Positivity for BSM

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Based on 2505.02910 and WIP

國家科學及技術委員會
NSTC National Science and Technology Council
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erc European Research Council
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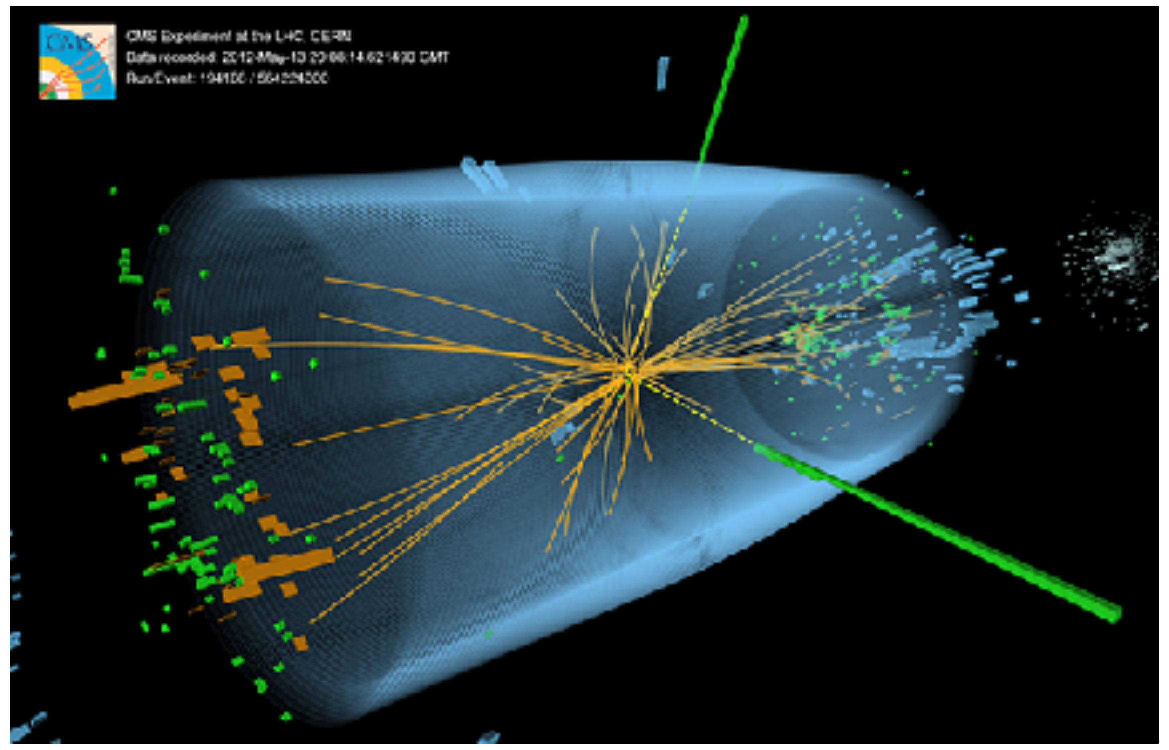
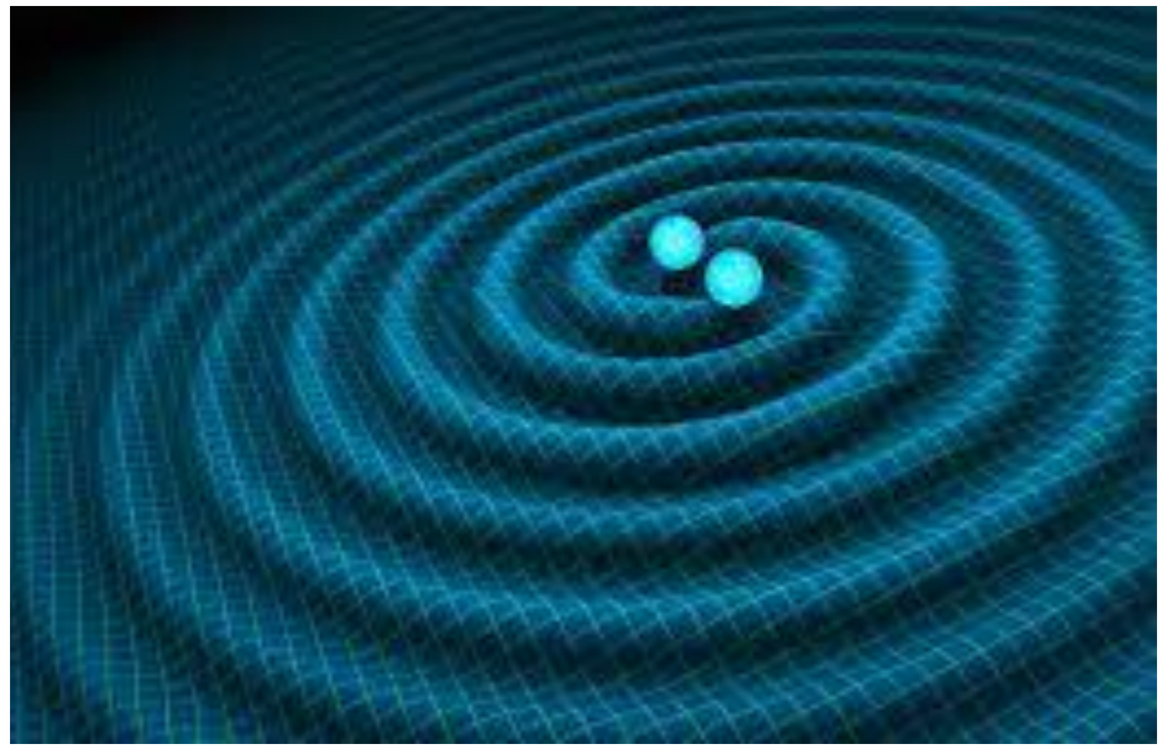
Question:

Can we apply bootstrap/positivity bounds to the real world?





IR Positivity from UV

$$\mathcal{L}_{\rm int} = \frac{c_8}{\Lambda^4}(\partial\phi)^4 + \dots$$

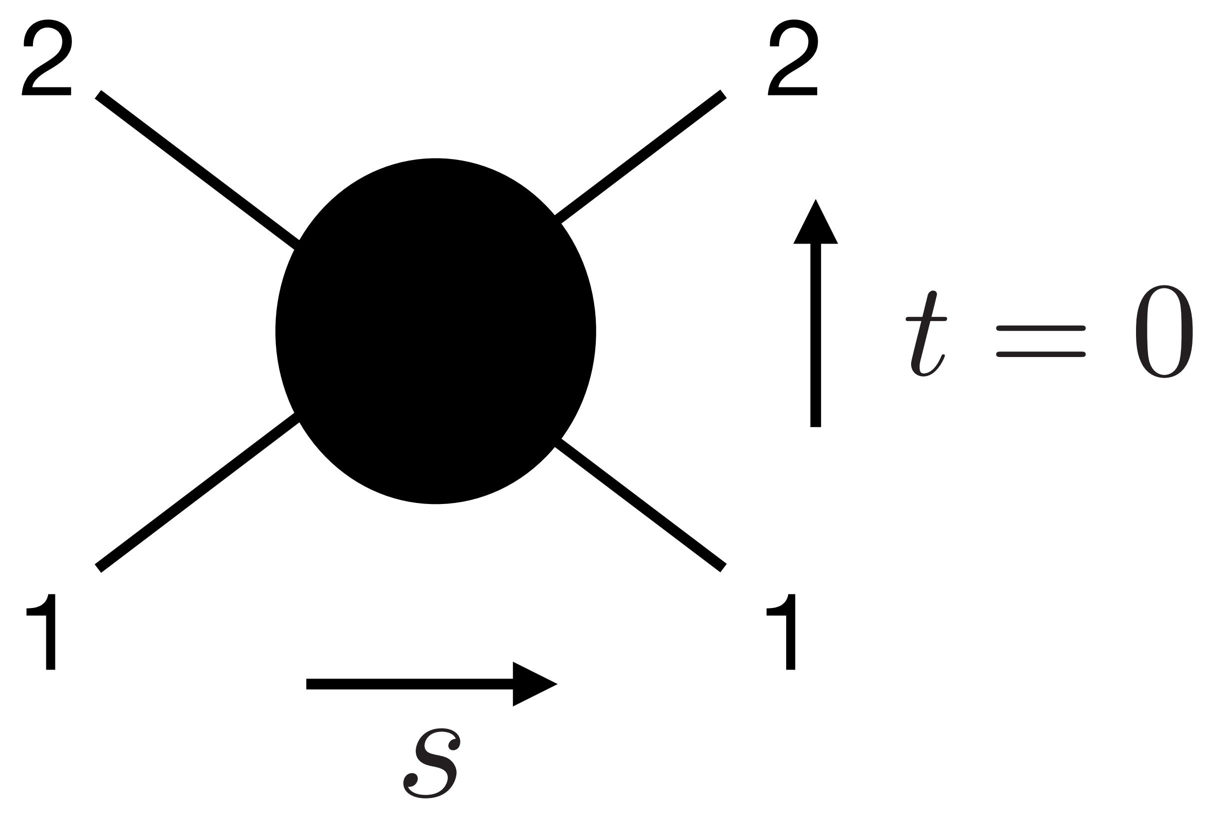
$$A \propto \frac{c_8}{\Lambda^4}(s^2 + t^2 + u^2) + \dots$$

IR Effective field theorists:

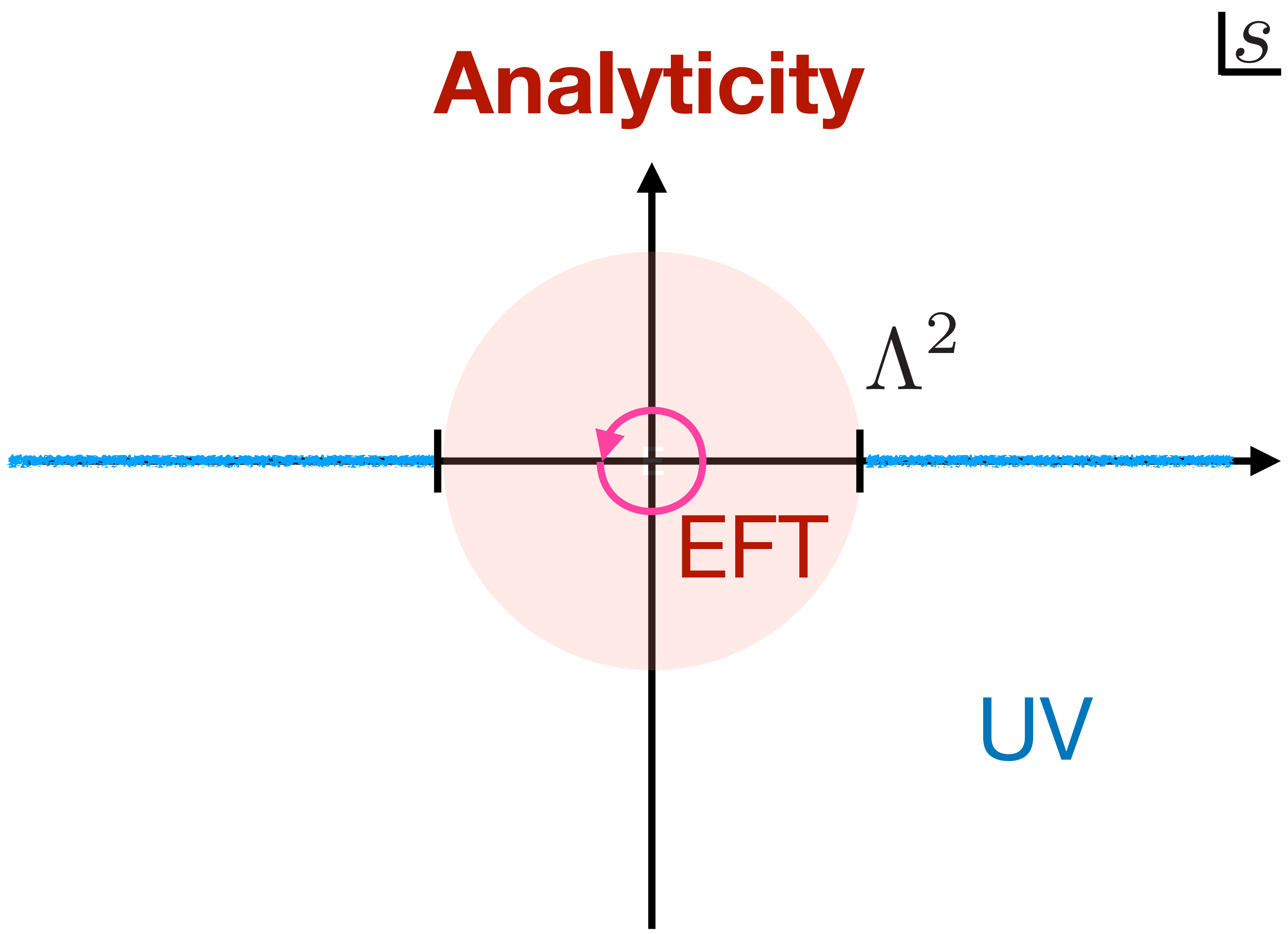
$$c_8 \sim \mathcal{O}(1)$$

IR Positivity from UV

[Pham, Truong, ’85] [Adams, Arkani-Hamed, Dubovsky, Nicholas, Rattazzi ’06, …]



$$A = \frac{c_8}{\Lambda^4}(s^2+t^2+u^2)+\dots$$



$$c_8 = \frac{1}{2\pi i}\oint \frac{ds'}{s'^3}A(s')$$

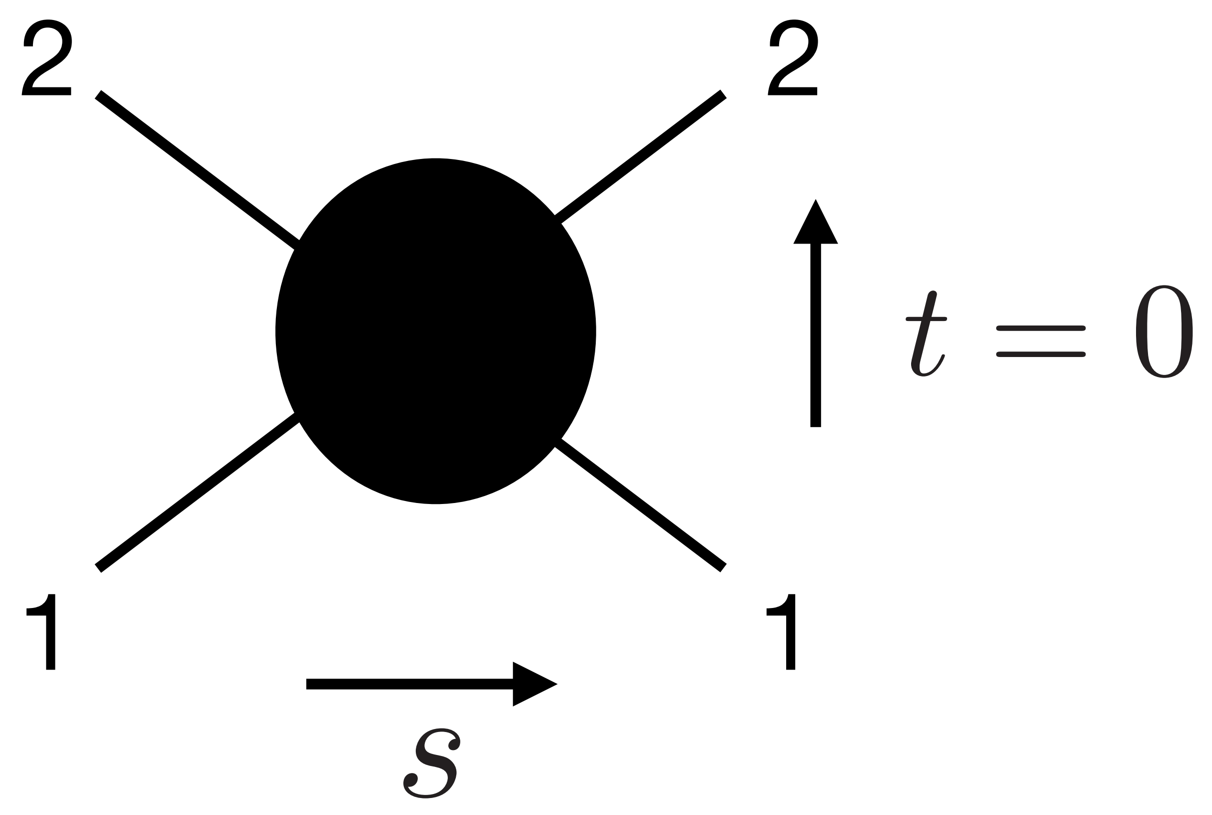
IR Positivity from UV

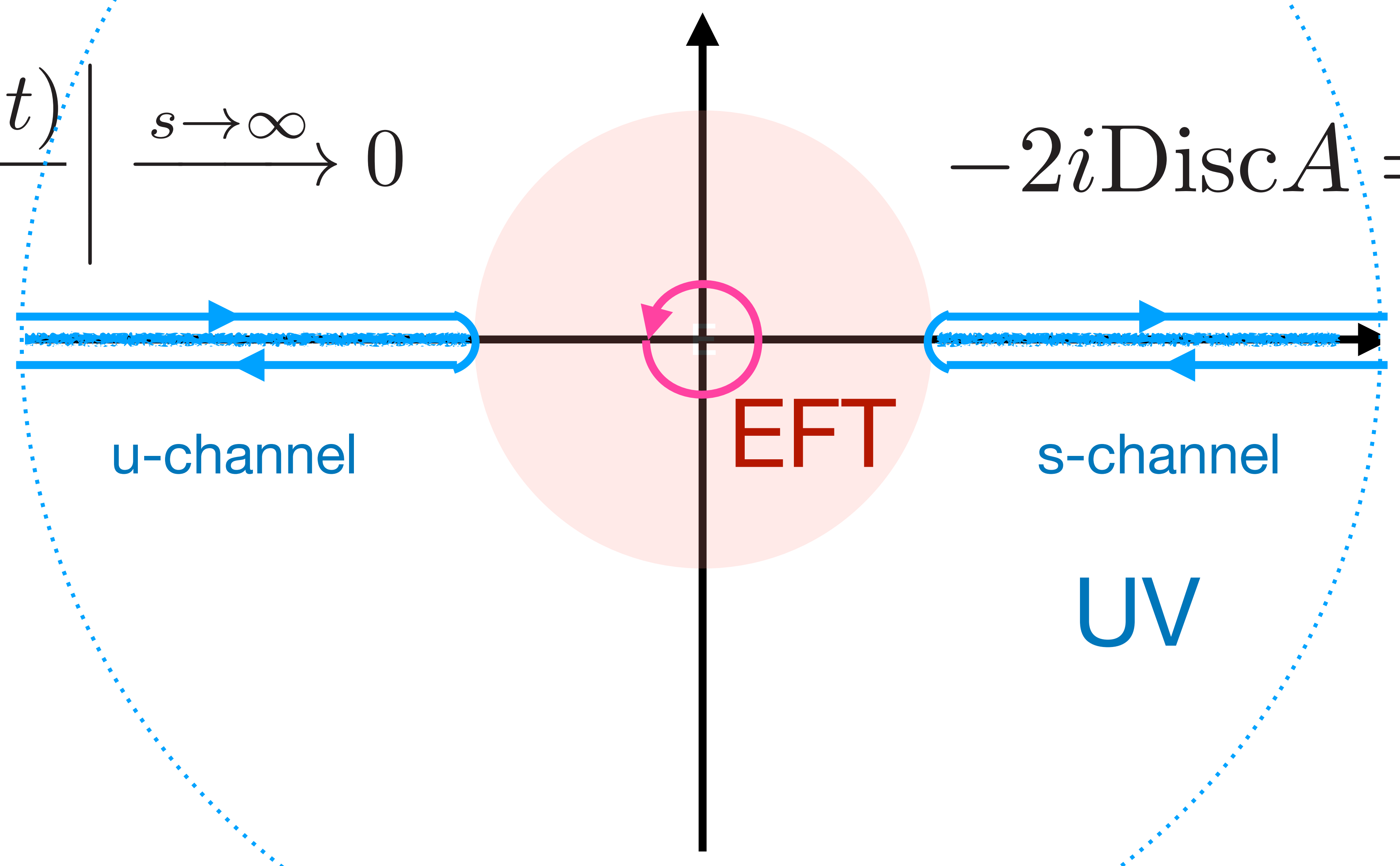
[Pham, Truong, ’85] [Adams, Arkani-Hamed, Dubovsky, Nicholas, Rattazzi ’06, …]

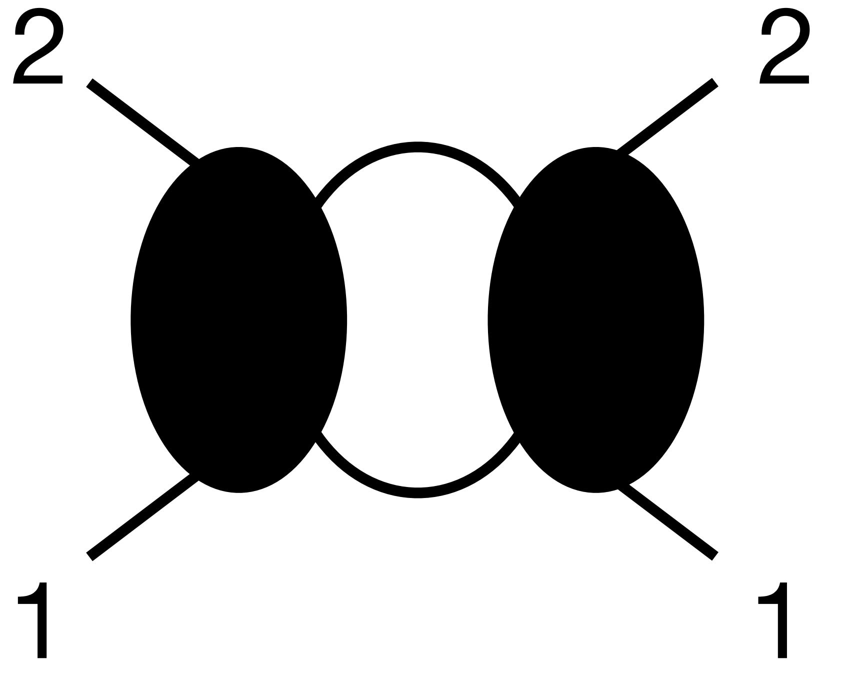
Causality **Analyticity** **Unitarity**

$$\left|\frac{A(s,t)}{s^2}\right| \xrightarrow{s\to\infty} 0$$

$$-2i\mathrm{Disc}A = A^\dagger A \geq 0$$







$$c_8 = \frac{1}{2\pi i}\oint \frac{ds'}{s'^3}A(s')$$

IR Positivity from UV

[Pham, Truong, ’85] [Adams, Arkani-Hamed, Dubovsky, Nicholas, Rattazzi ’06, …]

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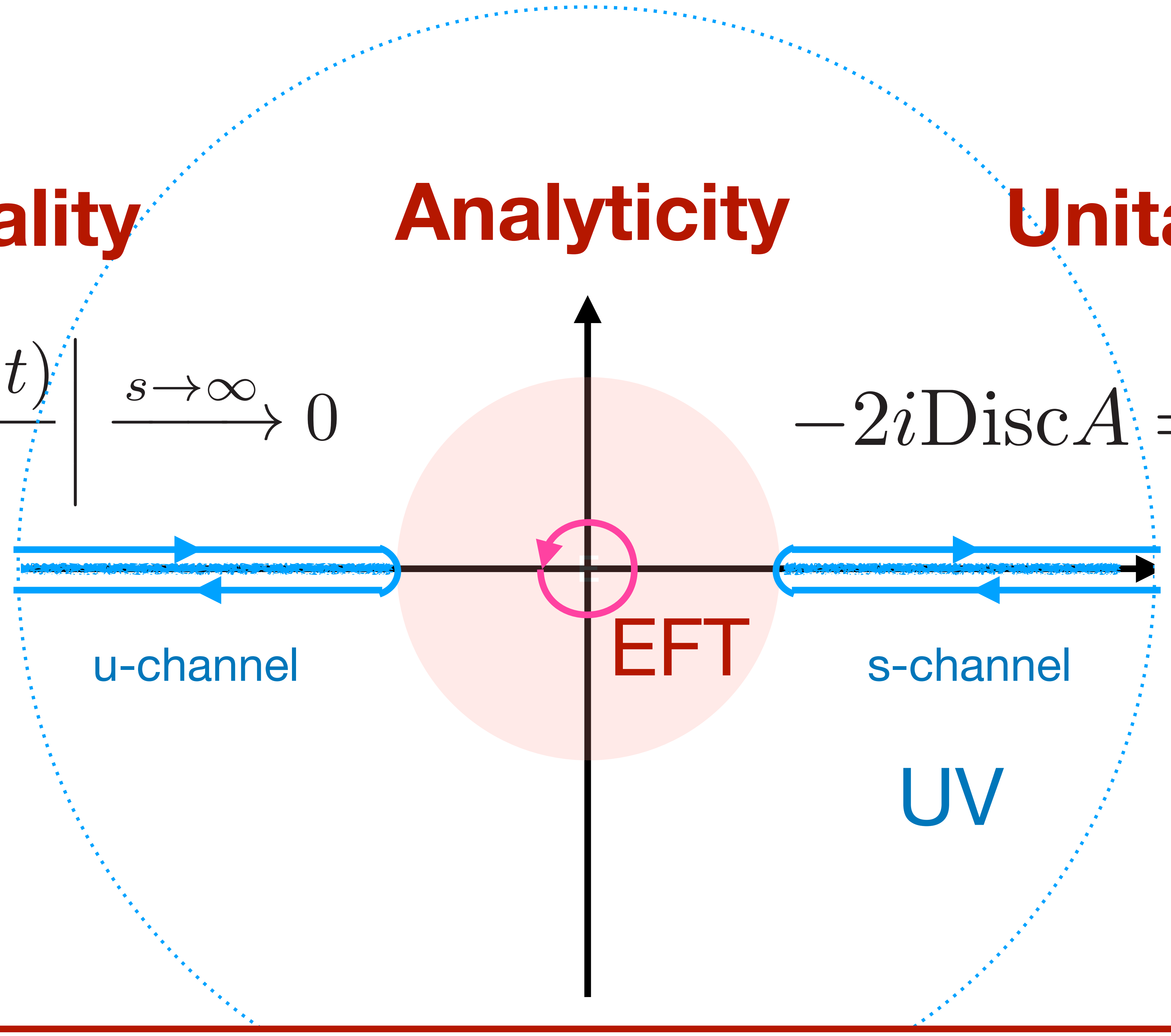
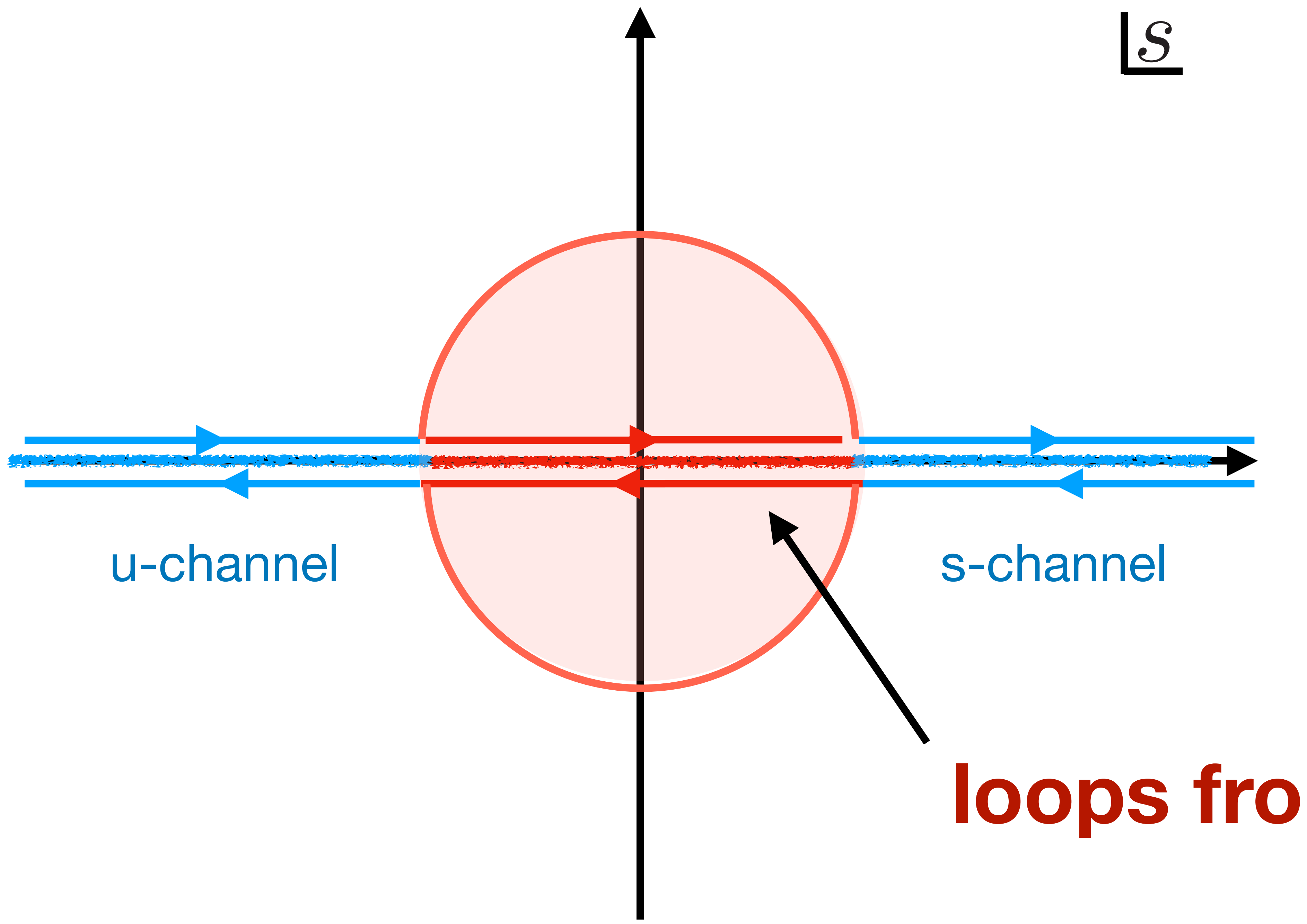


UV-IR relation: $$c_8 = \frac{1}{2\pi i}\oint \frac{ds'}{s'^3}A(s') = \int_{\Lambda^2}^{\infty}\frac{ds'}{\pi s'^3}|A(s')|^2 \geq 0$$

IR Positivity from UV

$$\mathcal{L}_{\rm int} = \lambda\phi^4 + \frac{c_6}{\Lambda^4}\,\partial^2\phi^4 + \frac{c_8}{\Lambda^4}\,(\partial\phi)^4$$

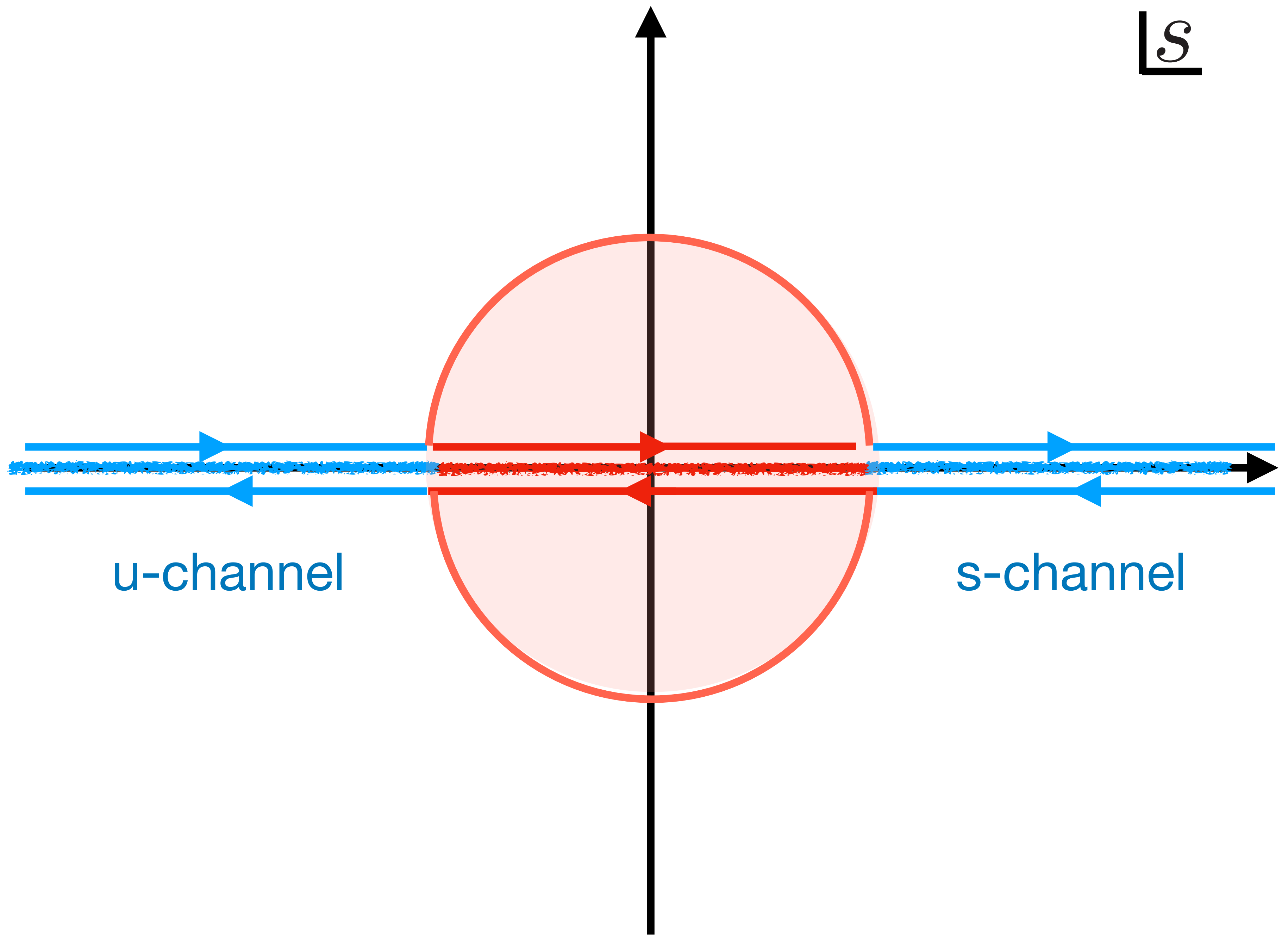


loops from light particles

Positivity for Real World?

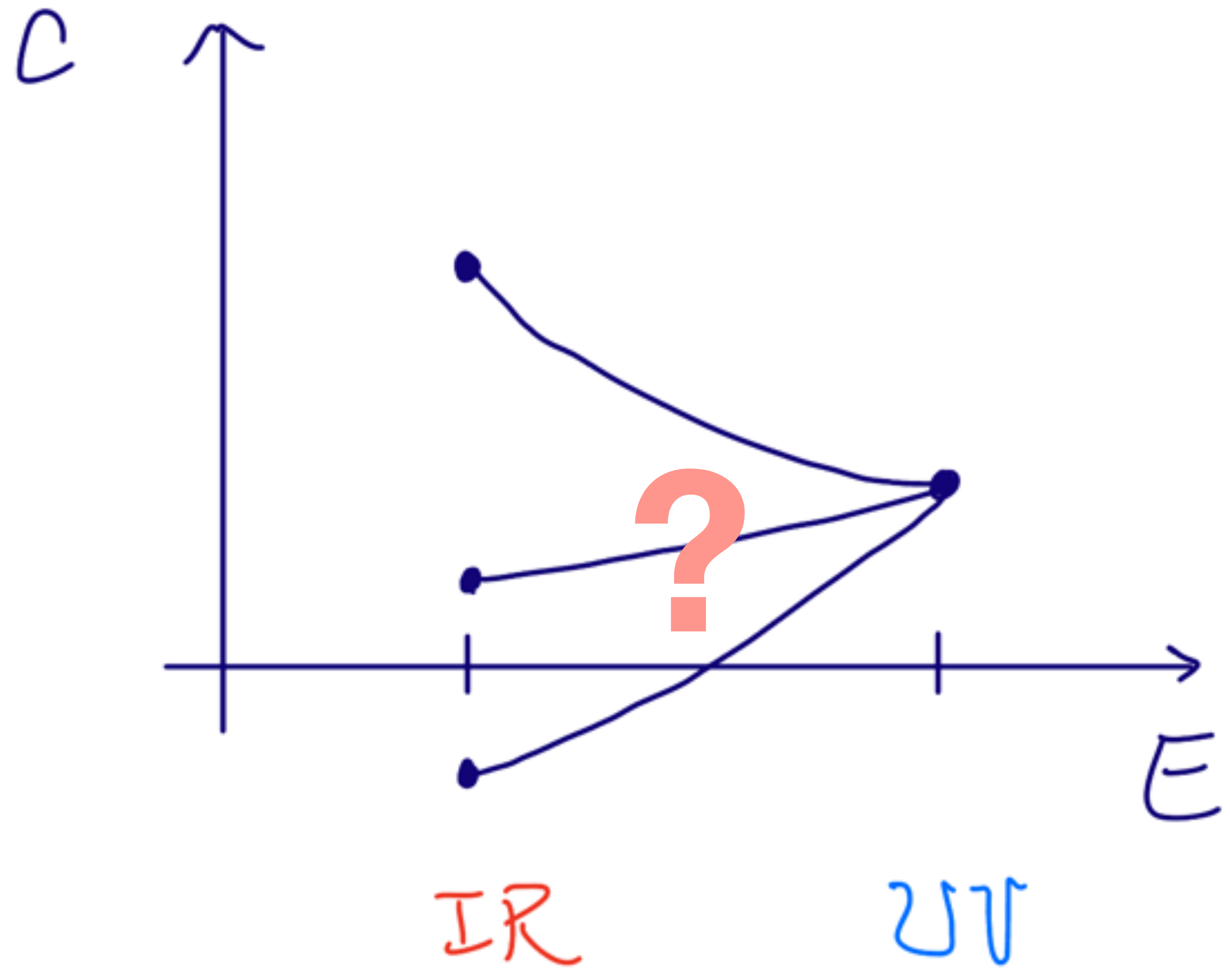
- Many real world theories (SM/GR) contain massless particles and lower order interaction which yield branch cuts in the IR
  - IR branch cuts correct positivity bounds
    Arkani-Hamed, Huang, Huang; Bellazzini, Riembau, Riva; Beadle, Isabella, Perrone, Ricossa, Riva; Chang, Parra-Martinez; Ye, Cao, Wu, Gu; …
  - IR branch cuts induce RG running

    not small for pions



Question:

When does RG preserve tree-level positivity?

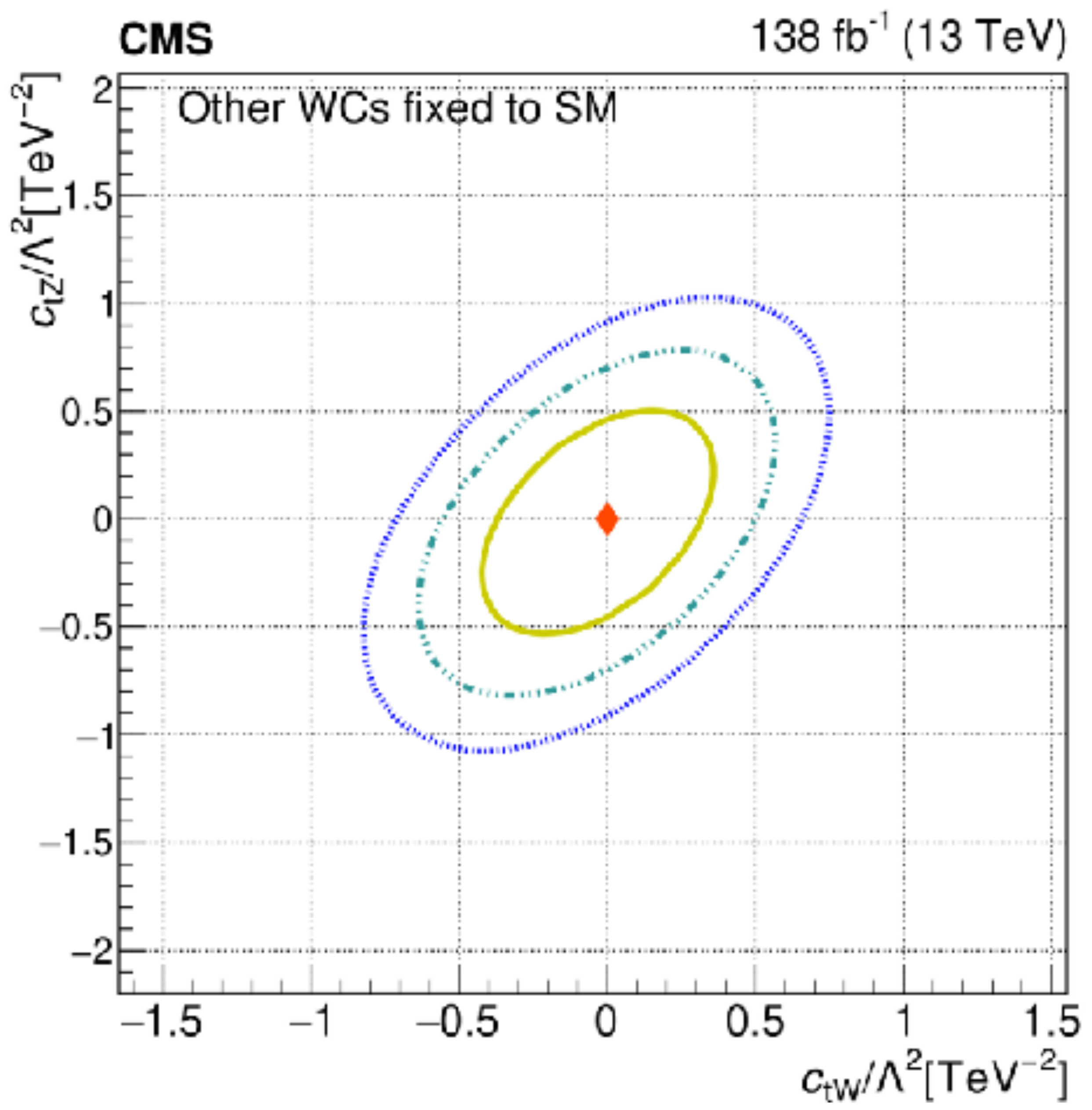


Standard Model EFT

Standard Model EFT (SMEFT):

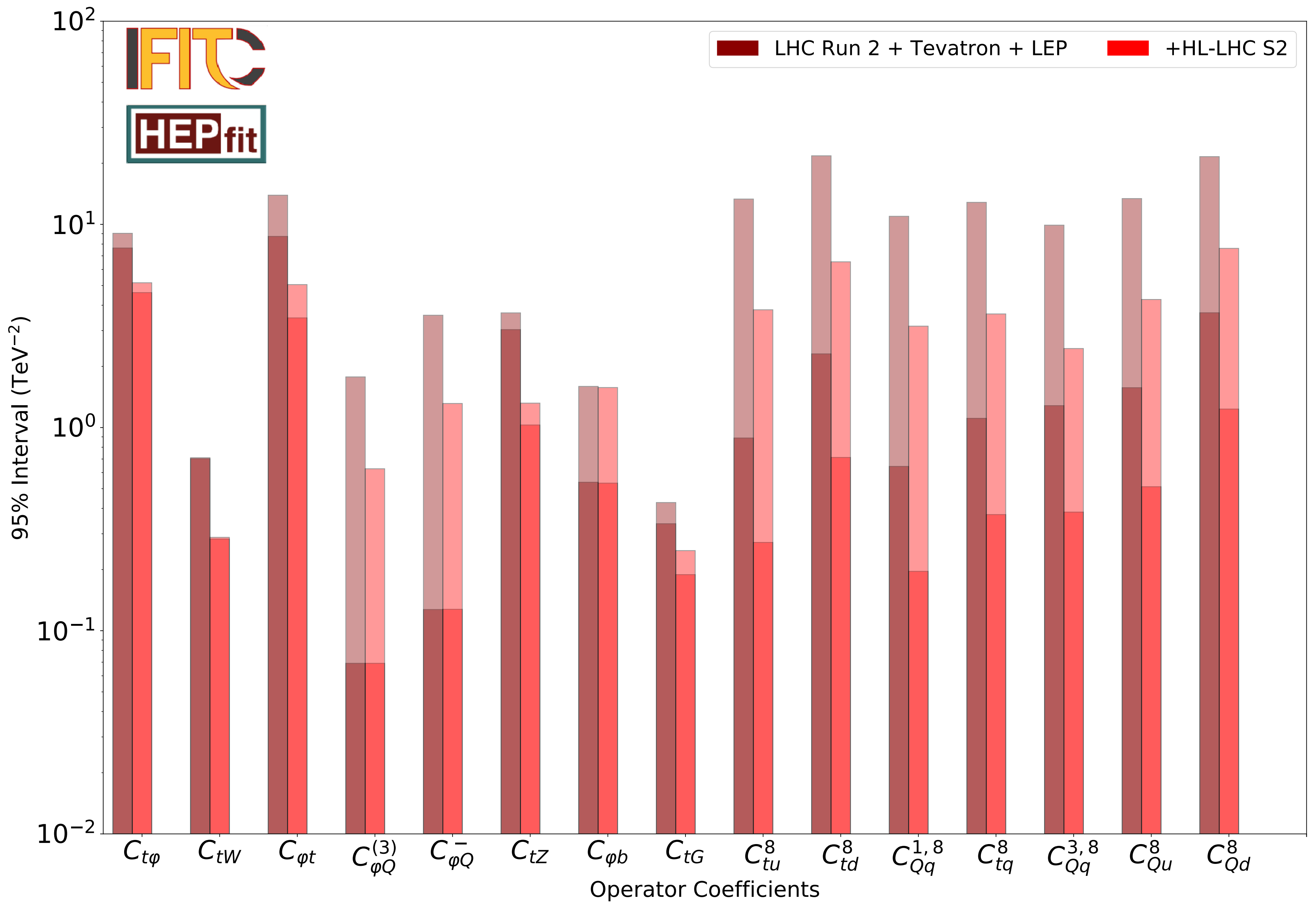
$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_{n,i} \frac{1}{\Lambda^{n-4}} c_i^{(n)} \mathcal{O}_i^{(n)}$$

Risen as a popular scenario for experimental searches



[CMS-CMS-TOP-22-006]

HI-LHC will improve the constraints by several factors



[2205.02140]

Challenge: Rapid growth of the complexity

Standard Model EFT (SMEFT): $\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_{n,i} \frac{1}{\Lambda^{n-4}} c_i^{(n)} \mathcal{O}_i^{(n)}$

Order	dim 4	dim 5	dim 6	dim 7	dim 8
	1	$1/\Lambda$	$1/\Lambda^2$	$1/\Lambda^3$	$1/\Lambda^4$
# of interactions	Standard Model	*2*	*84*	*30*	*993*

[Henning, Lu, Melia, Murayama]

Goal: find universal structures agnostic about UV details

Why going beyond tree-level and dim-6 operators?

[Snowmass white paper: Alioli et al., 2203.06771]

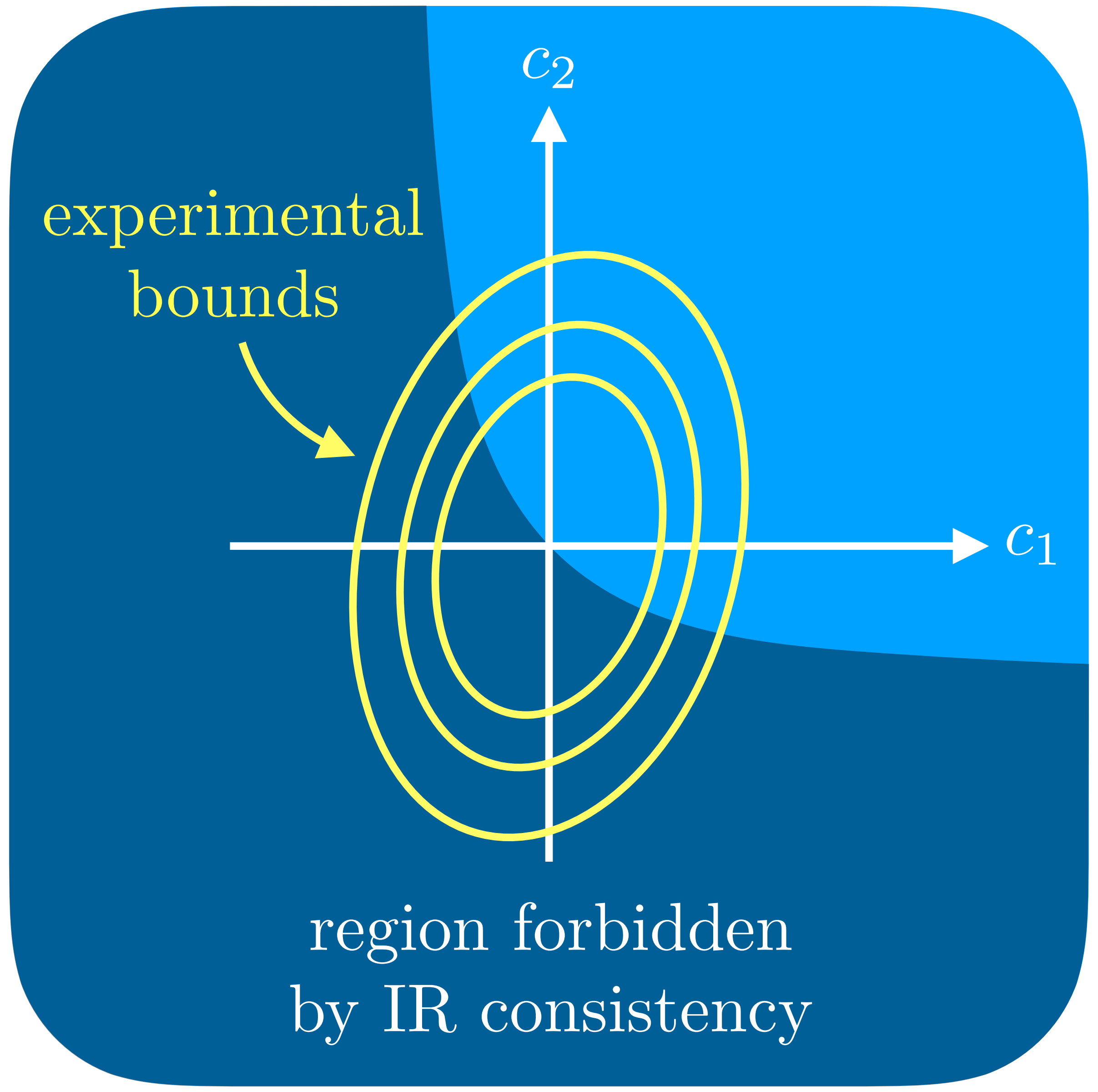
- Scale separation: comparing data from different scales
- High cutoff: the cutoff can be high bounded by low-energy precision measurements
- Observables: the leading correction may starts at dim-8 or loop level
  - Light-by-light scattering. EWPD. Higgs decay. [e.g. see Grojean, Guedes, Nepveu, Salla]
  - RG running can bring tree-level induced operators to these observables
- Interference: the dimension-6 operators may not interfere with SM

$$\sigma \sim (\text{dim-6})^2 + (\text{SM}) \times (\text{dim-8})$$

[Azatov, Contino, Machado, Riva]

Positivity for the SMEFT?

- Positivity for the SMEFT would be interesting for
  - further trimming down the parameter space
  - violation implies exotic UV completion



[Remmen, Rodd '19]

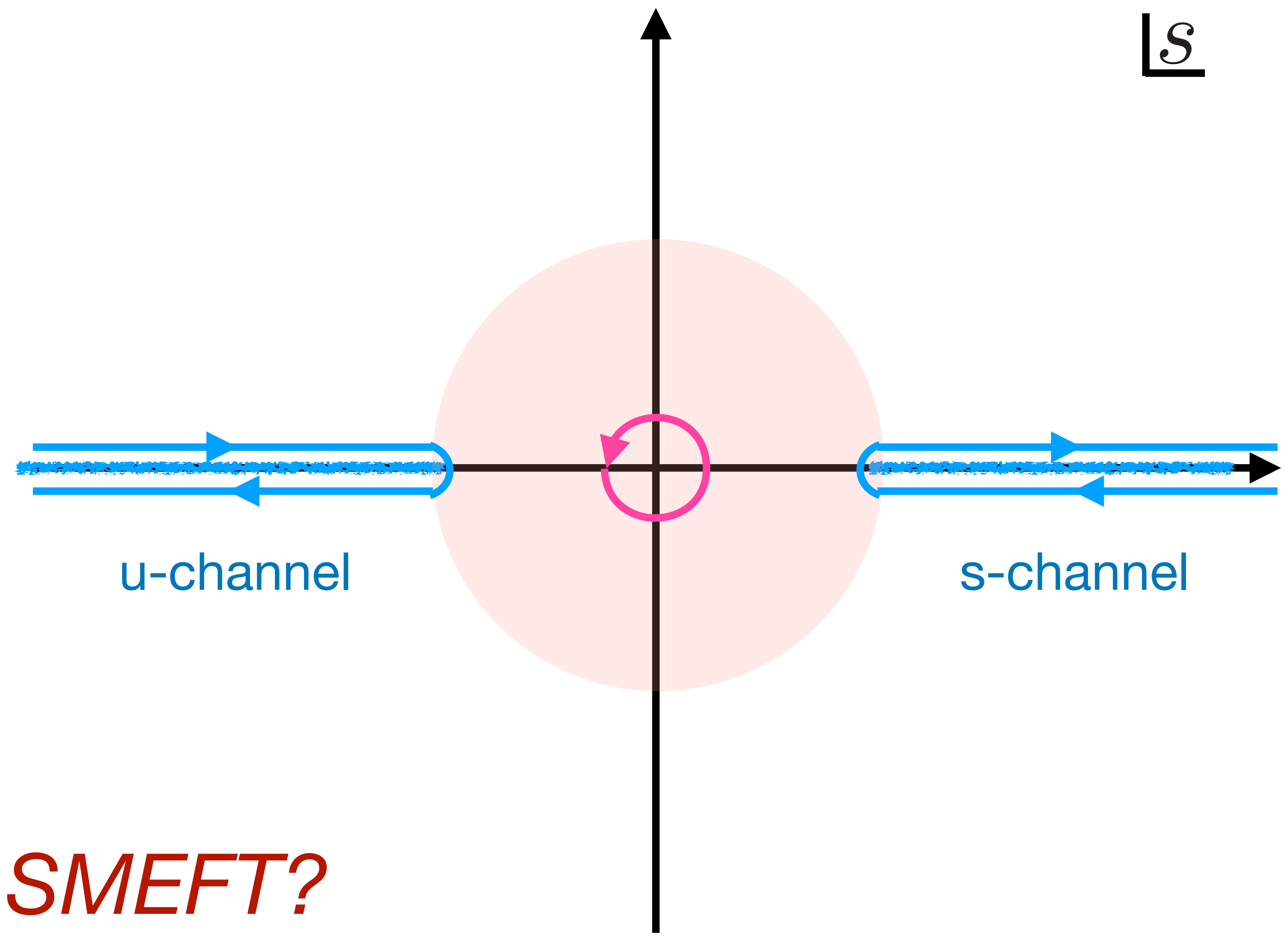
Positivity for the SMEFT?

- Assuming the same tree-level in the IR

[Remmen, Rodd ’19;….]

$\mathcal{O}_{8,1}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_{8,2}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_{8,3}$	$(D_\mu H^\dagger D^\mu H)^2$

$$c^{(2)}_{H^4D^4} > 0\,,$$
$$c^{(2)}_{H^4D^4} + c^{(1)}_{H^4D^4} > 0\,,$$
$$c^{(3)}_{H^4D^4} + c^{(2)}_{H^4D^4} + c^{(1)}_{H^4D^4} > 0$$



Are positivity bounds robust for the SMEFT?

This talk: classify the RG flow

Example: RG of dim-8 operators

$$\mathcal{L} \supset \mathcal{L}_{\rm SM} + \sum_i \frac{1}{\Lambda^2} c_{6,i} \mathcal{O}_{6,i} + \sum_i \frac{1}{\Lambda^4} c_{8,i} \mathcal{O}_{8,i} + \dots$$

$$\frac{dc_{8,i}}{d \ln \mu} = \underbrace{\gamma_{ij}(g) c_{8,j}}_{\text{SM x dim-8}} + \underbrace{\gamma_{ijk}(g) c_{6,j} c_{6,k}}_{\text{dim-6 x dim-6}}$$

Higgs Sector

$\boldsymbol{H^4 D^2}$	
$\mathcal{O}_{HD}$	$(H^\dagger D_\mu H)^* (H^\dagger D^\mu H)$
$\mathcal{O}_{H\Box}$	$(H^\dagger H) \Box (H^\dagger H)$

$\boldsymbol{H^4 D^4}$	
$\mathcal{O}^{(1)}_{H^4 D^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}^{(2)}_{H^4 D^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}^{(3)}_{H^4 D^4}$	$(D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$

Dim-8 RG from a pair of dim-6 operators.

“Simple” operators but not so simple structures

$$\dot{c}^{(1)}_{H^4D^4} = \frac{1}{3}\left(-16\,c^2_{H\Box} + 32\,c_{HD}c_{H\Box} - 11\,c^2_{HD}\right) - \left(\mathcal{G}_1 + 2\,\mathcal{G}_2\right) + \left(\mathcal{F}_1 - \mathcal{F}_2 - \mathcal{F}_3\right),$$

$$\dot{c}^{(2)}_{H^4D^4} = -\frac{1}{3}\left(16\,c^2_{H\Box} + 16\,c_{HD}c_{H\Box} + 5\,c^2_{HD}\right) - \mathcal{G}_1 - \left(\mathcal{F}_1 + \mathcal{F}_2\right),$$

$$\dot{c}^{(3)}_{H^4D^4} = \frac{1}{3}\left(-40\,c^2_{H\Box} - 16\,c_{HD}c_{H\Box} + 7\,c^2_{HD}\right) + \left(2\,\mathcal{G}_1 + \mathcal{G}_2 - \mathcal{G}_3\right) + \left(2\,\mathcal{F}_2 + \mathcal{F}_3\right).$$

scalar loop gauge boson loop fermion loop

$$\mathcal{G}_1 \equiv 6\,g_2^2\left(c^2_{W^3} + c^2_{\widetilde{W}W^2}\right),$$

$$\mathcal{G}_2 \equiv 8\left(c^2_{H^2WB} + c^2_{H^2\widetilde{W}B}\right),$$

$$\mathcal{G}_3 \equiv 16\left(c^2_{H^2B^2} + c^2_{H^2\widetilde{B}B} + 3\left(c^2_{H^2W^2} + c^2_{H^2\widetilde{W}W}\right) + 8\left(c^2_{H^2G^2} + c^2_{H^2\widetilde{G}G}\right)\right)$$

$$\mathcal{F}_1 \equiv \frac{8}{3}\left[3\,\mathrm{tr}\left((c_{Hd})^\dagger c_{Hd}\right) + \mathrm{tr}\left((c_{He})^\dagger c_{He}\right) + 2\,\mathrm{tr}\left((c^{(1)}_{HL})^\dagger c^{(1)}_{HL}\right) + 6\,\mathrm{tr}\left((c^{(1)}_{Hq})^\dagger c^{(1)}_{Hq}\right) + 3\,\mathrm{tr}\left((c_{Hu})^\dagger c_{Hu}\right)\right]$$

$$\mathcal{F}_2 \equiv \frac{16}{3}\left[\mathrm{tr}\left((c^{(3)}_{HL})^\dagger c^{(3)}_{HL}\right) + 3\,\mathrm{tr}\left((c^{(3)}_{Hq})^\dagger c^{(3)}_{Hq}\right)\right],$$

$$\mathcal{F}_3 \equiv 8\,\mathrm{tr}\left((c_{Hud})^\dagger c_{Hud}\right),$$

When we organize the operators by forward scattering

Combinations correspond to forward scattering

[Chala et al. 2106.05291, 2110.01624; Liao, Nepveu, CHS, 2505.02910]

$$\dot{c}^{(2)}_{H^4D^4} = -\frac{1}{3}\left(4\,(2\,c_{H\Box}+c_{HD})^2 + c^2_{HD}\right) - \mathcal{G}_1 - (\mathcal{F}_1+\mathcal{F}_2)\,, \qquad \leq 0$$

$$\dot{c}^{(1)}_{H^4D^4} + \dot{c}^{(2)}_{H^4D^4} = -\frac{1}{3}\left(28\,c^2_{H\Box} + 4\,(c_{H\Box}-2\,c_{HD})^2\right) - (2\,\mathcal{G}_1+2\,\mathcal{G}_2) - (2\,\mathcal{F}_2+\mathcal{F}_3)\,, \qquad \leq 0$$

$$\dot{c}^{(1)}_{H^4D^4} + \dot{c}^{(2)}_{H^4D^4} + \dot{c}^{(3)}_{H^4D^4} = -\left(24\,c^2_{H\Box} + 3\,c^2_{HD}\right) - (\mathcal{G}_2+\mathcal{G}_3)\,, \qquad \leq 0$$

Definite-sign of RG running for arbitrary dim-6 couplings

Universal behavior

Positivity in EFT Renormalization

[2505.02910 w/ You-Peng Liao, Jasper Roosmale Nepveu]

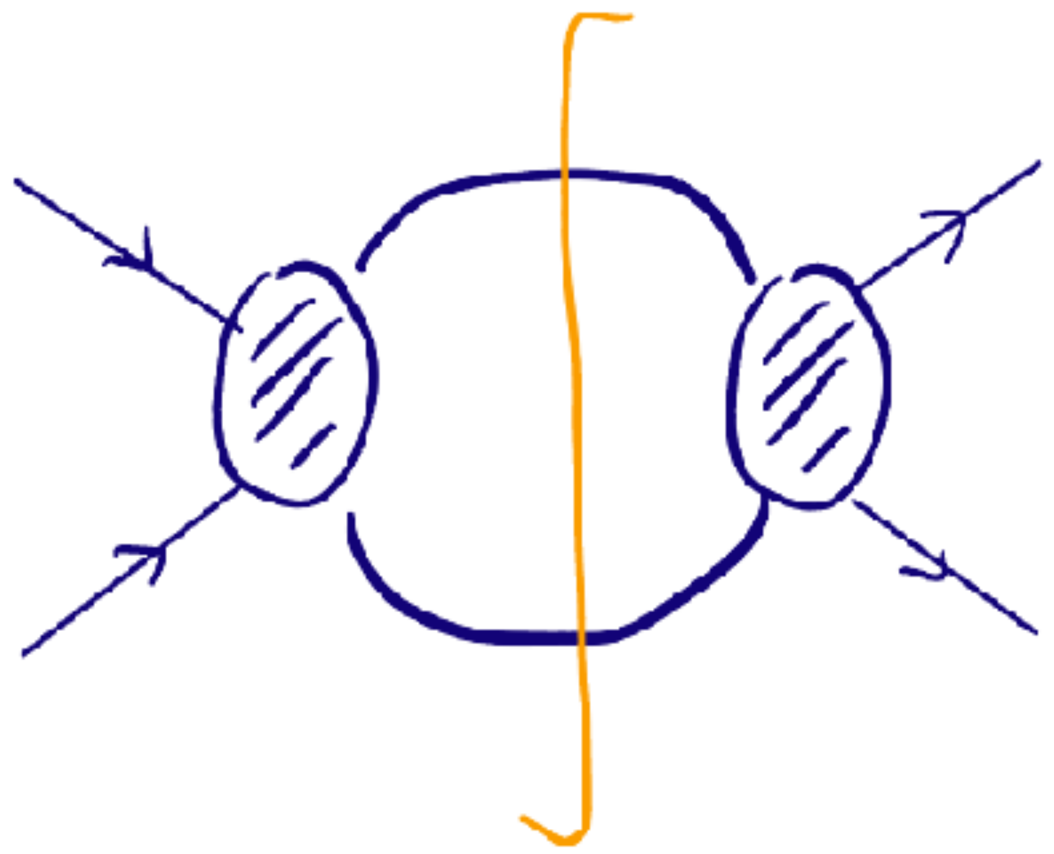
RG from on-shell amplitudes

unitarity & analyticity

$$\ln(s/\mu^2) = \ln s - \ln \mu^2$$

branch cut

renormalization



Consider no 3pt coupling to avoid UV-IR cancellation

$$\frac{d}{d\ln\mu} A^{\text{tree}} = -\frac{d}{d\ln\mu} A^{1-\text{loop}}$$

$$= -\frac{1}{2\pi} \sum_{s,t,u} \text{Im} A$$

optical theorem

$$= -\frac{1}{\pi} \sum_{s,t,u} \int d\text{LIPS} A A^\dagger$$ - IR div.

See full treatment by Caron-Huot, Wilhelm;
Bern, Parra-Martinez, Sawyer; Elias Miro, Ingoldby, Riembau;
Baratella, Fernandez, von Harling, Pomarol;…

RG from on-shell amplitudes

unitarity & analyticity

example:

$$\lambda\phi^4 : \qquad \frac{d\lambda}{d\ln\mu} = \frac{3\lambda^2}{16\pi^2}$$

important to sum over all channels!

t-channel branch cut can contribute

$$\frac{d}{d\ln\mu}A^{\text{tree}} = -\frac{d}{d\ln\mu}A^{1-\text{loop}}$$

$$= -\frac{1}{2\pi}\sum_{s,t,u} \text{Im}A$$

optical theorem

$$= -\frac{1}{\pi}\sum_{s,t,u}\int d\text{LIPS}AA^\dagger$$

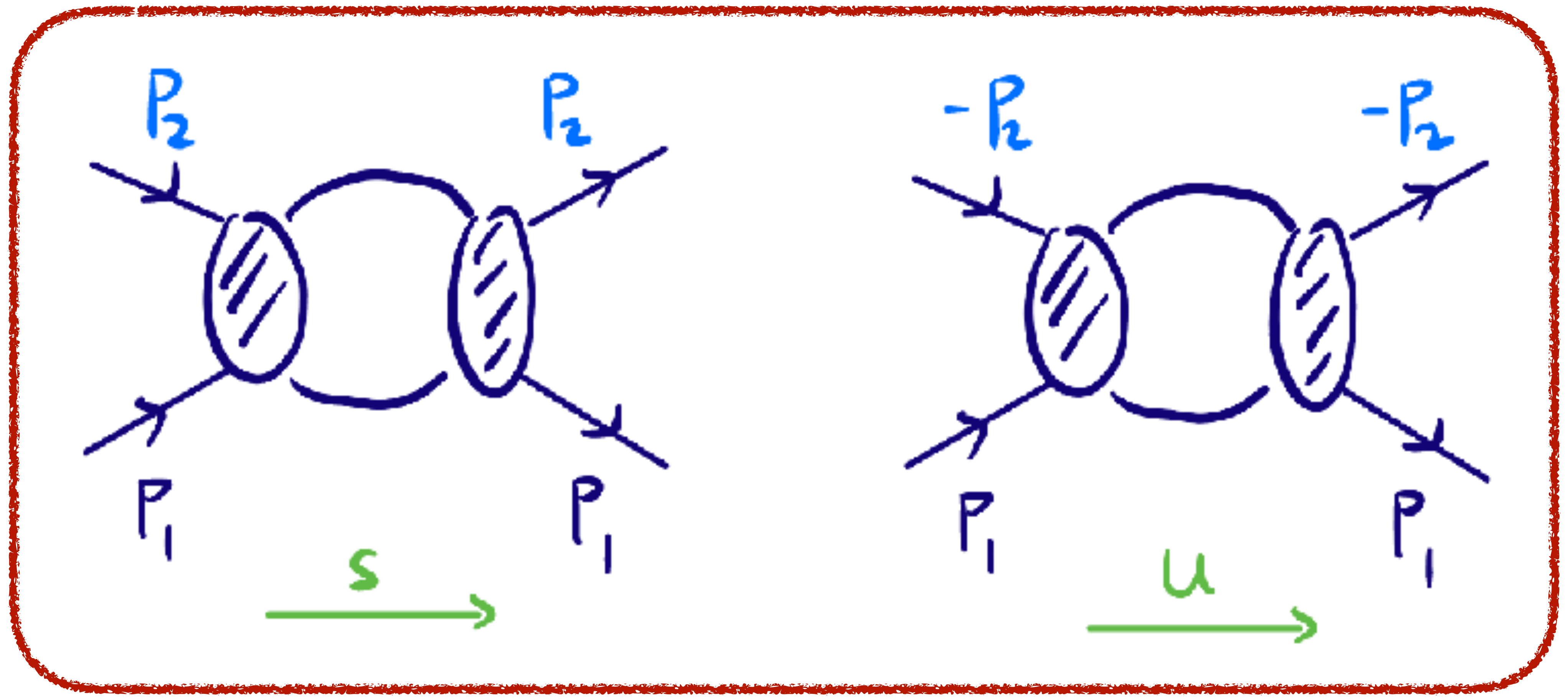
RG from on-shell amplitudes

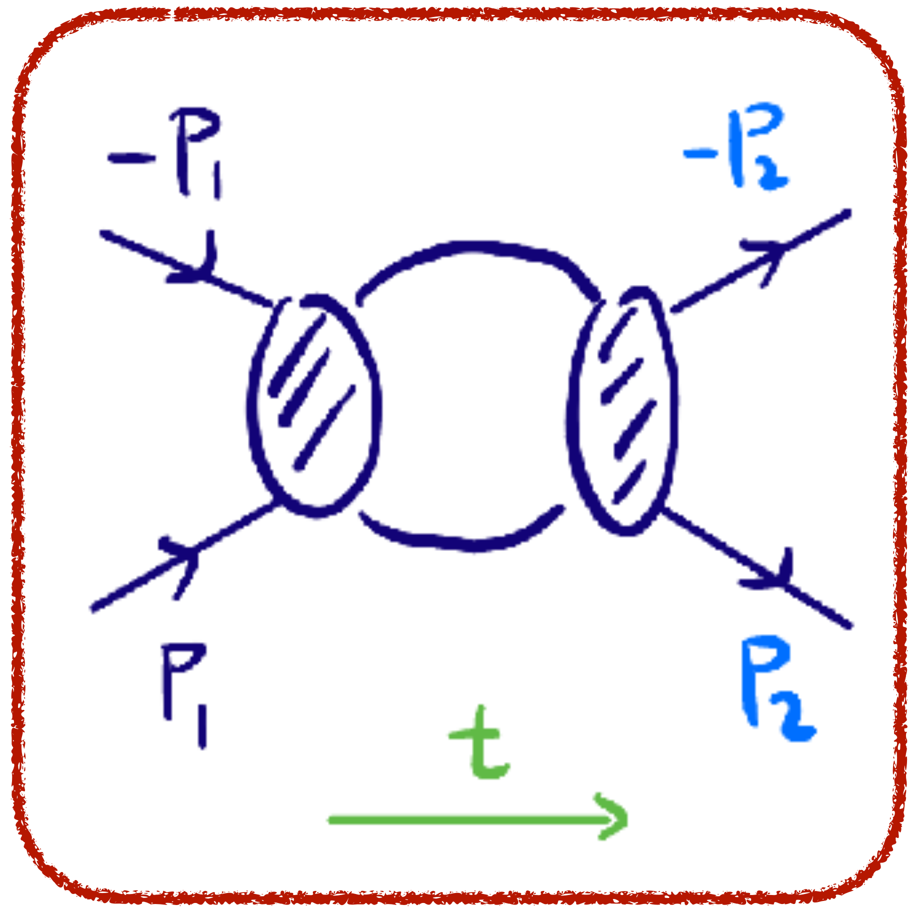
$$\frac{d}{d\ln\mu}A^{\text{tree}} = -\frac{1}{\pi}\sum_{s,t,u}\int d\text{LIPS}AA^{\dagger}$$

Forward kinematics: $t = (p_1' - p_1)^2 = 0$

$|i\rangle = |f\rangle$

$|i\rangle \neq |f\rangle$



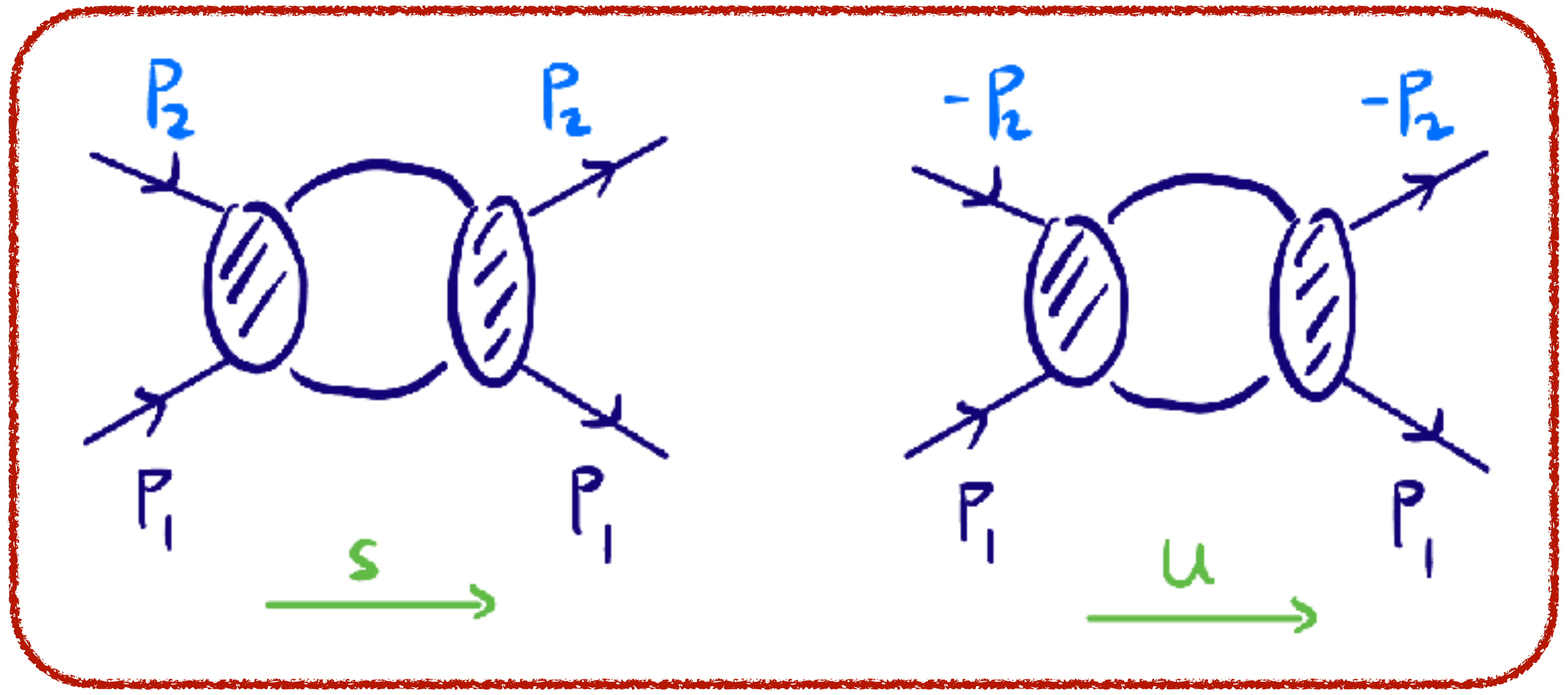


Forward channels

Non-forward channels

Forward Channels

$$|i\rangle = |f\rangle$$



Unitarity: $\langle i|AA^\dagger|i\rangle \geq 0$

Need to ensure:

1. truncation is still positive
2. the s and u channel have the same sign

Forward Channels

Unitarity: $\langle i|A_m A_m^\dagger|i\rangle \geq 0$

Positive truncation

dim-6 dim-6

Non-positive truncation

dim-4 dim-8

The RG induced by the interference term is not sign-definite

Forward Channels

s-channel (s>0)

$$\int d\mathrm{LIPS} A A^\dagger$$

$$= a_0 + a_1 s + a_2 s^2 + \ldots$$

$$t = 0: \quad s = -u$$

u-channel (u>0)

$$\int d\mathrm{LIPS} A A^\dagger$$

$$= b_0 + b_1 u + b_2 u^2 + \ldots$$

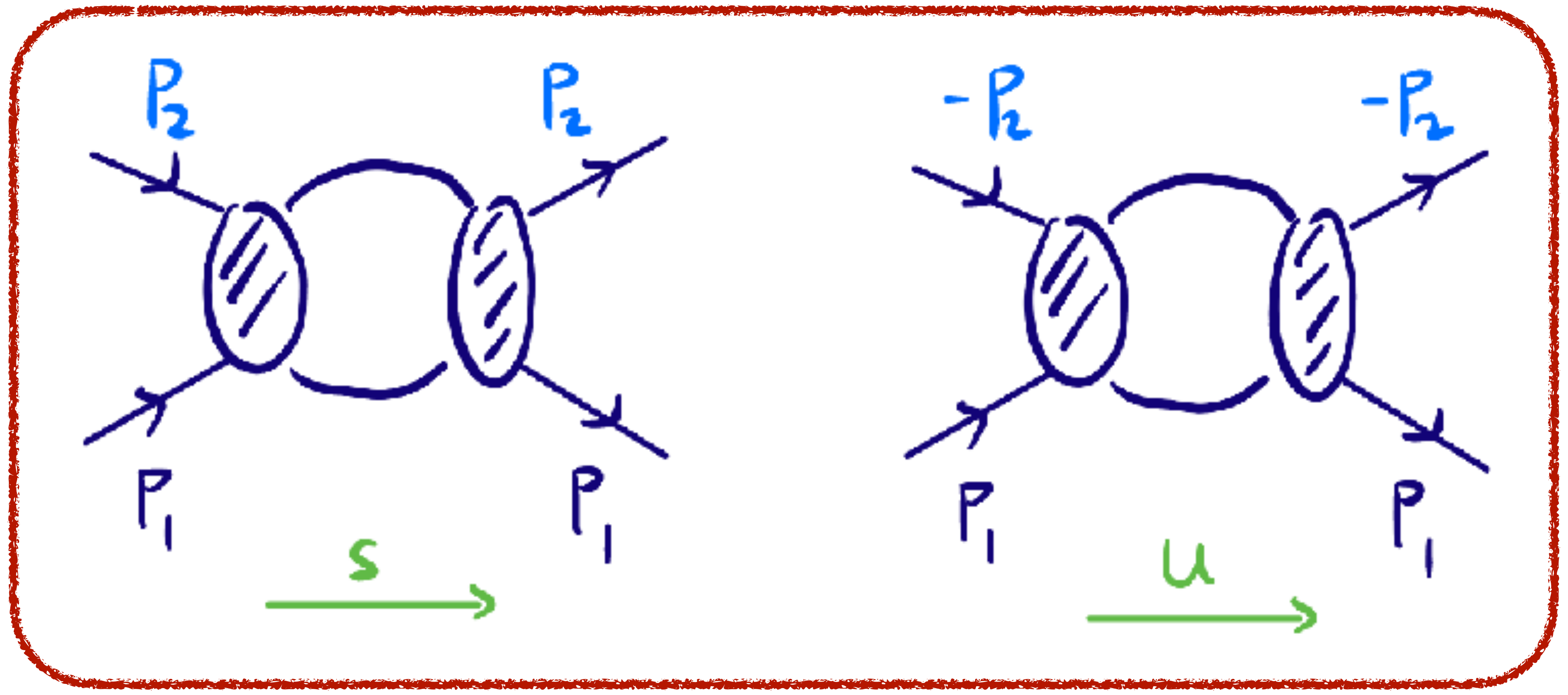
$$(a_0 + b_0)s^0 + (a_1 - b_1)s + (a_2 + b_2)s^2 + \ldots$$

dim-4 dim-6 dim-8

Same-sign contributions for s^{even}, or (dim-even)^2 insertions*

**special examples exist for dim-6 from (dim-5)^2*

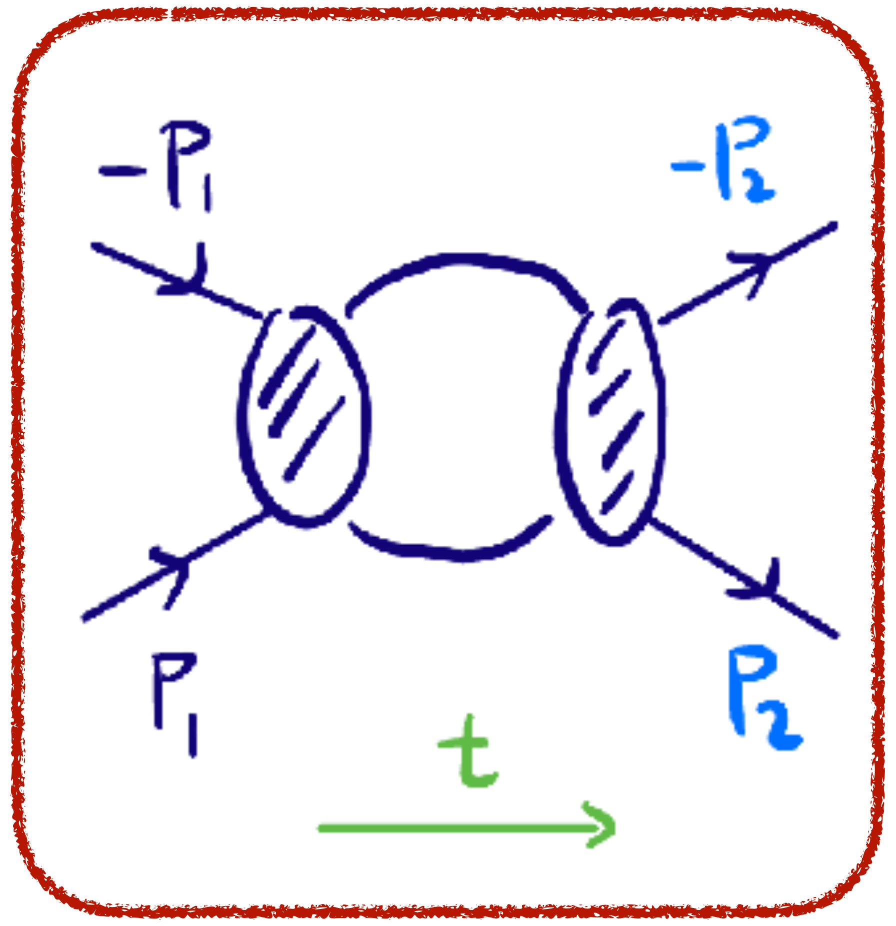
Forward Channels



Unitarity: $\langle i|AA^\dagger|i\rangle \geq 0$

Positivity holds when a pair of even-dimensional operators are inserted

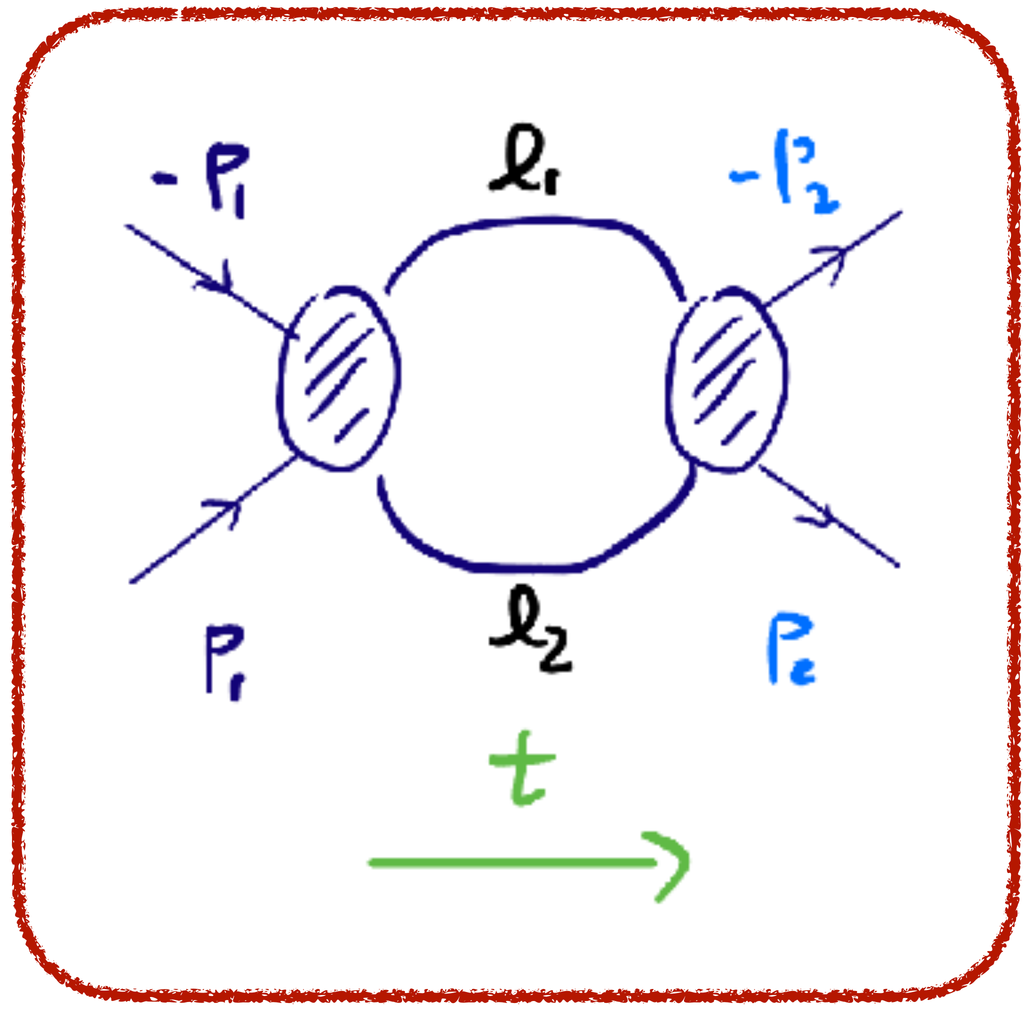
Non-forward Channels



$$|i\rangle \neq |f\rangle$$

Problem: the loop integrand is not positive definite

Non-forward Channels

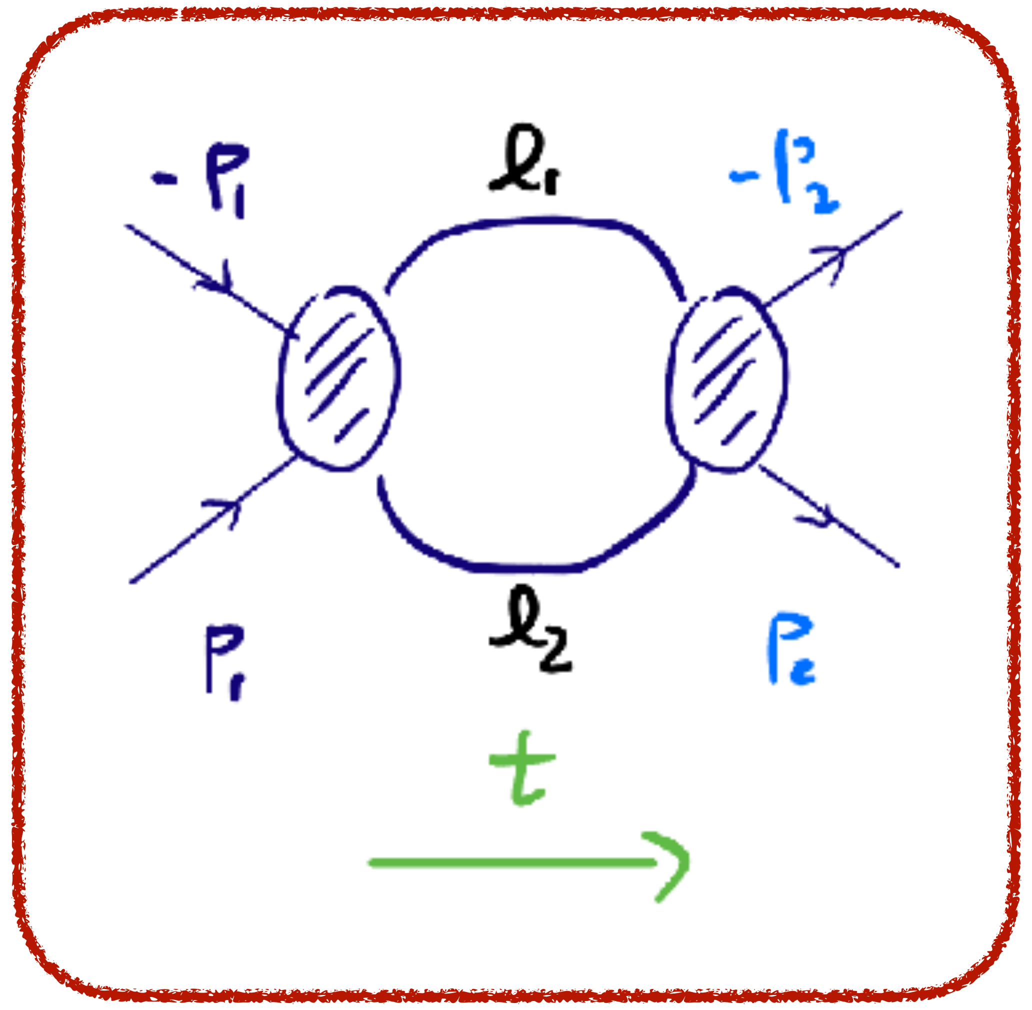


$$\mathrm{Disc}_t A = \frac{1}{\pi}\int d\mathrm{LIPS}\, A(1,1' \to \ell_1,\ell_2) A^*(2,2' \to \ell_1,\ell_2)$$

$$\propto \frac{1}{\pi}\int d\mathrm{LIPS}\, \left(|\ell_1\rangle[\ell_1|\right)^{2|h_1|+n_1} \left(|\ell_2\rangle[\ell_2|\right)^{2|h_2|+n_2}$$

helicity (spin) extra derivatives

Non-forward Channels



$$\mathrm{Disc}_t A = \frac{1}{\pi} \int d\mathrm{LIPS}\, A(1, 1' \to \ell_1, \ell_2) A^*(2, 2' \to \ell_1, \ell_2)$$

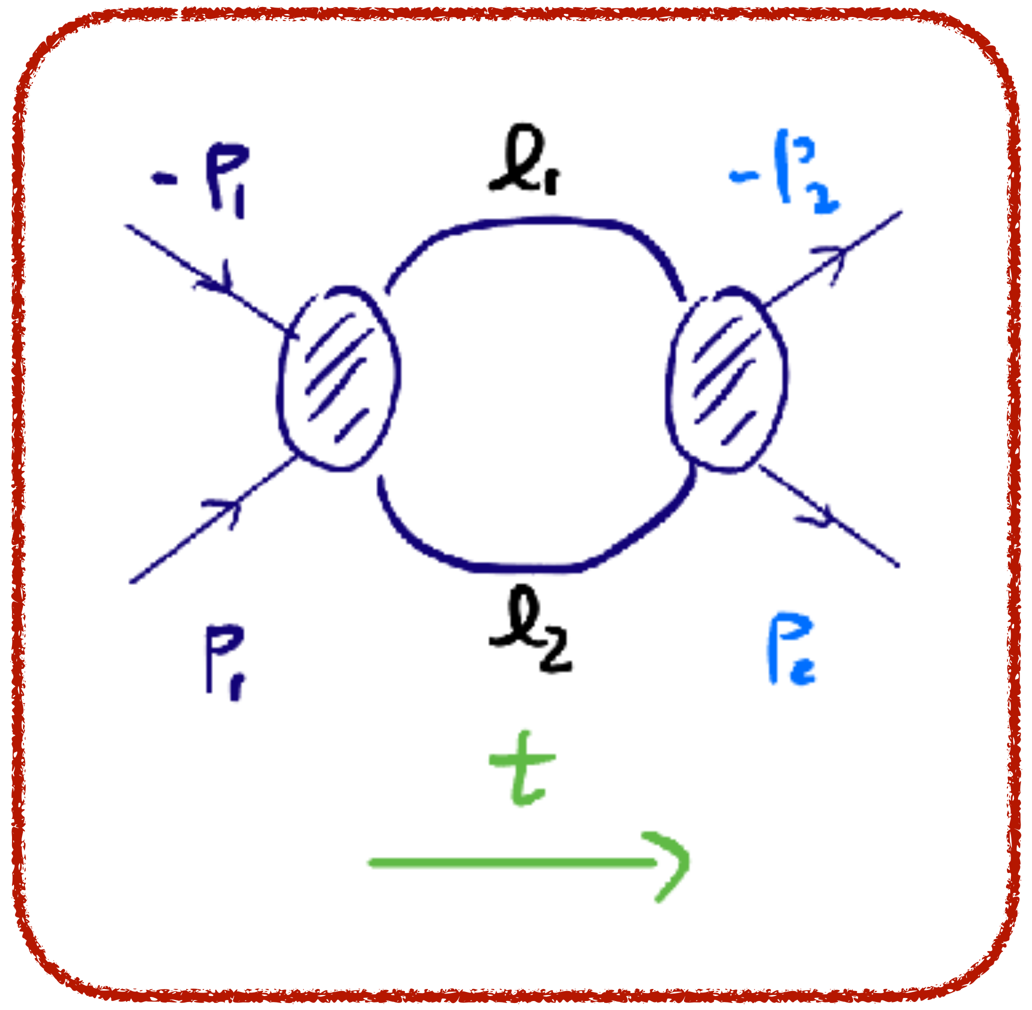
Single-scale int.
Lorentz invariance

$$\longrightarrow \propto (p_1 - p_1')^{2|h_1| + 2|h_2| + n_1 + n_2} \to 0$$

This cut vanishes except for $h_1 = h_2 = n_1 = n_2 = 0$

External fields cannot have spin or extra derivatives; $\lambda\phi^4$ is the only exception

Non-forward Channels



$$
\begin{aligned}
\mathrm{Disc}_t A =& \frac{1}{\pi}\int d\mathrm{LIPS}\, A(1,1'\to \ell_1,\ell_2)A^*(2,2'\to \ell_1,\ell_2) \\
\propto& (p_1 - p_1')^{2|h_1|+2|h_2|+n_1+n_2} \to 0
\end{aligned}
$$

The non-forward channel does not contributes in EFT

Corollary: a new non-renormalization theorem

Unitarity, Analyticity, and Lorentz invariance

$$\frac{d}{d\ln\mu}A^{\text{tree}} = -\frac{1}{\pi}\sum_{s,t,u}\int d\text{LIPS}\,AA^{\dagger}$$

unitarity & analyticity

no non-forward channel (Lorentz inv.)

$$= -\frac{1}{\pi}\sum_{s,u}\int d\text{LIPS}\,AA^{\dagger}$$

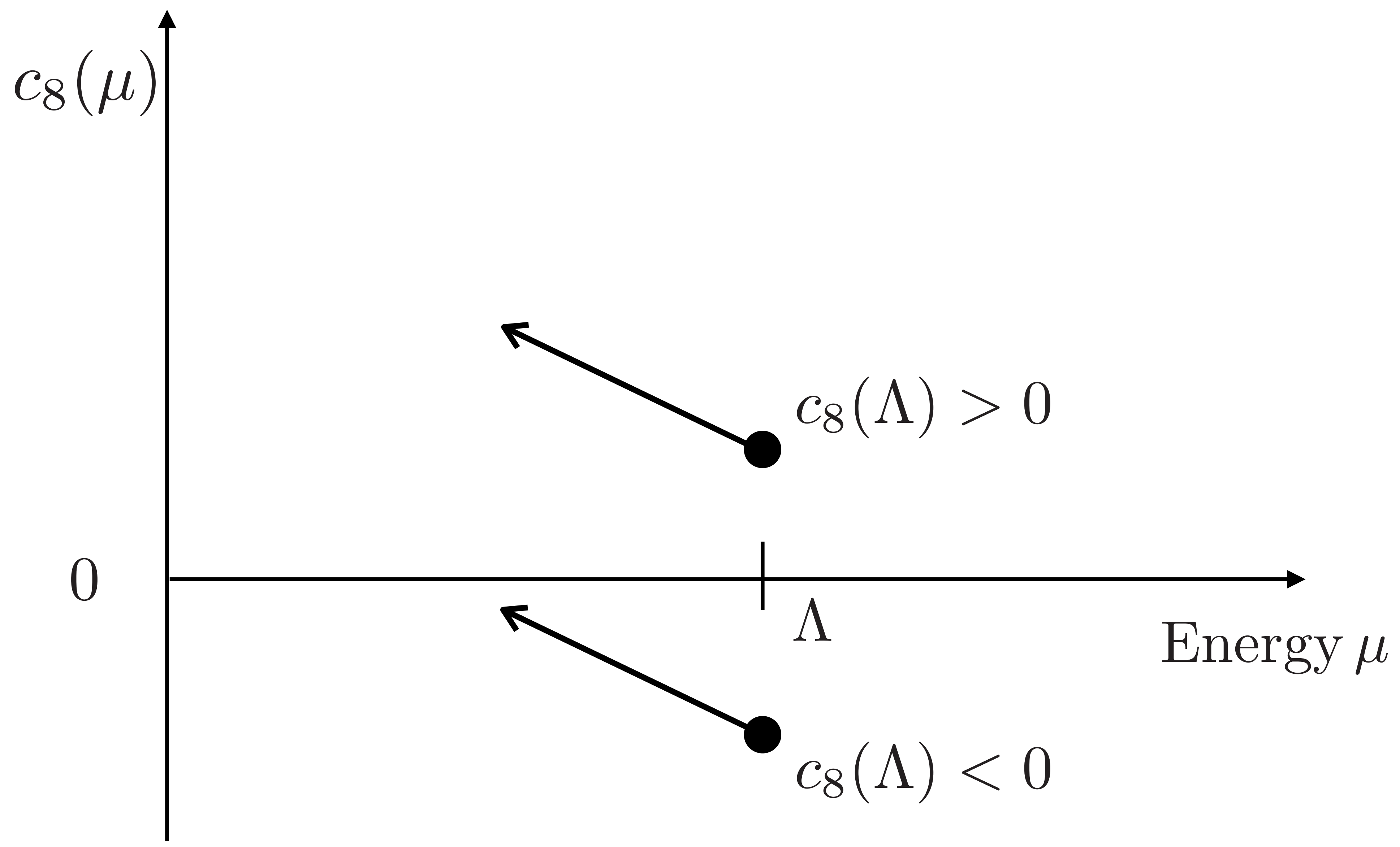
unitarity

$$= -\left(\beta_4 + \beta_6 s^1 + \beta_8 s^2 + \ldots\right)$$

$\beta_{4n} \geq 0$ *from a pair of dim-even operators insertions*

$$\text{IR unitarity:} \quad \left.\frac{d\,c_{4n}}{d\ln\mu}\right|_{(\text{dim-}(2n+2))^2} \leq 0$$

Positivity in EFT Renormalization



IR unitarity: $$\left.\frac{d\,c_{4n}}{d\ln\mu}\right|_{(\text{dim-}(2n+2))^2} \leq 0$$

[You-Peng Liao, Jasper Roosmale Nepveu, CHS]

Applications

Application: Pions

- The renormalization in chiral perturbation theory at O(p⁴) has the structure

$$\frac{dc_4}{d\ln\mu} = -\frac{4}{3}\frac{1}{16\pi^2} + \gamma(\alpha_{\rm EM})c_4$$

O(p⁴) ~ 0.17   [O(p²)]²   EM~ numerically very small

[Knecht, Urech]

- The renormalization from O(p²) dominates because the loops are not suppressed due to the strong coupling nature.

Naive Dimensional Analysis (NDA) [Georgi, Manohar]

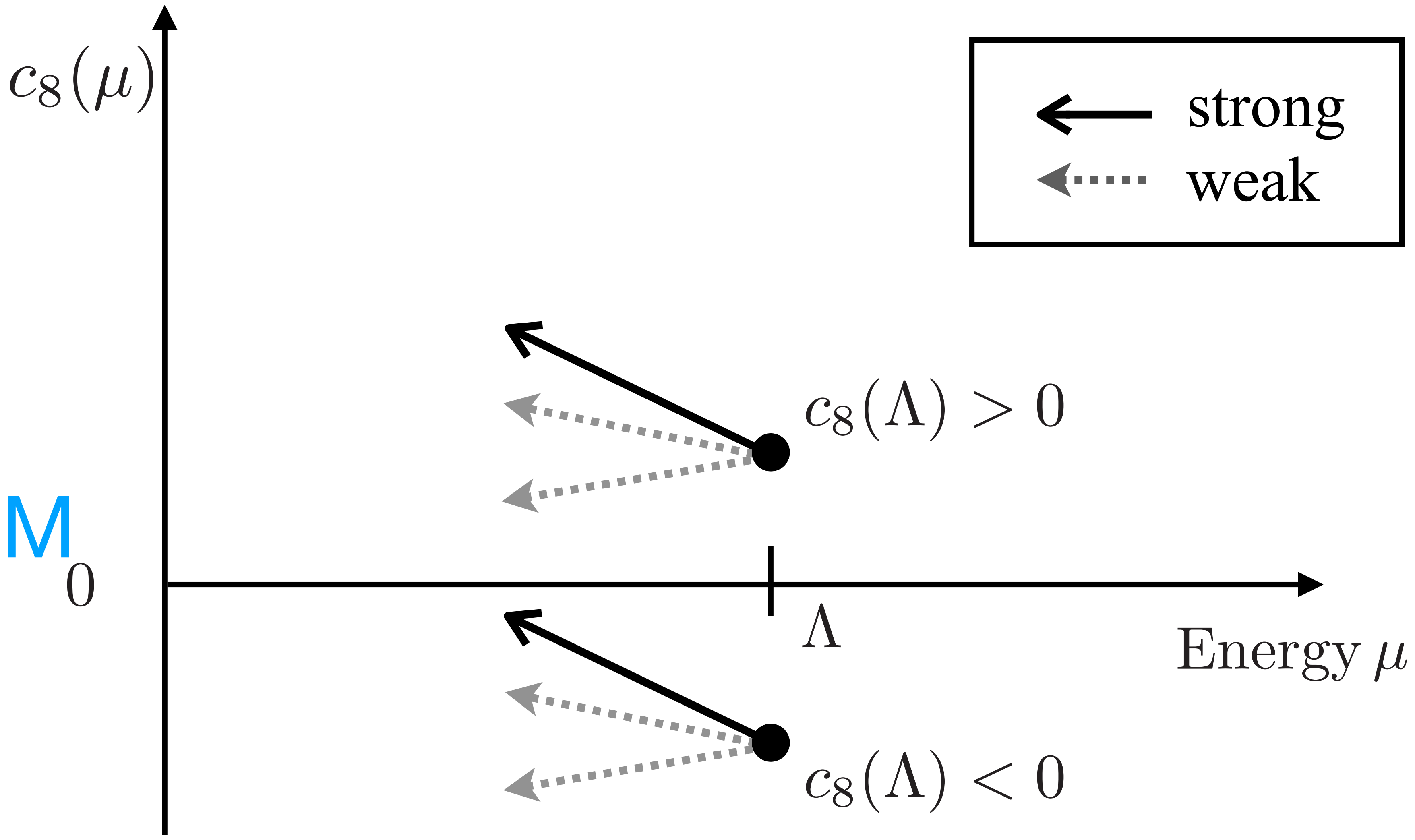
Application: SMEFT

- The analogy of ChPT suggests that (dim-6)² dominates when SMEFT is *"strong-coupled": EFT coefficients closer to NDA than SM couplings*

$$\frac{dc_{8,i}}{d\ln\mu} = \underbrace{\frac{\gamma_{ijk}}{16\pi^2}c_{6,j}c_{6,k}}_{} + \underbrace{\frac{\gamma_{ij}}{16\pi^2}c_{8,j}}_{}$$

(dim-6)²: negative due to our thm.

SM*dim-8: suppressed by weakly-coupled SM



Positivity "Violation" from RG

UV model:
heavy singlet S & light Higgs H

[modified from construction by Chala & Santiago]

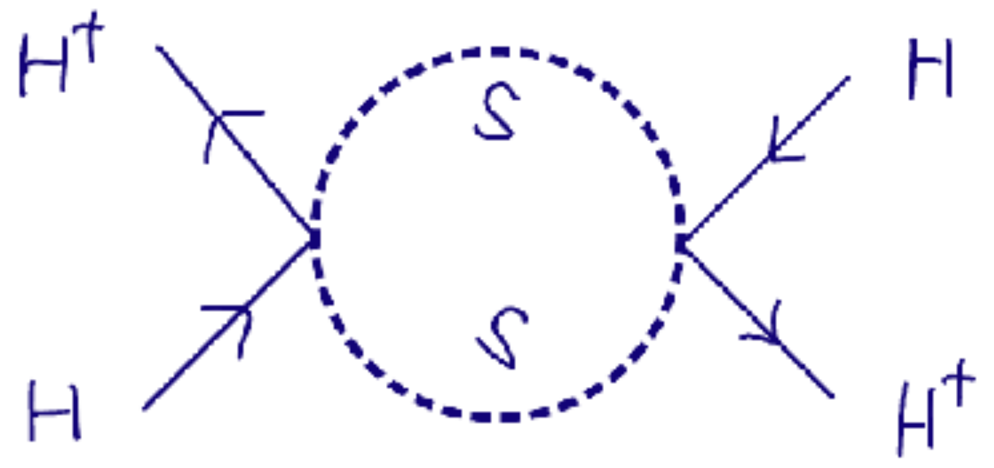
$$\mathcal{L} = \frac{1}{2}(\partial S)^2 - \frac{1}{2}M^2 S^2 + \partial H^\dagger \partial H - m^2 H^\dagger H - \lambda (H^\dagger H)^2 - \frac{g}{2} S^2 H^\dagger H$$

EFT:

$$\mathcal{L}_{\mathrm{EFT}} = D^\mu H^\dagger D_\mu H - \lambda (H^\dagger H)^2 + \sum_i \frac{1}{M^2} c_{6,i}\mathcal{O}_{6,i} + \sum_i \frac{1}{M^4} c_{8,i}\mathcal{O}_{8,i}$$

$\mathcal{O}_{6,1}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{6,2}$	$(H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$
$\mathcal{O}_{6,3}$	$(H^\dagger H)^3$

$\mathcal{O}_{8,1}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_{8,2}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_{8,3}$	$(D_\mu H^\dagger D^\mu H)^2$



At matching scale M:

$$(c_{8,1}, c_{8,2}, c_{8,3}) = \left(0, 0, \frac{1}{60}\frac{g^2}{16\pi^2}\right)$$

Tree-level positivity:

$$c_{8,2} > 0,$$
$$c_{8,2} + c_{8,1} > 0,$$
$$c_{8,2} + c_{8,1} + c_{8,3} > 0$$

$c_{8,1}$ and $c_{8,2}$ are at the critical point

Positivity “Violation” from RG

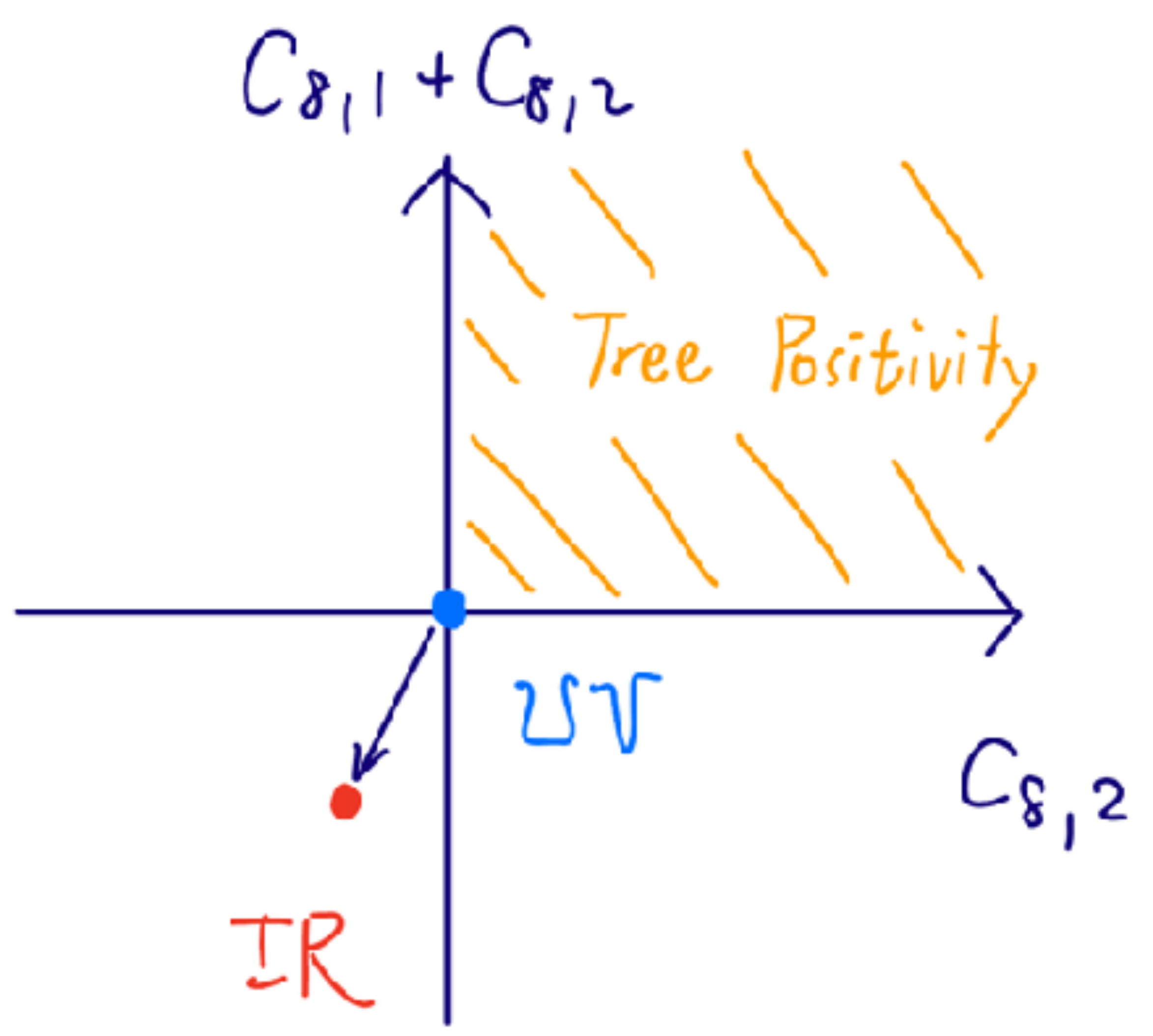
Weakly coupled new physics: $\lambda \gg g$ so SM*dim-8 dominates RG

At matching scale M:

$$(c_{8,1}, c_{8,2}, c_{8,3}) = \left(0, 0, \frac{1}{60}\frac{g^2}{16\pi^2}\right)$$

RG equations:

$$\begin{aligned}
\dot{c}_{8,2} &= \frac{8}{3}\lambda(c_{8,1} + 3c_{8,2} + c_{8,3}) > 0 \\
\dot{c}_{8,2} + \dot{c}_{8,1} &= \frac{16}{3}\lambda(2c_{8,1} + 2c_{8,2} + c_{8,3}) > 0 \\
\dot{c}_{8,2} + \dot{c}_{8,1} + \dot{c}_{8,3} &= \frac{16}{3}\lambda(5c_{8,1} + 4c_{8,2} + 6c_{8,3}) > 0
\end{aligned}$$



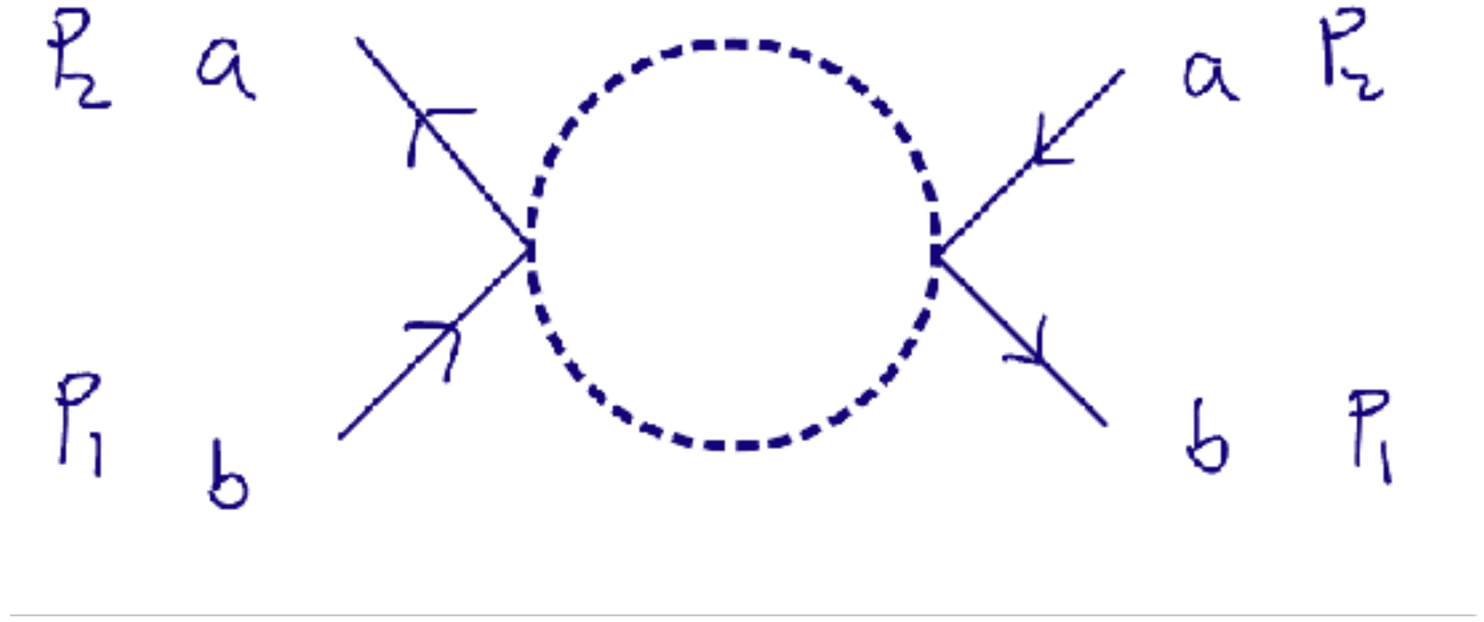
Tree-level positivity for $c_{8,1}$ and $c_{8,2}$ are violated in IR

Extensions

Dim-6 Operators

- If we can forbid one of the s and u channels, we can still apply the theorem
- Example:
  4-fermion operator renormalized by Weinberg-like operator

$$c_{ab}\,\bar{\psi}_a\psi_b\,\phi^2 \quad \to \quad c_{ijkl}\,(\bar{\psi}_i\psi_j)(\bar{\psi}_k\psi_l)$$



$$\dot{c}_{abba} = -2|c_{ab}|^2$$

New Non-renormalization Theorem

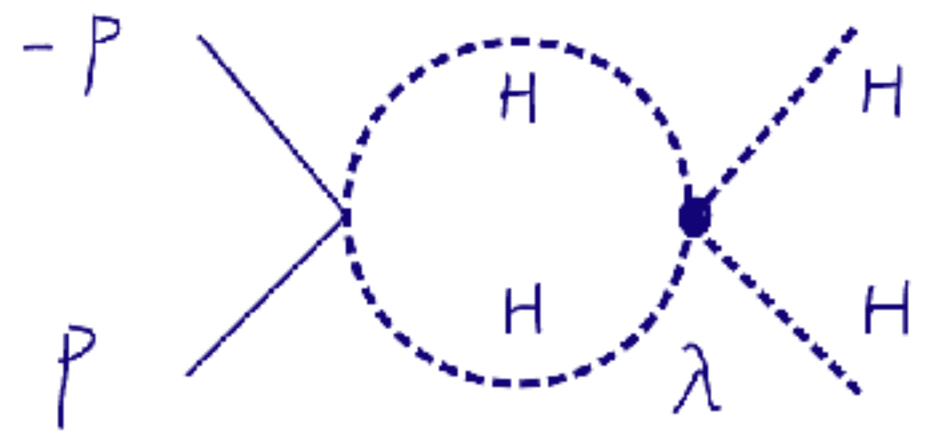
- As a corollary, our theorem implies

"If the operator can be measured by the forward limit and if the non-forward channel is the only channel, then the operator is not renormalized."

- Dim-6 Examples:

$F^2 H^\dagger H$ $\rightarrow$ $$\left.\frac{dc_{F^2H^2}}{d\ln\mu}\right|_\lambda = \frac{12\lambda}{16\pi^2} c_{F^2H^2}$$

zero in forward limit



$i\bar{\psi}\gamma^\mu\psi H^\dagger \overleftrightarrow{D}_\mu H$ $\rightarrow$ $$\left.\frac{dc_{\psi^2DH^2}}{d\ln\mu}\right|_\lambda = 0$$

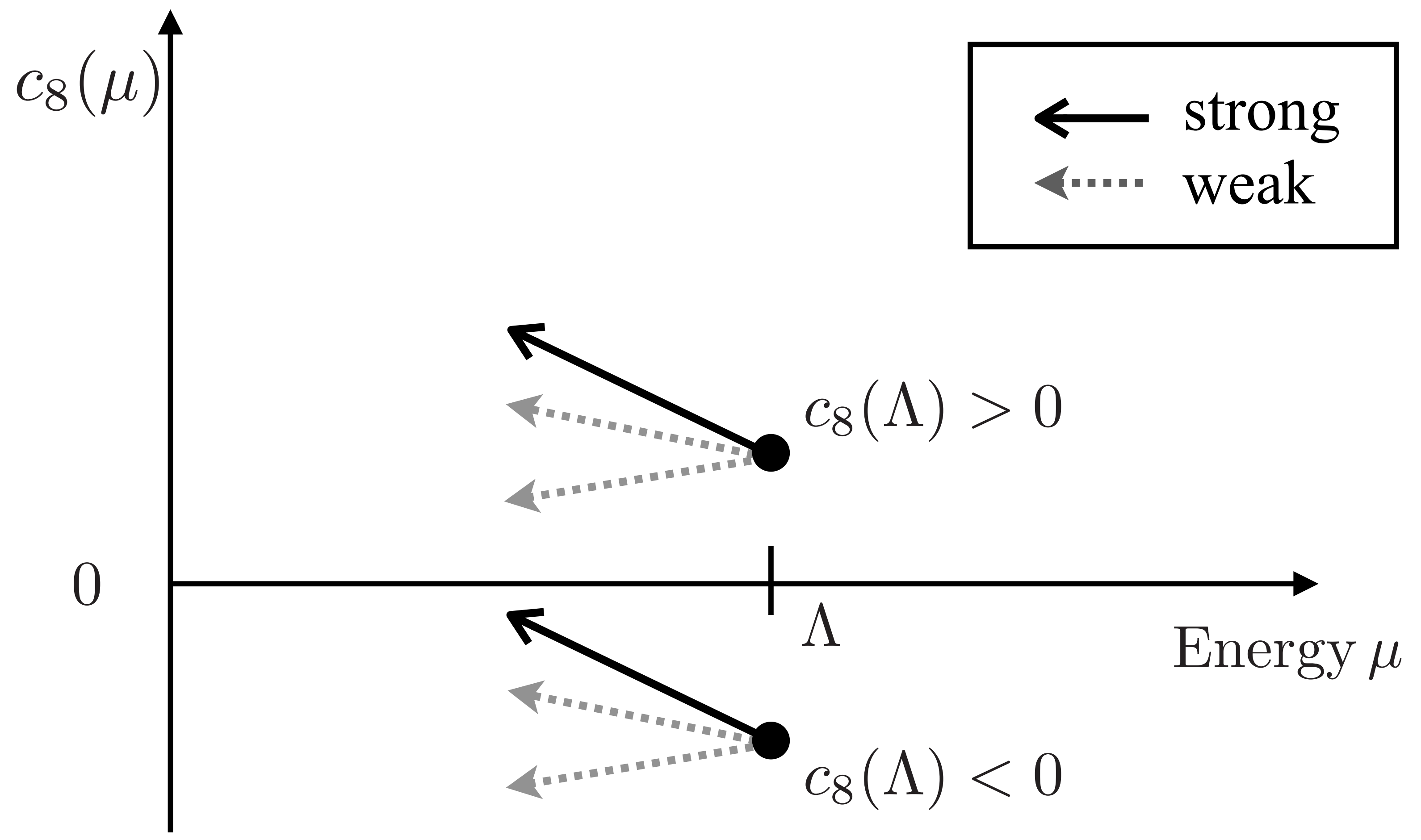
nonzero in forward limit

Summary & outlook

- RG flow in EFT can have universal structures from unitarity and causality
- We prove this for a very general EFT at one loop,

  with applications to the SMEFT and classical EFT for black holes

[2505.02910 w/ Liao, Roosmale Nepveu, CHS] [WIP, Chang, Hsu, CHS, Zhou]



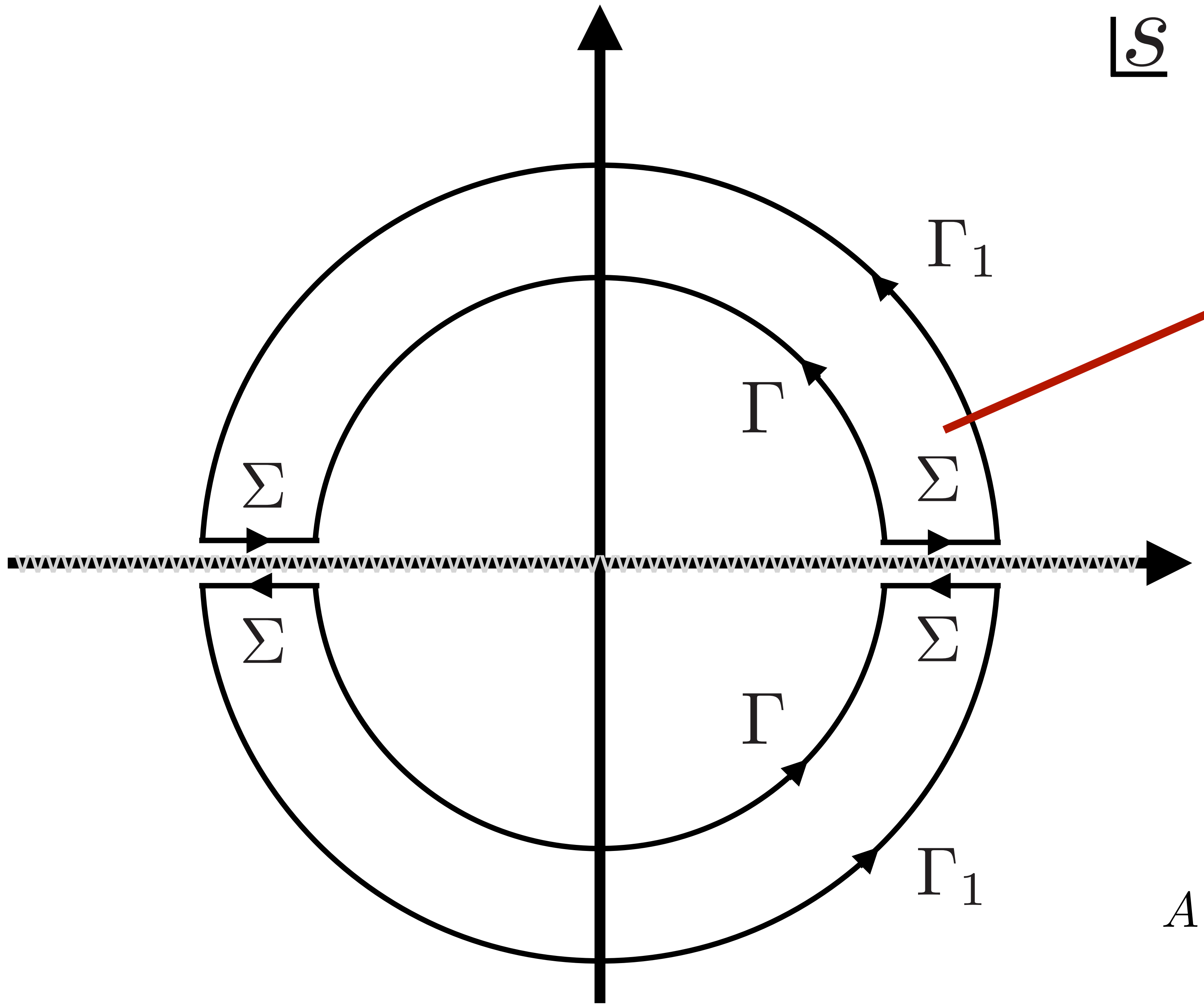
Summary & outlook

- Positivity in EFT Matching?
  [WIP w/ Ludvig Chen, Maria Derda, Roosmale Nepveu]
- Positivity in the presence of gauge coupling?
- Positivity for classical black holes and neutron starts
  [WIP w/ Chih-Hao Chang, Chia-Hao Hsu, Zihan Zhou]
- Beyond 4-point interaction
- Imprints in Dim-6 RG?

Thank you

Backup

IR Contour dominates by Renormalizable Parts



$$\frac{d}{dr}\mathrm{arc}(r) = -\frac{\beta_{\tilde{\lambda}}}{3\,r^3} + \frac{2}{\Lambda^4}\frac{\beta_8}{r} + \mathcal{O}\left(\frac{1}{\Lambda^6}\right) \leq 0$$

$$\beta_{\tilde{\lambda}} = \frac{3\lambda^2}{16\pi^2} \geq \frac{6r^2}{\Lambda^4}\beta_8 + \mathcal{O}\left(\frac{r^3}{\Lambda^6}\right)$$

$$\begin{aligned} A\,(s,t) = & \left[-\tilde{\lambda} - \frac{\beta_{\tilde{\lambda}}}{6}\left(\ln\left(\frac{-s}{\mu^2}\right) + \ln\left(\frac{-u}{\mu^2}\right) + \ln\left(\frac{-t}{\mu^2}\right) + 6\right)\right] \\ & + \left[\frac{2\,\tilde{c}_8}{\Lambda^4}(s^2+u^2+t^2) + \frac{\beta_8}{\Lambda^4}\left(s^2\ln\left(\frac{-s}{\mu^2}\right) + u^2\ln\left(\frac{-u}{\mu^2}\right) + t^2\ln\left(\frac{-t}{\mu^2}\right) + 2\,(s^2+t^2+u^2)\right)\right] + \ldots, \end{aligned}$$

Higher Points and Higher Loops

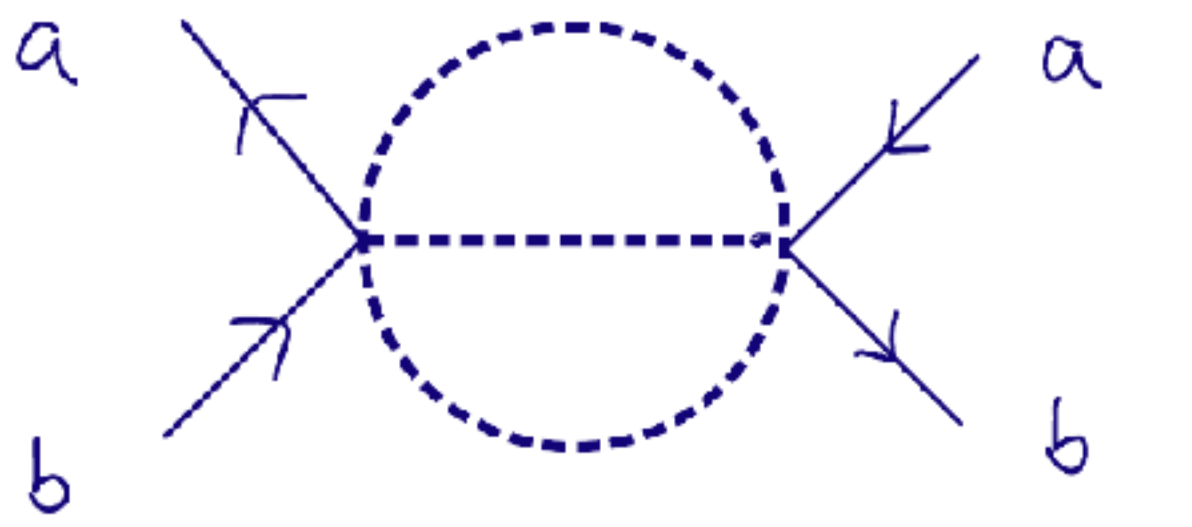
- The perturbative approach can be extended to higher points and higher loops

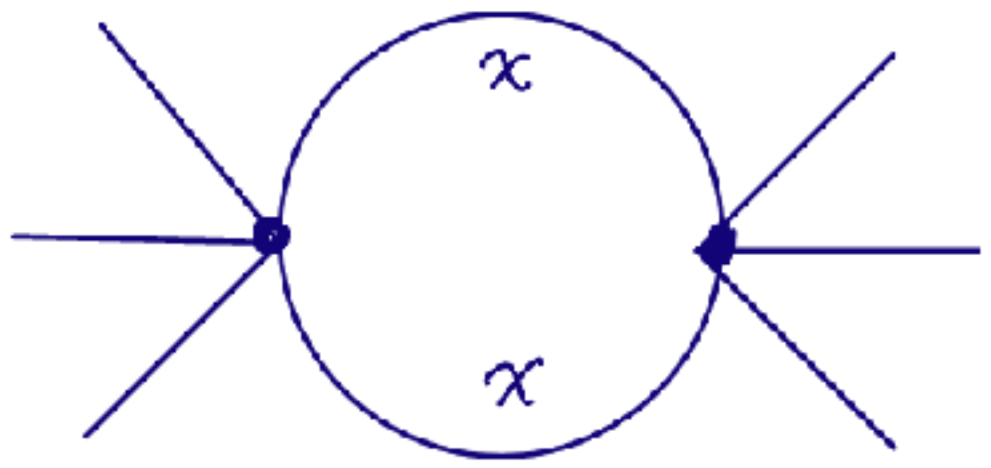
$$c_{ab}\,\bar{\psi}_a\psi_b\,\phi^3 \quad\to\quad c_{ijkl}\,(D\bar{\psi}_i D\psi_j)(\bar{\psi}_k\psi_l)$$

$$(D\phi D\chi)^2\phi^2\chi \quad\to\quad (D\phi D\phi)^2\phi^2$$

$$\frac{dc_{abba}}{d\ln\mu} = -18|c_{ab}|^2\frac{1}{(16\pi^2)^2}$$

$$\frac{dc_2}{d\ln\mu} = -16\frac{c_1^2}{16\pi^2}.$$





Comparison with Chala et al.

—2106.05291, 2110.01624, 2301.09995, 2309.16611

- Important observation that motivates this work
- Our proof only relies on ***IR*** properties, regardless of the ***UV*** completion
- Our results can be generalized to dim-6 operators, where positivity bounds are usually not available
- We prove the vanishing of t-channel contribution which is crucial
- Generalization to higher points and higher loops are accessible