Workshop on recent developments from QCD to nuclear matter, 2025/12/17-20, Academia Sinica, Taipei, Taiwan

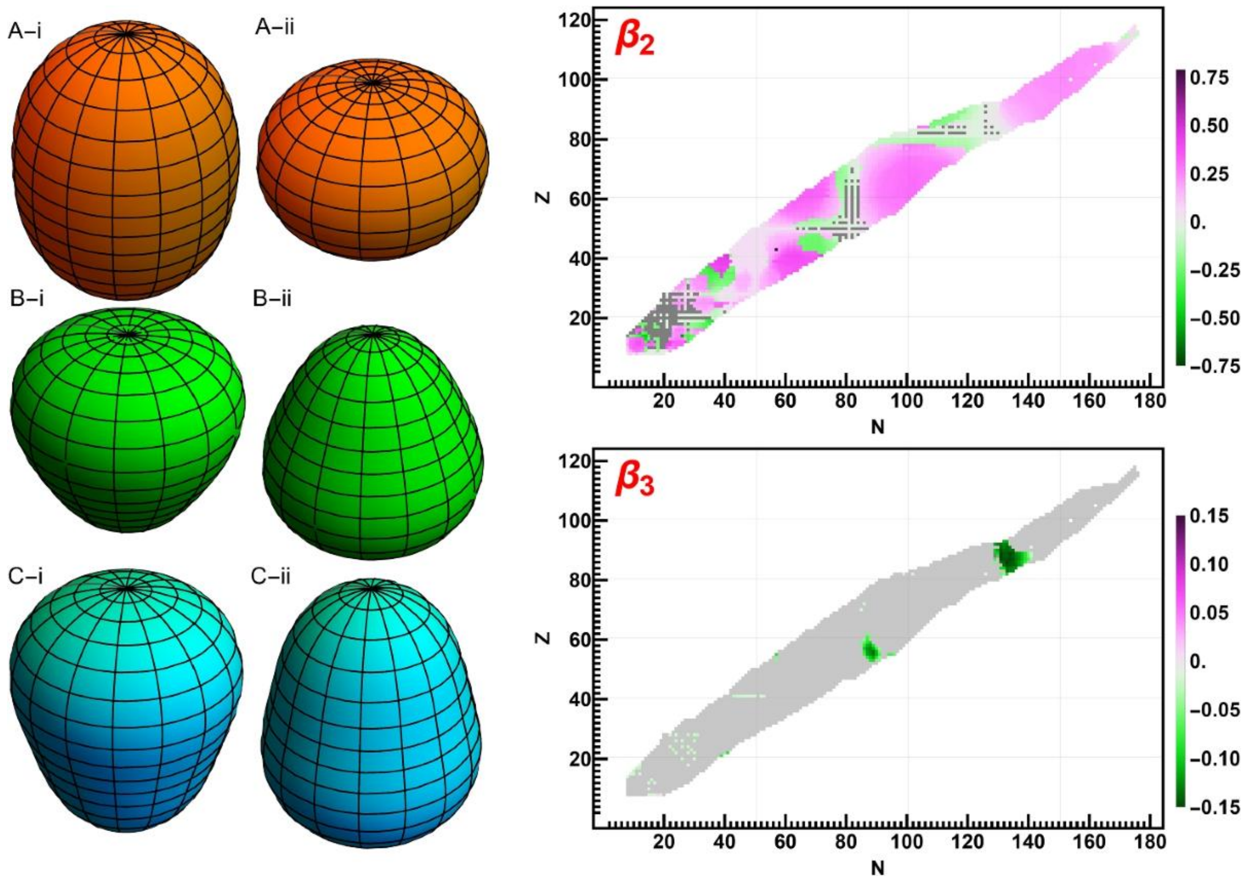
Probing nuclear shape and surface vibration in heavy-ion collisions

Masakiyo Kitazawa
(YITP, Kyoto)

Hagino, MK, PRC **112** (2025) L041901 [2508.05125];
MK, Esumi, Niida, Nonaka, arXiv:2510.18383.

Contents

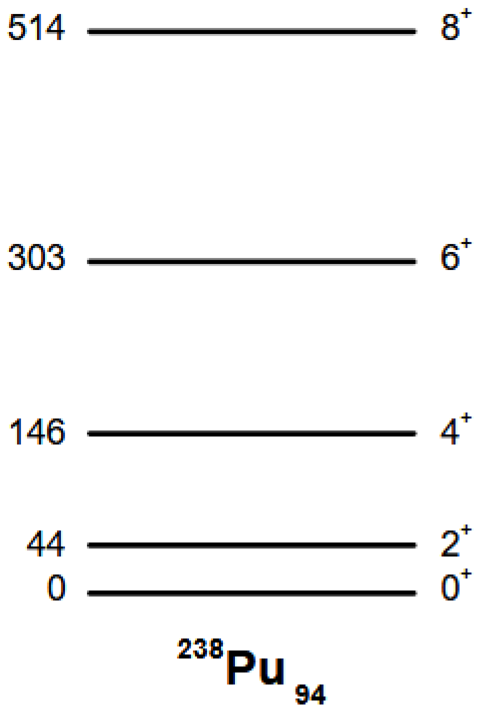
Deformation of Nucleus



doi.org/10.1007/s10751-024-01880-7

Who saw it?

Evidence 1: Rotational Bands



$$E_I = \frac{I(I+1)\hbar^2}{2J}$$

$E(4^+)/E(2^+) \sim 3.3$ for rotors

$E(4^+)/E(2^+) \sim 2.0$ for harmonic osc.

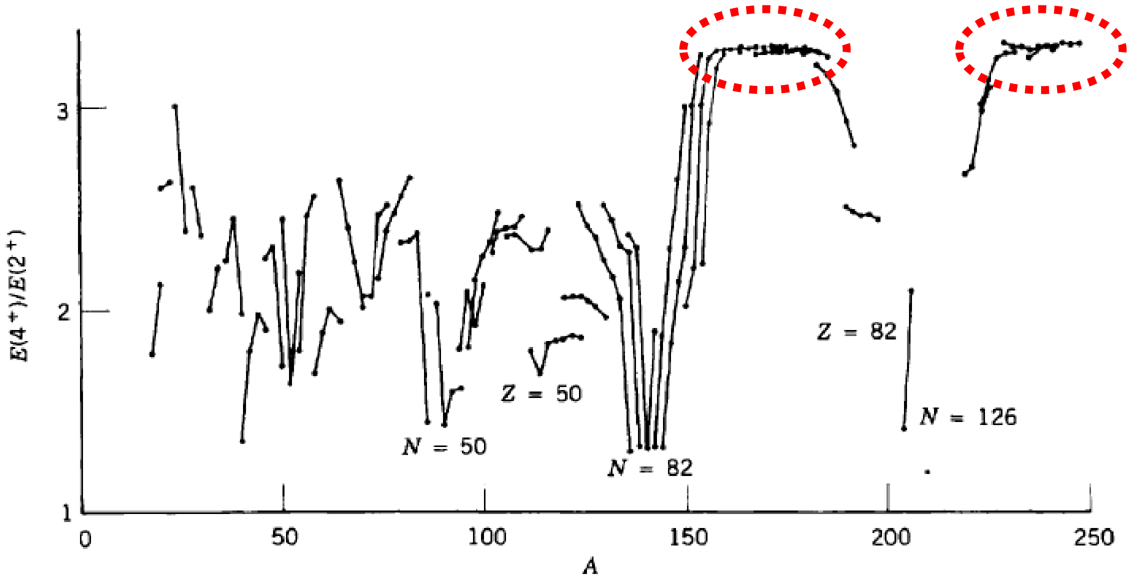
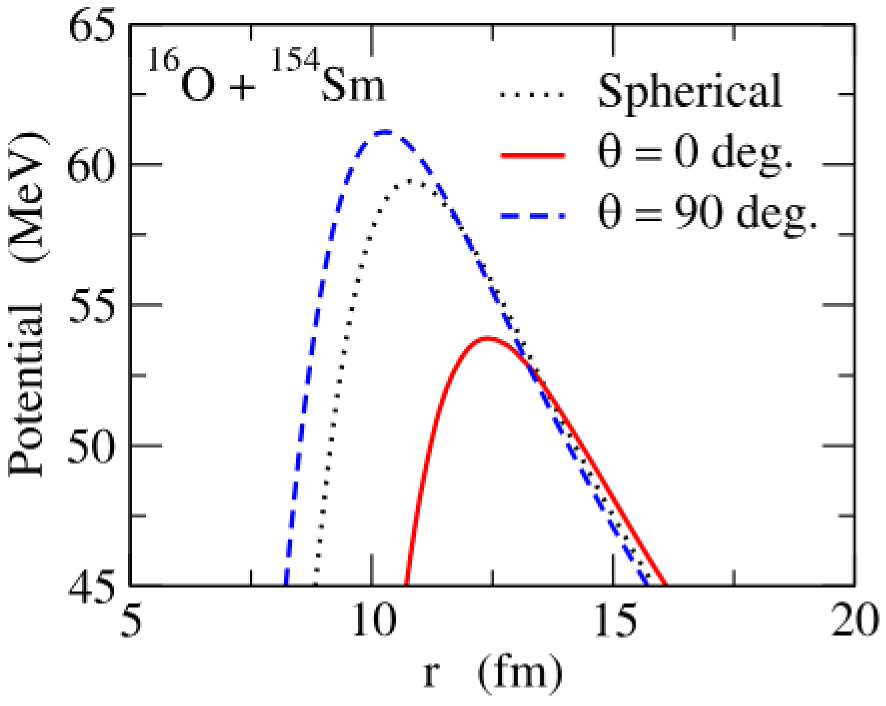


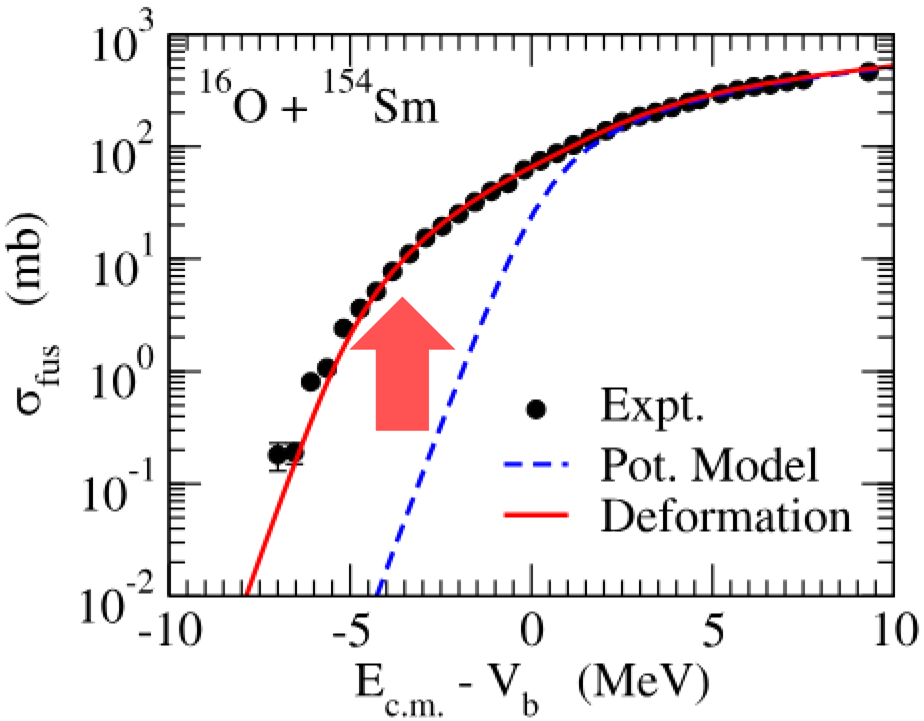
Figure 5.15b The ratio $E(4^+) / E(2^+)$ for the lowest 2^+ and 4^+ states of even-Z, even-N nuclei. The lines connect sequences of isotopes.

Energy spectrum of rigid body rotation → Existence of deformation

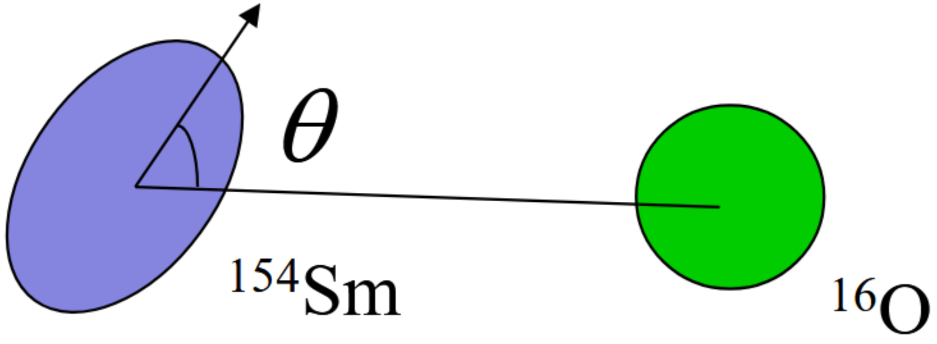
Evidence 2: Nuclear Fusion Reaction

Hagino, Takigawa, PTEP 128, 1061 (2012)

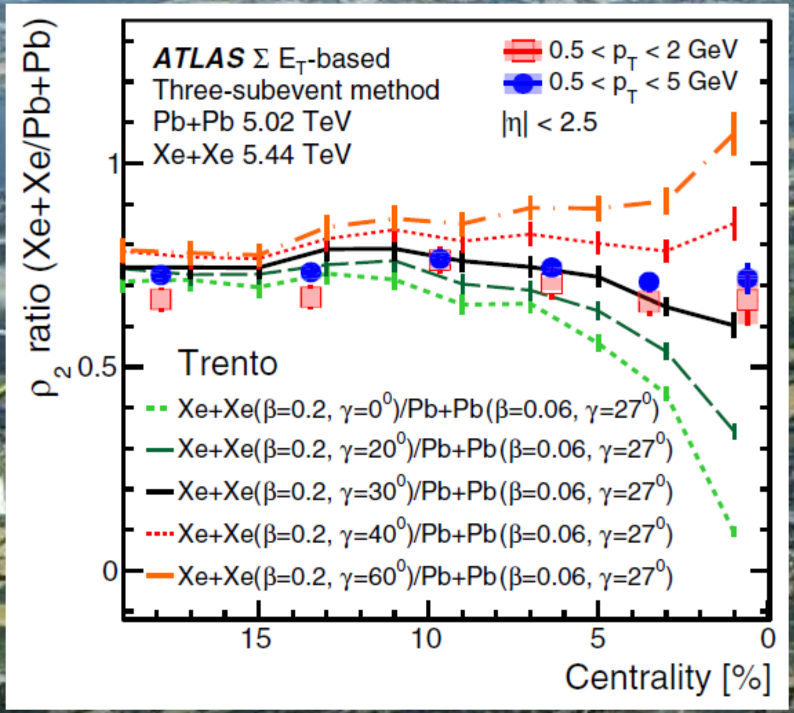




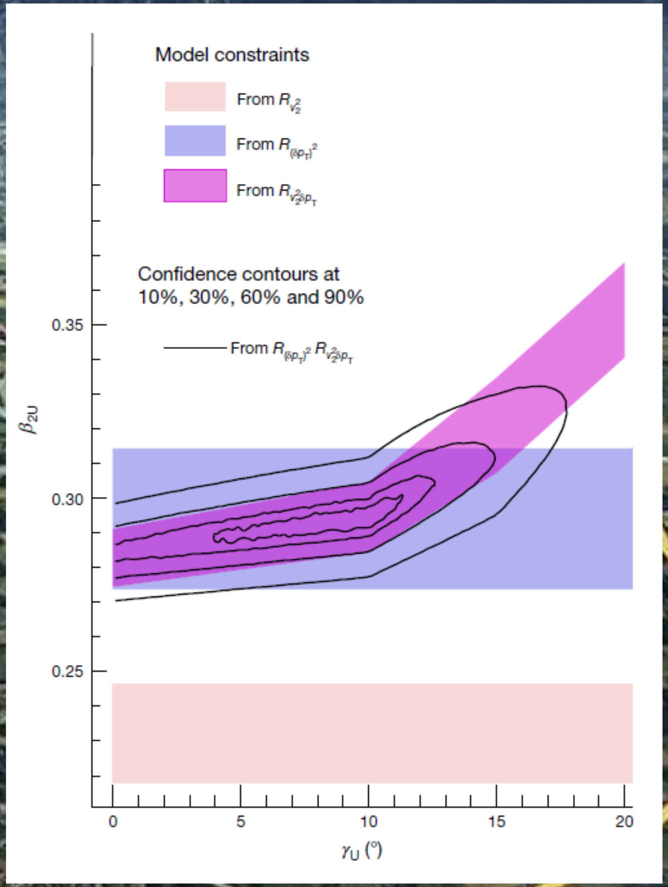
Fission barrier is lowered for deformed nuclei.



Using Relativistic Heavy-Ion Collisions for nuclear-shape studies?!

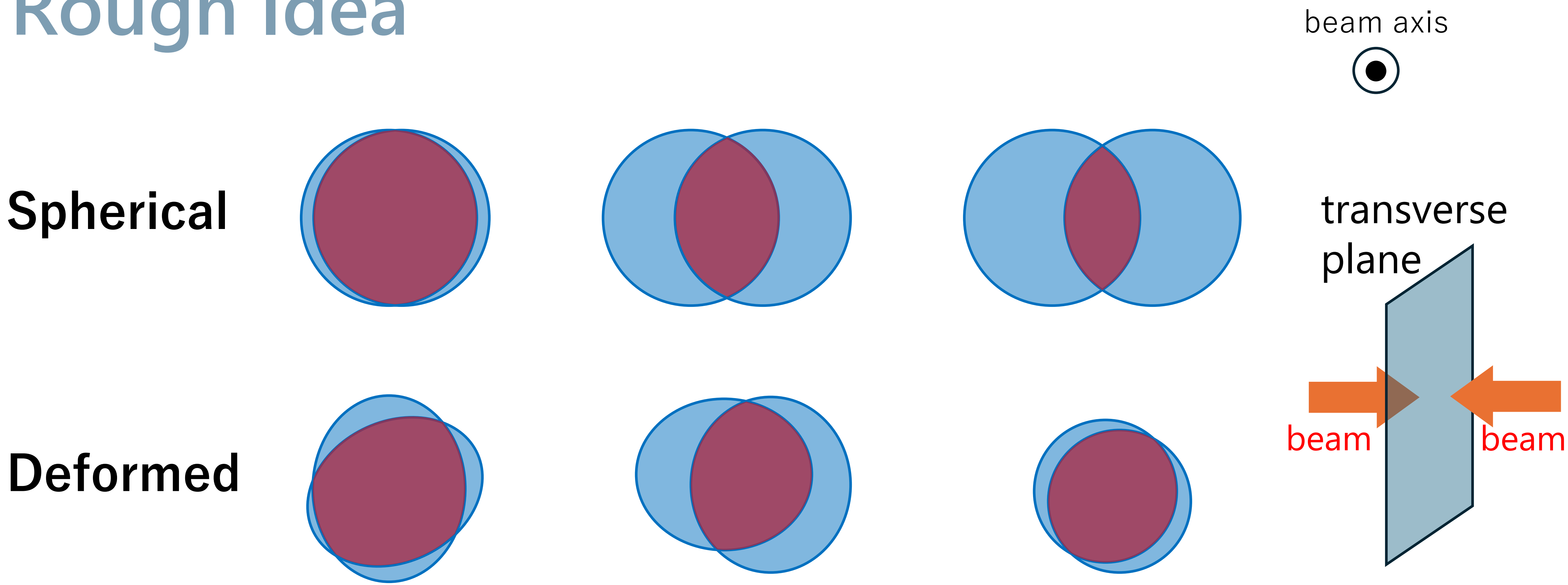


ATLAS, PRC 107, 054910 (2023)



STAR, Nature 635, 67 (2024)

Rough Idea



Different transverse shapes for the spherical and deformed nuclei.
Distribution is reflected into anisotropic flows in the final state.

High-energy collisions → snapshot of the overlapping region of intrinsic states

Ultra-Central Collisions (UCC)

UCC ➔ Almost all particles participate in the collisions

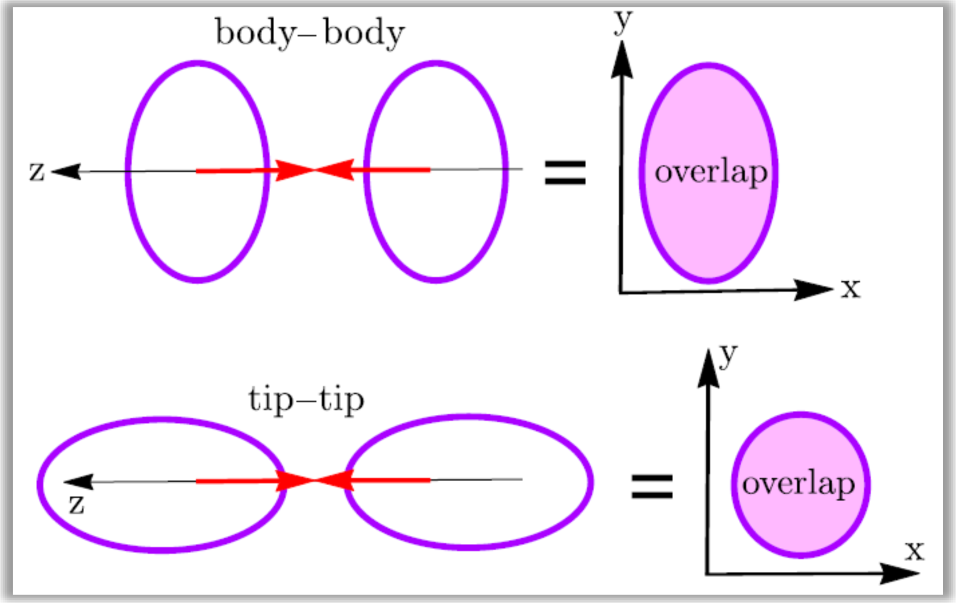
Collision of Prolate nuclei

	mean radius	anisotropy
tip-tip	small	small
body-body	large	large

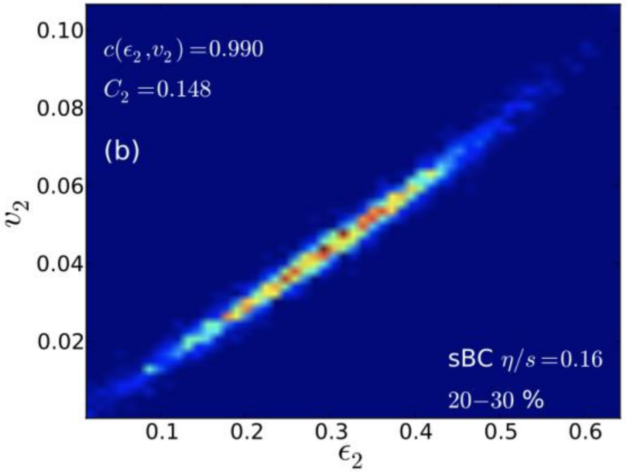
hydro. evolution

tip-tip	large $\overline{p}_T$ / small v_2
body-body	small $\overline{p}_T$ / large v_2

➔Inverse correlation of v_2 & $\overline{p}_T$



Giacalone, PRL ('20)



Niemi+, PRC87 ('13)

Experimental Result @STAR

tip-tip	large $\bar{p}_T$ / small v_2
body-body	small $\bar{p}_T$ / large v_2

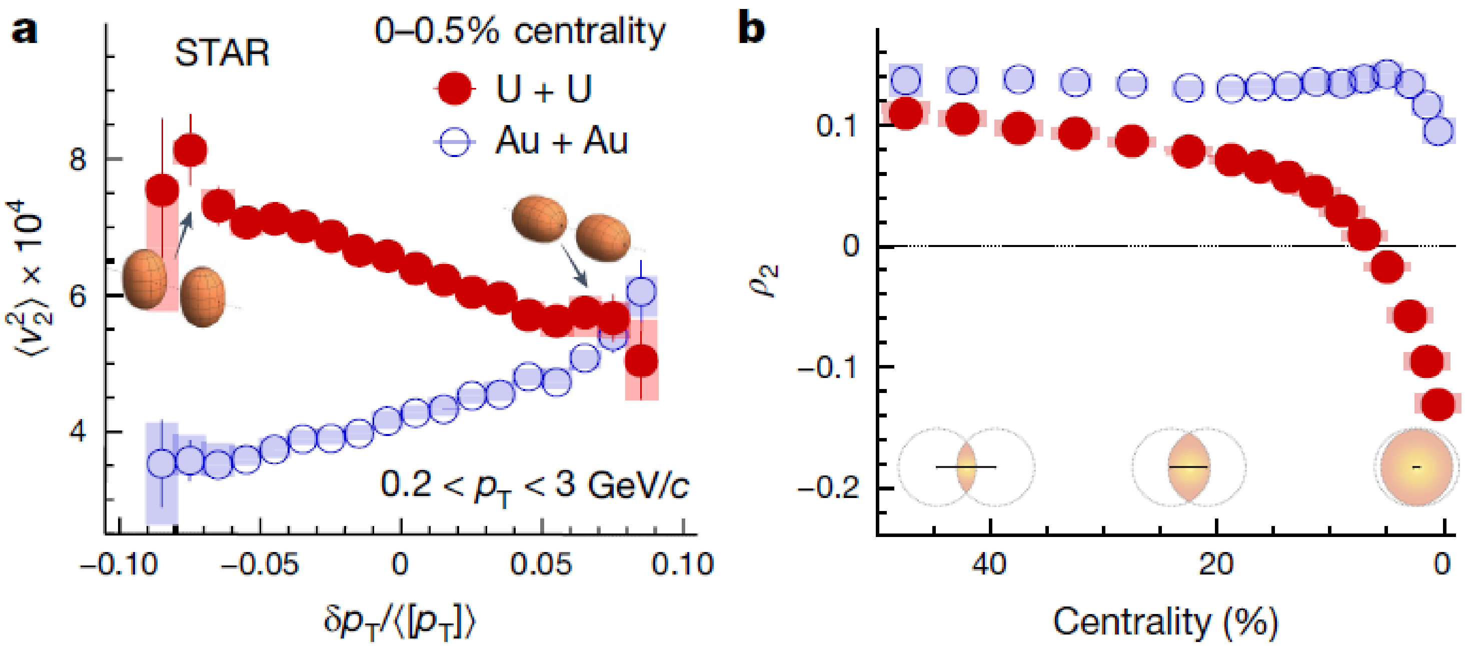
➔Inverse correlation of v_2 & $\bar{p}_T$

$v_n - p_T$ correlation:

$$\rho(v_n\{2\}^2, [p_\perp]) = \frac{\mathrm{cov}(v_n\{2\}^2, [p_\perp])}{\sqrt{\mathrm{Var}(v_n^2)_{\mathrm{dyn}} C_{p_\perp}}}$$

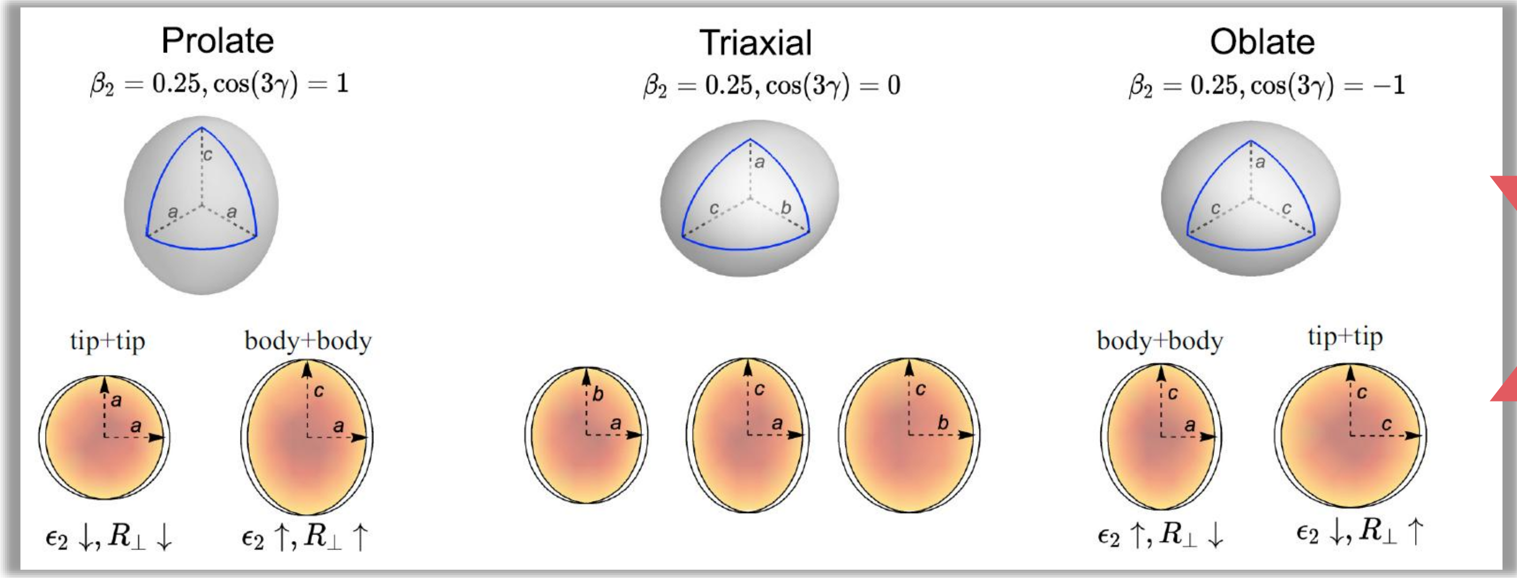
Bozek, PRC 93 ('16)

STAR, Nature 635 ('24)

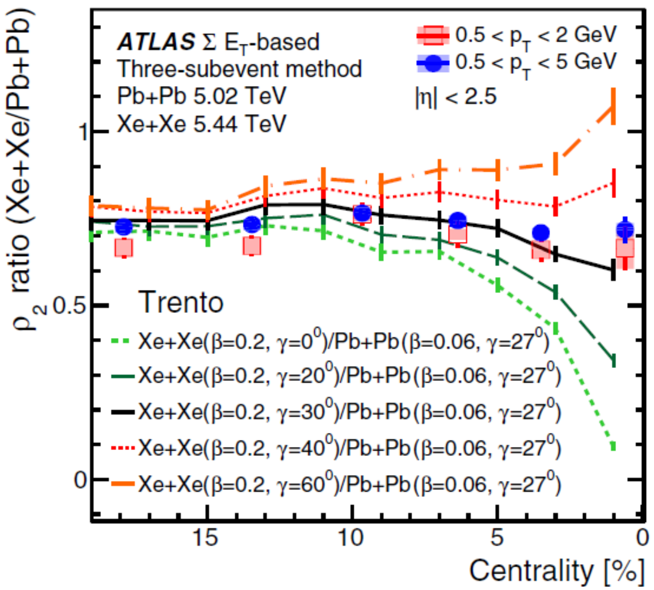


Triaxial Deformation

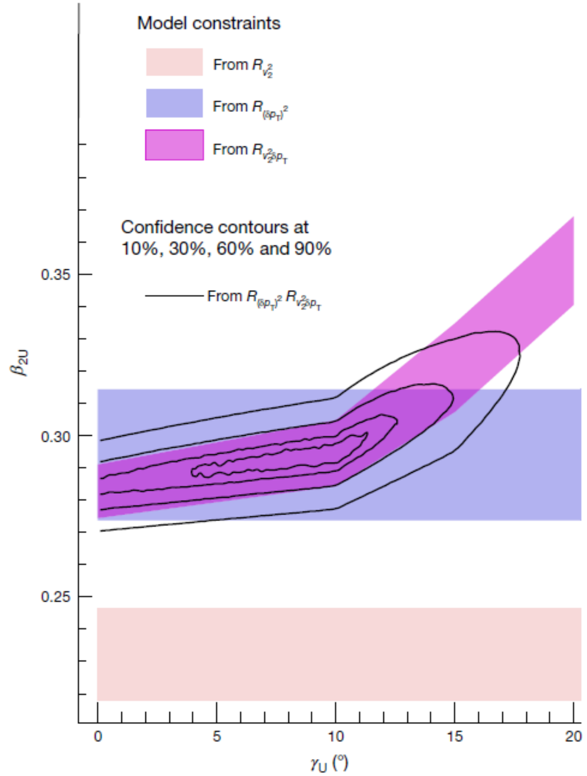
Fig.: Jia, PRC ('22)



ATLAS, PRC 107 (2023)



STAR, Nature 635 (2024)



- v_2-p_T correlation is sensitive to deform. param. γ.
- Other correlations sensitive to β_2 and γ.

$$\left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle = -\frac{3\sqrt{5}}{28\pi^{3/2}} \cos(3\gamma)\beta_2^3. \qquad \left\langle \left(\frac{\delta d_\perp}{d_\perp} \right)^2 \right\rangle = \frac{1}{16\pi}\beta_2^2$$

Further Extension

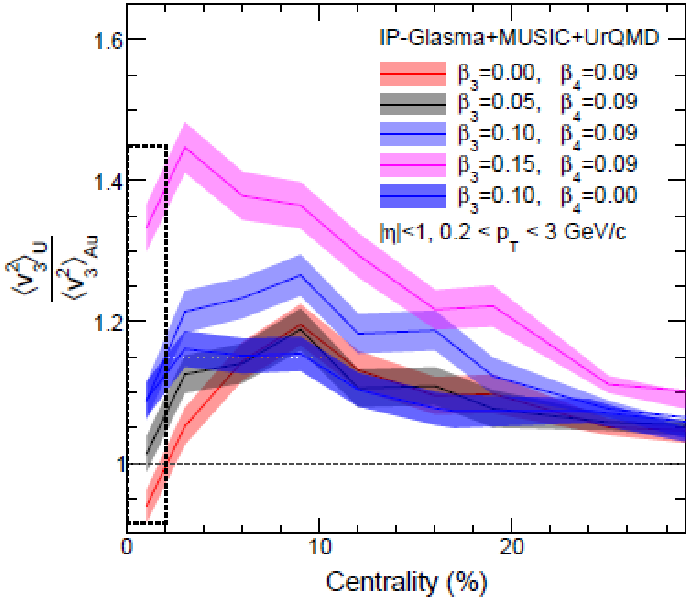
Probing

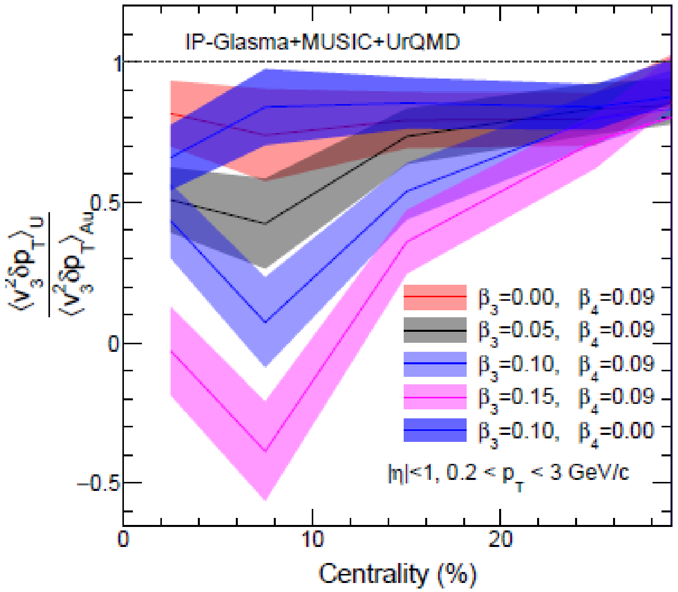
- Shape fluctuations

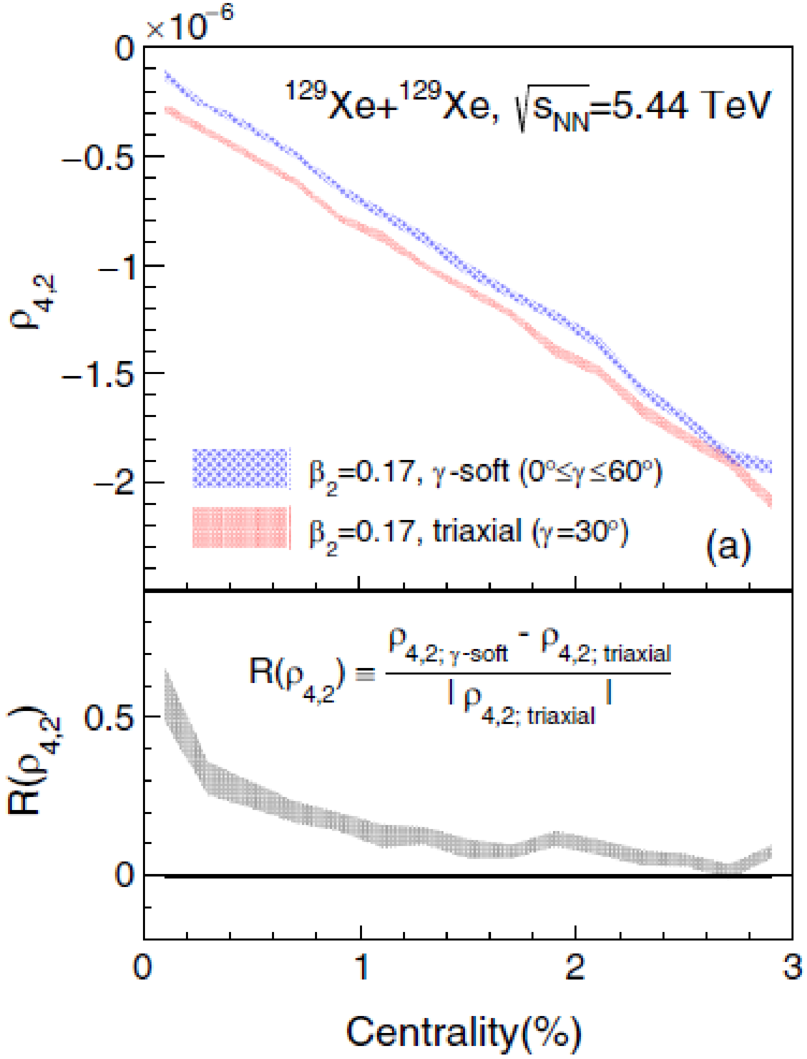
  Zhao, Xu, Zhou, Liu, Song, PRL ('24); Hagino, MK, PRC ('25); Liu+, 2509.09376; …

- Octupole/hexadecapole deformation

  Zhang+, 2504.15245; Xu+, 2504.19644; …

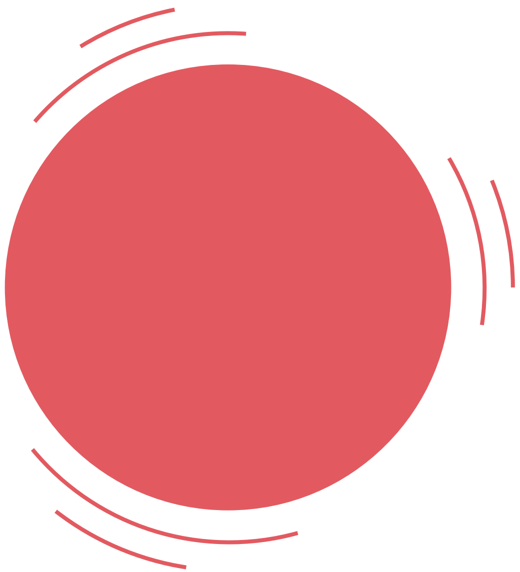






Quantum Surface Vibration

Hagino, MK, PRC ('25)

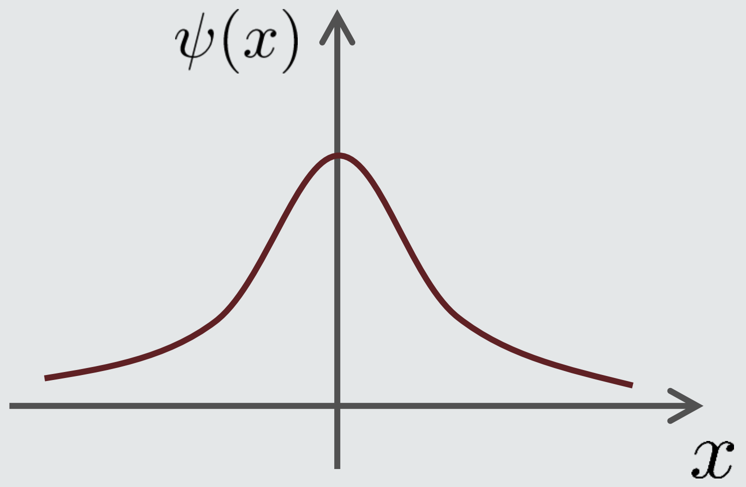


- Shape of a nucleus is **quantumly** vibrating even on the ground state.
- timescale of HIC $\gg$ surface vibration

HIC takes a snapshot of shape fluctuation.

Harmonic Oscillators

ground state



$\langle x \rangle = 0 \quad \langle x^2 \rangle \neq 0$

See also
Zhao, Xu, Zhou, Liu, Song ('24)
Xu, Xu, Zhao, Zhao, Song, Wang ('25)
Liu+, 2509.09376

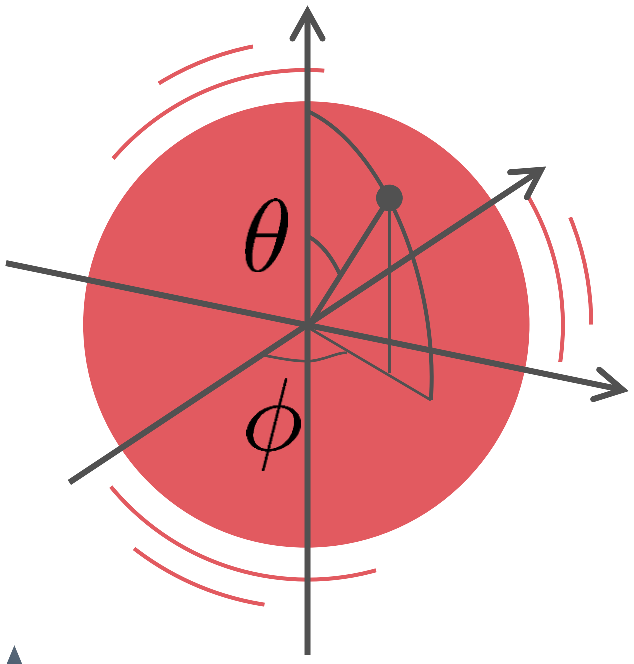
Spherical Nuclei

Hagino, MK, PRC ('25)

Space-fixed coordinates

$$\rho(\boldsymbol{r}) = \frac{\rho_0}{1 + e^{(r - R(\theta,\phi))/a}}$$

$$R(\theta,\phi) = R_0 \left(1 - \frac{1}{4\pi} \sum_{\lambda,\mu} |\alpha_{\lambda\mu}|^2 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\hat{\boldsymbol{r}}) \right)$$



Harmonic-oscillator model for surface vib.

$$H = \frac{1}{2} \sum_{\lambda,\mu} \left(B_\lambda |\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda |\alpha_{\lambda\mu}|^2 \right)$$

$$\left\langle \sum_{\mu} |\alpha_{\lambda\mu}|^2 \right\rangle = (\beta_\lambda)^2$$

More complicated in **body-fixed coordinates**

Constraint from low-E exp. of $B(E\lambda)$

$$\beta_\lambda = \frac{4\pi}{3ZR_0^\lambda} \sqrt{\frac{B(E\lambda)\uparrow}{e^2}}$$

Hagino, Takigawa ('12)
Hagino, Ogata, Moro ('22)

Treatment of surface vibration is apparent in the space-fixed coordinates.
Deformation params. β_λ can be constrained from transition probability.

Transverse Distribution

Hagino, MK, PRC ('25)

Initial Transverse Distr.

droplet full-overlap model

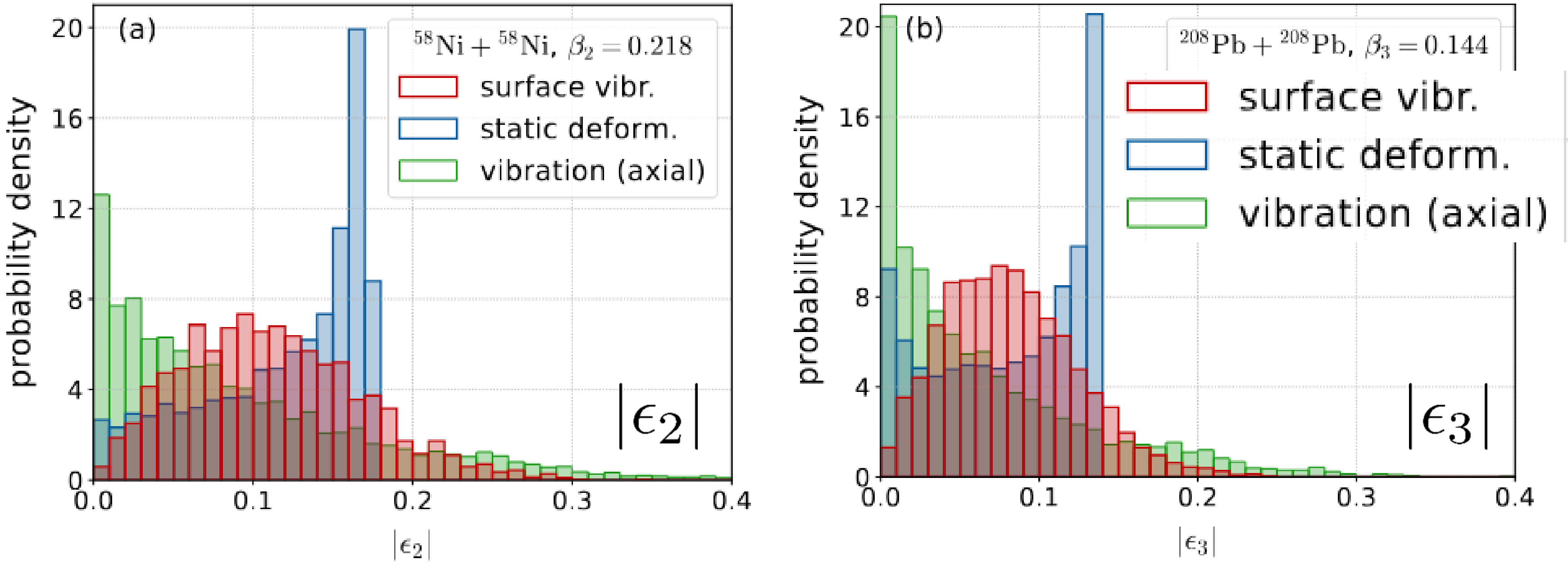
$$\rho^{(z)}(\boldsymbol{r}_\perp) = \int_{-\infty}^{\infty} dz\, \rho(\boldsymbol{r})$$

Jia ('22)

$$\epsilon_n = -\frac{\langle\langle (x-iy)^n \rangle\rangle}{\langle\langle (x^2+y^2)^{n/2} \rangle\rangle}$$

^{58}Ni, Quadrupole

^{208}Pb, Octupole



- Distribution differs significantly between the surface vibration and static deformation.
- Axial deformation is insufficient to describe the surface vibration.

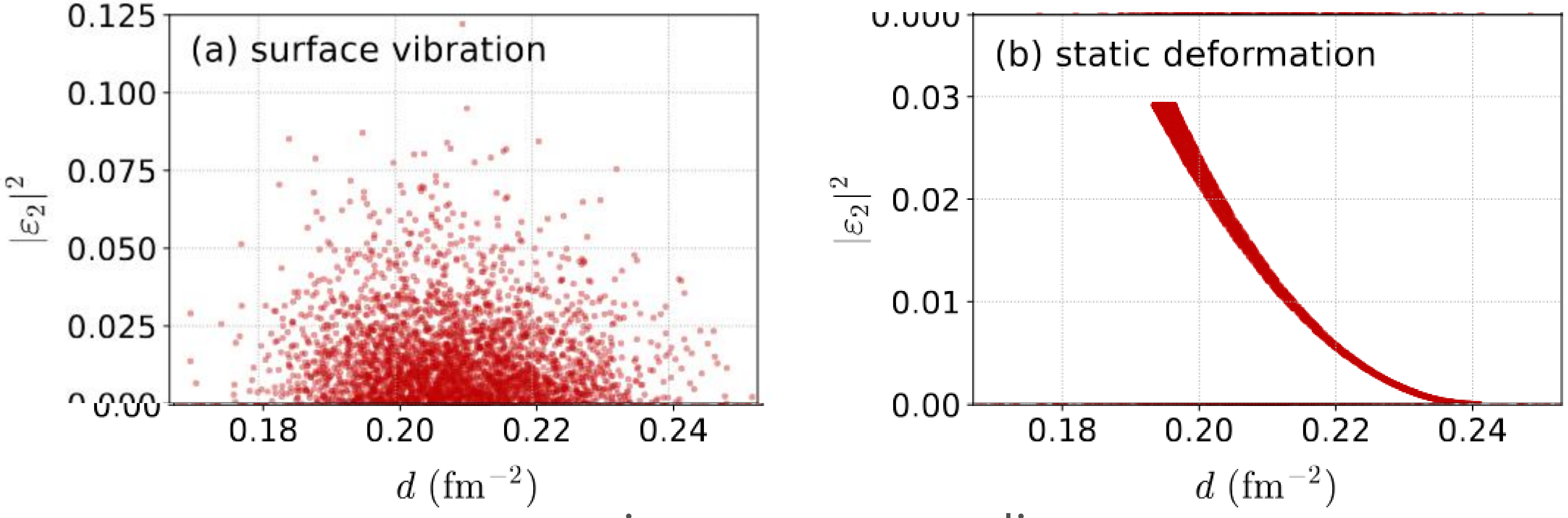
		mean	std. dev.	skewness	kurtosis
^{58}Ni, $\lvert\epsilon_2\rvert$	SV	0.112(1)	0.0554(7)	0.49(3)	-0.02(11)
	SD	0.119(1)	0.0500(5)	-0.79(3)	-0.62(6)
	SV-A	0.090(1)	0.0816(13)	1.22(4)	1.12(20)
^{208}Pb, $\lvert\epsilon_3\rvert$	SV	0.0822(8)	0.0416(5)	0.55(4)	0.15(11)
	SD	0.0821(8)	0.0461(4)	-0.38(3)	-1.29(3)
	SV-A	0.0650(12)	0.0649(11)	1.35(5)	1.49(22)

Transverse Distribution 2

Hagino, MK, PRC ('25)

^{58}Ni, Quadrupole, $\beta_2 = 0.218$

Surface Vibration **Static Deform.**



inverse mean radius

$$\epsilon_n = -\frac{\langle\langle (x - iy)^n \rangle\rangle}{\langle\langle (x^2 + y^2)^{n/2} \rangle\rangle},$$

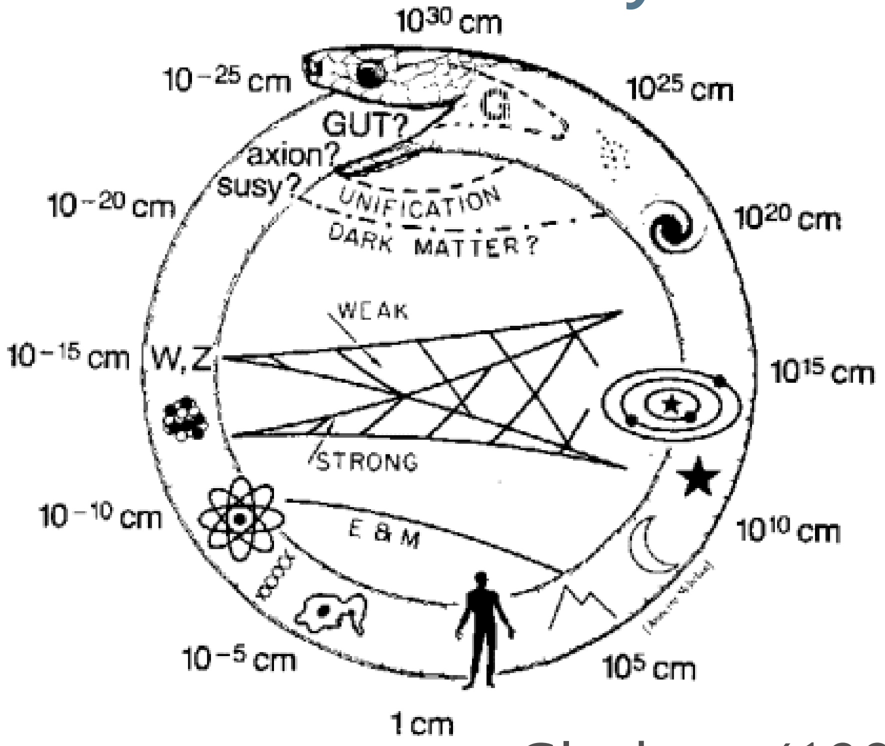
$$d = \frac{1}{\sqrt{\langle\langle x^2 \rangle\rangle \langle\langle y^2 \rangle\rangle}}$$

Short Summary

- Surface vibration and static deformation are discriminable through the distributions of ϵ_2, d.
- Space-fixed prescription is more convenient in treating the surface vibration of spherical nuclei.

Uroboros in Nuclear Physics

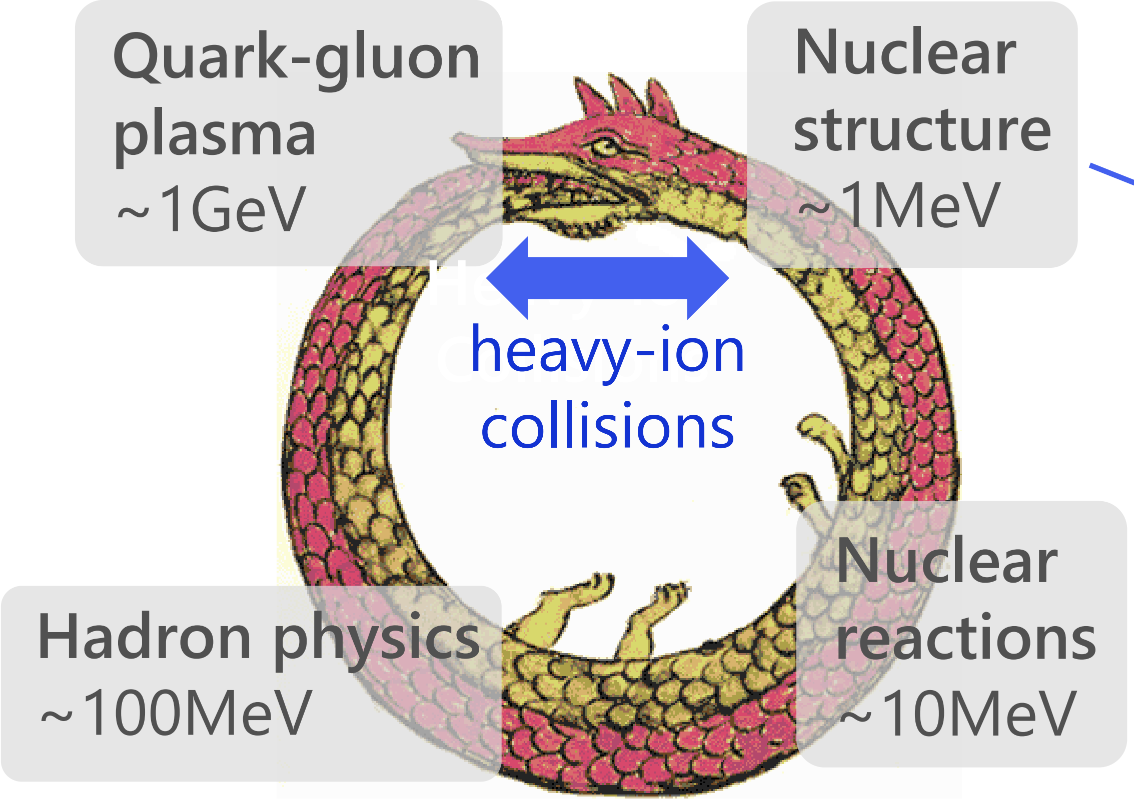
Uroboros of Physics



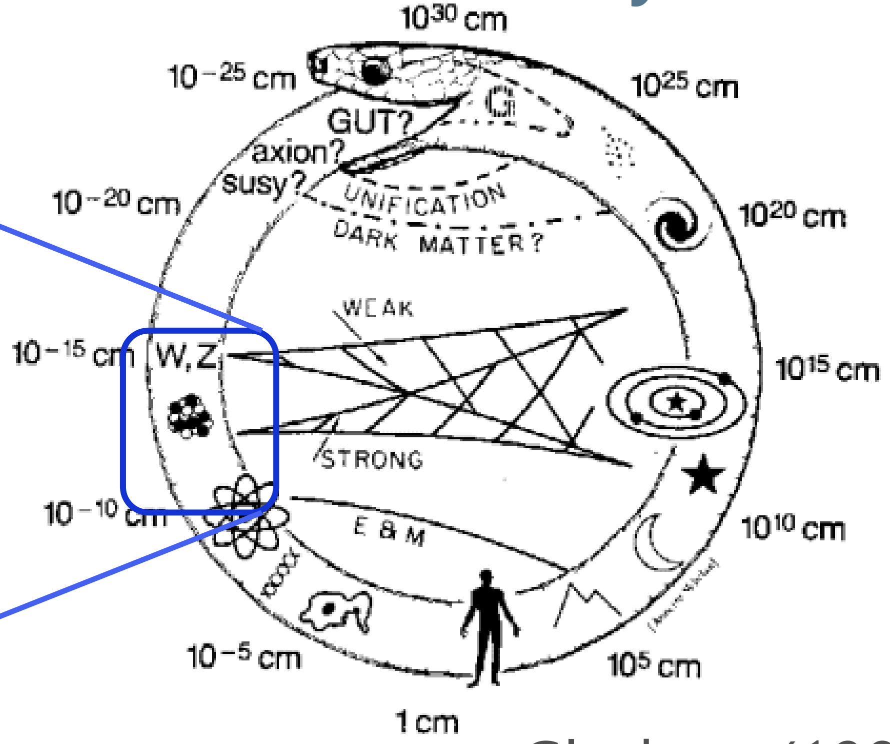
Glashow (1982)

- ◻ High-Energy HIC provides us with info. of nuclear structure.
- ◻ Nuclear structure is necessary for understanding Relativistic HIC.

Uroboros in Nuclear Physics



Uroboros of Physics



Glashow (1982)

- High-Energy HIC provides us with insights into nuclear structure.
- Nuclear structure is necessary for understanding relativistic HIC.

Workshop at YITP, April 2026

Contents

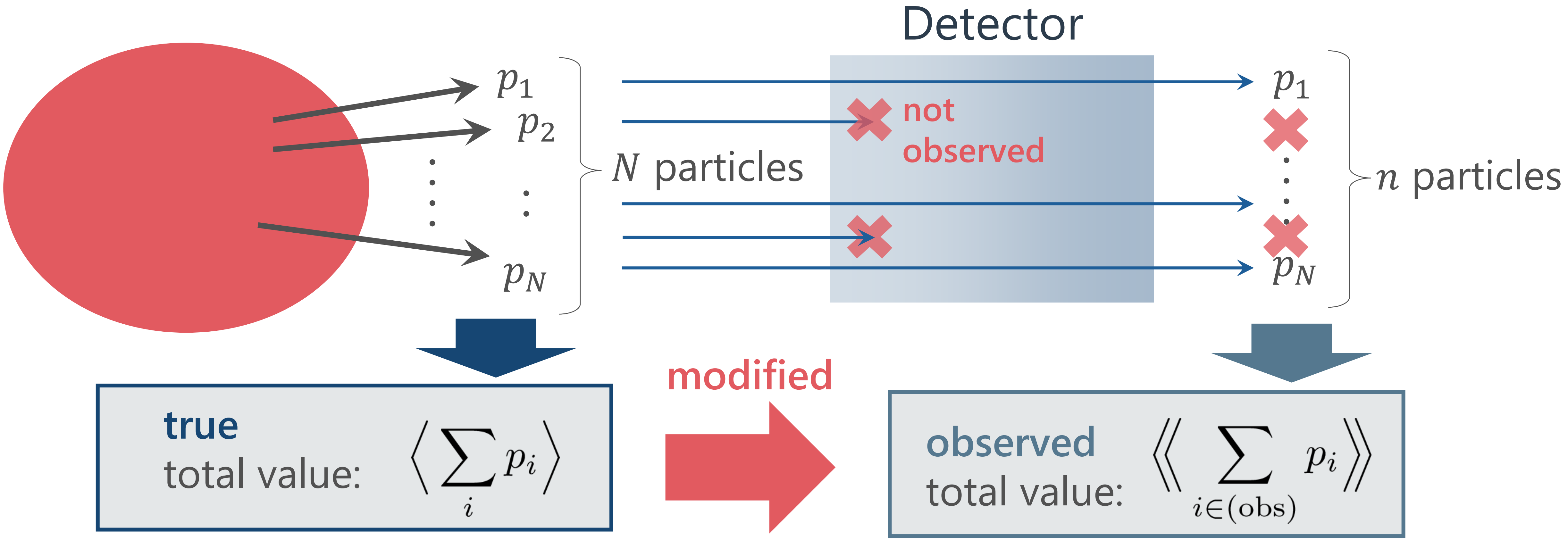
1. Probing Shape Fluctuations of Nuclei in HIC

   Hagino, MK, PRC112 (2025).

2. Efficiency Correction of Particle-averaged Quantities

   MK, Esumi, Niida, Nonaka, arXiv:2510.18383.

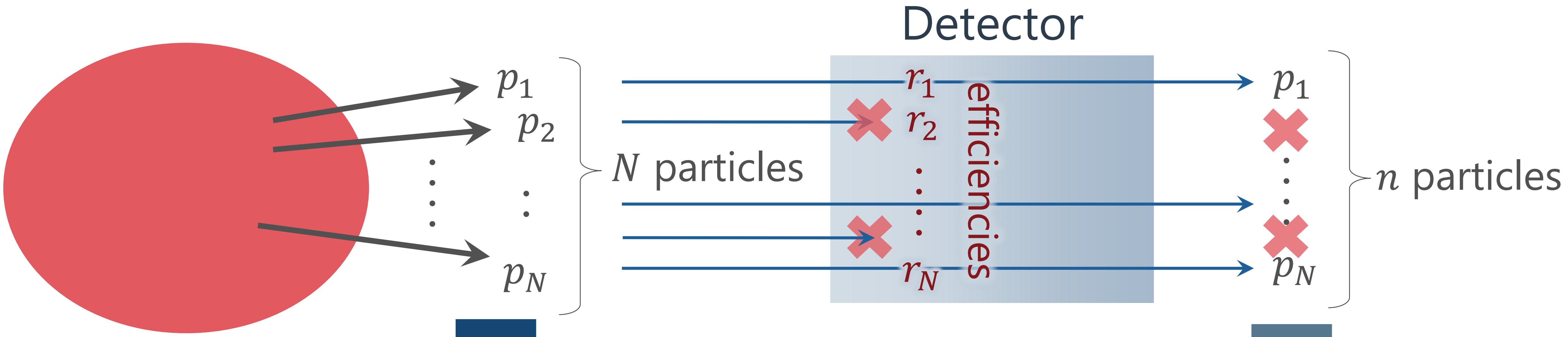
Detector Efficiency



Real detectors lose some particles

Observed results are modified.
Effects must be corrected to obtain the true result.

Efficiency Correction: Total Number



true total value: $\left\langle \sum_i p_i \right\rangle$

modified

observed total value: $\left\langle\!\left\langle \sum_{i\in(\mathrm{obs})} p_i \right\rangle\!\right\rangle$

Correction Formula: $$\left\langle \sum_i p_i \right\rangle = \left\langle\!\left\langle \sum_i \frac{p_i}{r_i} \right\rangle\!\right\rangle$$

Moments (Cumulants) of Total Number

$$\left\langle \left(\sum_i p_i \right)^n \right\rangle$$

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Correction Procedure:

Use factorial moments/cumulants

$$\left\langle \left(\sum_i p_i \right)^n \right\rangle_{\rm f} = \left\langle\!\left\langle \left(\sum_i \frac{p_i}{r_i} \right)^n \right\rangle\!\right\rangle_{\rm f}$$

Assumption: efficiencies of individual particles are independent

Nonaka, MK, Esumi ('17)
Asakwa, MK, PPNP ('16); MK, Luo ('17)

Moments (Cumulants) of Total Number

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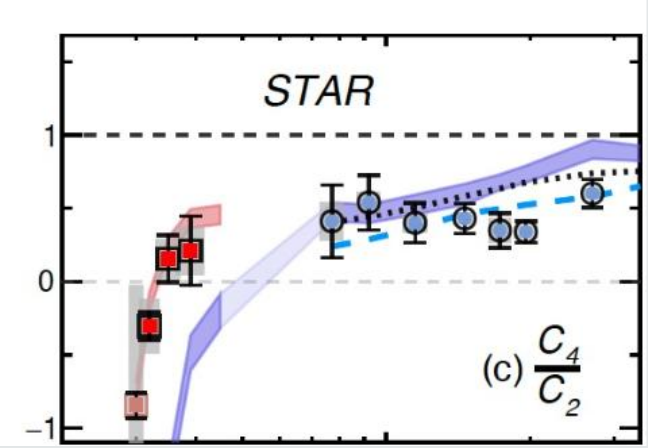
Nonaka, MK, Esumi ('17)
Asakwa, MK, PPNP ('16); MK, Luo ('17)

Note
Search for QCD-CP using conserved-charge fluctuations



Long history of efficiency correction:
MK, Asakawa ('12); Bzdak, Koch ('12,'15); Luo ('14); MK ('16); Nonaka+ ('16); Bzdak, Holtzman, Koch ('16); MK, Luo ('17); Nonaka, MK, Esumi ('17); ...

Particle-Averaged Quantities

$$\left\langle \tfrac{1}{N} \textstyle\sum_i p_i \right\rangle, \quad \left\langle \left(\tfrac{1}{N} \textstyle\sum_i p_i \right)^n \right\rangle, \quad \left\langle \frac{1}{N(N-1)} \textstyle\sum_{i \neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$

Particle-Averaged Quantities

$$\left\langle \tfrac{1}{N} \sum_i p_i \right\rangle, \quad \left\langle \left(\tfrac{1}{N} \sum_i p_i \right)^n \right\rangle, \quad \left\langle \tfrac{1}{N(N-1)} \sum_{i\neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$

Many fundamental observables in HIC are of this form!

mean p_T, flow anisotropy $v_n\{m\}$, $v_2 - p_T$ correlation, etc.

$$v_n\{2\} = \left\langle \frac{1}{N(N-1)} \sum_{i\neq j} e^{in(\phi_i - \phi_j)} \right\rangle$$

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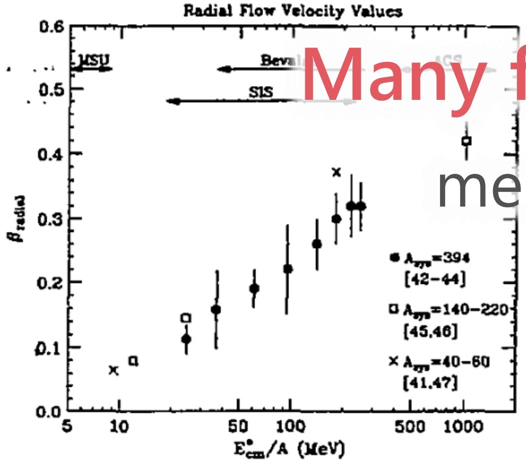
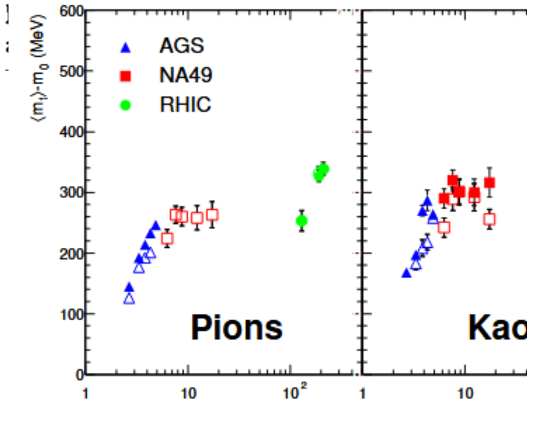
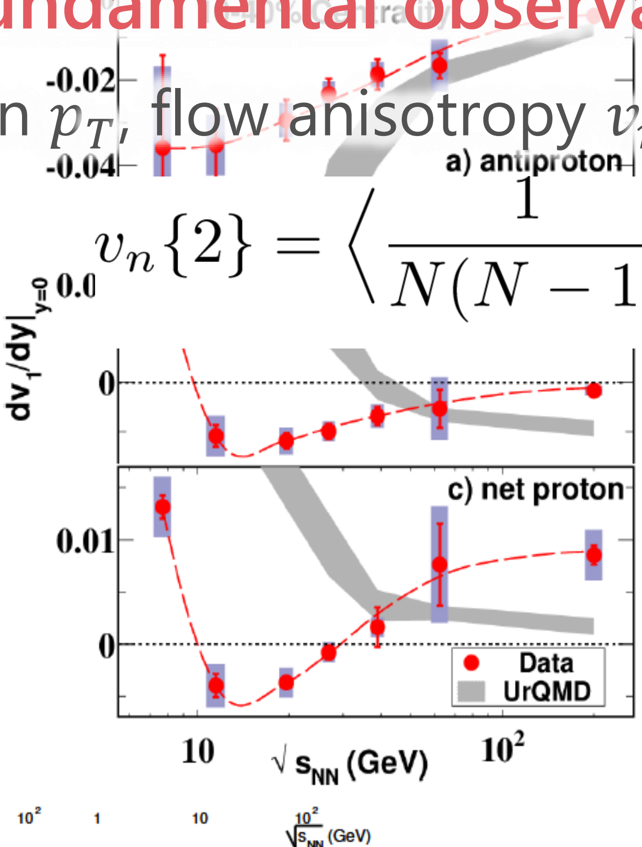
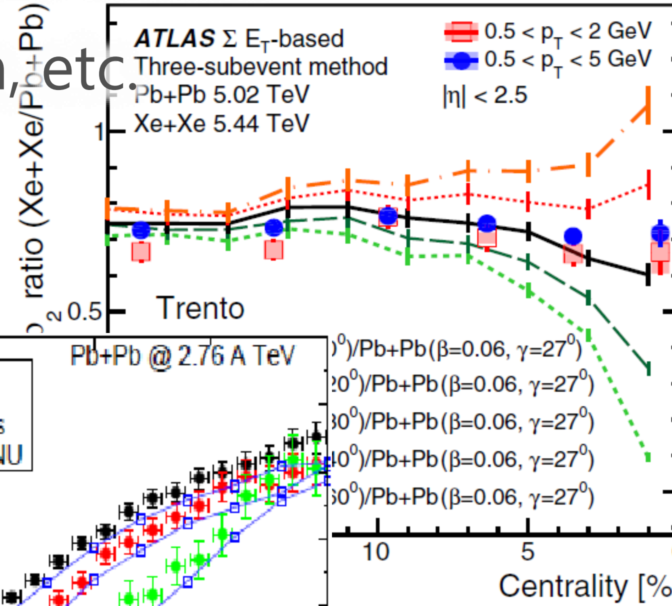
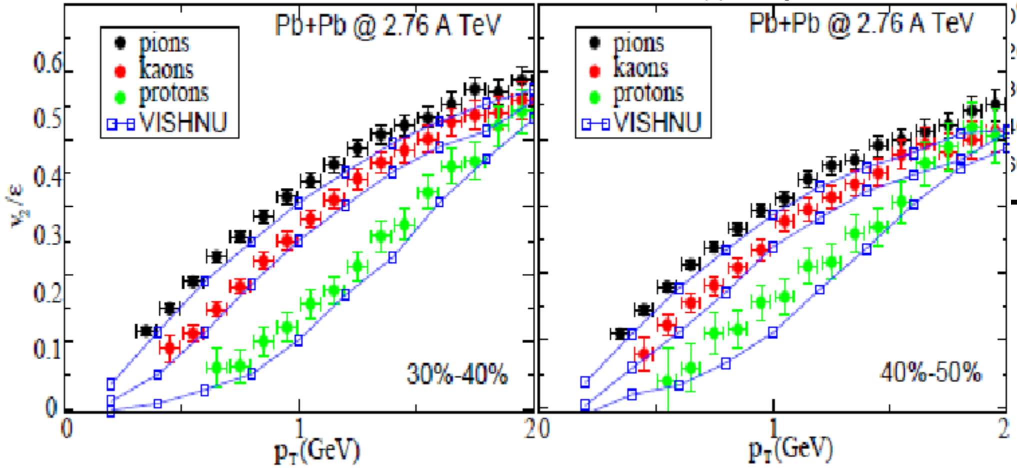










Particle-Averaged Quantities

$$\left\langle \tfrac{1}{N} \textstyle\sum_i p_i \right\rangle, \quad \left\langle \left(\tfrac{1}{N} \textstyle\sum_i p_i \right)^n \right\rangle, \quad \left\langle \frac{1}{N(N-1)} \textstyle\sum_{i \neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$

Many fundamental observables in HIC are of this form!

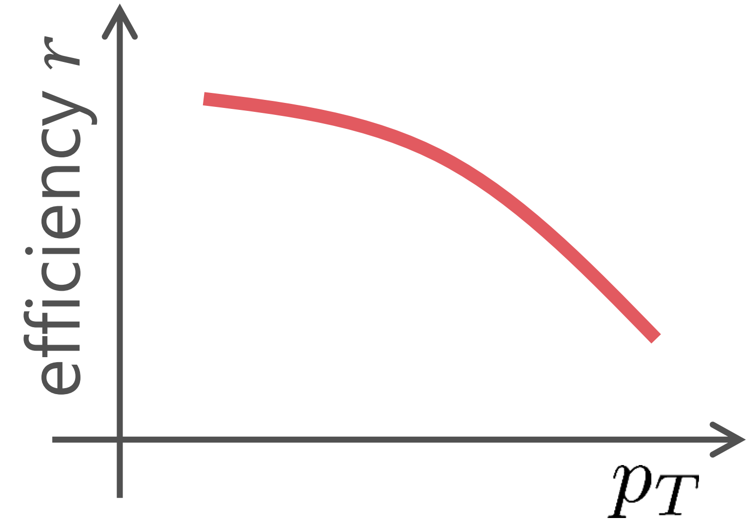
"Conventional" Correction Formulas

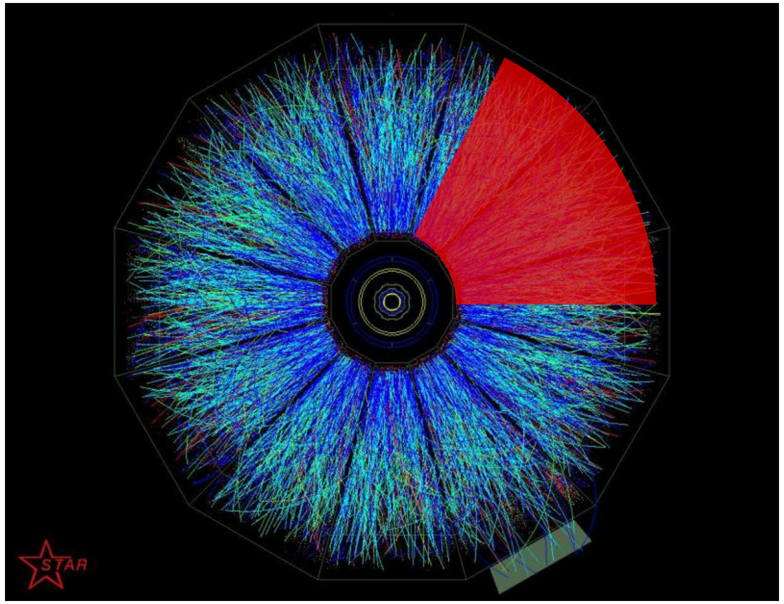
$$\left\langle \frac{1}{N(N-1)} \sum_{i \neq j} p_i^{(1)} p_j^{(2)} \right\rangle = \left\langle\!\!\left\langle \frac{\sum_{i \neq j} p_i^{(1)} p_j^{(2)} / r_i r_j}{\sum_{i \neq j} 1 / r_i r_j} \right\rangle\!\!\right\rangle$$

e.g. ATLAS, PRC**107**, 054910 ('23); STAR, Nature **635**, 67 ('24)

Question: Are these formulas correct?

Correction is Necessary!!





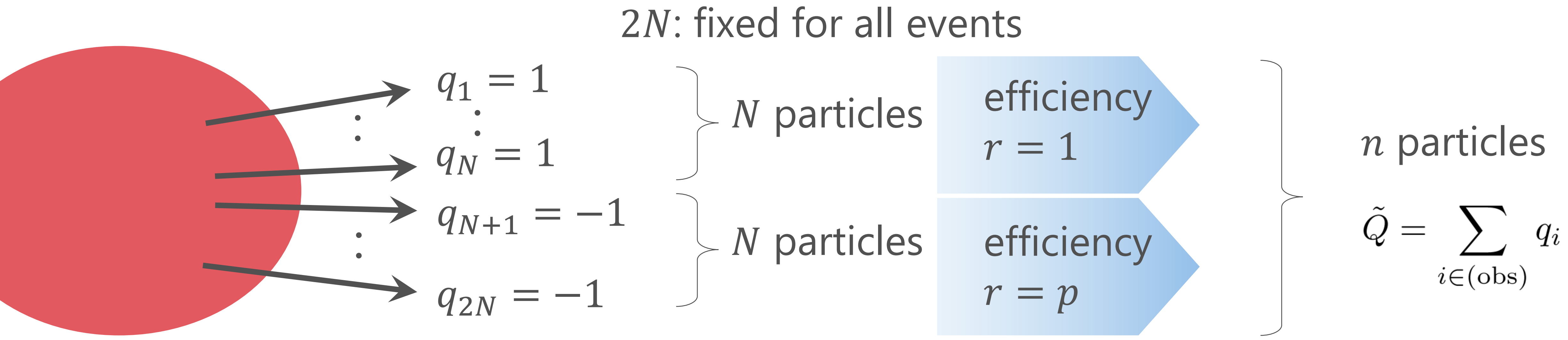
p_T-dependent efficiency → alter mean p_T

Azimuthally nonuniform efficiency → produce unphysical $v_n\{m\}$

More serious effects on higher-order correlations!

Check in a Simple Model

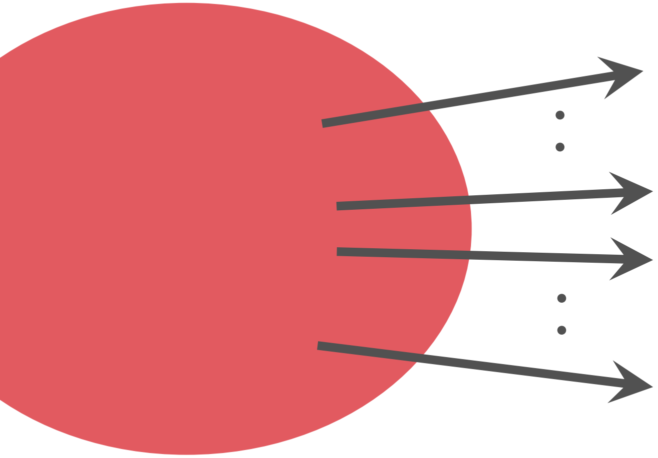
MK, Esumi, Niida, Nonaka, arXiv:2510.13838



Check in a Simple Model

MK, Esumi, Niida, Nonaka, arXiv:2510.13838

$2N$: fixed for all events



$q_1 = 1$
$\vdots$
$q_N = 1$ } N particles — efficiency $r = 1$

$q_{N+1} = -1$
$\vdots$
$q_{2N} = -1$ } N particles — efficiency $r = p$

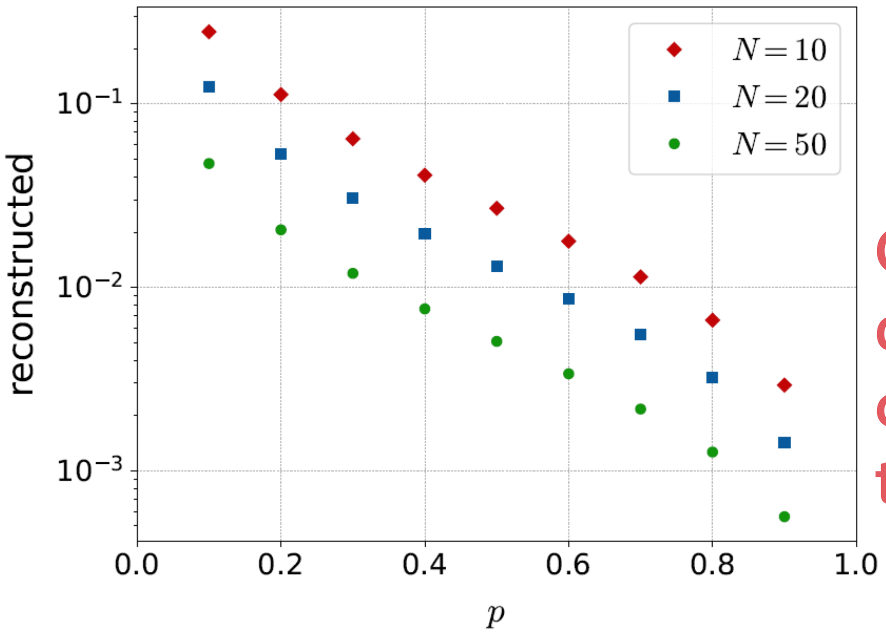
} n particles

$$\tilde{Q} = \sum_{i \in (\mathrm{obs})} q_i$$

Mean:

True result
$$\left\langle \frac{Q}{N} \right\rangle = 0$$

Reconstructed
$$\left\langle \frac{\sum_i q_i / r_i}{\sum_i 1/r_i} \right\rangle$$



Conventional formula does not reproduce the correct result even for the mean!!

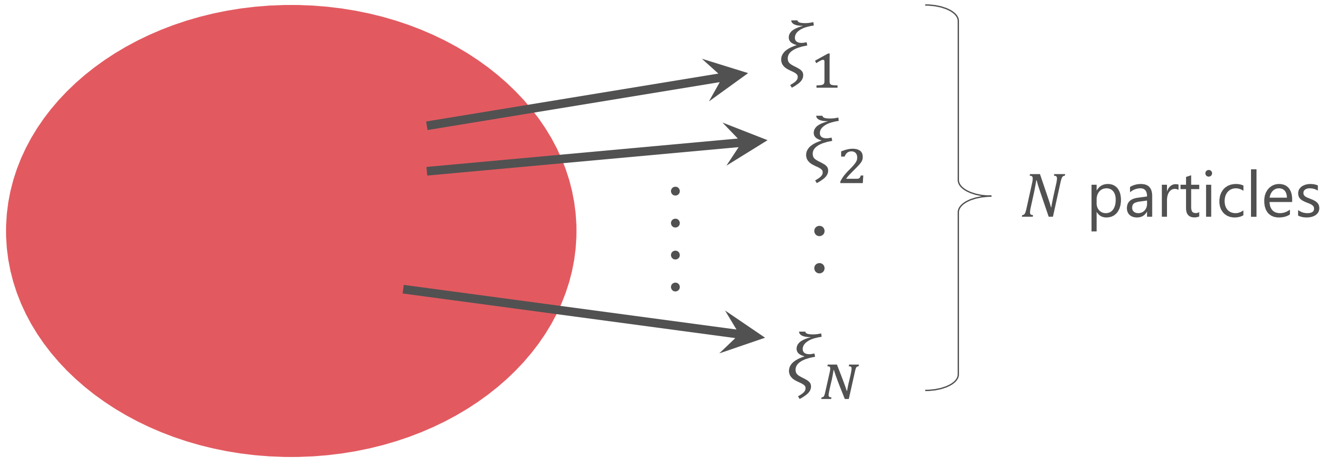
Derivation of Correction Formulas

Assumptions

1. Particle production is described by a classical prob. distr. func. $P(N;\vec{p}_T)$.
2. Probs. to observe individual particles are independent.
3. For each observed particle, the value of efficiency r_i can be specified.
4. Other detectors' effects are not considered.

True distr. func.

$$P(N;\vec{\xi})$$



Connecting True/Observed Distr. Funcs.

True distr. func.

$$P(N;\vec{\xi})$$

Observed distr. func.

$$\tilde{P}(n;q)$$

- n: observed particle number
- $q = \Sigma_{i\in(\mathrm{obs.})}\xi_i$: observed sum

Probability Distr. of Observed Quantities (uniform r)

$$\tilde{P}(n;q) = \sum_{N=1}^{\infty}\int d\vec{\xi}\sum_{\{b_i\}}\left[\prod_{i=1}^{N}(1-r)^{1-b_i}r^{b_i}\right]\delta_{n,\sum_i b_i}\delta(q-\Sigma_i b_i\xi_i)P(N;\vec{\xi})$$

$$b_i = 0,1$$

Generating Function

MK, Esumi, Niida, Nonaka, arXiv:25010.13838

Prob. distr. func: $\tilde{P}(n;q) = \sum_{N=1}^{\infty} \int d\vec{\xi} \sum_{\{b_i\}} \left[\prod_{i=1}^{N} (1-r)^{1-b_i} r^{b_i} \right] \delta_{n,\sum_i b_i} \delta(q - \Sigma_i b_i \xi_i) P(N;\vec{\xi})$

Generating func: $\tilde{G}(s,t) = \sum_n \int dq \tilde{P} s^n t^q = \sum_N \int d\vec{\xi} P \prod_i (1 - r + rst^{\xi_i})$

Represent the quantity that you want to express by the derivative of the generating function.
Then, represent it in terms of the observed variables.

$$\left\langle \frac{\sum_i \xi_i}{N} \right\rangle_{\mathrm{true}} = \int_\alpha^1 ds \frac{r}{s} \left[\partial_t \tilde{G}(s,t) \right]_{t=1} = \left\langle \frac{\sum_i \xi_i}{n} \left(1 - \alpha^n\right) \right\rangle_{\mathrm{obs}} \qquad \alpha = \frac{r-1}{r}$$

Note: $\left\langle \frac{\sum_i \xi_i}{N} \right\rangle_{\mathrm{true}} \neq \left\langle \frac{\sum_i \xi_i}{n} \right\rangle_{\mathrm{obs}}$ α^n term compensates the $n = 0$ contribution.

Results: Correction Formulas

MK, Esumi, Niida, Nonaka, arXiv:25010.13838

Mean

$$\left\langle \frac{Q}{N} \right\rangle = \left\langle\!\left\langle \sum_{i=1}^{n} \xi_i k_i \right\rangle\!\right\rangle_{n\neq 0}$$

$$k_i = \frac{1}{r_i} \int_0^1 d\sigma \prod_{j\neq i} \frac{\sigma + r_j \alpha_j}{r_j}$$

2nd Order

$$\left\langle \frac{\{Q_1 Q_2\}}{N(N-1)} \right\rangle = \left\langle\!\left\langle \sum_{i\neq j} q_{1,i} q_{2,j} k_{2;i,j} \right\rangle\!\right\rangle_{n\neq 0,1}$$

$$k_{2;i,j} = \frac{1}{r_i r_j} \int_0^1 d\sigma' \int_0^{\sigma'} d\sigma \prod_{l\neq i, l\neq j} \left(\frac{\sigma}{r_l} + \alpha_l \right)$$

- Correction formulas are written in forms including integral.
- This formula can reproduce the correct result for the previous simple model.

$$\{Q_{w_1} Q_{w_2}\} \equiv \sum_{i\neq j} \xi_i^{(w_1)} \xi_j^{(w_2)}$$

$$\alpha_i = \frac{1 - r_i}{r_i}$$

Summary

Efficiency Correction Formulas

$$\left\langle \frac{Q}{N} \right\rangle = \left\langle\!\left\langle \sum_{i=1}^{n} \xi_i k_i \right\rangle\!\right\rangle_{n\neq 0} \qquad \left\langle \frac{\{Q_1 Q_2\}}{N(N-1)} \right\rangle = \left\langle\!\left\langle \sum_{i\neq j} q_{1,i} q_{2,j} k_{2;i,j} \right\rangle\!\right\rangle_{n\neq 0,1},$$

$$k_i = \frac{1}{r_i} \int_0^1 d\sigma \prod_{j\neq i} \frac{\sigma + r_j \alpha_j}{r_j}, \qquad k_{2;i,j} = \frac{1}{r_i r_j} \int_0^1 d\sigma' \int_0^{\sigma'} d\sigma \prod_{l\neq i, l\neq j} \left(\frac{\sigma}{r_l} + \alpha_l \right),$$

These formulas reproduce the true value in simple models.

They will play crucial roles in experimental studies in HIC.

What I have not Understood

Relation of these formulas with the conventional ones.

More simplified formula / unified understanding of mathematical structure.

Should Self-correlations be Eliminated?

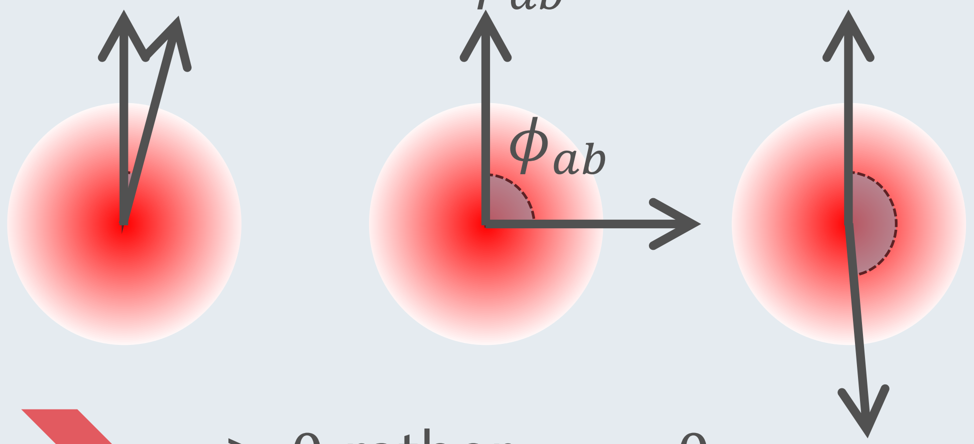
Flow correlations: $v_2^2 = \left\langle \frac{\sum_{i \neq j} e^{i(\phi_i - \phi_j)}}{N(N-1)} \right\rangle$

The "self correlation" terms are usually neglected. **Why?**

Argument 1:

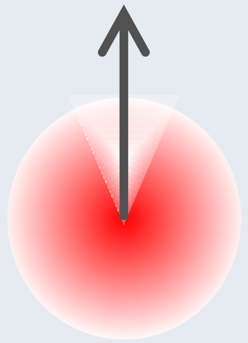
Emission of 2 independent particles

ϕ_{ab}: random



$v_2 > 0$ rather $v_2 = 0$

Argument 2:



Emission of a particle take away density

suppress probability to emit particles to same direction

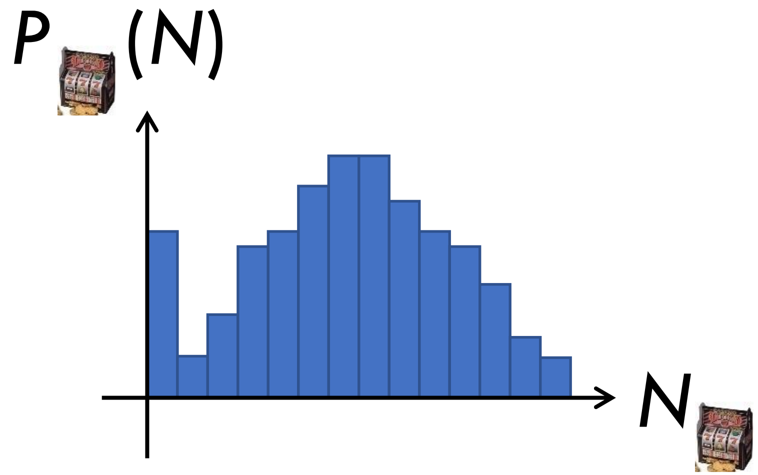
Simpler Example: Particle Number Fluc.

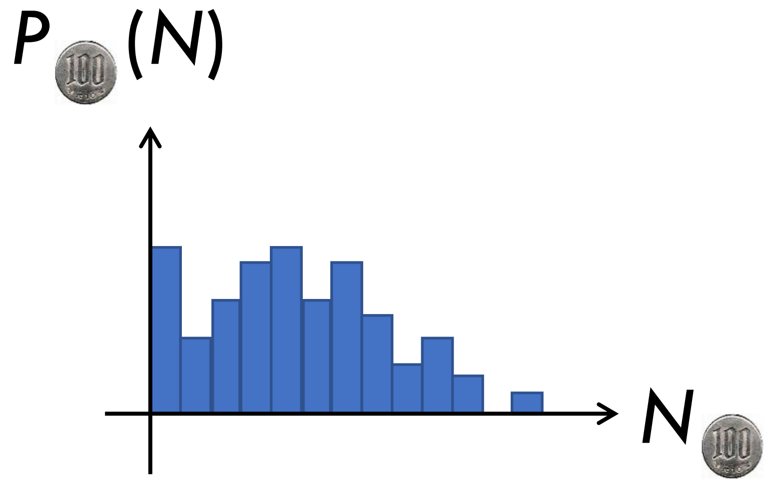
How can we obtain the cumulants of the true distribution only from observed information on $\tilde{P}(n)$?

MK, Asakawa, 2012

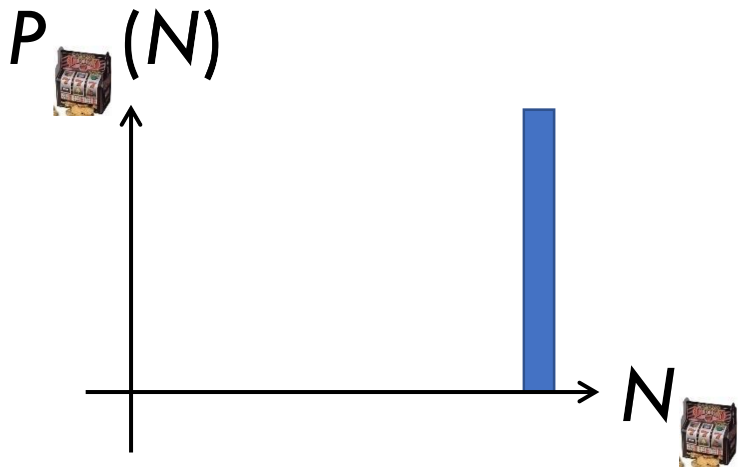
MK, Asakawa, 2012;2012

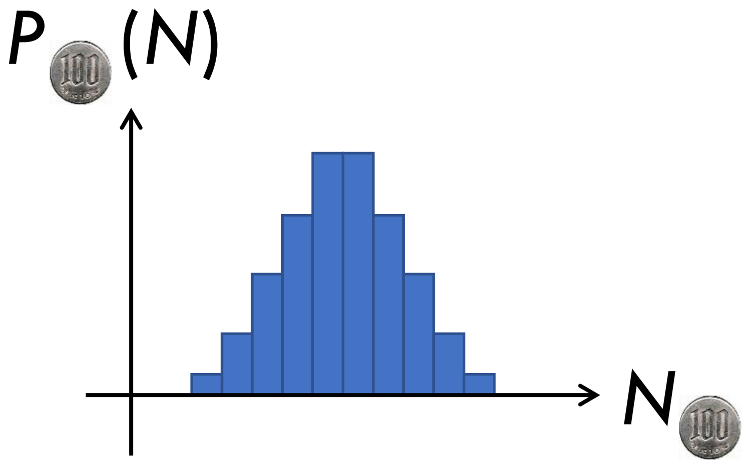
Slot Machine Analogy



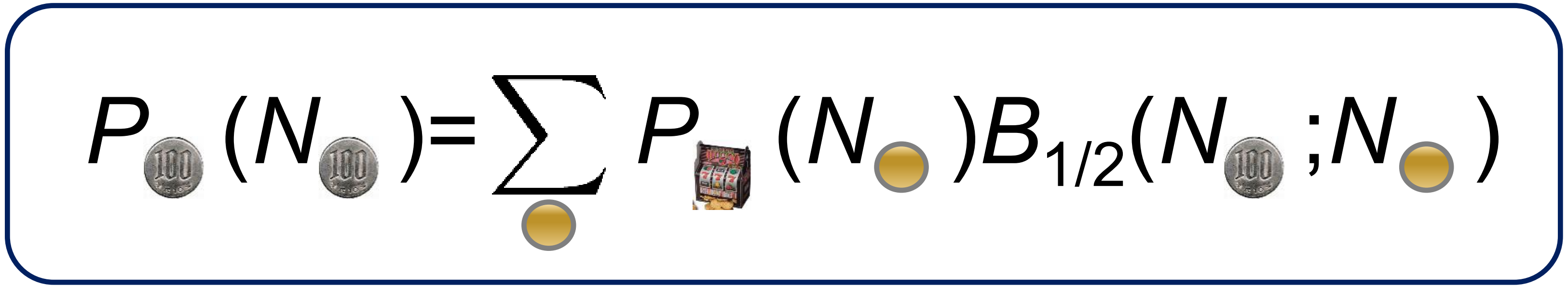


Fixed # of coins





Reconstructing Total Coin



Example

$$
\begin{cases}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots
\end{cases}
$$

genuine info. Poisson noise

Note: Higher order cumulants are more fragile.

MK, Asakawa, 2012;2012