Compton amplitudes and nucleon structure from lattice QCD

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QCDSF Collaboration

Workshop on PDFs in the EIC era
Institute of Physics, Academia Sinica
Taipei, Taiwan
17 June 2025

QCDSF Collaboration

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K. Utku Can
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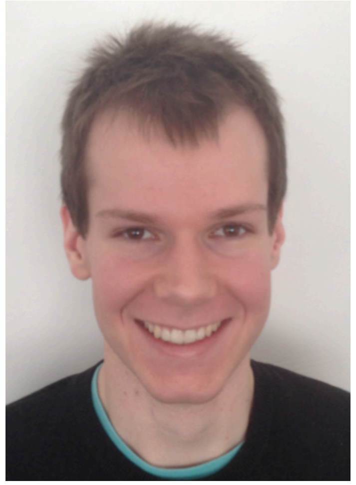
Student contributions in today's talk…



Alex Chambers
U.Adelaide
PhD 2018

Kim Somfleth
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PhD 2020

Mischa Batelaan
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PhD 2023

Alec Hannaford Gunn
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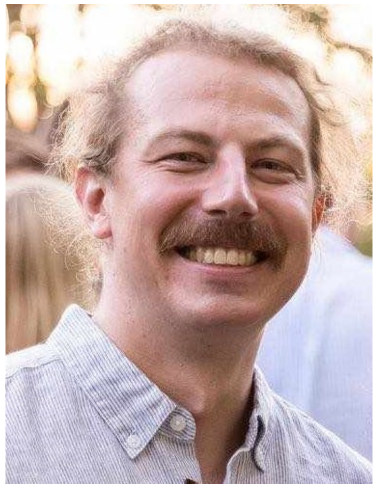
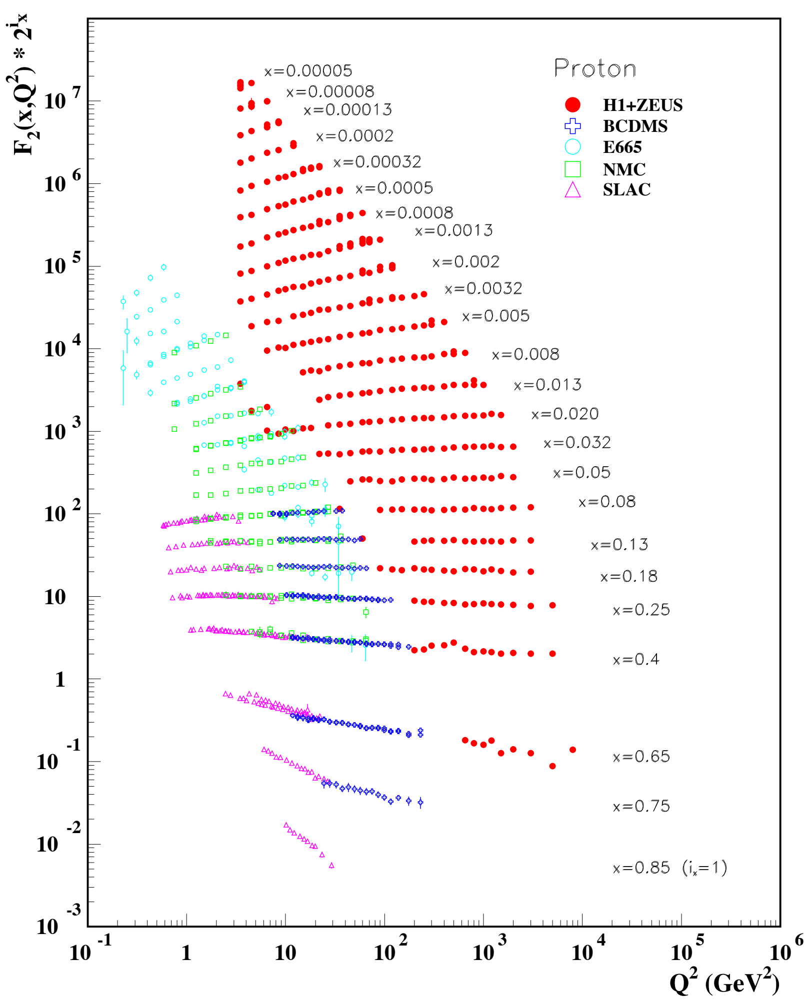
MPhil 2025

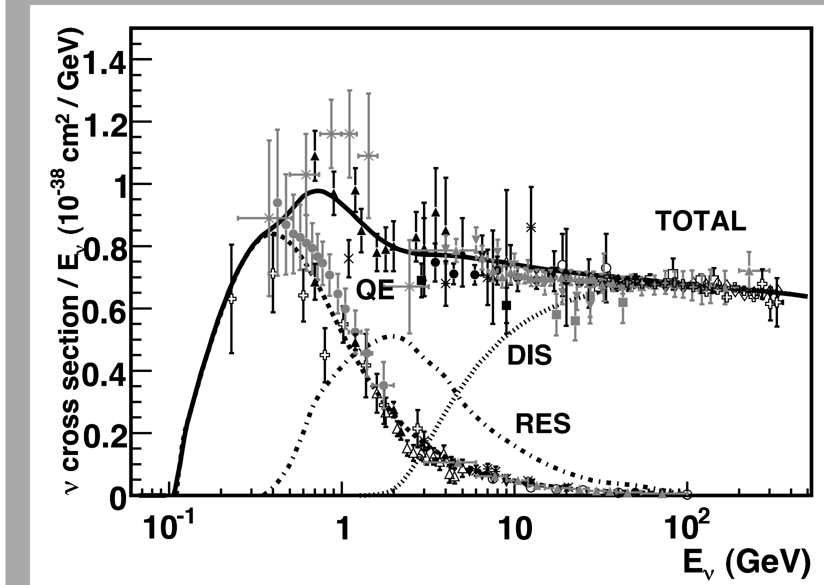


Joshua Crawford
U.Adelaide
PhD 2025 (!)

Motivation

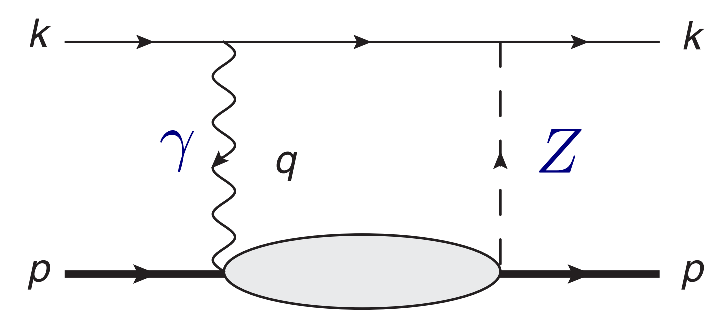
Power corrections
Theoretical foundations to inform Q^2 cuts of empirical parton fits.

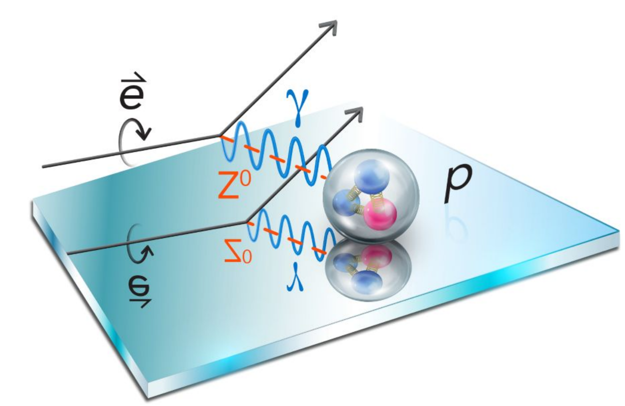




Neutrino-nucleus cross sections
Precise theoretical input required for next-generation neutrino oscillation program

Radiative corrections
Searches for new physics in the proton weak charge.
Require knowledge of gamma-*Z* interference structure functions.





Precision radiative corrections

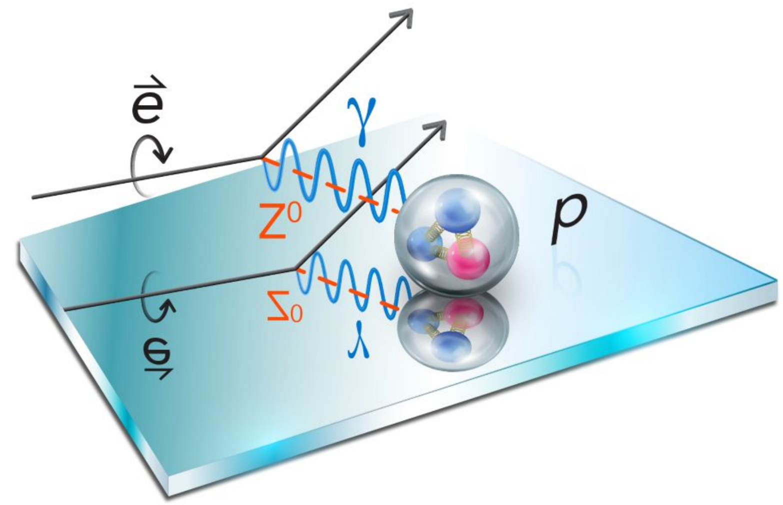
nature

Letter | Published: 09 May 2018

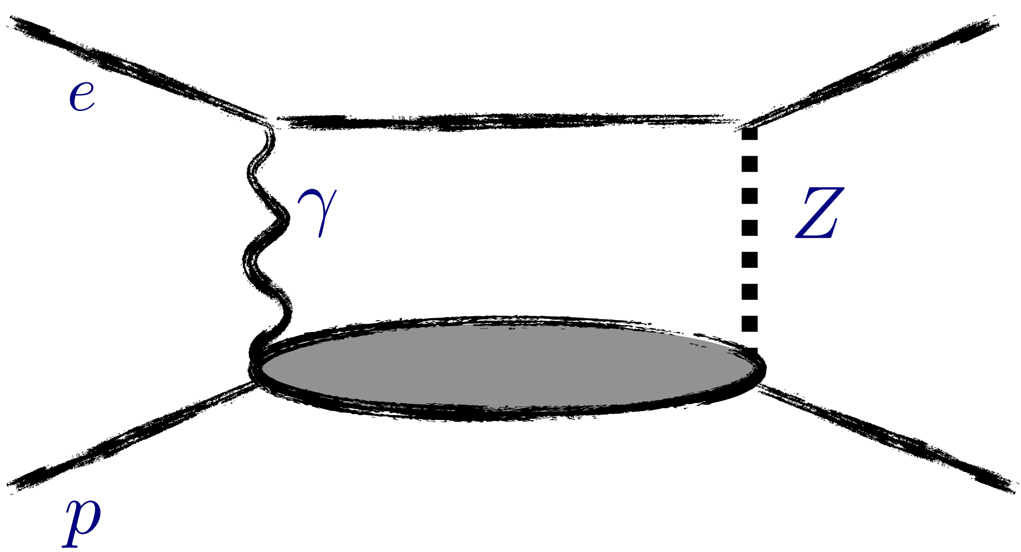
Precision measurement of the weak charge of the proton

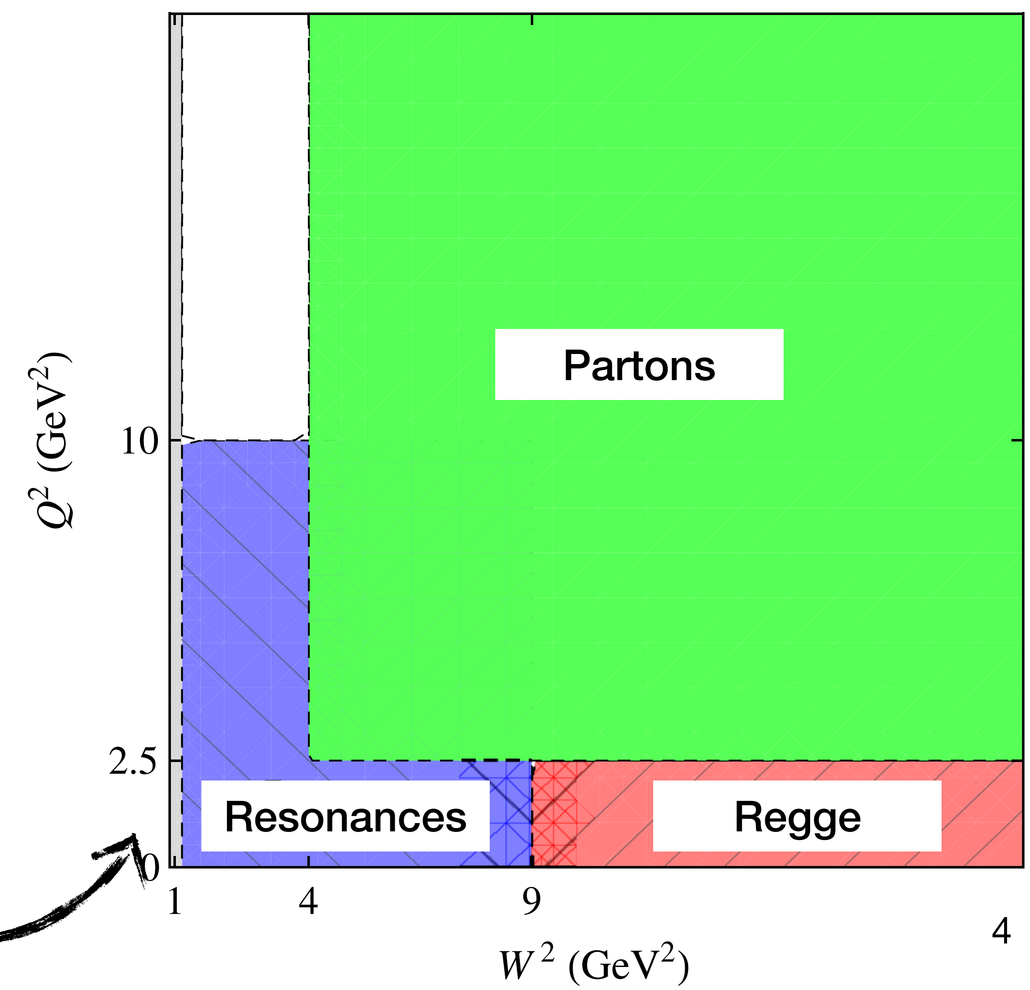
The Jefferson Lab Qweak Collaboration

Nature **557**, 207–211(2018) | Cite this article



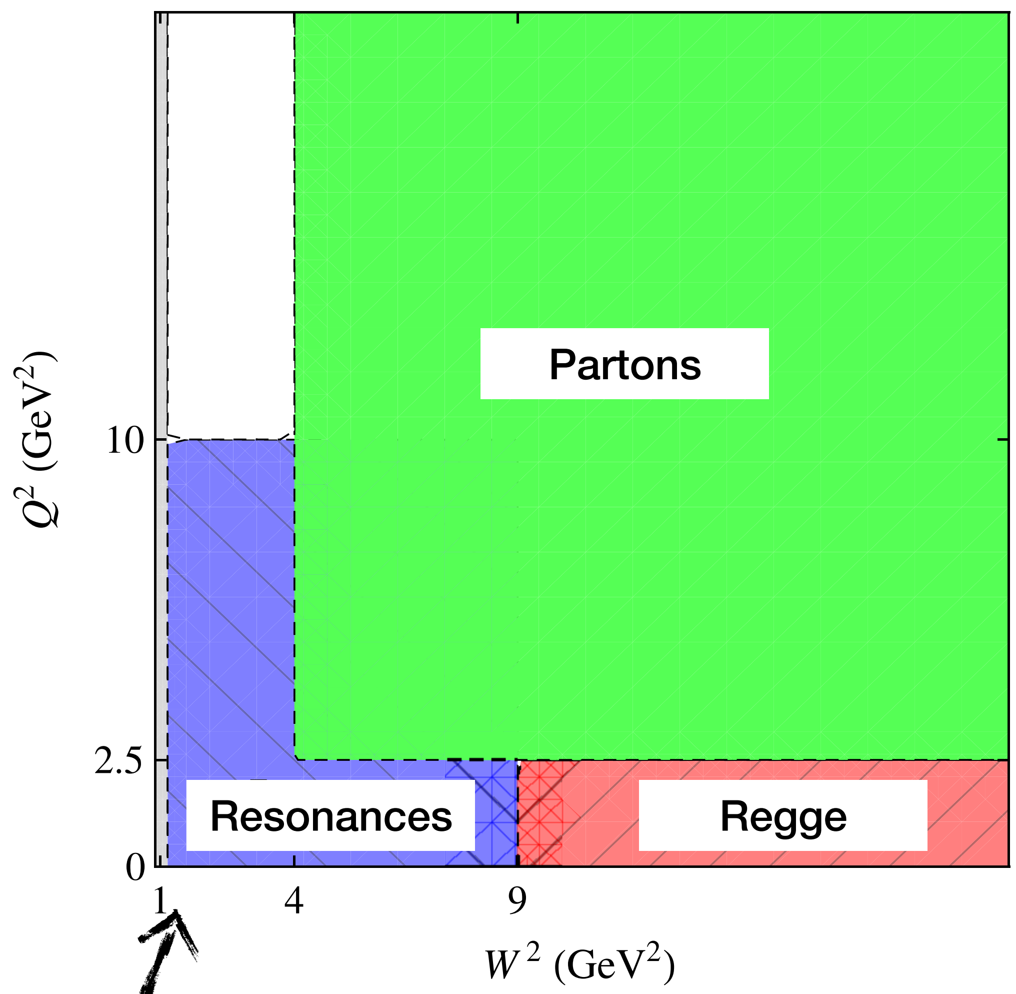
Need to correct for (energy-dependent) gamma-*Z* box.





Dispersive calculation of box requires knowledge of gamma-*Z* interference structure functions.

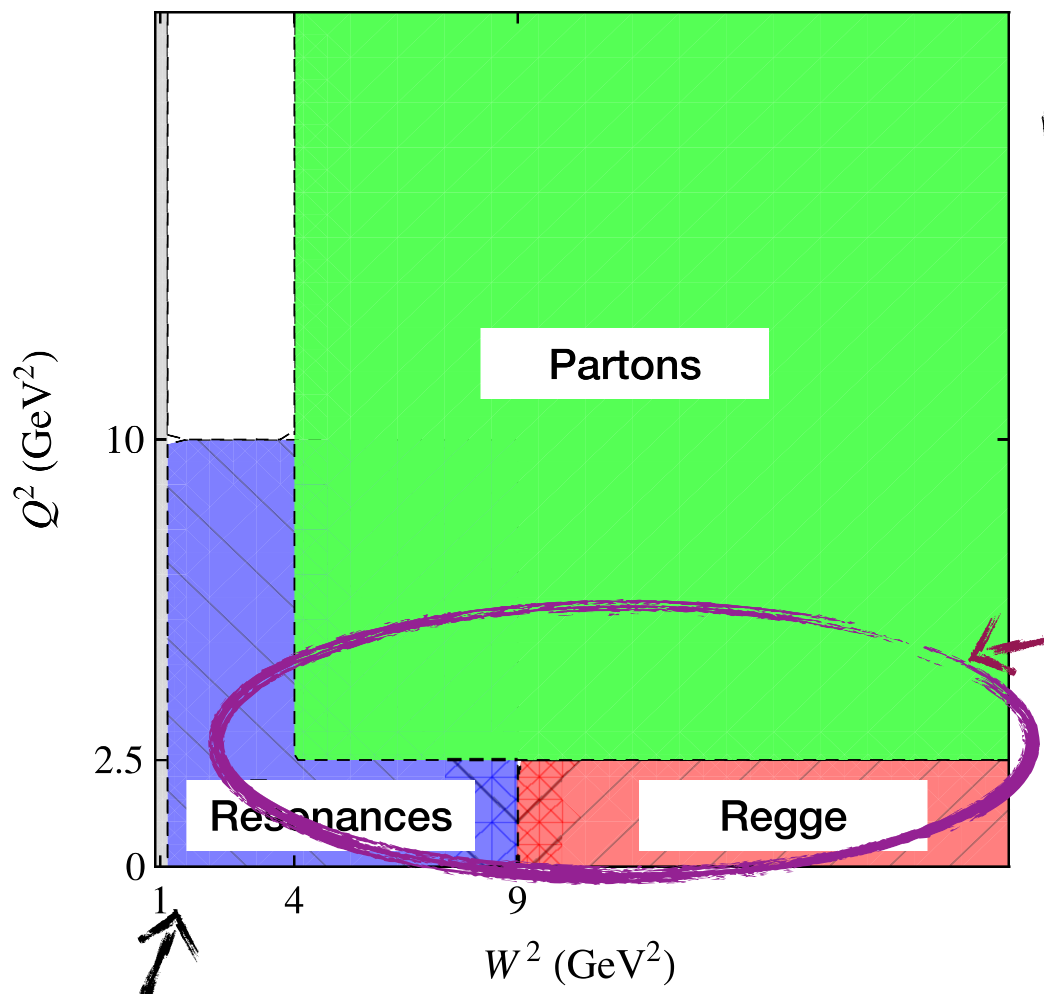
What can lattice do?



Matrix elements of local operators
→ Low(est) moments of parton distributions
e.g. $\langle x \rangle$

Resonance region with 2-body final state
$\langle N\pi | J | N \rangle$
(working towards 3)

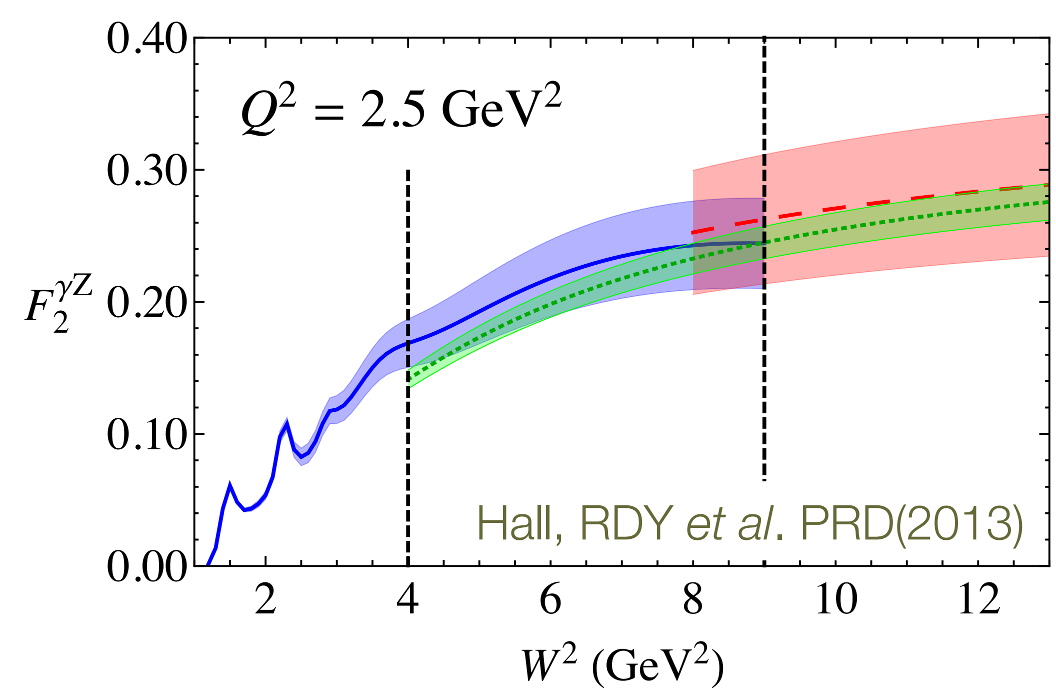
What can lattice do?



Matrix elements of local operators
→ Low(est) moments of parton distributions
e.g. $\langle x \rangle$

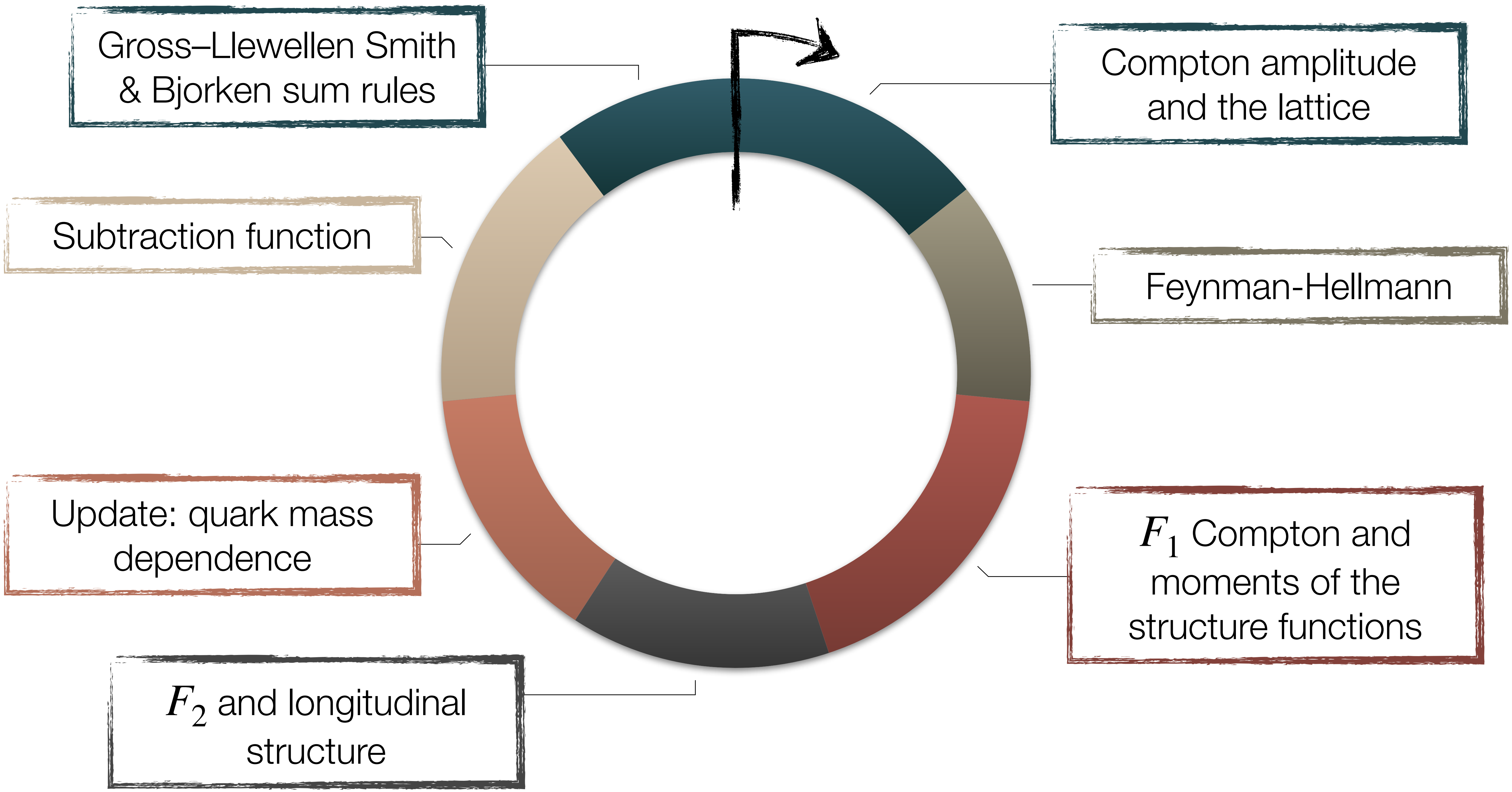
How do we gain constraint from this region from lattice QCD?

Resonance region with 2-body final state
$\langle N\pi | J | N \rangle$
(working towards 3)



Phenomenological estimate of γZ

Outline

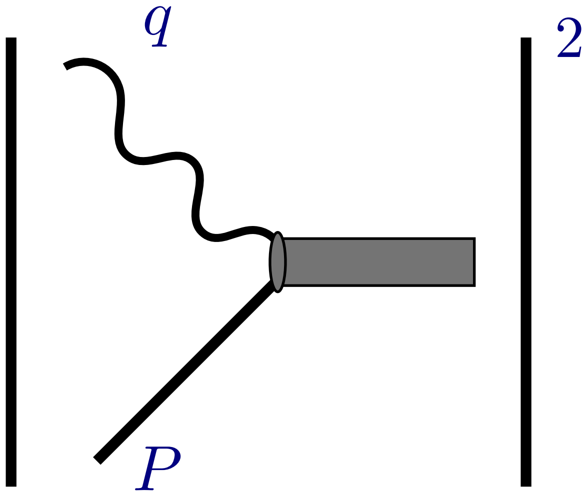


Compton amplitude and the lattice
Feynman-Hellmann
F_1 Compton and moments of the structure functions
F_2 and longitudinal structure
Update: quark mass dependence
Subtraction function
Gross–Llewellen Smith & Bjorken sum rules

Compton amplitude and structure functions

Optical theorem

Cross section ~ Hadron tensor



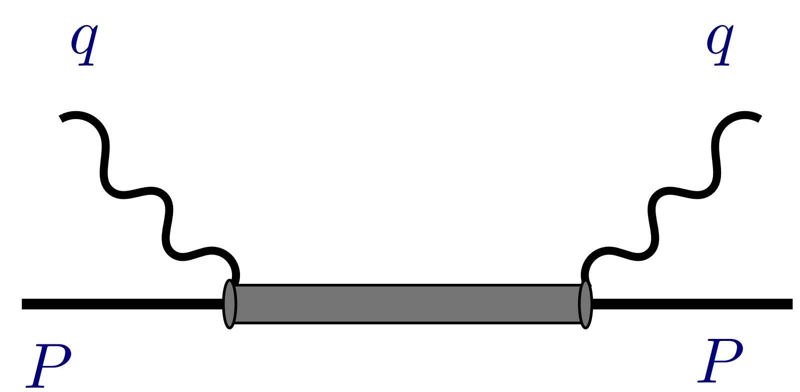
$$W_{\mu\nu} \sim \int d^4x \, e^{iq.x} \langle p| \left[J_\mu(x), J_\nu(0) \right] |p\rangle$$

Structure functions

$$F_{1,2}(P \cdot q, Q^2)$$

spin-averaged

Forward Compton **amplitude**



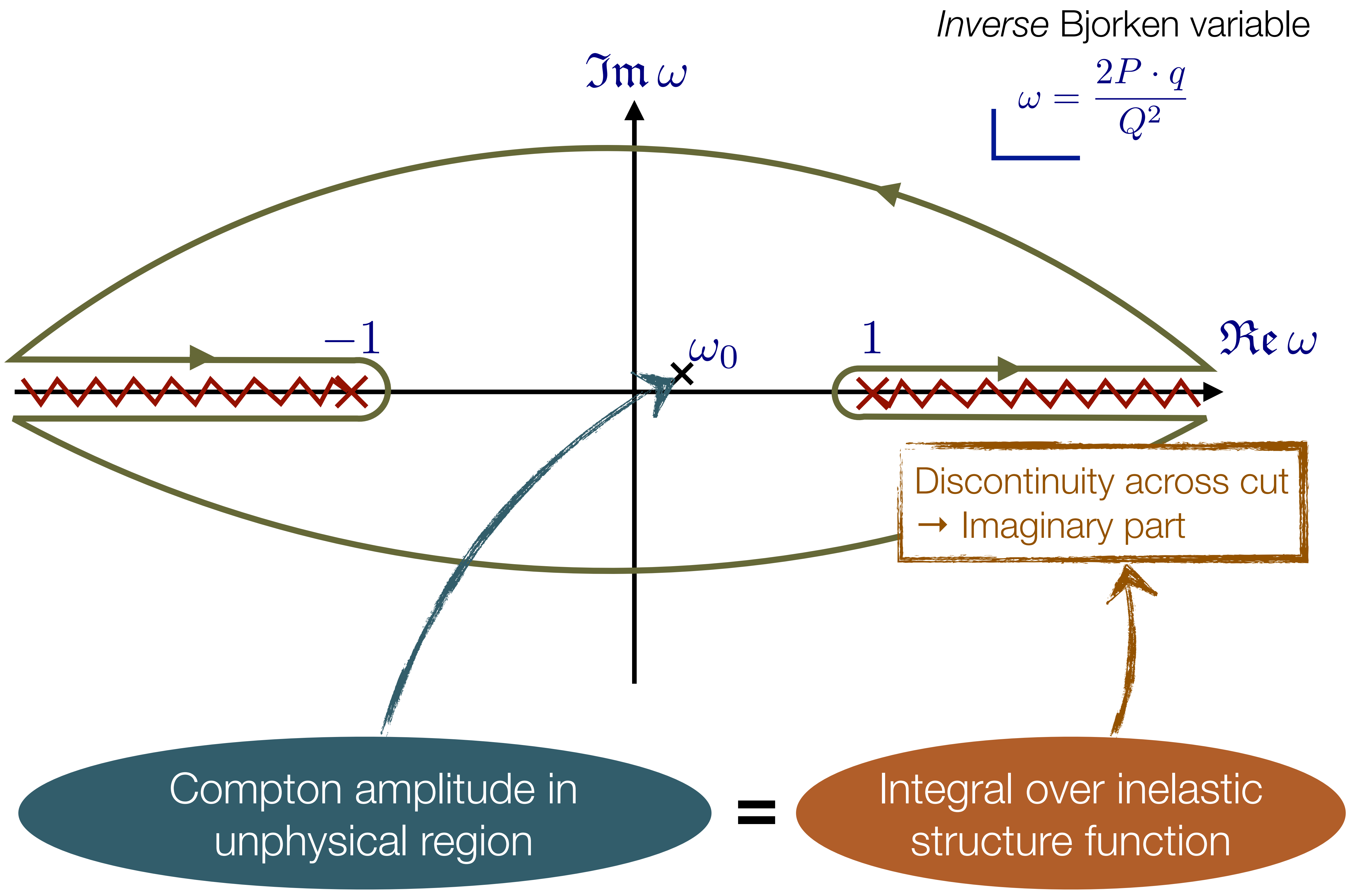
$$T_{\mu\nu} \sim \int d^4x \, e^{iq.x} \langle p| \mathrm{T} J_\mu(x) J_\nu(0) |p\rangle$$

(Compton) structure functions

$$\mathcal{F}_{1,2}(P \cdot q, Q^2)$$

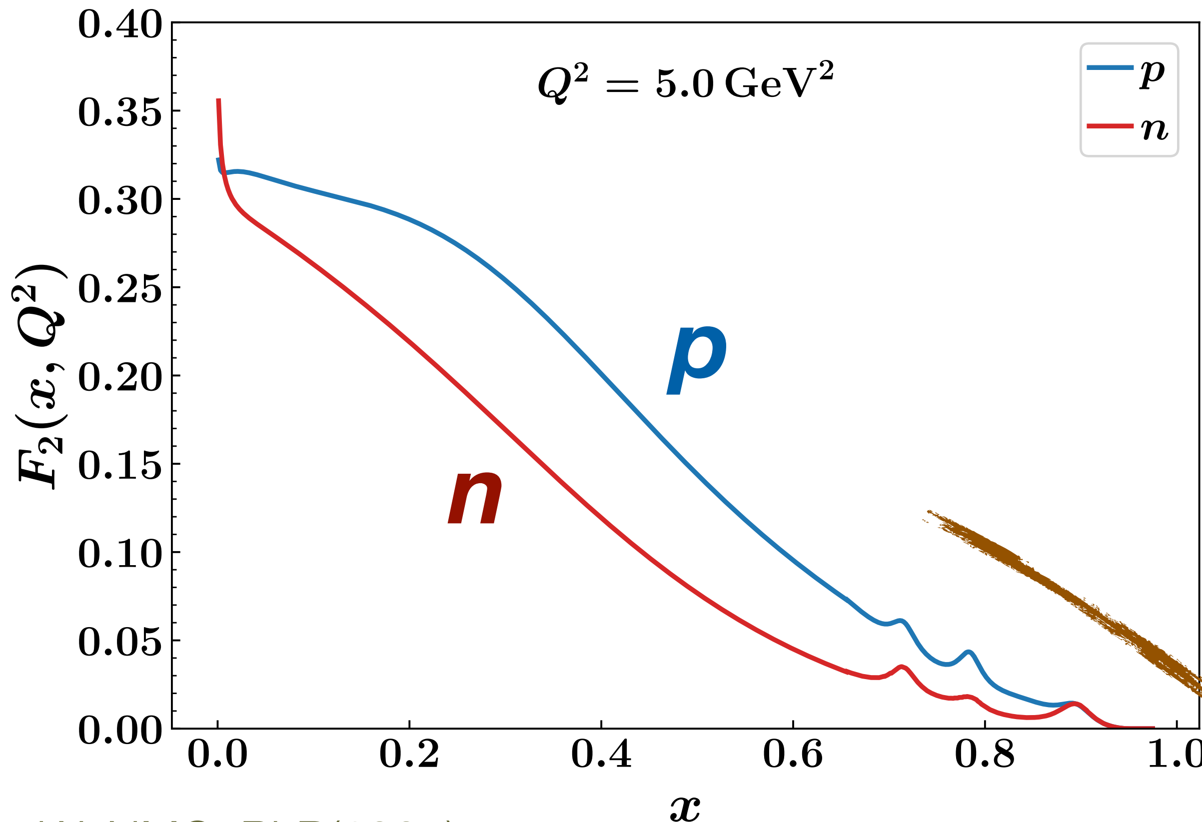
Optical theorem: $F_i = \dfrac{1}{2\pi} \mathrm{Im}\, \mathcal{F}_i$

Dispersion relation for Compton amplitude



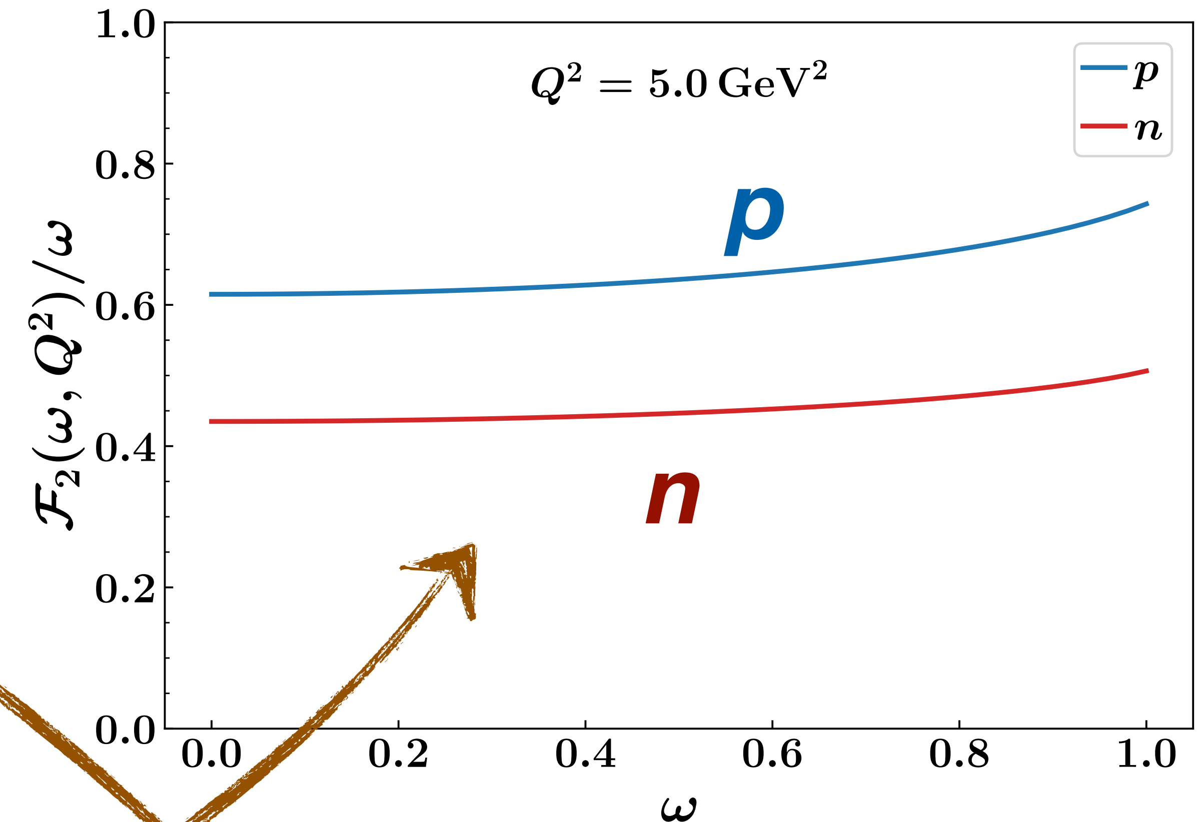
What does Compton look like? F_2

Structure functions



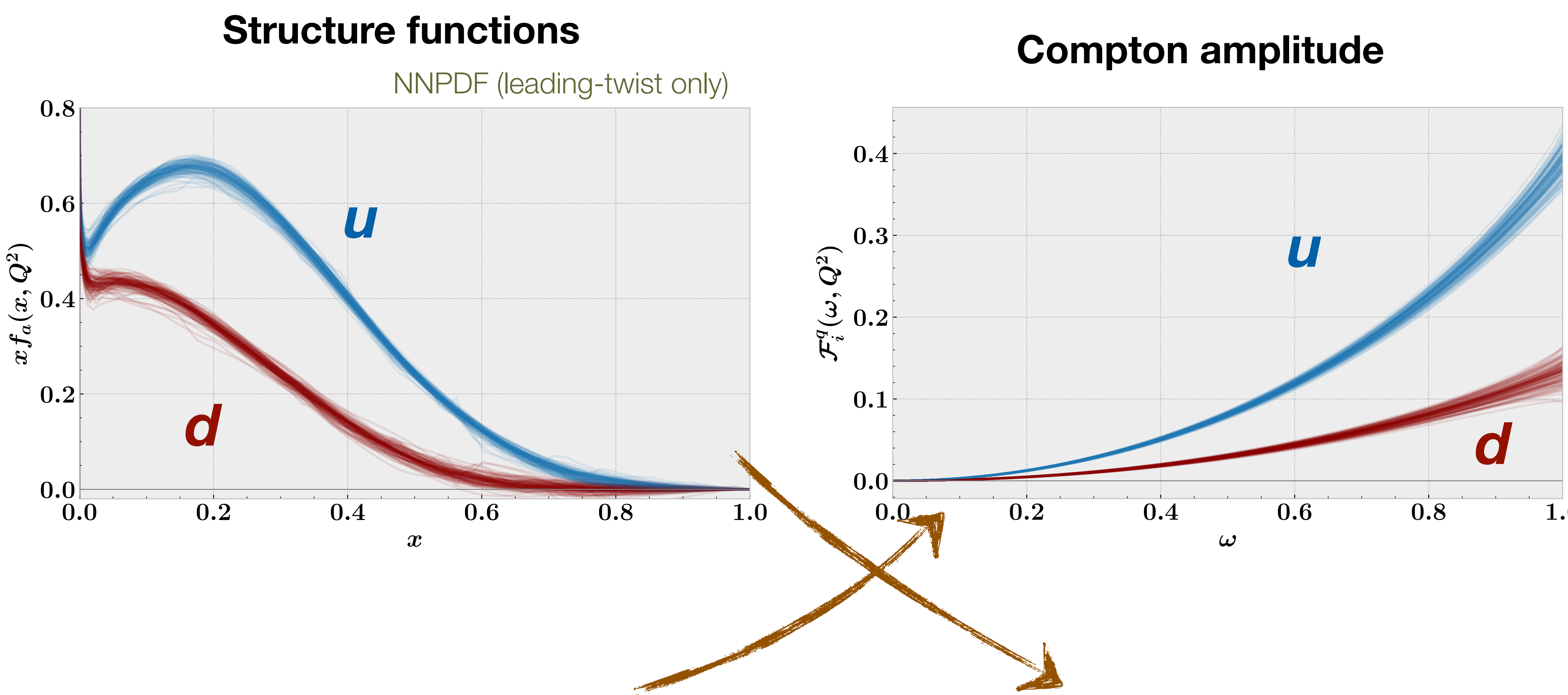
high-W: NMC, PLB(1995)
low-W: Bosted-Christy, PRC(2010)

Compton amplitude



$$\frac{\mathcal{F}_2(\omega, Q^2)}{\omega} = 4 \int_0^1 dx\, \frac{F_2(x, Q^2)}{1 - (x\omega)^2}$$

What does Compton look like? F_1



$$\overline{\mathcal{F}}_1(\omega, Q^2) = 2\omega^2 \int_0^1 dx \, \frac{2xF_1(x, Q^2)}{1-(x\omega)^2}$$

But the lattice is Euclidean??

Minkowski Compton

(spin, Lorentz suppressed)

$$T(p,q) = i\int d^4z\, e^{iq\cdot z}\langle p|\mathrm{T}\left\{J(z)J(0)\right\}|p\rangle$$

$$= \sum\!\!\!\!\!\!\int_X \int_0^\infty dt\, i\, e^{i(q_0+E_p-E_X+i\epsilon)t}\langle p|J(0)|X(\mathbf{p}+\mathbf{q})\rangle\langle X(\mathbf{p}+\mathbf{q})|J(0)|p\rangle + (q\to -q)$$

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Minkowski Compton

(spin, Lorentz suppressed)

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$$\frac{1}{(E_X - E_p - q_0 - i\epsilon)}$$

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$$\frac{1}{(E_X - E_p - q_0 - i\epsilon)}$$

Euclidean Compton

$$T^{\mathcal{E}}(p,q) = \int_0^\infty d\tau e^{q_0 \tau} \int d^3z e^{-i\mathbf{q}\cdot\mathbf{z}} \langle p | J(\mathbf{z},\tau) J(0) | p \rangle + (q \to -q)$$
$$= \sum\!\!\!\!\!\!\int_X \int_0^\infty d\tau\ e^{(q_0 + E_p - E_X)\tau} \langle p | J(0) | X(\mathbf{p}+\mathbf{q}) \rangle \langle X(\mathbf{p}+\mathbf{q}) | J(0) | p \rangle + (q \to -q)$$

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$$\frac{1}{(E_X - E_p - q_0)} \qquad \textbf{if } E_X > E_p + q_0$$

But the lattice is Euclidean??

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Euclidean Compton

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and if $E_X > E_p + q_0$ there are no singularities in $\int_{E_{\mathrm{th}}}^\infty dE_X$

if $E_{X(\mathbf{p}\pm\mathbf{q})} > E_p \pm q_0 \;\Rightarrow T^{\mathcal{E}}(p,q) = T(p,q)$

But the lattice is Euclidean??

Minkowski Compton
(spin, Lorentz suppressed)

$$T(p,q) = i\int d^4z\, e^{iq\cdot z}\langle p|\mathrm{T}\{J(z)J(0)\}|p\rangle$$

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$$\frac{1}{(E_X-E_p-q_0-i\epsilon)}$$

Euclidean hadron tensor
- *Unintegrated* form
- Inversion problem

Euclidean Compton

$$T^{\mathcal{E}}(p,q) = \int_0^\infty d\tau e^{q_0\tau}\int d^3z e^{-i\mathbf{q}\cdot\mathbf{z}}\langle p|J(\mathbf{z},\tau)J(0)|p\rangle + (q\to -q)$$

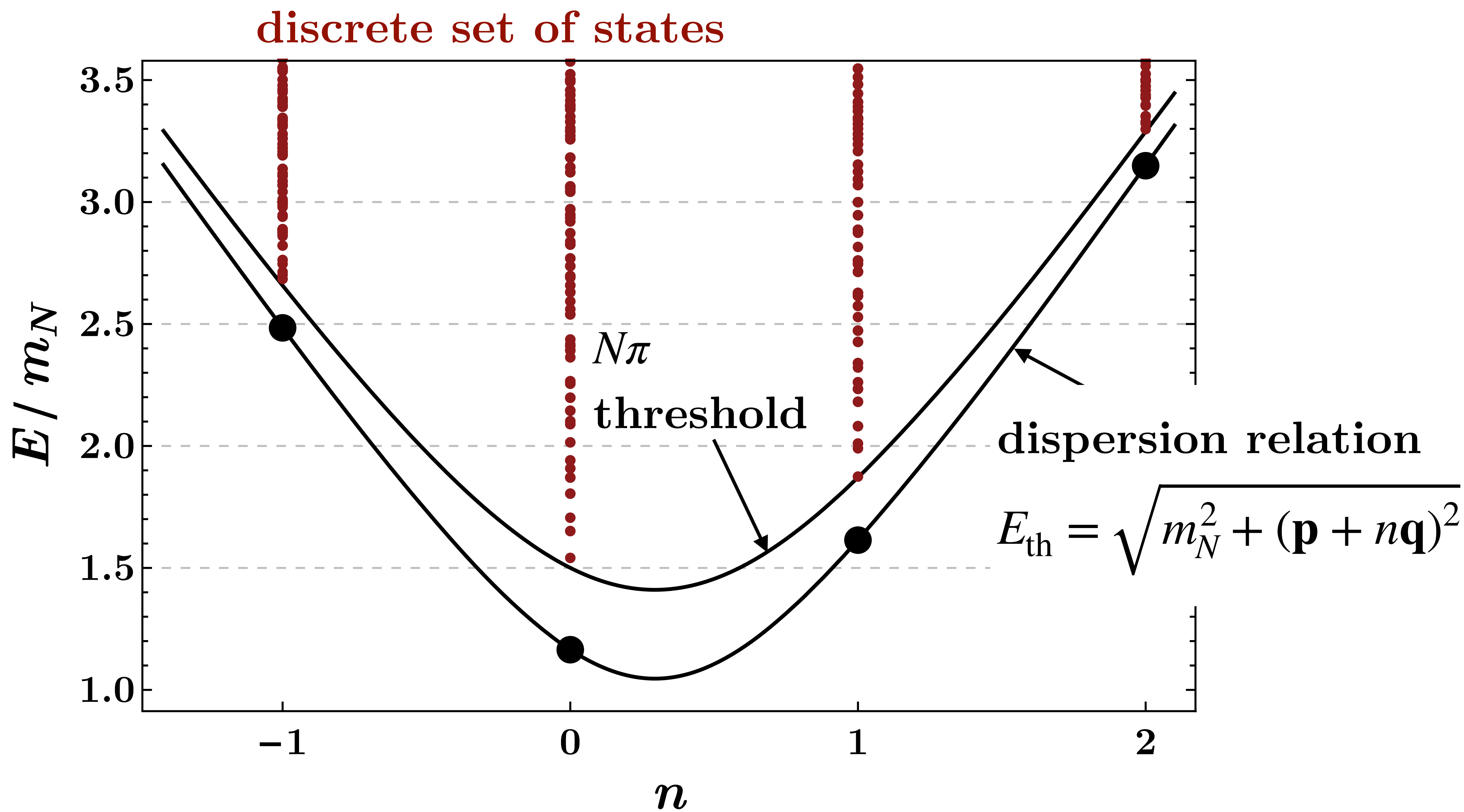
$$= \sum\!\!\!\!\!\!\int_X \int_0^\infty d\tau\ e^{(q_0+E_p-E_X)\tau}\langle p|J(0)|X(\mathbf{p}+\mathbf{q})\rangle\langle X(\mathbf{p}+\mathbf{q})|J(0)|p\rangle + (q\to -q)$$

$$\frac{1}{(E_X-E_p-q_0)} \qquad \textbf{if } E_X > E_p + q_0$$

and if $E_X > E_p + q_0$ there are no singularities in $\displaystyle\int_{E_{\mathrm{th}}}^\infty dE_X$

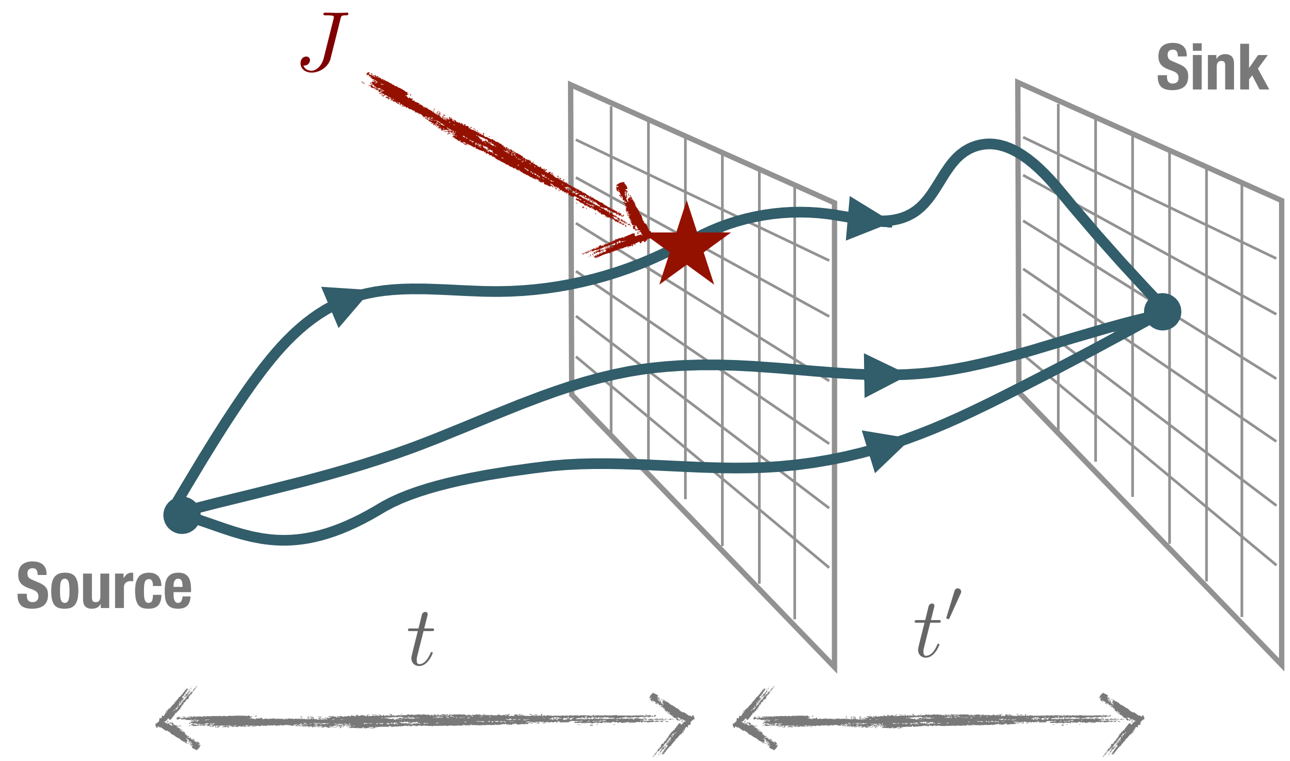
$$\textbf{if } E_{X(\mathbf{p}\pm\mathbf{q})} > E_p \pm q_0 \ \Rightarrow T^{\mathcal{E}}(p,q) = T(p,q)$$

Kinematic restriction



Must **only** consider nucleon momenta to correspond to lowest energy connected by discrete multiples of **q**

Feynman-Hellmann in lattice QCD

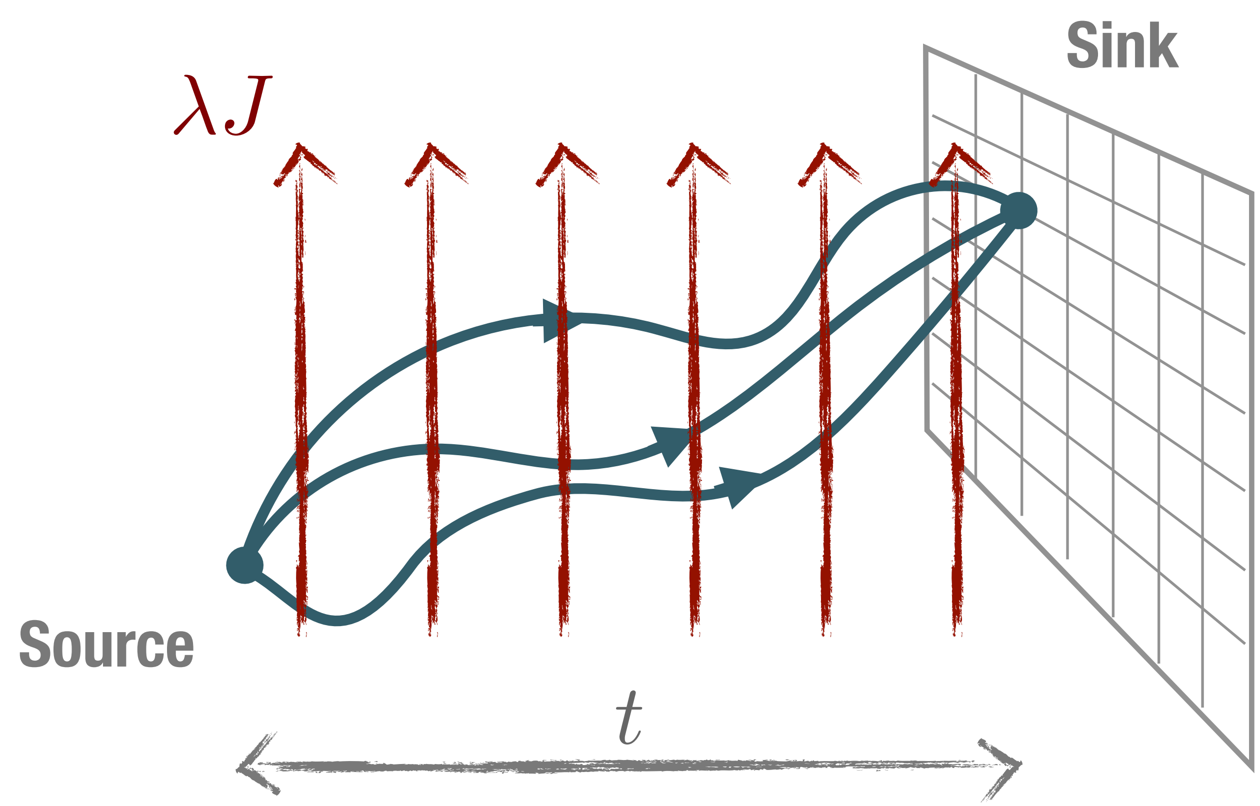


3-pt functions

$$t, t' \gg \frac{1}{\Delta E}$$ ← energy gap to lowest excitation

$$\frac{\langle C_3(t,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|J|N\rangle$$

Matrix elements on the lattice



Feynman–Hellmann

$$t \gg \frac{1}{\Delta E}$$

$$\left.\frac{\partial E}{\partial \lambda}\right|_{\lambda\to 0} \propto \langle N|J|N\rangle$$

Matrix elements from Feynman–Hellmann

- Feynman–Hellmann in quantum mechanics:

$$\frac{dE_n}{d\lambda} = \langle n|\frac{\partial H}{\partial \lambda}|n\rangle$$

  - matrix elements of the derivative of the Hamiltonian determined by derivative of corresponding energy eigenvalues

- Lattice: **evaluate energy shifts with respect to weak external fields**

- Modify action with external field:

$$S \to S + \lambda \int \mathrm{d}^4 x\, \mathcal{O}(x)$$

real parameter

local operator, e.g. $\bar{q}(x)\gamma_5\gamma_3 q(x)$

- Calculation of matrix element hadron spectroscopy [2-pt functions only]

$$\frac{\partial E_H(\lambda)}{\partial \lambda} = \frac{1}{2E_H(\lambda)}\langle H|\mathcal{O}|H\rangle$$

Spin content [connected]

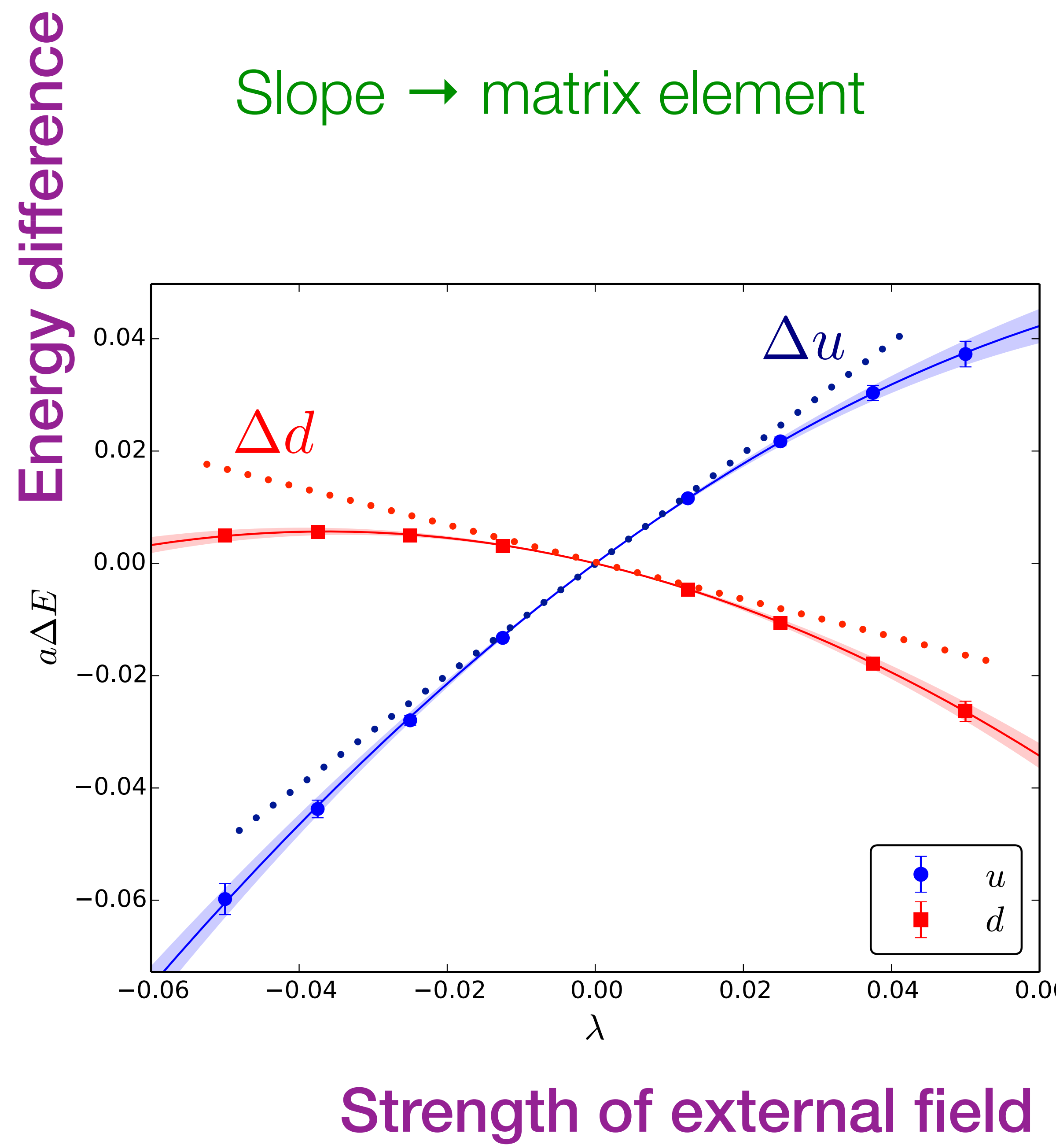
- Modify action

$$S \to S + \lambda \sum_x \bar{q}(x) i\gamma_5\gamma_3 q(x)$$

- Nucleon energy shift isolates spin content

$$\frac{\partial E_N(\lambda)}{\partial \lambda} = \frac{1}{2M_N}\langle N|\bar{q}i\gamma_5\gamma_3 q|N\rangle$$
$$= \Delta q$$

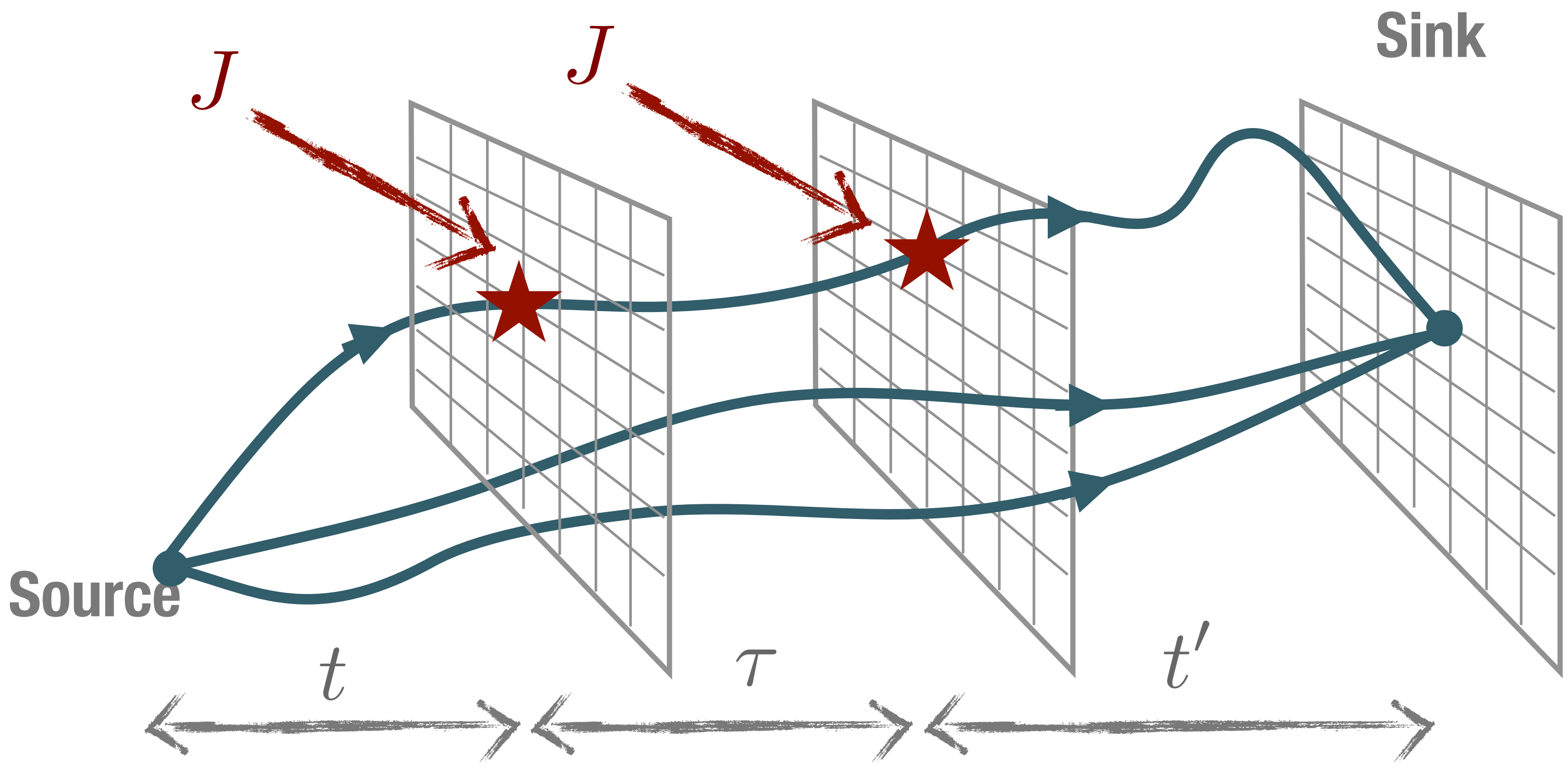
Slope → matrix element



[Chambers *et al*. PRD(2014)]

3-pt function → 2-pt function

4-pt functions



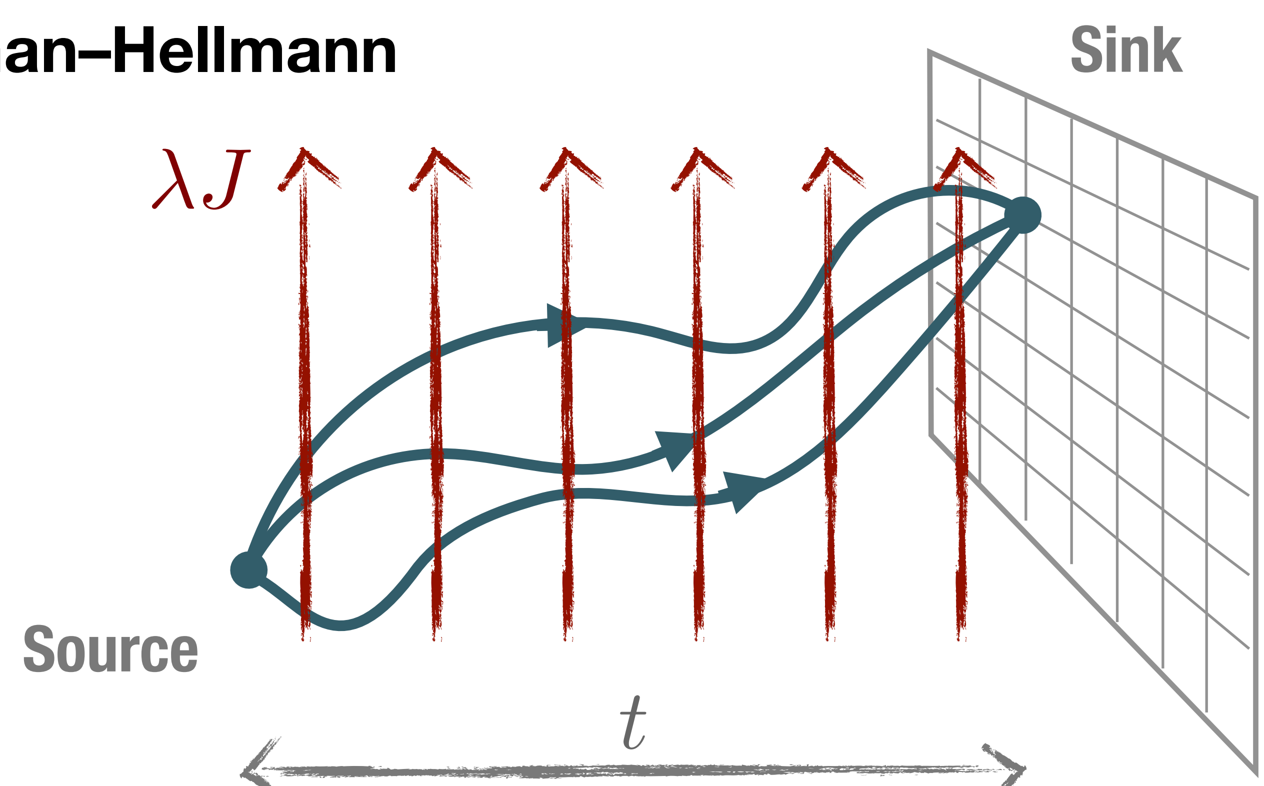
$$t, t' \gg \frac{1}{\Delta E}$$

$$\frac{\langle C_4(t,\tau,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|J(\tau_E)J|N\rangle$$

$$\int_0^\infty d\tau_E \to \langle N|JJ|N\rangle$$

Compton on the lattice

Feynman–Hellmann



$$t \gg \frac{1}{\Delta E}$$

$$\left.\frac{\partial^2 E}{\partial \lambda^2}\right|_{\lambda\to 0} \propto \langle N|JJ|N\rangle$$

Feynman–Hellman (2nd order)

- Field theory version of 2nd order perturbation theory:

$$E = E_0 + \lambda\langle N|V|N\rangle + \lambda^2 \sum_{X\neq N} \frac{\langle N|V|X\rangle\langle X|V|N\rangle}{E_0 - E_X} + \dots$$

Only get a linear term for elastic case $\omega=1$

$E_0 < E_X$
Intermediate states cannot go on-shell for $\omega<1$

- Final result. We study second-order perturbation on the lattice

$$\frac{\partial^2 E_{\mathbf{p}}}{\partial\lambda^2} = -\frac{1}{2E_{\mathbf{p}}}\int d^4\xi \left(e^{iq.\xi} + e^{-iq.\xi}\right)\langle\mathbf{p}|\mathrm{T}J(\xi)J(0)|\mathbf{p}\rangle$$

see Can, RDY *et al*. PRD(2020)

(First) numerical results: $\mathscr{F}_1 \rightarrow$ moments of F_1

Chambers, RDY *et al*. PRL(2017)
Can, RDY *et al*. PRD(2020)

Compton on the lattice

- Forward spin-averaged Compton amplitude

$$\omega = \frac{2p\cdot q}{Q^2}$$

$$T^{\mu\nu}(p,q) = \rho_{ss'} \int d^4x\, e^{iq\cdot x} \langle p, s' | \mathrm{T}\left\{ J^\mu(x) J^\nu(0) \right\} | p, s \rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) \mathcal{F}_1(\omega, Q^2) + \frac{1}{p\cdot q} \left(p^\mu - \frac{p\cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p\cdot q}{q^2} q^\nu \right) \mathcal{F}_2(\omega, Q^2)$$

- Choose simplest kinematics to directly isolate F1

$$J^3 J^3, \text{ and } q_3 = p_3 = 0$$

$$T^{33}(p,q) \to \mathcal{F}_1(\omega, Q^2)$$

Obligatory slide on lattice specs

QCDSF/UKQCD configurations

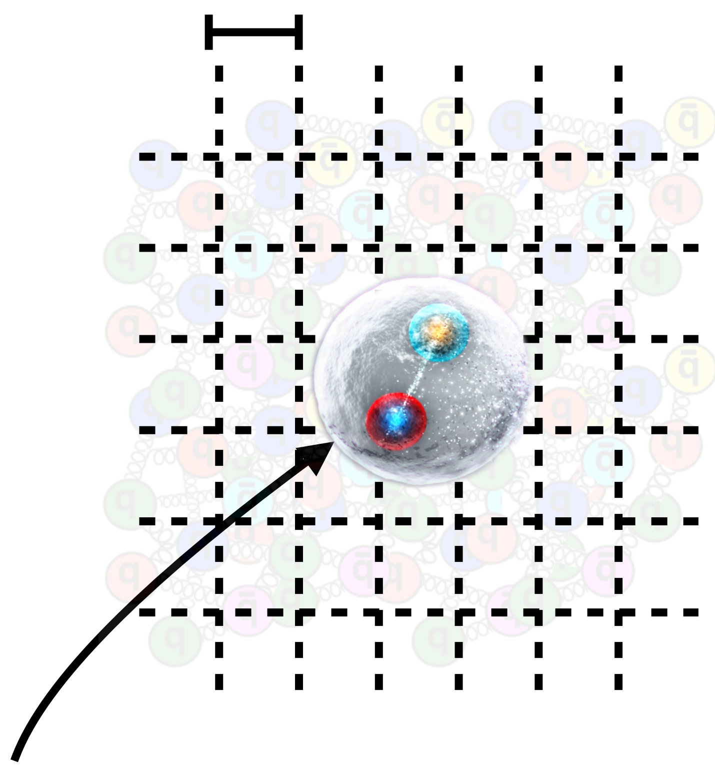
$\begin{pmatrix} 32^3 \times 64 \\ 48^3 \times 96 \end{pmatrix}$, 2+1 flavor (u/d+s)

$\beta = \begin{pmatrix} 5.50 \\ 5.65 \end{pmatrix}$, NP-improved Clover action

Phys. Rev. D 79, 094507 (2009), arXiv:0901.3302 [hep-lat]

$$m_\pi \sim \begin{bmatrix} 470 \\ 420 \end{bmatrix} \text{MeV, ~SU(3)}$$

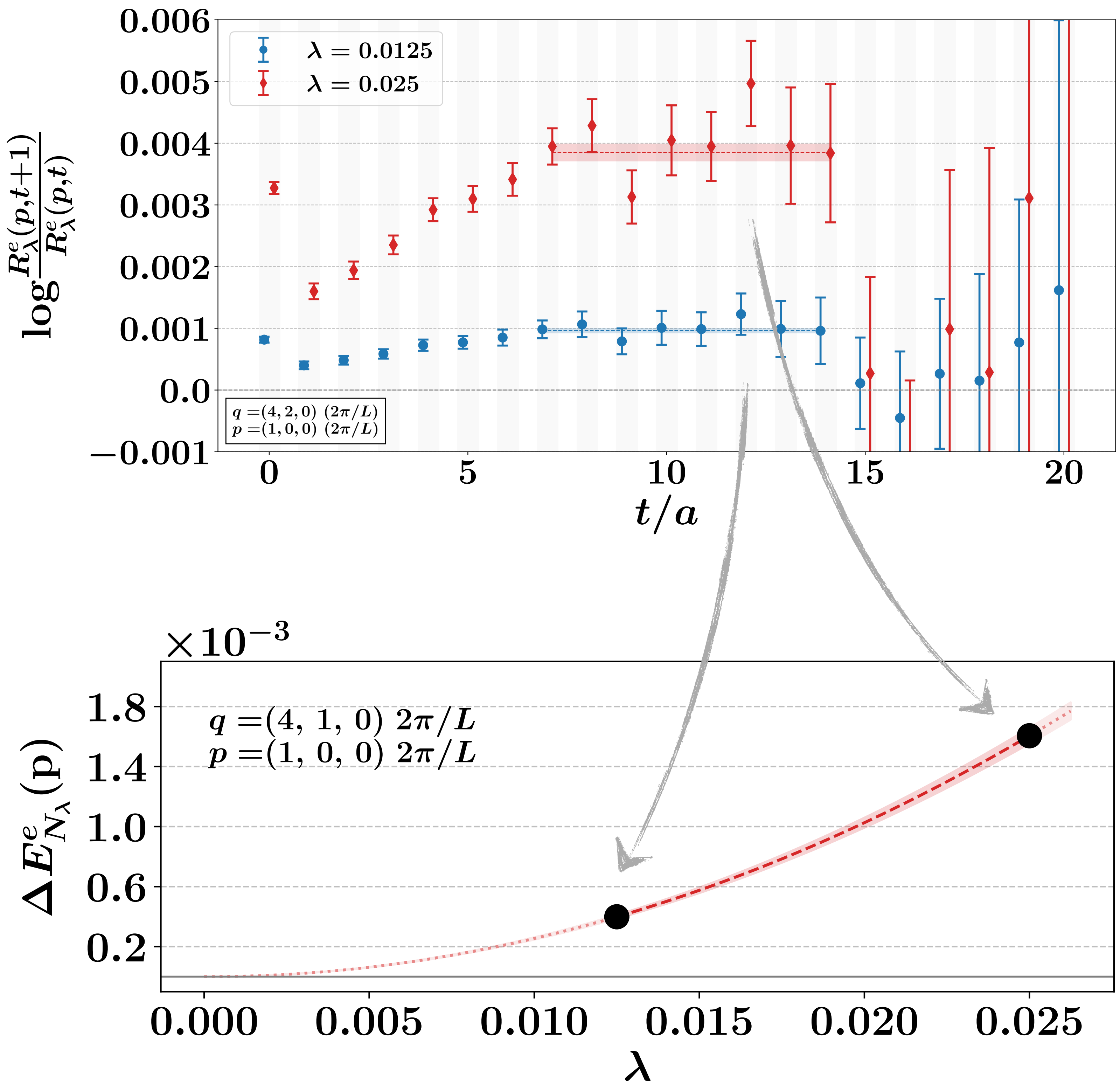
$$m_\pi L \sim \begin{bmatrix} 5.6 \\ 6.9 \end{bmatrix} \qquad a = \begin{bmatrix} 0.074 \\ 0.068 \end{bmatrix} \text{ fm}$$



Unmodified QCD background

- Local EM current insertion, $J_\mu(x) = Z_V \bar{q}(x)\gamma_\mu q(x)$ (valence only)
- Feynman–Hellmann propagators at 4 field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Up to $\mathcal{O}(10^4)$ measurements for each pair of Q^2 and λ

Energy shifts



Effective energies

2 external field strengths

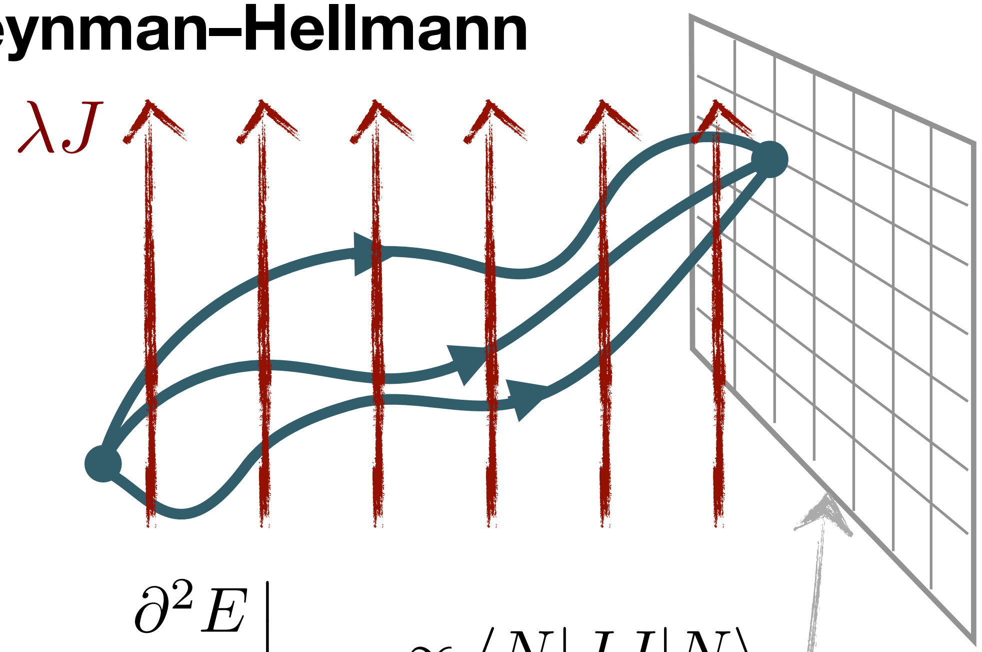
$$\Delta E = E(\lambda) - E_0$$

Isolate 2nd derivative
(almost "exact" quadratic)

$$\Delta E = \tfrac{1}{2}\lambda^2 \frac{\partial^2 E}{\partial \lambda^2} + \ldots$$

Kinematic coverage

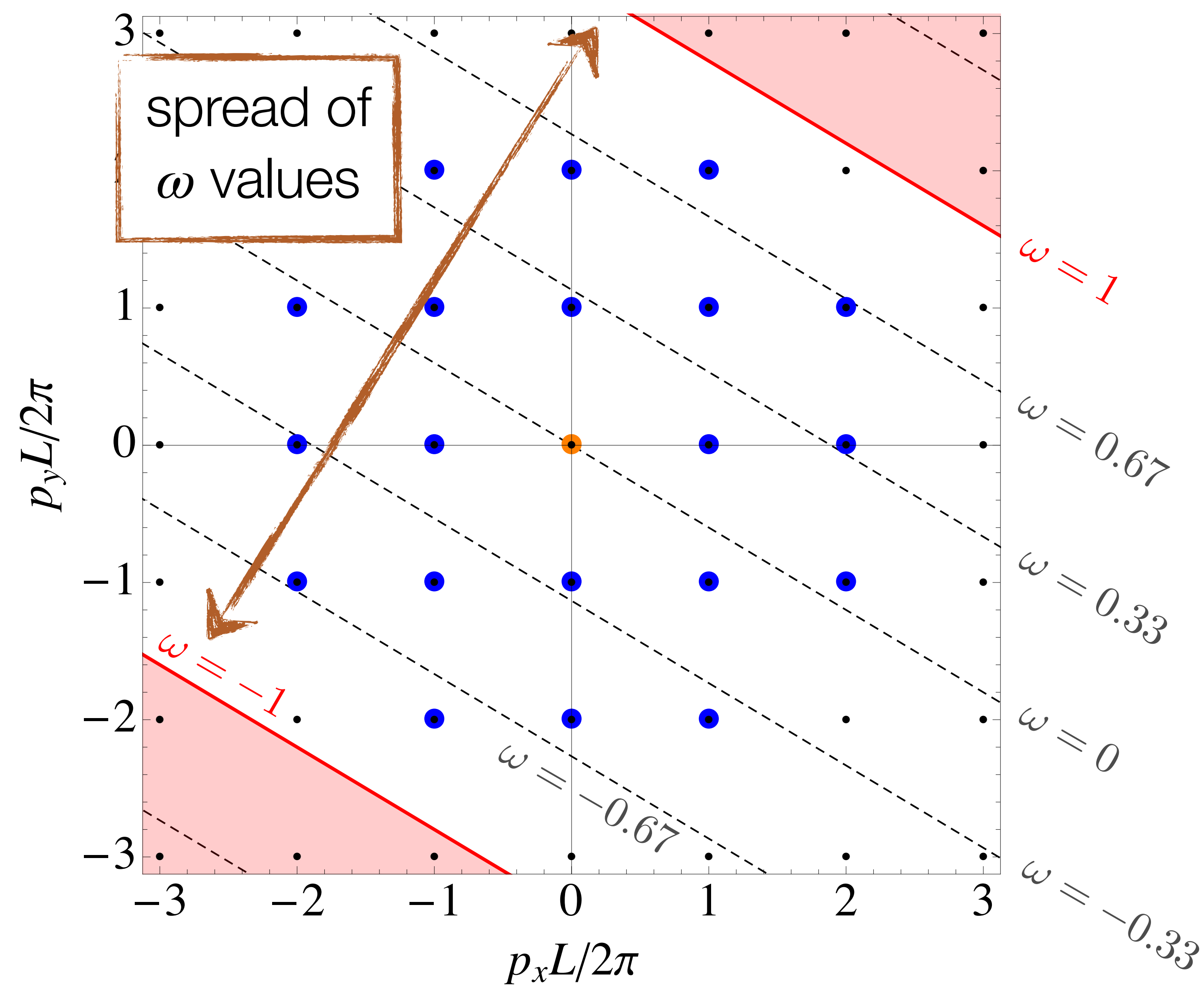
Feynman–Hellmann



$$\left.\frac{\partial^2 E}{\partial \lambda^2}\right|_{\lambda\to 0} \propto \langle N|JJ|N\rangle$$

$\mathbf{q}$ fixed for each propagator evaluation (costly)

freedom to choose Fourier projection at hadron sink (cheap)

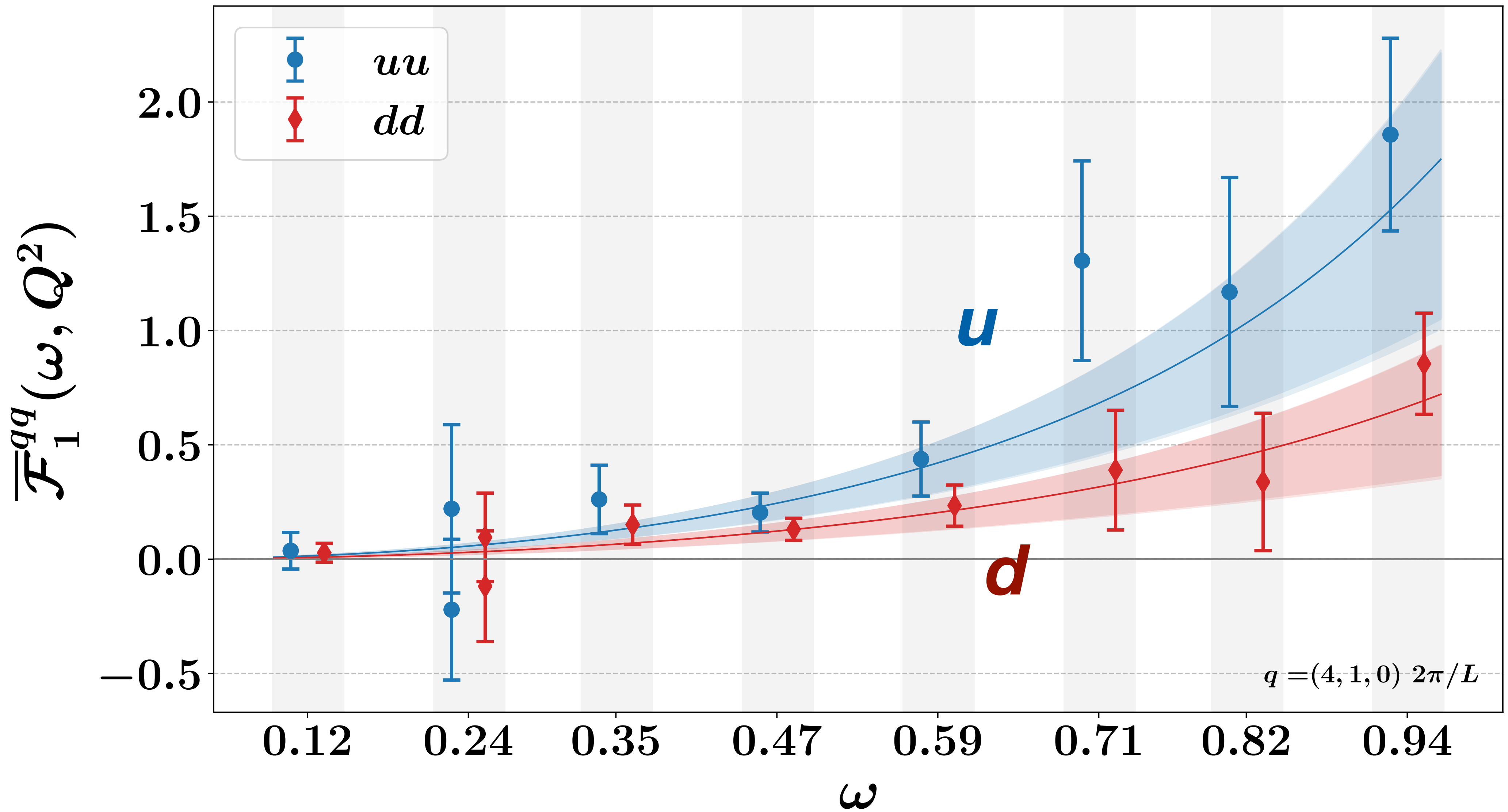


$$\omega = \frac{2p\cdot q}{Q^2} = \frac{2\mathbf{p}\cdot\mathbf{q}}{\mathbf{q}^2}$$

$$\mathbf{q} = \frac{2\pi}{L}(3,5,0)$$

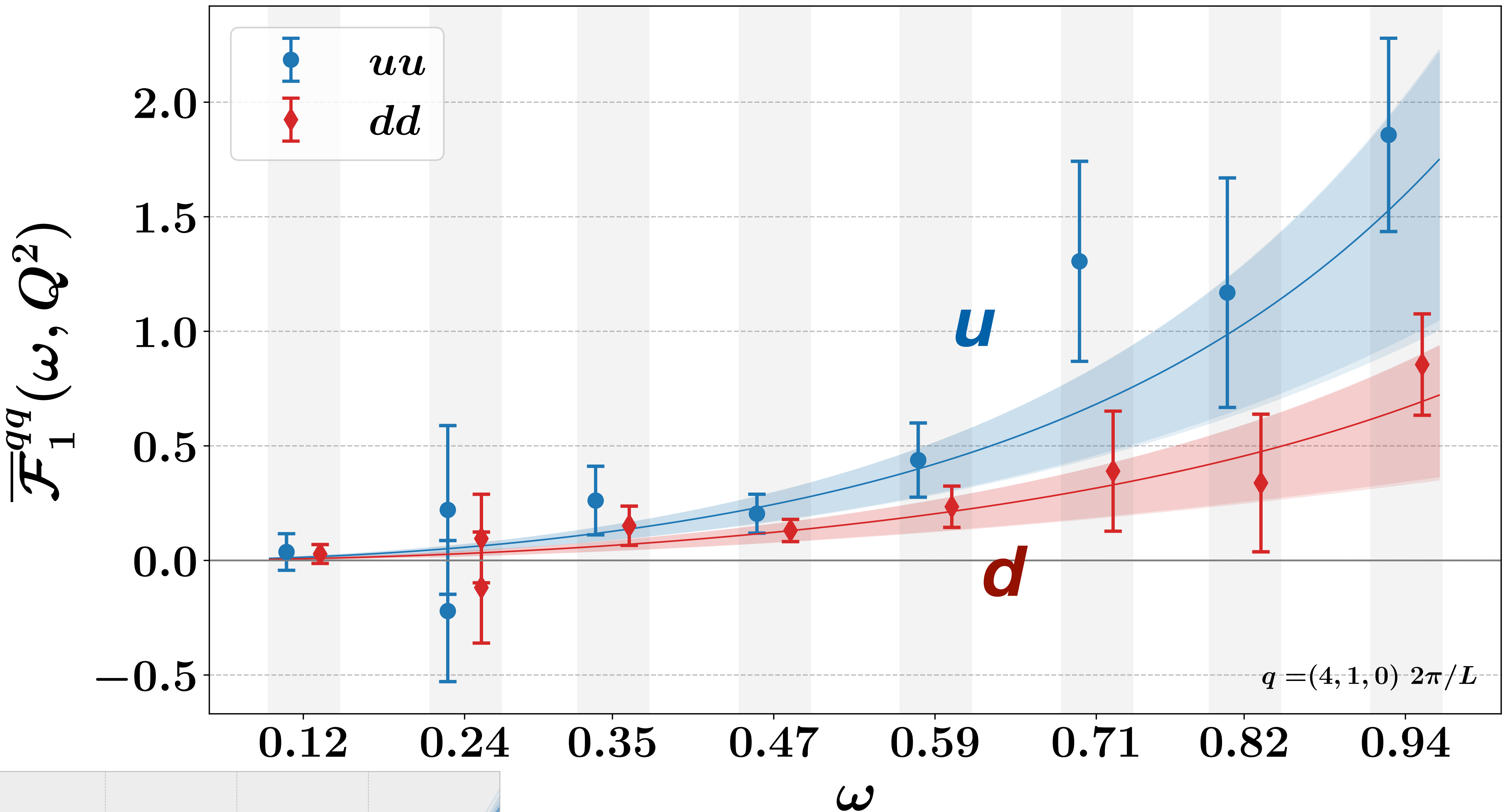
$$q_4 = 0$$

Compton

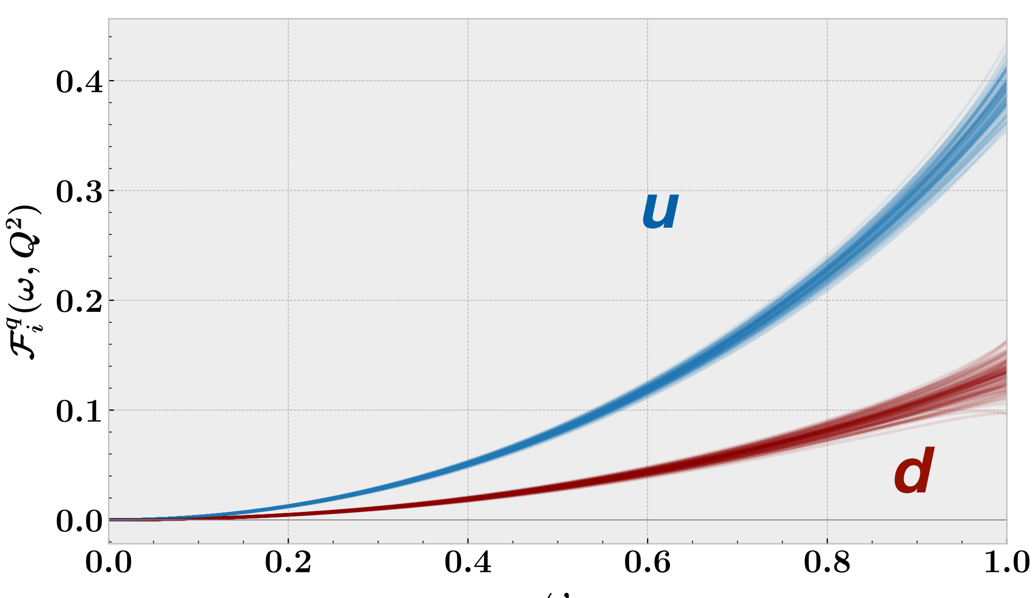


uu
dd
u
d
2.0
1.5
1.0
0.5
0.0
−0.5
$\overline{\mathcal{F}}_1^{qq}(\omega, Q^2)$
$q = (4, 1, 0)\ 2\pi/L$
0.12
0.24
0.35
0.47
0.59
0.71
0.82
0.94
ω

Compton



uu
dd
u
d
$q = (4, 1, 0)\ 2\pi/L$
$\overline{\mathcal{F}}_1^{qq}(\omega, Q^2)$
2.0
1.5
1.0
0.5
0.0
−0.5
0.12
0.24
0.35
0.47
0.59
0.71
0.82
0.94
ω



u
d
$\mathcal{F}_i^q(\omega, Q^2)$
0.4
0.3
0.2
0.1
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Moments

- Recall dispersion integral:

$$\overline{\mathcal{F}}_1(\omega, Q^2) = 2\omega^2 \int_0^1 dx \, \frac{2xF_1(x, Q^2)}{1-(x\omega)^2} = 2\sum_{n=1}^{\infty} \omega^{2n} M_{2n}^{(1)}(Q^2)$$

Moments

$$M_{2n}^{(1)}(Q^2) = 2\int_0^1 dx \, x^{2n-1} F_1(x, Q^2)$$

- Positivity constraint:

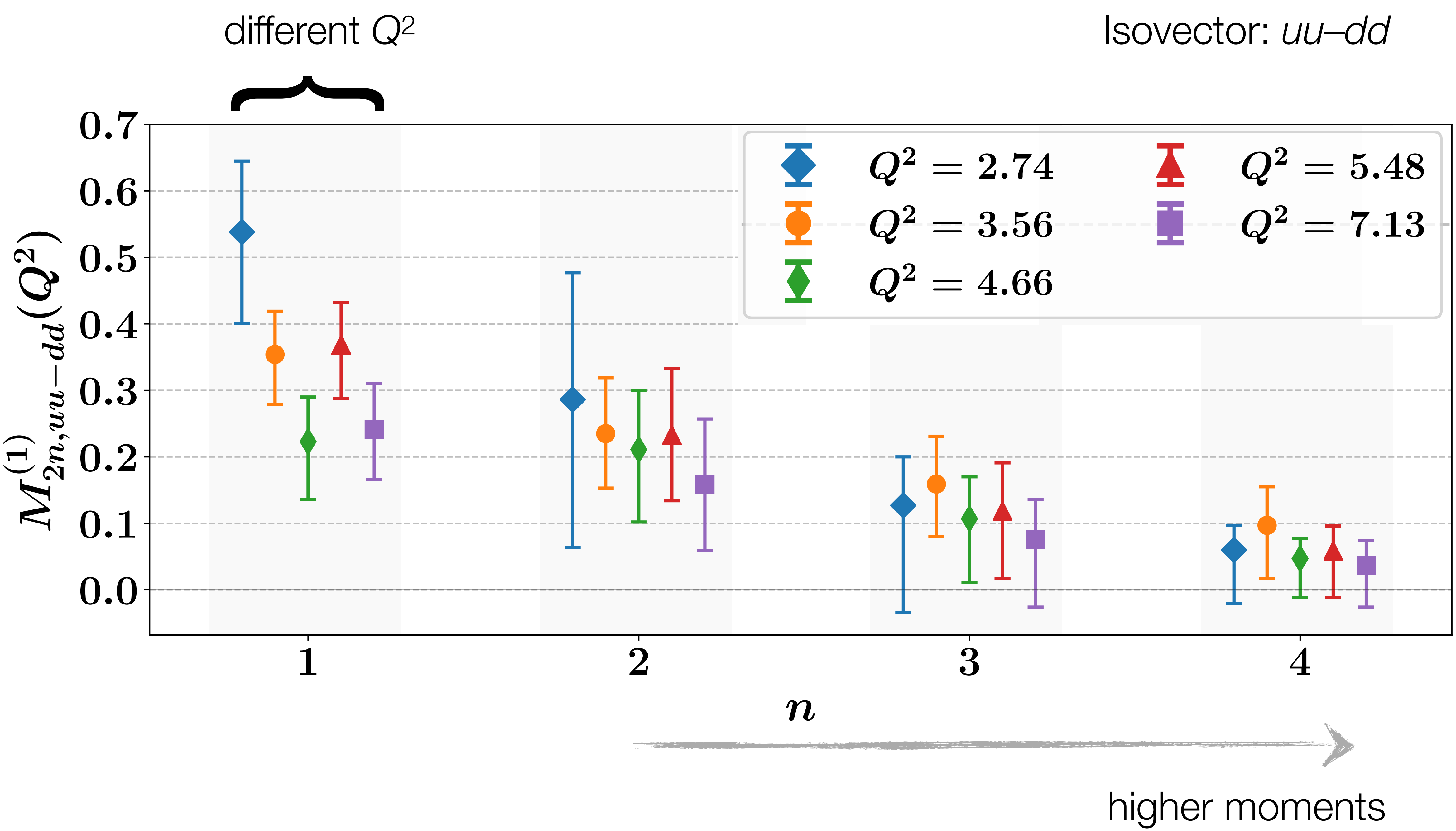
$$M_2 \geq M_4 \geq M_6 \geq M_8 \geq M_{10} \geq \ldots > 0$$

- Use Bayesian fit enforcing monotonicity of moments

Priors: $M_{2n+2} \in [0, M_{2n}]$ (uniform sampling)

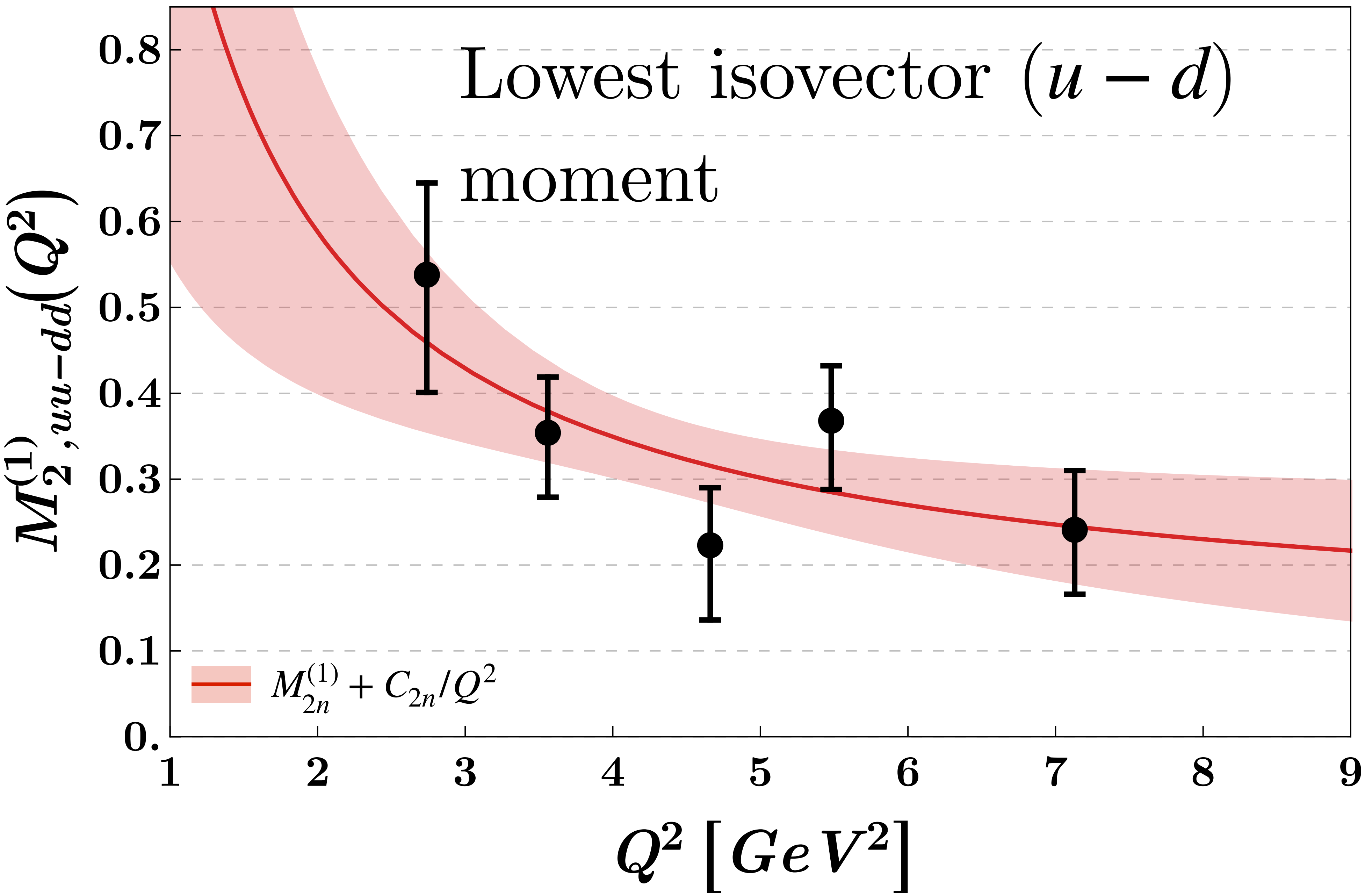
low moments insensitive to truncation order

Low moments



different Q^2
Isovector: *uu–dd*
$Q^2 = 2.74$
$Q^2 = 3.56$
$Q^2 = 4.66$
$Q^2 = 5.48$
$Q^2 = 7.13$
$M^{(1)}_{2n,uu-dd}(Q^2)$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
2
3
4
n
higher moments

A hint of power



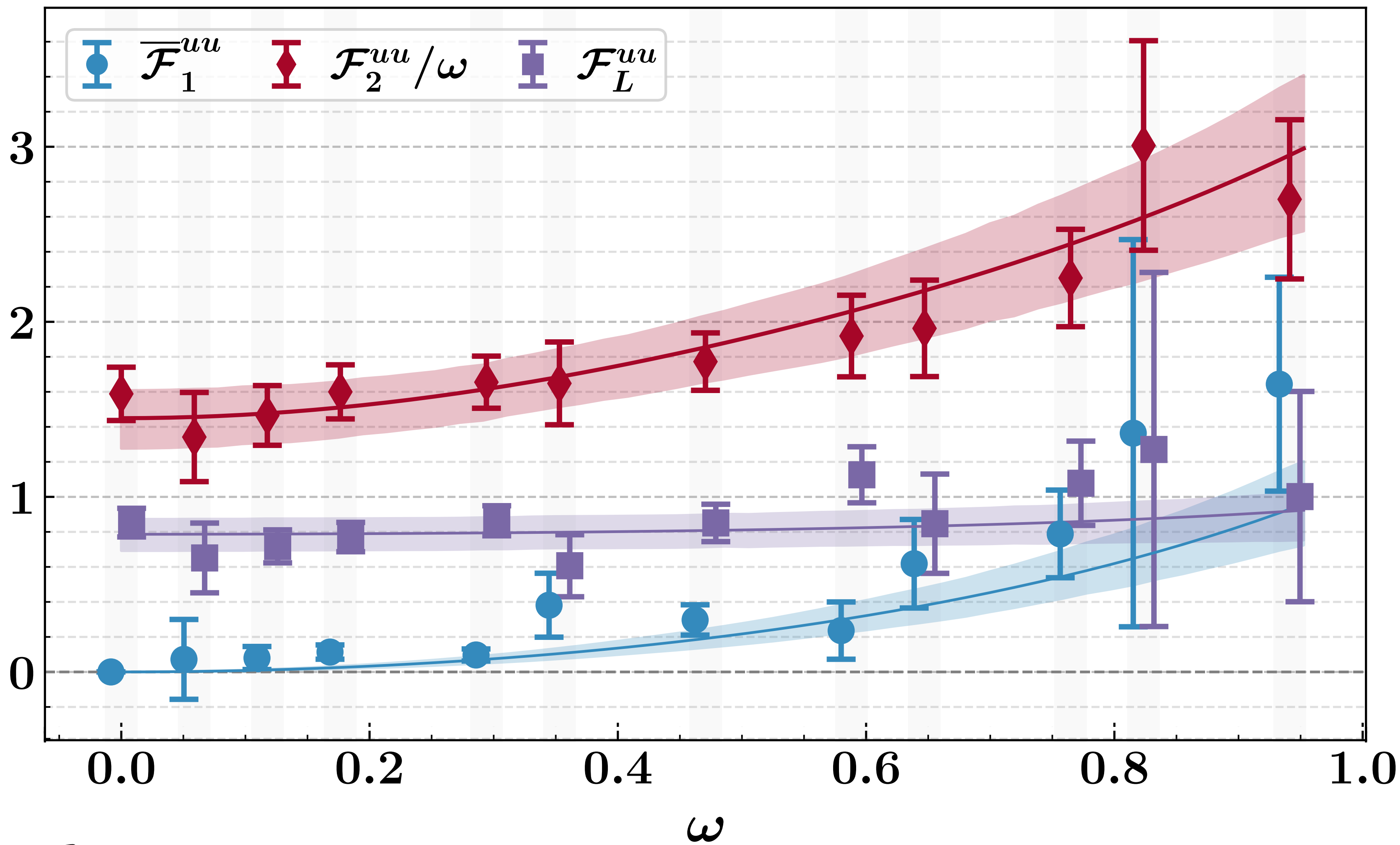
squint hard enough and there's a power correction in (lowest moment of) $\mathscr{F}_1$

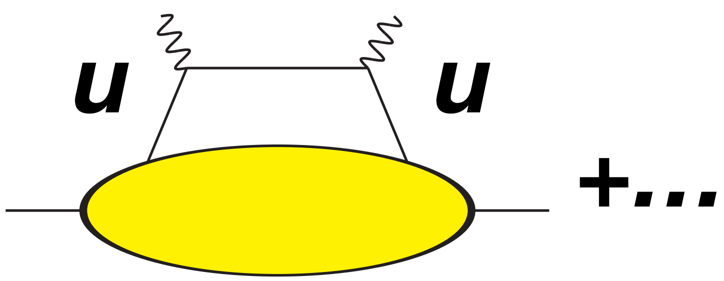
$\mathscr{F}_2$ and the longitudinal structure function

Can, RDY *et al*. Phys.Rev.D 107 (2023) 5, 054503

Compton structure functions

48^3x96, 2+1 flavour
$a = 0.068$ fm
$m_\pi \sim 420$ MeV
$Q^2 = 4.9\, GeV^2$





Compton structure functions

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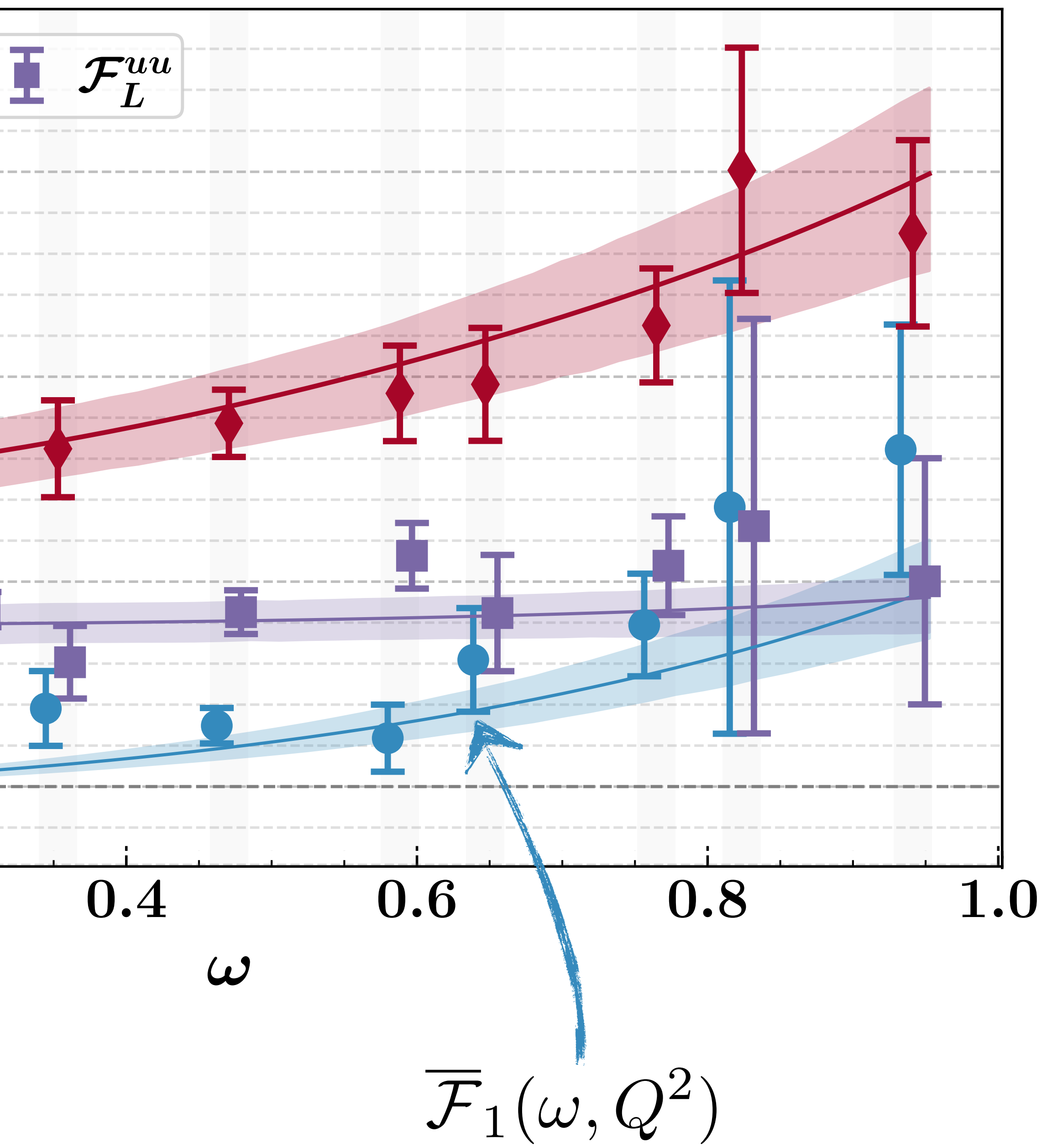

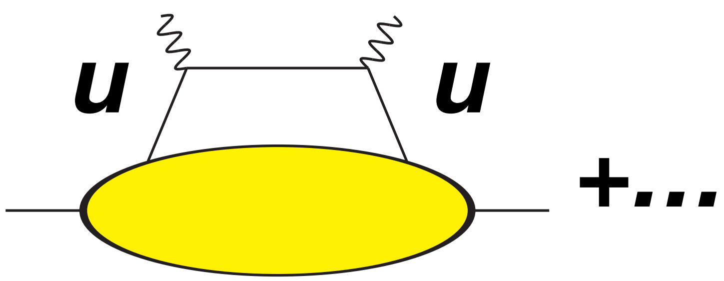

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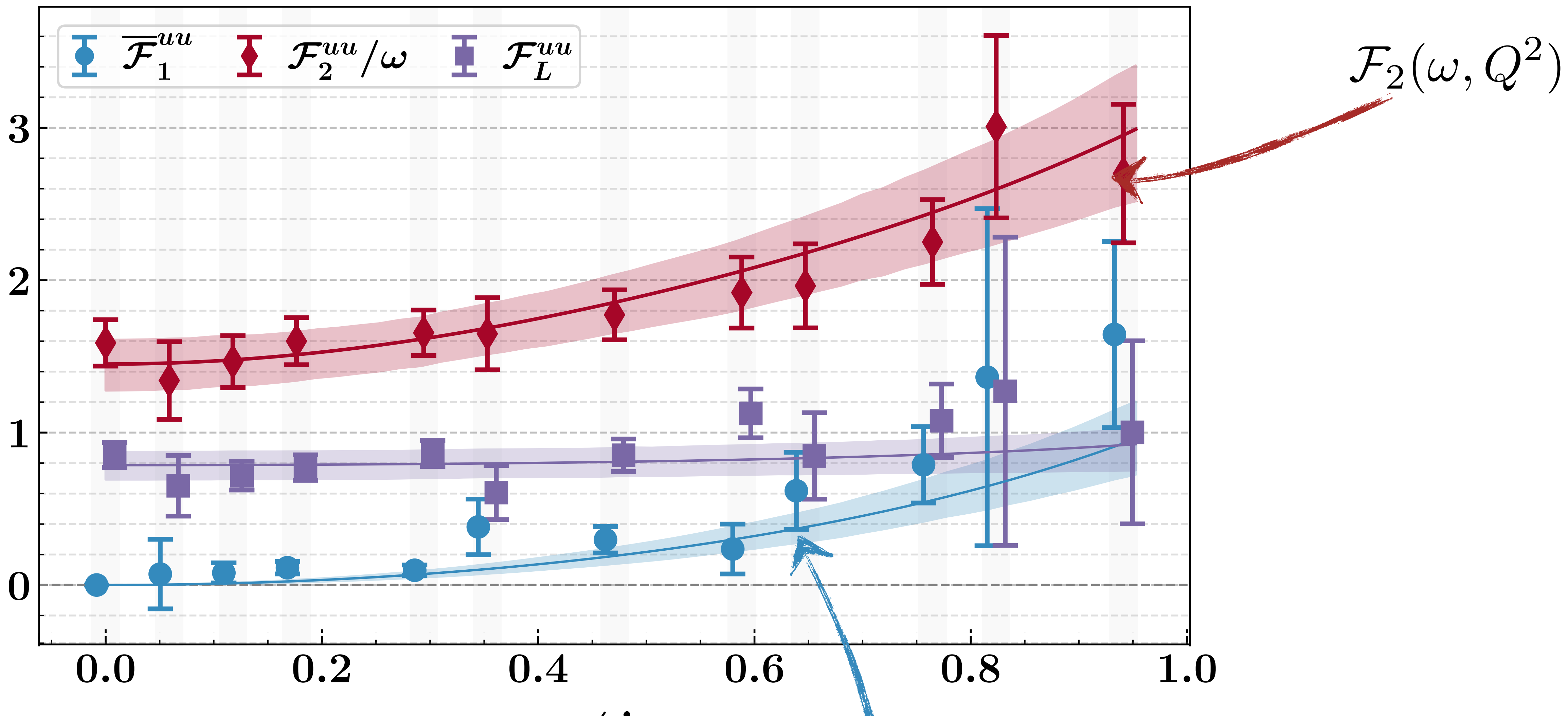

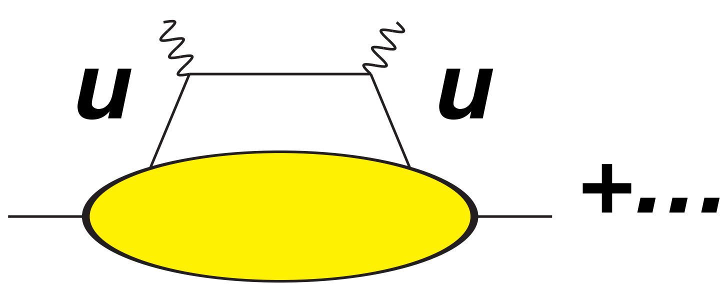

Compton structure functions

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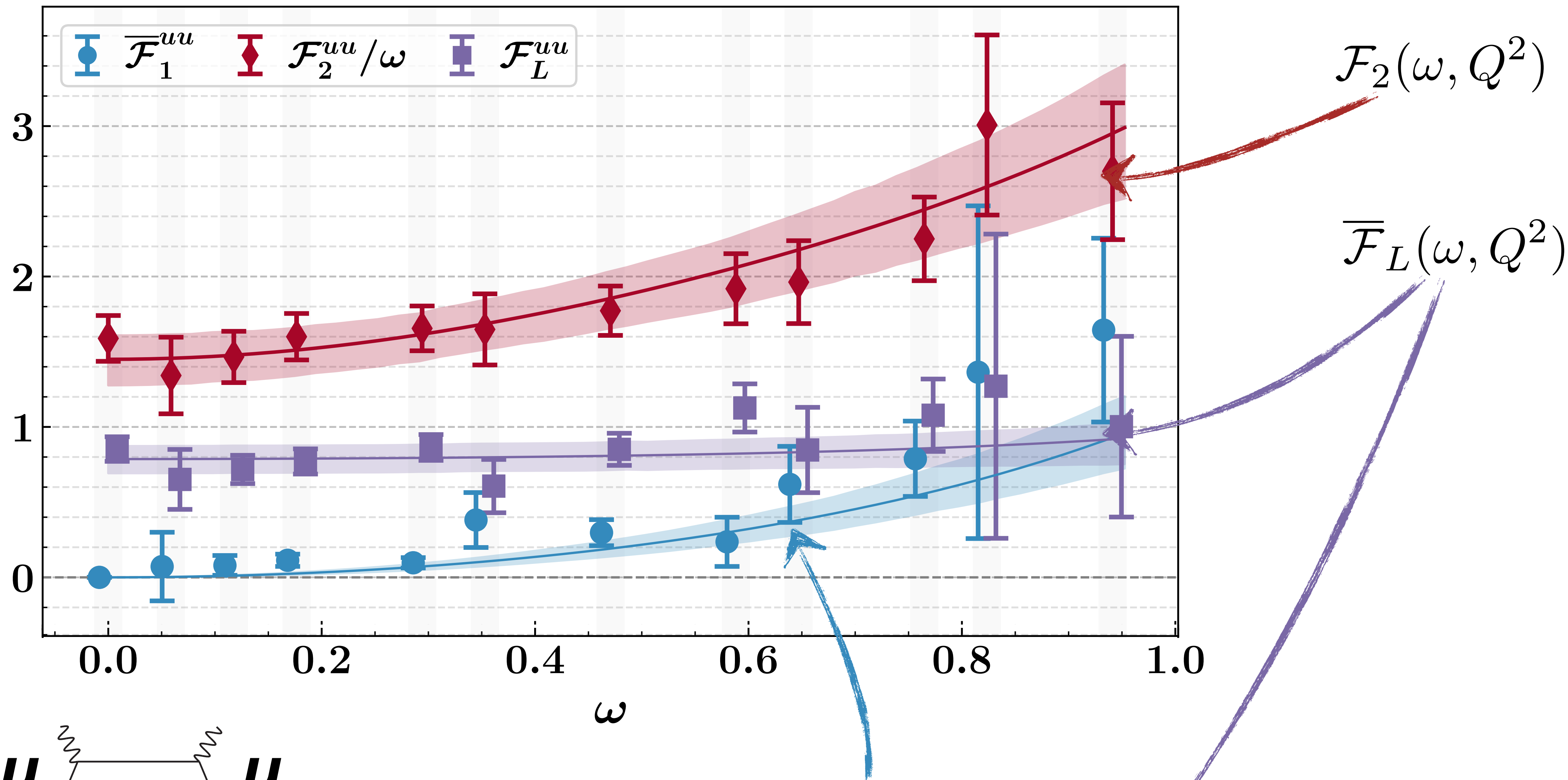
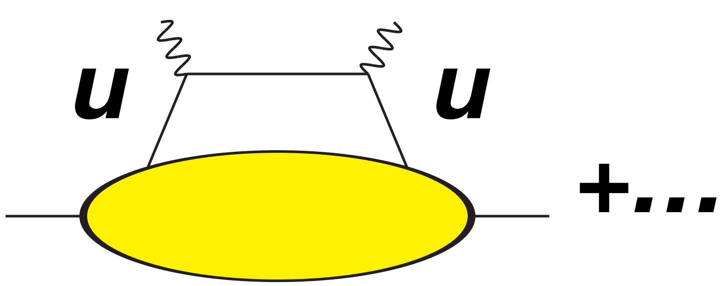




$$\overline{\mathcal{F}}_L(\omega, Q^2) = -\overline{\mathcal{F}}_1(\omega, Q^2) + \left(\frac{\omega}{2} + \frac{2M_N^2}{\omega Q^2}\right)\mathcal{F}_2(\omega, Q^2)$$

Longitudinal Compton amplitude

Moments: Simultaneous fits

- Dispersion relation for F_L:

$$\overline{\mathcal{F}}_L(\omega, Q^2) = \frac{8M_N^2}{Q^2}\int_0^1 dx\, F_2(x, Q^2) + 2\omega^2 \int_0^1 dx \frac{F_L(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}$$

- Parameterise in terms of moments of F_1 and F_L

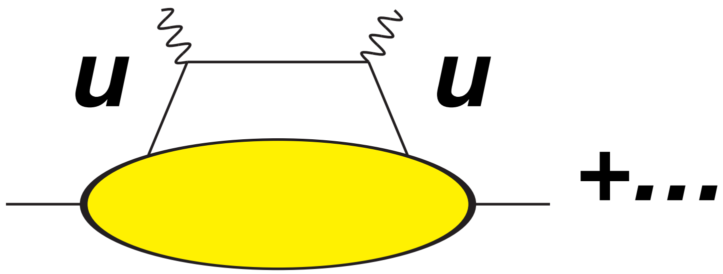
$$M_2^{(1)}, M_4^{(1)}, M_6^{(1)}, \ldots,$$
$$M_0^{(L)}, M_2^{(L)}, M_4^{(L)}, \ldots$$

independently positive definite

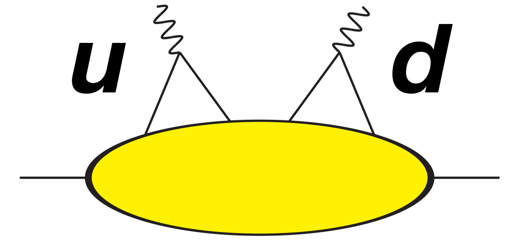
- Fit to two independent amplitudes F_1 and F_2

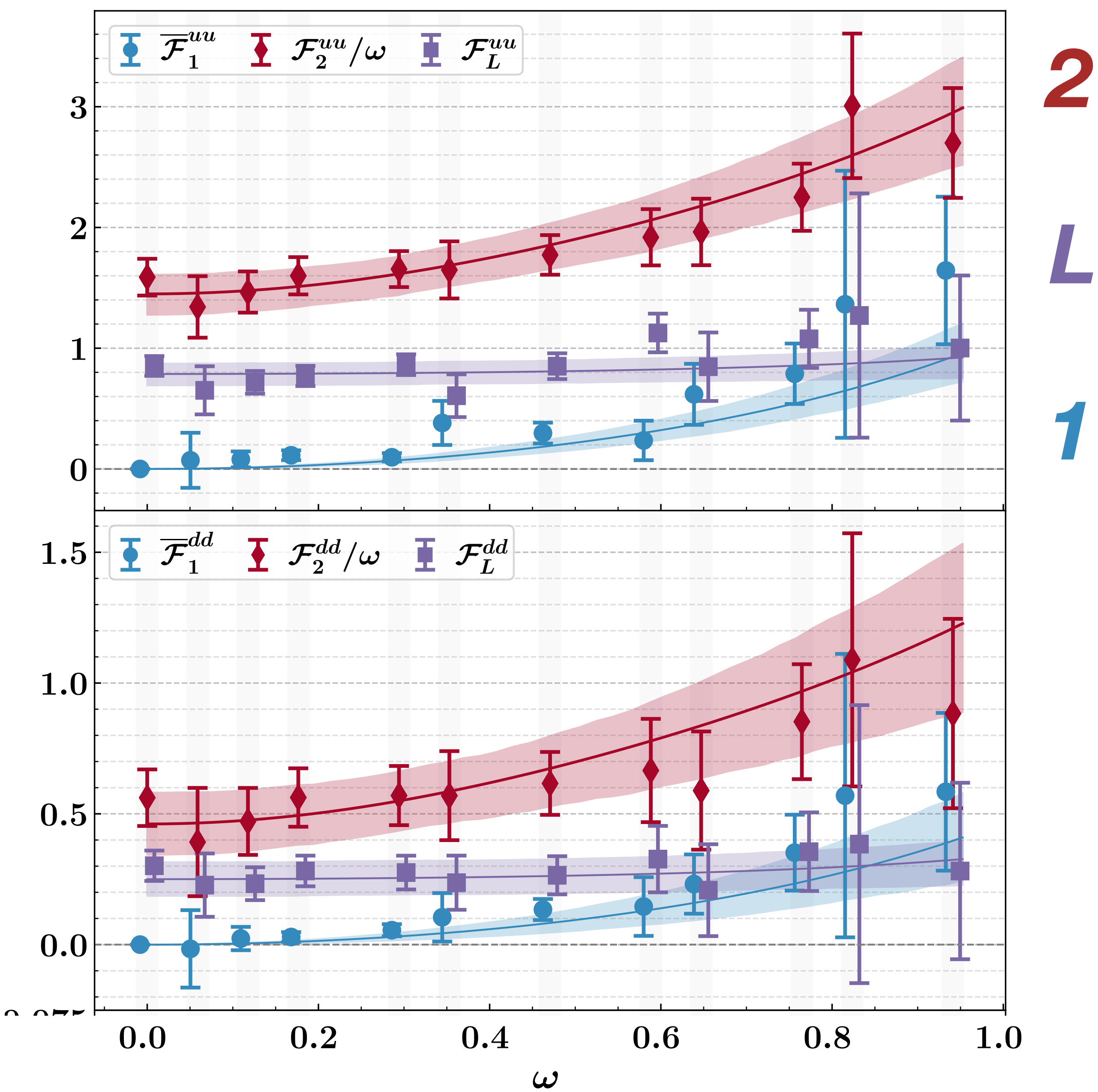
$$\overline{\mathcal{F}}_1(\omega, Q^2) = 2\sum_{n=1} \omega^{2n} M_{2n}^{(1)}(Q^2)$$

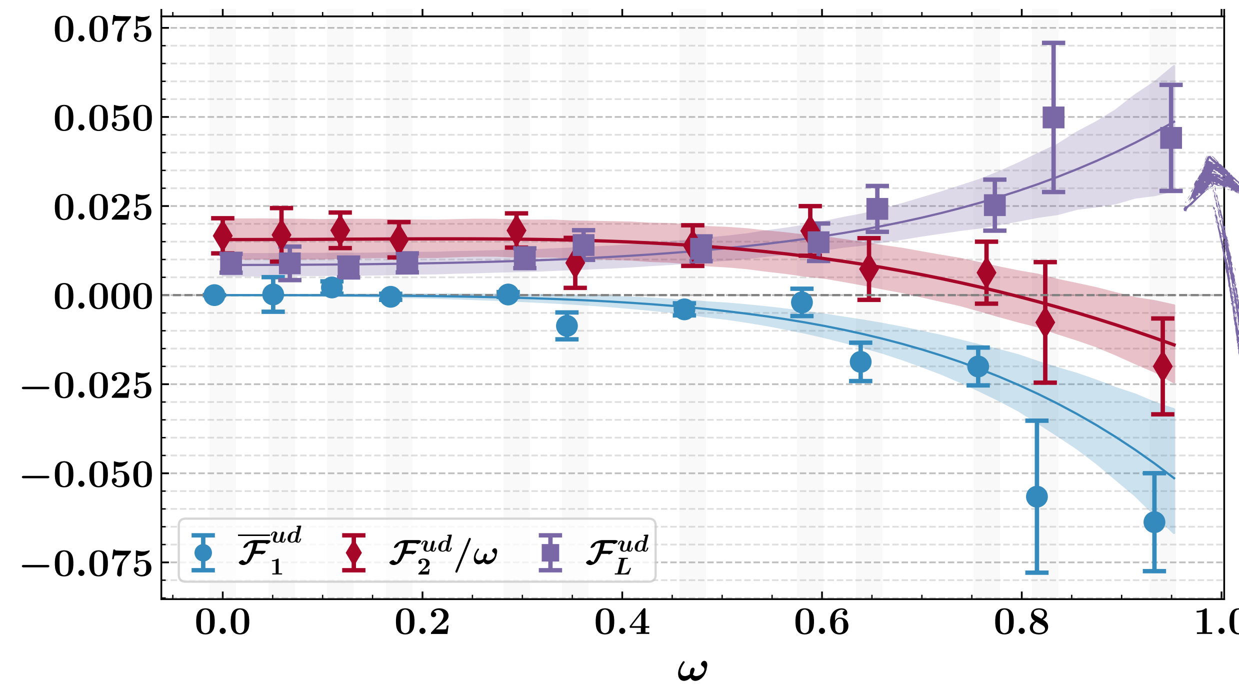
$$\frac{\mathcal{F}_2(\omega, Q^2)}{\omega} = \frac{\tau}{1 + \tau\omega^2}\sum_{n=0} 4\omega^{2n}\left[M_{2n}^{(1)} + M_{2n}^{(L)}\right]$$



$Q^2 = 4.9\,\mathrm{GeV}^2$



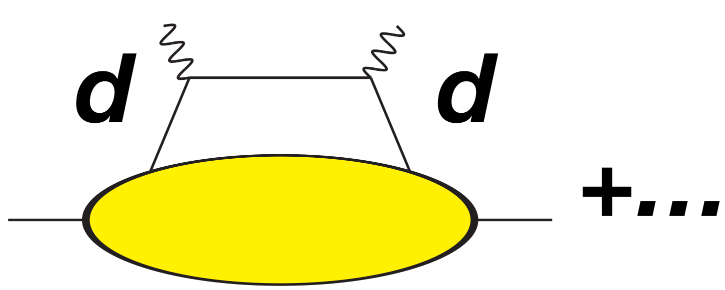




flavour-interference structure functions

* *small* in magnitude

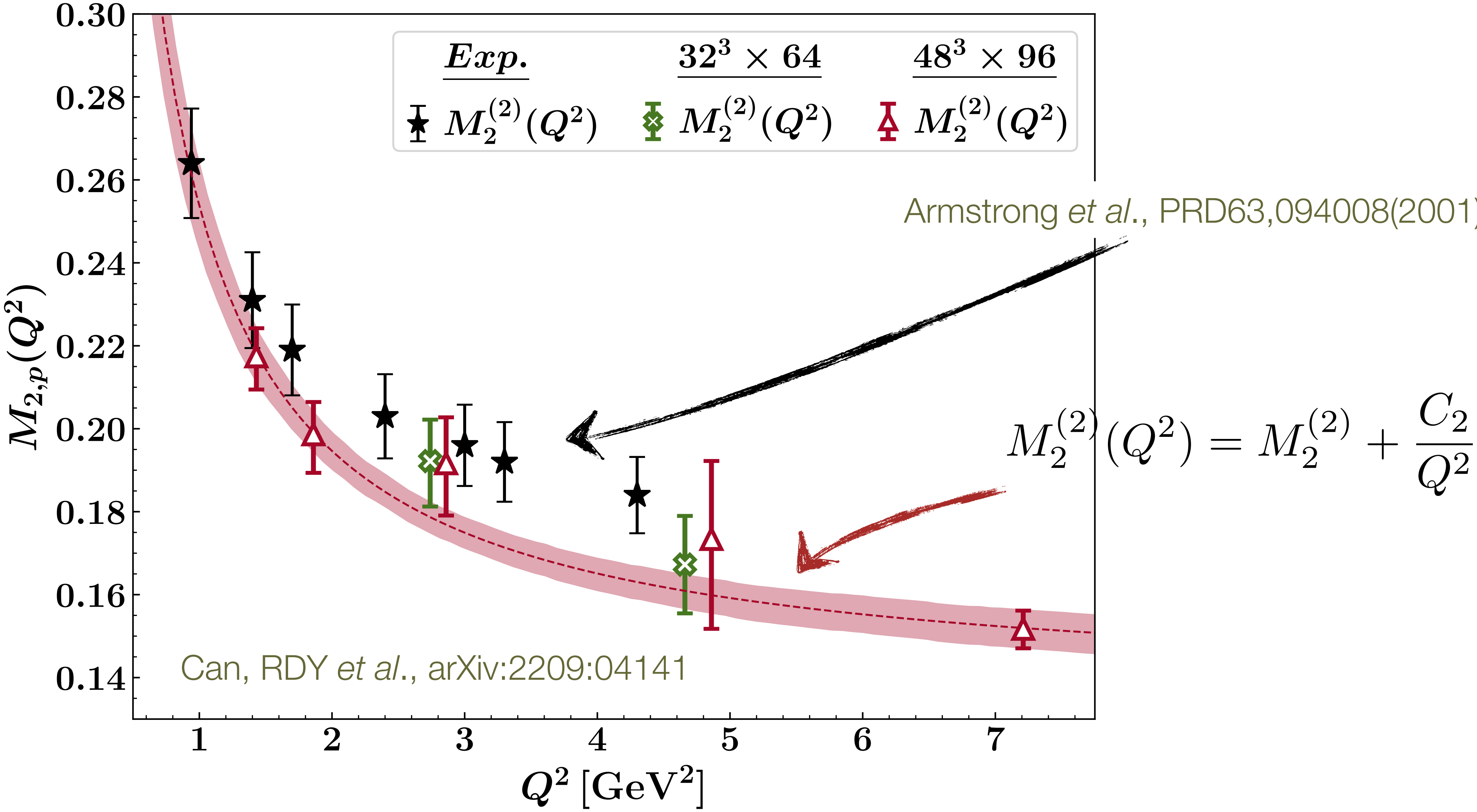
* non-trivial signal for longitudinal structure



Lowest moment of F_2 (proton)

Comparison with experiment

48³x96, 2+1 flavour
$a = 0.068\,\text{fm}$
$m_\pi \sim 420\,\text{MeV}$

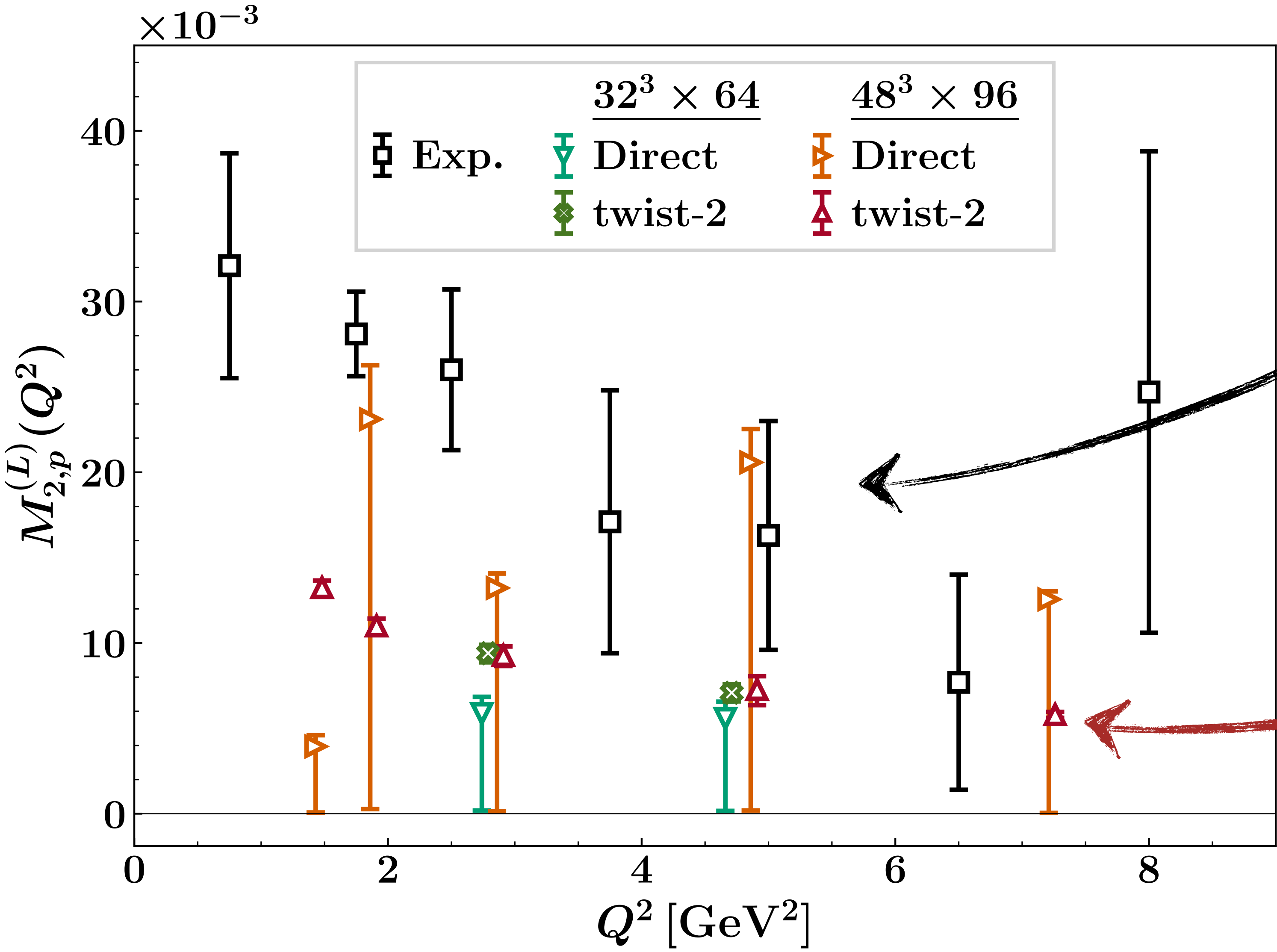


$$M_2^{(2)}(Q^2) = M_2^{(2)} + \frac{C_2}{Q^2}$$

Clear evidence for power corrections!

Compatible with phenomenological trend

Longitudinal moments

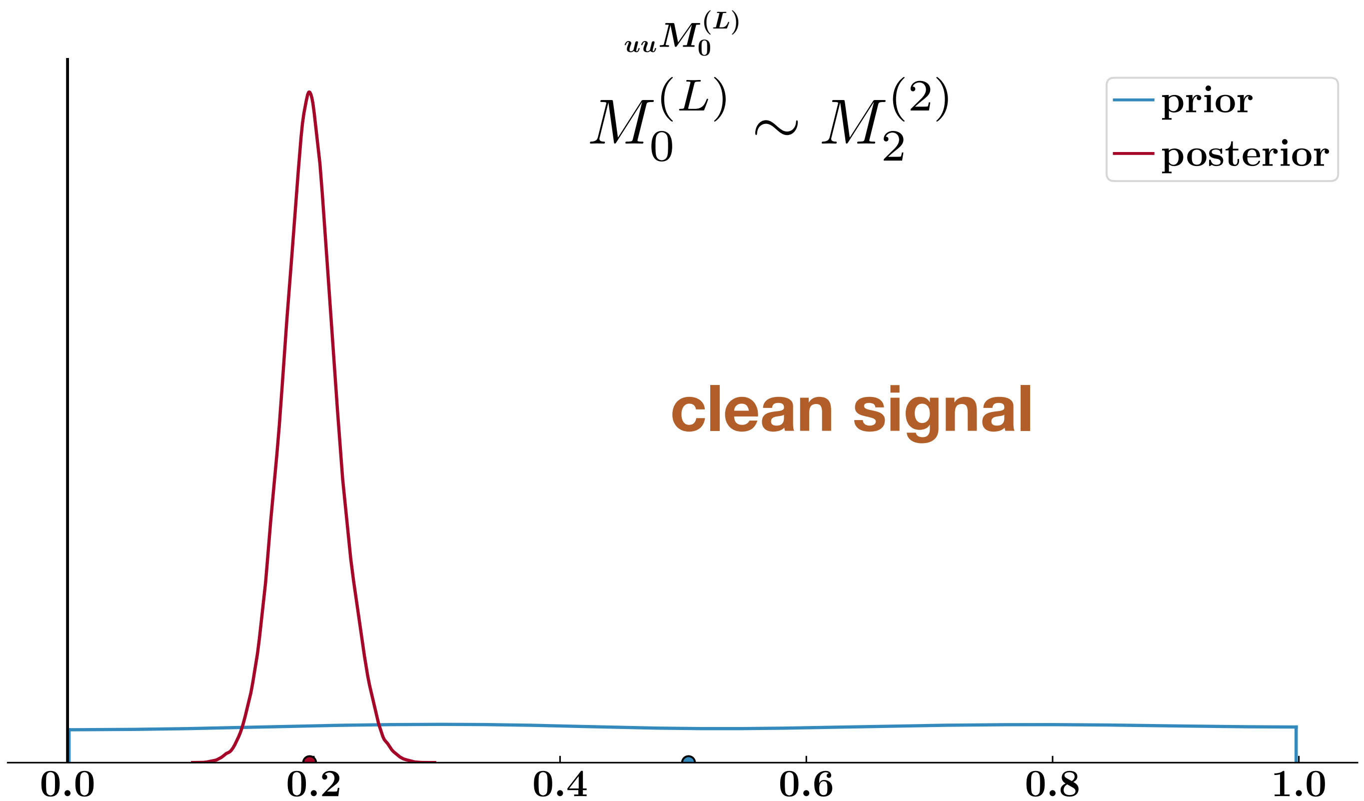


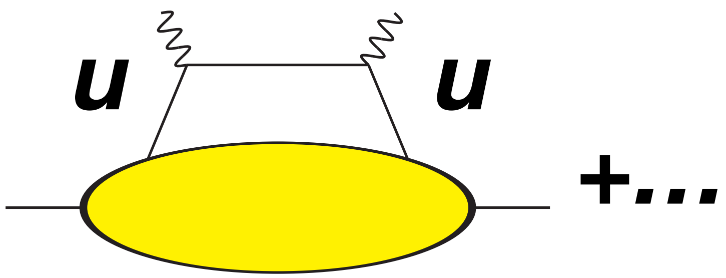
Monaghan *et al.*, PRL110,152002(2013)

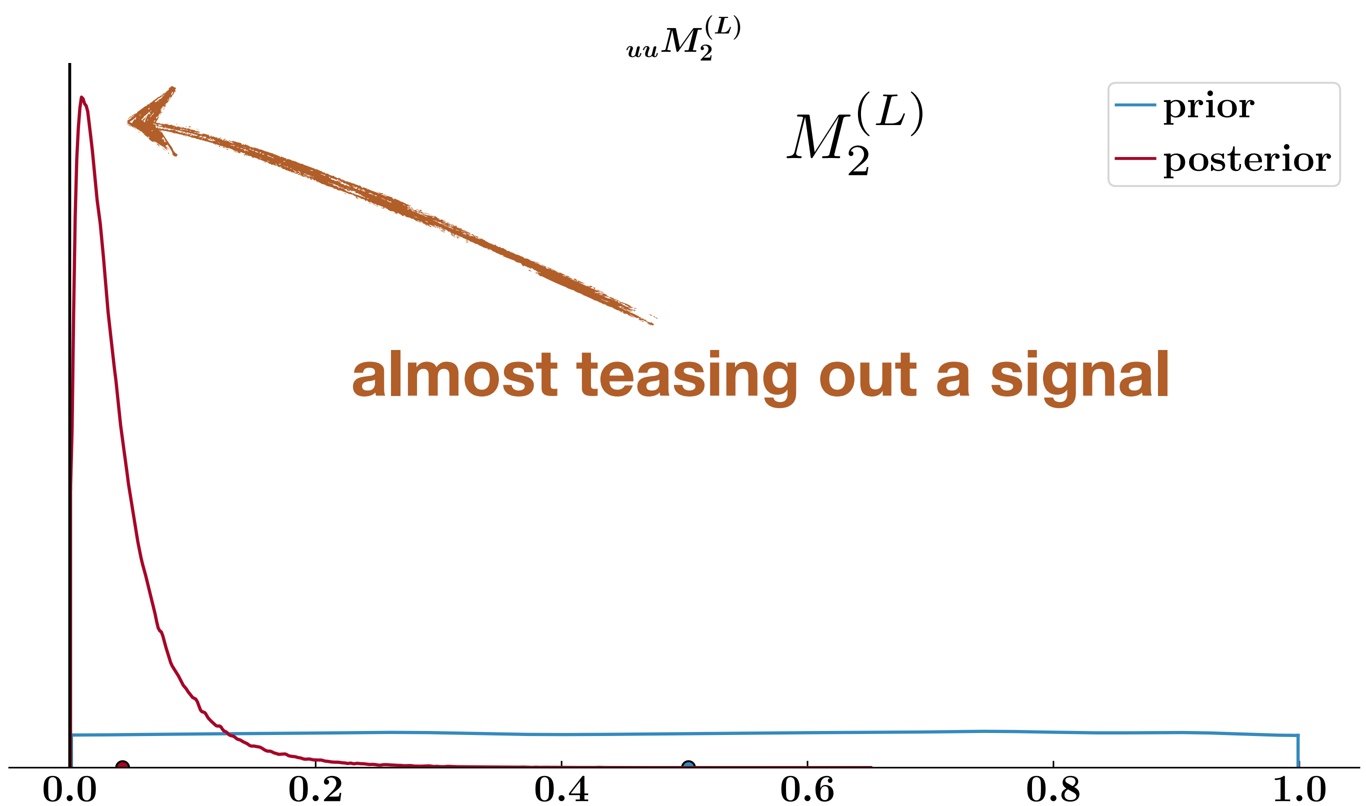
Leading-twist part

$$M_{2,p}^{(L),\text{twist-2}} = \frac{4}{9\pi}\alpha_s M_{2,p}^{(2),\text{twist-2}}$$

Moment posteriors — longitudinal SF



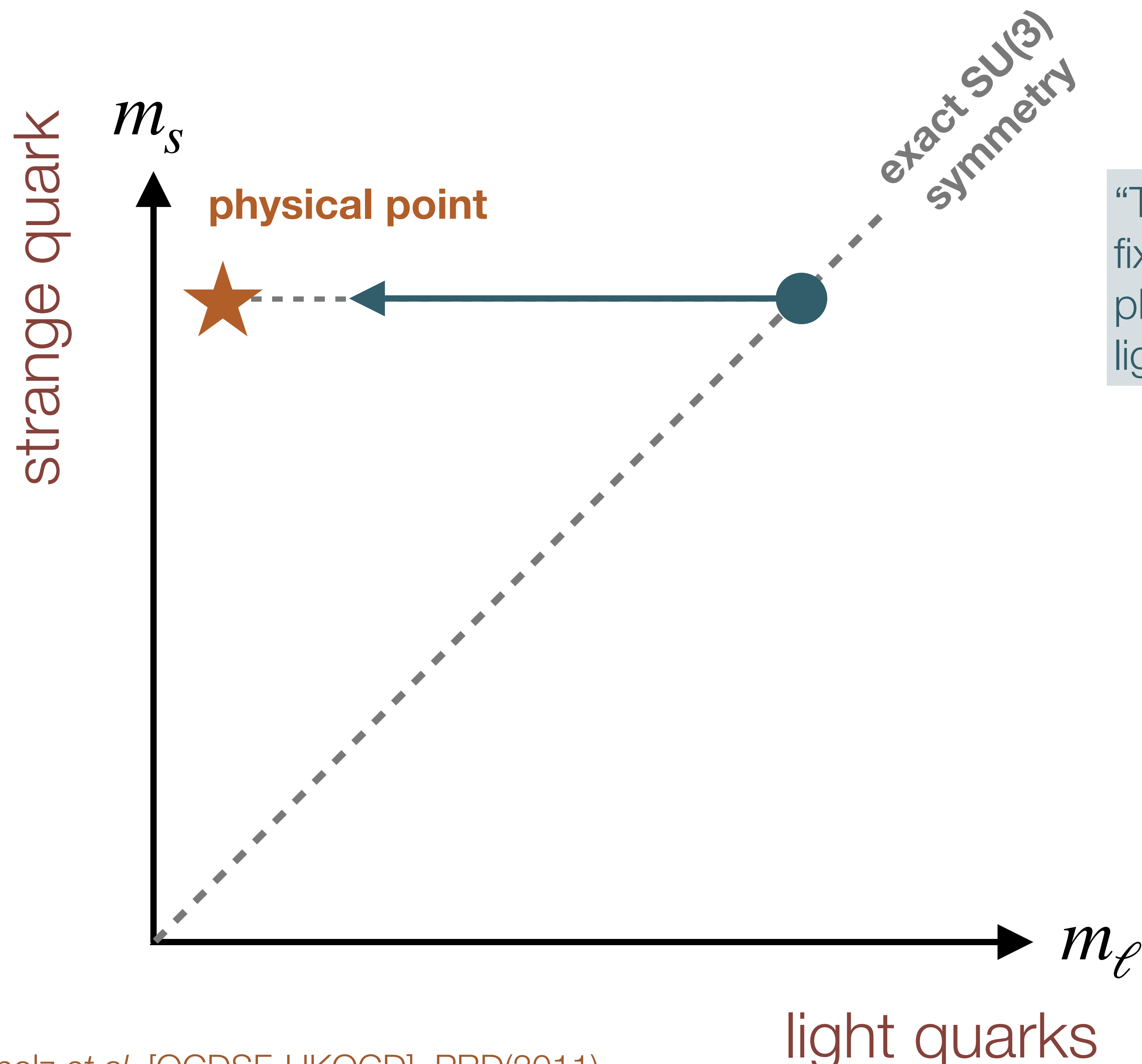




Preliminary update: Quark mass dependence

Can, RDY *et al.*, 2505.04033 (conf proc.)

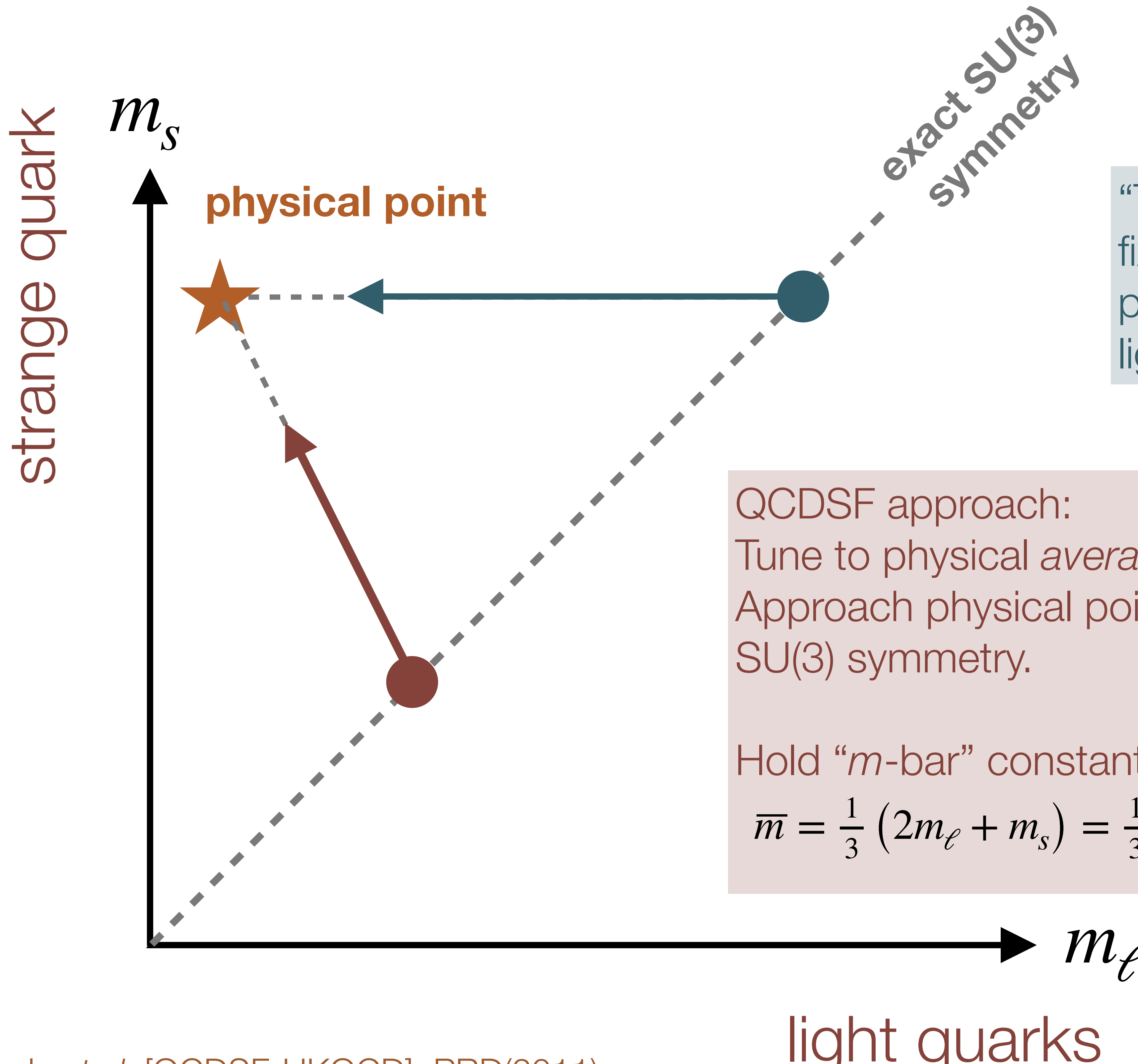
2+1-flavour quark-mass plane



"Typical" calculation:
fix strange quark mass to physical point and lower light quark mass

Bietenholz *et al*. [QCDSF-UKQCD], PRD(2011)

2+1-flavour quark-mass plane



"Typical" calculation:
fix strange quark mass to physical point and lower light quark mass

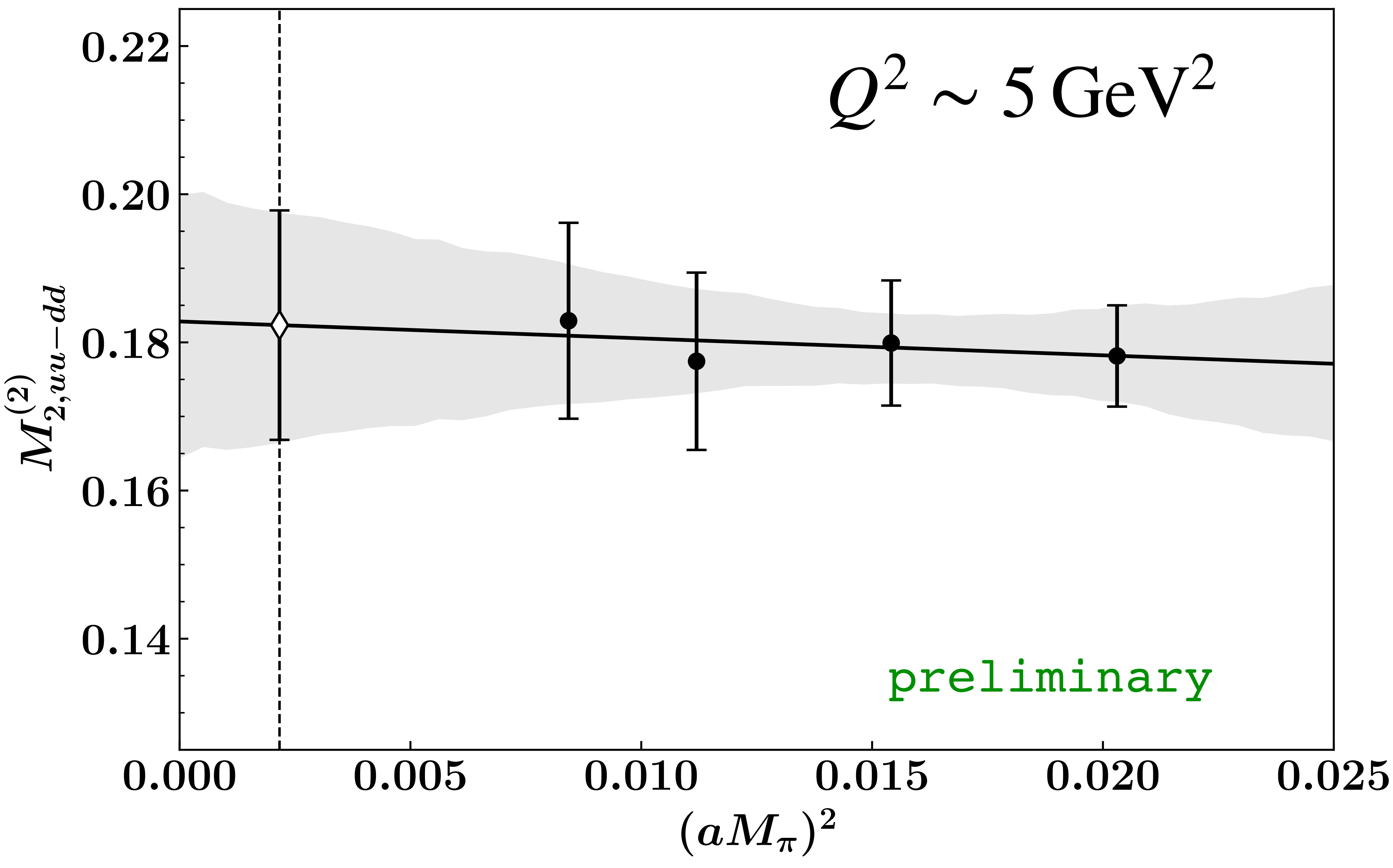
QCDSF approach:
Tune to physical *average* quark mass.
Approach physical point by breaking SU(3) symmetry.

Hold "*m*-bar" constant:

$$\overline{m} = \frac{1}{3}\left(2m_\ell + m_s\right) = \frac{1}{3}\left(2m_\ell^{\text{phys}} + m_s^{\text{phys}}\right)$$

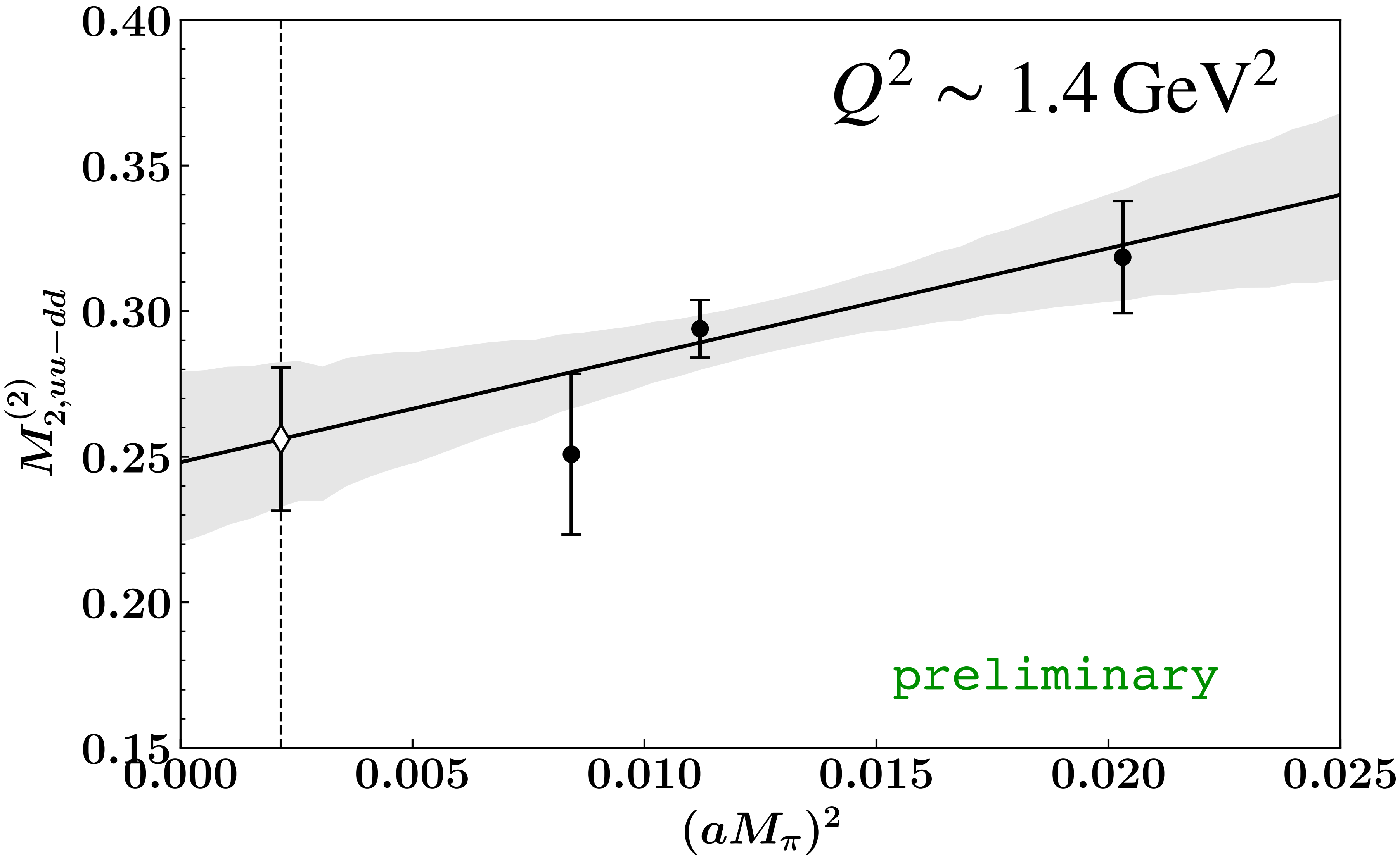
Bietenholz *et al*. [QCDSF-UKQCD], PRD(2011)

Lowest moment of F2



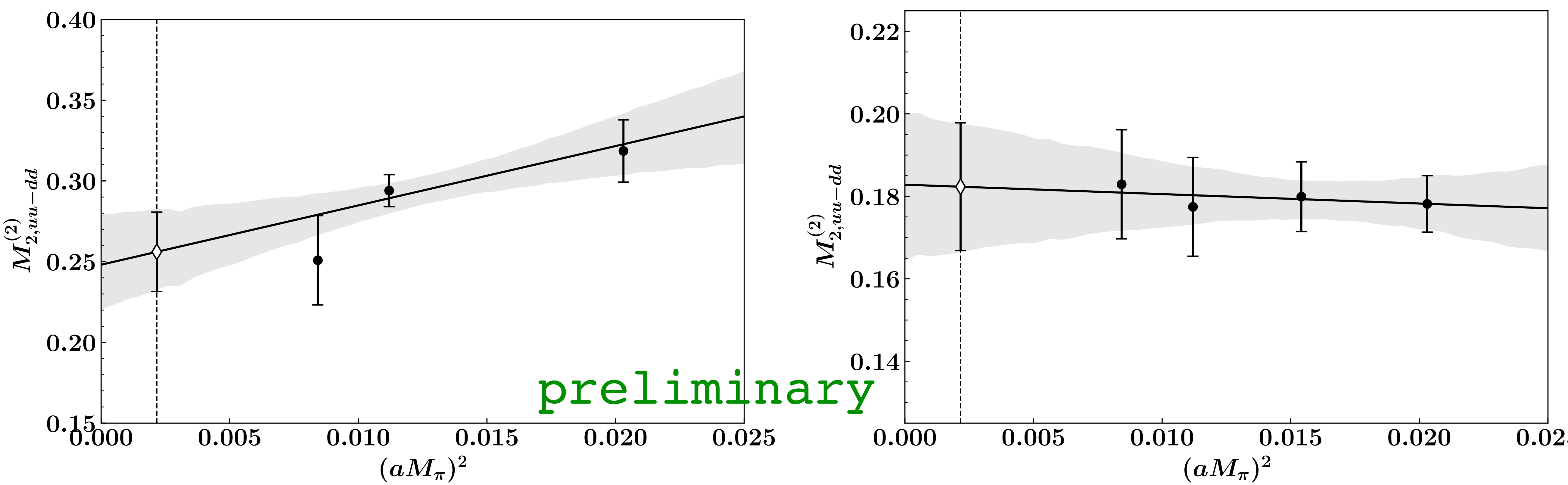
Isovector moment appears to show weak quark mass dependence

Lowest moment of F2

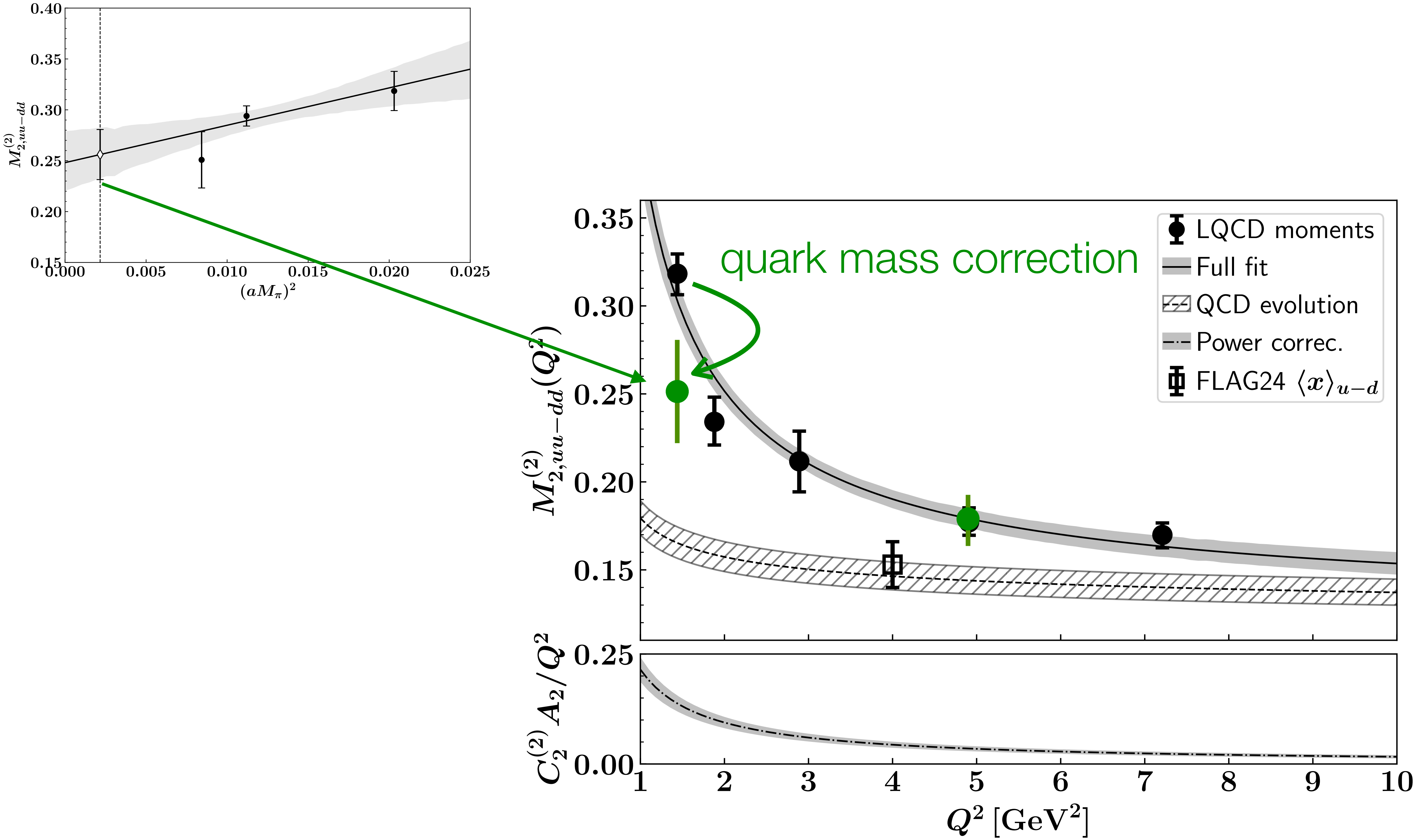


Mild quark mass dependence at lower Q^2

$Q^2 \sim 1.4\,\mathrm{GeV}^2$ $Q^2 \sim 5\,\mathrm{GeV}^2$



Early interpretation: Stronger quark mass dependence seen in power (higher-twist) corrections


$M^{(2)}_{2,uu-dd}$
$(aM_\pi)^2$
0.40
0.35
0.30
0.25
0.20
0.15
0.000
0.005
0.010
0.015
0.020
0.025
quark mass correction
LQCD moments
Full fit
QCD evolution
Power correc.
FLAG24 $\langle x \rangle_{u-d}$
$M^{(2)}_{2,uu-dd}(Q^2)$
$C^{(2)}_2 A_2/Q^2$
0.25
0.00
Q^2 [GeV^2]
1
2
3
4
5
6
7
8
9
10

The Compton subtraction function

Can, **Schar**, RDY *et al*. Phys.Rev.D 111 (2025) 9, 094513

Compton subtraction function

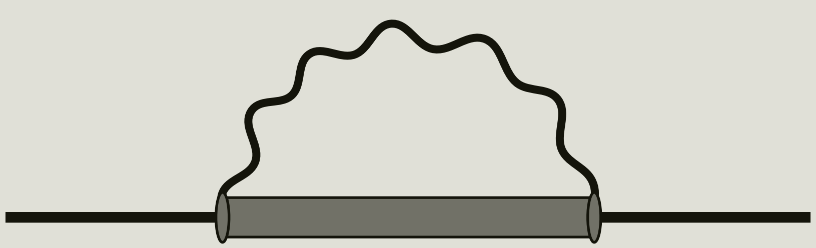
$$\mathscr{F}_1(\omega, Q^2) = \mathscr{F}_1(0, Q^2) + \frac{2\omega^2}{\pi}\int_1^\infty d\omega' \frac{\mathrm{Im}\mathscr{F}_1(\omega', Q^2)}{\omega'(\omega'^2 - \omega^2 - i\epsilon)}$$

The subtraction function:

$$\mathscr{F}_1(0, Q^2) \equiv S_1(Q^2)$$

Cottingham formula

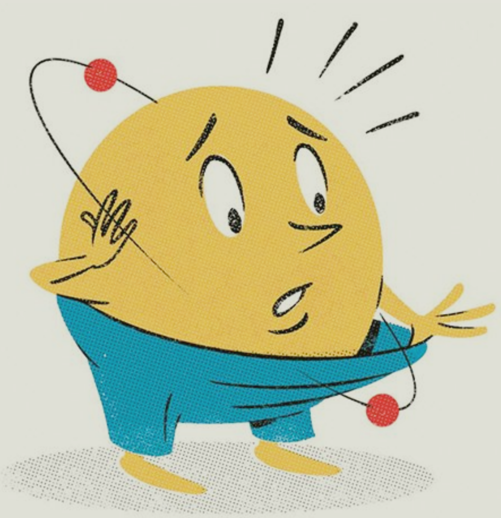
Electromagnetic self energy



$(M_n - M_p)^{\mathrm{EM}}$?

Muonic hydrogen

Proton charge radius



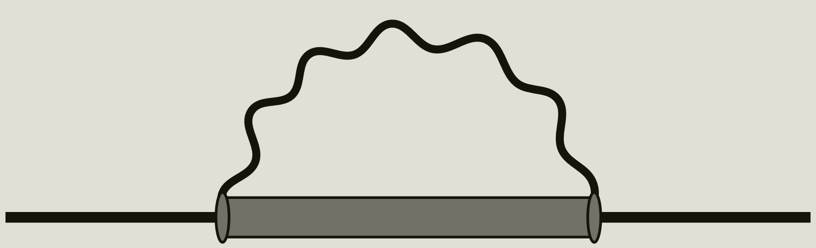
Compton subtraction function

$$\mathcal{F}_1(\omega, Q^2) = \mathcal{F}_1(0,Q^2) + \frac{2\omega^2}{\pi}\int_1^\infty d\omega' \frac{\mathrm{Im}\mathcal{F}_1(\omega', Q^2)}{\omega'(\omega'^2 - \omega^2 - i\epsilon)}$$

Results above for $\mathcal{F}_1$ were only for the dispersive part… a subtracted dispersion relation $\overline{\mathcal{F}}_1(\omega, Q^2)$

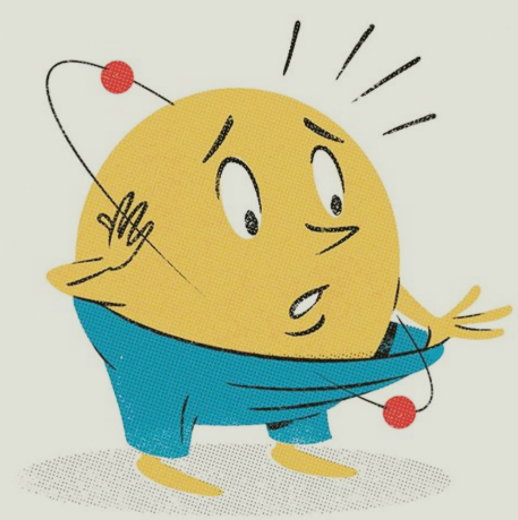
The subtraction function:
$\mathcal{F}_1(0,Q^2) \equiv S_1(Q^2)$

Cottingham formula
Electromagnetic self energy



$(M_n - M_p)^{\mathrm{EM}}$?

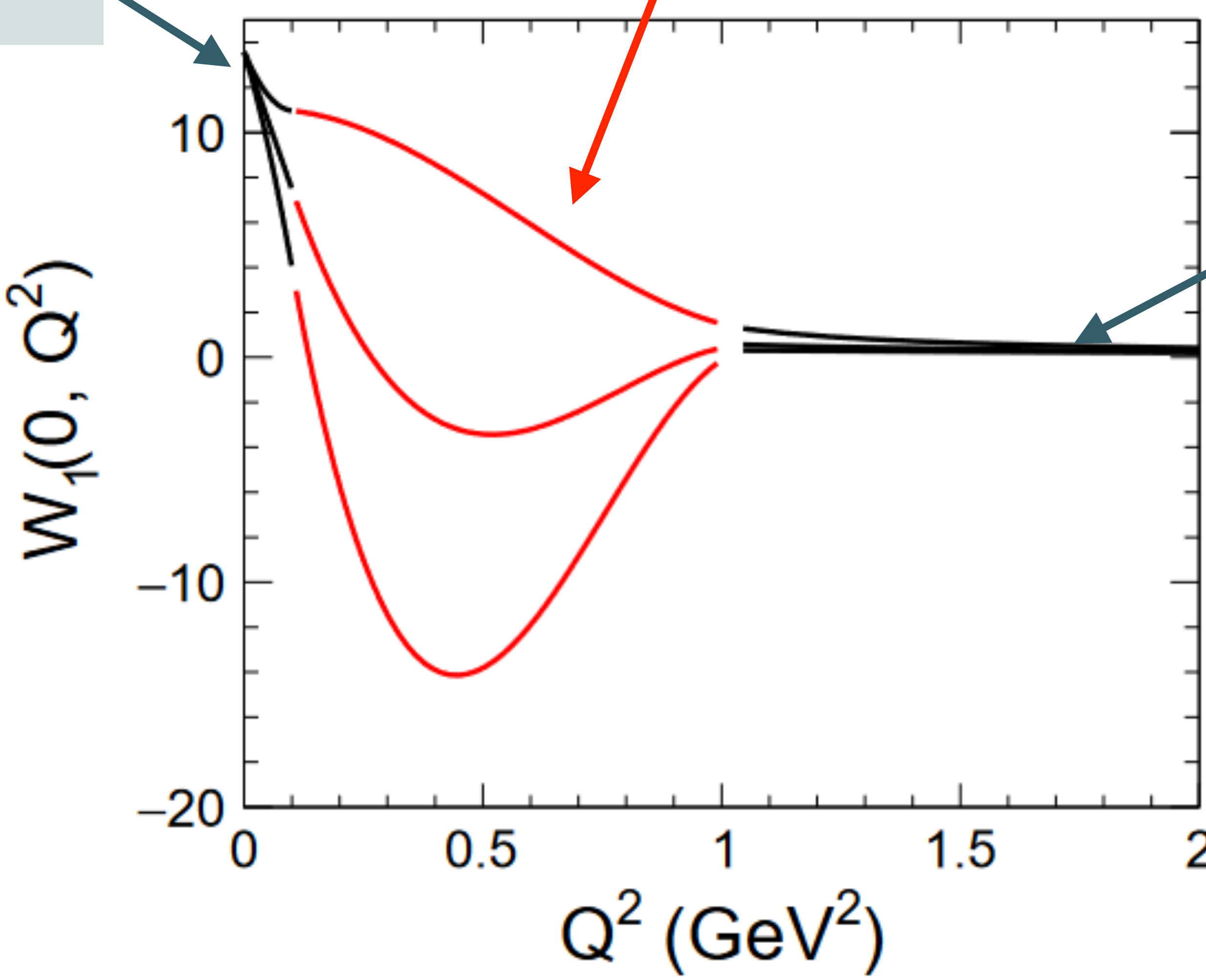
Muonic hydrogen
Proton charge radius



Uncertainty in subtraction function

What happens in between?

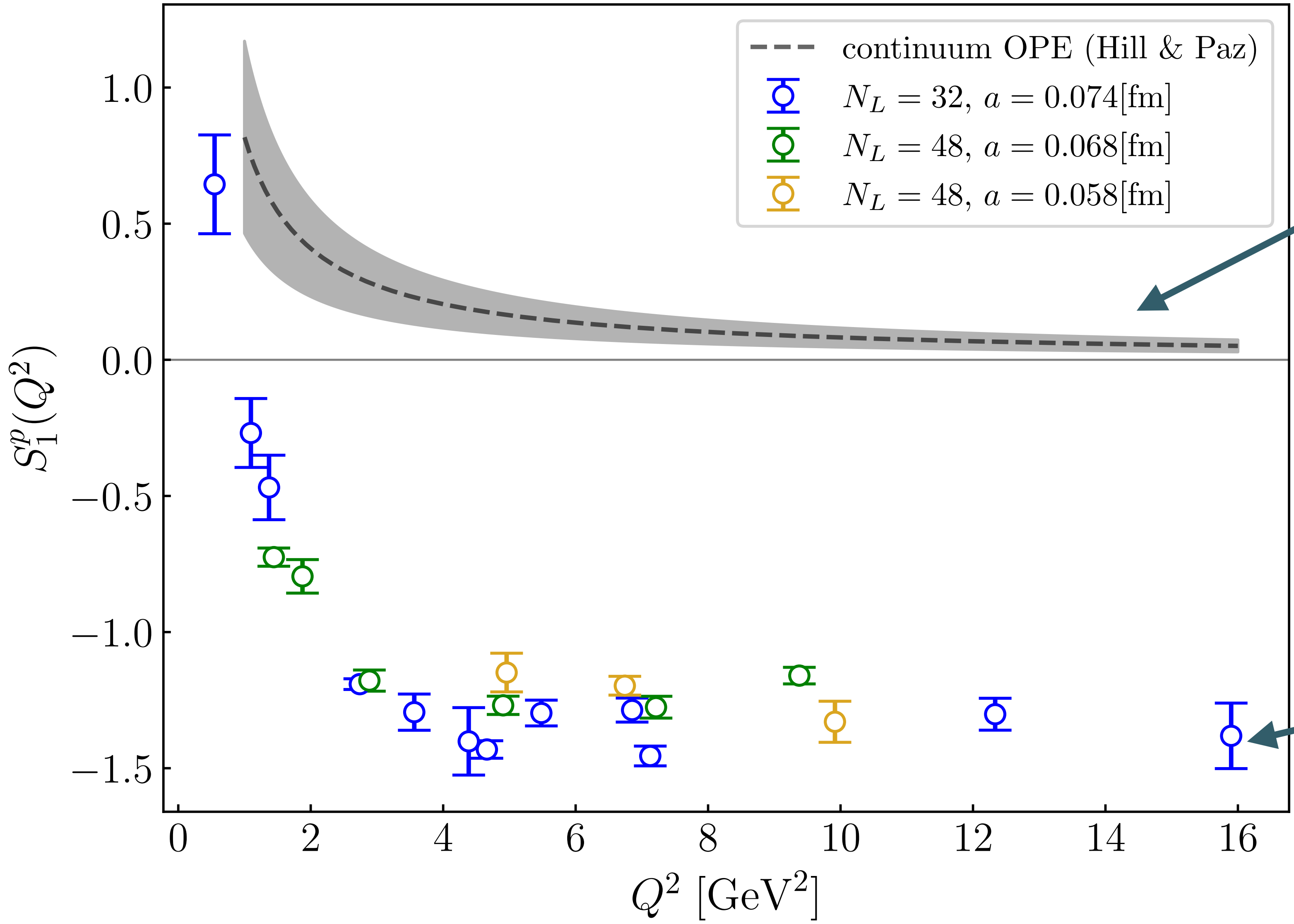
Low energy constrained by chiral EFT, Birse & McGovern EPJA(2012)



High energy constrained by OPE

Hill & Paz, PRD(2017)

Subtraction from lattice Feynman–Hellmann

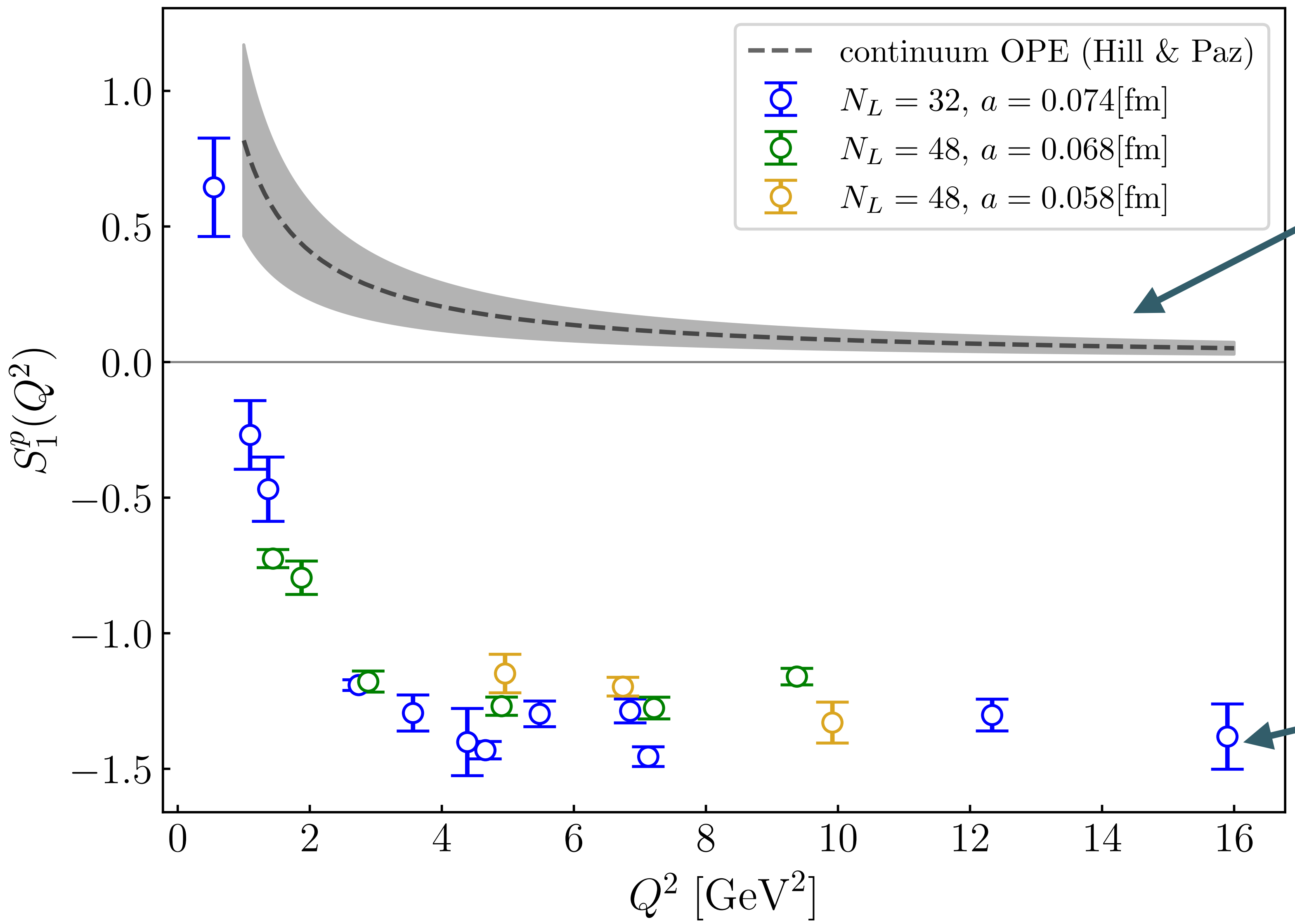


OPE prediction $\sim 1/Q^2$
(with matching lattice inputs)

😱

Lattice results
approaching non-zero constant

Subtraction from lattice Feynman–Hellmann



OPE prediction $\sim 1/Q^2$
(with matching lattice inputs)

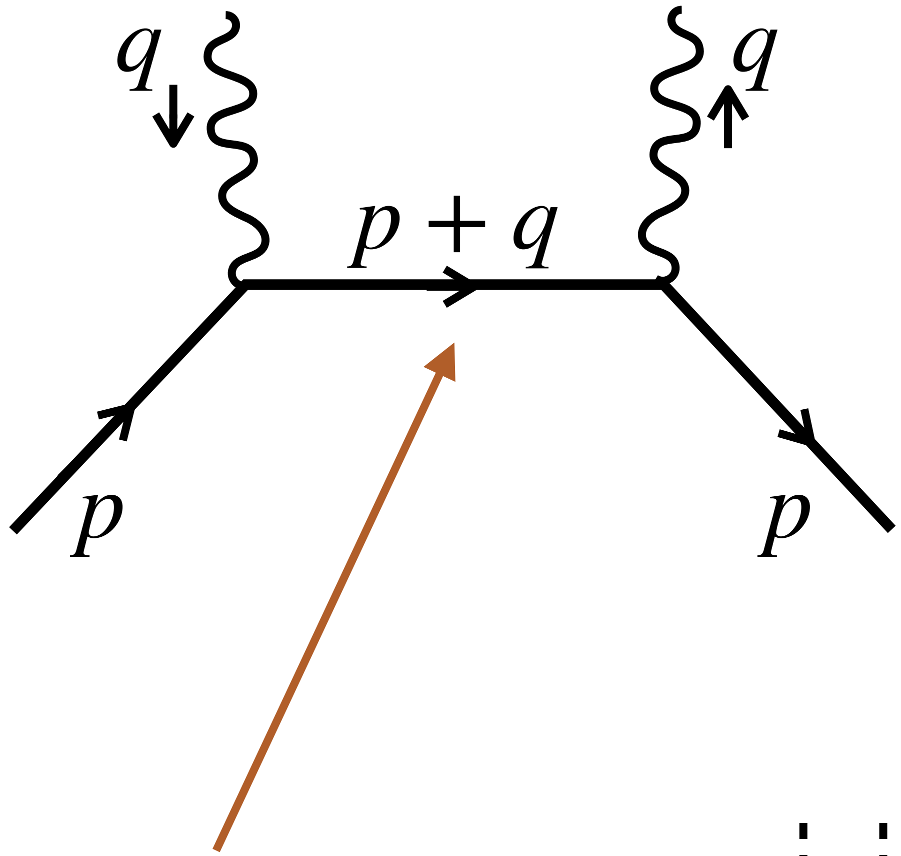
Lattice results
approaching non-zero constant

2 Possible scenarios

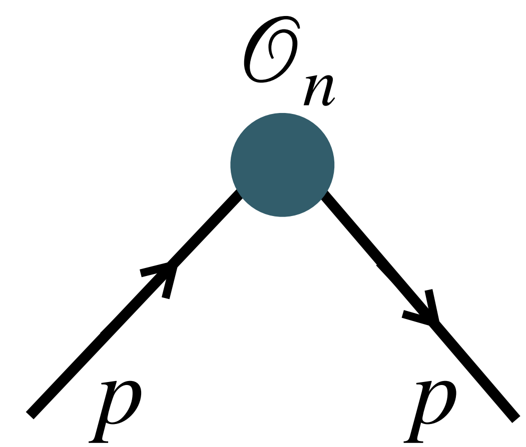
A: Yikes!
Massive lattice discretisation artefacts!!

B: Exciting!
Evidence for fixed pole [suggested by Regge phenomenology]

Lattice OPE

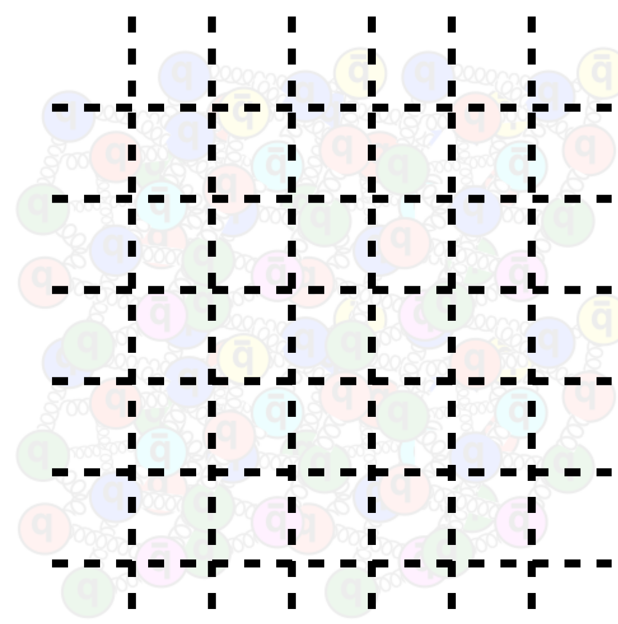


$$\simeq \sum_n C_n(q) \times$$



Wilson propagator

$$S_W(k) = a\frac{M(k) - i\gamma_\mu \sin(ak_\mu)}{M(k)^2 + \sum_\mu \sin^2(ak_\mu)},$$



Note that we're on a lattice:
careful with limits

Not $q \to \infty$,

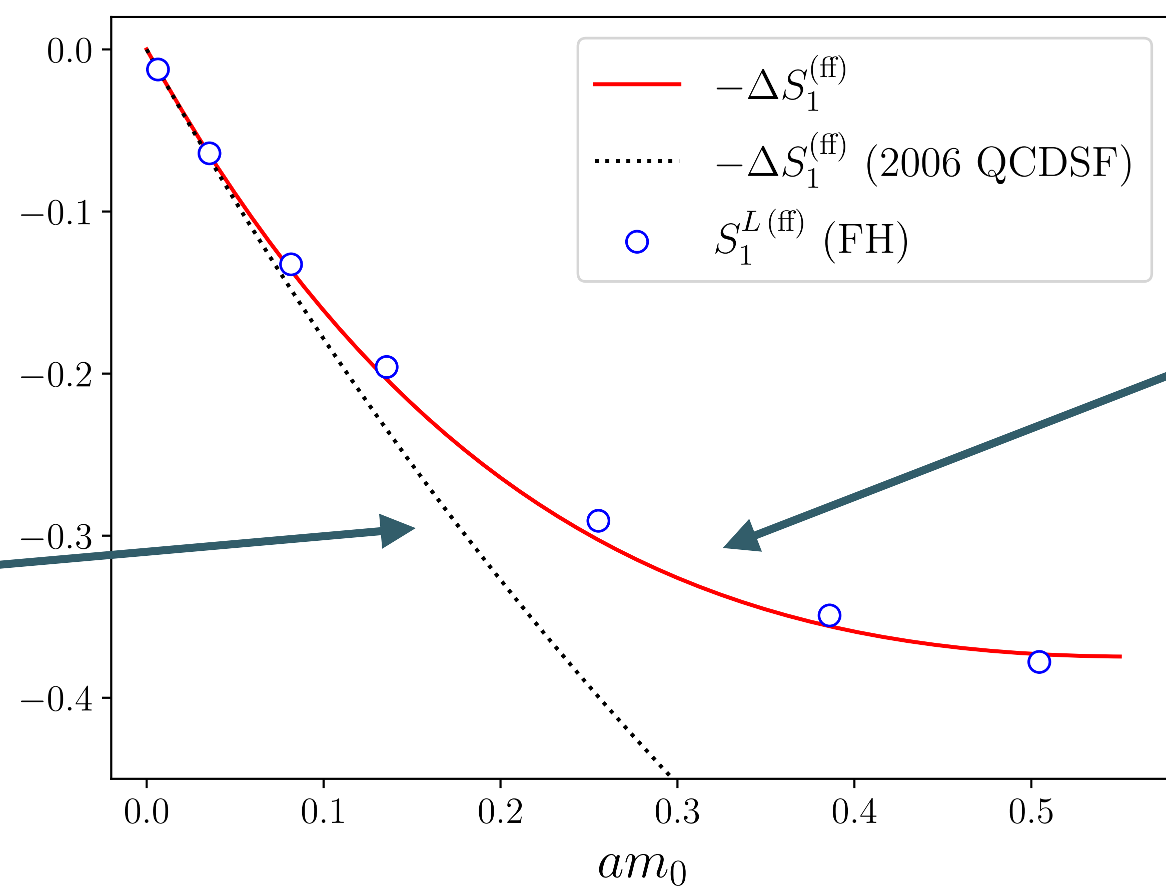
but $p_\mu \ll \sqrt{-q^2}$

Free fermion

- Subtraction function vanishes for free fermion $S_1^{\text{free}}(Q^2) = 0$
- On the lattice, this is 100% discretisation artefact

$$\Delta S_1 = C_W(q, m_0)\, Z_V^2 \langle p \,|\, \overline{\psi}\psi \,|\, p \rangle$$

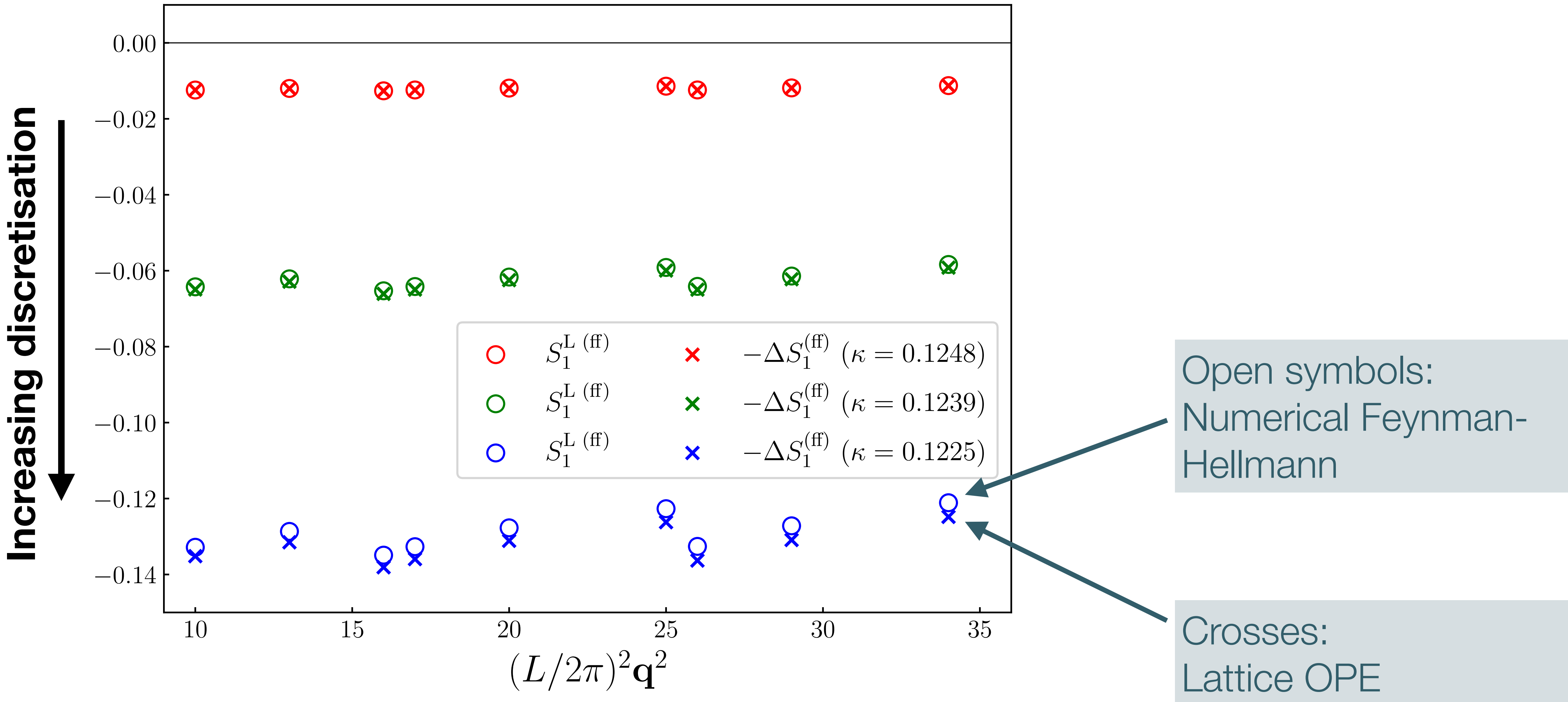
$$C_W(q, m_0) = \frac{2r \sum_\rho \left[1 - \cos(aq_\rho)\right]}{\sum_\rho \sin^2(aq_\rho) + a^2 M^2(q, m_0)}$$



Leading Lattice OPE
Göckeler *et al*. QCDSF
PoS(2006)

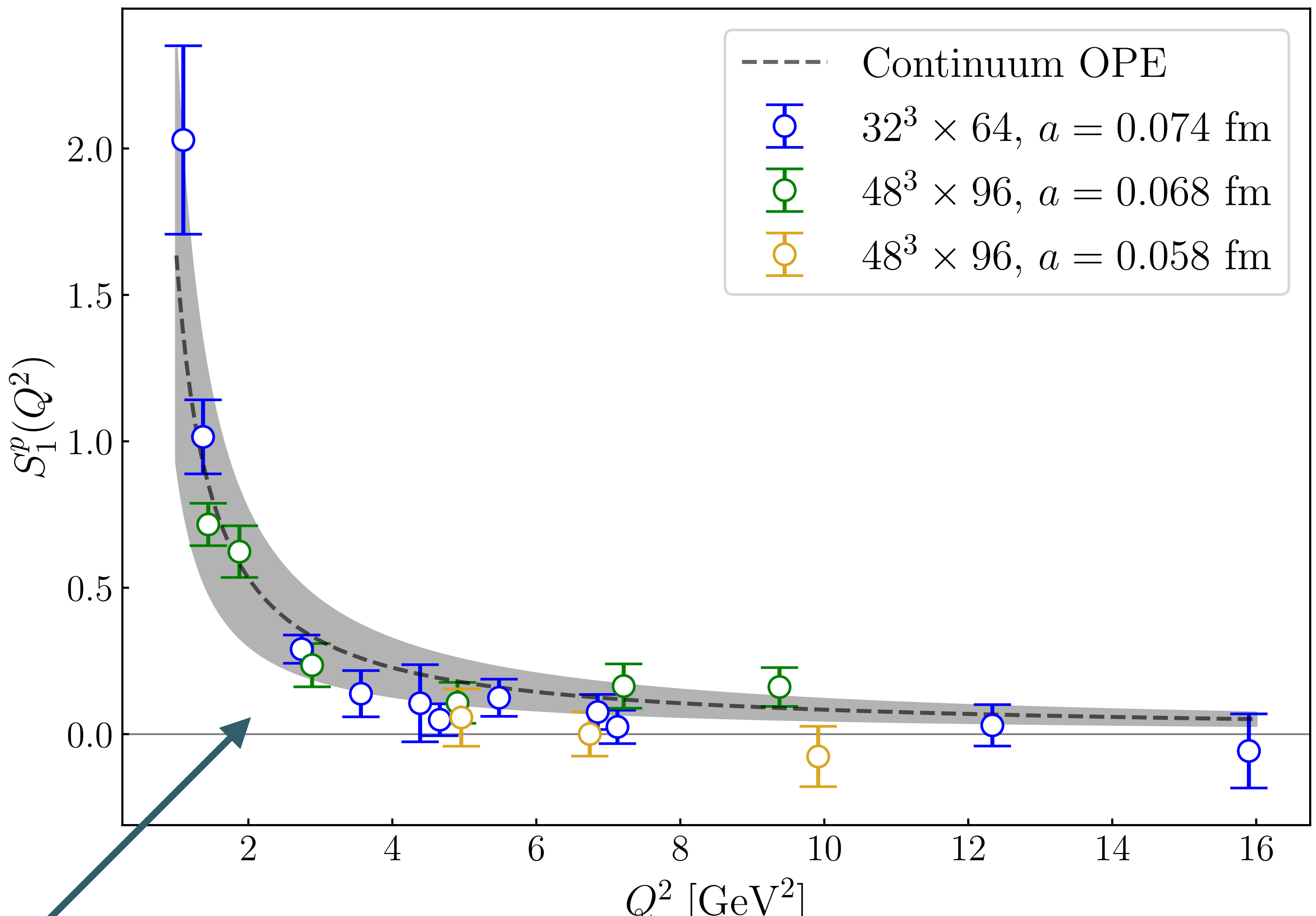
This calculation

Free fermion (*cont.*)



Apply correction to proton S_1

- Numerically evaluate correction for proton subtraction function
  - Requires evaluation of scalar matrix element (compute from 3-pt function)



Curve is **NOT** a fit: **PREDICTION** of continuum OPE

- No Regge fixed pole!
- Can get $S_1(Q^2)$ from lattice

Sum rules: Gross–Llewellen Smith, Bjorken

Can, RDY *et al*., Phys.Rev.D 111 (2025) 11, 114505

Crawford, RDY *et al*., (for Josh’s thesis, soon!)

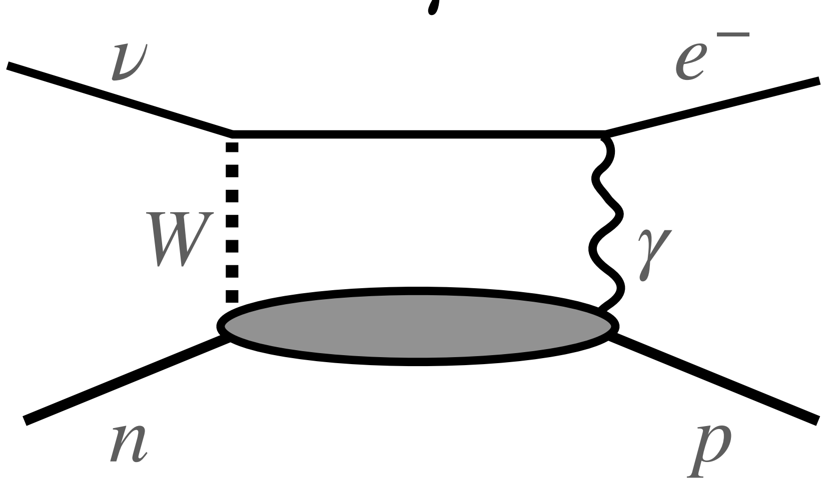
Precision beta decay

- Neutron lifetime:

Czarnecki, Marciano & Sirlin, PRD(2004)

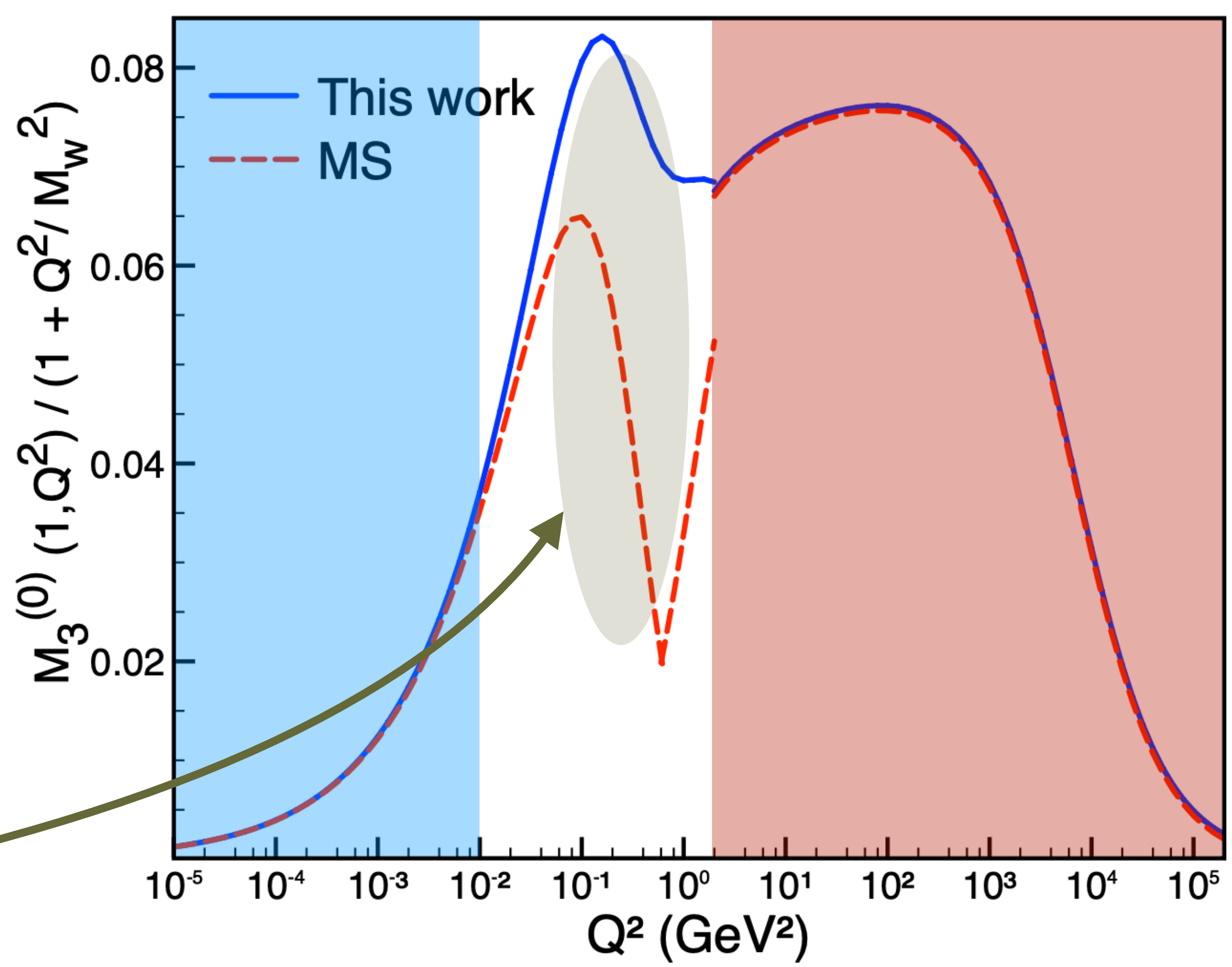
$$\frac{1}{\tau_n} = \frac{G_\mu^2 \left|V_{ud}\right|^2}{2\pi^3} m_e^5 \left(1 + 3g_A^2\right)\left(1 + \mathrm{RC}\right) f,$$

Includes γW box



$$\square_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2 + Q^2}\int_0^1 dx\, C_N(x, Q^2)\, F_3^{(0)}(x, Q^2)$$

Seng *et al*. PRL(2018)



Form factors **pQCD**

Low moment of F_3 structure function required from lattice QCD
$0.01 \lesssim Q^2 \lesssim 2\ \mathrm{GeV}^2$

Parity-violating structure

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\,\mathcal{T}\{J_\mu(z)J_\nu(0)\}\,|p,s\rangle$$

$$= -g_{\mu\nu}\mathcal{F}_1(\omega,Q^2) + \frac{p_\mu p_\nu}{p\cdot q}\mathcal{F}_2(\omega,Q^2) + i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathcal{F}_3(\omega,Q^2)$$

$$+\frac{q_\mu q_\nu}{p\cdot q}\mathcal{F}_4(\omega,Q^2) + \frac{p_{\{\mu}q_{\nu\}}}{p\cdot q}\mathcal{F}_5(\omega,Q^2) + \frac{p_{[\mu}q_{\nu]}}{p\cdot q}\mathcal{F}_6(\omega,Q^2)$$

Compute low moment from lattice FH: $M_1^{(3)}(Q^2) = \int_0^1 dx F_3(x,Q^2) = \left.\frac{\mathcal{F}_3(\omega,Q^2)}{4\omega}\right|_{\omega=0}$

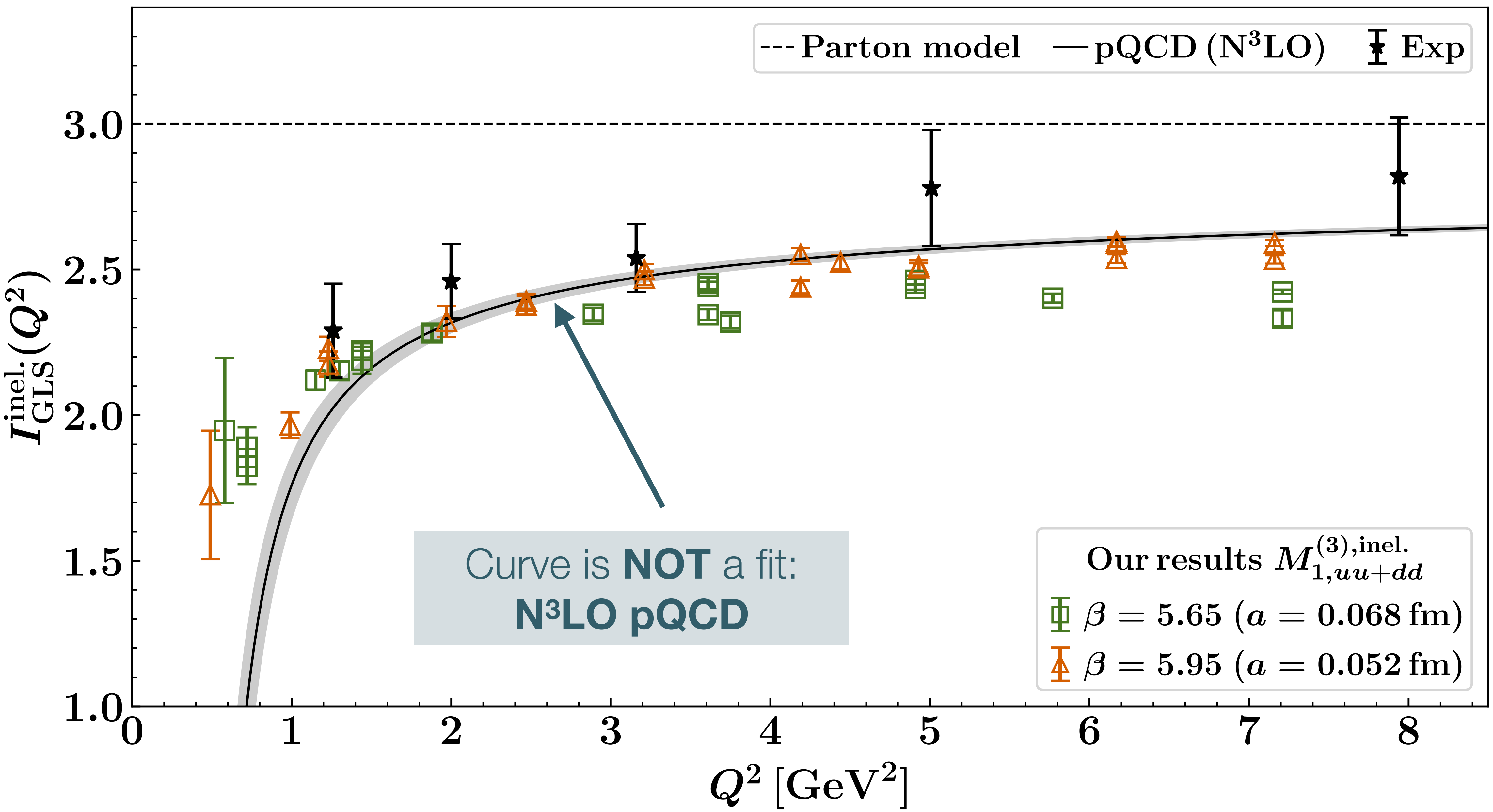
Gross–Llewellyn Smith sum rule:

$$M_{1,uu}^{(3)}(Q^2) = \int_0^{1^-} dx F_3(x,Q^2) = 2\left(1+\sum_{i=1}^{3} a_s^i\, c_i(n_f)\right) + \frac{\Delta_{HT}}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

Leading-twist matrix element is just vector charge!!

Clean opportunity for lattice to resolve perturbative correction

Isoscalar $(uu + dd)$ GLS sum rule
Elastic pole subtracted



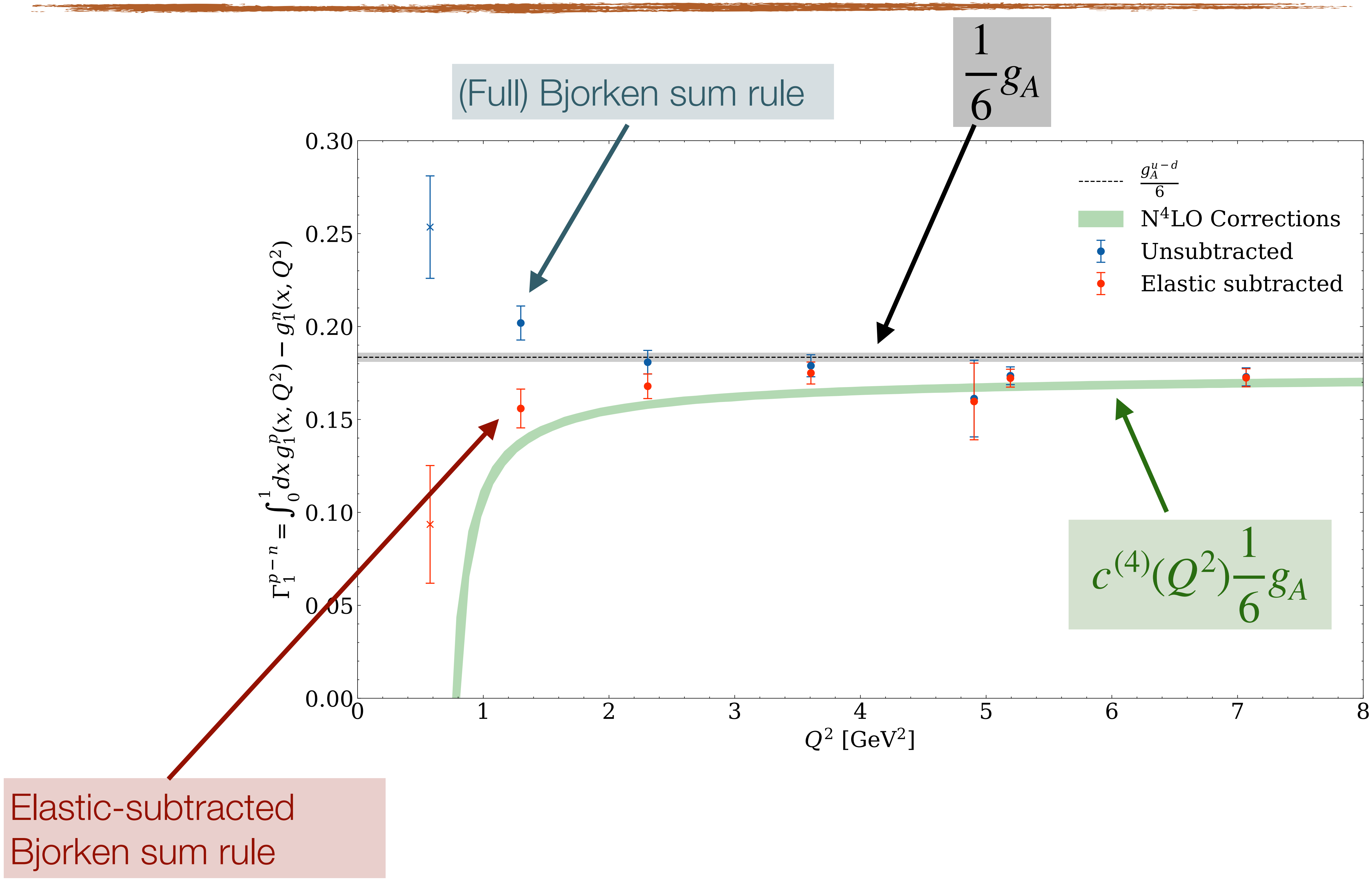
Conservative outlook: We can constrain γW box uncertainties

Ambitious outlook: We could determine α_s from GLS sum rule!

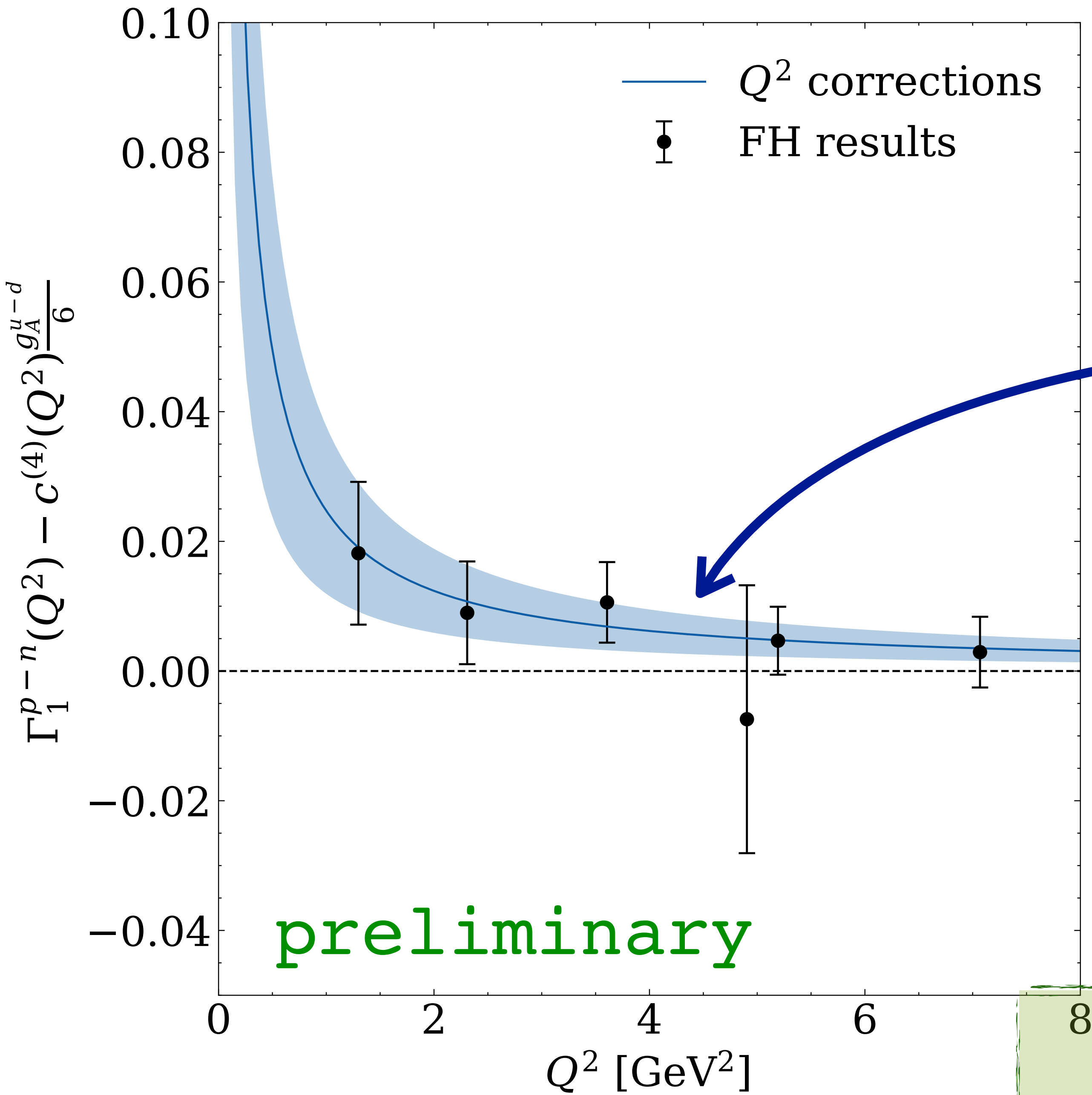
Bjorken sum rule

Bjorken sum rule

preliminary



Power corrections in Bjorken sum rule



Twist expansion

From 3-pt functions **NEW!**

$$\frac{M^2}{9Q^2}\frac{1}{3}\left(a_2 + 4d_2 + 4f_2\right)$$

Twist 2
$\langle p, s \,|\, \mathcal{O}^5_{\{\rho\mu\nu\}} \,|\, p, s\rangle$

Twist 3
$\langle p, s \,|\, \mathcal{O}^5_{[\rho\{\mu]\nu\}} \,|\, p, s\rangle$

Twist 4
$\langle p, s \,|\, \bar{q}\tilde{G}^{\mu\nu}\gamma_\nu q \,|\, p, s\rangle$

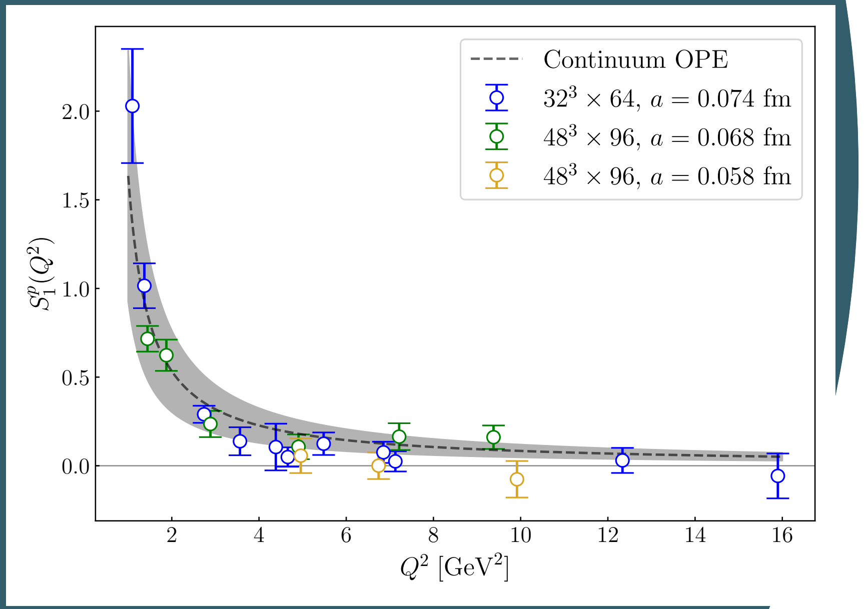
$$\mathcal{O}^5_{\rho\mu\nu} \sim \bar{q}\gamma_\rho\gamma_5 D_\mu D_\nu q$$

Colour Lorentz forces (*see Zanotti*)
d_2 and f_2 allows separation of chromo-electric and -magnetic components!

Perspective

Compton on the lattice provides clean determination of integrated quantities

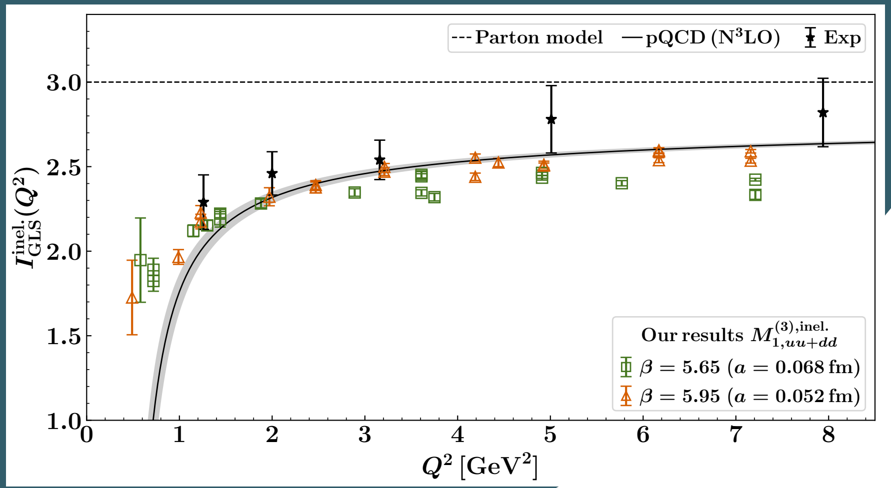
$$\frac{\mathcal{F}_2(\omega, Q^2)}{\omega} = 4\int_0^1 dx\, \frac{F_2(x, Q^2)}{1-(x\omega)^2}$$



Moments provide useful benchmarking tool

Weak quark-mass dependence at leading twist; stronger at low Q^2

Lattice OPE helps resolve discrepancy with subtraction function



GLS and Bjorken sum rules offer potential to resolve pQCD and higher twist

Lattice Compton

Backup slides

Proton subtraction correction