Multi-step Electroweak Phase Transitions and Gravitational Waves in the Inverted Type-I 2HDM

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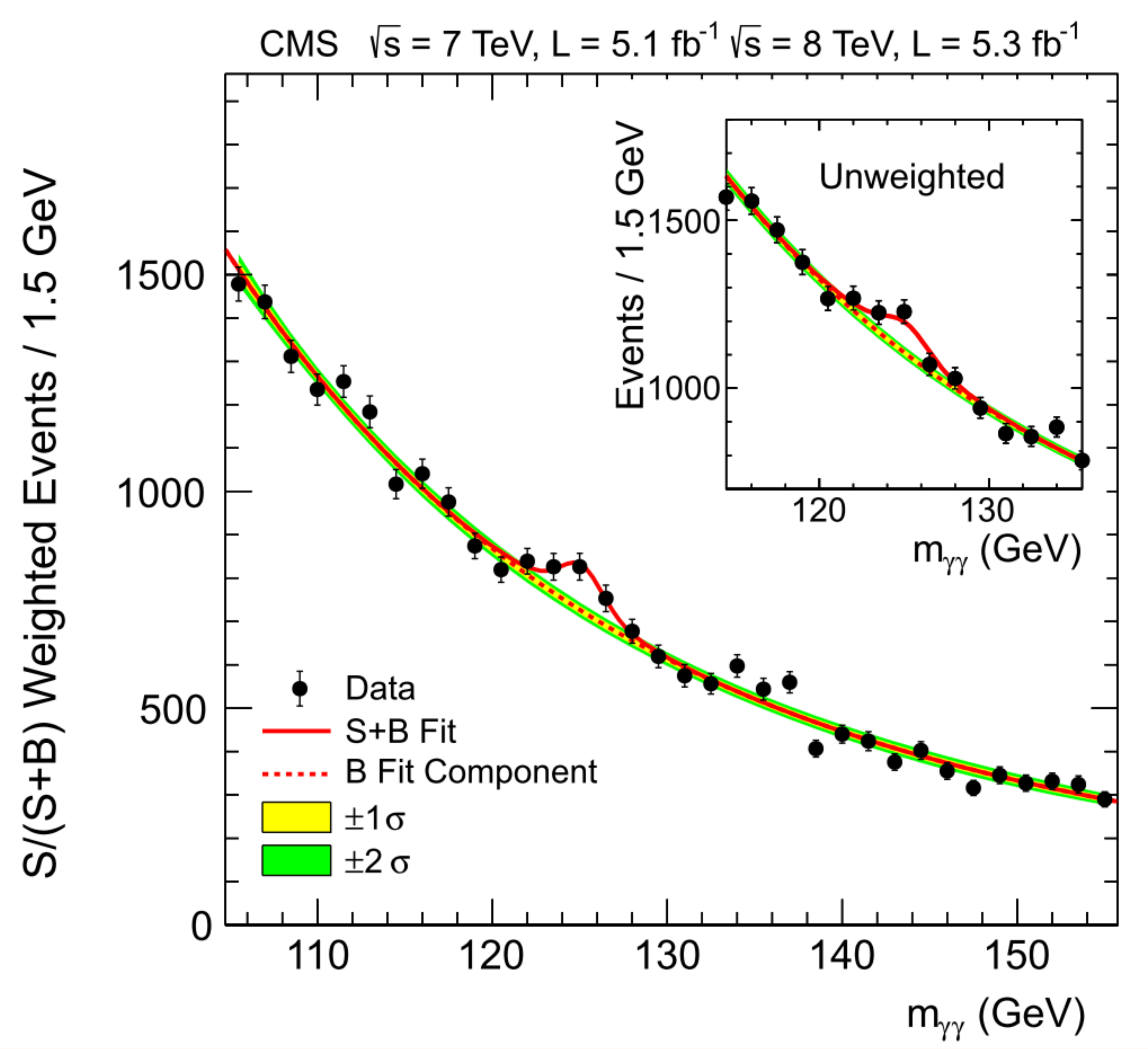
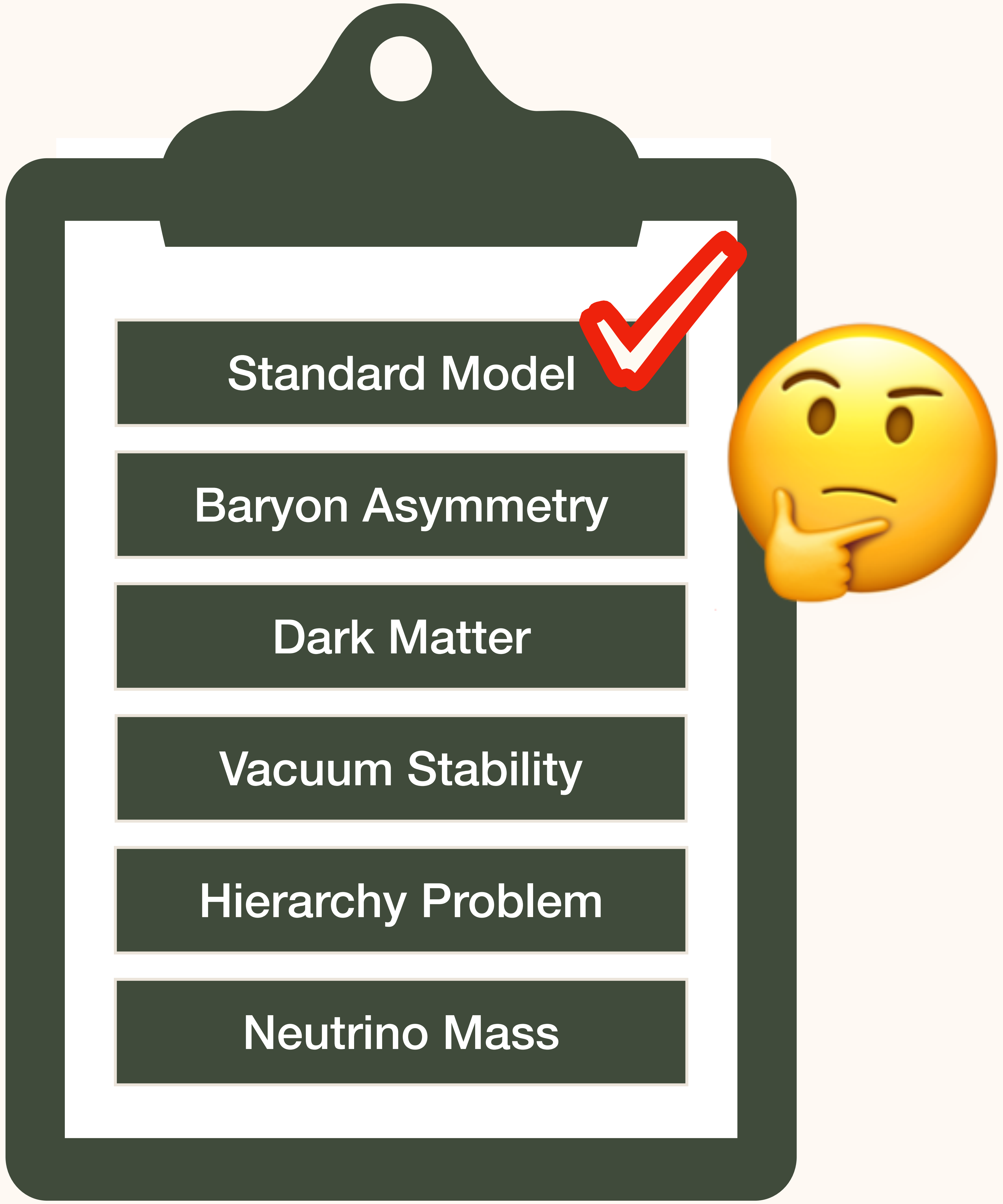
1. Introduction

The Standard Model: Success and Open Questions



CMS Collaboration / Physics Letters B 716 (2012) 30–61 35



Baryon Asymmetry of the Universe

Baryon-to-Photon Ratio (η_B): $\eta_B \equiv \dfrac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6 \times 10^{-10}$

SM Prediction: $\eta_B^{\mathrm{SM}} \sim 10^{-20}$

Sakharov Conditions (1967) for Baryogenesis:

1. Baryon Number (B) Violation
2. C and CP Violation
3. Departure from Thermal Equilibrium

Why “Strong First-Order”?

Sakharov Condition #3. Departure from Thermal Equilibrium

$\Rightarrow$ **First-Order Phase Transition (FOPT)**

: Discontinuous transition $\rightarrow$ non-equilibrium process

Bubble Dynamics

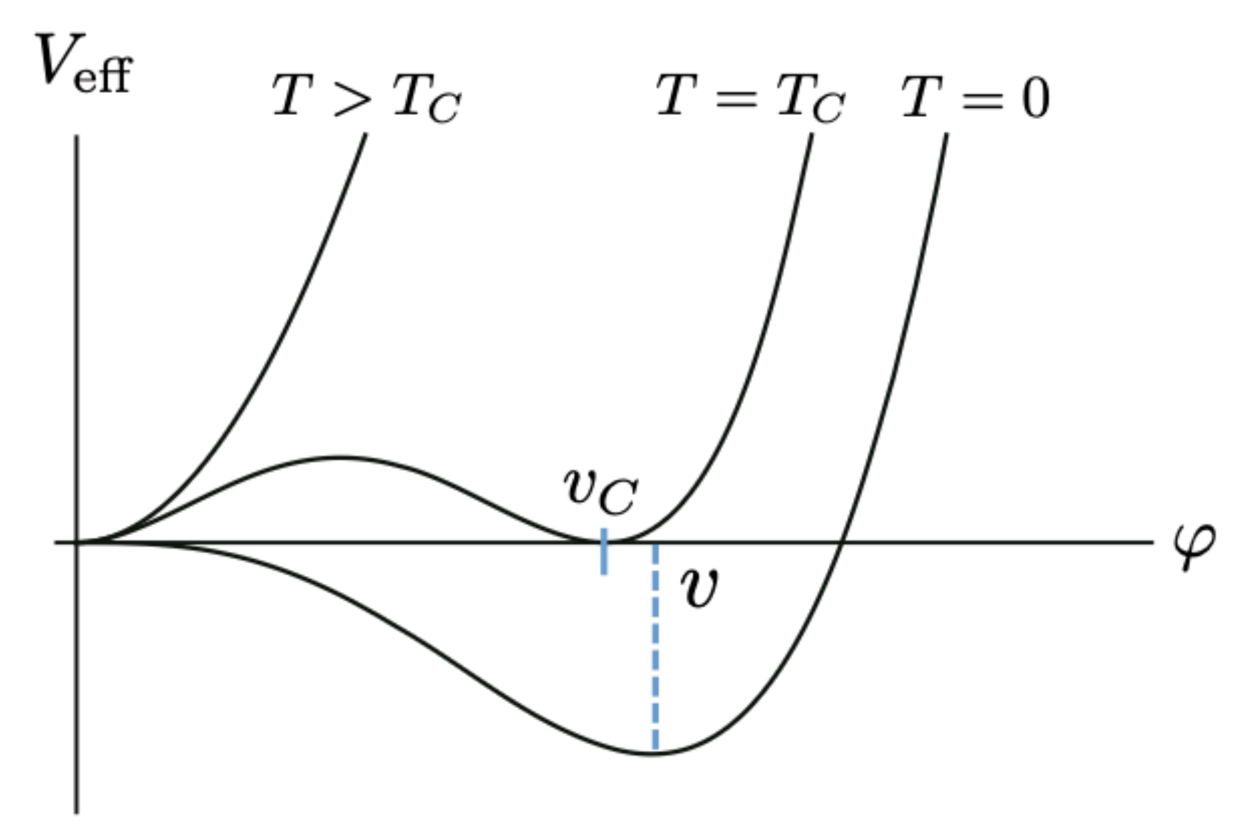
Broken phase bubbles nucleate and expand within Symmetric phase

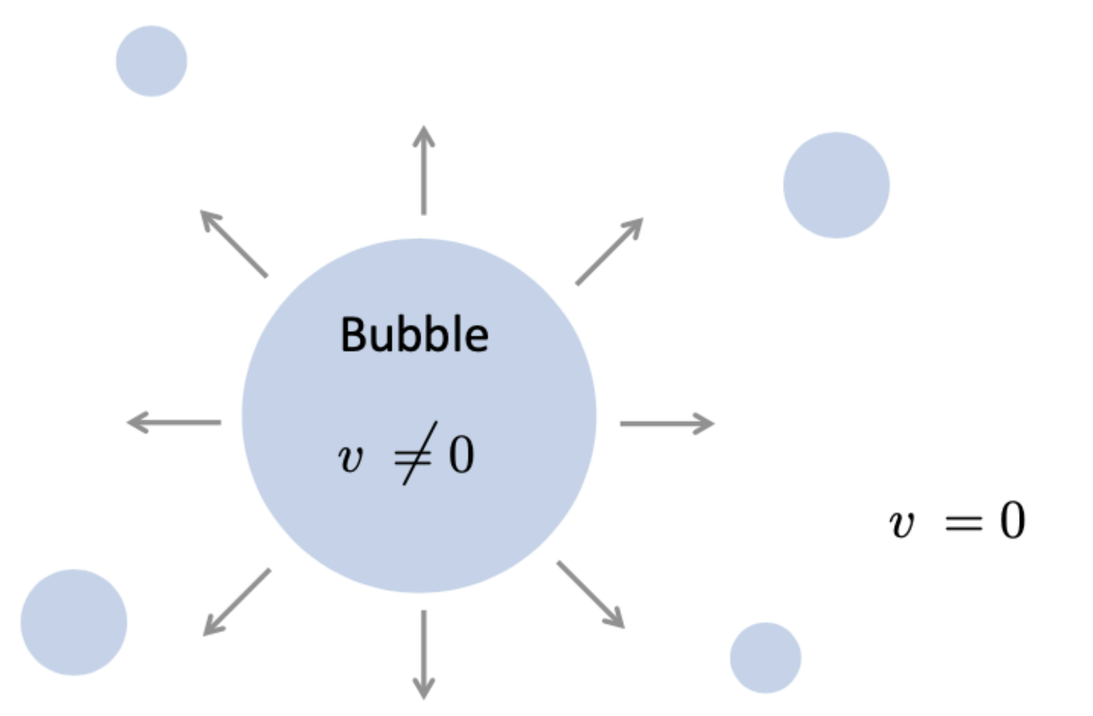
Requirement for "Strong" Transition

Suppression of Sphaleron washout in the broken phase

: $\Gamma_{\text{sph}} < H(T) \;\Rightarrow\; \dfrac{v_c}{T_c} \gtrsim 1$

Fuyuto, Kaori, ”Electroweak Baryogenesis”



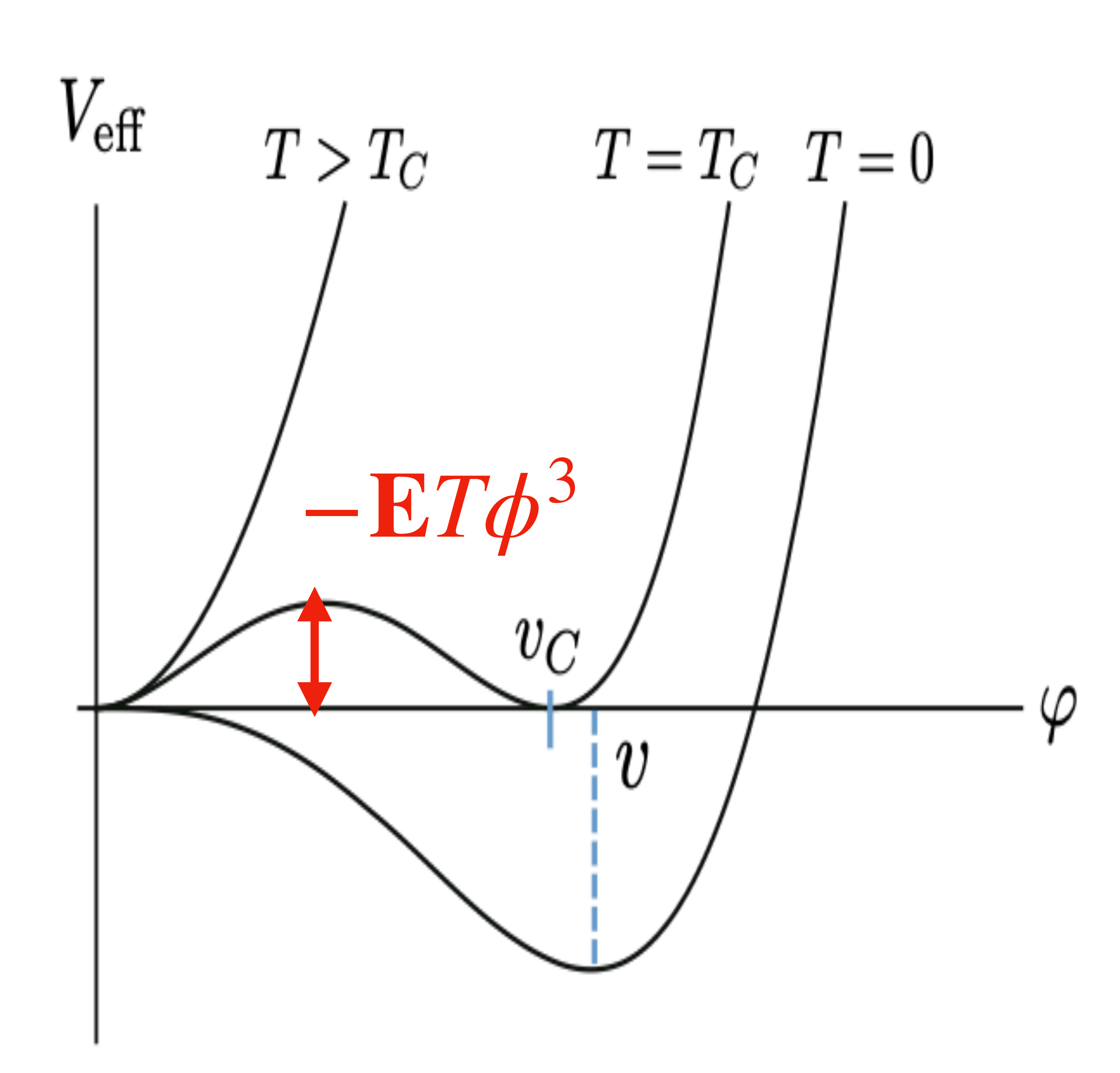


Why an "extended Higgs sector"?

Standard Model: $m_H = 125$ GeV Higgs results in a **Crossover** EWPT.
→ Consequence: Sphaleron Washout of Baryon Asymmetry

Enhancing the Cubic Term
a large negative Cubic term ($-\mathbf{E}T\phi^3$) in the thermal effective potential.
→ Deep potential barrier for a SFOPT requires

The BSM Solution
An **Extended Higgs Sector** → new scalar bosons
⇒ can significantly **enhance the crucial coefficient 'E'.**
⇒ One of the simplest: **Two-Higgs-Doublet Model (2HDM)**



$$V(\phi_h, T) \approx (DT^2 - \mu^2)\phi_h^2 - ET\phi_h^3 + \frac{\lambda}{4}\phi_h^4$$

Two-Higgs-Doublet Model

Branco et al. arXiv:1106.0034

CP-Conserving 2HDM with a softly broken Z_2 symmetry

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \dfrac{v_1 + \rho_1 + i\eta_1}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \dfrac{v_2 + \rho_2 + i\eta_2}{\sqrt{2}} \end{pmatrix}$$

- Z_2 symmetry: $\Phi_1 \to +\Phi_1, \quad \Phi_2 \to -\Phi_2$
- $\tan\beta \equiv v_2/v_1$
- $v_1^2 + v_2^2 = v^2 \simeq (246\ \text{GeV})^2$

$$\begin{aligned} V(\Phi_1, \Phi_2) = {} & m_{11}^2 (\Phi_1^\dagger \Phi_1) + m_{22}^2 (\Phi_2^\dagger \Phi_2) - m_{12}^2 \left[(\Phi_1^\dagger \Phi_2) + \text{h.c.} \right] \\ & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ & + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2} \left[(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right] \end{aligned}$$

Normal vs. Inverted Scenario

Branco et al. arXiv:1106.0034

CP-Conserving 2HDM with a softly broken Z_2 symmetry

CP-even mixing:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

CP-odd mixing:

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

Charged mixing:

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

Five physical Higgs bosons:

h	lighter CP-even Higgs
H	heavier CP-even Higgs
A	CP-odd Higgs
$H^\pm$	Charged Higgs bosons

Normal Scenario:

$h \simeq 125$ **GeV**

→ h : **SM-like Higgs**

Vs.

Inverted Scenario:

$H \simeq 125$ **GeV**

→ H : **SM-like Higgs**

Why Inverted 2HDM?

$$V(\phi_h, T) \approx (DT^2 - \mu^2)\phi_h^2 \boxed{- ET\phi_h^3} + \frac{\tilde{\lambda}}{4}\phi_h^4$$

$$E = \frac{1}{12\pi}\left[\frac{6m_W^3}{v^3} + \frac{3m_Z^3}{v^3} + \frac{m_h^3}{v^3}\right] + E_{(H/A/H^\pm)}$$

$$m_\alpha^2(\phi_h) = M^2 + \lambda_\alpha \phi_h^2$$

where λ_α: a linear combination of the λ_i (I=1~5)

and $M^2 = \dfrac{m_{12}^2}{\sin\beta\cos\beta}$

$$E_{(\alpha)} \approx \begin{cases} \frac{1}{12\pi}\lambda_\alpha^{3/2} = \frac{1}{12\pi}\frac{m_\alpha^3}{v^3}, & M^2 \ll \lambda_\alpha \phi_h^2, \\ 0, & M^2 \gg \lambda_\alpha \phi_h^2 \end{cases}$$

Too heavy m_α (thermal decoupling)

Why the "Inverted" Scenario?

$M^2 \gg \lambda_\alpha \phi_h^2 \Rightarrow E \simeq 0$ → **No SFOPT**

In the normal scenario, $m_h = 125$ GeV
$H, A, H^\pm$ are heavier and M^2 is also large.
→ Weak EWPT

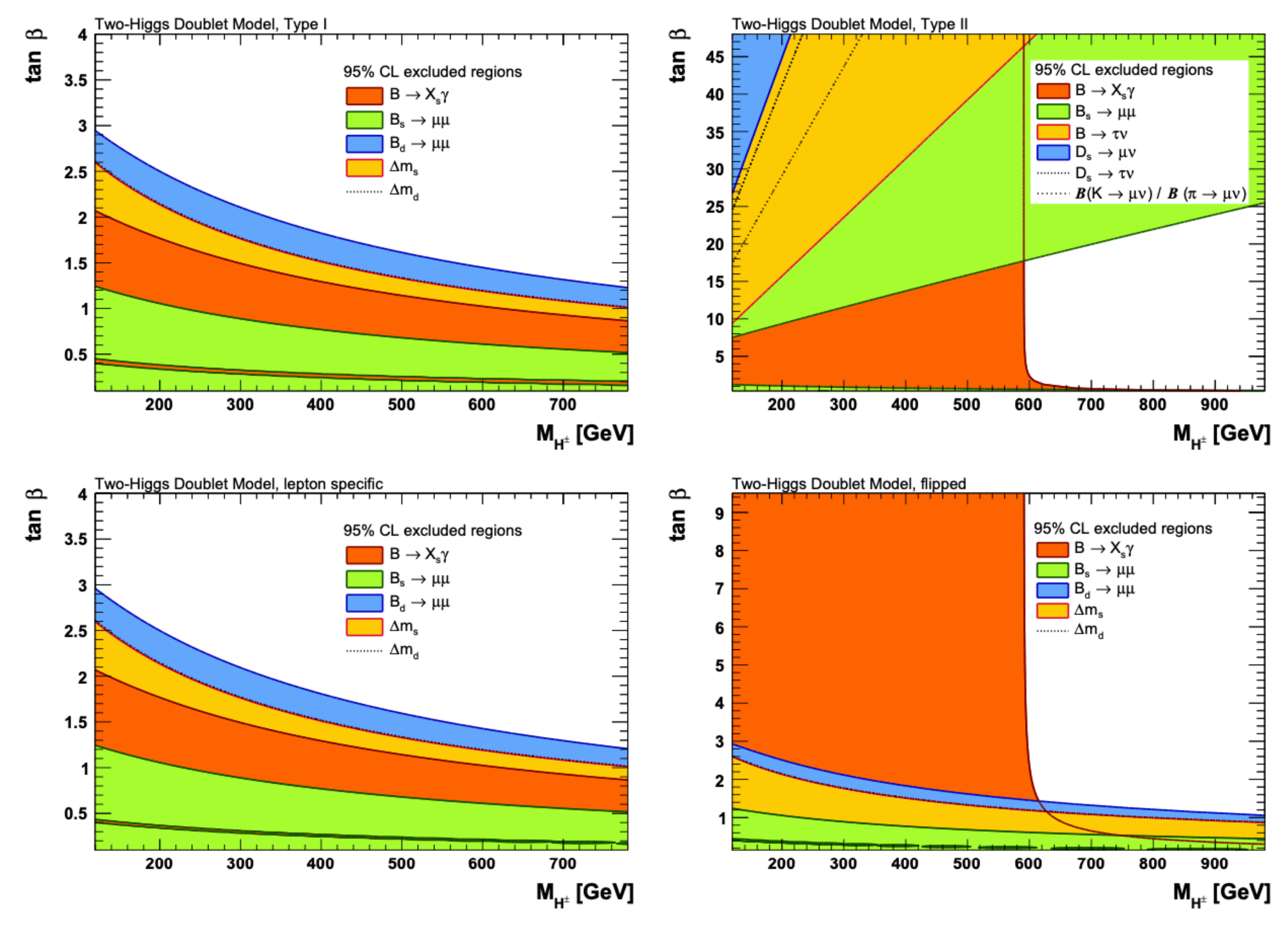
In the inverted scenario, $m_H = 125$ GeV
$h, A, H^\pm$ are lighter and M^2 is also small.
→ Enhance the cubic term and enable SFOEWPT

Why Type-I 2HDM?

Haller et al. arXiv:1803.01853

Type-I
Much weaker bounds
→ **the largest viable parameter space**
for EWPT studies

Type-X
lepton-specific
leading to stronger constraints from leptonic observables.



Type-II / Type-Y

Strong B-physics constraints:

→ $m_{H^\pm} \gtrsim 600$ GeV

Our Model: Inverted Type-I 2HDM

Let's Focus on "Type-I Inverted 2HDM"!

Model parameters

$m_H = 125$ GeV (inverted),

$m_h\,,\ m_A\,,\ m_{H^\pm},$

$m_{12}^2\,,\ \ \sin(\beta-\alpha)\,,\ \ \tan\beta$

Yukawa Type

Type-I

One-loop effective potential at finite temperature

classical background field configurations:

$$\Phi_1^c = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ w_1 \end{pmatrix}, \quad \Phi_2^c = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ w_2 + i w_3 \end{pmatrix},$$

$$\vec{w} \equiv (w_1, w_2, w_3) \sim (w_1, w_2)\,.$$

One-loop effective potential at finite temperature:

$$V_{\text{eff}}(\vec{w}, T) = \underbrace{V_{\text{tree}}(\vec{w})}_{\text{Tree-level}} + \underbrace{V_{\text{CW}}(\vec{w})}_{\text{One-loop}} + \underbrace{V_{\text{CT}}(\vec{w})}_{\text{Counterterm}} + \underbrace{\boxed{V_T(\vec{w}, T)}}_{\text{Thermal correction}}\,.$$

One-loop effective potential at finite temperature

· The one-loop Coleman-Weinberg (CW) potential:

$$V_{\text{CW}}(\vec{w}) = \frac{1}{64\pi^2}\sum_i n_i m_i^4(\vec{w})\left[\ln\left(\frac{m_i^2(\vec{w})}{\mu^2}\right) - c_i\right],\ (i = h, H, A, H^\pm, G^0, G^\pm, W^\pm, Z, f)$$

· The counter-term potential:

$$V_{\text{CT}}(\vec{w}) = \delta m_{11}^2\frac{w_1^2}{2} + \delta m_{22}^2\frac{w_2^2+w_3^2}{2} - \delta m_{12}^2 w_1 w_2 + \frac{\delta\lambda_1}{8}w_1^4 + \frac{\delta\lambda_2}{8}(w_2^2+w_3^2)^2 + \frac{\delta\lambda_3+\delta\lambda_4}{4}w_1^2(w_2^2+w_3^2) + \frac{\delta\lambda_5}{4}w_1^2(w_2^2-w_3^2),$$

· The thermal correction:

$$V_T(\vec{w}, T) = \frac{T^4}{2\pi^2}\sum_{i_B} n_{i_B} J_B\left(\frac{m_{i_B}^2(\vec{w})}{T^2}\right) + \frac{T^4}{2\pi^2}\sum_{j_F} n_{j_F} J_F\left(\frac{m_{j_F}^2(\vec{w})}{T^2}\right) - \frac{T^4}{12\pi}\sum_{k_L} n_{k_L}\left[\left(\frac{\tilde{m}_{k_L}^2(\vec{w}, T)}{T^2}\right)^{3/2} - \left(\frac{m_{k_L}^2(\vec{w})}{T^2}\right)^{3/2}\right],$$

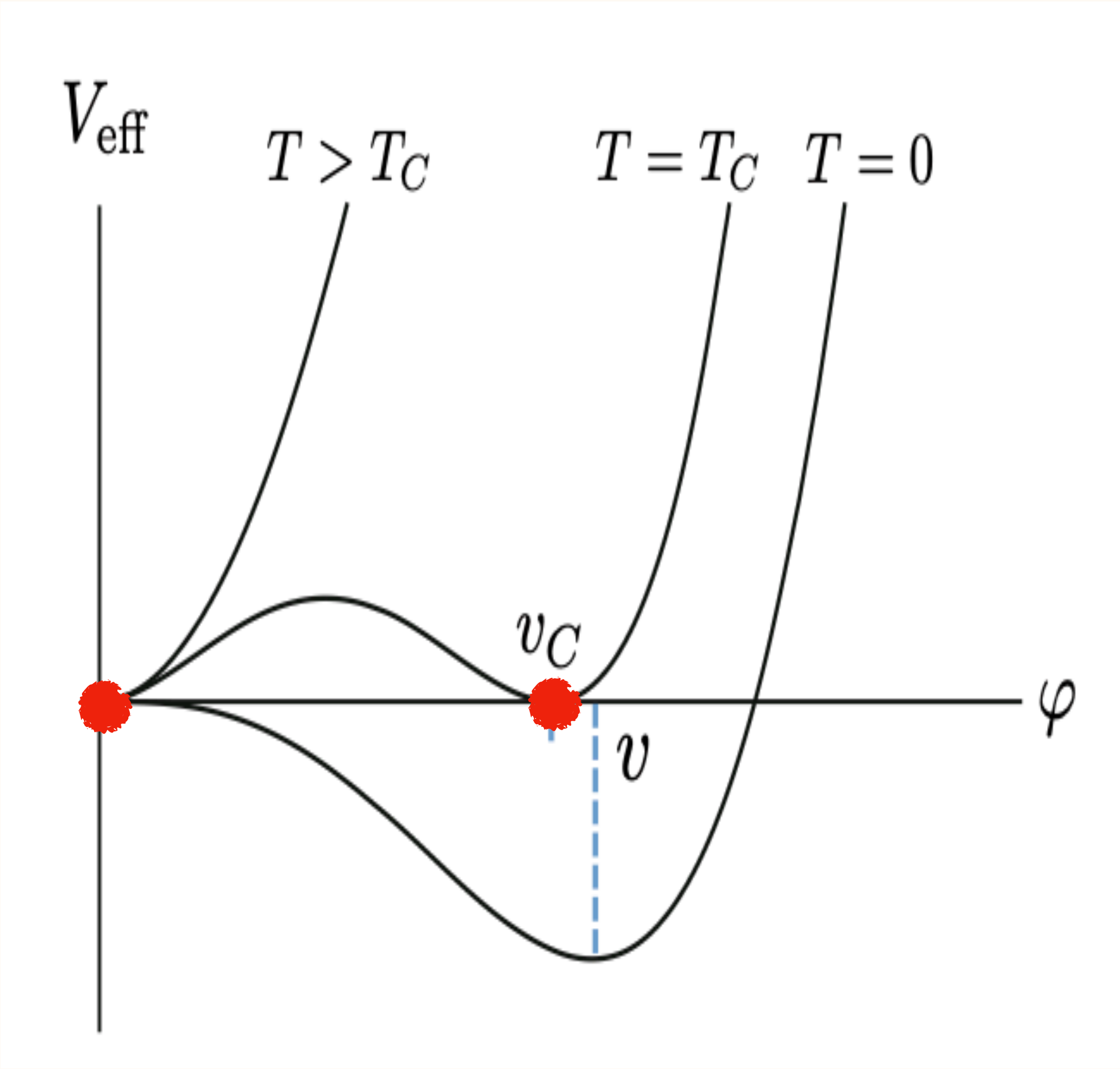
· The thermal functions $J_B(x)$ for bosons and $J_F(x)$ for fermions:

$$J_B(x) = \int_0^\infty dk\, k^2 \ln\left[1 - e^{-\sqrt{k^2+x}}\right],\quad J_F(x) = \int_0^\infty dk\, k^2 \ln\left[1 + e^{-\sqrt{k^2+x}}\right].$$

where $\tilde{m}_k^2(\vec{\omega}, T)$ is the temperature-dependent thermal (Debye) masses appearing in the daisy resummation term

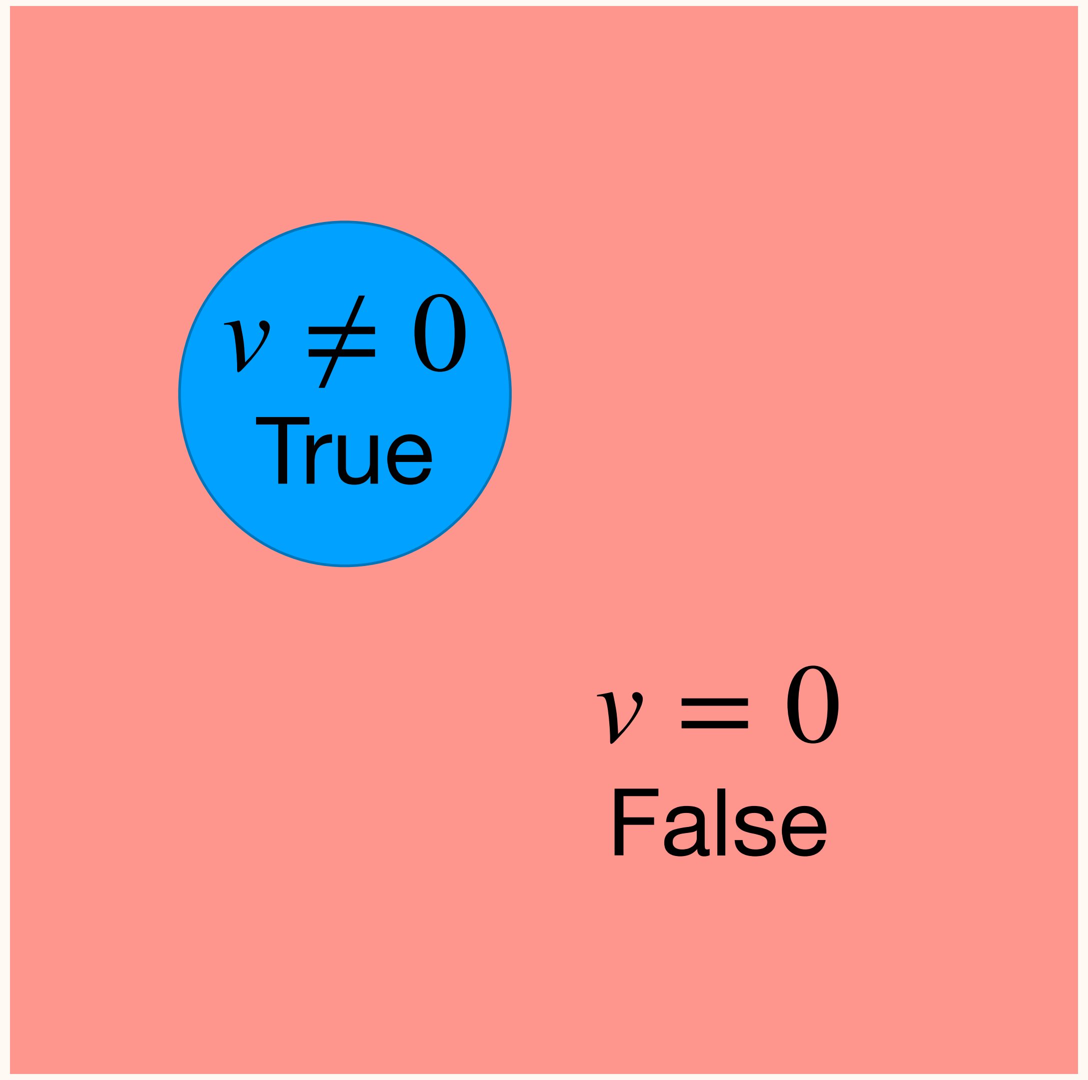
Characteristic Temperature

P_{true}: ture vacuum fraction



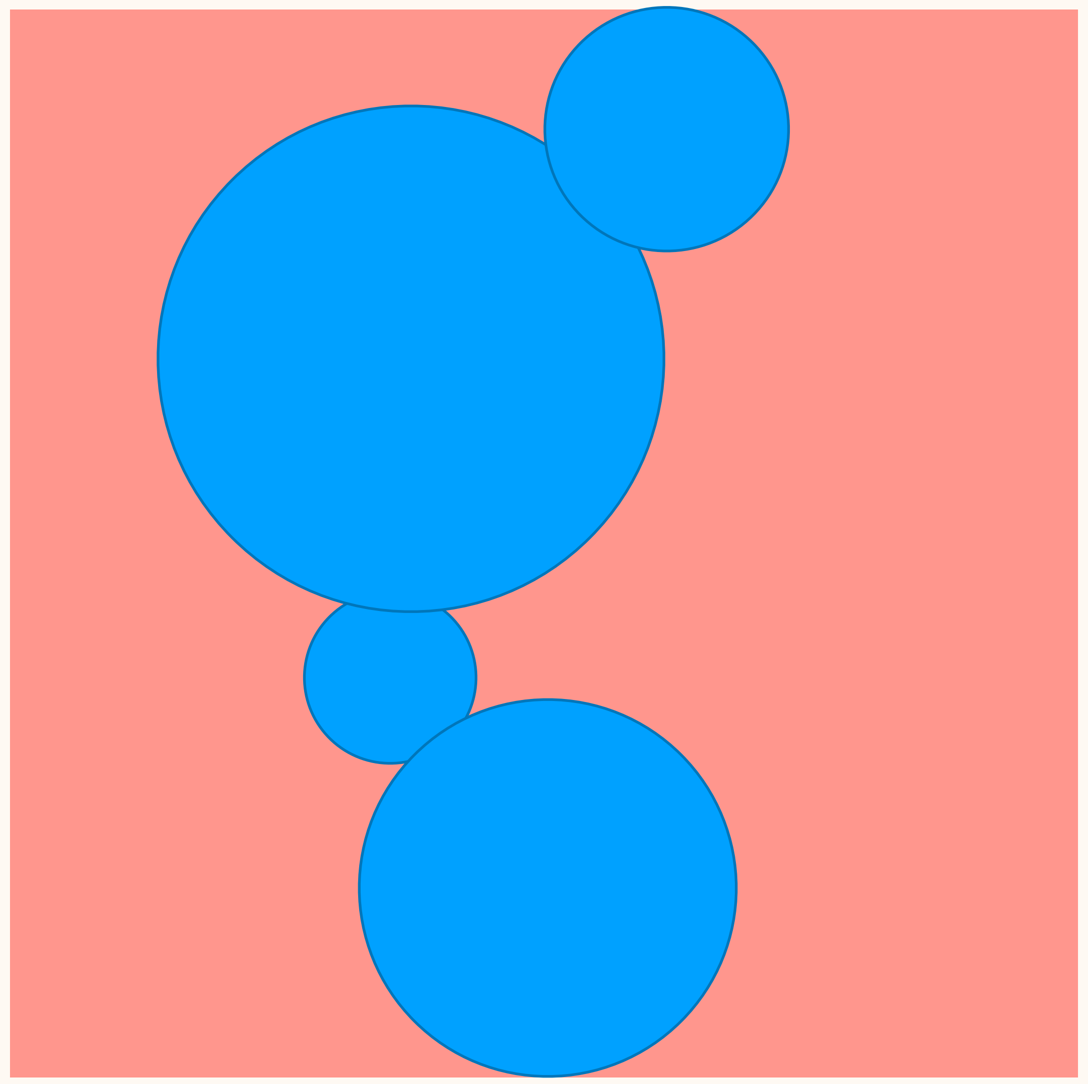
Critical temperature

$$V_{\text{eff}}(0, T_C) = V_{\text{eff}}(v_C, T_C)$$



Nucleation temperature

$$\frac{\Gamma(T_n)}{H^4(T_n)} = 1$$



Percolation temperature

$$P_{\text{true}}(T_p) = 0.29$$

Completion temperature

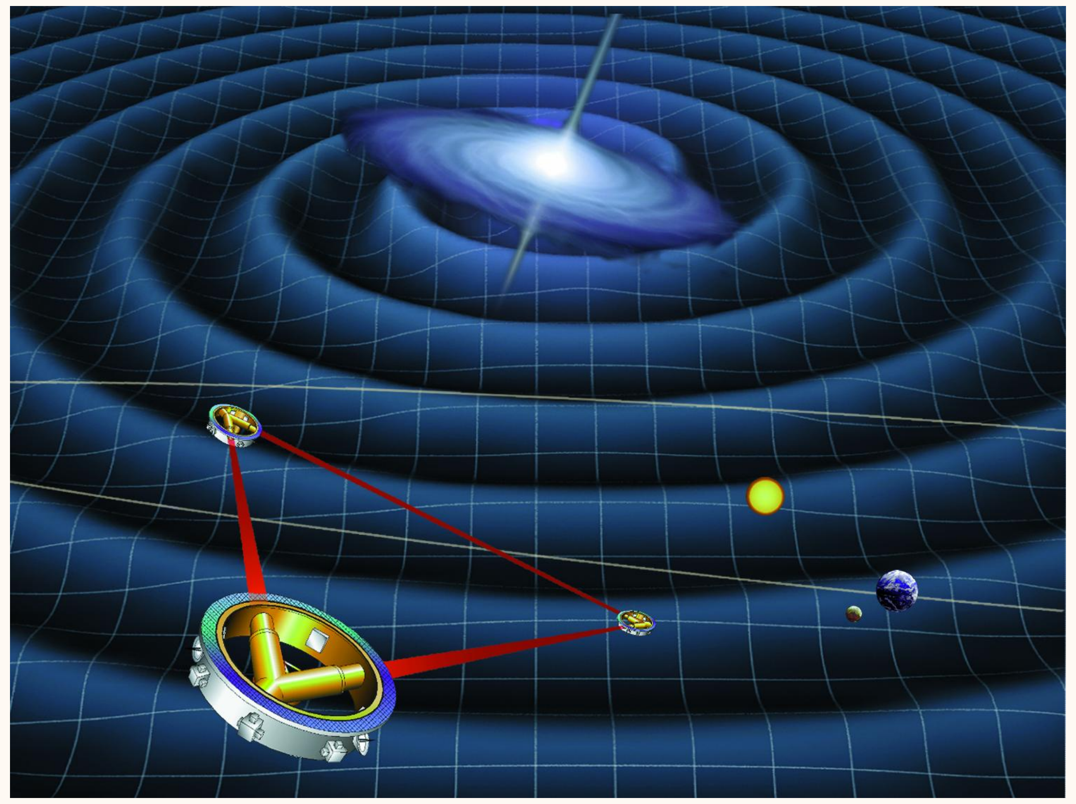
$$P_{\text{true}}(T_f) = 0.99$$

BSMPTv3 (https://arxiv.org/pdf/2404.19037) calculates
one-loop daisy-resummed finite-temperature effective potential and tracks its minima.

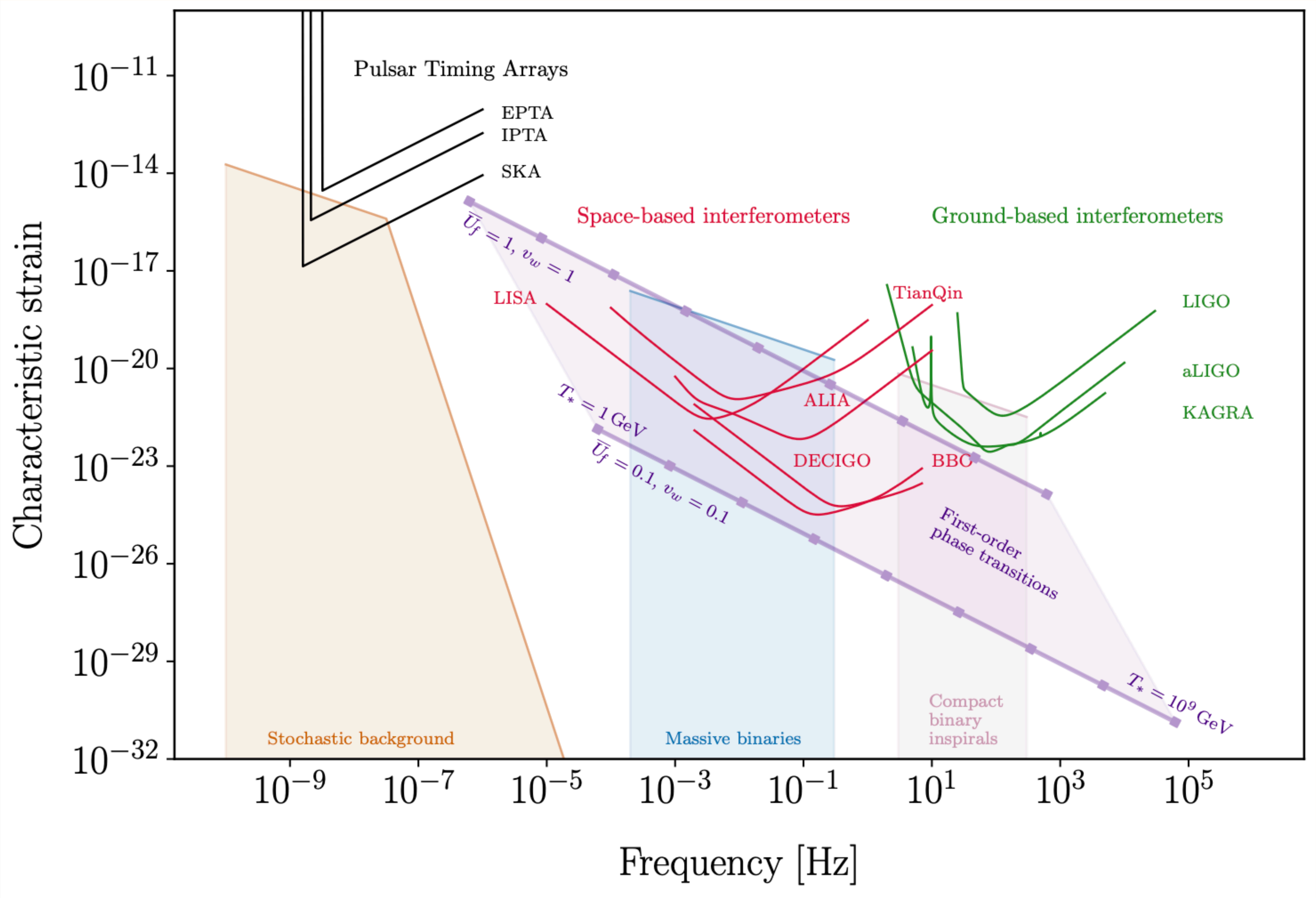
Gravitational Wave from SFOPT

Three Main Sources
of GW from SFOPT:

1. **Bubble Collisions**
2. **Sound Waves**
3. **Turbulence**

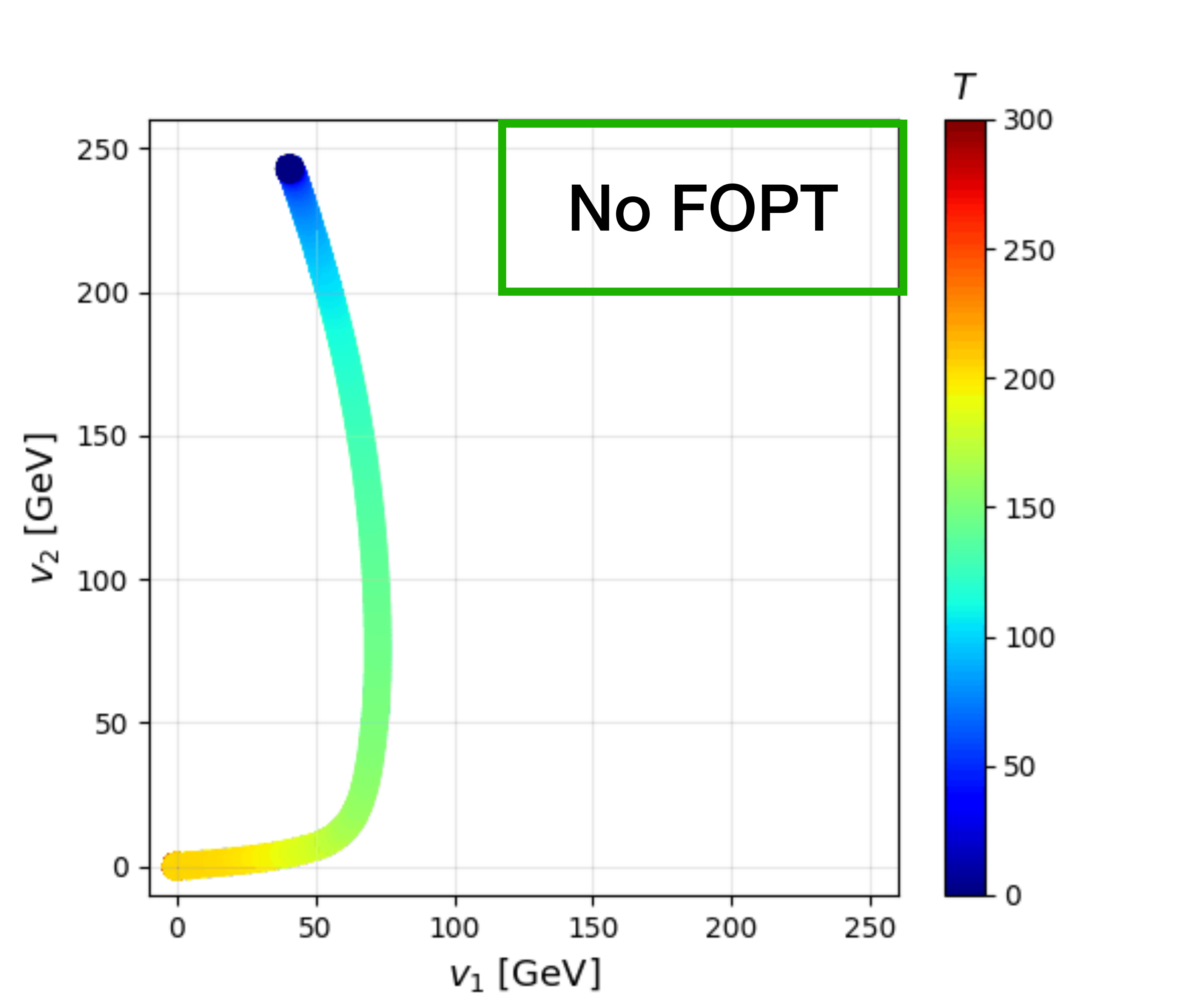


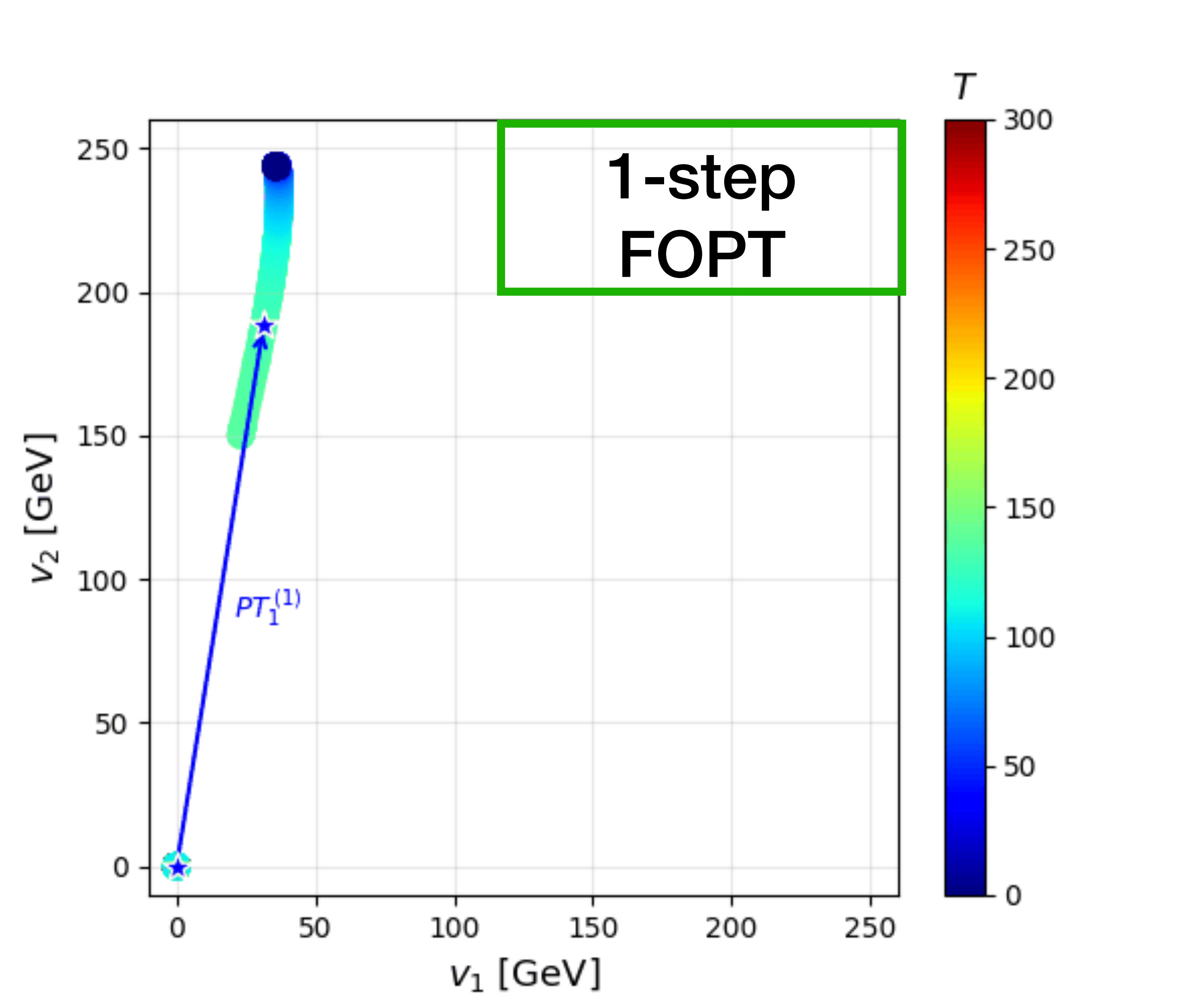
LISA
(Laser Interferometer Space Antenna)



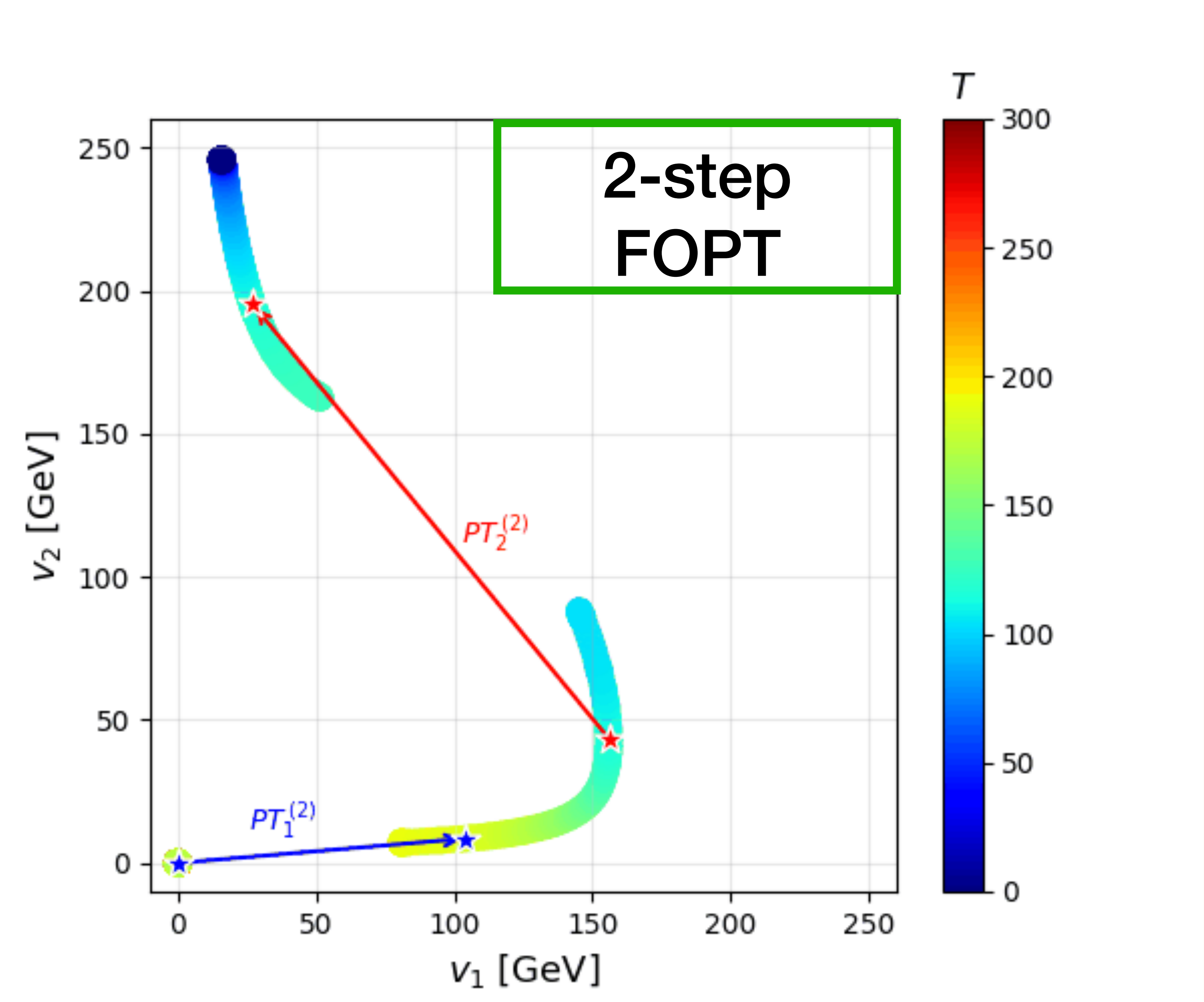
2. Multi-Step EWPTs in 2HDM

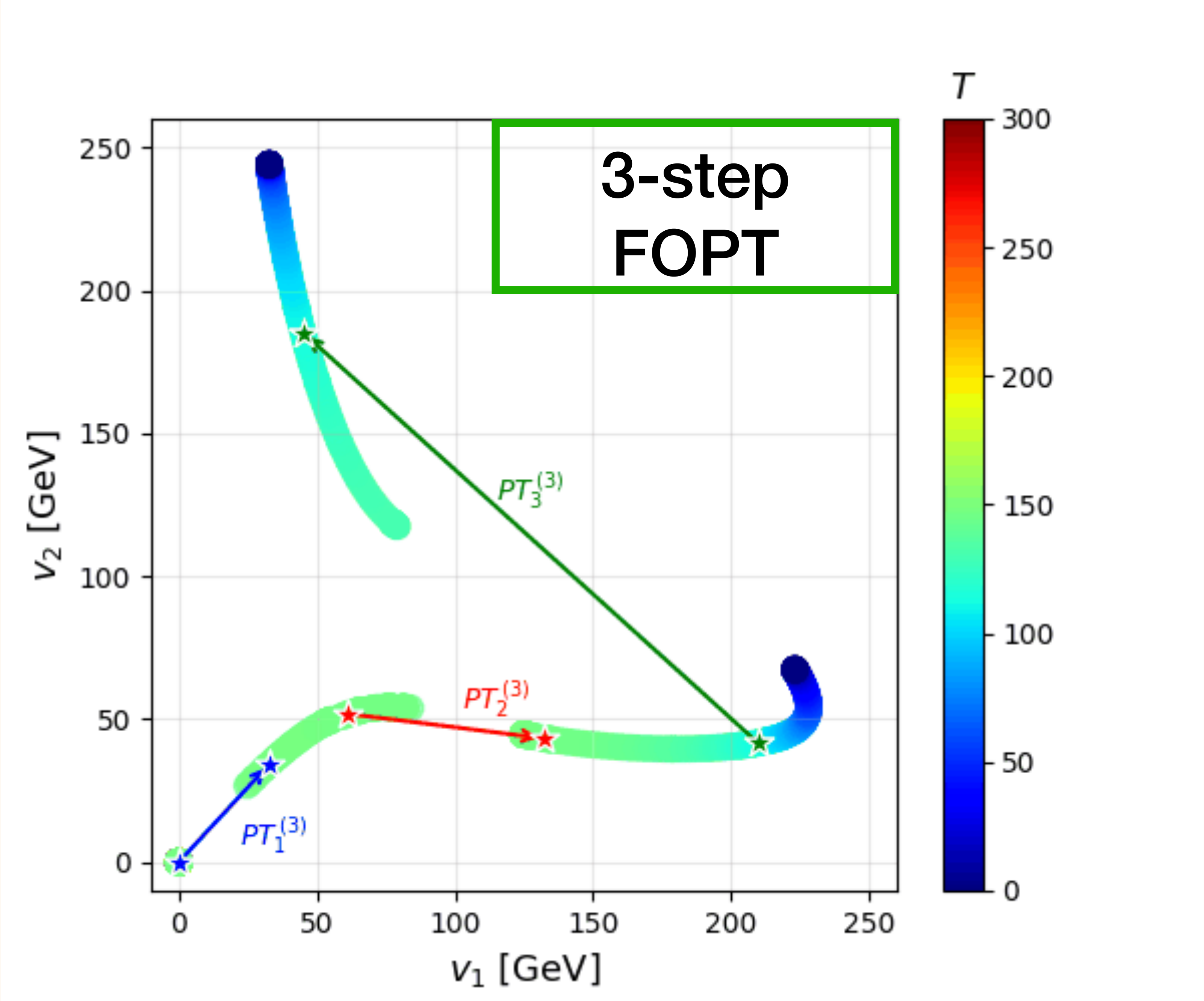
Minima Tracer - FOPT



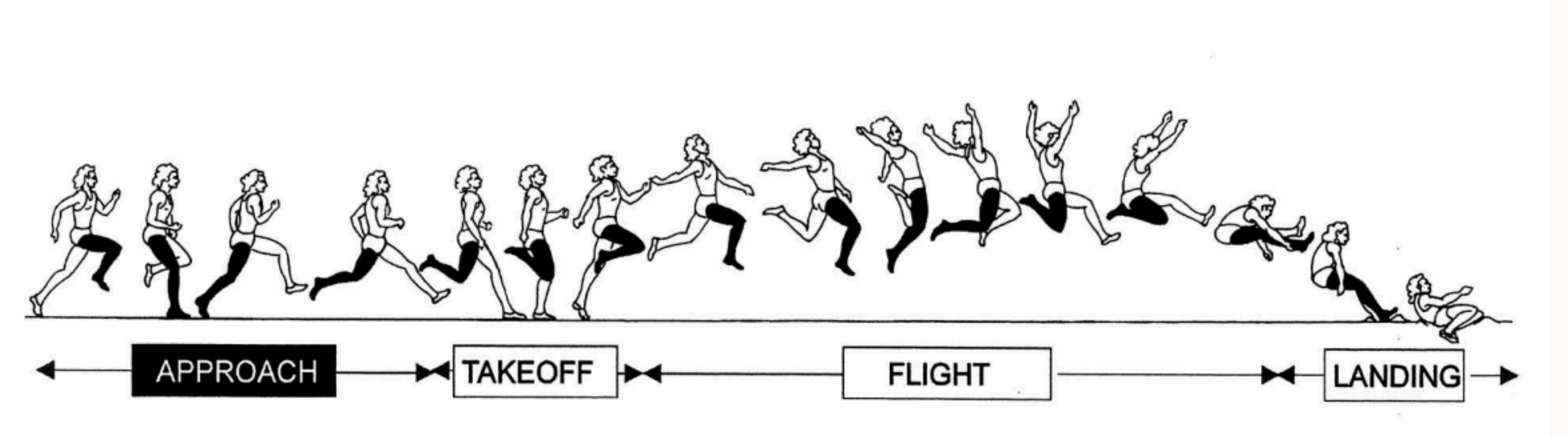


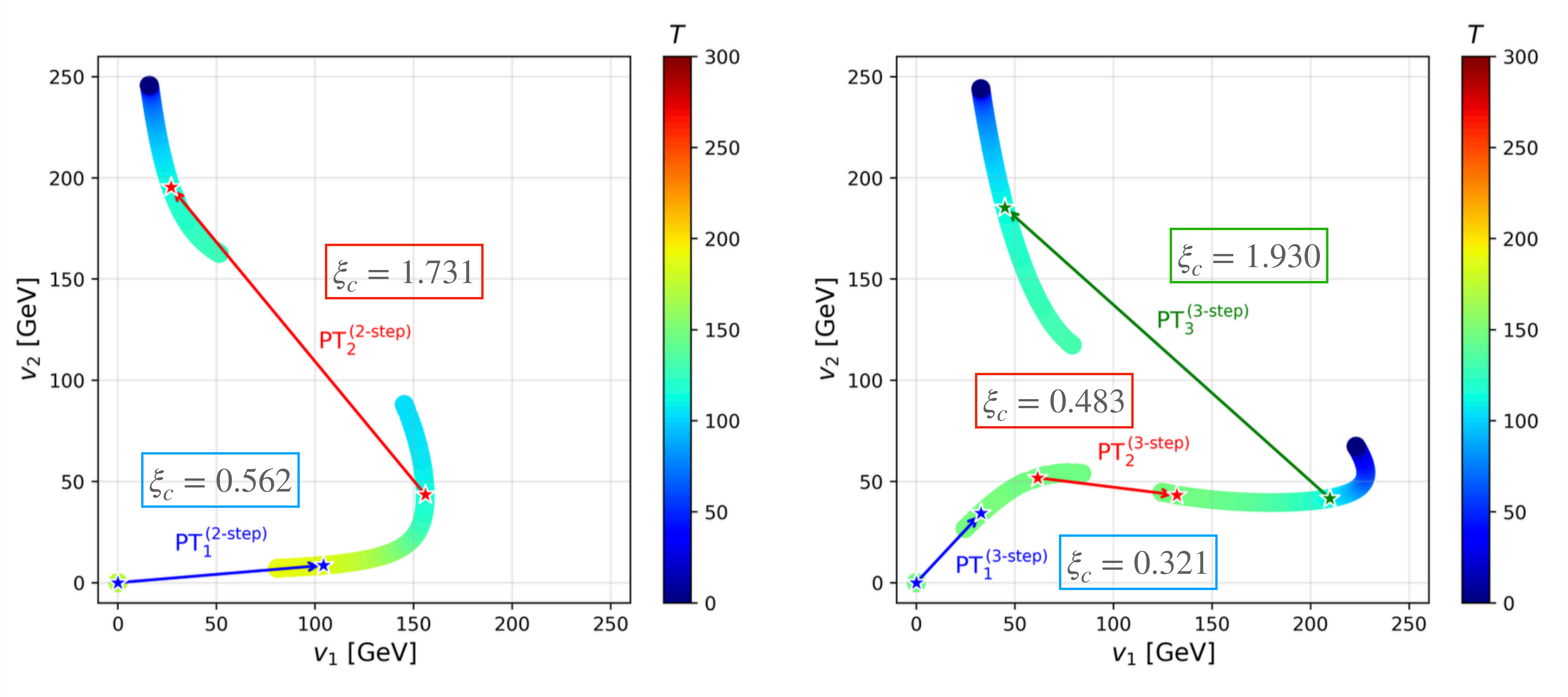
What is Multi-Step FOPT?





What is Multi-Step FOPT?





$$V_{\text{eff}}\left(v_{1(i-)}^{T_c}, v_{2(i-)}^{T_c}, T_c\right) = V_{\text{eff}}\left(v_{1(i+)}^{T_c}, v_{2(i+)}^{T_c}, T_c\right)$$

$$\xi_c = \frac{\sqrt{\left(v_{1(i+)}^{T_c} - v_{1(i-)}^{T_c}\right)^2 + \left(v_{2(i+)}^{T_c} - v_{2(i-)}^{T_c}\right)^2}}{T_c}$$

Scan Model Parameters

Scan Range of Model Parameters

$$m_H = 125 \text{ GeV},$$

$$m_h \in [30{,}120] \text{ GeV}, \quad m_A \in [30{,}700] \text{ GeV}, \quad m_{H^\pm} \in [80{,}700] \text{ GeV},$$

$$s_{\beta-\alpha} \in [-0.5{,}0.5], \quad t_\beta \in [1{,}50], \quad m_{12}^2 \in [0{,}2\times 10^4] \text{ GeV}^2 .$$

Theoretical Constraints

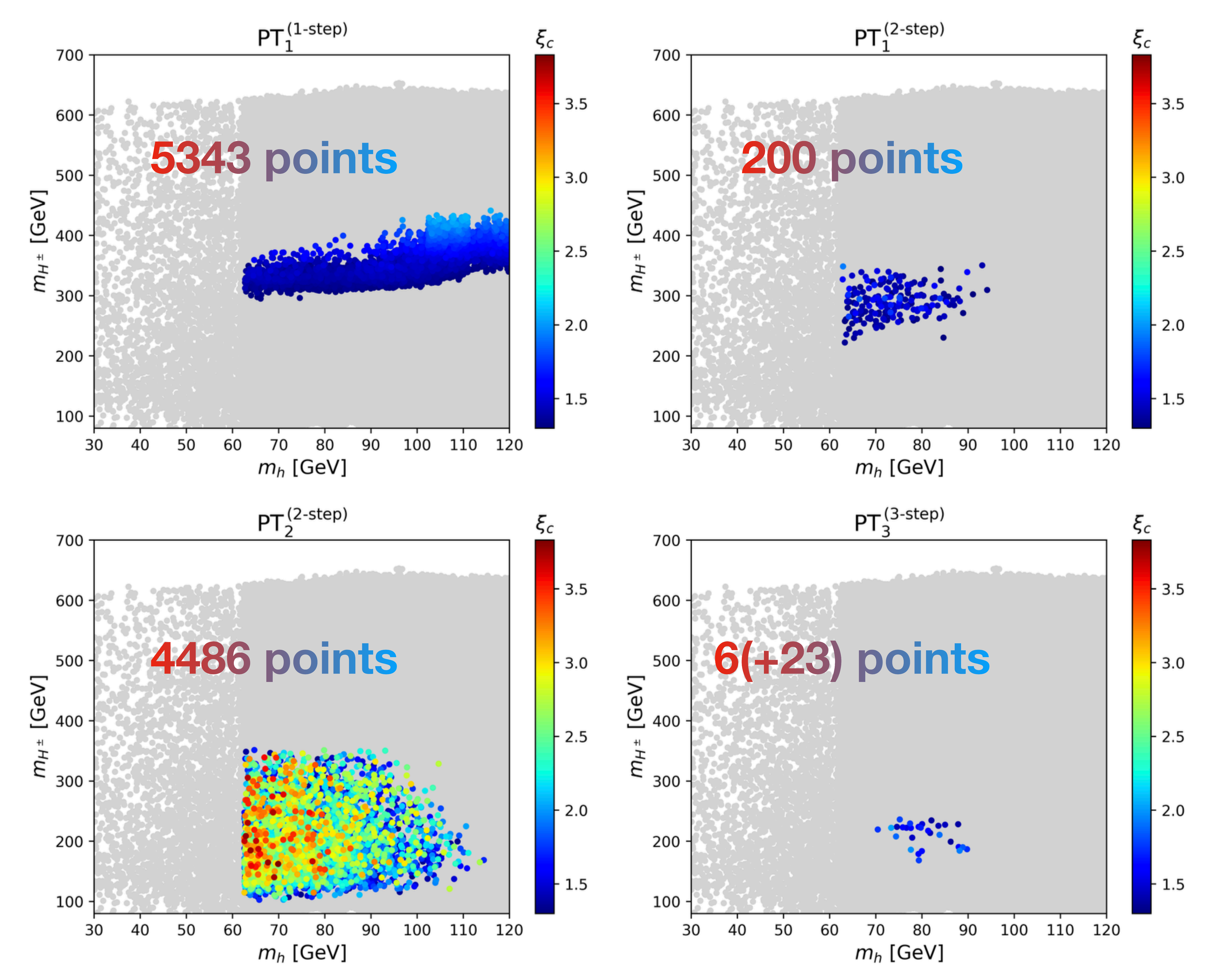
1. Bounded-from-below
2. Unitarity at tree-level
3. Perturbativity
4. Vacuum stability

Experimental Constraints

1. B-physics / FCNC
2. Electroweak Precision Data (EWPD)
3. Direct searches (LEP, Tevatron, LHC)
4. Higgs precision data

Multi-step PTs in 2HDM?

Out of 2M Physical parameter points,



Physical parameter points:

Parameter points that satisfy all the theoretical requirements and current experimental constraints.

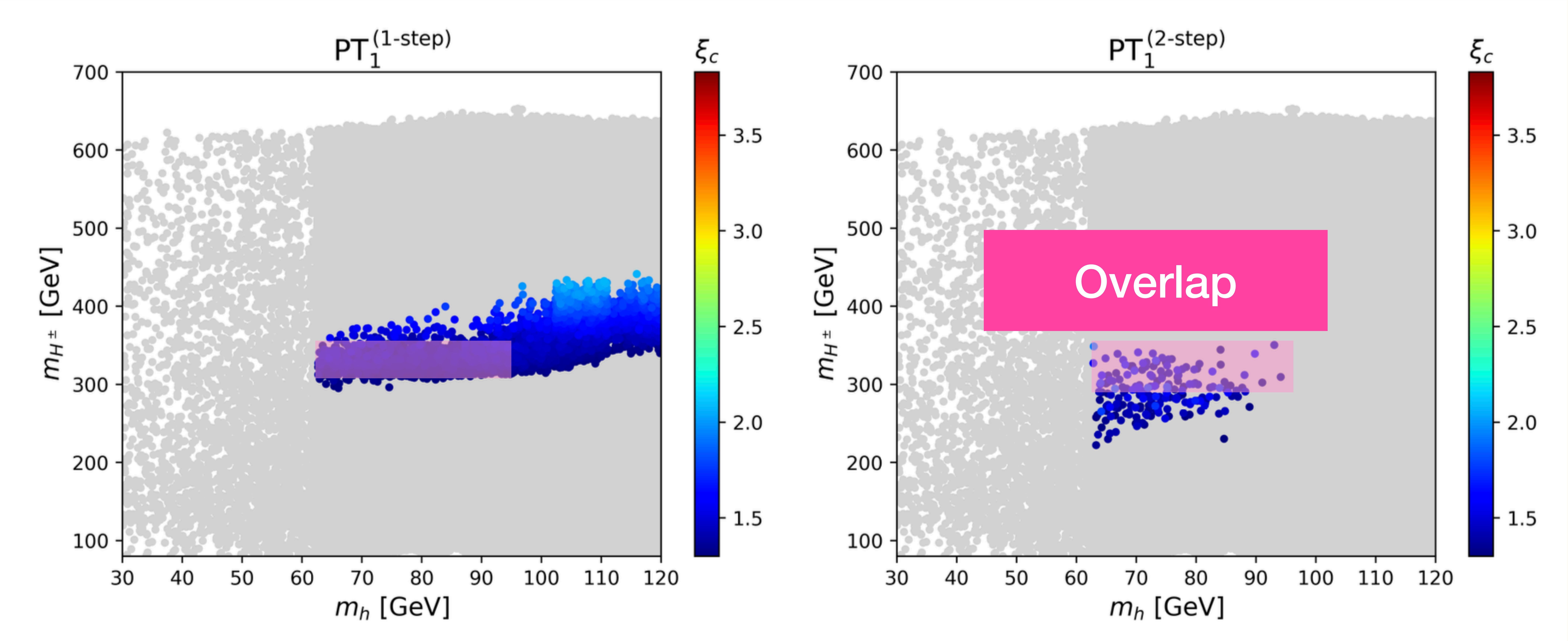
SFOEWPT parameter points:

Physical parameter points that additionally satisfy the SFOEWPT condition, $\xi_c > 1.3$.

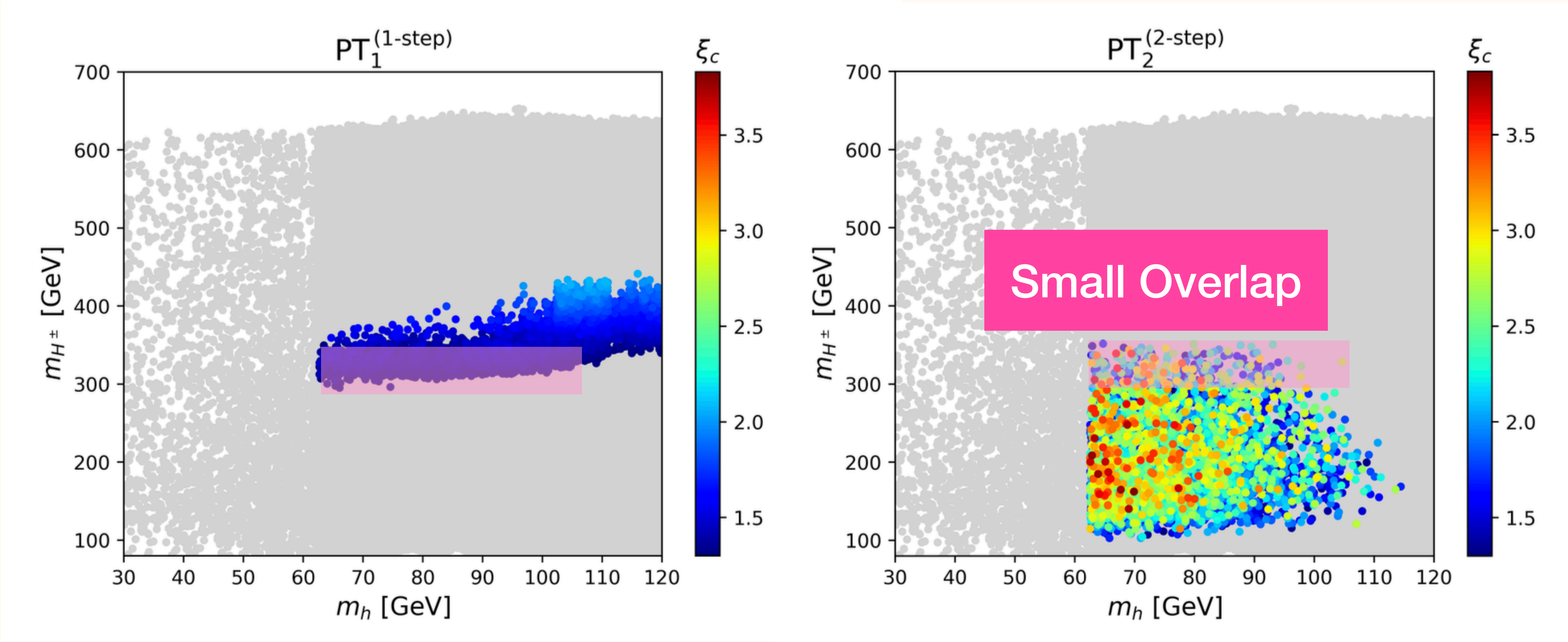
$\mathrm{PT}_i^{(n\text{-step})}$:

the i-th transition in an n-step SFOPT.

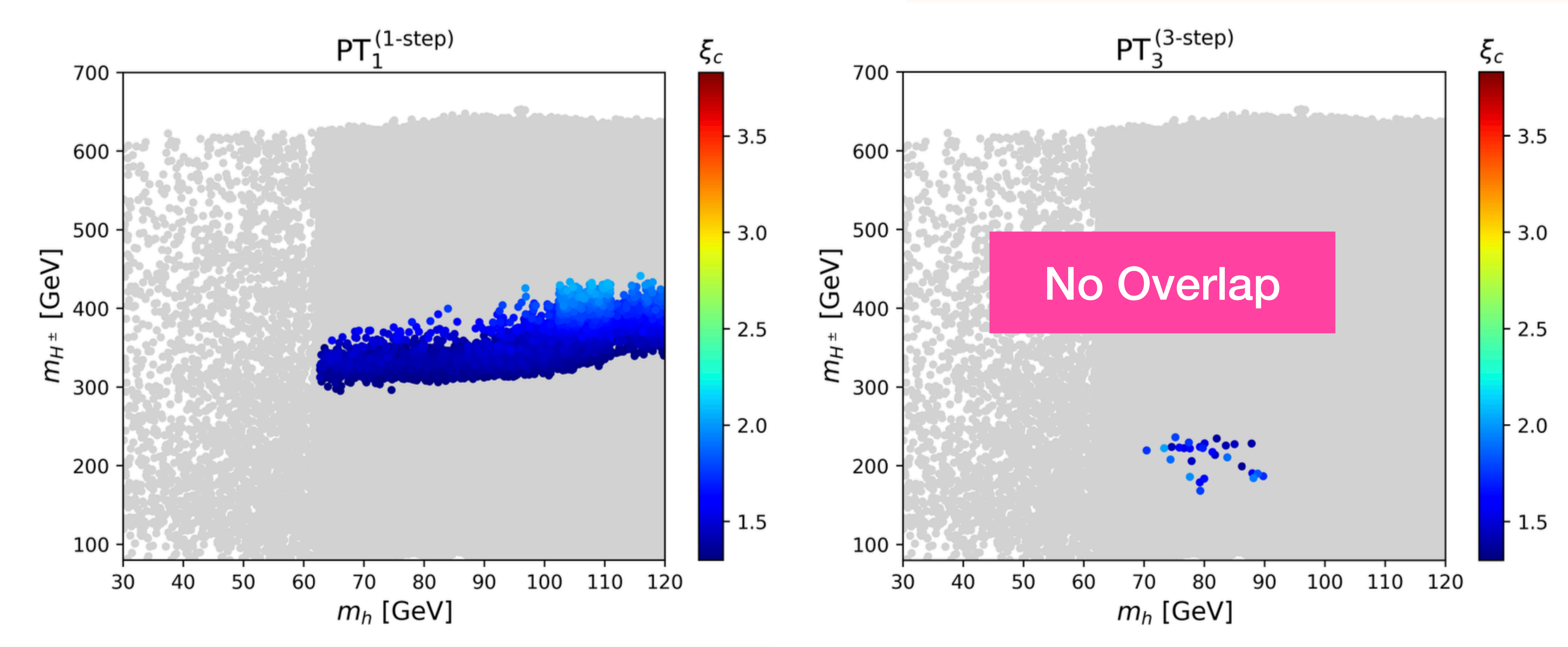
PT_1(1-step) vs. PT_1(2-step)



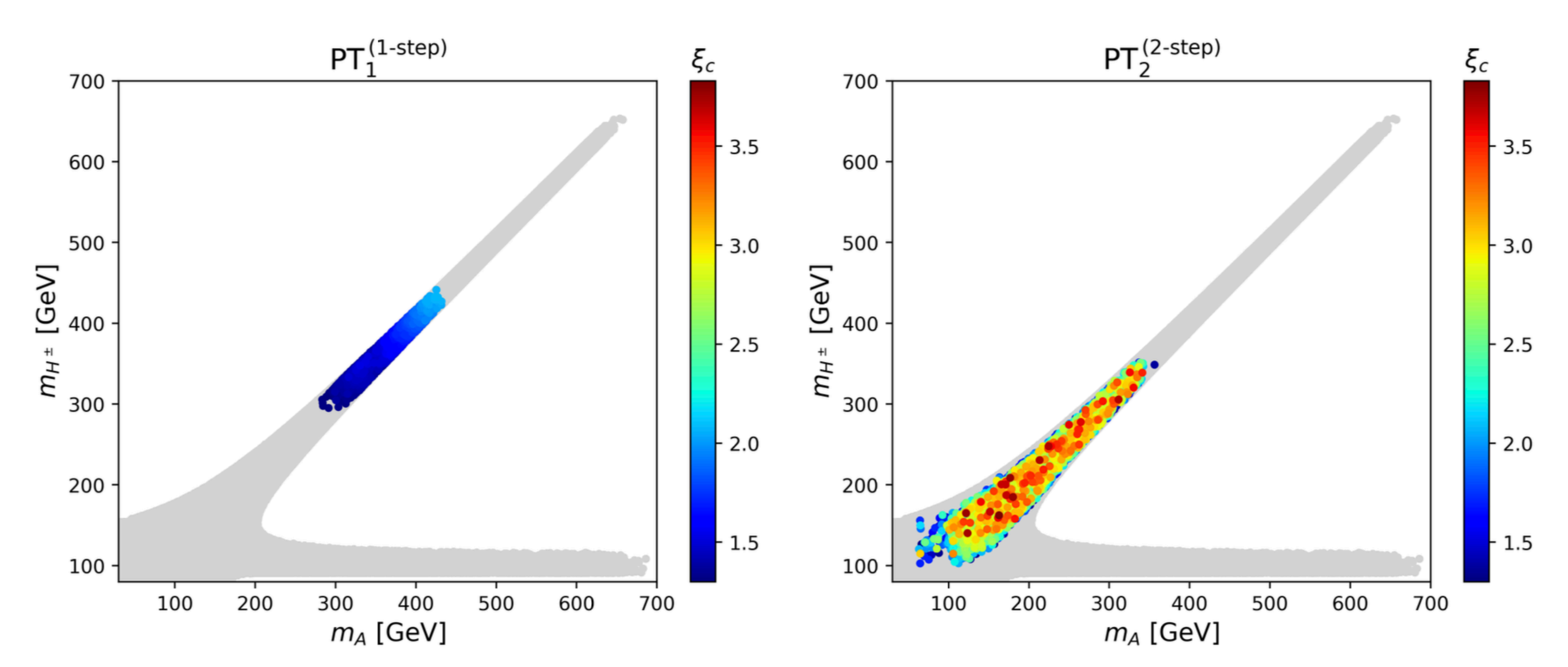
PT_1(1-step) vs. PT_2(2-step)



PT_1(1-step) vs. PT_3(3-step)



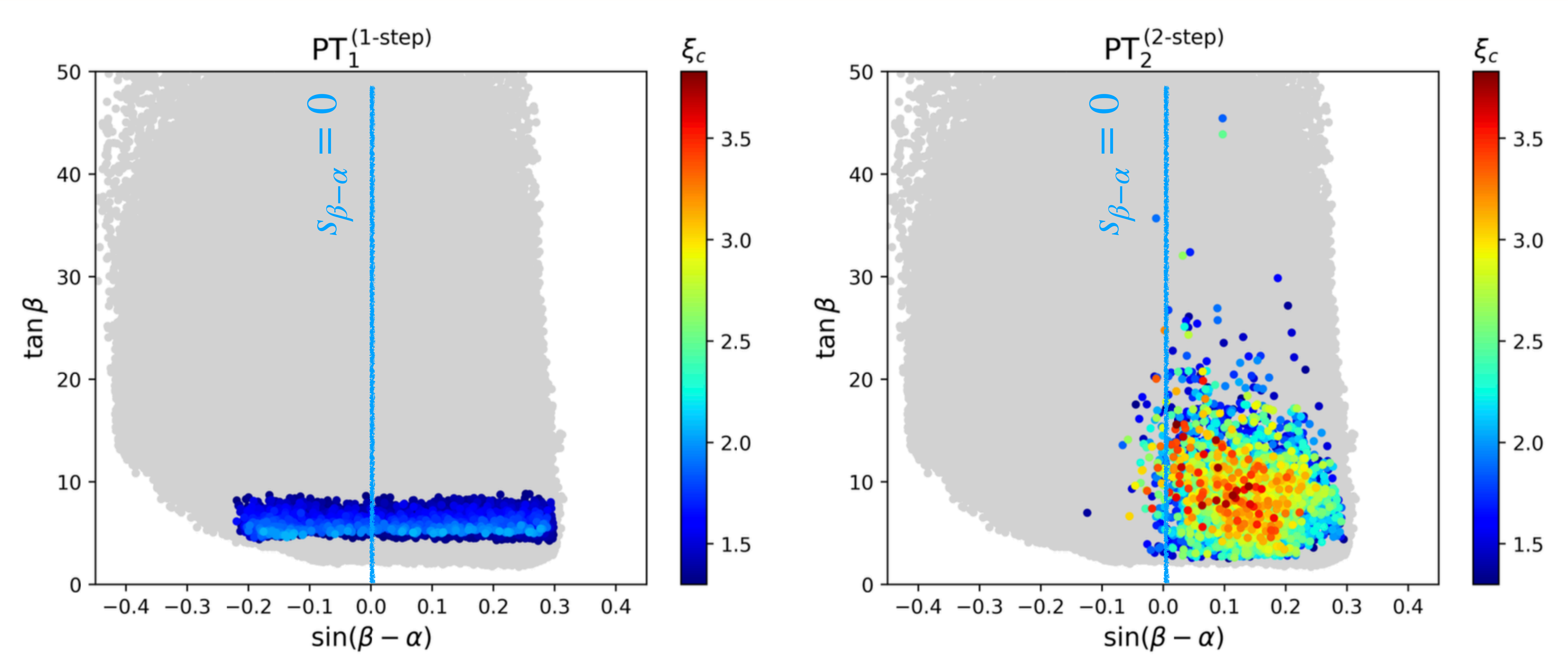
Mass Degeneracy



The region with $m_h \sim m_{A,H^\pm}$ is excluded in 1-step

The region with $m_h \sim m_{A,H^\pm}$ is allowed in 2-step

$\tan\beta$ and $\sin(\beta-\alpha)$



In the 1-step, $\sin(\beta-\alpha)<0$ is allowed

Larger $\tan\beta \Rightarrow$ Weaker Yukawa Couplings

In the 2-step, $\sin(\beta-\alpha)>0$ is dominant

3. Gravitational Waves from SFOPT

Gravitational Wave from SFOPT

- SFOPT: A Violent Process in the Early Universe

- True Vacuum Bubbles nucleate, expand, and collide.

- Three Main Sources of Gravitational Waves
  1. **Bubble Collisions**
  2. **Sound Waves**
  3. **Turbulence**

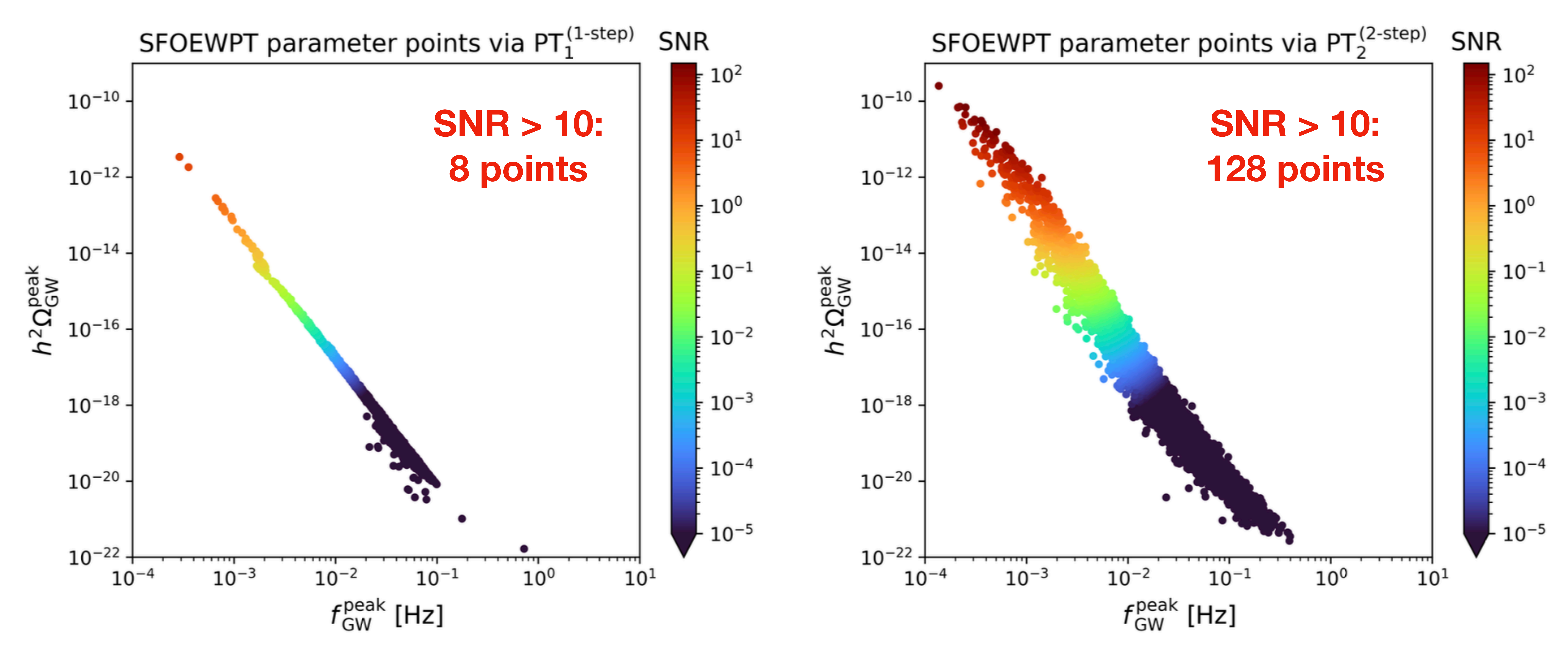
  $\Rightarrow$ **Stochastic Gravitational Wave Background (SGWB).**

- This GW signal is a key target for future detectors, e.g., LISA.

Detectable by LISA?

** SNR > 10 for $PT_3^{(3\text{-step})}$: 2 points

Color: SFOPT points



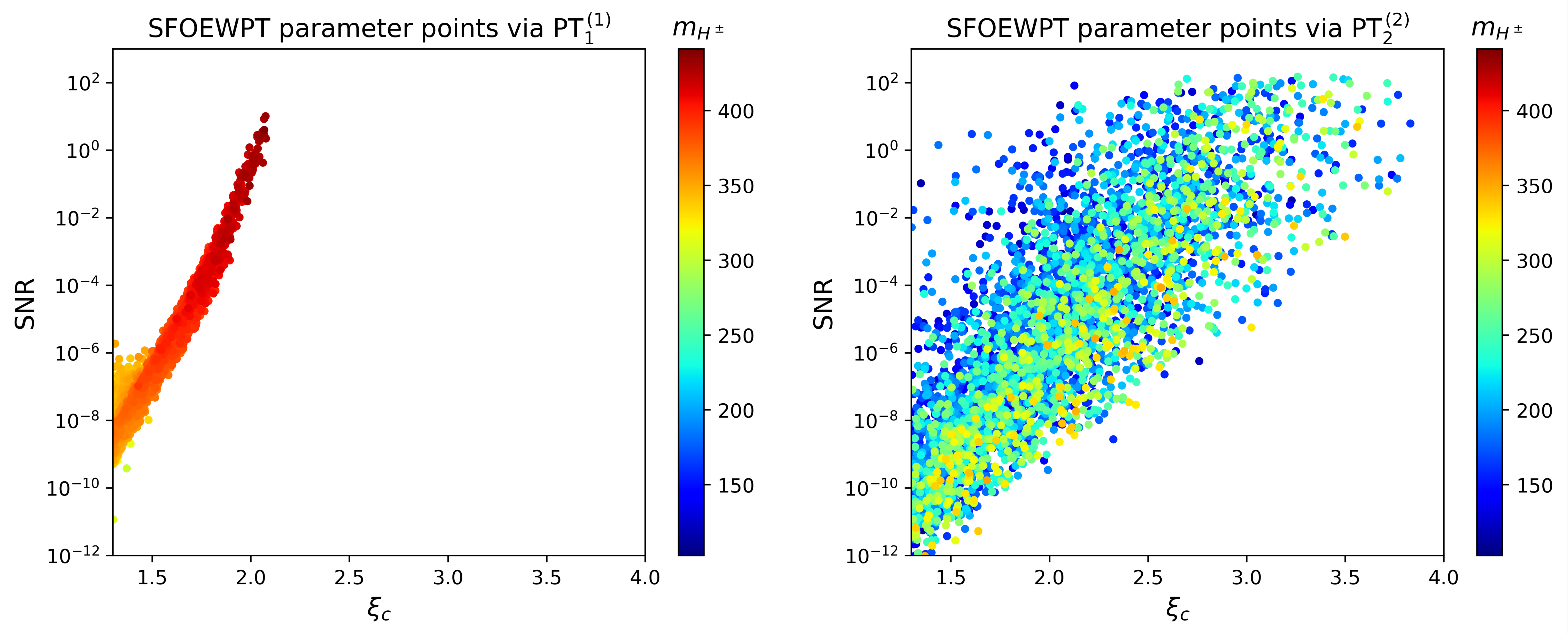
$$h^2\Omega_{GW}(f) \simeq h^2\Omega_{GW}^{SW,peak}\left(\frac{4}{7}\right)^{-\frac{7}{2}}\left(\frac{f}{f_{GW}^{SW,peak}}\right)^3 \times \left[1+\frac{3}{4}\left(\frac{f}{f_{GW}^{SW,peak}}\right)^2\right]^{-\frac{7}{2}}$$

signal-to-noise ratios:

$$\mathrm{SNR} = \sqrt{\mathcal{T}\int_{f_{min}}^{f_{max}} df \left[\frac{h^2\Omega_{GW}(f)}{h^2\Omega_{Sens}(f)}\right]^2}$$

Order Parameters and SNR

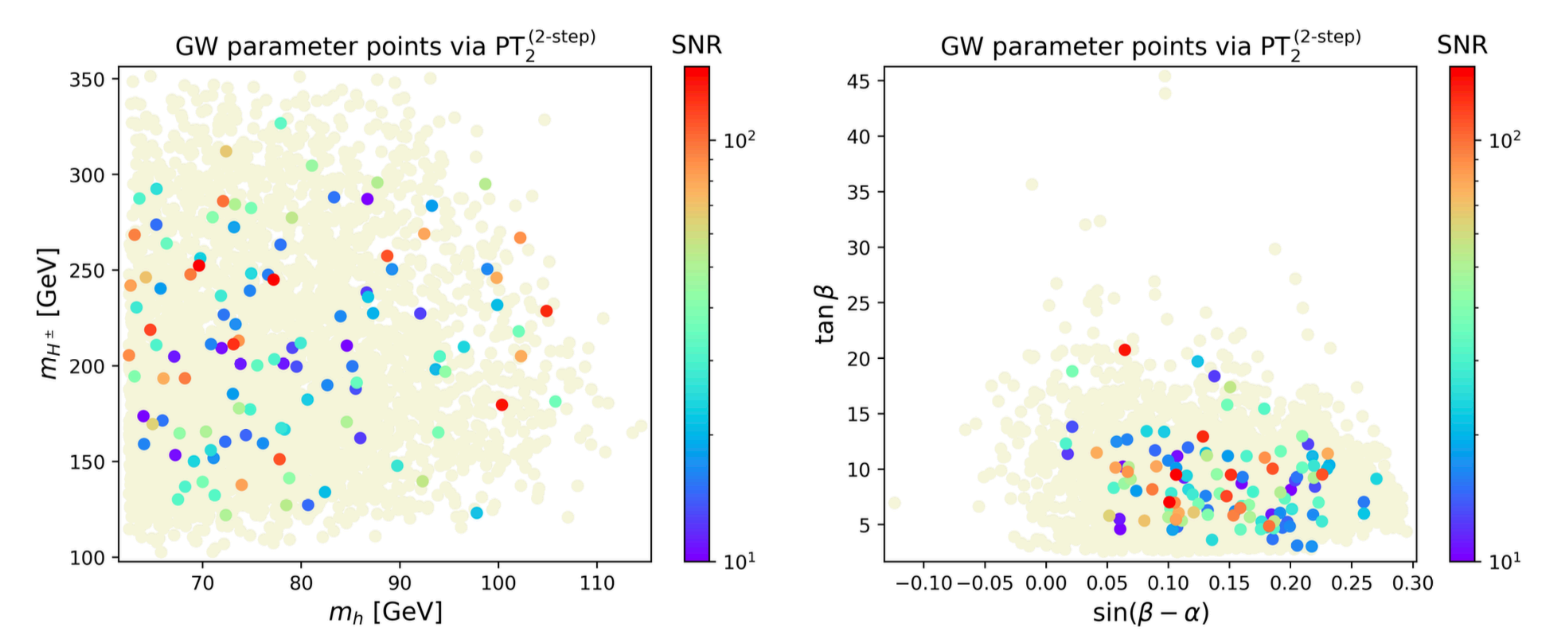
Color: SFOPT points



GW parameter points

beige: SFOEWPT parameter points

Color: GW parameter points
a LISA GW signal with an SNR > 10.



SFOEWPT and GW parameter regions are **not clearly separated**

The GW detection points are broadly distributed.

5. Conclusion

Conclusion

1. **Inverted Type-I 2HDM** provides a comprehensive framework where Multi-step SFOEWPT scenarios.
2. **Multi-Step SFOEWPT**: Successfully supports SFOEWPT via 1-step, 2-step, and 3-step phase transition. Multi-step transitions often have the potential to produce stronger phase transitions.
3. **GW Signatures**: Multi-step transitions are the predominant source of detectable Gravitational Waves. A few points in the Inverted Type-I 2HDM have SNR > 10.

Thank you

Vacuum uplifting in One-Step PT

Vacuum Energy Density Difference at T=0 (Depth of the Potential):

$$\mathscr{F}_0 = V(T=0)(v) - V(T=0)(0) < 0$$

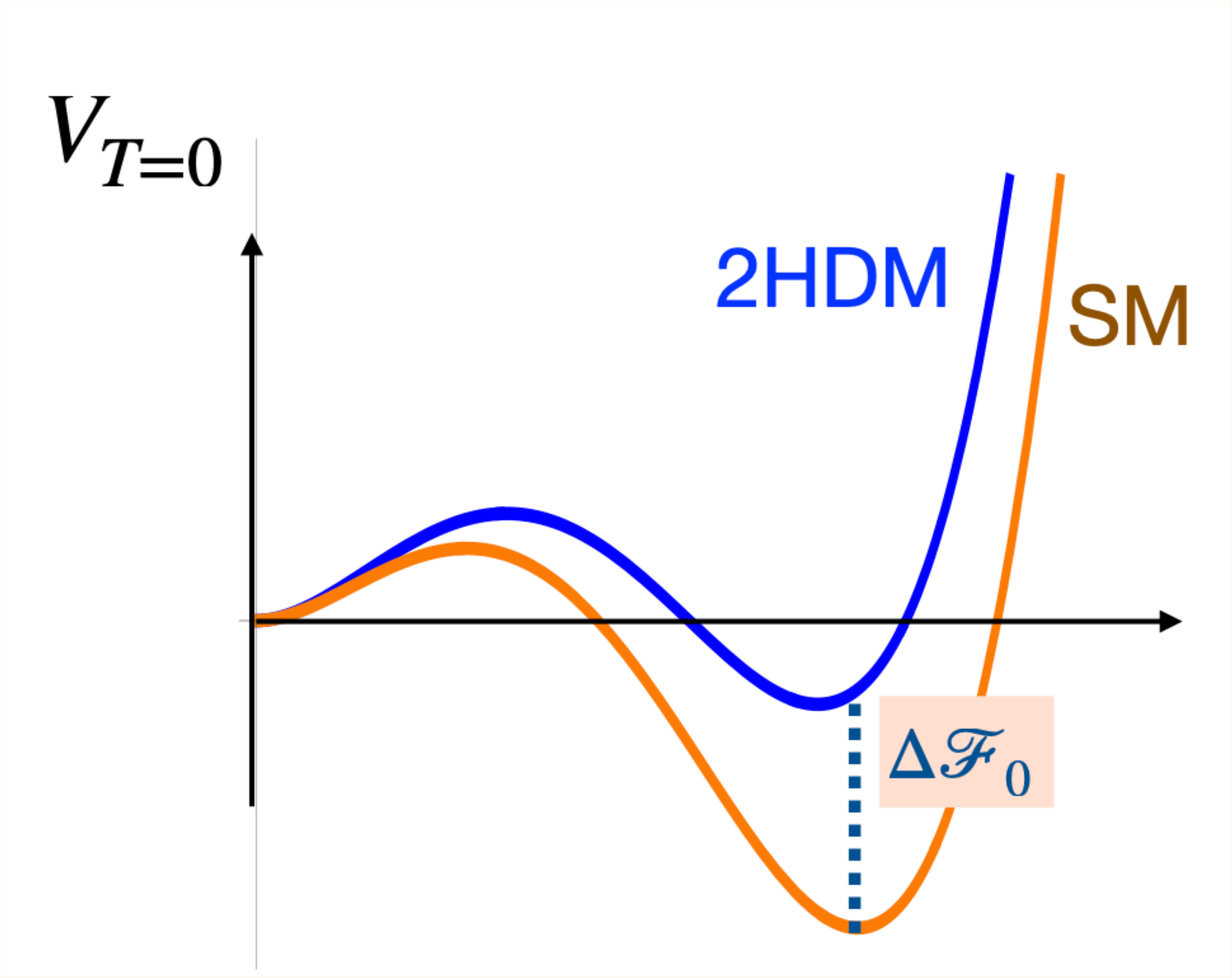
The Uplifting Measure (Ref: Dorsch:2017nza):

$$\Delta\mathscr{F}_0 = \mathscr{F}_0^{\text{2HDM}} - \mathscr{F}_0^{\text{SM}}$$

$\Delta\mathscr{F}_0 > 0$ indicates strong BSM field transformation of the Higgs potential.

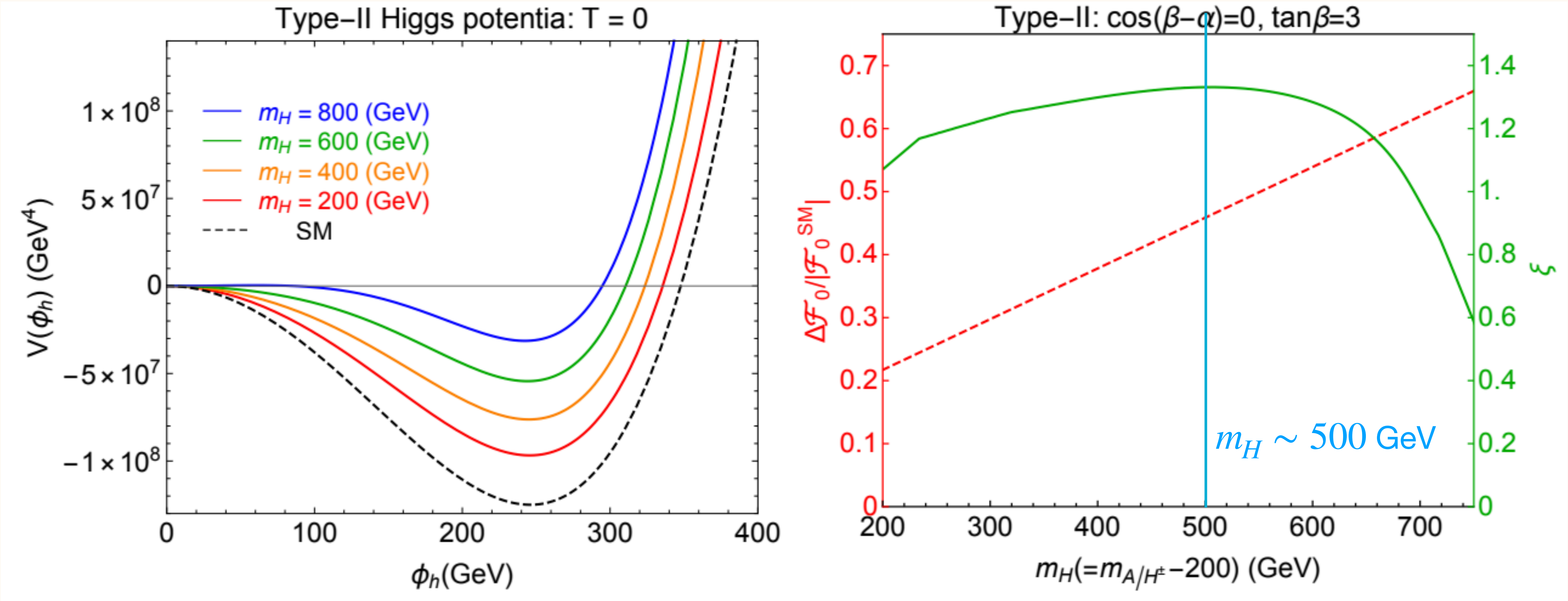
SFOPT Connection:

This strong transformation creates the necessary high energy barrier at T_c, making the SFOPT condition ($\xi_\mathbf{c} = \mathbf{v_c}/\mathbf{T_c}$) easier to satisfy.



Ref: Jeonghyeon Song's talk in HPNP2025

Vacuum uplifting in One-Step PT



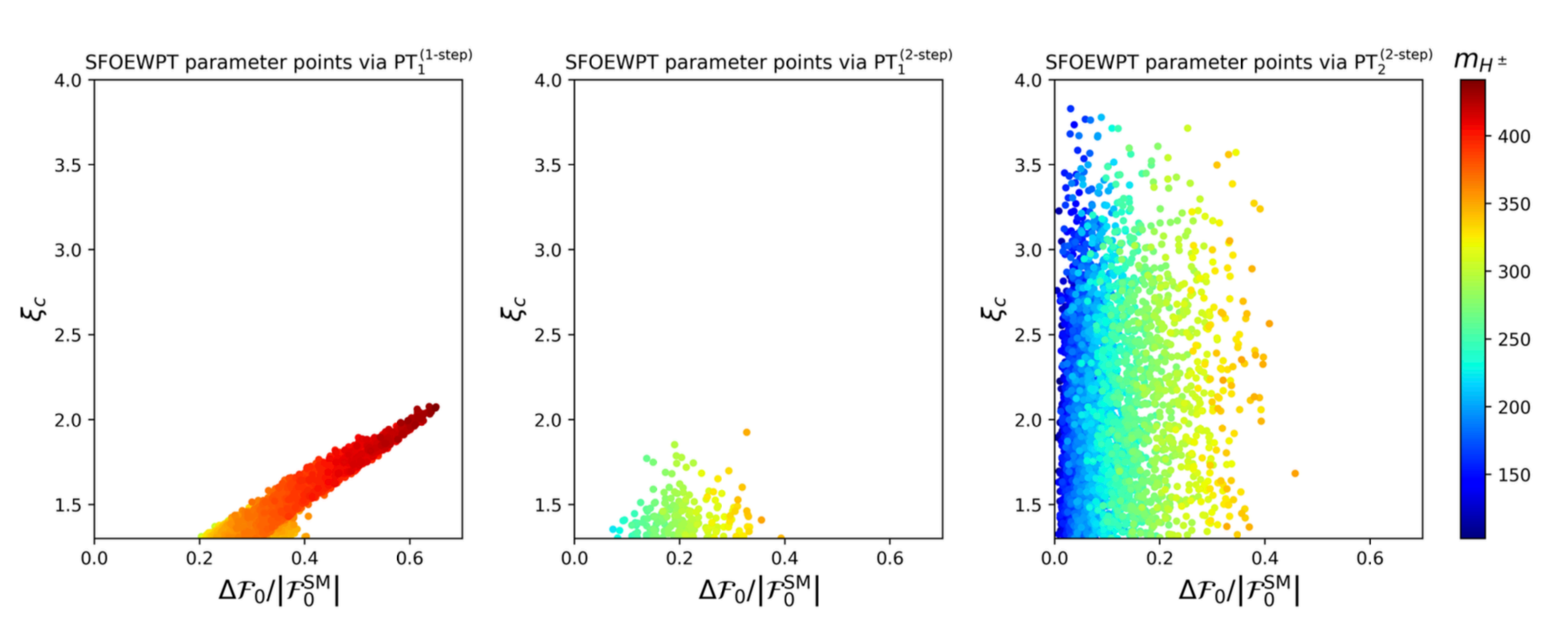
Ref: Wei Su, Su:2020pjw

There is a study show a strong positive correlation between the vacuum uplifting ratio ($\boldsymbol{\Delta\mathcal{F}_0}/|\mathcal{F}_\mathbf{0}^{\text{SM}}|$) and the SFOPT strength ($\xi_\mathbf{c}$) (up to $m_H \sim 500$ GeV).

Question:

Does the strong $\Delta\mathcal{F}_0 - \xi_c$ correlation remain valid for Multi-step Phase Transitions?

Vacuum uplifting in Multi-Step PT

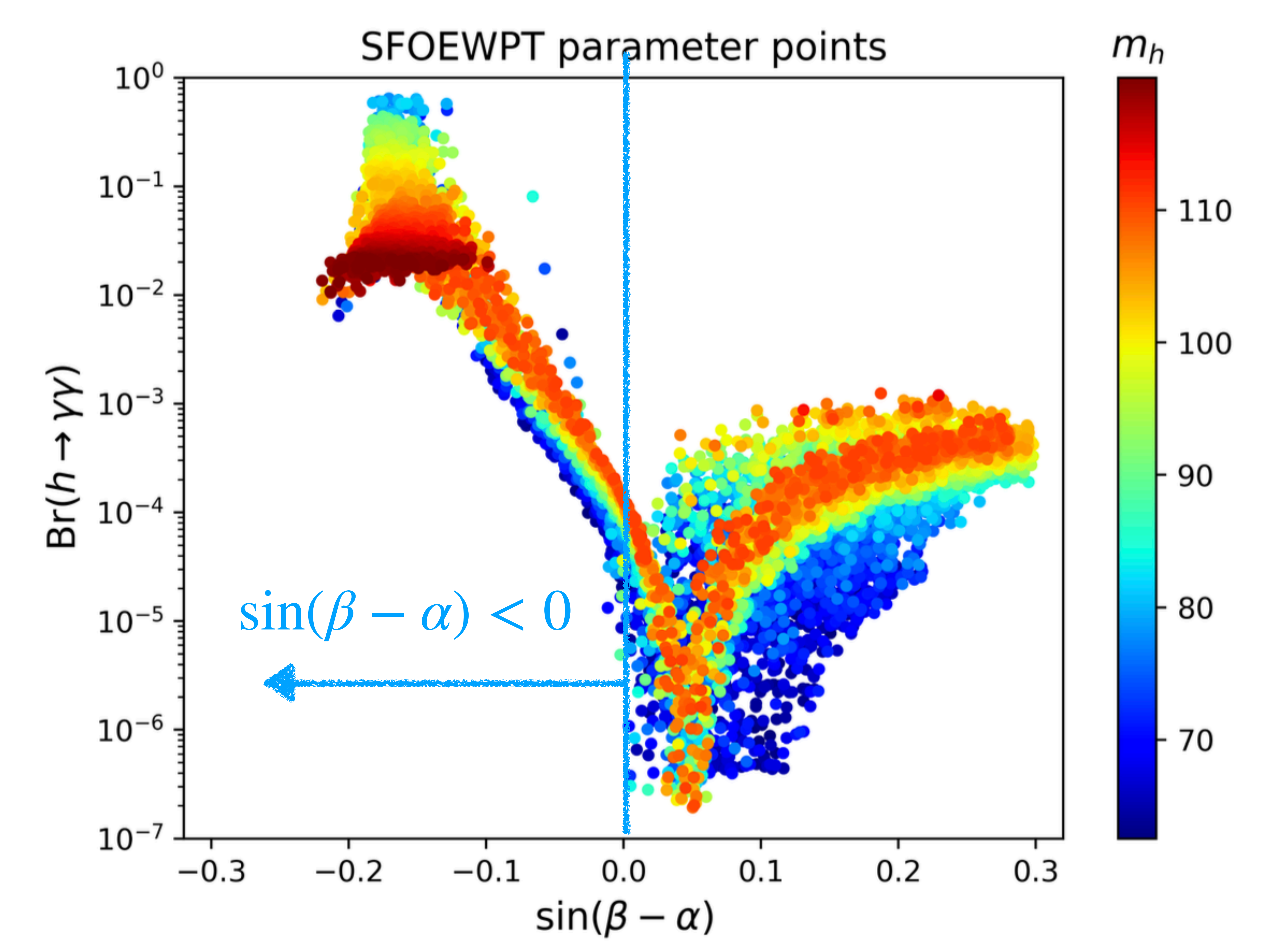


Answer: No!

There is no correlation between $\Delta\mathscr{F}_0/|\mathscr{F}_0^{\mathrm{SM}}|$ and ξ_c in Multi-step PTs.

+) Collider Signatures

Decay modes of h



Dominant decay modes of h

1. Most case:
(1) $h \to bb$, (2) $h \to \tau\tau$

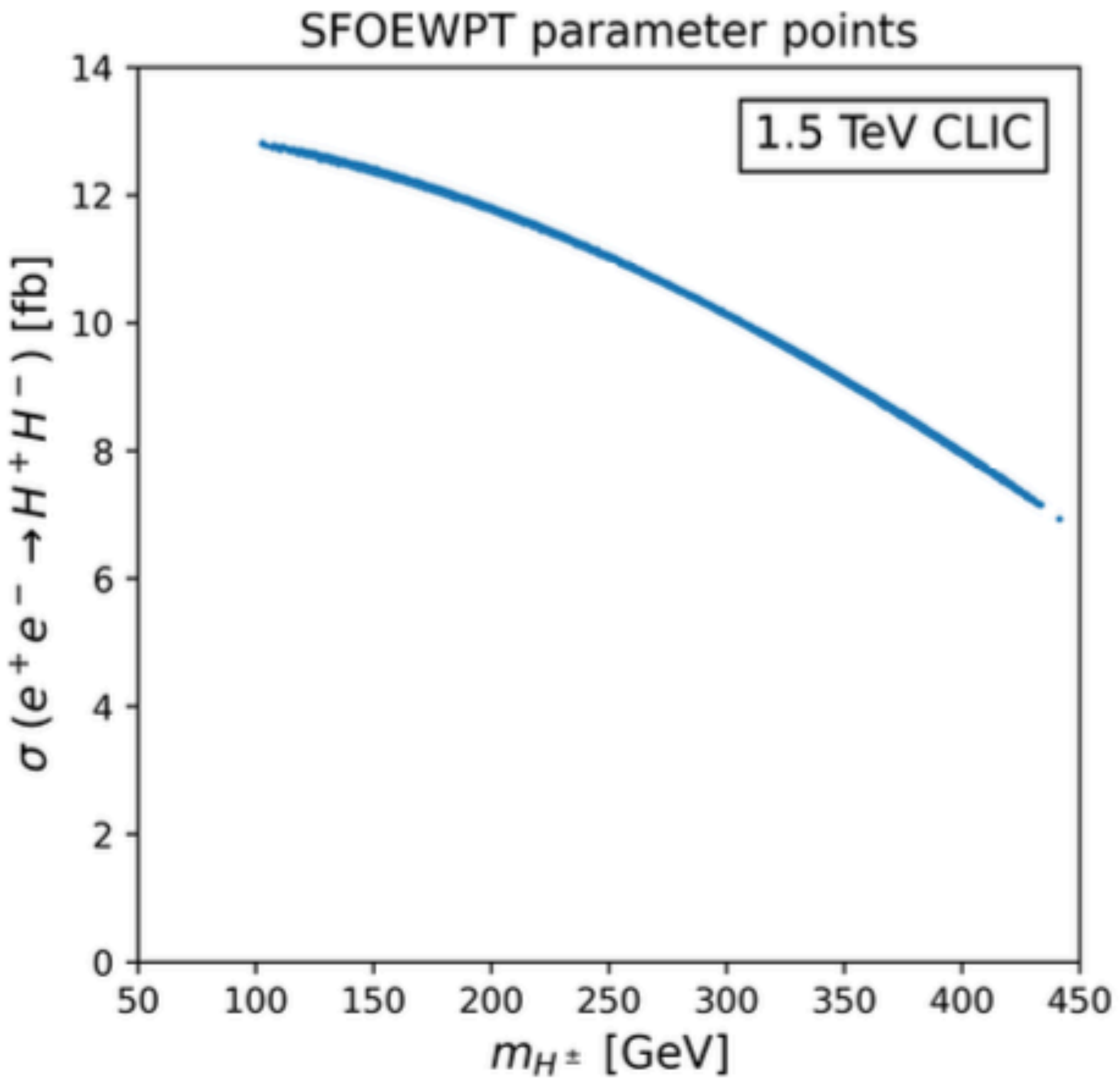
2. Part of region with $\sin(\beta - \alpha) < 0$:
(1) $h \to \gamma\gamma$, (2) $h \to bb$

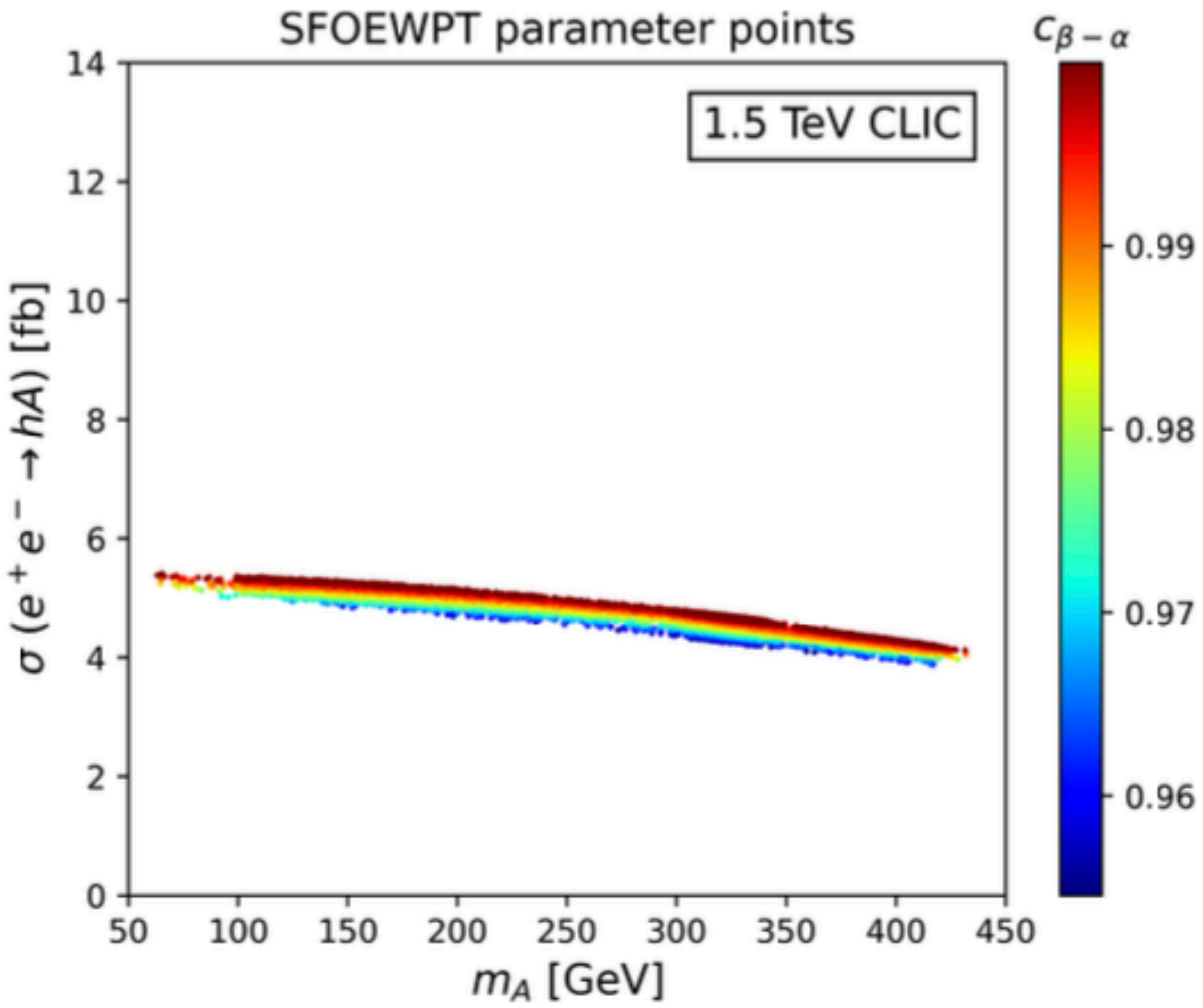
$$\xi_f^h = \sin(\beta - \alpha) + \frac{\cos(\beta - \alpha)}{\tan \beta}$$

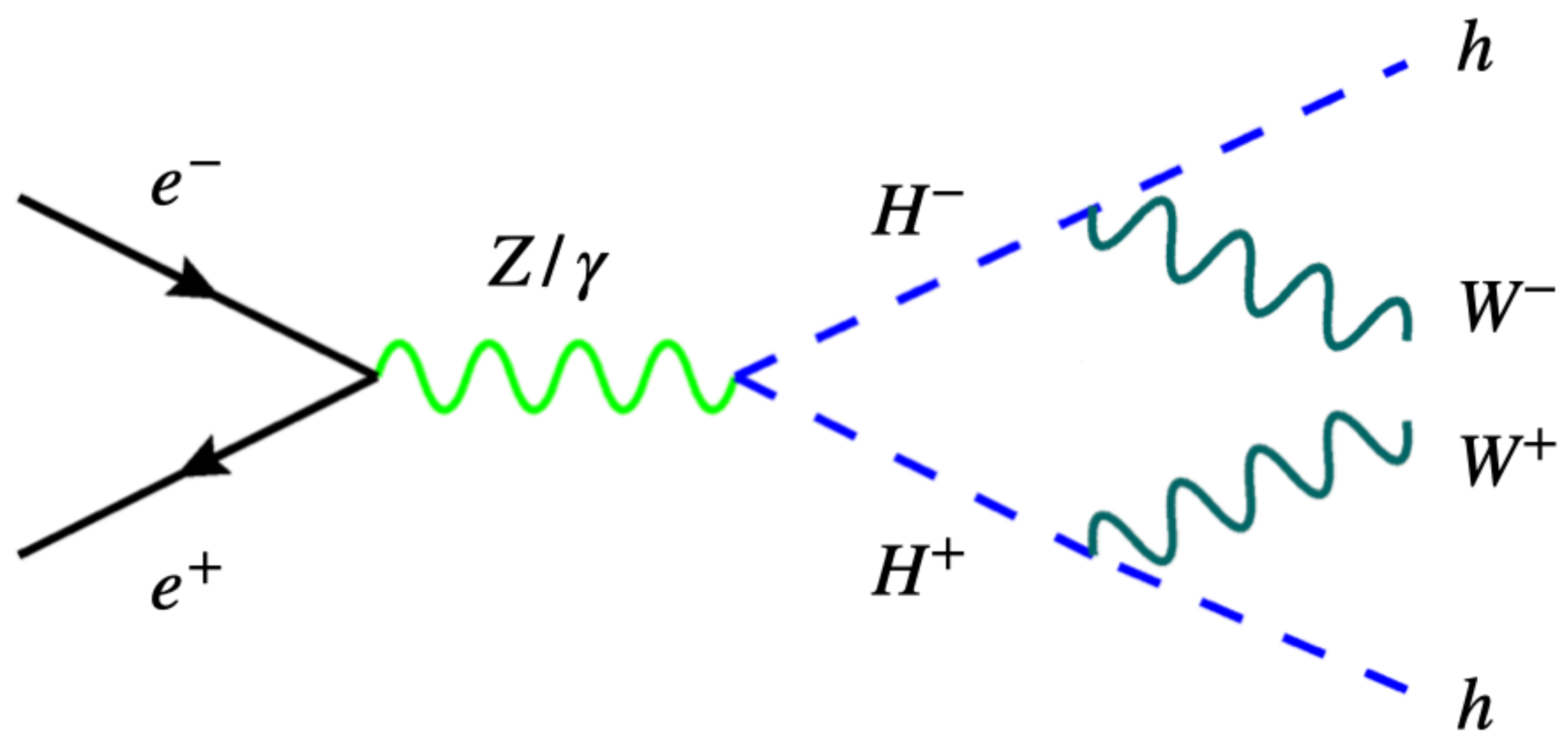
$\sin(\beta - \alpha) < 0 \Rightarrow$ cancellation $\Rightarrow$ smaller ξ_f^h

$\Rightarrow$ larger $\mathrm{Br}(h \to \gamma\gamma)$

Production Processes







(1) $bb\tau\tau$ mode: $e^+e^- \to H^+H^- \to W^+W^-hh \to W^+W^-b\bar{b}\tau^+\tau^- \to jjb\bar{b}\tau^+\tau^-$

(2) $bb\gamma\gamma$ mode: $e^+e^- \to H^+H^- \to W^+W^-hh \to W^+W^-b\bar{b}\gamma\gamma \to jjb\bar{b}\gamma\gamma$

Benchmark points

BP	$m_{H^\pm}$	m_h	m_A	$s_{\beta-\alpha}$	t_β	$\overline{M}$	$\mathrm{Br}(h \to bb)$	$\mathrm{Br}(h \to \gamma\gamma)$	Stage	ξ_c
BP-1	104.6	71.3	107.5	0.022	11.8	72.0	8.42×10^{-1}	3.00×10^{-6}	$\mathrm{PT}_2^{(2\text{-step})}$	2.23
BP-2	199.1	70.4	168.7	0.151	4.5	68.6	8.42×10^{-1}	3.50×10^{-5}	$\mathrm{PT}_2^{(2\text{-step})}$	1.31
BP-3	199.4	101.2	174.3	0.184	12.6	95.9	8.02×10^{-1}	6.39×10^{-4}	$\mathrm{PT}_2^{(2\text{-step})}$	1.69
BP-4	301.3	71.2	300.7	0.037	8.6	51.2	8.42×10^{-1}	3.00×10^{-6}	$\mathrm{PT}_2^{(2\text{-step})}$	1.47
BP-5	309.2	97.7	313.5	0.217	9.9	90.4	8.09×10^{-1}	5.07×10^{-4}	$\mathrm{PT}_2^{(2\text{-step})}$	2.80
BP-6	345.5	78.0	343.8	-0.171	5.7	52.6	3.52×10^{-2}	6.42×10^{-1}	$\mathrm{PT}_1^{(1\text{-step})}$	1.41
BP-7	351.1	79.8	338.7	0.200	6.8	61.4	8.33×10^{-1}	1.82×10^{-4}	$\mathrm{PT}_2^{(2\text{-step})}$	2.57
BP-8	395.1	102.7	402.5	0.004	5.8	85.6	8.06×10^{-1}	9.10×10^{-5}	$\mathrm{PT}_1^{(1\text{-step})}$	1.77
BP-9	401.8	95.6	399.2	-0.186	5.6	70.6	1.90×10^{-1}	2.02×10^{-1}	$\mathrm{PT}_1^{(1\text{-step})}$	1.77
BP-10	403.1	119.8	393.6	-0.178	4.6	102.6	1.53×10^{-1}	1.86×10^{-2}	$\mathrm{PT}_1^{(1\text{-step})}$	1.77

Event selections

For the $bb\tau\tau$ mode:

$e^+e^- \to \ell\nu jjb\bar{b}\tau^+\tau^-$

Lepton Multiplicity: $N_{\ell^\pm} = 1$

Jet Multiplicity: $N_j = 2,\, N_b = 2,\, N_\tau = 2$

Invariant Mass Cuts: $m_{b\bar{b}} < m_h,\, m_{\tau_h\tau_h} < m_h$

For the $bb\gamma\gamma$ mode:

$e^+e^- \to \ell\nu jjb\bar{b}\gamma\gamma$

Lepton Multiplicity: $N_{\ell^\pm} = 1$

Jet Multiplicity: $N_j = 2,\, N_b = 2$

Photon Multiplicity: $N_\gamma = 1$

Invariant Mass Cuts: $m_{b\bar{b}} < m_h,\, m_{\gamma\gamma} < m_h$

The Background is **negligible**

Number of Signal Events

Number of signal events for $e^+e^- \to H^+H^-$ at the 1.5 TeV CLIC with $\mathcal{L}_{\rm tot} = 4\,\mathrm{ab}^{-1}$					
Benchmark	$bb\tau\tau$ mode	$bb\gamma\gamma$ mode	Benchmark	$bb\tau\tau$ mode	$bb\gamma\gamma$ mode
BP-1	68.49	0.00	BP-6	0.06	36.14
BP-2	46.64	0.05	BP-7	29.64	0.26
BP-3	68.50	0.98	BP-8	34.10	0.12
BP-4	38.55	0.00	BP-9	1.53	54.16
BP-5	42.38	0.75	BP-10	1.17	4.35

The SFOEWPT region offers **high discovery potential** at the 1.5 TeV CLIC via Charged Higgs pair production.