Magnetic helicity, monopoles, and baryon asymmetry

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arXiv:2507.01576, 2509.23858 w/ K. Mukaida (KEK, SOKENDAI), F. Uchida (IBS)
arXiv:2509.25734 + H. Fukuda (Tokyo), K. Kamada (UCAS)

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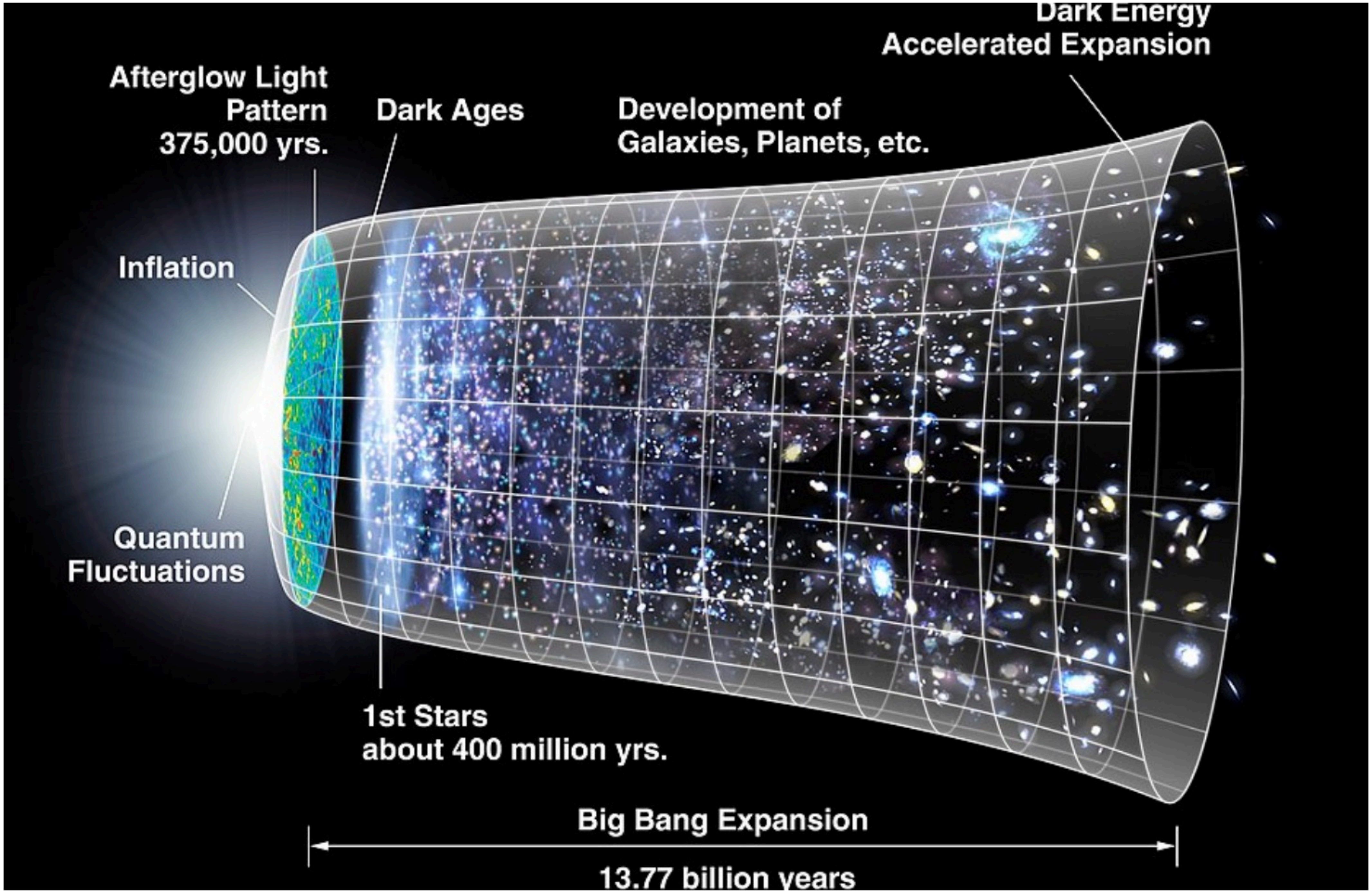

This talk: Cosmological Magnetic Field.

Interplay to baryon asymmetry of universe.

Motivation: hint for InterGalactic Magnetic Fields (IGMF).

Helicity

The cosmological magnetic field is characterized by

- Energy
- Helicity
- Correlation length

I focus on the helicity $\mathscr{H}$ in this talk:

$$\mathscr{H} = \int d^3x \, \overrightarrow{A} \cdot \overrightarrow{B}.$$

Time Evolution of $\mathscr{H}$

$$\mathscr{H} = \int d^3x\, \overrightarrow{A} \cdot \overrightarrow{B}.$$

Taking time derivative, we get

$$\frac{\partial \mathscr{H}}{\partial t} = \int d^3x\, \overrightarrow{E} \cdot \overrightarrow{B} \qquad \rightarrow \qquad \Delta \mathscr{H} = \int F \wedge F \sim \int d^4x\, \overrightarrow{E} \cdot \overrightarrow{B}.$$

However, this is modified in the presence of magnetic monopoles.

This has implications on baryogenesis scenario from magnetic helicity.

Talk Plan

1. $U(1)$ gauge theory
2. Georgi-Glashow
3. Implications on baryogenesis

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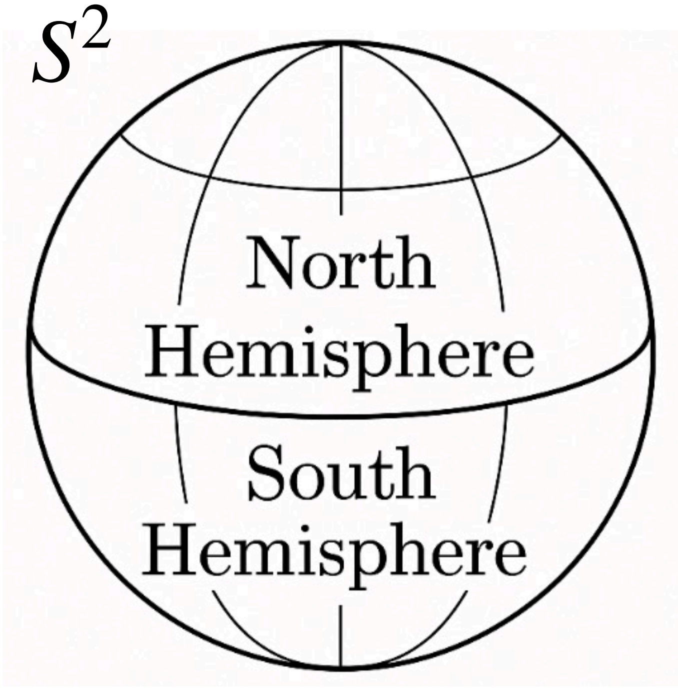
Monopole

Monopole of $U(1)$ gauge field A,

characterized by configuration of A around monopole.

$\mathbb{R}^{1,3} = \mathbb{R}^{1,1} \times \mathbb{R}_{\geq 0} \times S^2$

Equator S^1



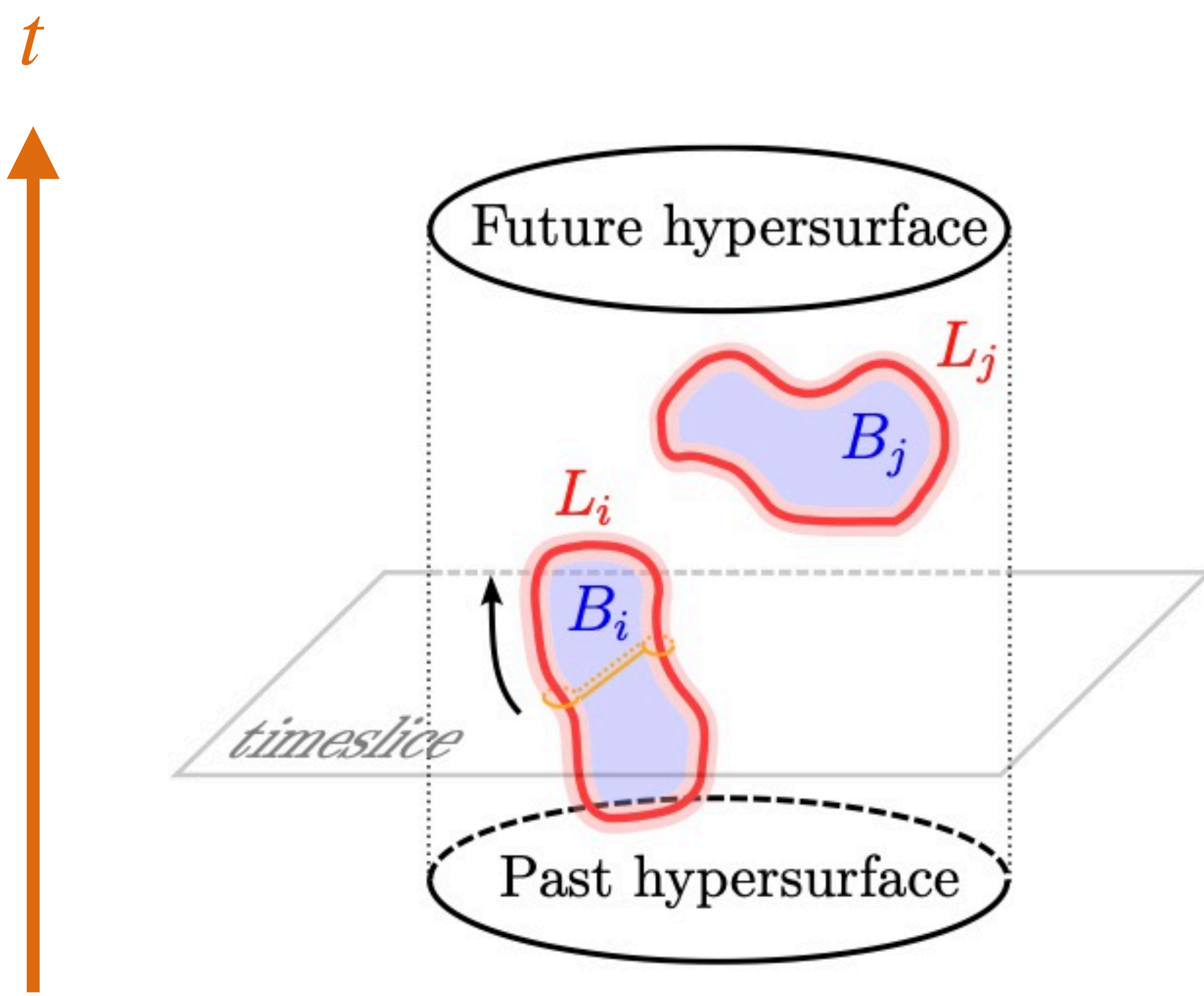
North patch A_N

South patch A_S

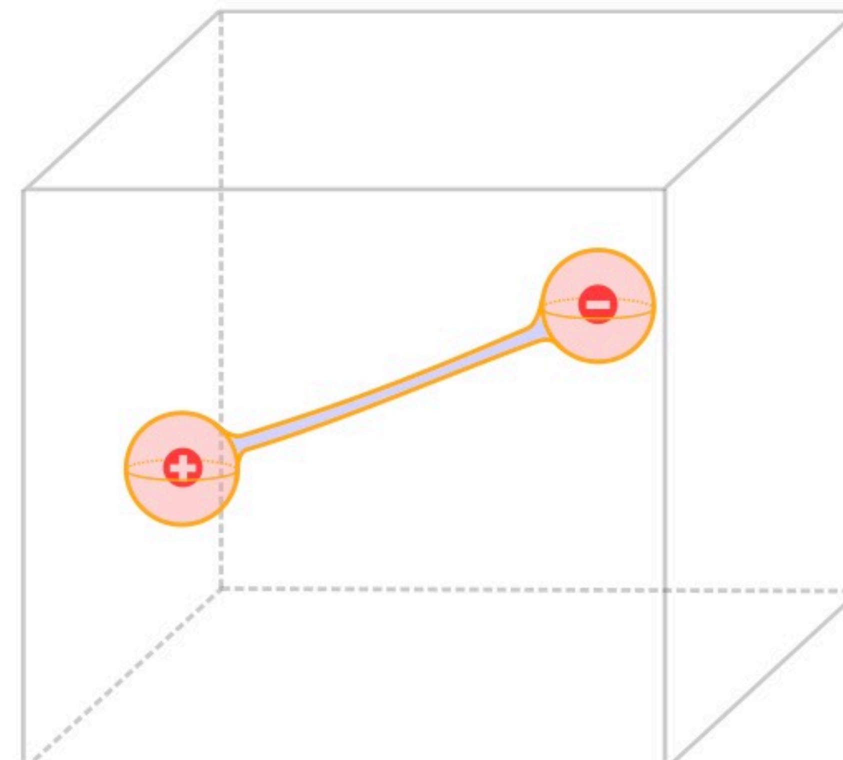
A_N and A_S are related by gauge tr. $A_N - A_S = d\Lambda$.

$\pi_1(U(1)) = \mathbb{Z}$.

In the presence of monopoles, A is not defined globally.



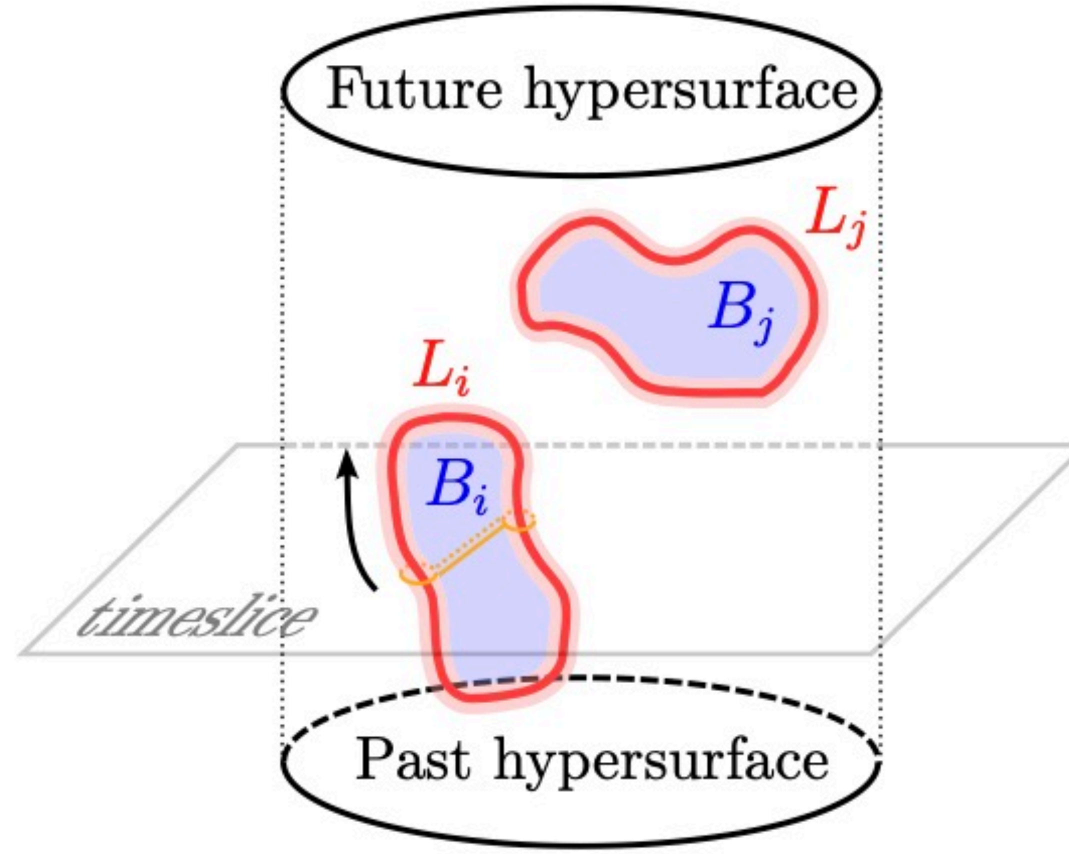
t = const. slice



L_i: world line of monopole.

A is well defined in spacetime without L_i and B_i.

M'': Spacetime without L_i and B_i.



$$\int_{M''} F \wedge F = \mathscr{H}_{\text{future}} - \mathscr{H}_{\text{past}} + \underline{\sum_i \int_{S_i} A \wedge F}$$

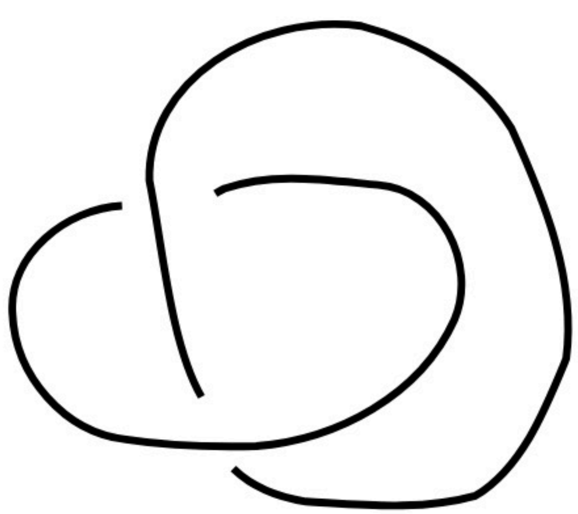
Contribution from additional bry.

[Fukuda, YH, Kamada, Mukaida, Uchida '25]

Example: Helicity Change in Higgs Phase

Let us assume that $U(1)$ is Higgsed, and flux tube is formed.

Configuration has helicity $\mathscr{H}_{\text{past}} = 2\left(\frac{2\pi}{e}\right)^2$



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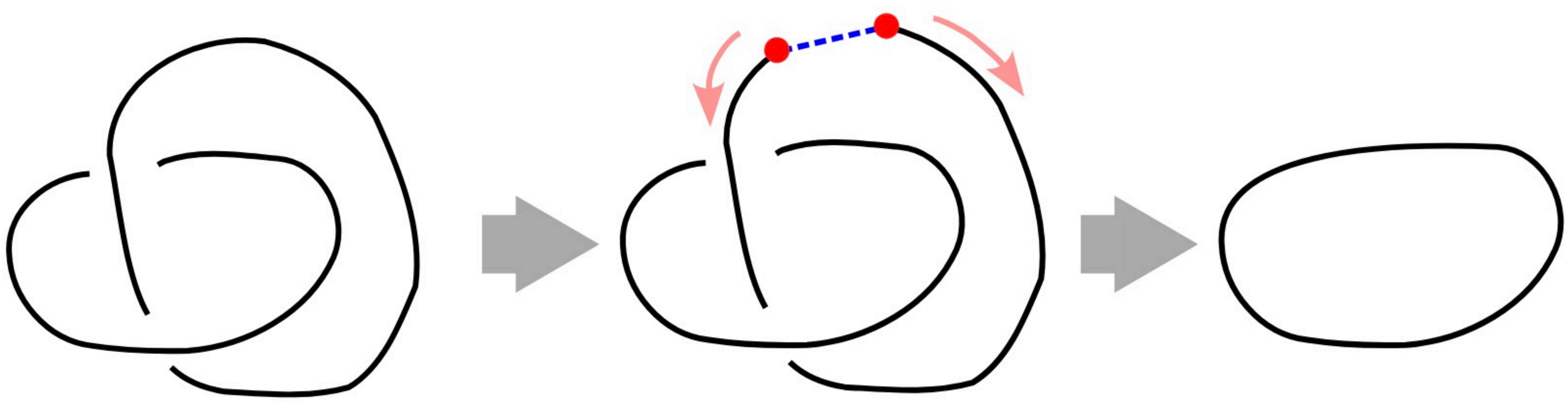

Tube may disappear by monopole pair creation.

Example: Helicity Change in Higgs Phase

During this process, $F \wedge F = 0$ as there are no electric field in Higgs phase.

On the other hand, $\int_{S_i} A \wedge F = -\frac{4\pi}{e}$.

$$\mathcal{H}_{\text{future}} = \mathcal{H}_{\text{past}} + \int_{M'} F \wedge F + \frac{4\pi}{e} \int_{\Sigma} F$$

$$= 2\left(\frac{2\pi}{e}\right)^2 - 0 - \frac{4\pi}{e} \cdot \frac{2\pi}{e} = 0.$$

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Georgi-Glashow

$SU(2)$ gauge theory with adjoint Higgs field, Φ.

Gauge symmetry breaking:

$SU(2) \to U(1)$ by $\langle \Phi \rangle \neq 0$.

The low energy theory is $U(1)$ gauge theory
with ’t Hooft Polyakov monopole.

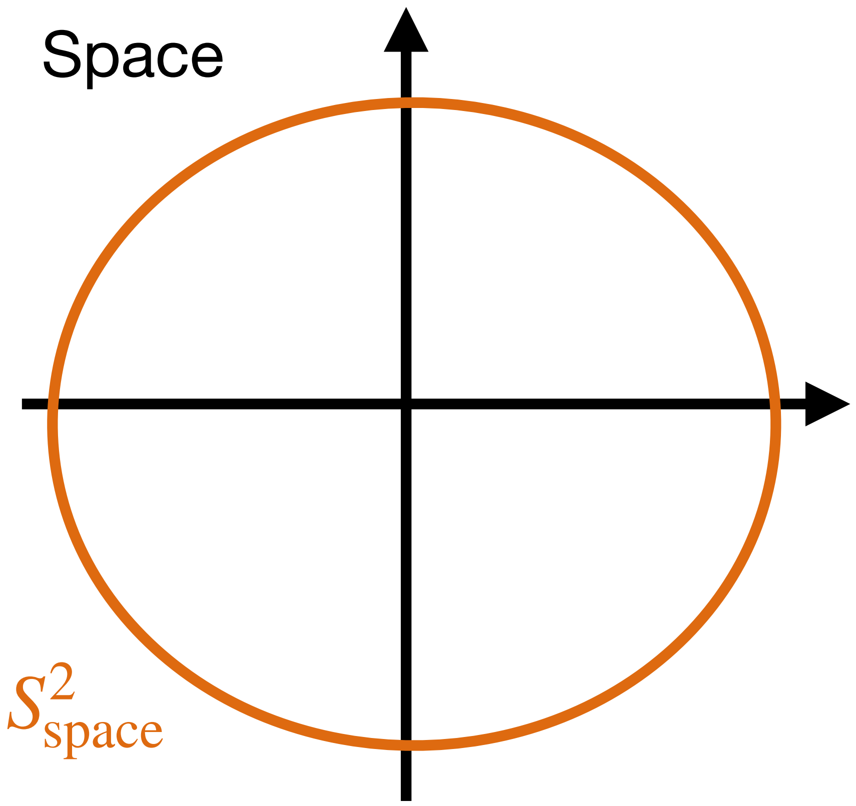
Viewed as a UV completion of $U(1)$ gauge theory.

’t Hooft Polyakov monopole

Higgs potential is $V = \lambda \left(\Phi^2 - v^2\right)^2$.

The minimum is at $\Phi^2 = \left(\Phi_1^2 + \Phi_2^2 + \Phi_3^2\right)^2 = v^2$.

Labelled by the map: $S^2_{\text{space}} \rightarrow S^2_{\text{min}}$



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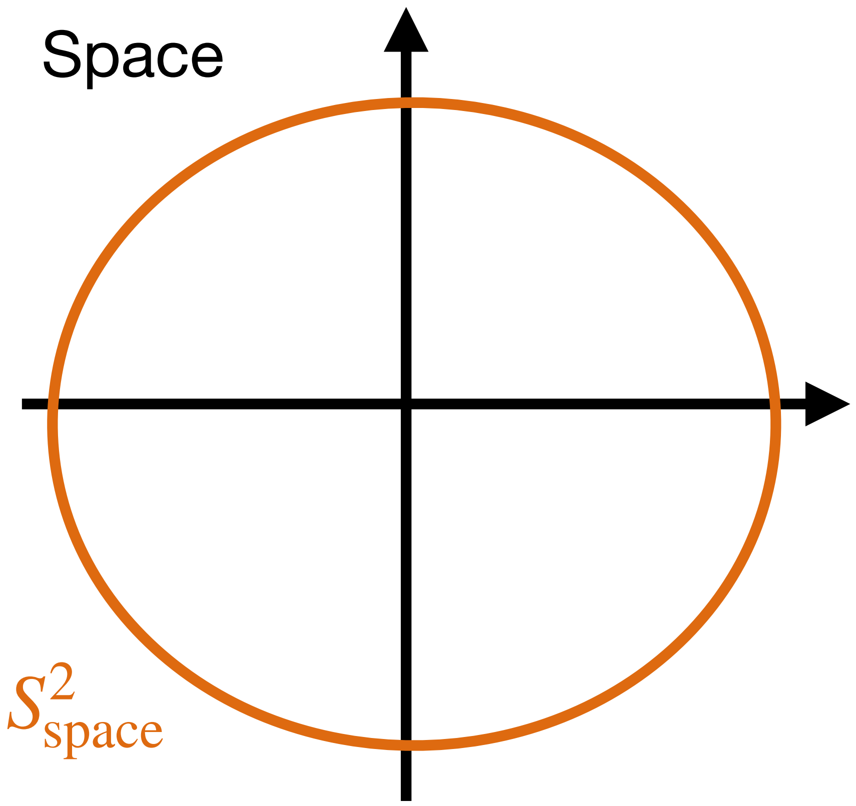

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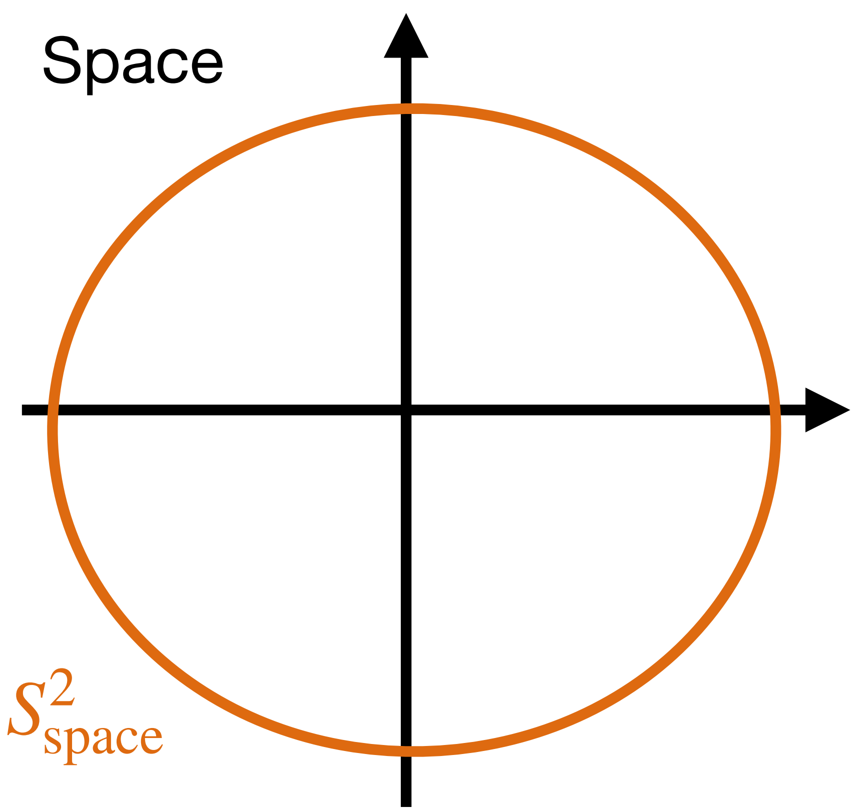

Potential minimum is S^2_{min}

Topological class of map $S^2_{\text{space}} \to S^2_{\text{min}}$

is classified by integers (magnetic charge).

’t Hooft tensor

How can be define low energy $U(1)$ field strength?

Naively, $A_\mu^{U(1)} = A_\mu^a n^a$, $\qquad F_{\mu\nu}^{U(1)} = \partial_\mu A_\nu^{U(1)} - \partial_\nu A_\mu^{U(1)}$,

where $n^a = \Phi^a / \sqrt{|\Phi|^2}$.

However, $F_{\mu\nu}^{U(1)}$ is NOT gauge invariant.

The invariant definition is known as ’t Hooft tensor:

$$\mathcal{F}_{\mu\nu}^{U(1)} = \partial_\mu (n^a A_\nu^a) - \partial_\nu (n^a A_\mu^a) - \frac{1}{g} \epsilon^{abc} n^a \partial_\mu n^b \partial_\nu n^c.$$

This satisfies Bianchi identity: $\epsilon^{\mu\nu\rho\sigma} \partial_\rho \mathcal{F}_{\mu\nu}^{U(1)} = 0$.

Definition of helicity

Using ’t Hooft tensor, we can define

$$\mathscr{H}_{U(1)} = \int \mathscr{A} \wedge \mathscr{F}.$$

In low energy (massless excitation only), we find

$$\mathscr{H}_{U(1)} = \frac{16\pi^2}{g^2}\left(N_{\rm CS} + N_H\right)$$

$N_{\rm CS}$: $SU(2)$ Chern Simons Number.

N_H: Higgs winding. Define U by $UnU^\dagger = T^3$,

and then $N_H = \frac{1}{24\pi^2}\int \mathrm{Tr}(UdU^\dagger \wedge UdU^\dagger \wedge UdU^\dagger)$.

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Baryon Asymmetry

The conversion of magnetic field is from chiral to non-chiral.

High T: $\mathrm{U}(1)_Y$ is massless. Low T: $\mathrm{U}(1)_{\mathrm{em}}$ is massless.

SSB of $\mathrm{U}(1)_{\mathrm{M}}^{[0]} \to$ NG mode in 3d.

ABJ anomaly tells us $\Delta Q_{B+L} = 6\left(\Delta N_{\mathrm{CS}}^{\mathrm{SU(2)_L}} - \Delta H_Y\right)$.

Primordial magnetic field leads to baryon asymmetry [Kamada, Long '16].

non-zero for chiral magnetic field

zero for non-chiral magnetic field

Toy Model

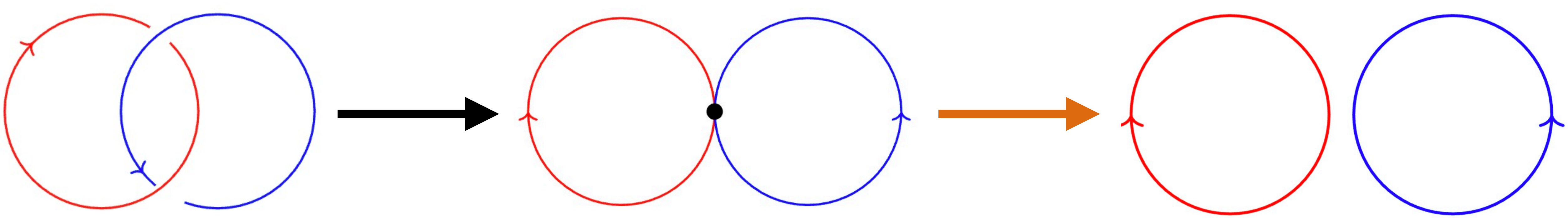
$U(1)$ gauge theory with massless fermion and Higgs boson.

$\Delta Q_{\text{chi}} = 2\Delta N_{\text{CS}}$.

SSB: $U(1) \overset{\text{Higgs VEV}}{\rightarrow}$ Nothing.

Suppose there is $U(1)$ magnetic field before SSB.

After SSB, we have



Loops of
Massive magnetic fields.

Chirality is generated
through $\Delta N_{\text{CS}} \neq 0$.

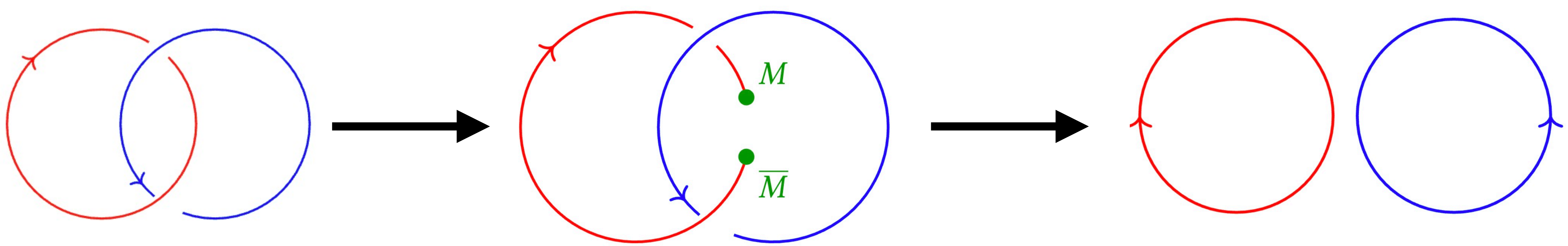
[YH, Mukaida, Uchida '25]

Subtlety

Helicity: $\mathscr{H}_{U(1)} = \frac{16\pi^2}{g^2}\left(N_{\rm CS} + N_H\right),$

Asymmetry generation: $\Delta Q_{\rm chi} = 2\Delta N_{\rm CS}$

Monopole production



No asymmetry.

Helicity can relax from nonzero to zero w/o changing $N_{\rm CS}$.

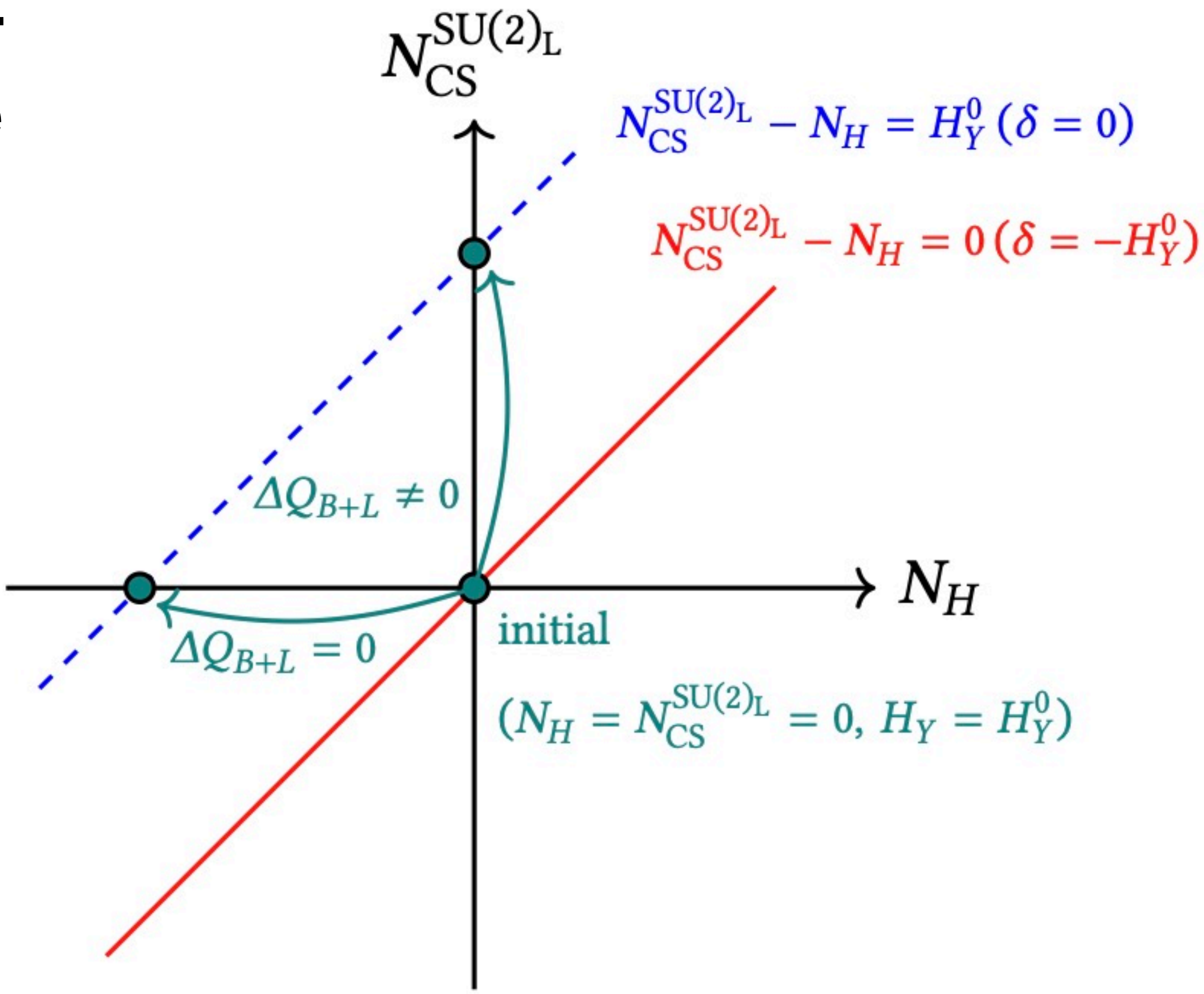
[YH, Mukaida, Uchida '25]

Standard Model case

Similar argument applies to SM.
Two ways to disappear massive magnetic field.

(1) $N_{\rm CS}^{\rm SU(2)_L}$ changes.
Baryon is generated.

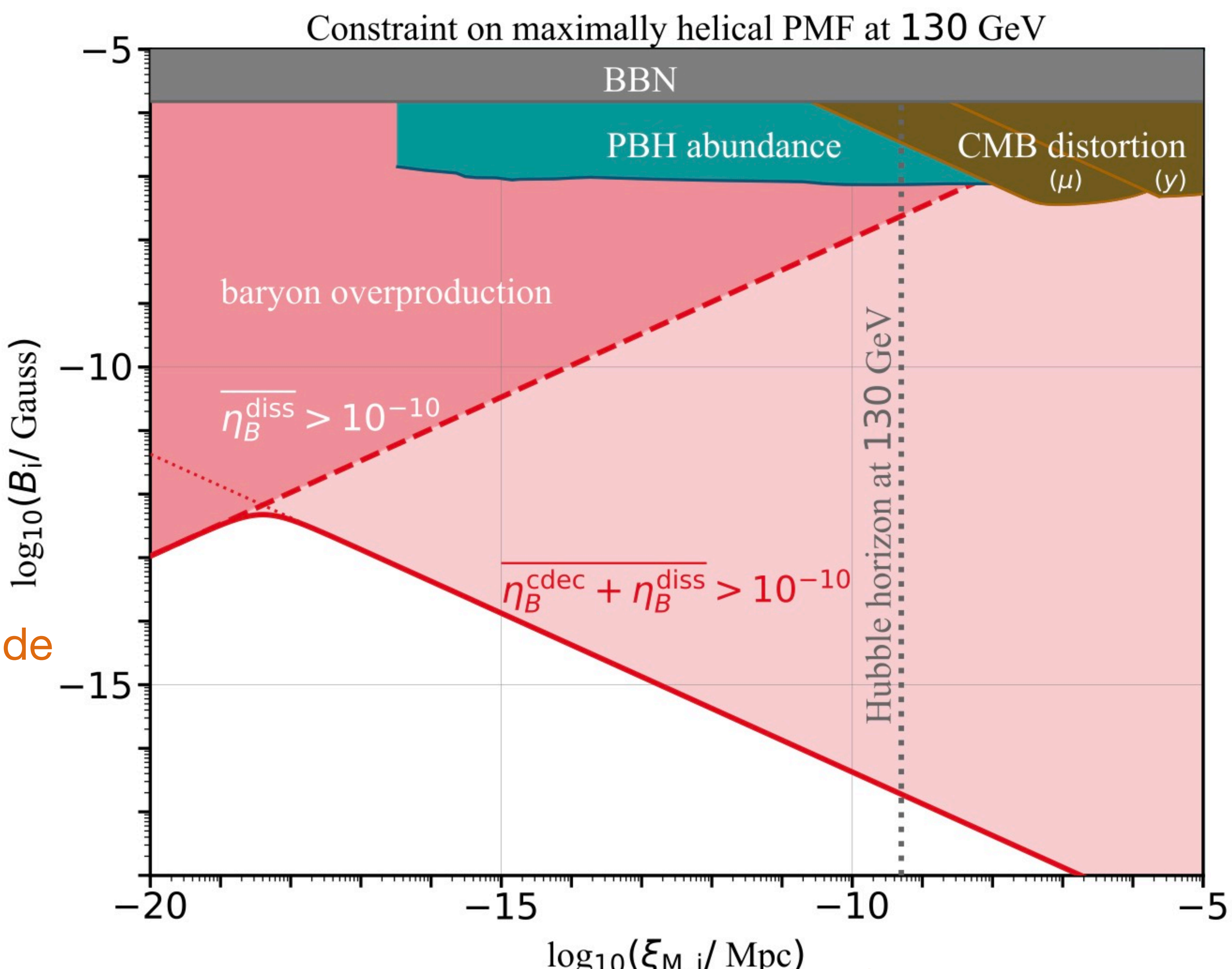
(2) N_H changes.
Baryon is NOT generated.



[YH, Mukaida, Uchida '25]

Light colored region: Excluded if N_H change is inefficient.

Dark colored region: Excluded if N_H change is efficient.



Amplitude

Correlation length

Colored regions are excluded.

[YH, Mukaida, Uchida '25]

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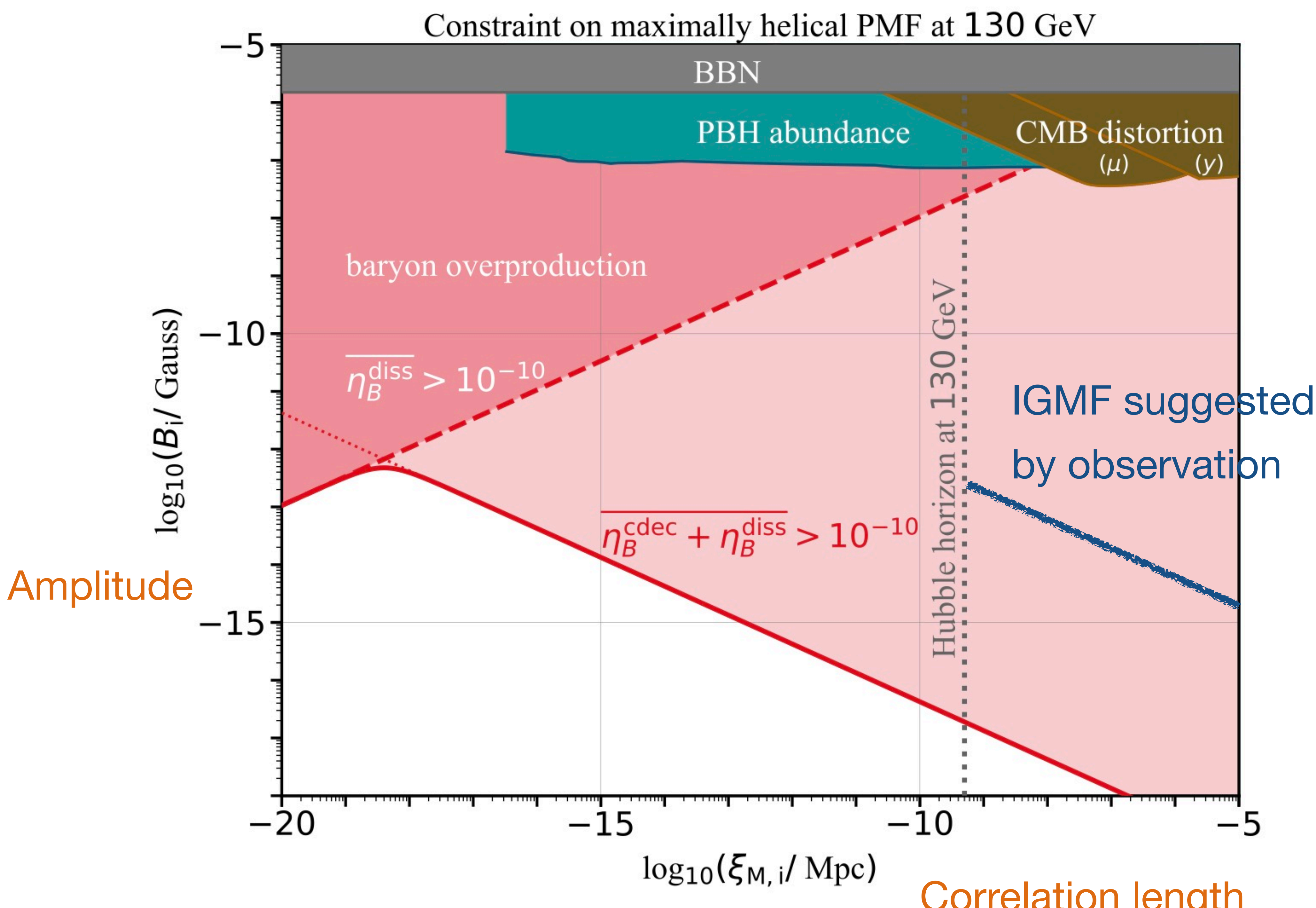

Colored regions are excluded.

If N_H change is inefficient, IGMF is inconsistent with baryon asymmetry.

If N_H change is efficient, IGMF is consistent with baryon asymmetry.

More detailed study is necessary to settle the situation.

Summary

- Hint of cosmological magnetic field.
- Helicity change in the presence of monopoles.
- Implications for baryon asymmetry.