A recent global extraction of TMD distributions

ART25

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with I. Scimemi, A. Vladimirov, P. Zurita
based on: [2503.11201]

PDFs in EIC era **Academia Sinica, June 2025**

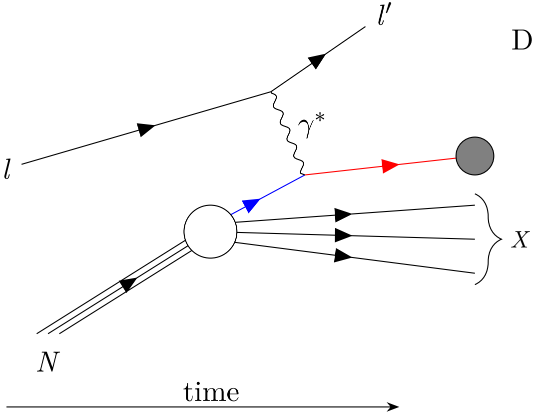
Evolution of Extractions

	DY		DY+SIDIS
N2LO	**SV19** $\chi^2 = 1.17$	+SIDIS (FFs) →	**SV19** $\chi^2 = 1.16 \oplus 0.95$
	↓	flavour dependent ansatz improved uncertainty progression more perturbative precision more high-energy DY data	↓
N4LO	**ART23** $\chi^2 = 1.00$	+SIDIS (FFs) → **tune model**	**ART25** $\chi^2 = 1.17 \oplus 0.92$

Transverse Scale I: SIDIS

Semi-inclusive DIS specifies the struck quark's final state.
Requires knowlegde of 2 hadronic functions

$$l(k) + N(P) \longrightarrow l'(k') + h(p_h) + X$$



Describe process with:

- PDF $f(x)$
- FFs $d(z)$ $\sim$ probability for struck quark f to hadronize into state of energy E_h

$q_T : q \perp \langle P, q \rangle$

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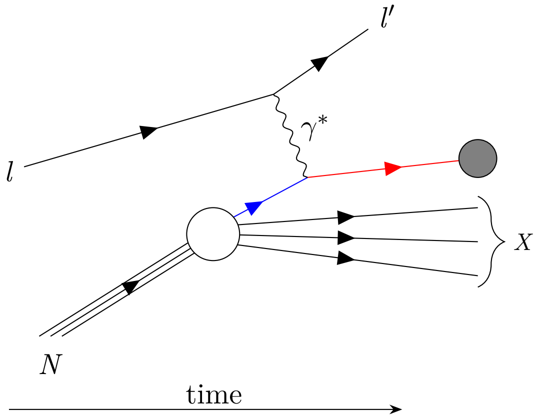


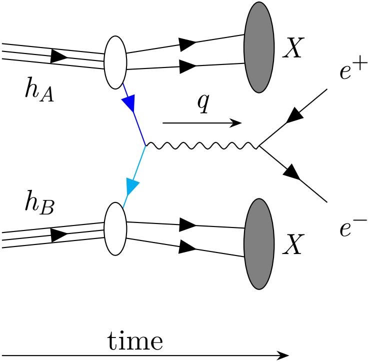
Kinematics in SIDIS

$$q = k - k'$$
$$Q^2 = -q^2$$
$$x_B = \frac{Q^2}{2P \cdot q}$$
$$z = \frac{P \cdot p_h}{P \cdot q}$$

Transverse Scale II: DY

A *clean* process:

$$h_A + h_B \longrightarrow e^+ + e^- + X$$



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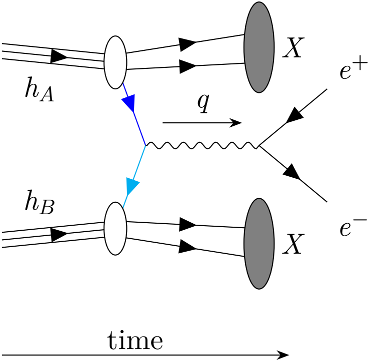
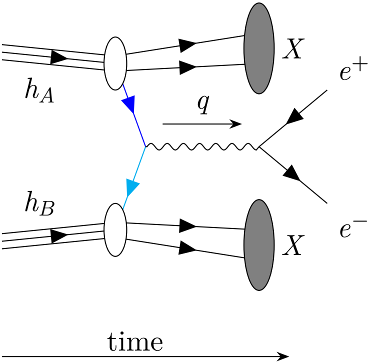


- only PDFs involved
- measure final state lepton pair
- $q_T : q \perp \langle P_A, P_B \rangle$

Transverse Scale II: DY

A *clean* process:

$$h_A + h_B \longrightarrow e^+ + e^- + X$$



Kinematics in DY

$$Q^2 = q^2$$
$$y = \frac{1}{2}\ln\left(q_+/q_-\right)$$
$$x_{1,2} = \sqrt{\frac{q_+ q_-}{s}}\, e^{\pm y}$$

TMD regime: in a nutshell (or two)

examine events in which the scale $\lambda = \frac{q_T}{Q}$ is **small**
[The field modes can be separated (Power counting) and ordered]

$$p_{hc} = Q\left(1, \lambda^2, \lambda\right)$$
$$p_{\overline{hc}} = Q\left(\lambda^2, 1, \lambda\right)$$

[and the effective current determined]
(determines which dirac structures contribute!)
(state of the art extractions still in LP factorisation!)

$$J^\mu(z)J^\nu(0) = \overline{\xi}_n(z_- n + b)\gamma_\perp^\mu \xi_{\overline{n}}(z_+\overline{n} + b)\overline{\xi}_{\overline{n}}(0)\gamma_\perp^\nu \xi_n(0) + \ldots$$

TMD regime: in a nutshell (or two)

$$f(x, b/k_T)$$

factorisation: separate high energy modes from slow modes $[\mu]$

$$f(x, b/k_T; \mu_F)$$

in TMD case:
require additional factorisation scale $[\zeta]$ to disentangle b and Q

$$f(x, b/k_T; \mu_F, \zeta)$$

TMD DY cross-section (γ^*)

$$\frac{d\sigma}{dQ^2\,dy\,dq_T^2} = \frac{2\pi\alpha_{\rm em}^2(Q)}{3N_c\,s\,Q^2}\left(1+\frac{q_T^2}{2\,Q^2}\right)\sum_f C_{\rm DY}(Q,\mu_F)$$
$$\times\int_0^\infty db\; b\; J_0(bq_T)\left(\frac{Q^4}{\zeta_1\zeta_2}\right)^{-\mathcal{D}(b,\mu_F)}$$
$$\times f_{1,f}(x_1,b;\mu_F,\zeta_1)f_{1,\bar{f}}(x_2,b;\mu_F,\zeta_2)$$

Phenomenology:
what you HAVE to do: model

- $f_f(x,b)$ at reference scale (μ_F,ζ_i)
- $\mathcal{D}(b,\mu_F)$

Evolution equations

$$\mu^2 \frac{d}{d\mu^2} f(x, b; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} f(x, b; \mu, \zeta)$$
$$\zeta \frac{d}{d\zeta} f(x, b; \mu, \zeta) = -\mathcal{D}(b, \mu) f(x, b; \mu, \zeta)$$

μ evolution is perturbative ✓
ζ evolution is not:
Parametrisation Collins-Soper kernel ($\mathcal{D}$)

$$\mathcal{D}(b, \mu) = \mathcal{D}_{\text{small-b}}(b^*, \mu) \qquad + \mathcal{D}_{\text{NP}}(b)$$

Form of TMDPDF

Parametrisation of TMDPDF:

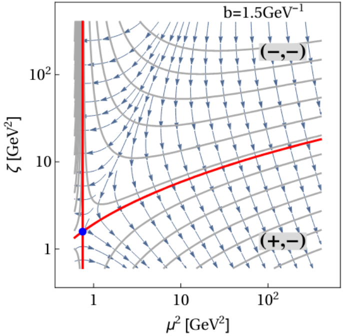
$$f_{1,f}(x,b) = \int_x^1 \frac{dy}{y} \sum_{f'} C_{f\to f'}(y,\mathbf{L},a_s) q_{f'}\left(\frac{x}{y}\right) f_{\mathrm{NP}}^{f}(x,b)$$

$$\lim_{b\to 0} f_{\mathrm{NP}}^{f}(x,b) = 1$$

Phenomenology:
what you CAN do:
use boundary condition for TMDs & PDFs

Scale dependence of TMDs: in a plane

$$\mu^2 \frac{d}{d\mu^2} f(x, b; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} f(x, b; \mu, \zeta)$$
$$\zeta \frac{d}{d\zeta} f(x, b; \mu, \zeta) = -\mathcal{D}(b, \mu) f(x, b; \mu, \zeta)$$



To evolve $(\mu, \zeta) \to (Q, Q^2)$

- define TMDPDF at saddle point
- evolve in μ
- evolve in ζ

TMD DY cross-section (γ^*)

$$\frac{d\sigma}{dQ^2\,dy dq_T^2} = \frac{2\pi\alpha_{\text{em}}^2(Q)}{3N_c s Q^2}\left(1 + \frac{q_T^2}{2\,Q^2}\right)\sum_f C_{\text{DY}}(Q)$$
$$\times \int_0^\infty db\, b\, J_0(bq_T)\left(\frac{Q^2}{\zeta(b)}\right)^{-2\mathcal{D}(b,Q)}$$
$$\times f_{1,f}(x_1, b) f_{1,\bar{f}}(x_2, b)$$

- $\mu_F = Q$
- $f_f(x, b)$ at saddle point scale $(\mu(b), \zeta(b))$
- decouple PDF and evolution

Evolution of Extractions

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N4LO	**ART23** $\chi^2 = 1.00$	+SIDIS (FFs) → **tune model**	**ART25** $\chi^2 = 1.17 \oplus 0.92$

N2LO → N4LO:

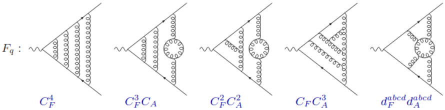
flavour dependent ansatz

improved uncertainty progression

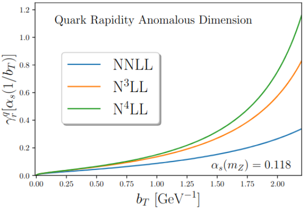
more perturbative precision

more high-energy DY data

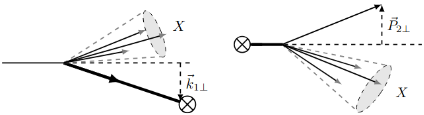
Perturbative input



"Quark and gluon form factors in four-loop QCD" [2202.04660]
$\rightarrow C(Q^2)$

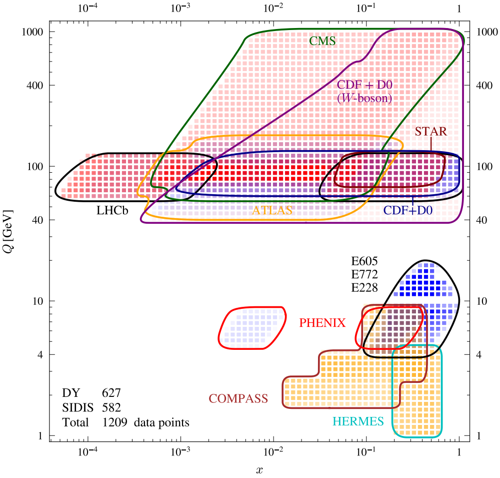


Rapidity anomalous dimension at N4LL [2205.02242]
$\rightarrow \mathcal{D}_{\text{pert}}(b, \mu)$



Splitting functions at N3LL [1908.03831]
$\rightarrow C_{f\rightarrow f'}(y, \mu_{\text{OPE}})$

Kinematic range of included data, datasets



Features:

- large range of resolution scale: 1 GeV → 1 TeV
- *W* production in DY
- $\frac{q_T}{Q} < 0.25$ imposed cut
- More high-energy DY:
  - SV19: 457 + 582
  - ART25: 627 + 582

Parametrisation: SV19

PDF

$$f_{\mathrm{NP}}^{f}(x,b) = \exp\left(-\frac{(\lambda_1 \bar{x} + \lambda_2 x + \lambda_3 x\bar{x})}{\sqrt{1+\lambda_4 x^{\lambda_5} b^2}} b^2\right)$$

FF

$$d_{\mathrm{NP}}^{f}(z,b) = \exp\left(-\frac{(\eta_1 z + \eta_2 \bar{z})}{\sqrt{1+\eta_3 (b/z)^2}} \frac{b^2}{z^2}\right)\left(1+\eta_4 \frac{b^2}{z^2}\right)$$

RAD

$$\mathcal{D}_{\mathrm{model}}(b,\mu) = \mathcal{D}_{\mathrm{resum}}(\mu, b^*) + c_0 b b^*$$

$$f_{\mathrm{NP}}^{f}(x,b) \sim e^{-\alpha b} \quad \text{for} \quad b \gg 1$$

CS kernel

Parametrisation of TMD Evolution:

$$\mathcal{D}(b,\mu) = \mathcal{D}_{\text{small-b}}(b^*,\mu^*) + \int_{\mu^*}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\text{cusp}}(\mu') + \mathcal{D}_{\text{NP}}(b)$$

- Ansatz for NP part:

$$\mathcal{D}_{\text{NP}}(b) = c_0 b b^* + c_1 b b^* \ln\left(\frac{b^*}{B_{\text{NP}}}\right)$$

- log term brings sensitivity to moderate b region, determined by high energy DY data!
- 3 parameters for TMDPDF scale evolution

TMDPDF model: ART23/5

Parametrisation of TMDPDF:

$$f_{\mathrm{NP}}^{f}(x,b) = \cosh^{-1}\left(\left(\lambda_1^f (1-x)^{\lambda_3} + \lambda_2^f x\right) b\right)$$

$$f \in \left\{u, \overline{u}, d, \overline{d}, sea\right\}$$

ART23

$$\lambda_3 = 1$$

ART25

$$\lambda_1^q = \lambda_1^{\overline{q}}$$

10 indep. parameters for PDFs

$$f_{\mathrm{NP}}^{f}(x,b) \sim e^{-\alpha b} \quad \text{for} \quad b \gg 1$$

TMDFF model: ART25

Parametrisation of TMDFF:

$$d_{\mathrm{NP}}^{f,h}(z,b) = \cosh^{-1}\left(\eta_0^h \frac{b}{z}\right)\left(1 + \eta_1^{h,f}\frac{b^2}{z^2}\right)$$

$$h \in \left\{\pi^\pm, K^\pm\right\}$$

$$f \in \left\{u, \overline{d}, \overline{u}, sea\right\} \text{ for } h = \pi^+$$

10 indep. parameters for FFs

$$d_{\mathrm{NP}}^{f,h}(x,b) \sim e^{-\alpha b} \quad \text{for} \quad b \gg 1$$

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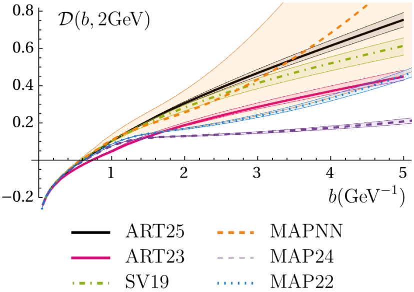
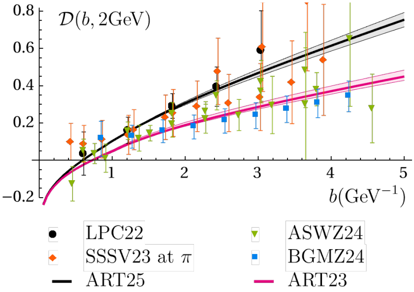
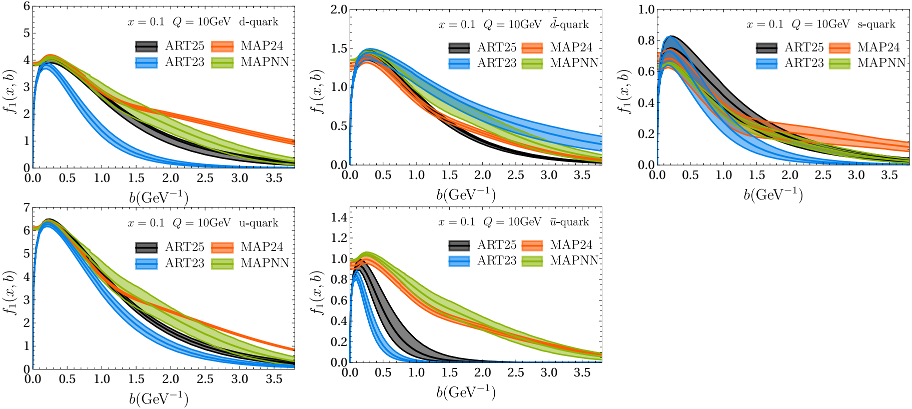
Extracted function: CS kernel

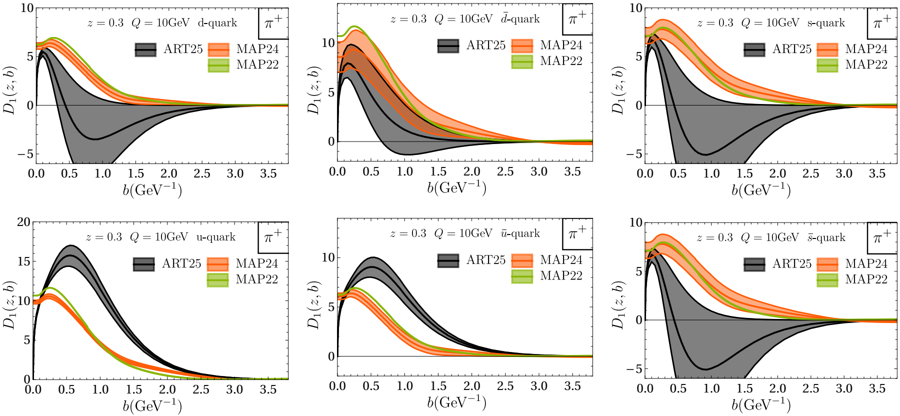




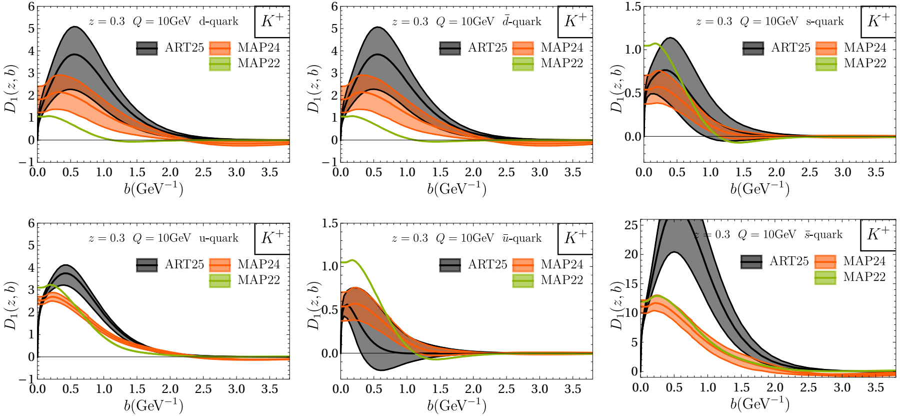
Extracted function: TMDPDF



Extracted function: TMDPDF



Extracted function: TMDPDF



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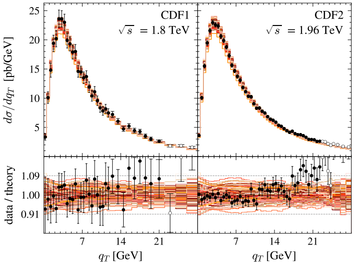
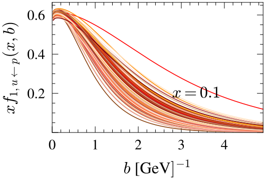
Problem I: Uncertainty processing fit

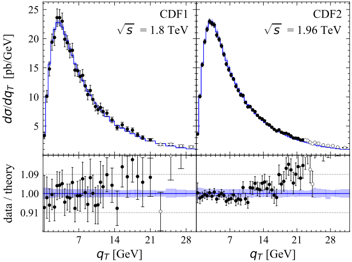
replica of data + replica of PDF $\xrightarrow{\text{fit}}$ TMDPDF replica

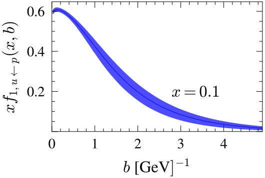
ensemble of replicas



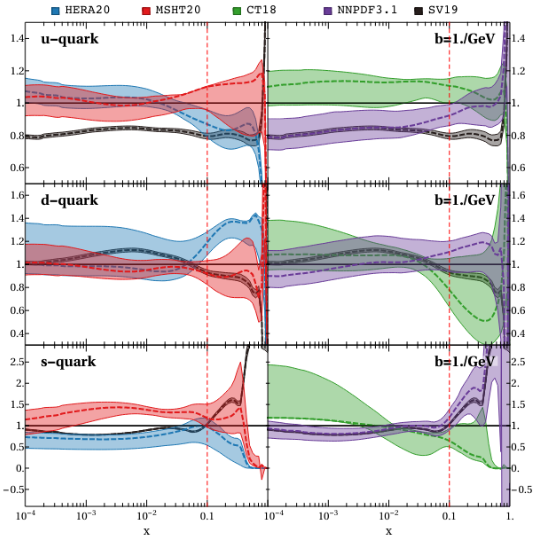


average value and 68% CI





Problem I: collinear PDF choice

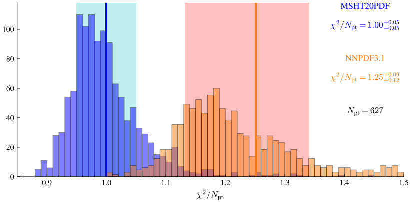


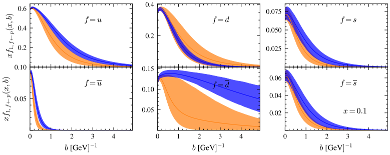
Param.	MSHT20	HERA2.0	NNPDF3.1	CT18
κ_1^u	0.12	0.11	0.28	0.05
κ_2^u	0.32	8.15	2.58	0.9

- obtained parameters stronly depend on PDF
- collinear PDF is base layer of TMDPDF
- we choose MSHT20 as the strongest candidate in JHEP 10 (2022) 118

Problem I: Consistency and Limitations

Effect of collinear PDF on the extraction:





Extractions using MSHT20 and NNPDF3.1

- impact of PDF is significant!
- even CS kernel is affected at moderate b
- additional, systematic uncertainty: estimate?
- Solution(?): independent TMDPDF fit w.o. constraint due to PDF
- at this precision core hours become a problem.

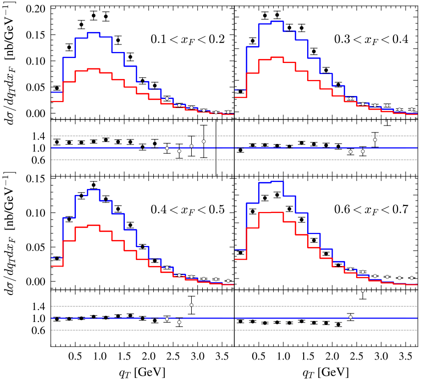
Problem II: fixed target DY (&Pion induced DY)

Table of χ^2 results
for fixed target DY data

Experiment	σ_{norm}	χ^2/N_{pt}	sys. shift	#dat
E228-200	25%	0.547	20%	43
E228-300	25%	0.683	26 %	53
E228-400	25%	1.241	29 %	79
E772	10%	1.233	20 %	35
E605	15%	0.357	38 %	35
PHE200	12%	0.386	-5%	3
A13-norm	0%	1.274	0 %	5

Origin: Higher Twist corrections?

Also for pi-DY
($Q \sim 10\,\text{GeV}$, fixed target)
predictions without shift

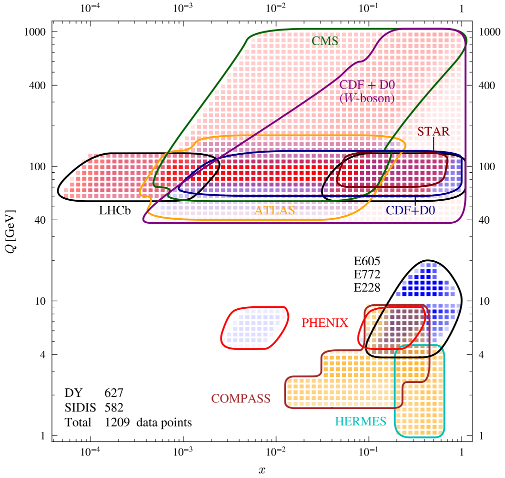


What do I want from the EIC

- SIDIS data $\sim Q \gg 1$

- access polarised distributions

Kinematic range of included data, datasets



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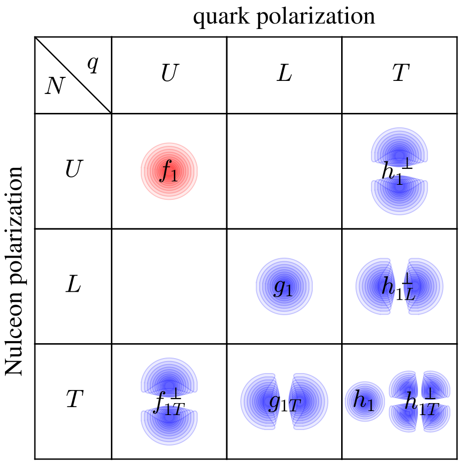
8 TMD distributions



Parametrised forms of TMDs include 8 functions.

Polarisation of quark
∼ spinor operator (Γ)

Polarisation of hadron
∼ exterior state

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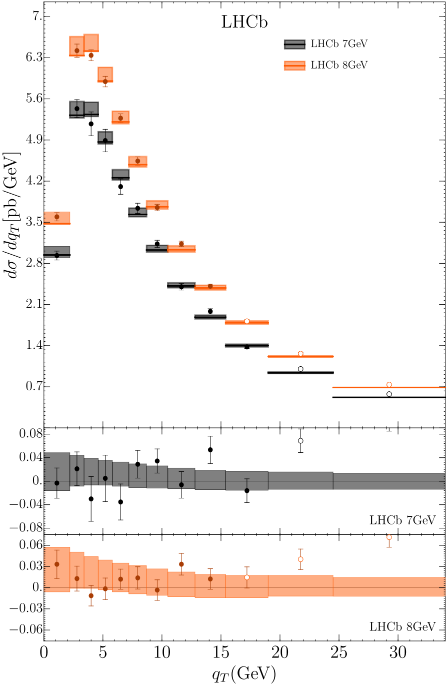
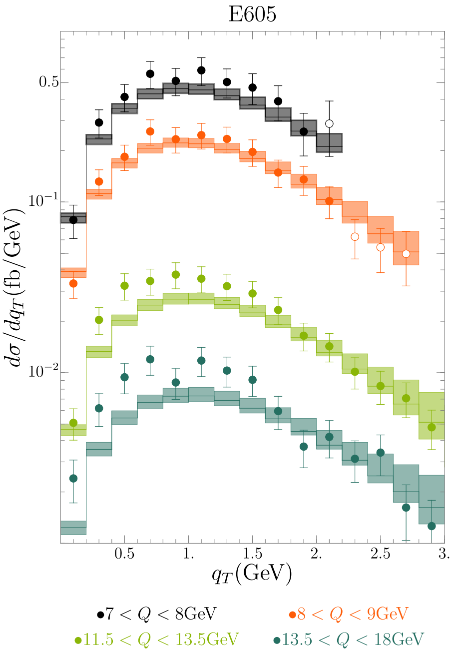
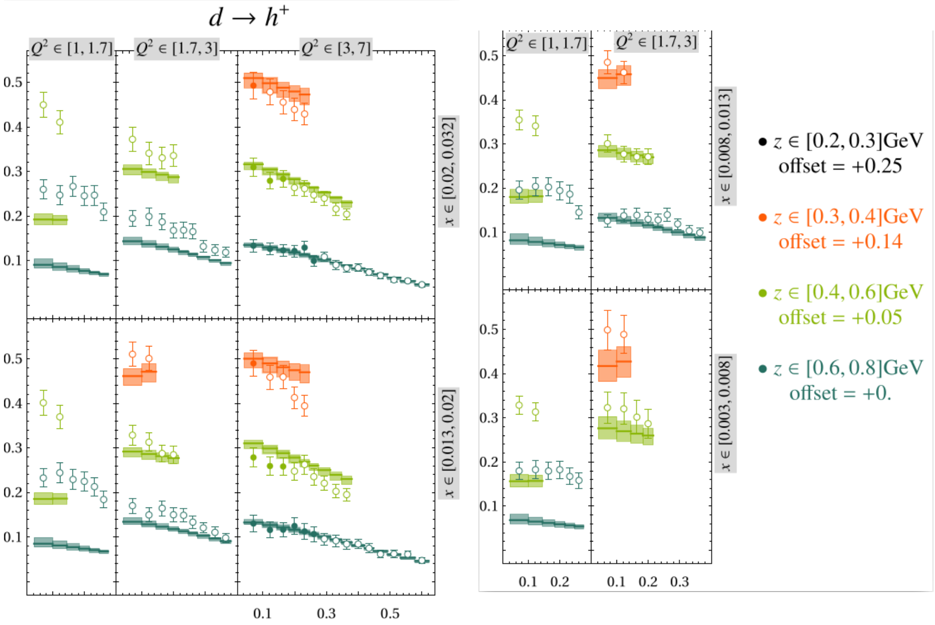

LHCb
LHCb 7GeV
LHCb 8GeV
$d\sigma/dq_T$[pb/GeV]
q_T(GeV)
LHCb 7GeV
LHCb 8GeV


E605
$d\sigma/dq_T$(fb/GeV)
q_T(GeV)
•$7 < Q < 8$GeV
•$8 < Q < 9$GeV
•$11.5 < Q < 13.5$GeV
•$13.5 < Q < 18$GeV



$d \to h^+$
$Q^2 \in [1, 1.7]$
$Q^2 \in [1.7, 3]$
$Q^2 \in [3, 7]$
$x \in [0.02, 0.032]$
$x \in [0.013, 0.02]$
$x \in [0.008, 0.013]$
$x \in [0.003, 0.008]$
0.1 0.2 0.3 0.4 0.5
0.1 0.3 0.5
0.1 0.2
0.1 0.2 0.3
• $z \in [0.2, 0.3]$GeV offset = +0.25
• $z \in [0.3, 0.4]$GeV offset = +0.14
• $z \in [0.4, 0.6]$GeV offset = +0.05
• $z \in [0.6, 0.8]$GeV offset = +0.