Workshop on parton distribution functions in the EIC era
Jun 16–18, 2025
Institute of Physics, Academia Sinica

# Gravitational form factors of the Nambu-Goldstone bosons from chiral effective models

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# Introduction

# Chiral symmetry breaking and the Nambu-Goldstone bosons

Chiral symmetry of strong interaction is spontaneously broken: eg. N(1/2+, 940) vs N(1/2-,1535), …

$\langle \bar{\psi}\psi \rangle \neq 0 \rightarrow$ massless Nambu-Goldstone boson

Explicit chiral symmetry breaking by current quark masses $m$ → Nambu-Goldstone bosons acquire mass $M$

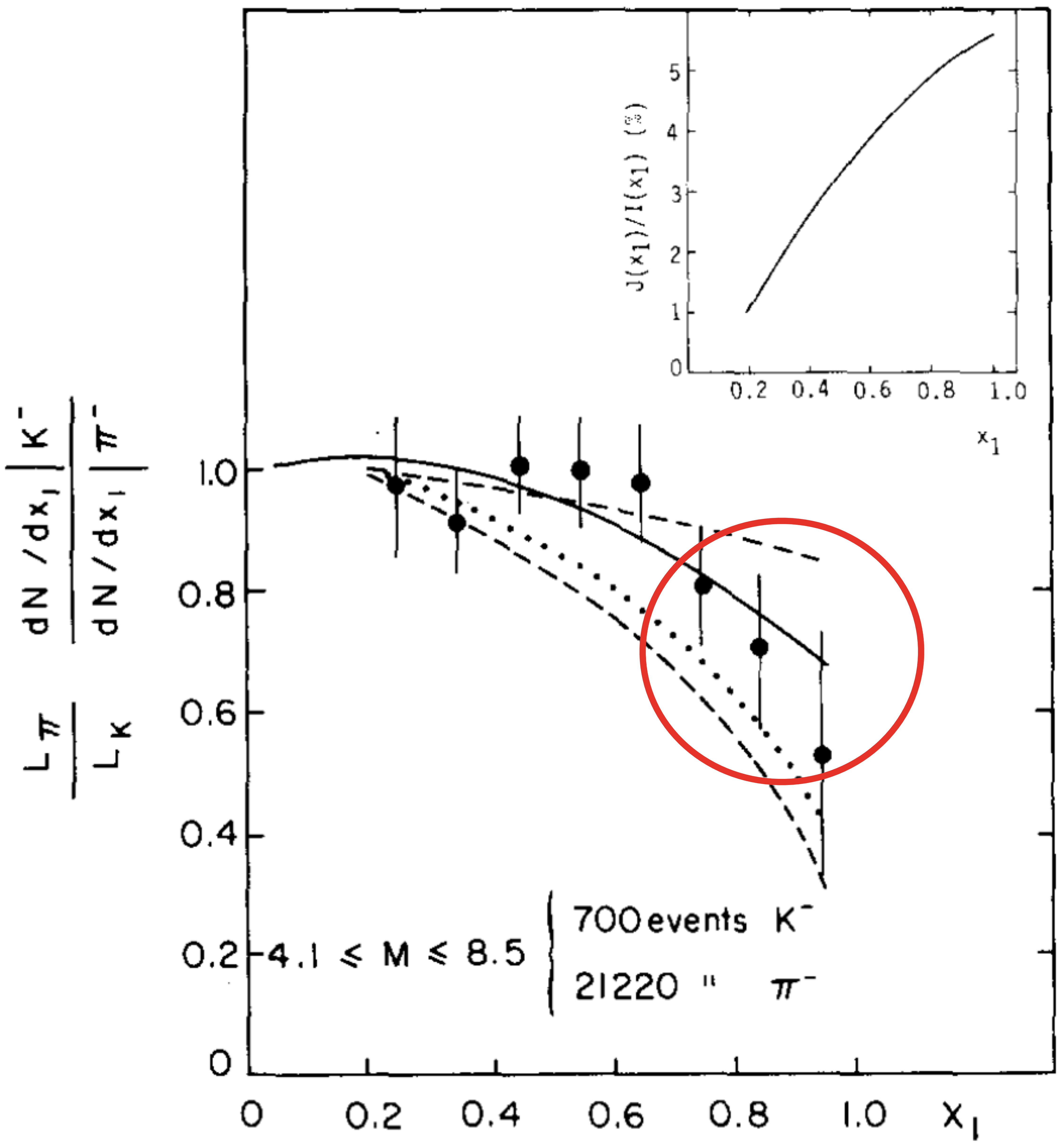
Gell-Mann - Oakes - Renner relation

$$M^2F^2 = -m\langle \bar{\psi}\psi \rangle + \mathcal{O}(m^2)$$

Including strangeness ($m_s \ll \Lambda$), SU(3)$_f$: $\pi$, $K$, $\eta$

Breaking SU(3)$_f$ with $m_s \approx 100$ MeV may require significant correction in $\mathcal{O}(m^2)$

**Explicit chiral symmetry breaking should differentiate the kaon and pion quark structure**




[Badier et al. *Phys.Lett.B* 93 (1980) 354-356]

Ratios for $K^-$ and $\pi^-$ -induced Drell-Yann cross section

Proportional to $\bar{u}_{K^-}(x)/\bar{u}_{\pi^-}(x)$

Dropping at x > 0.6 :

☞ s quark carries more momentum than u quark in a Kaon

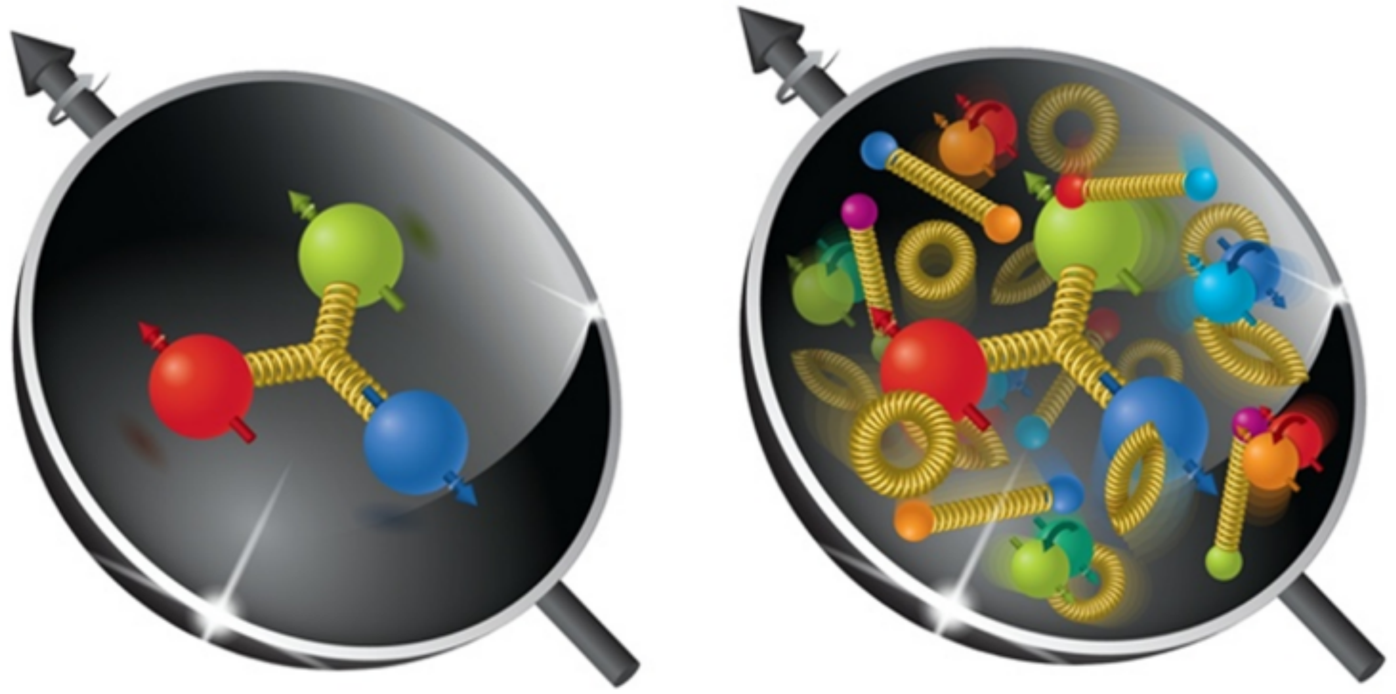
☞ u quark in Kaon is softer than u quark in pion

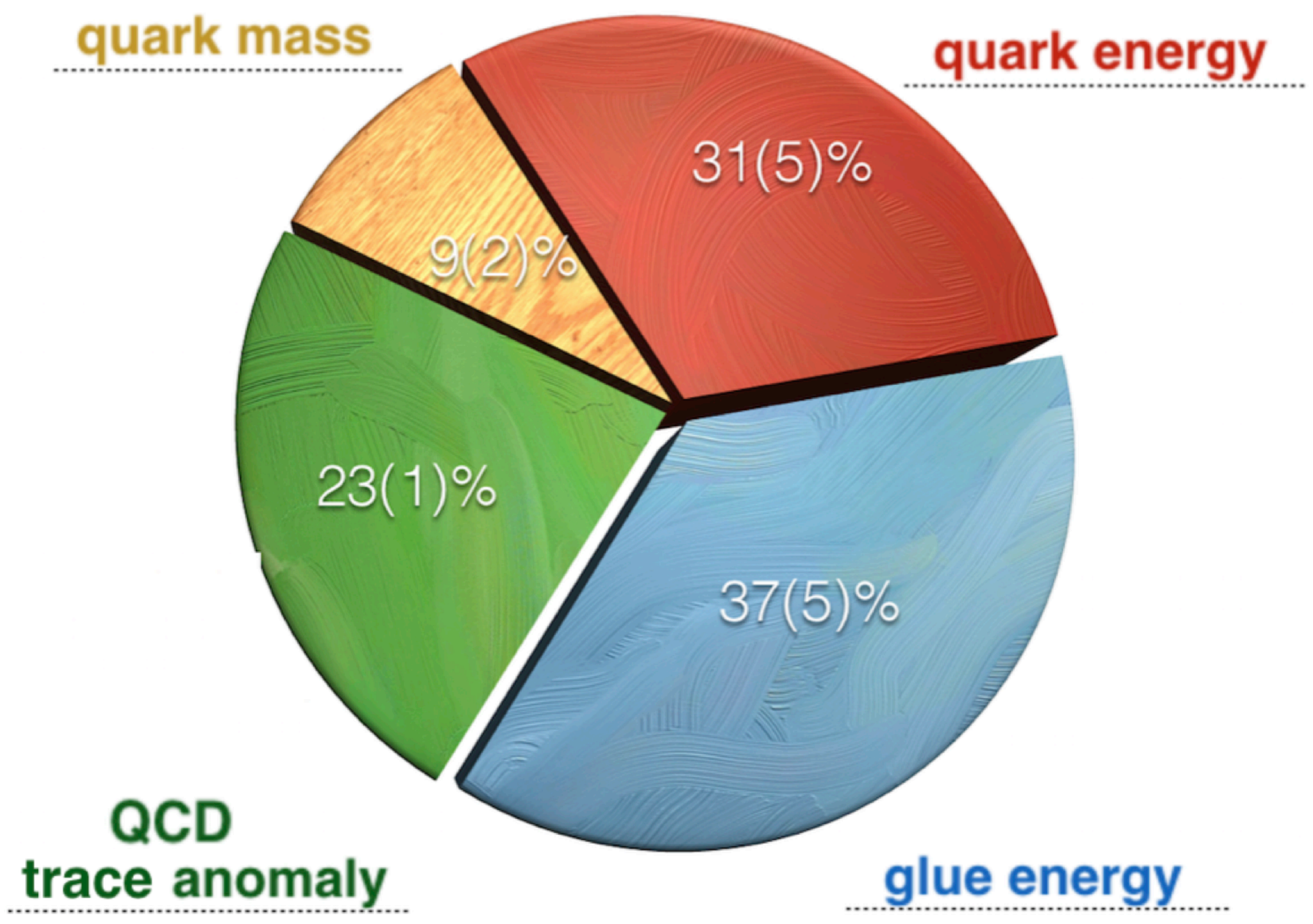
+ J/ψ production data can constrain the kaon PDFs

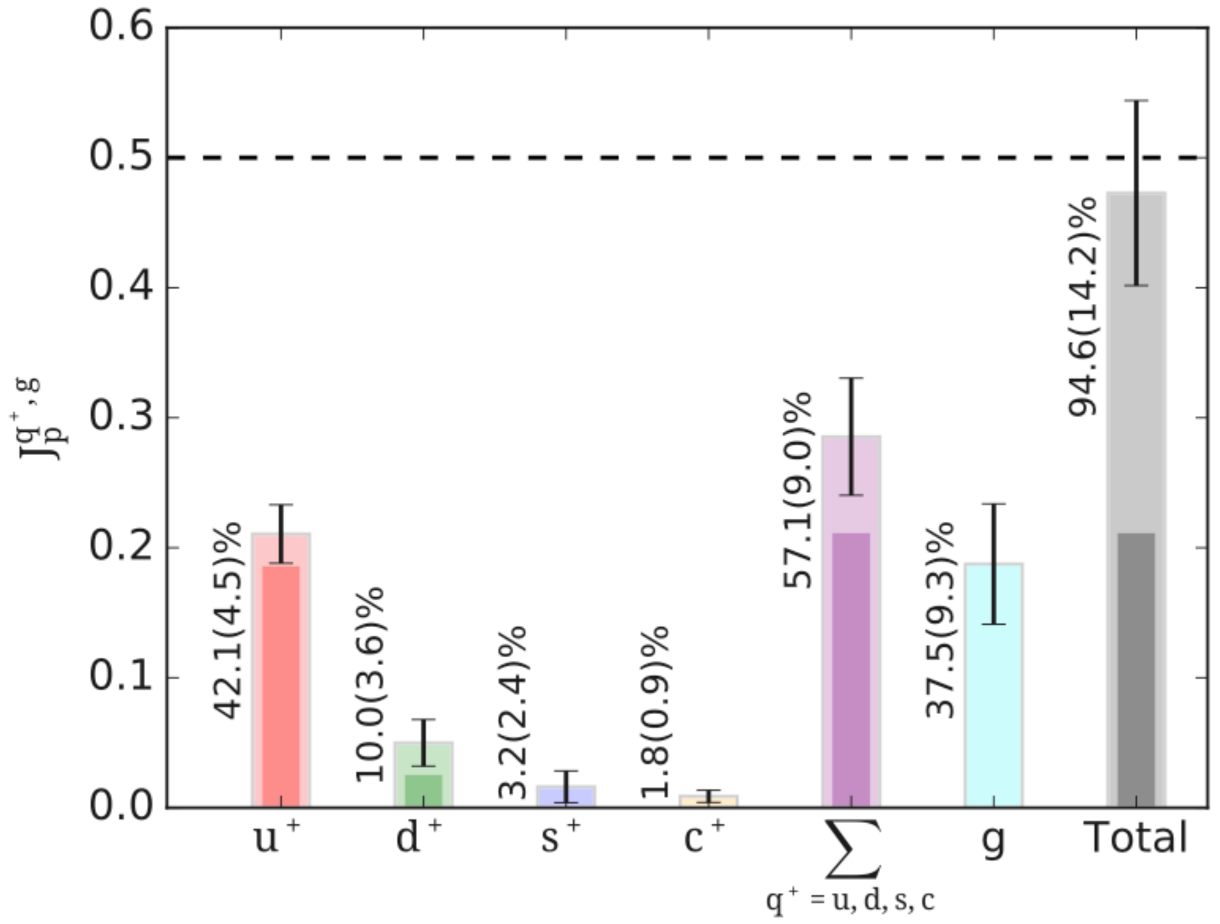
Details in Wen-Chen's talk on Tuesday

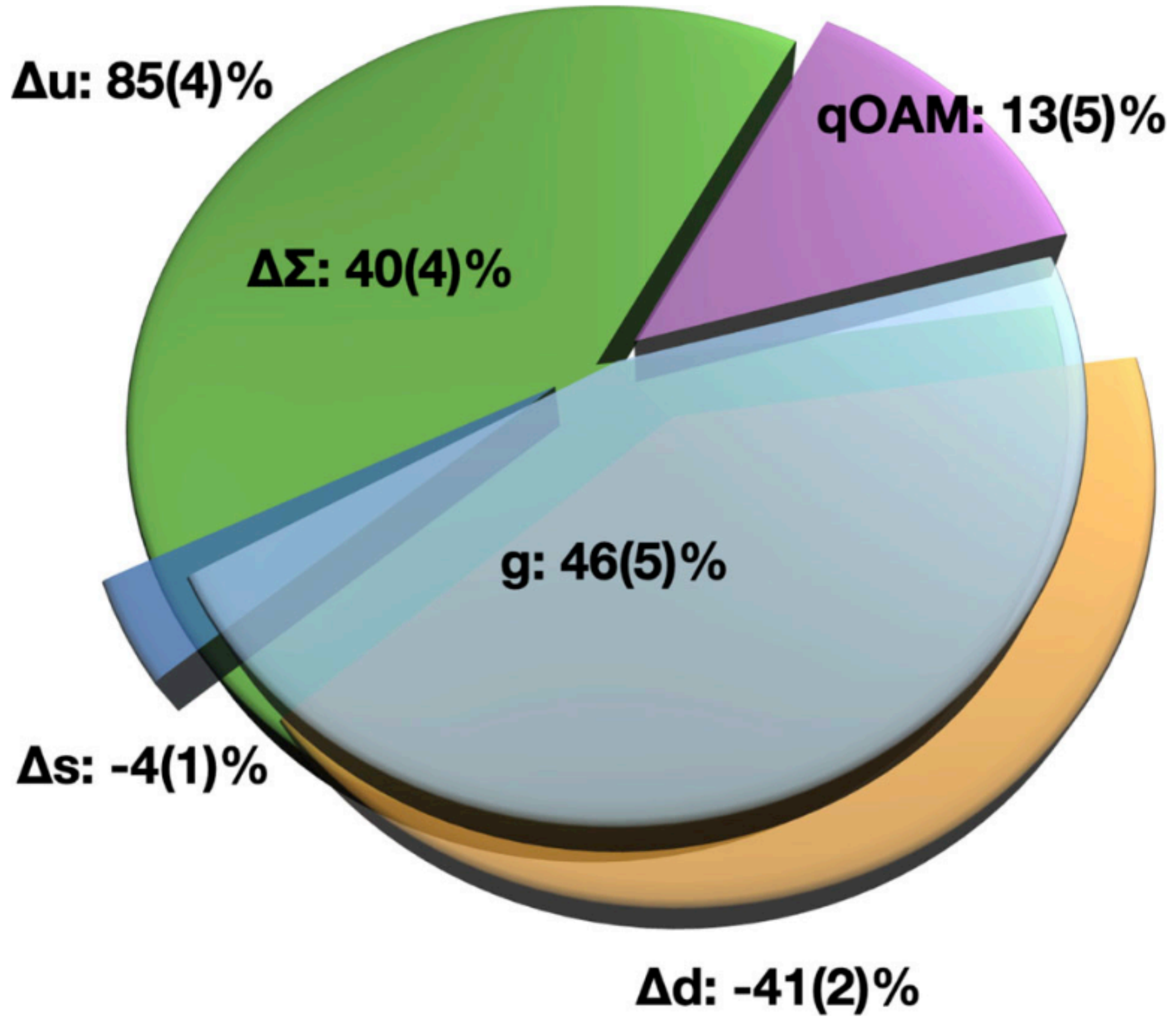
# New excitement about the pion structure

EIC aims to address the hadron mass and spin decomposition puzzles





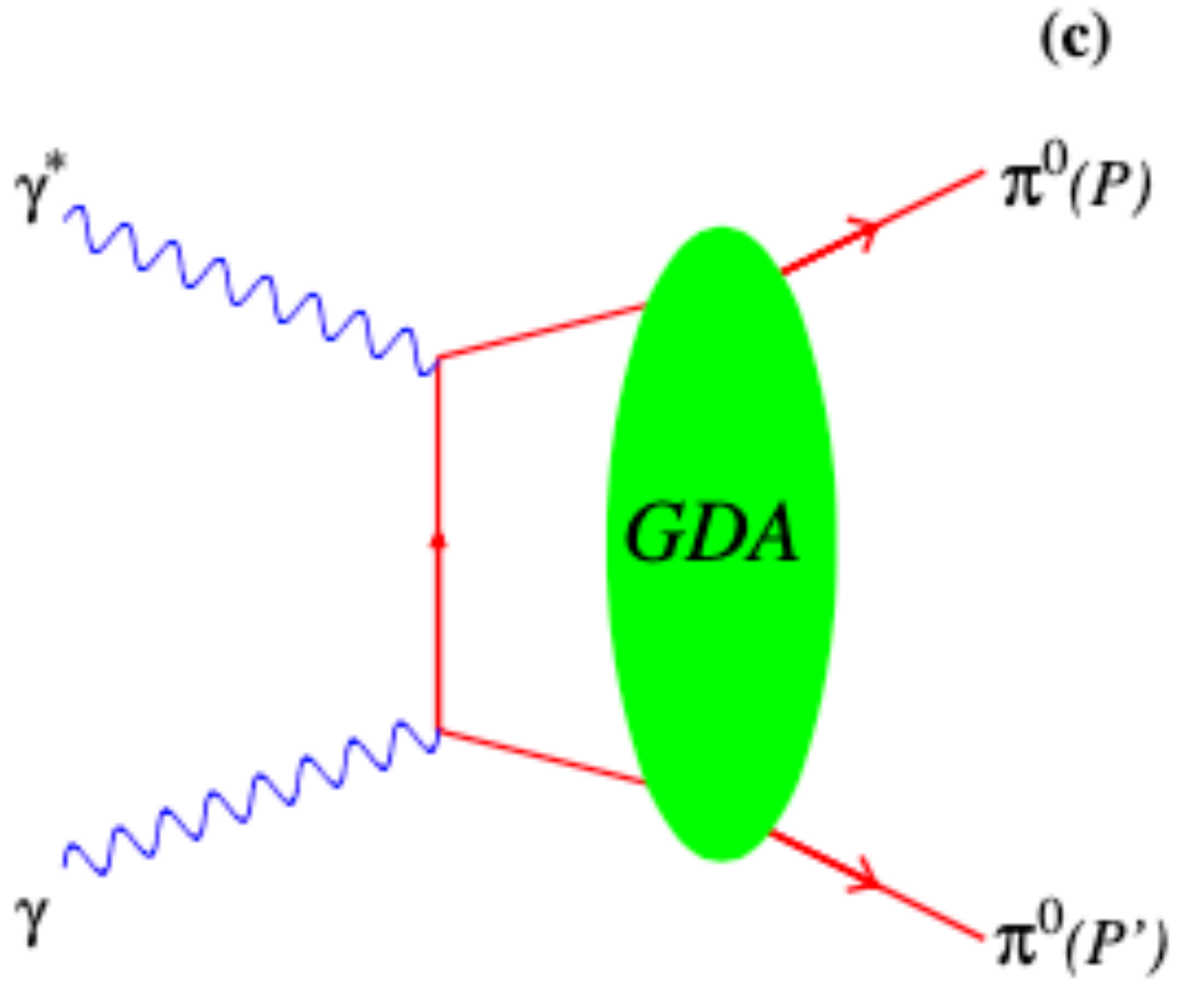




***Generalized parton distributions (GPDs) and gravitational form factors (GFFs)***

# New excitement about the pion structure

Pion gravitational form factors from s-t channel crossing relation between GDAs and GPDs




**Belle data** $\gamma\gamma^* \to \pi^0\pi^0$ [Masuda et al, PRD 93 (2016)]

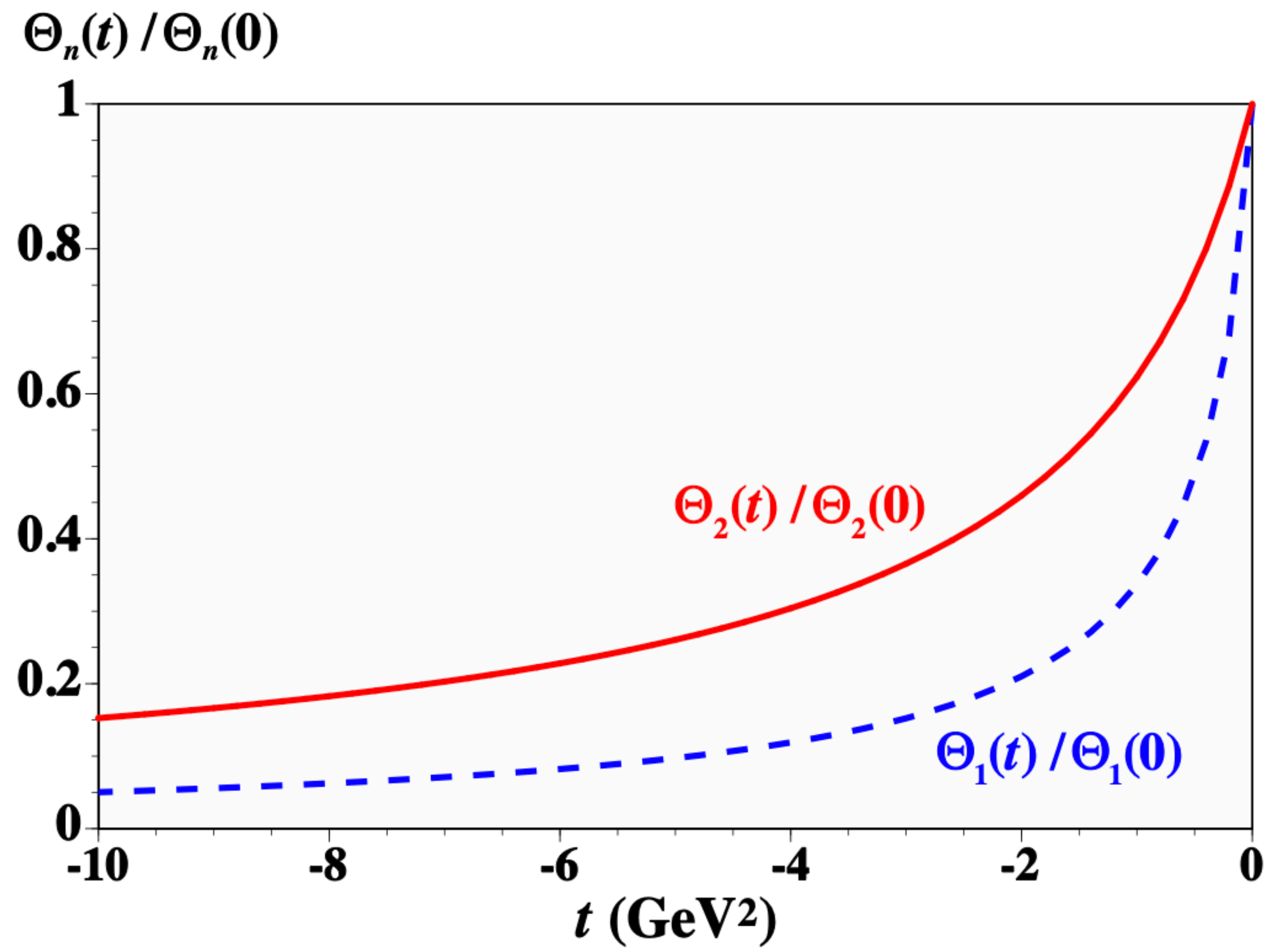


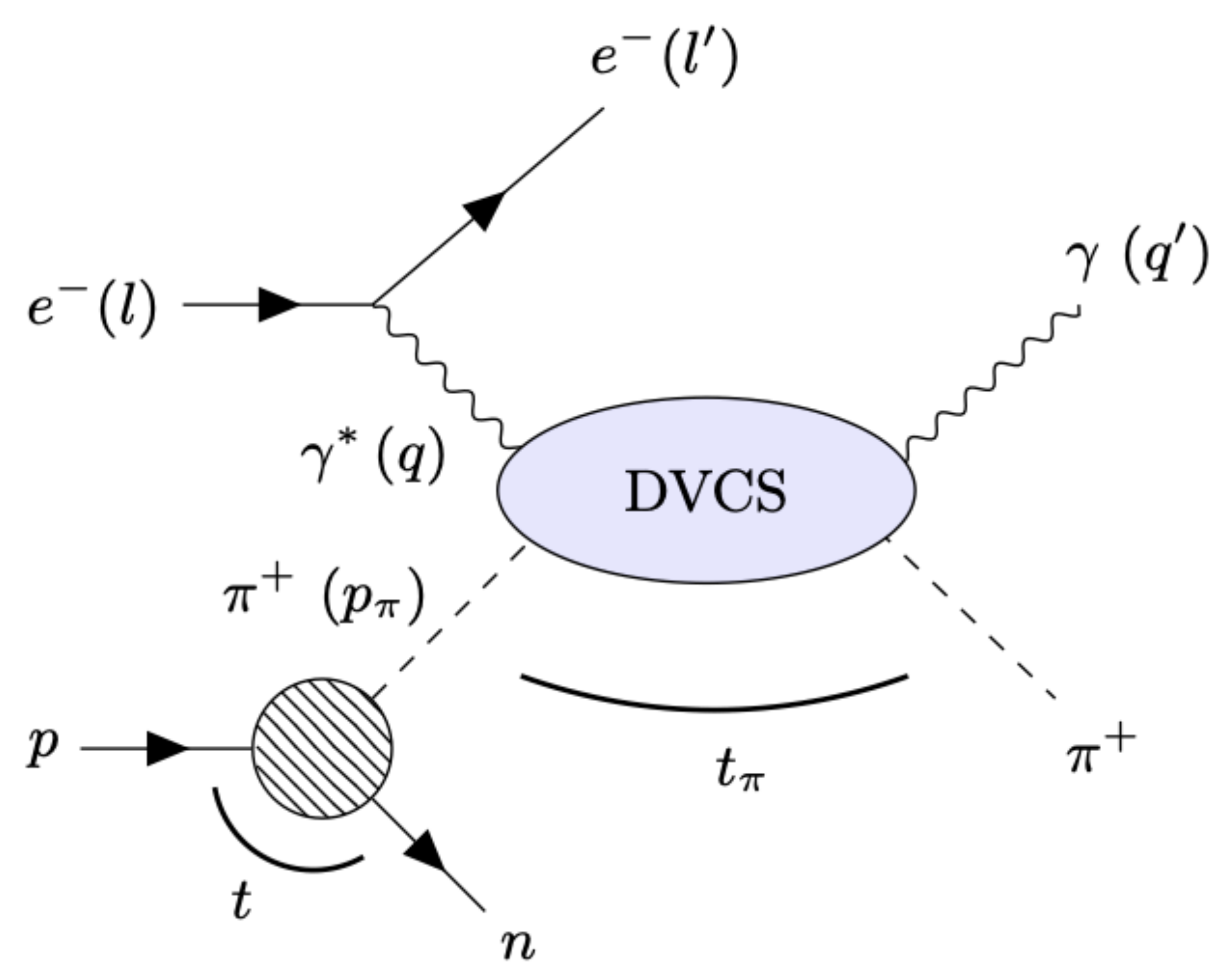

FIG. 20. Spacelike gravitational form factors normalized to their values at $t = 0$. [Kumano, Song, Teryaev ,PRD97 (2018)]

**Very large D'(0)!**

# New excitement about the pion structure

Accessing pion GPDs from Sullivan-DVCS process

[Chavez et al., *Phys.Rev.Lett.* 128 (2022) 20, 202501]




[Amrath et al. *Eur.Phys.J.C* 58 (2008) 179-192]

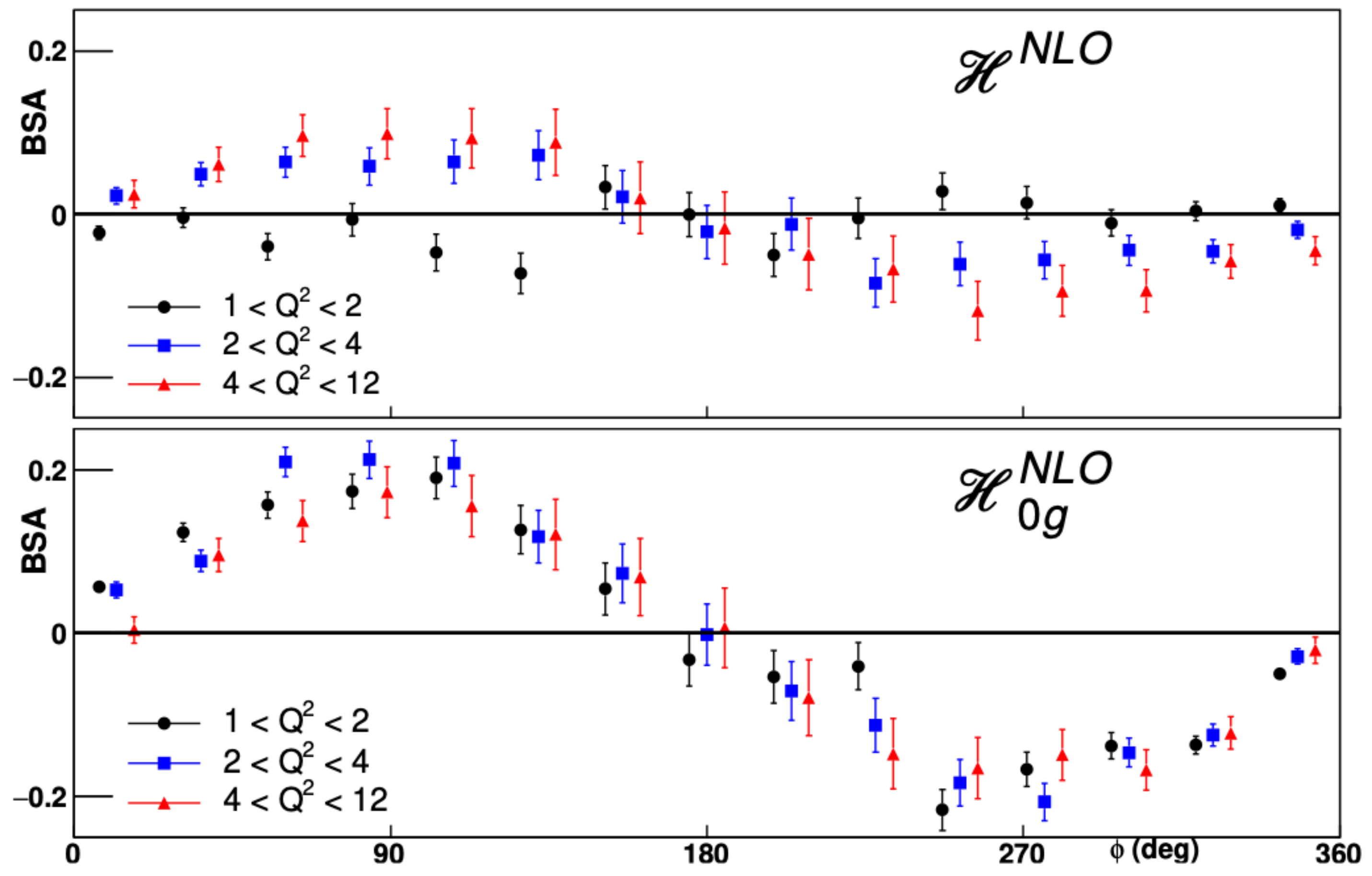



FIG. 4. Expected beam-spin asymmetries as function of $\phi$ with $\mathcal{H}^{NLO}$ (top) and $\mathcal{H}^{NLO}_{0g}$ (bottom) from EicC for $x_B^\pi \in [0.1; 0.5]$ and three different $Q^2$-bins: black circles for $Q^2$ between 1 and 2 GeV$^2$, blue squares between 2 and 4 GeV$^2$, and red triangles between 4 and 12 GeV$^2$.

Gluon GPDs (NLO) contributes to BSA

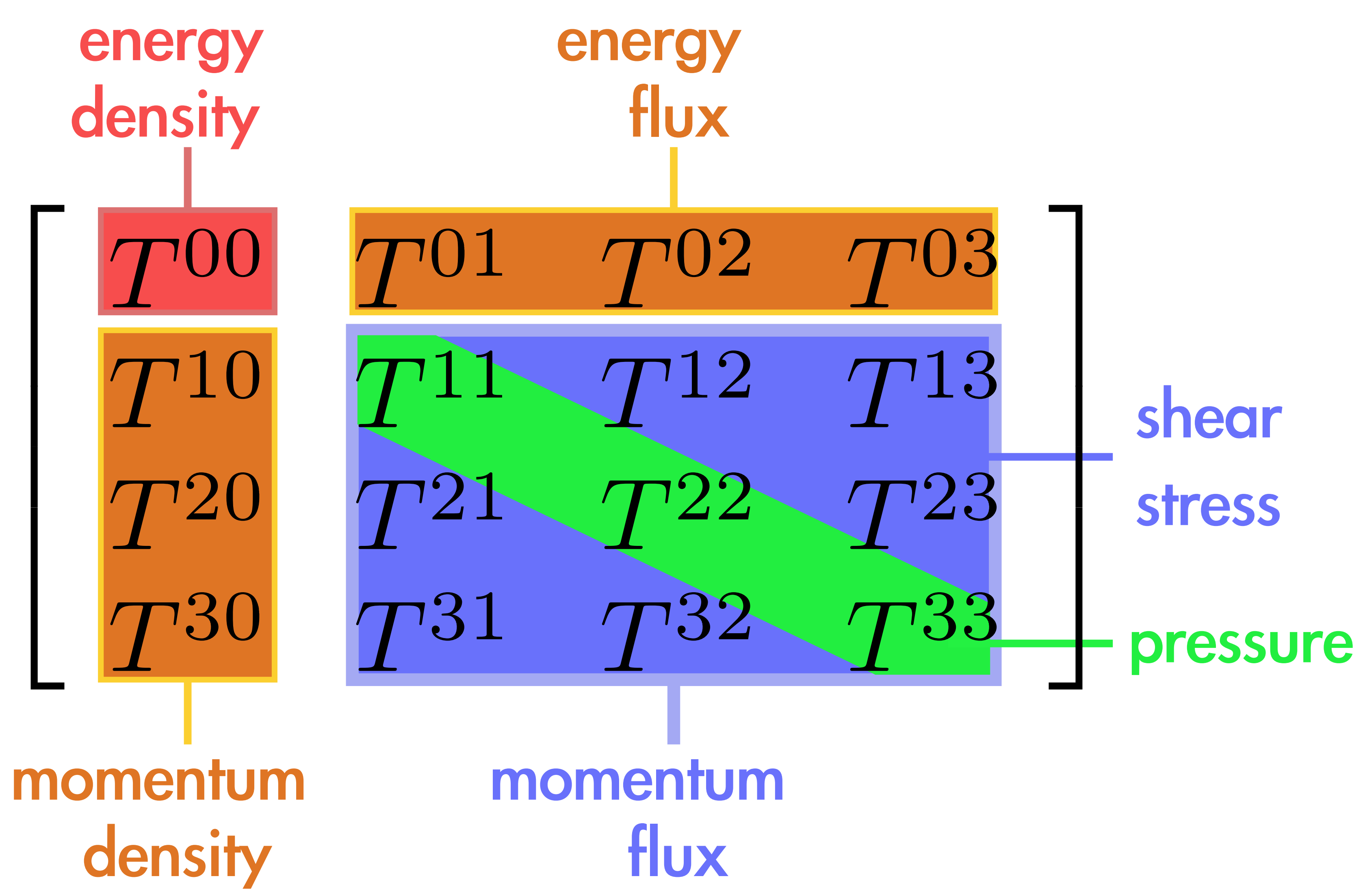
Gluonic GFFs in pion via Sullivan - J/ψ photoproduction at threshold

[Hatta and Schoenleber, arXiv:2502.12061v1]

## How does explicit chiral symmetry breaking affect the 3D quark structure and mechanical properties within the pion and kaon?

# QCD energy-momentum tensor and hadron matrix elements

# Energy-momentum tensor




Hilbert-Einstein action of a curved space-time (g) with matter (M)   (+ - - -)

$$S = -\frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\;R + \int d^4x\,\sqrt{-g}\;L_M$$

Variation of the matter-action (M) with respect to the metric tensor g

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g_{\mu\nu}(x)}$$

Flux of μ momentum across a surface of constant ν

Conservation of the EMT $\partial_\mu T^{\mu\nu} = 0$ : space-time translational invariance (Poincaré)

Symmetry μ ↔ ν (no torsion)

Matter part of the Einstein eq. as source of curvature (+cosmological constant term)

# Energy-momentum tensor operator of QCD

Quark $\hat{T}_q^{\mu\nu} = \frac{1}{4}\bar{\psi}_q\left(-i\overleftarrow{\mathcal{D}}^\mu\gamma^\nu - i\overleftarrow{\mathcal{D}}^\nu\gamma^\mu + i\overrightarrow{\mathcal{D}}^\mu\gamma^\nu + i\overrightarrow{\mathcal{D}}^\nu\gamma^\mu\right)\psi_q - \eta^{\mu\nu}\bar{\psi}_q(i\overleftrightarrow{\not{D}}/2 - m_q)\psi_q$

Gluon $\hat{T}_g^{\mu\nu} = -F^{\mu\alpha}F^\nu_\alpha + \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$

Symmetric ($\mu \leftrightarrow \nu$), gauge invariant (not in the canonical derivation)

Not conserved separately (renormalization scale dependent), but total operator $\hat{T}^{\mu\nu} = \hat{T}_q^{\mu\nu} + \hat{T}_g^{\mu\nu}$ is conserved

Trace anomaly: renormalized trace operator $\hat{T}^\mu{}_\mu = \frac{\beta(g)}{2g}F^2 + (1+\gamma_m)m\bar{\psi}\psi$ non-vanishing in the chiral limit

Hadronic matrix elements of the energy-momentum tensor (EMT) follow physical interpretation of the EMT

# Gravitational form factors of the spin-0 hadron

$$\langle p' | \hat{T}^a_{\mu\nu}(0) | p \rangle = 2P_\mu P_\nu \, A^a(t) + \frac{1}{2}(\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2)\, D^a(t) + \eta^{\mu\nu} 2M^2 \bar{c}^a(t)$$

$P = (p + p')/2$
$\Delta = q = p' - p,\ t = -\Delta^2$

$A^a(t)$

Mass distribution of the quarks and gluons inside the pion and kaon

At t=0, second Mellin moment of the unpolarized PDF

Normalization $A^q(0) + A^g(0) = 1$

$D^a(t)$ (D-term)

Fundamental, but not related to an obvious symmetry

Internal pressure and shear distributions

Negative for hadrons to satisfy the stability conditions

Dispersion relation of the DVCS (and DVMP) amplitudes (GPD model independent)

[Polyakov, Shuvaev hep-ph/0207153]
[Polyakov PLB555 (2003)]
[Polyakov, Schweitzer IJMPA33 (2018)]

$\bar{c}^a(t)$

Non-conservation of quark and gluon parts of EMT ~ $\eta_{\mu\nu}$

Contributes to the mass(00) and the pressure(ii)

Mass decomposition can constraint this quantity at t=0, pQCD, $\bar{c}^q_\pi(0) = -0.04 \pm 0.02\ (\mu = 1\ \text{GeV})$ [Tanaka, JHEP03 (2023)]]

$\sum_q \bar{c}^q + \bar{c}^g = 0$, Smallness of $\sum_q \bar{c}^q(0)$ at low scale, suppressed by instanton packing fraction (in tension with other studies)

[M. Polyakov, HDS, JHEP 156 (2018)]

# Gravitational form factors of the pion and kaon from a chiral effective model

# Quark one-loop effective action in the large Nc limit

$$\mathcal{S}_{\text{eff}} = \int \frac{d^4k}{(2\pi)^4} \bar{\psi}(k)(\not{k} - \hat{m})\psi(k) - \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} \bar{\psi}_f(p)\sqrt{M_f(p)}\, U^{\gamma_5}_{fg}(p-k)\sqrt{M_g(k)}\psi_g(k)$$

$$M(k) = MF^2(k), \qquad U^{\gamma_5}(x) = \exp\left[\frac{i}{F_{\mathcal{M}}}\gamma^5\lambda^a\mathcal{M}^a\right], \qquad \hat{m} = \text{diag}(m_u, m_d, m_s).$$

Inspired by the liquid instanton model at low-renormalization point $\mu \sim 1/\bar{\rho}$

$M(0) = 350$ MeV, computed by the gap equation from the instanton vacuum

Nonlinear chiral field for $SU(3)_f$ is introduced as well as the current quark masses ($m_u = m_d = 5$ MeV, ms = 100 MeV)

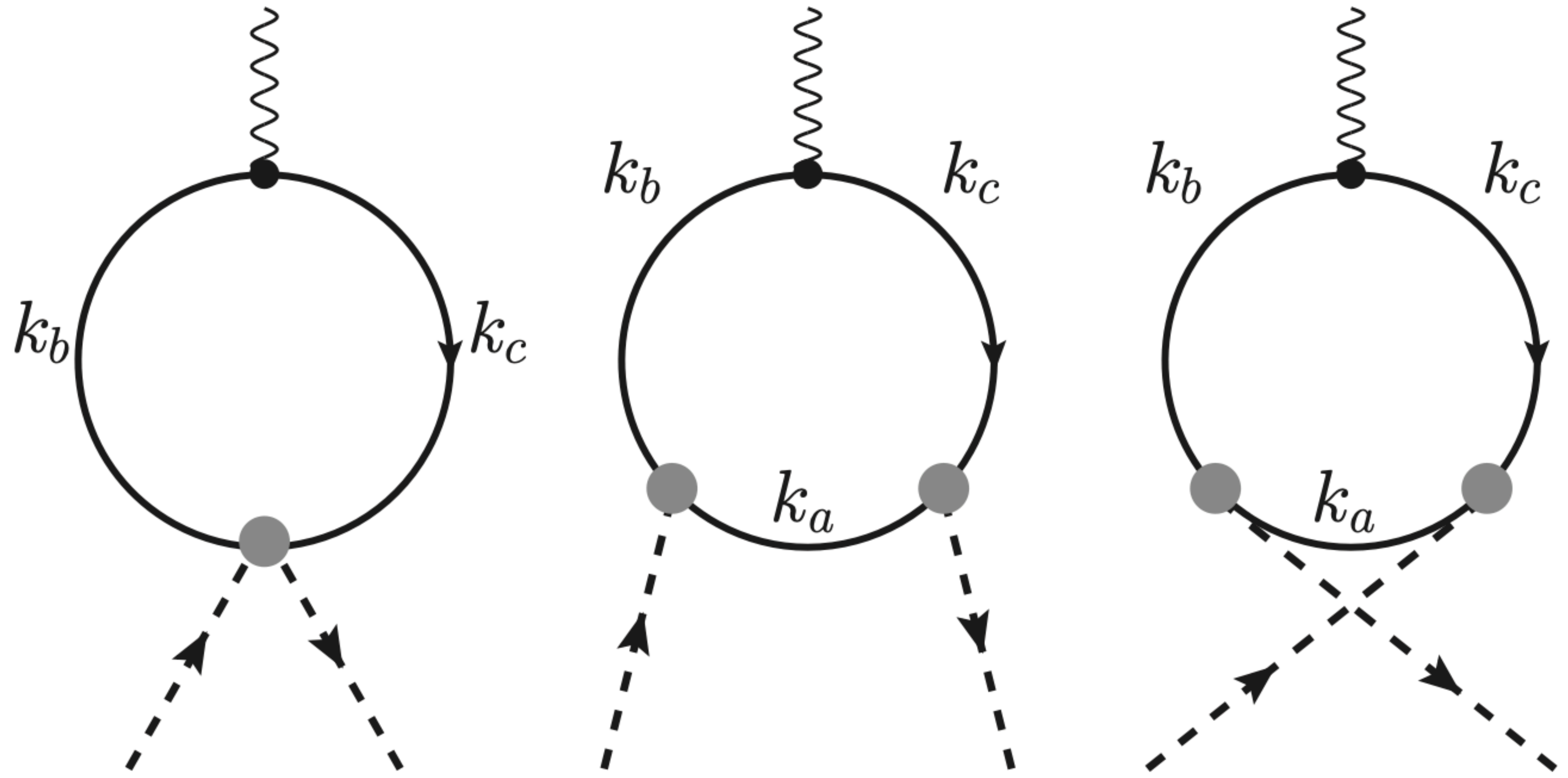
Analytic continuation to Minkowski space is assumed, [Praszalowicz and Rostworowski]

with n-pole type quark form factor: $F(k) = \left(\frac{1}{1 - k^2/\Lambda^2 - i\epsilon}\right)^n$, (instanton form factor n = 3/2)

EMT operator: $\hat{T}_{\mu\nu} = \bar{\psi}(-i\overleftarrow{\partial}_\mu\gamma_\nu - i\overleftarrow{\partial}_\nu\gamma_\mu + i\overrightarrow{\partial}_\mu\gamma_\nu + i\overrightarrow{\partial}_\nu\gamma_\mu)\psi$

# Meson matrix elements of the EMT operator

$$\langle p' | \hat{T}_{\mu\nu}(0) | p \rangle = 2P_\mu P_\nu \, A(t) + \frac{1}{2}(\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2) \, D(t)$$




| | | |
|---|---|---|
| Divergences: Quadratic | Quadratic, Logarithmic | Quadratic, Logarithmic |

*Quadratic divergences cancel out, leaving physical quantities logarithmically divergent!*

# von Laue stability condition of the pion

[HDSon and H.-Ch. Kim, PRD 90 (2014)]

Conservation of the EMT operator requires

$$P = \sum_{i=1}^{3} \langle \pi(p) \,|\, T^{ii}(0) \,|\, \pi(p) \rangle = 0$$

Analytic expression for P obtained (F(k)= 1)

$$P/3 = -\frac{2}{F^2}\frac{N_c M m}{4\pi^2}\int_0^\infty du\,\frac{1}{u^2}e^{-u\bar{M}^2} - \frac{2}{F^2}(p^2)\frac{N_c M^2}{4\pi^2}\int_0^1 dx\,x\int_0^\infty \frac{du}{u}e^{-u[x(1-x)p^2+\bar{M}^2]}$$

$$\sim \frac{2}{F^2}\left[\, m\langle\bar{\psi}\psi\rangle + m_\pi^2 F_\pi^2 + \mathcal{O}(m^2) \,\right]$$

*von Laue condition is guaranteed by the Gell-Mann – Oakes – Renner relation*

*+ correction constraining the value of m*

# Kaon?

$$P/3 = -\frac{1}{F^2}\frac{N_c M m_s}{4\pi^2}\int_0^\infty du\,\frac{1}{u^2}e^{-u\bar{M}_s^2}$$
$$-\frac{1}{F^2}\frac{N_c M m_q}{4\pi^2}\int_0^\infty du\,\frac{1}{u^2}e^{-u\bar{M}_q^2}$$
$$-\frac{1}{F^2}(p^2+(m_s-m_q)^2)\frac{N_c M^2}{4\pi^2}\int_0^1 dx\,x\int_0^\infty \frac{du}{u}e^{-u[x(1-x)p^2+x\bar{M}_q^2+(1-x)\bar{M}_s^2]}$$
$$-\frac{1}{F^2}(p^2+(m_s-m_q)^2)\frac{N_c M^2}{4\pi^2}\int_0^1 dx\,x\int_0^\infty \frac{du}{u}e^{-u[x(1-x)p^2+x\bar{M}_s^2+(1-x)\bar{M}_q^2]}$$

**GMOR relation for Kaon:** $F_K^2 m_K^2 = (m_s + m_q)\langle \bar{s}s + \bar{u}u\rangle/2$,

$$P \propto 0\ (\mathrm{GMOR}) + \mathcal{O}(\Delta m_{su}^2/m_K^2, m_u/m_s, m_s/M)$$

*Numerically, the correction deviates ~10% from exact GMOR relation for the kaon*

*the model parameter $m_s$ can be constrained*

# Gravitational form factors of the Kaon

$$\langle K^+(p')\,|\,\hat{T}_{\mu\nu}(0)\,|\,K^+(p)\rangle = \left[2P_\mu P_\nu\, A(t) + \frac{1}{2}(\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2)\, D(t)\right]$$

χPT result (O(p²)) [Donoghue and Leutwyler, ZPC52 (1991)]

$$A(t) = 1 - 2\, L_{12}^r \frac{t}{F^2}$$

'Gravitational' LECs: $L_{11}$, $L_{12}$, $L_{13}$

$$-D(t) = 1 + 2\frac{t}{F^2}(4L_{11}^4 + L_{12}^r)$$

$$-16\frac{m_K^2}{F^2}(L_{11}^4 - L_{13}^r) + \frac{3t}{4F^2}I_\pi(t) + \frac{3t}{2F^2}I_K(t) + \frac{9t - 8m_K^2}{12F^2}I_\eta(t)$$

$$I(q^2) = \frac{1}{48\pi^2}\left[\ln\frac{\mu^2}{m^2} - 1 + \frac{q^2}{5m^2}\right] + \mathcal{O}(q^4)$$

A + D at zero momentum transfer proportional to meson mass corrections (0 in the chiral limit)

[Hudson and Schweitzer, Phys. Rev. D 96, 114013 (2017)]

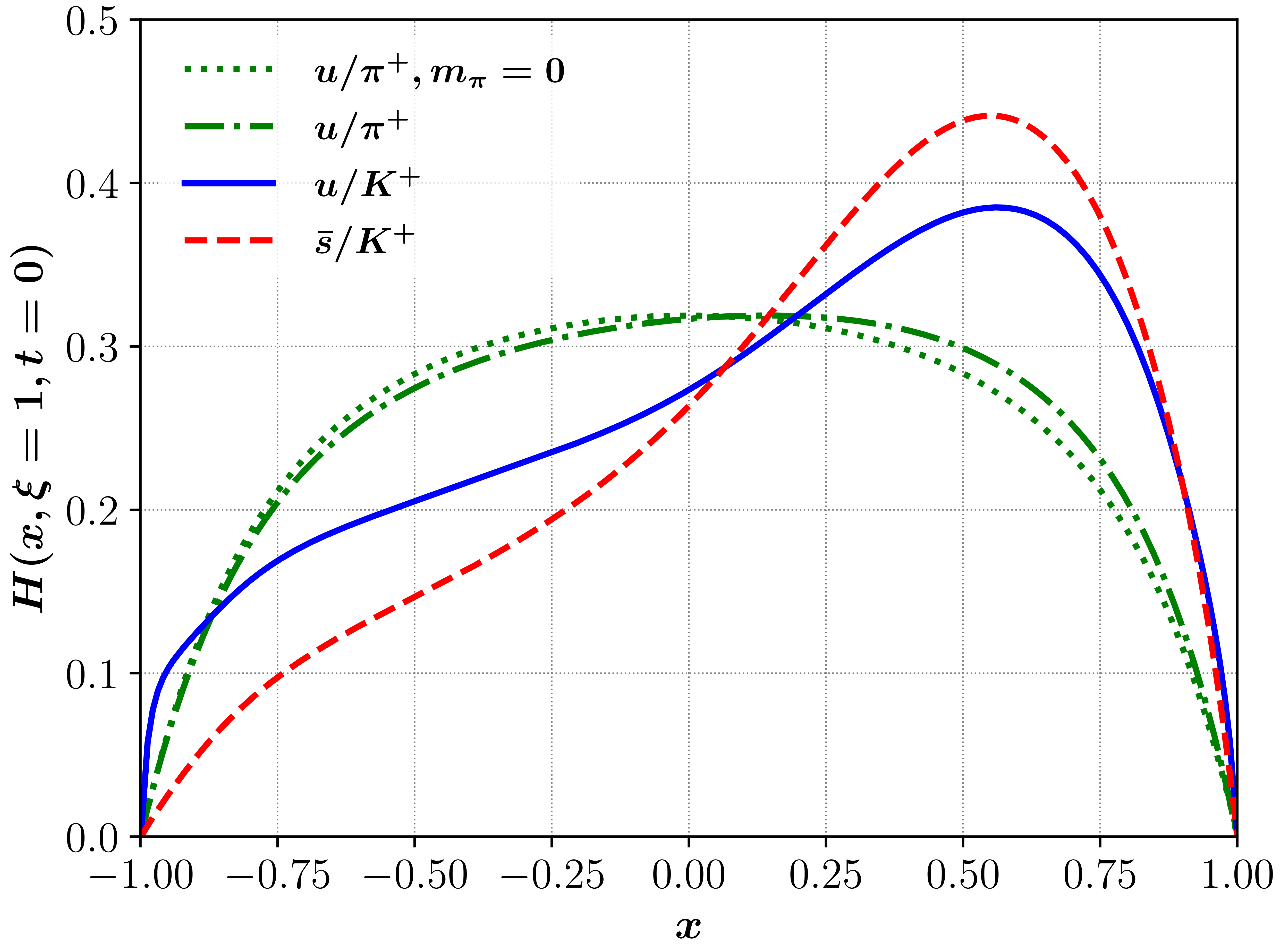
$$A(0) + D(0) = \boxed{\frac{16m_K^2}{F^2}(L_{11}^\mu - L_{13}^\mu)} + \frac{m_K^2}{72\pi^2F^2}\left[\ln\frac{\mu^2}{m_\eta^2} - 1\right] + \ldots \approx 0.23 \pm 0.15$$

**Leading Nc result in the quark model, magnitude is amplified by larger kaon mass (vs. A+D=0.03 for the pion)**

# Results in various theoretical studies

| | $A_\pi(0) + D_\pi(0)$ | $A_K(0) + D_K(0)$ | $A_{\bar{s}/K^+}(0)/A_{u/K^+}(0)$ | $D_{\bar{s}/K^+}(0)/D_{u/K^+}(0)$ |
|---|---|---|---|---|
| **ChQM** [Son & Hutauruk, 2025] | 0.04 | 0.36 | 1.26 | 1.10 |
| **ChPT** [Donoghue & Leutwyler 1991] | 0.03 | 0.23 | - | - |
| **LFWFs** [Raya et al, 2021] | - | - | 1.1 | 1.25 |
| **BSE-NJL** [Adhikari et al., 2021] | - | - | 1.32 | - |
| **DSE** [Y. Xu et al. 2023] | 0.03 | 0.23 | 1.56 | 1.25 |
| **BSE-NJL** [P. Hutauruk et al. 2016] | - | - | 1.38 | - |
| **MIT-Lattice** [Hackett et al. 2023] | ~0.10 | - | - | - |
| **ETMC-Lattice** [Delmar et al. 2024] | - | - | ~1.3 | - |

# Valence quark GPDs at $\xi = 1$ and explicit chiral symmetry breaking




**Chiral symmetry requires H(x,ξ=1,t=0) to be even in x**

[M. Polyakov and C. Weiss, Phys.Rev.D 60 (1999) 114017]

……. Pion ($m_\pi = 0$): exactly symmetric

-·-·- Pion ($m_\pi = 140$ MeV): slightly distorted

-------- $\bar{s}$ in $K^+$ ——— $u$ in $K^+$

Second Mellin moment: Gravitational form factors

$$\int_{-1}^{1} dx\, x\, 2H(x, \xi = 1, t = 0) = A(0) + D(0) = 0 + \mathcal{O}(m^2_{\pi,K})$$

Correction: $m_K^2/m_\pi^2 \approx 12$

*Size of A(0)+D(0) ~ size of asymmetry in xH(x,χ=1,t=0) ~ O($m^2$)*

# $\bar{c}^q$ of the pion and kaon

**pQCD ($N_f$=3, MS-bar, NNLO)** [Tanaka, JHEP03 (2023)]]

$$\bar{c}^q_\pi(t=0, \mu = 1\ \mathrm{GeV}) = -\,0.04 \pm 0.02$$

**Isospin & charge symmetry:** $\bar{c}^u_\pi = \bar{c}^{\bar{d}}_\pi = \bar{c}^{\bar{u}}_\pi = \bar{c}^d_\pi$

**In a constituent quark model:** $\bar{c}^u_\pi = \bar{c}^{\bar{d}}_\pi = \bar{c}^{\bar{u}}_\pi = \bar{c}^d_\pi = 0$

**Kaon:** $\bar{c}^s_{K^+}(t) + \bar{c}^u_{K^+}(t) = 0$ (constituent quark picture)

But $\bar{c}^s_{K^+}(t) \neq \bar{c}^u_{K^+}(t) \neq 0$ in general

# $\bar{c}^u(t)$, $\bar{c}^d(t)$, and $\bar{c}^s(t)$ of proton

[H. Won, H.-Ch. Kim, and J. Kim, 2307.00740]

- Nucleon as a quark-soliton in the large Nc meanfield:

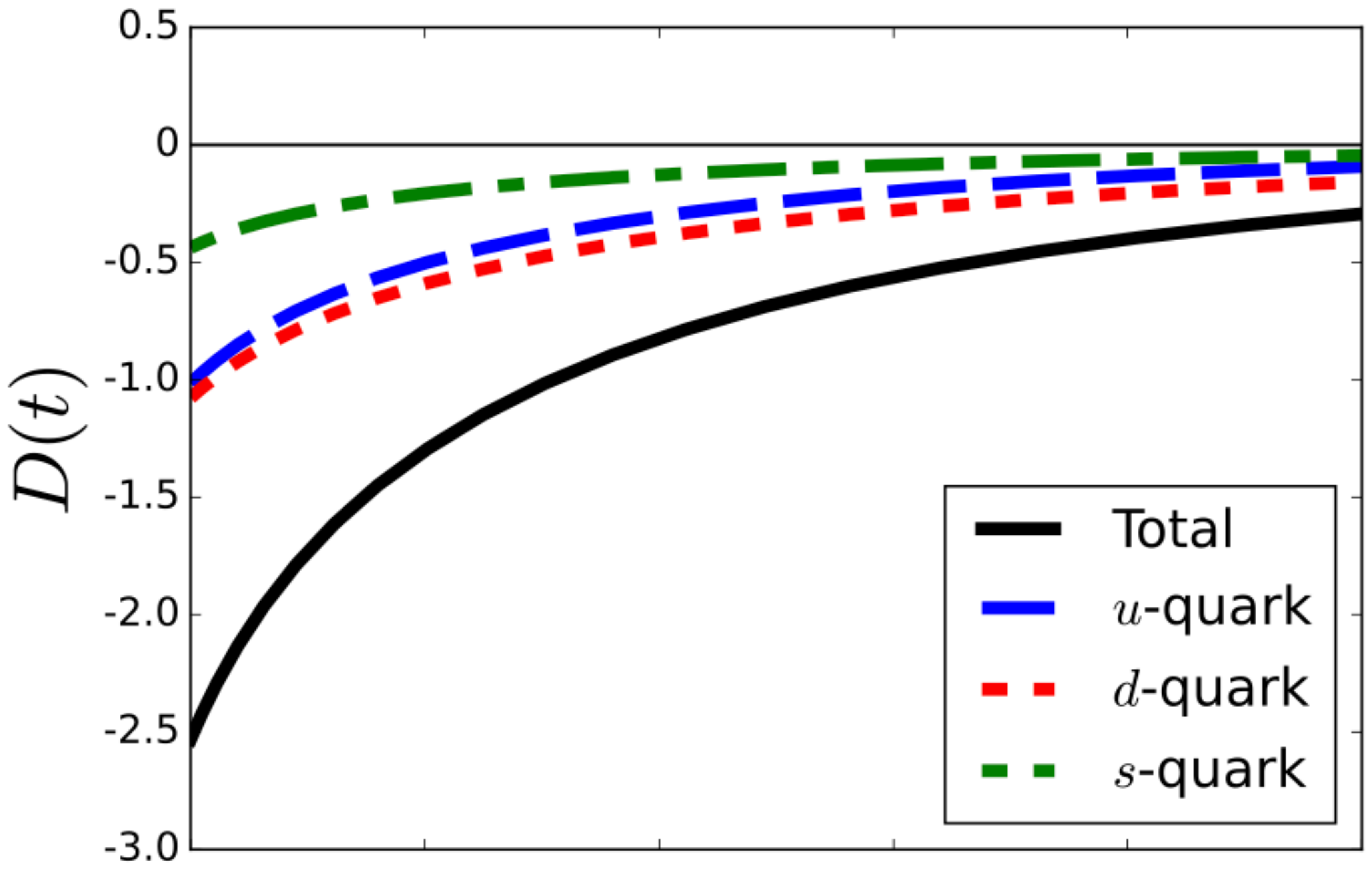
(constituent) quarks are only effective degrees of freedom

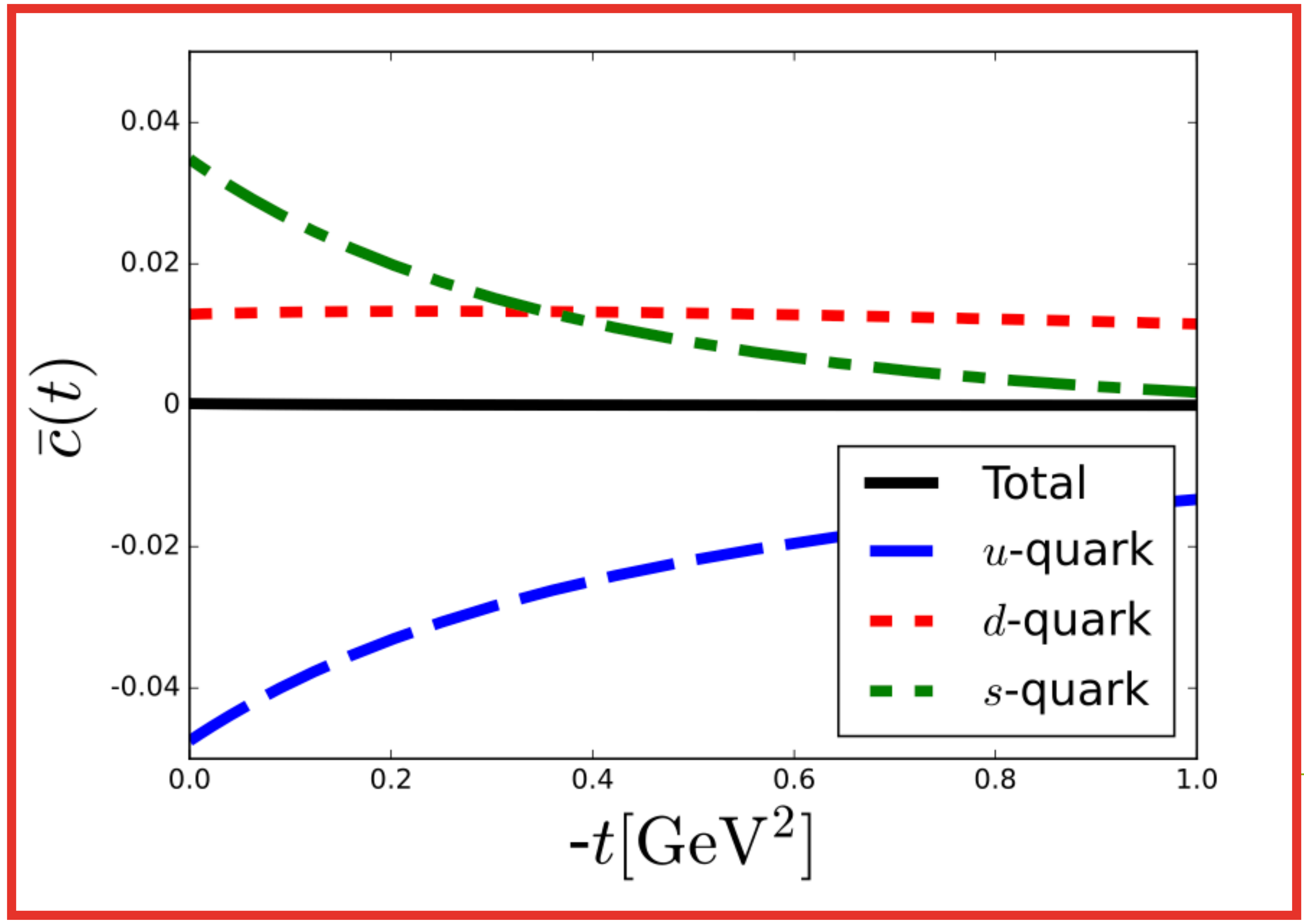
- $\sum_q \bar{c}^q(t) = 0$ is a mandatory condition for the conserved quark EMT operator.

- Nontrivial cancellation between the $\bar{c}^u(t)$ and $\bar{c}^d(t)$+$\bar{c}^s(t)$ is observed.

  (vs. in $\pi^+$, $\bar{c}^u(t) = \bar{c}^{\bar{d}}(t) = 0$, due to isospin symmetry)

- Contribution to the pressure distribution $\bar{c}^q$ is sizable



# Summary and outlook

**Observations**

Pion and kaon EMT matrix elements and GFFs from a chiral quark model

Stability of the pion and kaon relies on the pattern of chiral symmetry breaking (GMOR)

Explicit chiral symmetry breaking differentiates D-term of kaon and pion ~ ERBL region of GPDs

Quark D-terms in Kaon $D_{\bar{s}/K^+}(0)/D_{u/K^+}(0) = 1.1,\ D_{\bar{s}/K^+}(0) + D_{u/K^+}(0) = 0.64$

can be compared with the ChPT prediction ~0.77

**Questions**

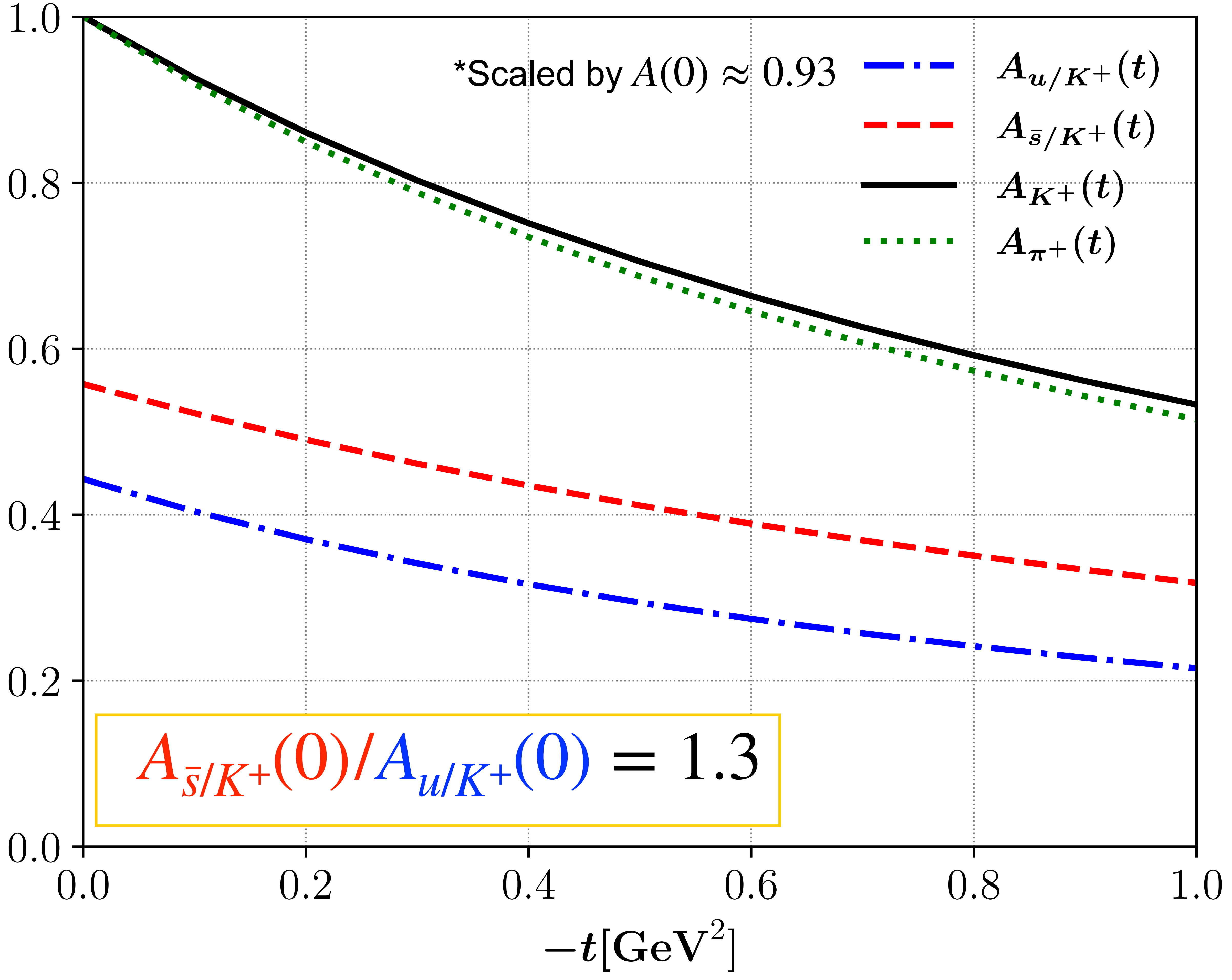
Kaon Sullivan-DVCS process in EIC?

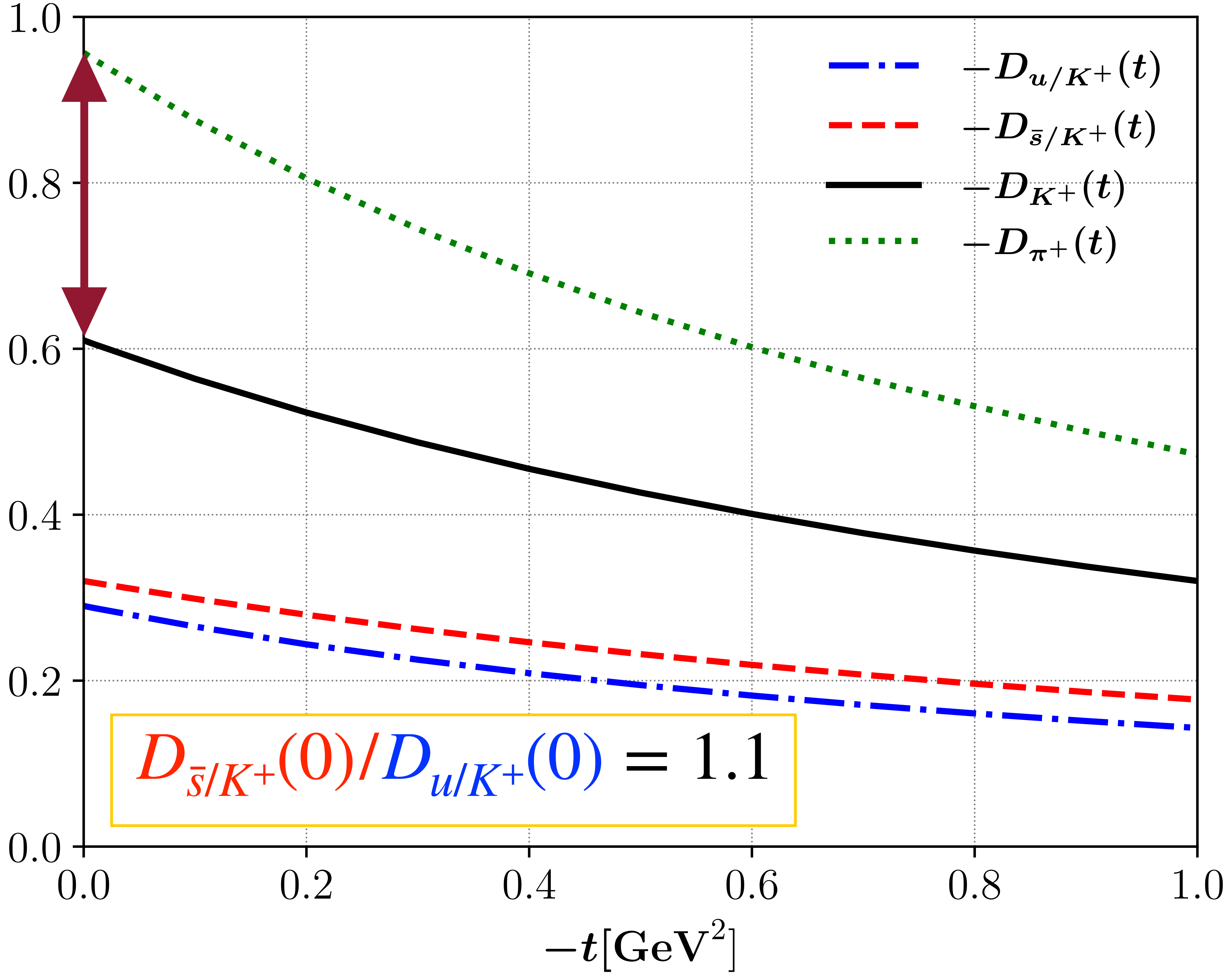
Gravitational form factors of the kaon, $\bar{c}^{u,s}_{K^+}(t)$?

# Thank you very much!

# Gravitational form factors from n=2 Mellin moments

$A(t)$ : Distribution of mass
$D(t)$ :Pressure and Shear



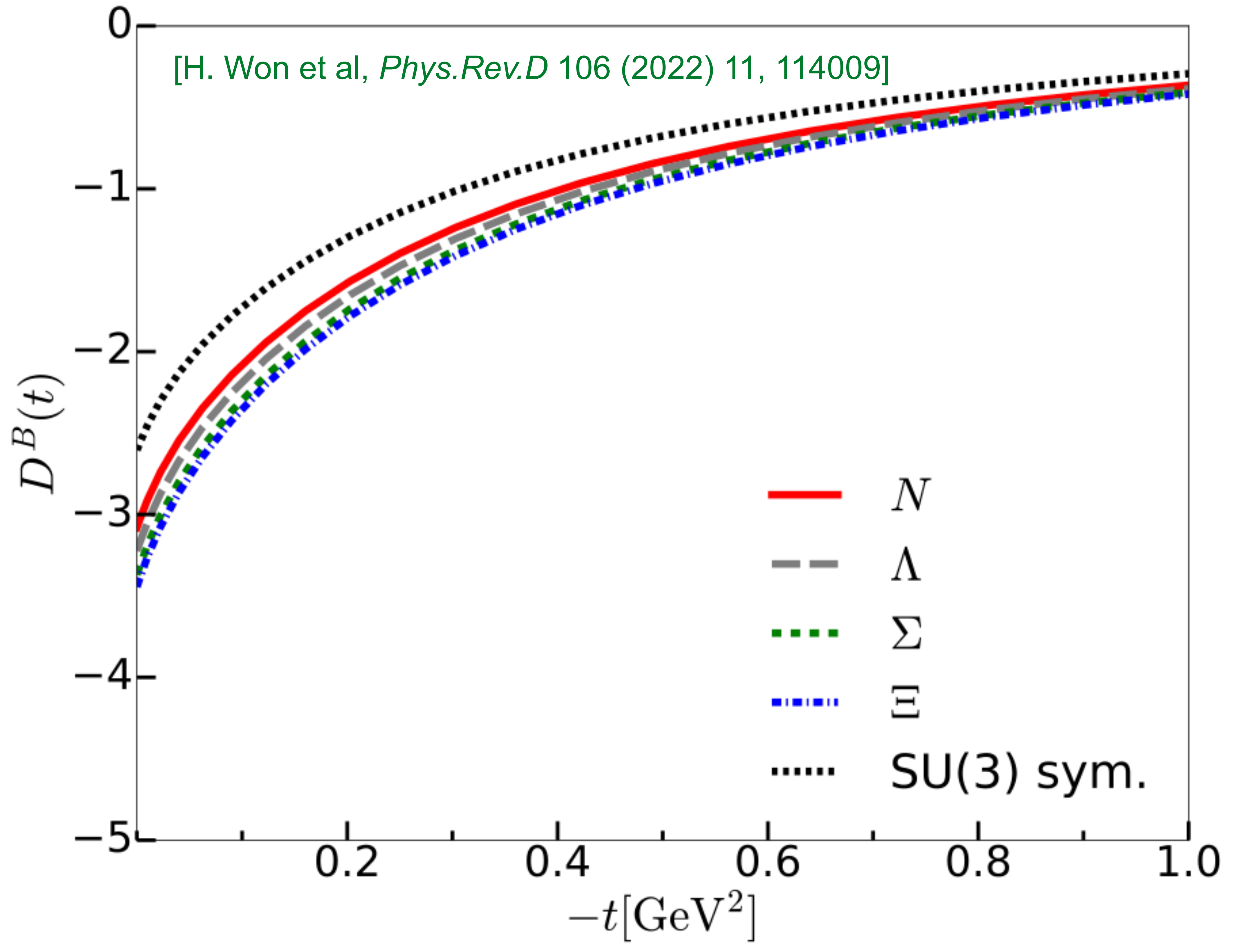



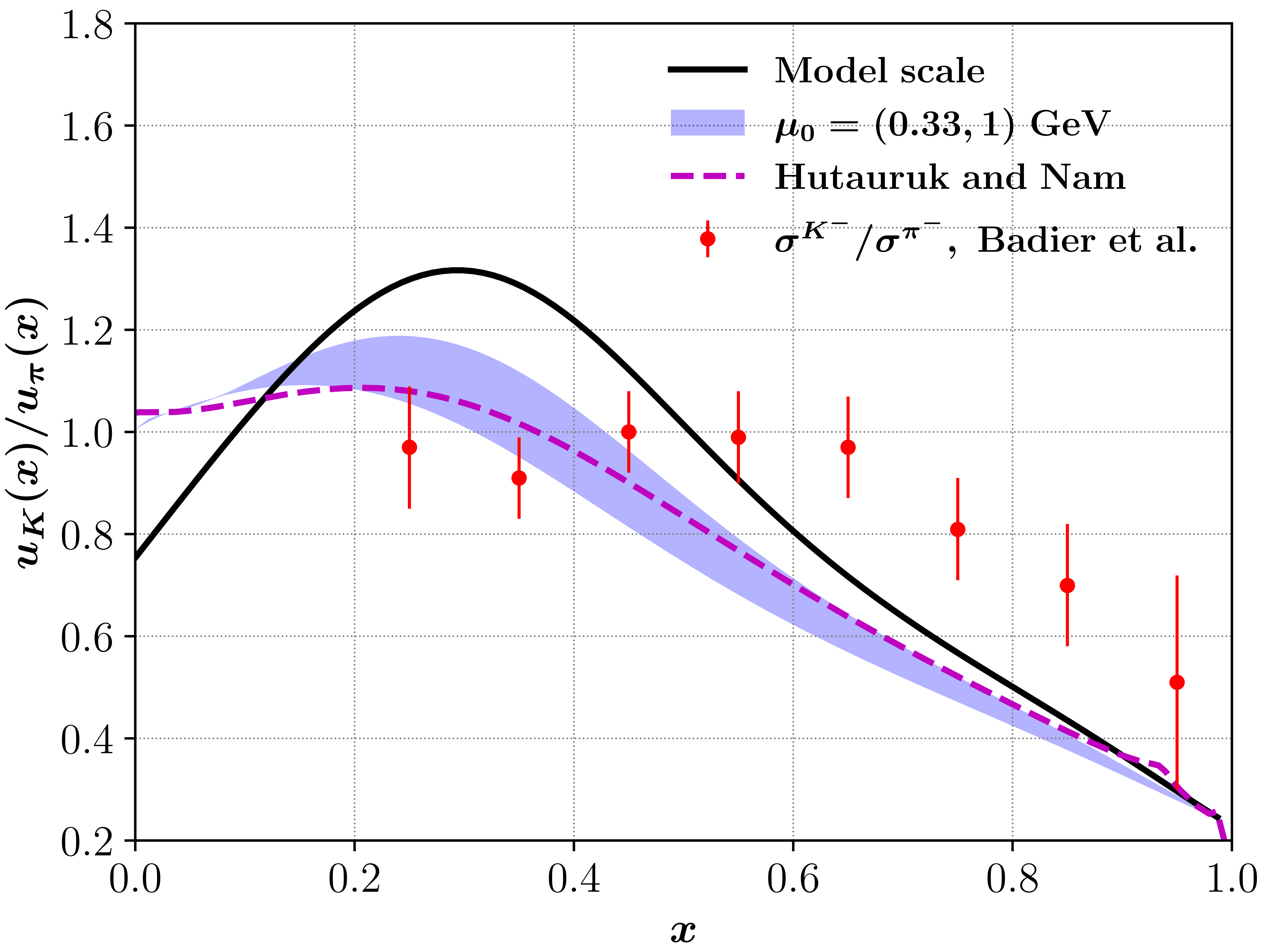

$$A_K(0) + D_K(0) = 0.36 \text{ vs. } A_\pi(0) + D_\pi(0) = 0.04$$

*D-term receives a meson mass correction of order* $m^2_{K,\pi}$

# vs D-term of Baryon octet from χQSM

| $B$ | $\varepsilon^B(0)$ GeV/fm$^3$ | $\langle r_\varepsilon^2\rangle_B$ fm$^2$ | $2J^B(0)$ | $g_A^{0,B}$ | $2L^B$ | $p^B(0)$ GeV/fm$^3$ | $(r_0)_B$ fm | $D^B(0)$ | $\langle r^2_{\rm mech}\rangle_B$ fm$^2$ |
|---|---|---|---|---|---|---|---|---|---|
| $N$ | 2.85 | 0.31 | 1.00 | 0.48 | 0.52 | 0.42 | 0.57 | -3.08 | 0.53 |
| $\Lambda$ | 3.12 | 0.26 | 1.00 | 0.40 | 0.60 | 0.44 | 0.57 | -3.22 | 0.53 |
| $\Sigma$ | 3.40 | 0.20 | 1.00 | 0.53 | 0.47 | 0.46 | 0.57 | -3.37 | 0.52 |
| $\Xi$ | 3.53 | 0.17 | 1.00 | 0.38 | 0.62 | 0.47 | 0.57 | -3.45 | 0.52 |
| SU(3) sym. | 1.89 | 0.54 | 1.00 | 0.46 | 0.54 | 0.35 | 0.57 | -2.60 | 0.55 |




Model scale
$\mu_0 = (0.33, 1)$ GeV
Hutauruk and Nam
$\sigma^{K^-}/\sigma^{\pi^-}$, Badier et al.
$u_K(x)/u_\pi(x)$
$x$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

# Thoughts on $\bar{c}(t)$

# $\bar{c}^u(t)$, $\bar{c}^d(t)$, and $\bar{c}^s(t)$ of proton

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**(constituent) quarks are only effective degrees of freedom**

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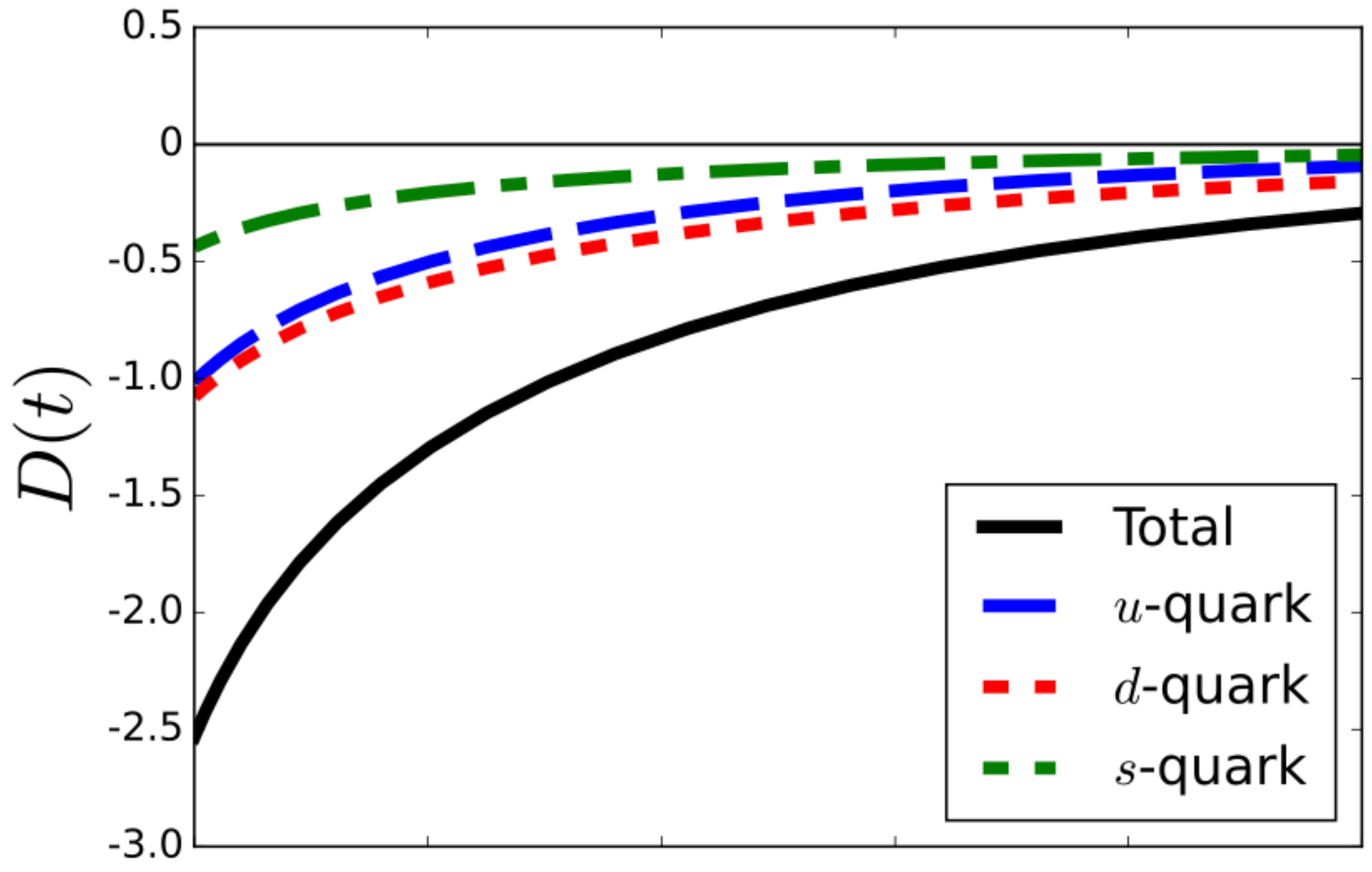
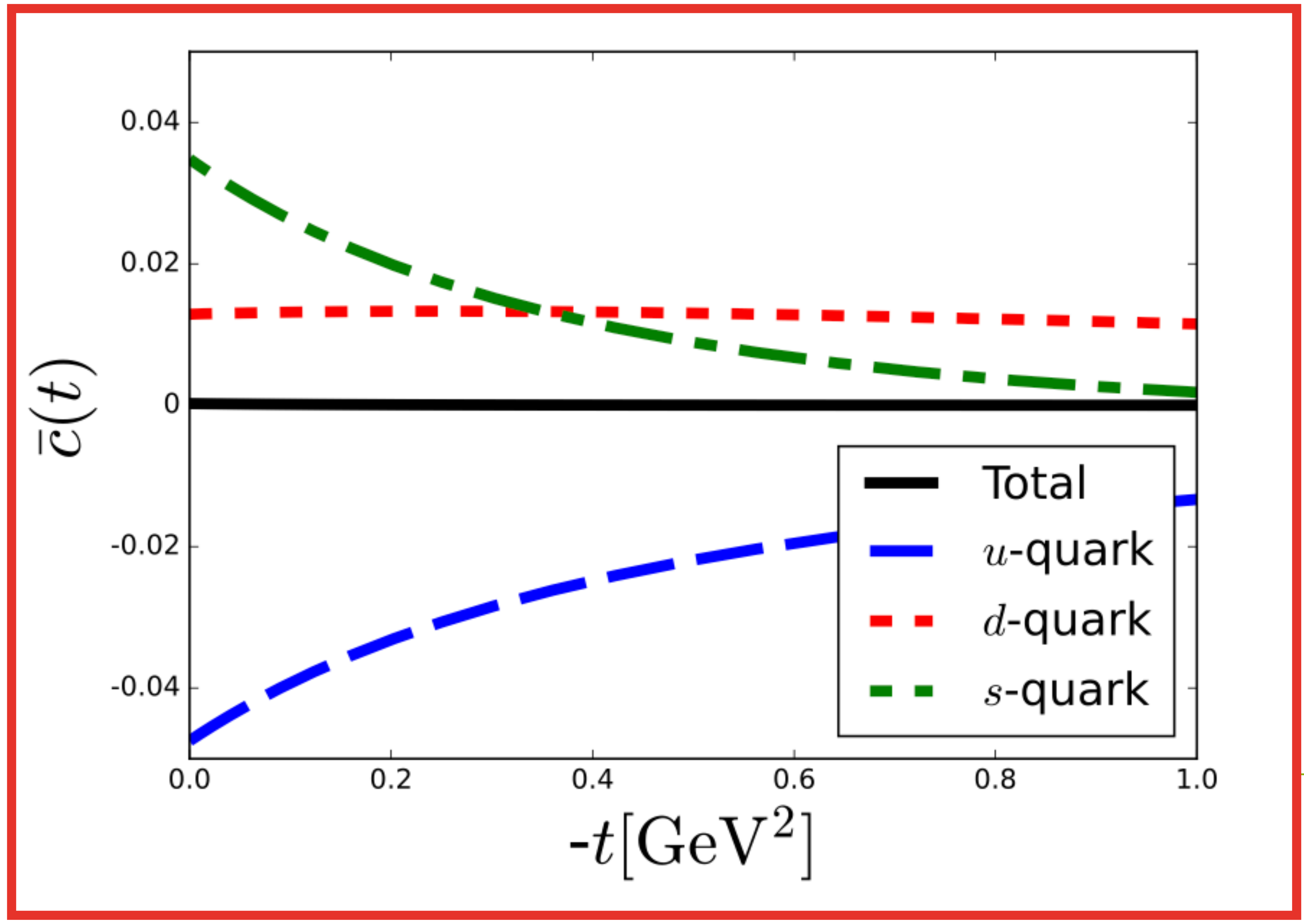
- **Nontrivial cancellation between the $\bar{c}^u(t)$ and $\bar{c}^d(t)$+$\bar{c}^s(t)$ is observed.**

  **(vs. in $\pi^+$, $\bar{c}^u(t) = \bar{c}^{\bar{d}}(t) = 0$, due to isospin symmetry)**

- **Contribution to the pressure distribution $\bar{c}^q$ is sizable**



# $\bar{c}$ from instantons and force btw quark and gluon subsystems

**QCD equation of motion & Effective operator from instantons** [Polyakov, HDS, JHEP156 (2018)]

$\rightarrow \bar{c}^q(0)$ is suppressed by small instanton packing fraction ~ $\mathcal{O}(\bar{\rho}^4/\bar{R}^4)$

$\rightarrow$ Estimation of the form factor at zero momentum transfer

$$: \bar{c}^Q(0) \equiv \sum_q \bar{c}^q(0) \approx 1.4 \times 10^{-2} \, (\mu = 1/\bar{\rho} = 600\text{MeV})$$

$\rightarrow \bar{c}^Q(t)$ contribution to the pressure inside the nucleon ~ up to 20% that of D(t)

Other studies $\bar{c}^q(0) =$

-1/4(MIT) bag model [Ji, Melnitchouk, Song, PRD56 (1997)]

-0.14 ($\mu = \infty$) pQCD [Hatta JHEP12 (2018)]

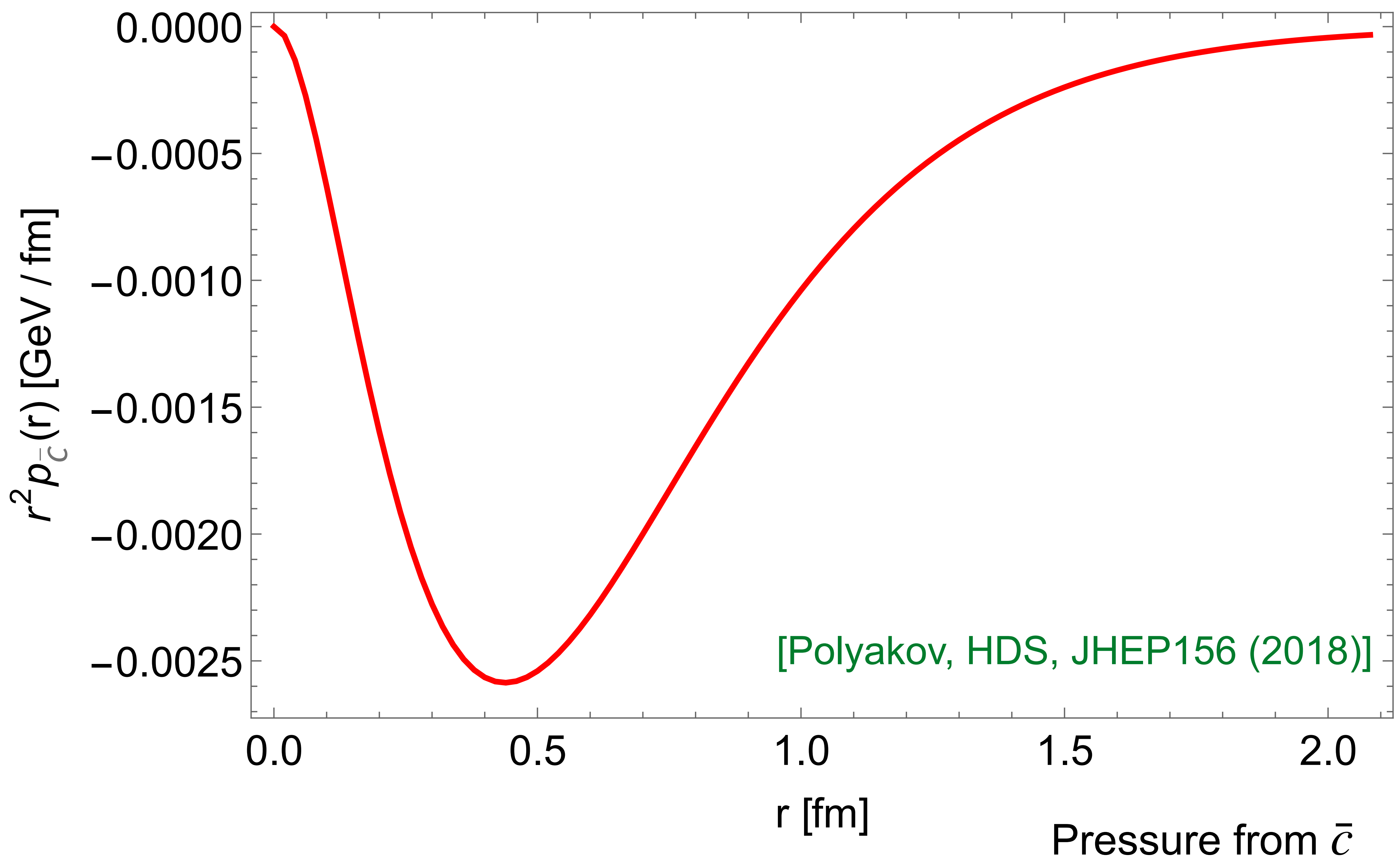
-0.124(6)($\mu = 2$ GeV) [Liu, PRD104 (2021)]

-0.18(3)($\mu = 1$ GeV) NNLO pQCD [Tanaka, JHEP03 (2023)]]

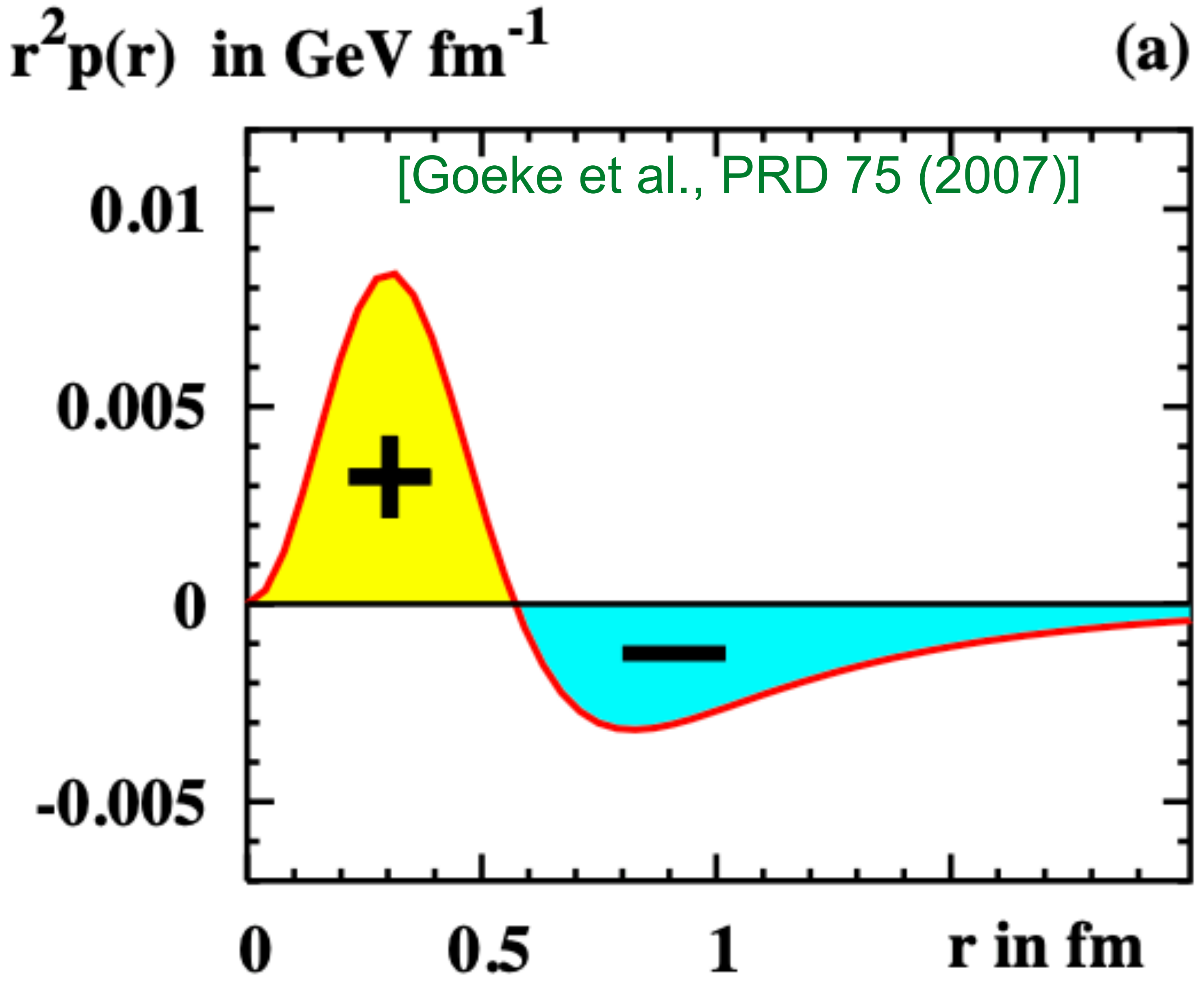
Renormalization scheme dependence [Metz, Pasquini, Rodini, PRD102 (2021)]

# Contribution of $\bar{c}$ to the pressure

$$p^a(r) = \frac{1}{6M_N}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\int\frac{d^3\Delta}{(2\pi)^3}\,e^{-i\vec{\Delta}\cdot\vec{r}}\,D^a(t) - M_N\int\frac{d^3\Delta}{(2\pi)^3}\,e^{-i\vec{\Delta}\cdot\vec{r}}\,\bar{c}^a(t)$$




Pressure from $\bar{c}$




Pressure from D

Smaller in size compared to the contribution of the D-term (right figure)

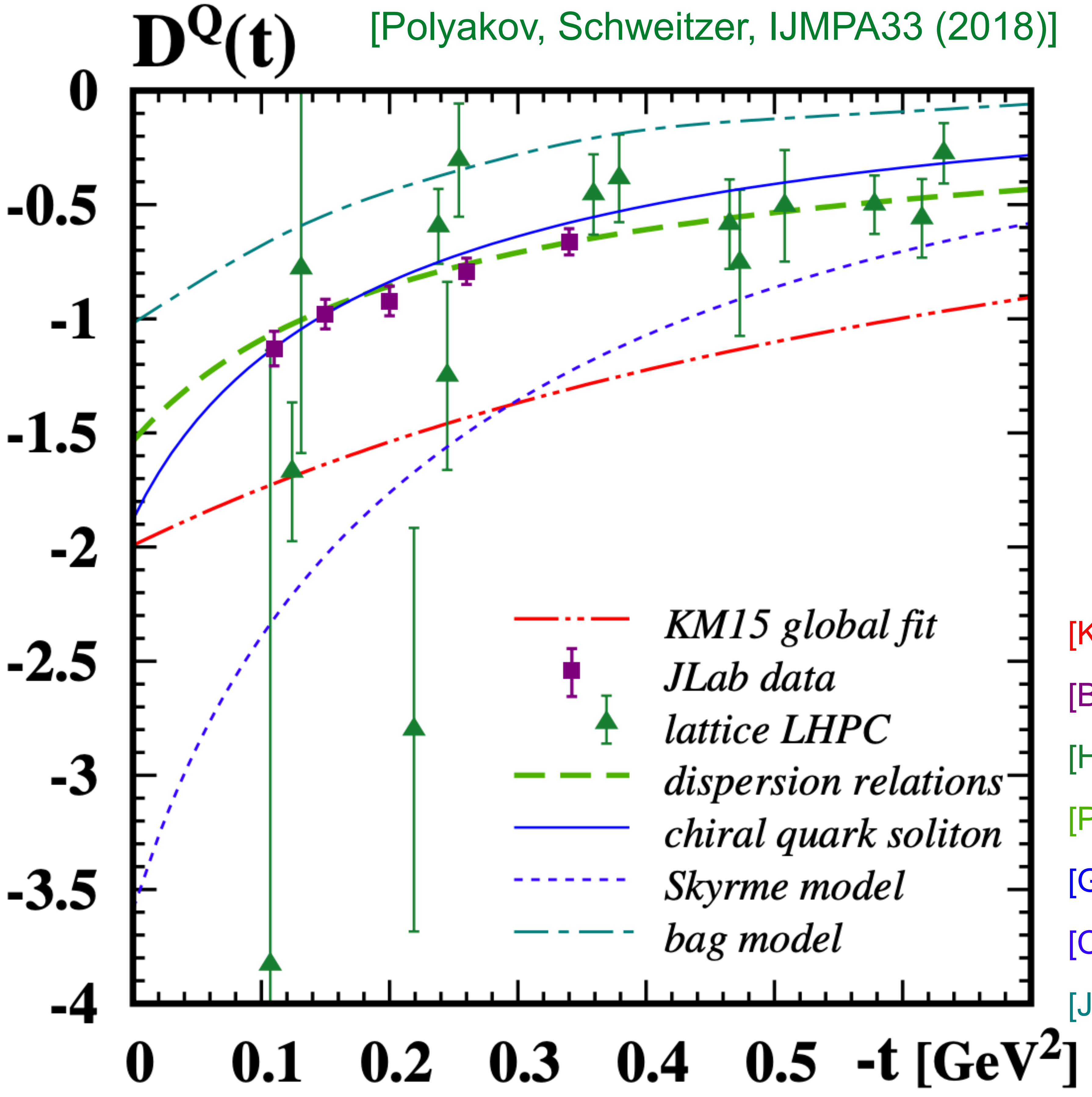
If large, contribution of $\bar{c}$ > D-term to the pressure distribution.

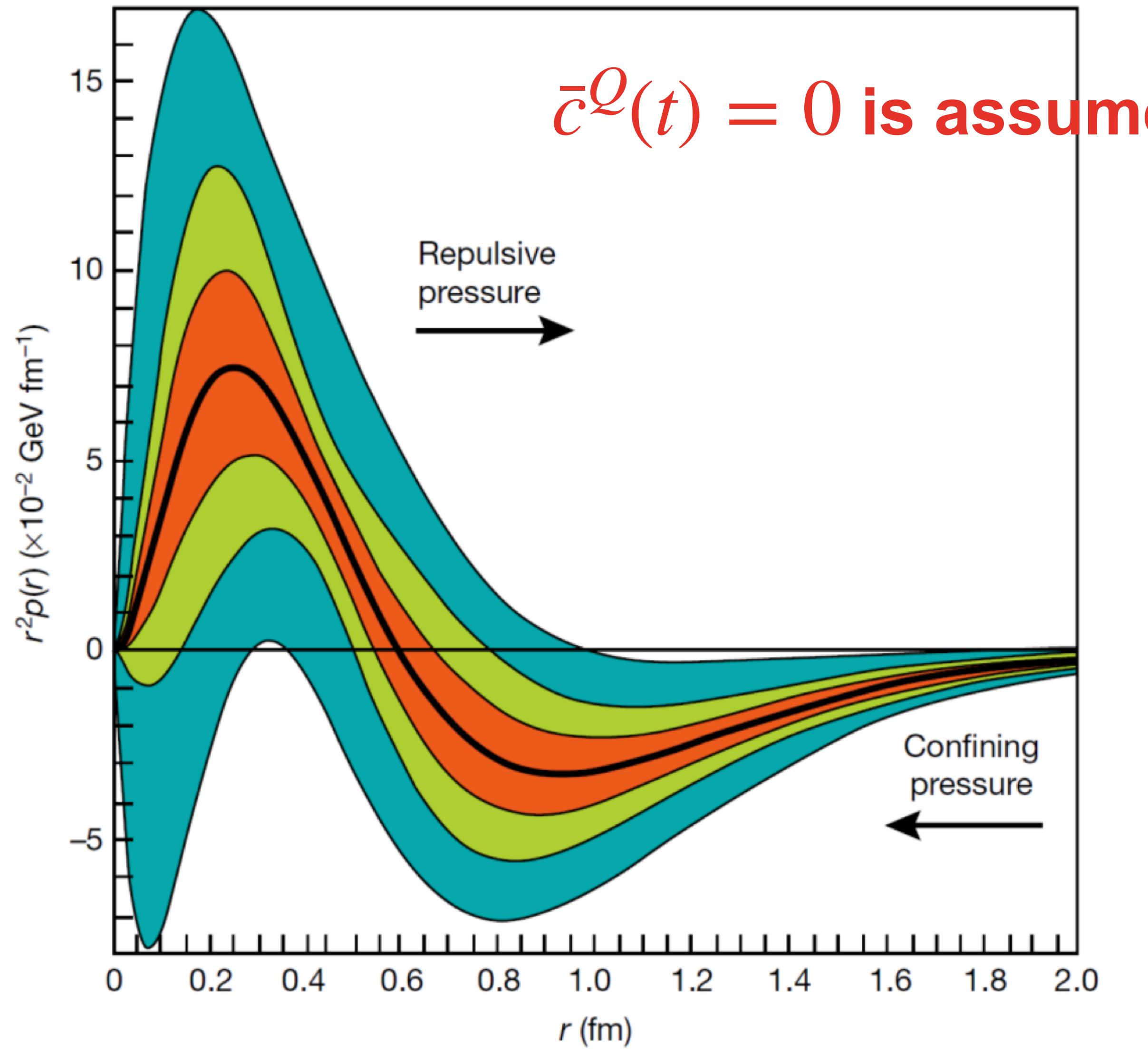
Constituent quark picture at the low energy?

Smallness assumed by phenomenological studies, eg. [K. Kumericki, Nature 570 (2019)] [Burkert,Elouadrhiri,Girod, Nature557(2018)]

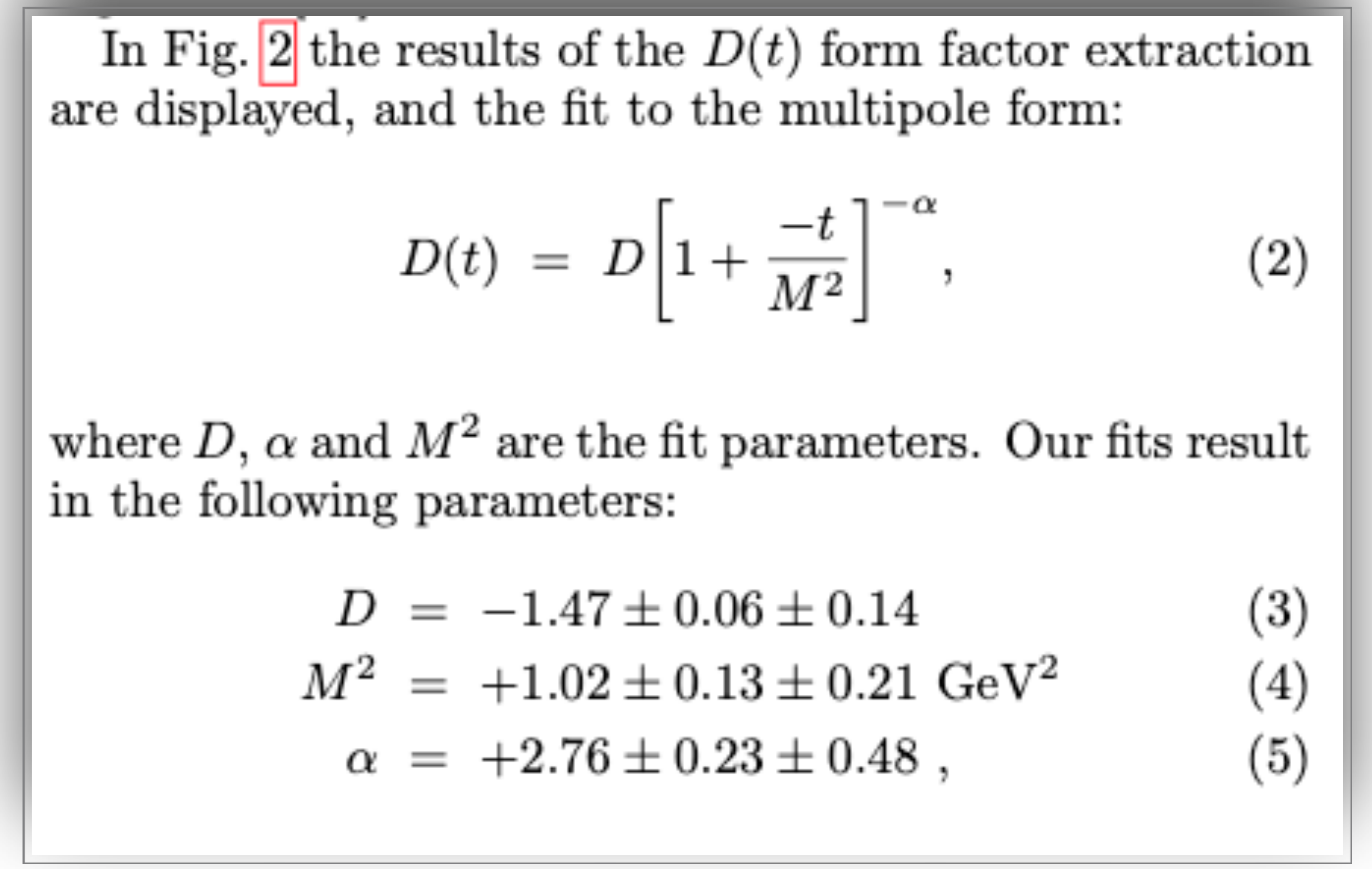
# Pressure distribution of the proton?

[Polyakov, Schweitzer, IJMPA33 (2018)]




$\bar{c}^Q(t) = 0$ **is assumed**



In Fig. 2 the results of the $D(t)$ form factor extraction are displayed, and the fit to the multipole form:

$$D(t) = D\left[1 + \frac{-t}{M^2}\right]^{-\alpha}, \qquad (2)$$

where $D$, $\alpha$ and $M^2$ are the fit parameters. Our fits result in the following parameters:

$$D = -1.47 \pm 0.06 \pm 0.14 \qquad (3)$$
$$M^2 = +1.02 \pm 0.13 \pm 0.21 \text{ GeV}^2 \qquad (4)$$
$$\alpha = +2.76 \pm 0.23 \pm 0.48\ , \qquad (5)$$

[Burkert,Elouadrhiri,Girod, Nature 557(2018)]

[Kumerički, Müller, EPJ.Web.Conf.112 (2016)]

[Burkert,Elouadrhiri,Girod, Nature557(2018)]

[Hägler et al. PRD77 (2008)]

[Pasquini, Polyakov, Vanderhaeghen, PLB739 (2014)]

[Goeke et al. PRD75 (2007)]

[Cebullar et al, NPA 794 (2007)]

[Ji,Melnitchouk,Song PRD56(1997)]

**Some remarks on the systematics of this analysis**

[K. Kumericki, Nature 570 (2019)]

[Dutrieux, Lorce, et al. (2021)]