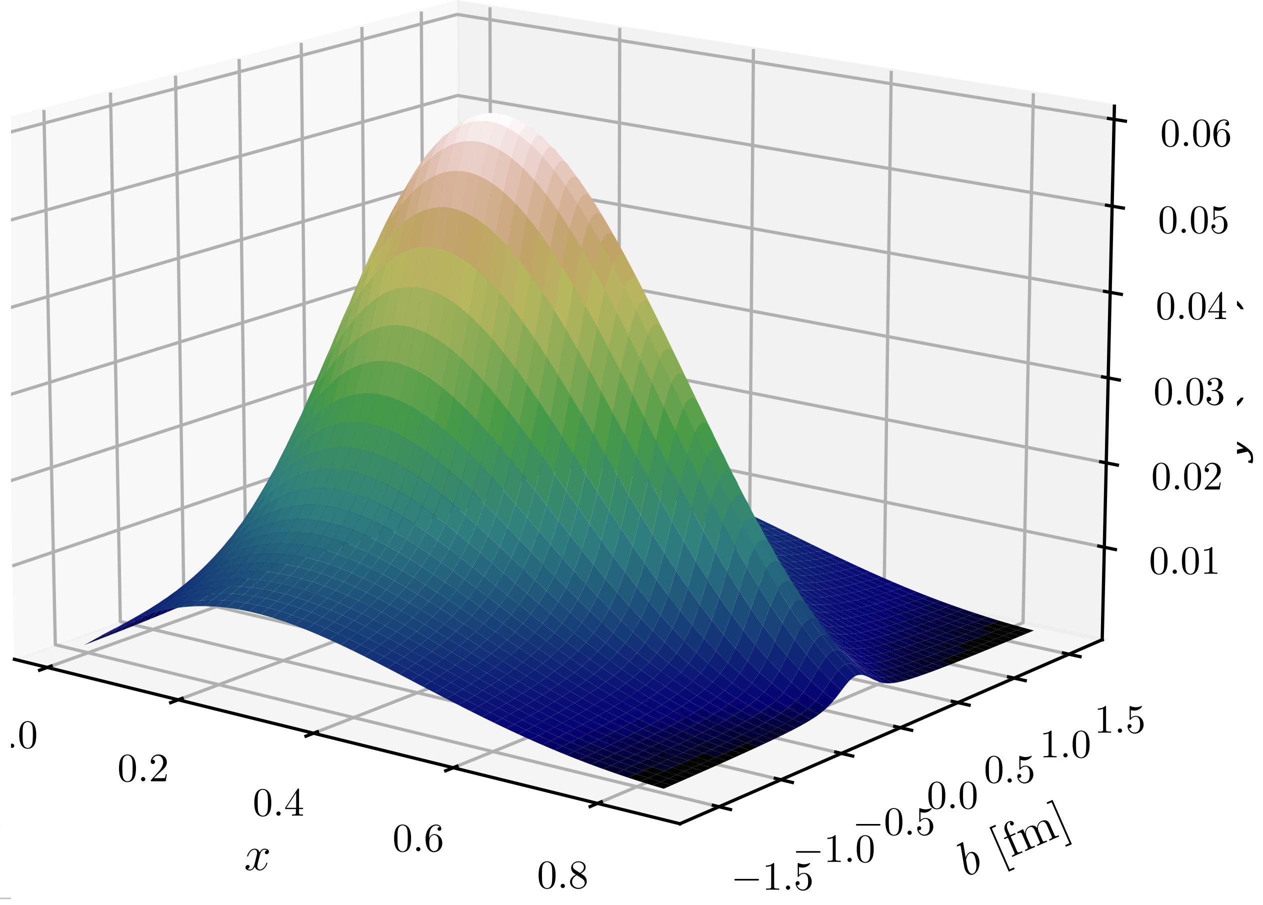


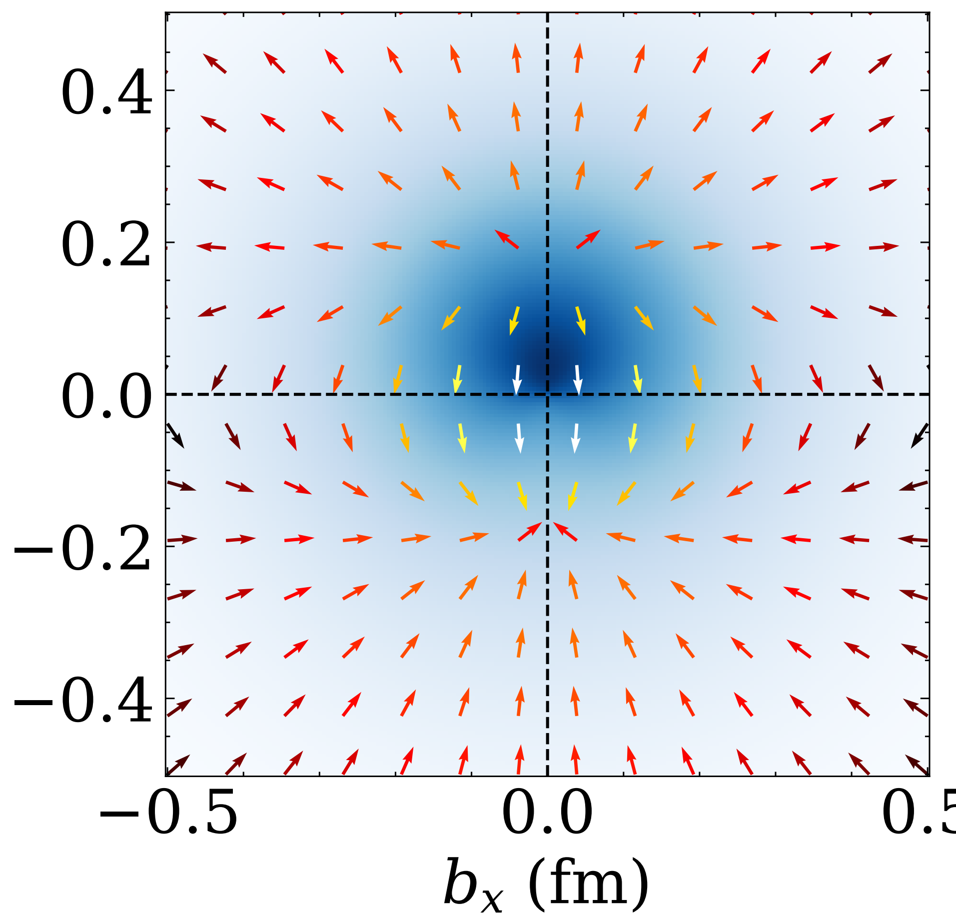
Sketch magnitude of force along b_x (fixed $b_y = 0$)

Summary and outlook







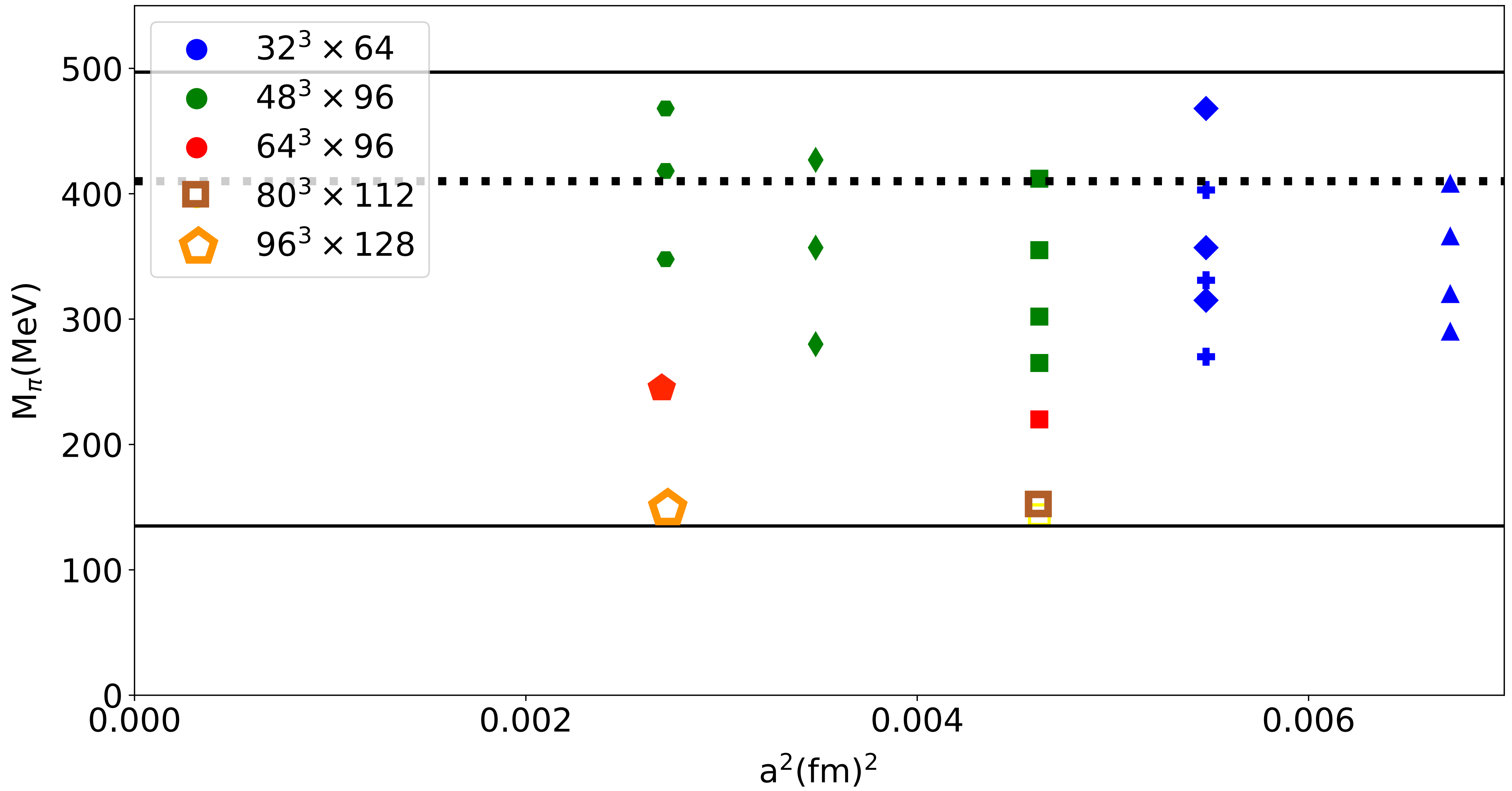


Precision results for hadronic structure observables now becoming possible

Feynman-Hellmann offers competitive alternative to n-point functions for some observables

Twist-3 matrix elements offer new opportunities to probe interaction dynamics of partons

Future improvements: ensembles with near-physical quark masses and $4 \lesssim m_\pi L$



BACKUP

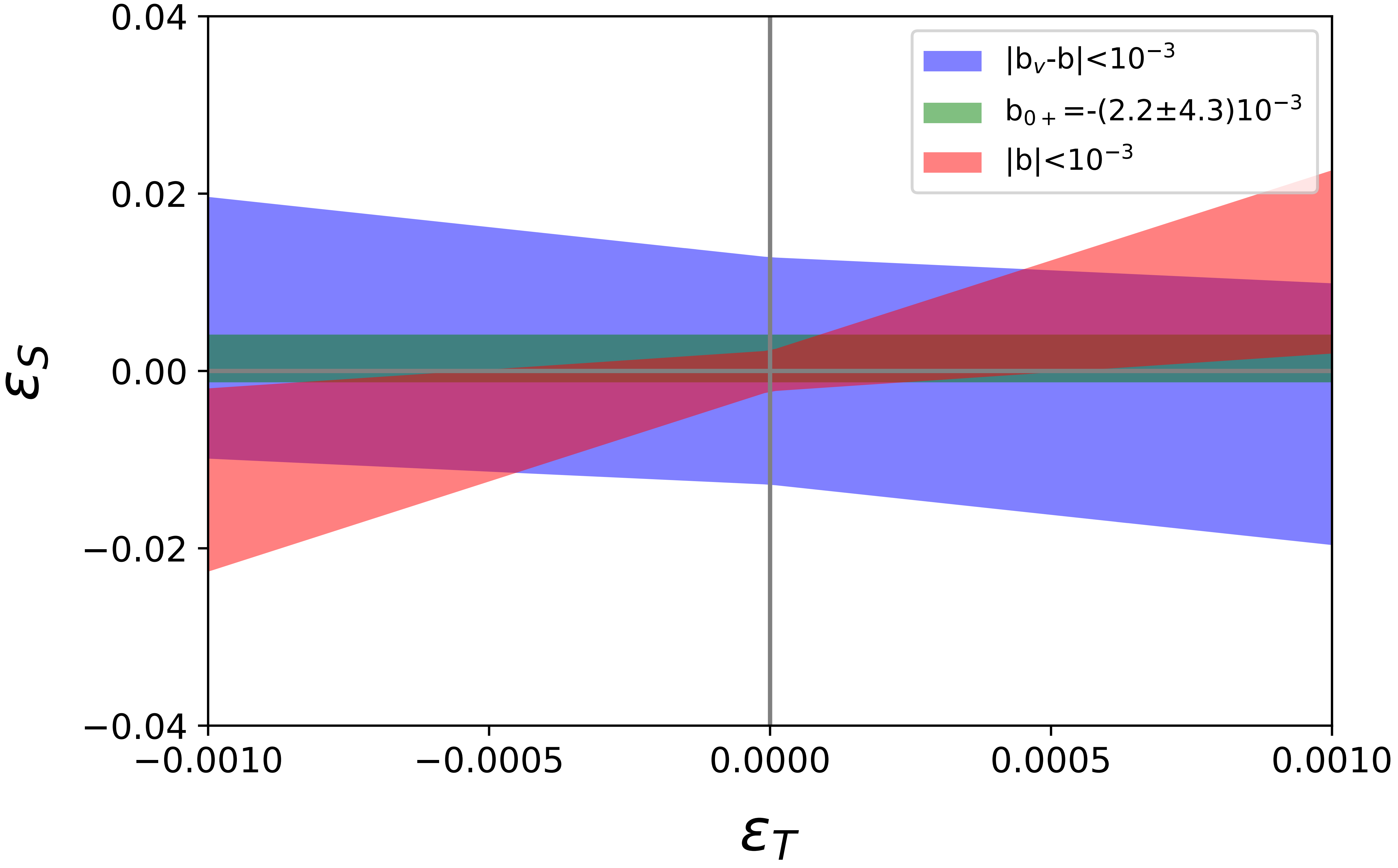
Impact on phenomenology

Following Bhattacharya et al., PRD, 2012

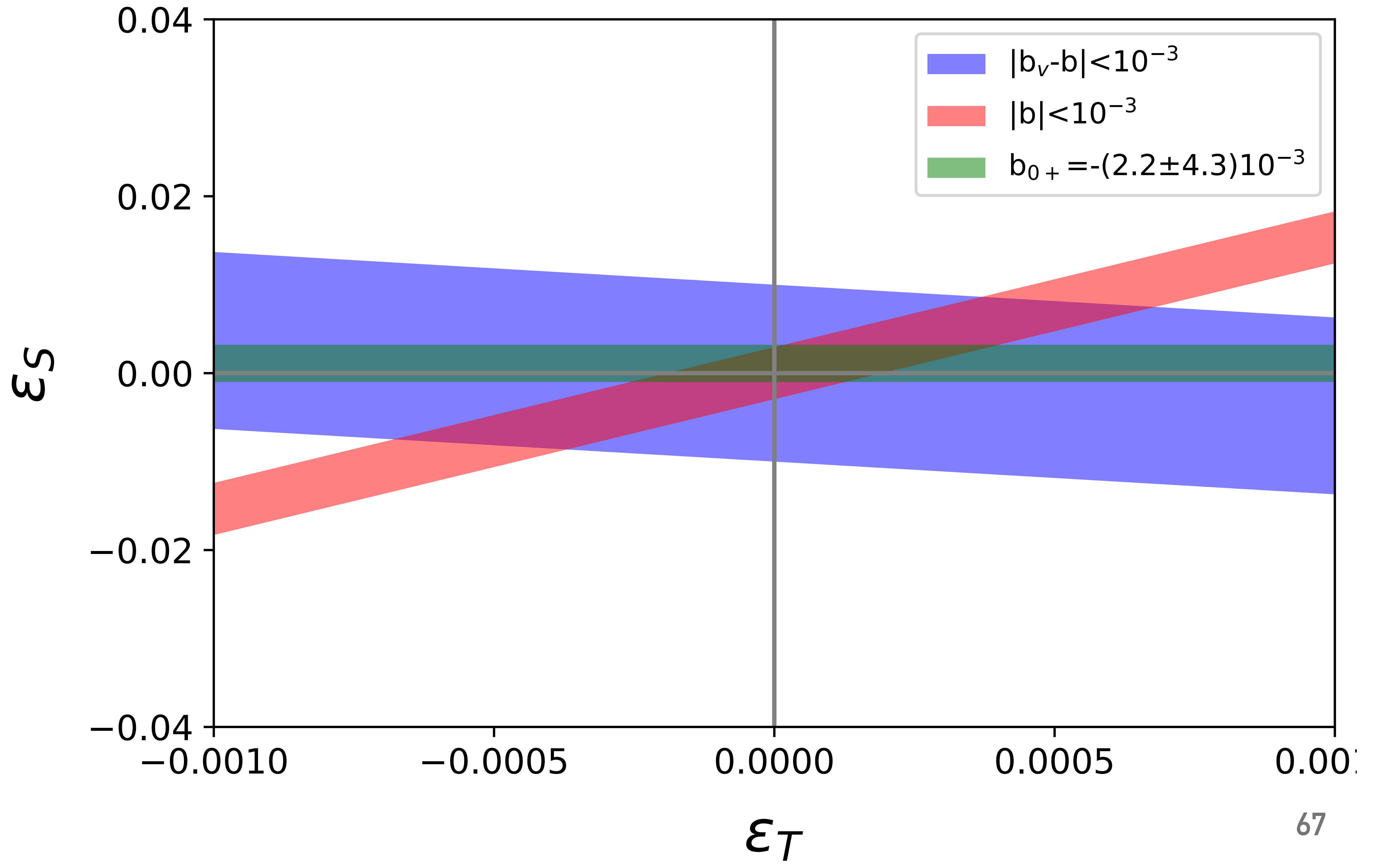
Experimental rates sensitive to product of

(Tensor and scalar charges: g_T/g_S) X (new-physics effective couplings: ϵ_T/ϵ_S)

Current and projected experimental limits with g_T/g_S (this work)

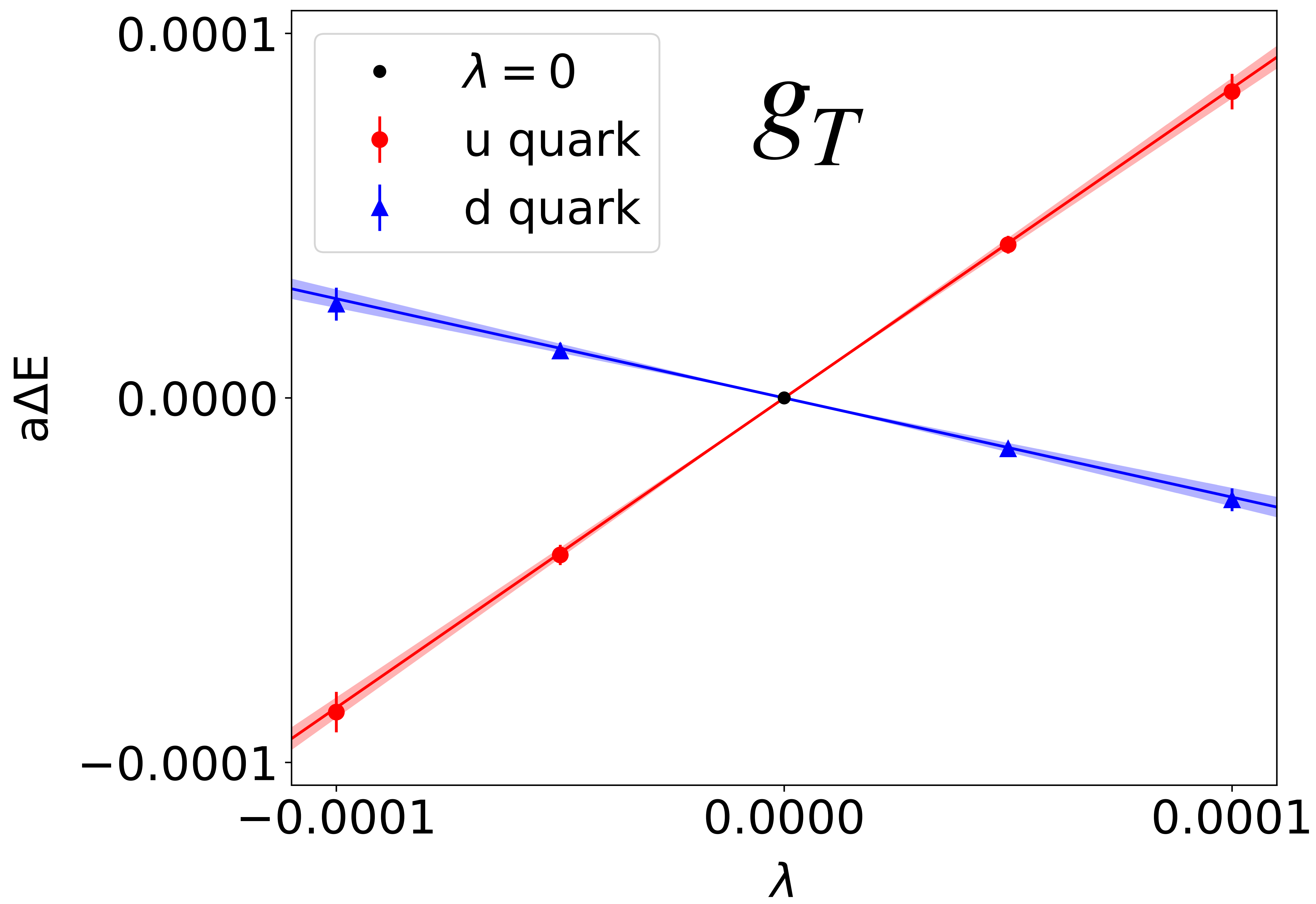


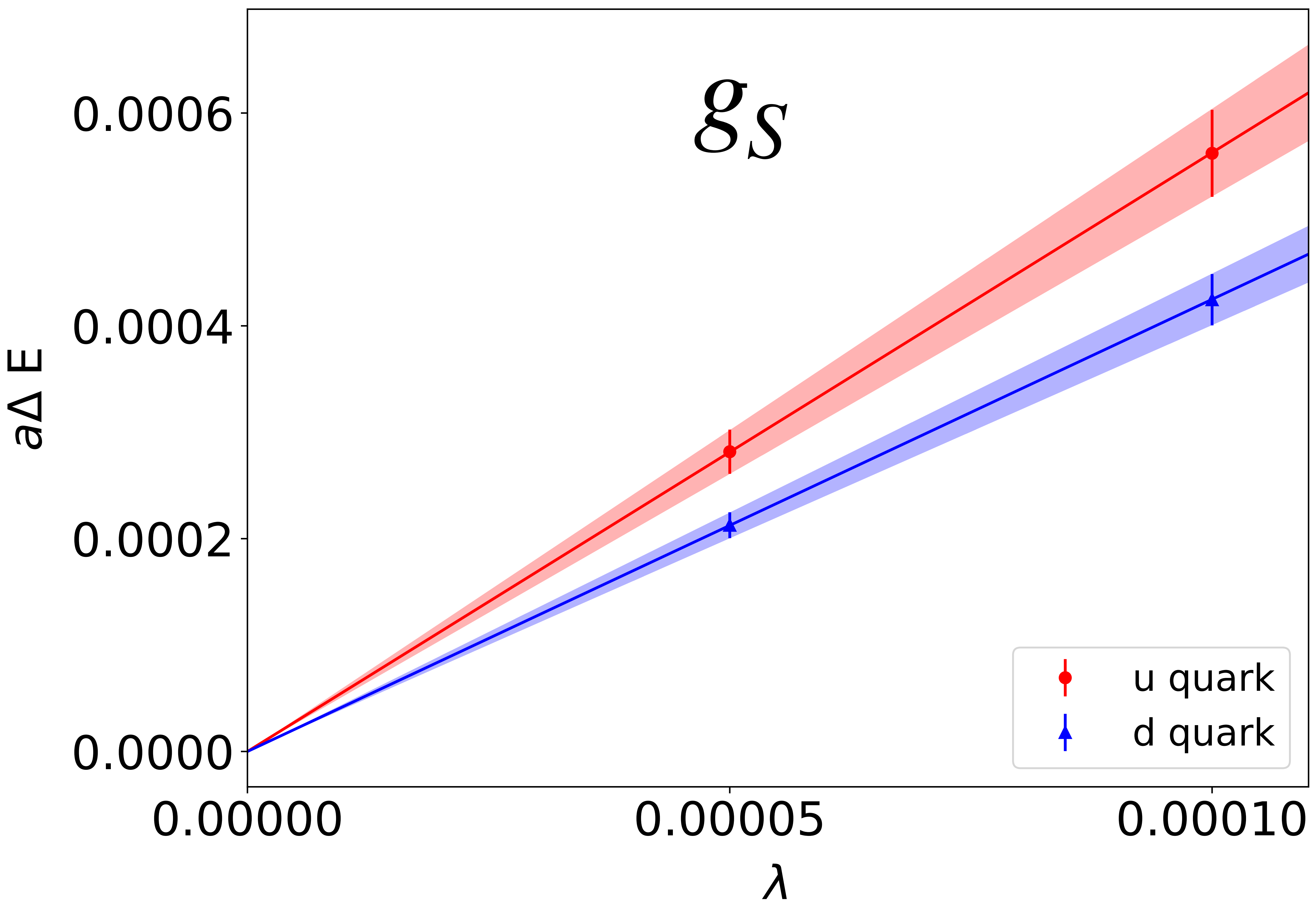
With $g_T = g_S = 1$ (no error)



Lambda dependence

$m_\pi \approx 265\,\mathrm{MeV},\ \mathrm{a} = 0.068\,\mathrm{fm},\ \mathrm{V} = 48^3 \times 96$





Spin-dependent:

$$\left.\frac{\partial E^{\uparrow}(\lambda)}{\partial\lambda}\right|_{\lambda=0} = +\,g_T^q \qquad \left.\frac{\partial E^{\downarrow}(\lambda)}{\partial\lambda}\right|_{\lambda=0} = -\,g_T^q$$

$$E(\lambda) = E(0) \pm \lambda g_T^q + \mathcal{O}(\lambda^2)$$

Spin-independent:

$$\left.\frac{\partial E(\lambda)}{\partial\lambda}\right|_{\lambda=0} = +\,g_S^q$$

$$E(\lambda) = E(0) + \lambda g_S^q + \mathcal{O}(\lambda^2)$$

Global fits

Want result

- in continuum and infinite volume limits
- at physical quark masses

Global fit

$$\delta m_l \to \delta m_l = \frac{m_\pi^2 - X_\pi^2}{X_\pi^2}$$

- Include $O(a)$ or $O(a^2)$ terms in X (singlet) and slope parameters

$$X_{D,F} = X^*_{D,F}(1 + c_1 \frac{1}{3}[f_L(m_\pi) + 2f_L(m_\pi)]) + c_2 a + c_3 \delta m_l^2$$

e.g. $\tilde{D}_1 = 1 - 2(\tilde{r}_1 + \tilde{b}_1 a)\delta m_l + \tilde{d}_1 \delta m_l^2$

- Free parameter to encode leading finite-volume correction on singlet:

$$f_L(m) = \left(\frac{m}{X_\pi}\right)^2 \frac{e^{-mL}}{\sqrt{mL}}$$

[functional form from chiral EFT, see Beane & Savage PRD(2004)]

- Work to $O(\delta m_l^2)$ in flavour expansion

Results - Hyperon charges

Not in FLAG, but recent results by RQCD [PRD108(2023)]

This work	*RQCD*
$g_T^{\Sigma} = 0.805(15)$	$g_T^{\Sigma} = 0.798(26)$
$g_T^{\Xi} = -0.1952(75)$	$g_T^{\Xi} = -0.1872(72)$
$g_A^{\Sigma} = 0.876(28)$	$g_A^{\Sigma} = 0.875(49)$
$g_A^{\Xi} = -0.206(21)$	$g_A^{\Xi} = -0.267(18)$
$g_S^{\Sigma} = 2.80(25)$	$g_S^{\Sigma} = 3.98(33)$
$g_S^{\Xi} = 1.59(12)$	$g_S^{\Xi} = 2.57(16)$

some tension

Momentum fractions extra

In quenched QCD with heavy quark masses reveals for both π and N $\langle x \rangle_q \sim 0.5 - 0.6, \quad \langle x \rangle_g \sim 0.4 - 0.5$

Currently generating dynamical ensembles with:

- $n_f = 2$ NP Clover fermions with $m_\pi \sim 600\,\mathrm{MeV}$
- 3 values each of λ_q and λ_g
- Z matrix more complicated:

$$\begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_u^{con} \\ \langle x \rangle_d^{con} \\ \langle x \rangle_u^{dis} \\ \langle x \rangle_d^{dis} \end{pmatrix}^R = \begin{pmatrix} Z_{gg} & Z_{gq} & Z_{gq} & Z_{gq} & Z_{gq} \\ 0 & Z_a - Z_b & 0 & 0 & 0 \\ 0 & 0 & Z_a - Z_b & 0 & 0 \\ Z_{qg} & Z_b & Z_b & Z_a & Z_b \\ Z_{qg} & Z_b & Z_b & Z_b & Z_a \end{pmatrix} \begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_u^{con} \\ \langle x \rangle_d^{con} \\ \langle x \rangle_u^{dis} \\ \langle x \rangle_d^{dis} \end{pmatrix}^{lat}$$

$$Z_{qq}^{NS} = Z_a - Z_b$$

$$Z_{qq}^{S} = Z_{qq}^{NS} + n_f Z_b$$

(slide from T.Howson, Cairns, Australia, 2022)

TROUBLE WITH THE GLUE

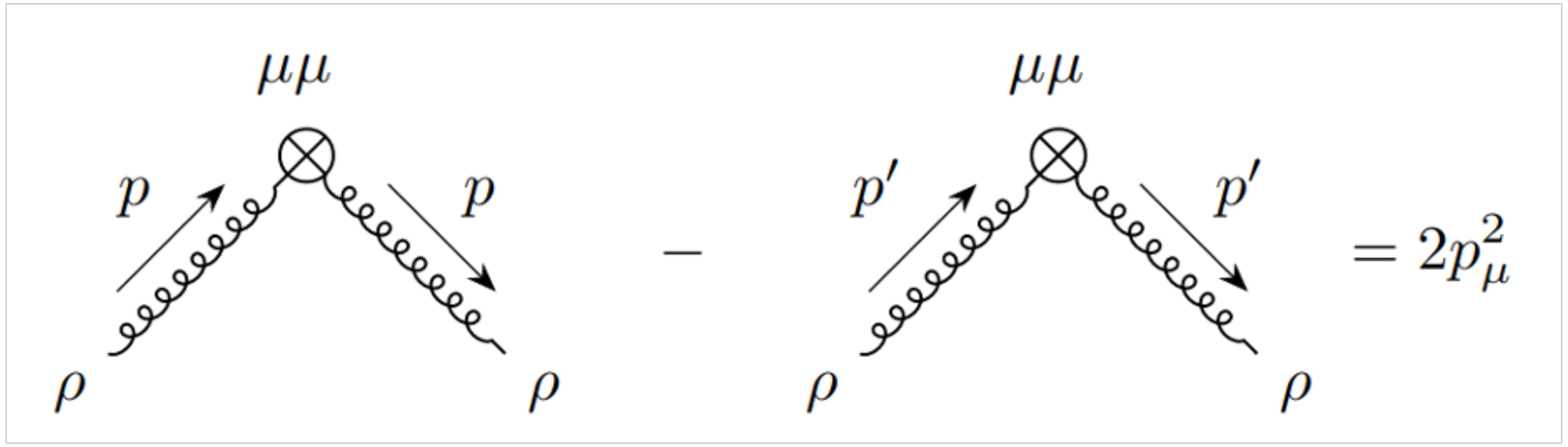
We can write out the gluon 3-point function from the EMT,

$$\langle A_{\sigma'}|\,\overline{T}^{g}_{\mu\nu}\,|A_\sigma\rangle = D_{\sigma'\rho}(p)\times$$

Gauge Dependent Terms, Will Mix

$$\Big(2p_\mu p_\nu\delta_{\rho\tau} - p_\mu p_\rho\delta_{\nu\tau} - p_\tau p_\nu\delta_{\rho\mu}) - p_\rho p_\nu\delta_{\mu\tau} + p^2(\delta_{\rho\nu}\delta_{\nu\tau} + \delta_{\mu\tau}\delta_{\rho\nu}) + \delta_{\mu\nu}(p_\rho p_\tau - p^2\delta_{\rho\tau})\Big) D_{\tau\sigma}(p)$$

Will want to extract the gauge independent term, vanish all other terms.



$$\rho \neq \mu,\quad p_\mu \neq 0,\quad p'_\mu = 0,\quad p'_\rho = p_\rho,\quad p^2 = p'^2$$