Nuclear Collision Dynamics and Pion Production

Natsumi Ikeno (Kobe University)

Akira Ono, Yasushi Nara, Akira Ohnishi, Che Ming Ko

- Heavy-ion collisions at intermediate energies (Sn+Sn at E/A~300 MeV, neutron-rich systems)
- Nuclear equation of state (EOS) and constraints on the symmetry energy
- Cluster and Pion production within AMD+(s)JAM transport model

- N. Ikeno, A. Ono, Y. Nara and A. Ohnishi, PRC93, 044612 (2016), PRC97, 069902(E) (2018), PRC101, 034607 (2020)
- N. Ikeno and A. Ono, PRC 108, 044601 (2023)
- N. Ikeno, A. Ono, and C. M. Ko, in preparation
- TMEP collaboration (Review paper), Prog. Part. Nucl. Phys. 125, 103962 (2022)

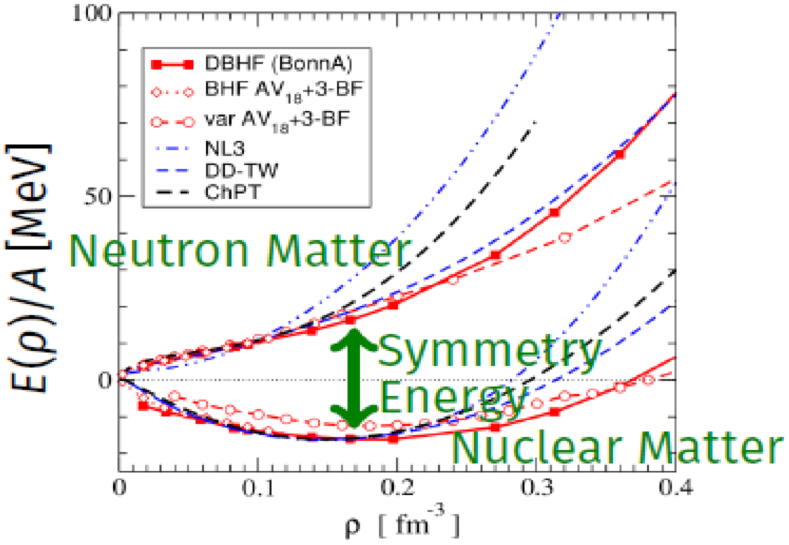
Investigation of the Nuclear Equation of State (EOS)

Nuclear matter EOS

$$\frac{E}{A}(\rho_p,\rho_n) = \left(\frac{E}{A}\right)_0(\rho) + E_{sym}(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = (\rho_n - \rho_p)/\rho$$

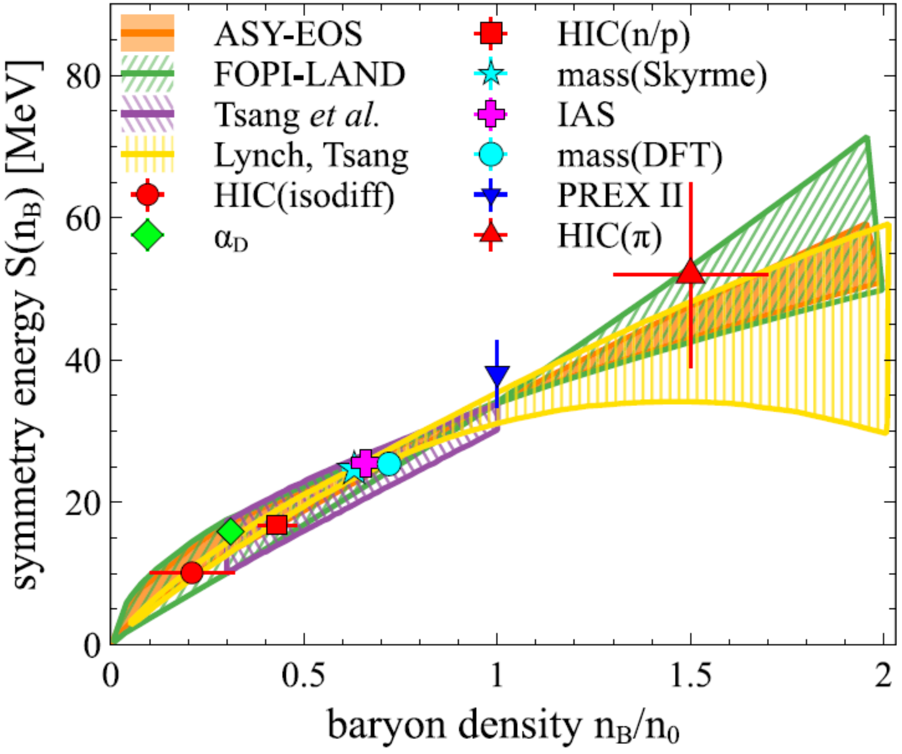
$$S_0 = E_{sym}(\rho_0), \quad L = 3\rho_0(dE_{sym}/d\rho)_{\rho=\rho_0}$$



Fuchs and Wolter, EPJA 30, 5 (2006)

Constraints on the symmetry energy $E_{sym}(\rho)$

A. Sorensen et al. Prog. Part. Nucl. Phys. 134, 104080 (2024)



- Low-density region: Many constraints
- High-density region ($\rho_0<\rho$) Still large uncertainties

How can we constrain $E_{sym}(\rho)$ at $\sim 2\rho_0$?

- Pion production in HICs (This talk)

- Isospin dependence of collective flow

Nuclear structure

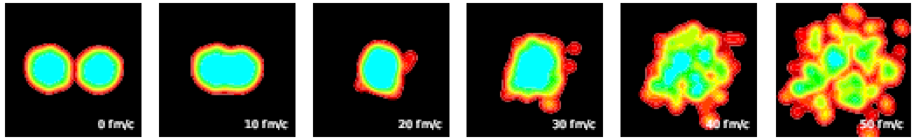
Neutron star mergers

Heavy-ion collisions

Pion production in Heavy-ion collisions

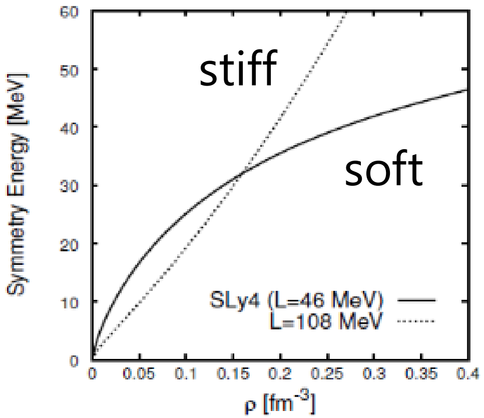
$^{132}Sn + ^{124}Sn$, $E/A = 300$ MeV, $b \sim 0$

AMD calculation

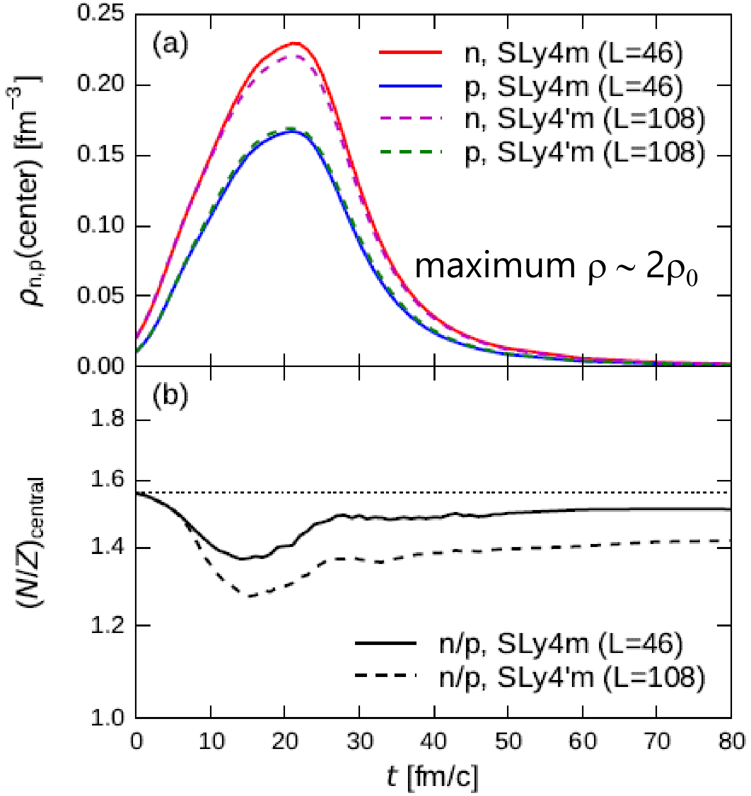


Symmetry energy soft / stiff	⟷ (Nucleon dynamics)	Nucleon N/Z	⟷ $(NN \leftrightarrow N\Delta)$	Δ Particle Δ^-/Δ^{++}	⟷ $(\Delta \leftrightarrow N\pi)$	Pion π^-/π^+

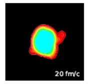
Effective interaction:
Skyrme force



Interest:
High density $\rho \sim 2\rho_0$



Clear difference of N/Z in high ρ due to different $E_{sym}(\rho)$

Formation in NN collisions 

π^- production

$$nn \to p\Delta^-$$
$$\Delta^- \to n\pi^-$$

π^+ production

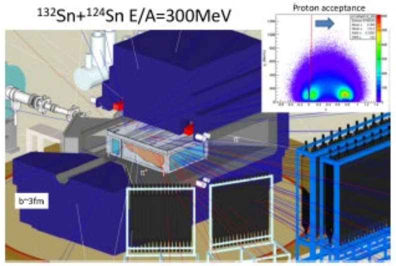
$$pp \to n\Delta^{++}$$
$$\Delta^{++} \to p\pi^+$$

Simple expectation : $\frac{\pi^-}{\pi^+} \simeq \left(\frac{N}{Z}\right)^2$ @high density

Considered to be sensitive to symmetry energy at high density B. A. Li, PRL 88, 192701 (2002)

Experimental study:
SπRIT project @RIBF

Sn+Sn, E/A=270 MeV

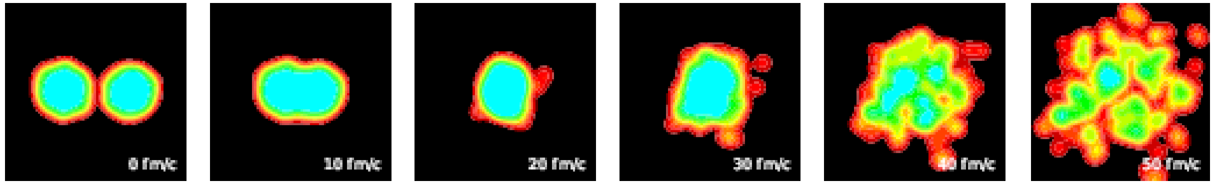


Transport equation for heavy-ion collisions

Transport models are used as the main method to obtain physics information from HICs by solving the time evolution of the collision reaction

$^{132}\mathrm{Sn}+{}^{124}\mathrm{Sn}$, $E/A = 300$ MeV, $b \sim 0$

AMD calculation



- Transport equation for one-body distribution function $f_\alpha(\mathbf{r}, \mathbf{p}, t)$ (α = n, p, Δ^-, Δ^0, Δ^+, Δ^{++}, π^-, π^0, π^+)

  BUU eq.

H. Wolter et al. [TMEP], (Review paper), Prog. Part. Nucl. Phys. 125, 103962 (2022)

$$\frac{\partial f_\alpha(\boldsymbol{r},\boldsymbol{p},t)}{\partial t} = \frac{\partial h_\alpha[f]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_\alpha}{\partial \boldsymbol{p}} - \frac{\partial h_\alpha[f]}{\partial \boldsymbol{p}} \cdot \frac{\partial f_\alpha}{\partial \boldsymbol{r}} + \int |\boldsymbol{v}| \frac{d\sigma}{d\Omega} \{f_3 f_4 (1-f)(1-f_2) - f f_2 (1-f_3)(1-f_4)\} \frac{d\boldsymbol{p}_2\, d\Omega}{(2\pi\hbar)^3}$$

Mean-field propagation term

Collision term
(NN $\leftrightarrow$ NN , NN $\leftrightarrow$ NΔ, $\Delta \leftrightarrow$ Nπ)

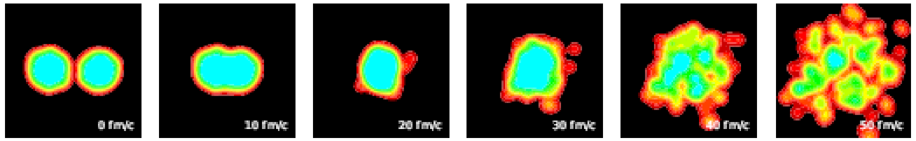
Single-particle Hamiltonian ($\Leftarrow$ EOS): $h_\alpha(\boldsymbol{r},\boldsymbol{p};f) = \dfrac{\delta E}{\delta f_\alpha(\boldsymbol{r},\boldsymbol{p})} = \dfrac{\boldsymbol{p}^2}{2m_\alpha} + U_\alpha(\boldsymbol{r},\boldsymbol{p};f)$

- Potentials U_a enter in both the mean-field propagation and the collision term (in principle)

What factors influence pion production, and to what extent?

^{132}Sn + ^{124}Sn, $E/A = 300$ MeV, $b \sim 0$

AMD calculation



Symmetry energy soft / stiff ⟷ (Nucleon dynamics) ⟷ Nucleon N/Z ⟷ ($NN \leftrightarrow N\Delta$) ⟷ Δ Particle Δ^-/Δ^{++} ⟷ ($\Delta \leftrightarrow N\pi$) ⟷ Pion π^-/π^+

✓ Symmetry energy at high density

- N. Ikeno, A. Ono, Y. Nara and A. Ohnishi, PRC93, 044612 (2016)

✓ Cluster correlation

- A. Ono, Prog. Part. Nucl. Phys. 105, 139 (2019)
- N. Ikeno, A. Ono, Y. Nara and A. Ohnishi, PRC93, 044612 (2016), PRC97, 069902(E) (2018)

✓ Pauli blocking

- N. Ikeno, A. Ono, Y. Nara and A. Ohnishi, PRC101, 034607 (2020)

✓ Momentum dependence of mean-field potential

- N. Ikeno and A. Ono, PRC 108, 044601 (2023)

✓ Delta and Pion potential effects

- N. Ikeno and A. Ono, PRC 108, 044601 (2023), - N. Ikeno, A. Ono, and C. Ko, in preparation

✓ Other unexpected effects

.... etc.

✓ **Details of transport codes:** Transport model evaluation project (TMEP)

- H. Wolter et al. (N.I.) [TMEP], (Review paper), Prog. Part. Nucl. Phys. 125, 103962 (2022)

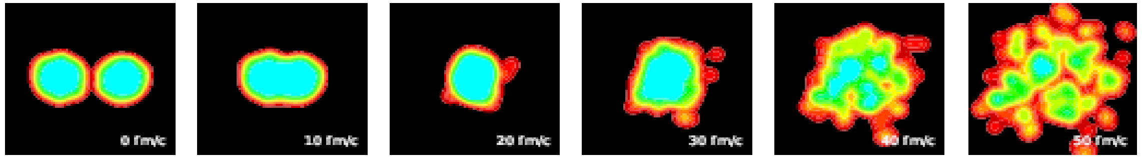
Importance of Cluster correlations

and

AMD+JAM Transport model

Importance of clusters

A. Ono, Prog. Part. Nucl. Phys. 105, 139 (2019)

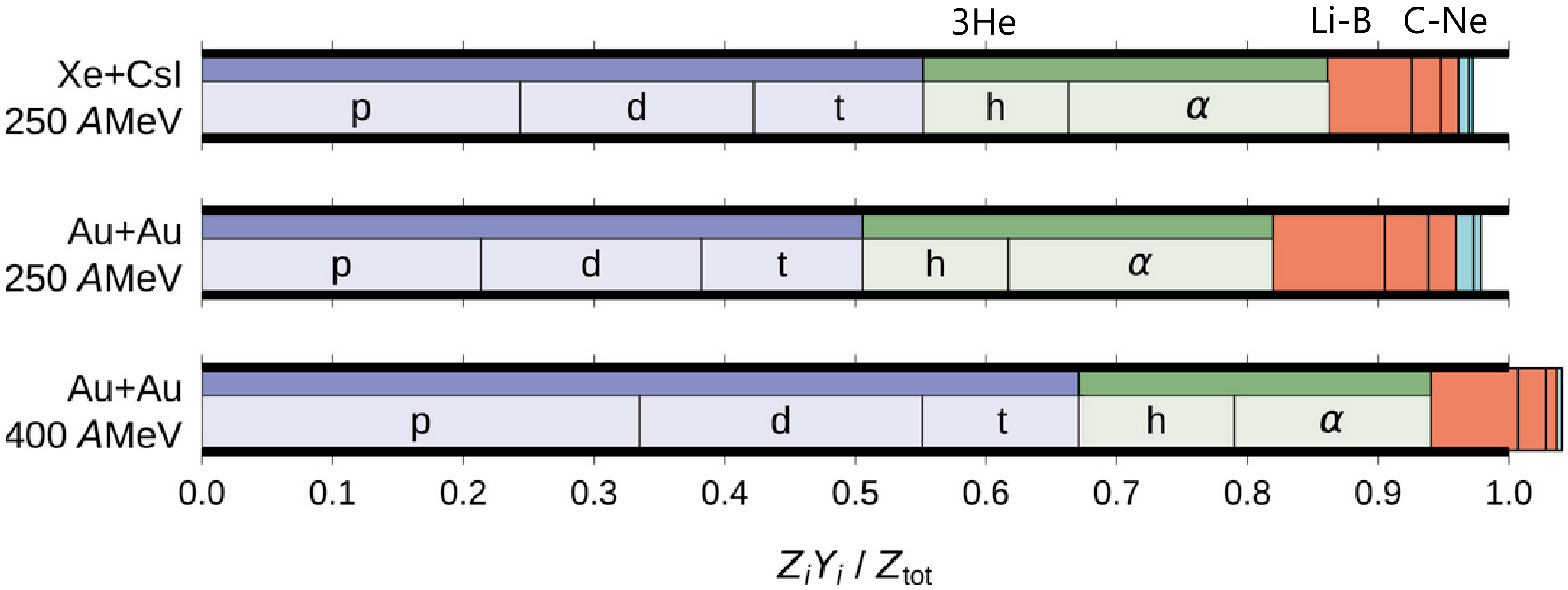


In the final stage, clusters and fragments are produced

➢ Decomposition of protons into final products:

Exp. data of FOPI

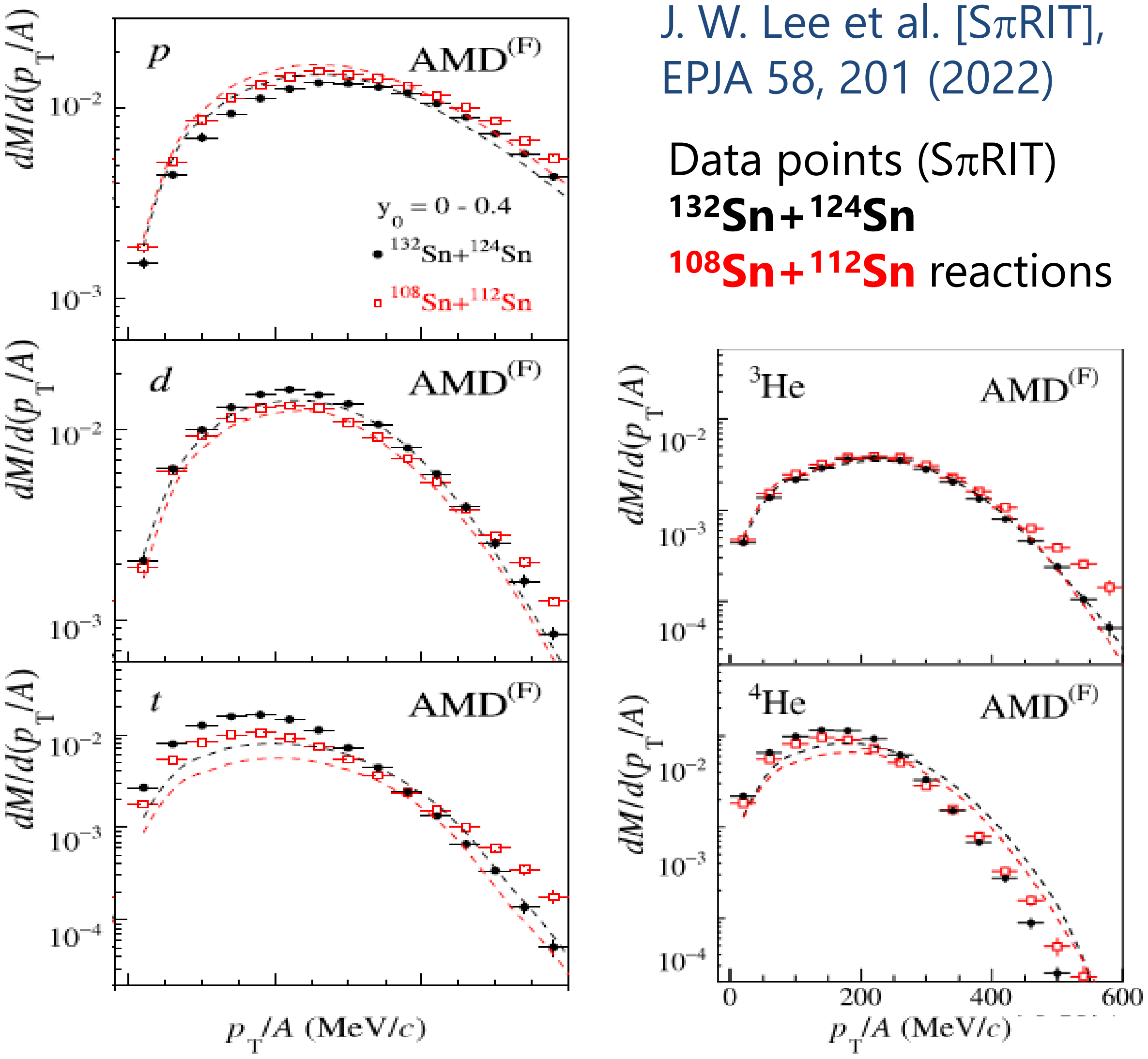
Reisdorf *et al.*, NPA 848 (2010) 366.



Only about 20-30% of the total protons are emitted as free protons. All the other protons are bound in light clusters and heavier fragments.

=> Cluster correlations can not be ignored in calculations

Cluster and comparison with the SπRIT data



J. W. Lee et al. [SπRIT], EPJA 58, 201 (2022)

Data points (SπRIT)
^{132}Sn+^{124}Sn
^{108}Sn+^{112}Sn reactions

AMD: Antisymmetrized Molecular Dynamics

A. Ono, H. Horiuchi, T. Maruyama, and A. Ohnishi, PTP87 (1992) 1185: A. Ono, Prog. Part. Nucl. Phys. 105 (2019)139

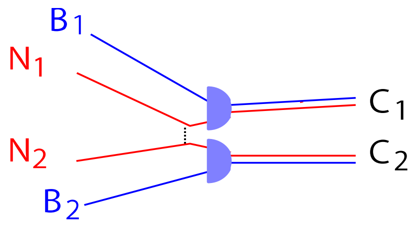
- AMD wave function

$$|\Phi_{\mathrm{AMD}}\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\boldsymbol{r}_j - \frac{\boldsymbol{Z}_i(t)}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right].$$

- Effective interaction: SLy4, SkM* etc.

- NN collisions with cluster correlations

N1 + B1 + N2 + B2 -> C1 + C2



N1, N2: Colliding nucleons
B1, B2: Spectator nucleons/clusters
C1, C2: N, (2N), (3N), (4N) (up to α cluster)

$$\frac{d\sigma(C_1,C_2)}{d\Omega} = P(C_1,C_2,p_{\mathrm{f}},\Omega)\frac{p_{\mathrm{i}}}{v_{\mathrm{i}}}\frac{p_{\mathrm{f}}}{v_{\mathrm{f}}}|M|^2\frac{p_{\mathrm{f}}}{p_{\mathrm{i}}}$$

More discussion within AMD M. Kaneko et al. [SπRIT], PLB 822, 136681 (2021): M. Kurata-Nishimura et al. [SπRIT], PLB871, 139970 (2025)

Transport model

N. Ikeno, A. Ono, Y. Nara, A. Ohnishi, PRC93(2016) 044612; PRC97(2018) 069902(E)

- Coupled equations for $f_\alpha(\mathbf{r}, \mathbf{p}, t)$ ($\alpha = N, \Delta, \pi$)

$$\frac{\partial f_N}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N}{\partial \boldsymbol{p}} = I_N[f_N, f_{\Delta,\pi}]$$

$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N, f_{\Delta,\pi}]$$

$I_N[f_N, f_{\Delta,\pi}]$:

- $N N \to N N$
- $N N \to N \Delta$
- $N \Delta \to N N$
- $\Delta \to N \pi$
- $N \pi \to \Delta$

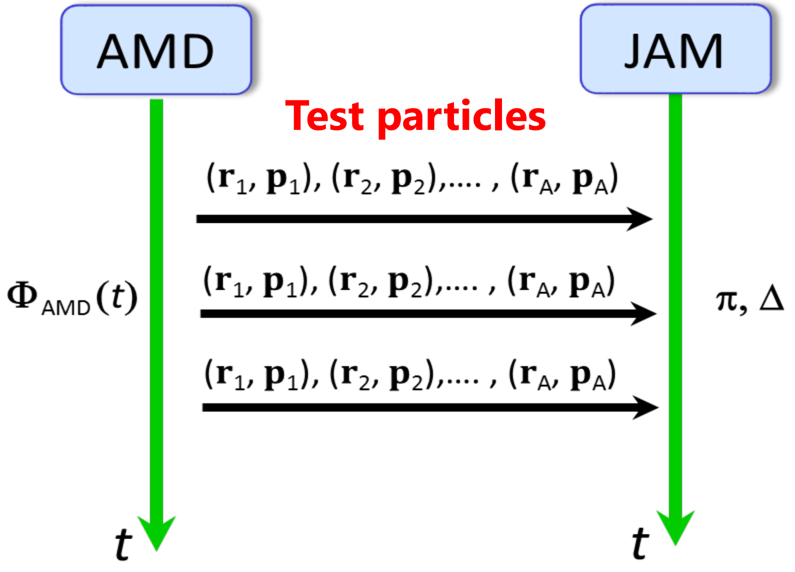
Our model: JAM coupled with AMD

Perturbative treatment of pion and Δ particle production

$$I_N = I_N^{\mathrm{el}}[f_N, 0] + \lambda I'_N[f_N, f_{\Delta,\pi}]$$

$f_{\Delta,\pi} = O(\lambda)$: Δ and pion productions are rare

$f_N = f_N^{(0)} + \lambda f_N^{(1)} + \ldots$



- **Nucleon f_N : Zeroth order equation**

$$\frac{\partial f_N^{(0)}}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N^{(0)}, 0]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{p}} = I_N^{\mathrm{el}}[f_N^{(0)}, 0]$$

Solved by AMD

- **Δ particle f_Δ and pion f_π : First order equation**

$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]$$

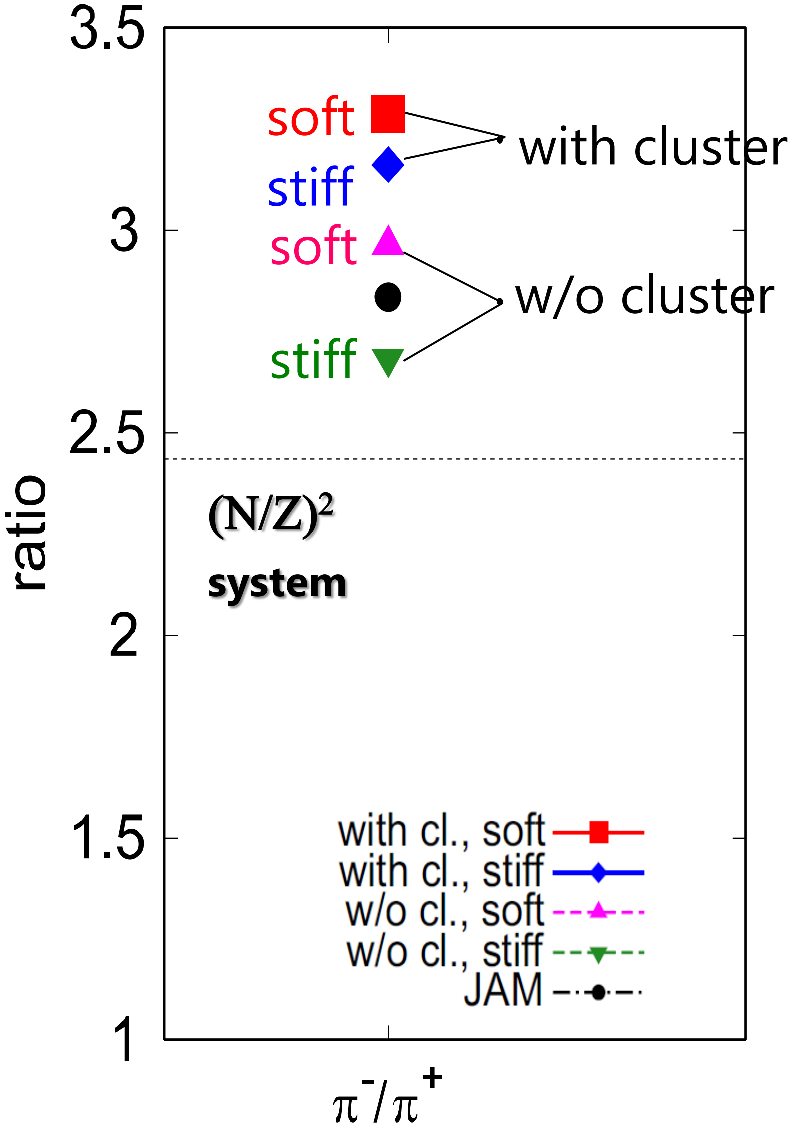
Solved by JAM for given $f_N^{(0)}$

Y. Nara, N. Otuka, A. Ohnishi, K. Niita, S. Chiba, PRC61 (2000) 024901

Cluster effect on the π^-/π^+ ratio

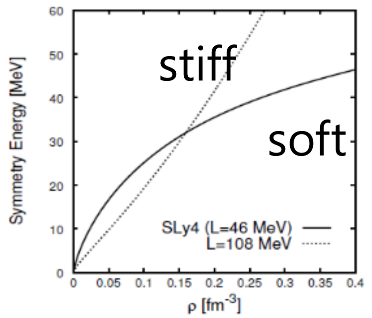
N. Ikeno, A. Ono, Y. Nara, A. Ohnishi,
PRC93(2016) 044612; PRC97(2018) 069902(E)

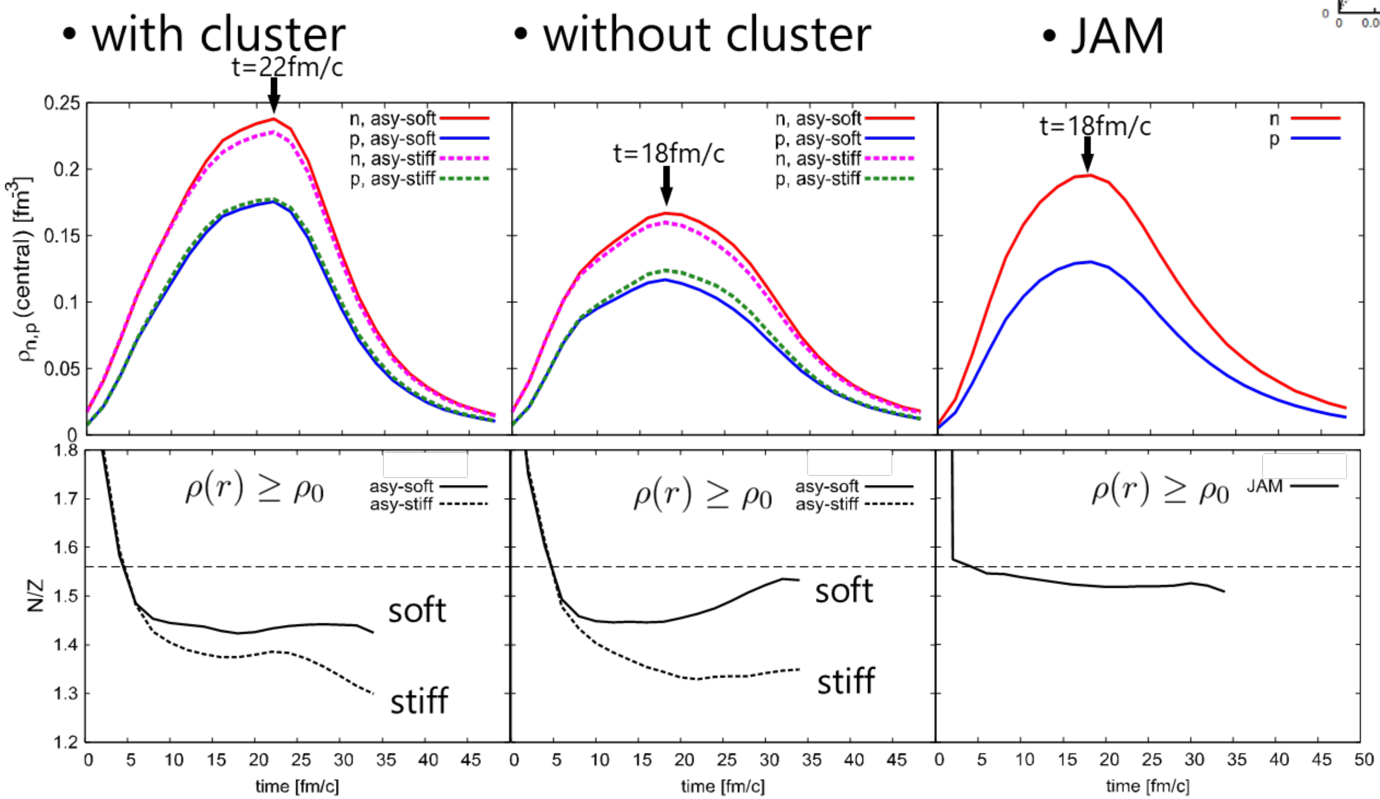
- AMD+JAM model



$^{132}Sn+^{124}Sn$, E/A=300 MeV

Effective interaction:
Skyrme force





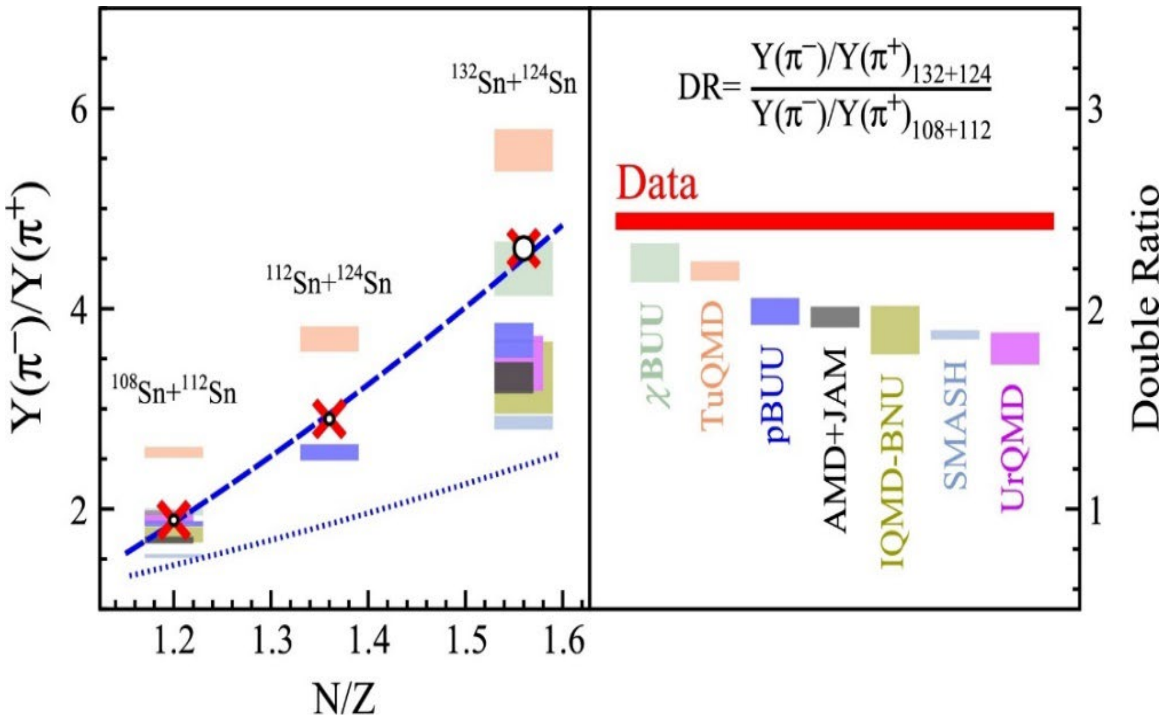
Nucleon dynamics is directly affected by Pion

⇒ **Cluster correlation is very important**

Most codes do not include cluster correlation

TMEP project and comparison with the SπRIT data

- Transport model evaluation project (TMEP) [Before exp. data were available]

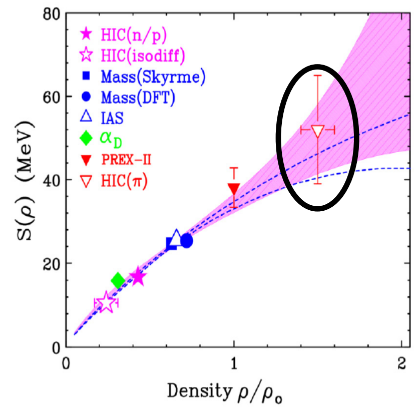


G. Jhang et al. [SπRIT, TMEP], PLB 813, 136016 (2021)

- ✓ Most codes consistently underestimate the data
- ✓ The band for each model: different L effect

In most codes, potentials were not taken into account in the collision term ($NN \leftrightarrow N\Delta$, $\Delta \leftrightarrow N\pi$)

W. G. Lynch, M. B. Tsang, PLB830, 137098 (2021)

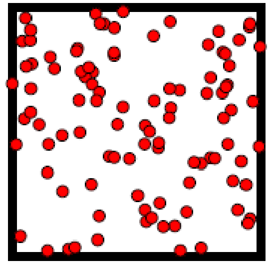


- SπRIT experiment @RIBF J. Estee et al. [SπRIT], PRL26, 162701 (2021)
  Slope of the symmetry energy is reported to be **42 <L < 117 MeV** with dcQMD(TuQMD)

Box Pion Comparison in Transport Model Evaluation Project (TMEP)

Compared 10 transport codes under controlled conditions of a system confined in a box with periodic boundary

A. Ono et al., [TMEP] PRC 100 (2019)

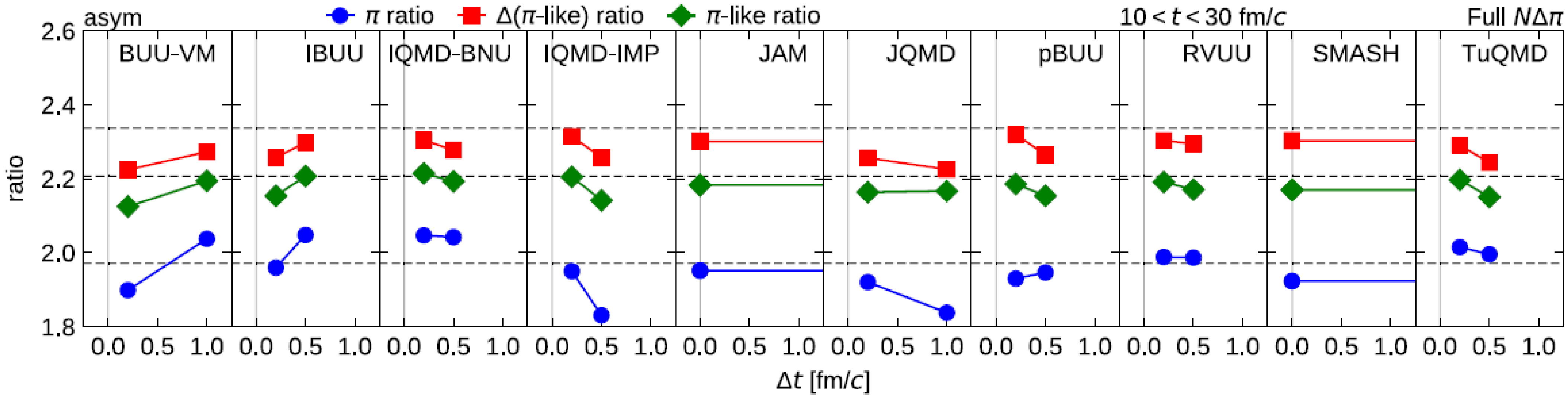


- V=(20 fm)^3 (ρ_B=0.16 fm^{-3})
- **No mean-field**, No Pauli-blocking
- NN ↔ NN, NN ↔ NΔ, Δ ↔ Nπ

Nucleons ⇔ pions, deltas in equilibrium

$$\frac{N}{Z}=\frac{\pi^-}{\pi^0}=\frac{\pi^0}{\pi^+}=\frac{\Delta^-}{\Delta^0}=\frac{\Delta^0}{\Delta^+}=\frac{\Delta^+}{\Delta^{++}}$$

=>Thermal and chemical equilibrium



π-like ratio = $\frac{\pi^- + \Delta^- + \frac{1}{3}\Delta^0}{\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+}$

π ratio = $\frac{\pi^-}{\pi^+}$

Agreement of final π^-/π^+ ratio is not so bad.
Uncertainty in the π^-/π^+ ratio is within 5%

Momentum dependence of the mean-field potentials

and

Improved transport model: AMD+sJAM

Transport equation with collision term under potentials

In our study (N. Ikeno and A. Ono, PRC 108, 044601 (2023)) :

✓ Improve the AMD+sJAM model to properly take into account such potentials consistently

- Potentials enter both the mean-field propagation and the collision term (in principle)

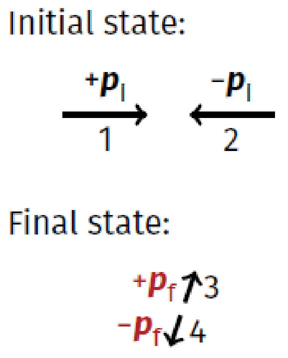
$$\frac{\partial f_\alpha(\boldsymbol{r},\boldsymbol{p},t)}{\partial t} = \underbrace{\frac{\partial h_\alpha[f]}{\partial \boldsymbol{r}}\cdot\frac{\partial f_\alpha}{\partial \boldsymbol{p}} - \frac{\partial h_\alpha[f]}{\partial \boldsymbol{p}}\cdot\frac{\partial f_\alpha}{\partial \boldsymbol{r}}}_{\text{Mean-field propagation term}} + \underbrace{\int |\boldsymbol{v}|\frac{d\sigma}{d\Omega}\{f_3 f_4(1-f)(1-f_2) - f f_2(1-f_3)(1-f_4)\}\frac{d\boldsymbol{p}_2\, d\Omega}{(2\pi\hbar)^3}}_{\text{Collision term}}$$

Mean-field propagation term

Collision term (NN ↔ NN , NN ↔ NΔ, Δ ↔ Nπ)

Collision rate/cross-section: σ (**p**$_1$, **p**$_2$: environment) for 1+2→3+4

$$v_i \frac{d\sigma}{d\Omega} \propto \int |M|^2 \delta(E_f - E_i) p_f^2\, dp_f = |M|^2 \frac{p_f^2}{v_f}, \qquad v_f = \frac{dE_f}{dp_f}$$



Potentials at the space-time point of the collision enter in the energies:

$$E_i = \sqrt{m_1^2 + p_1^2} + U_1 + \sqrt{m_2^2 + p_2^2} + U_2, \quad E_f = \sqrt{m_3^2 + p_3^2} + U_3 + \sqrt{m_4^2 + p_4^2} + U_4$$

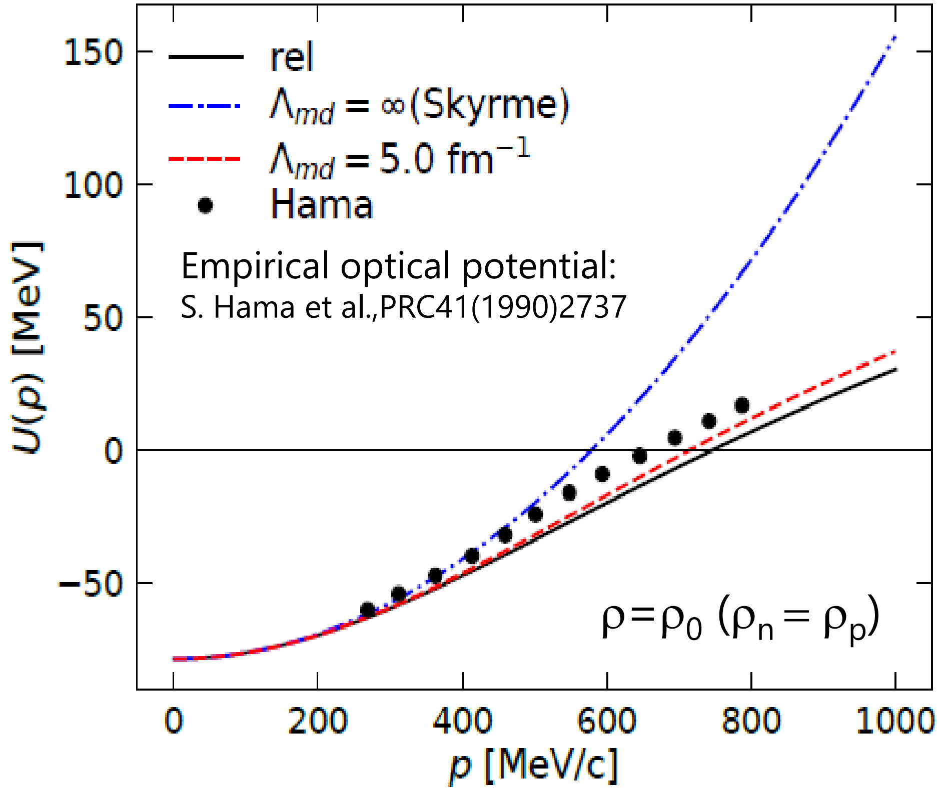
We must solve

$E_f(p_f) = E_i$

Assume energy conservation at each collision.

- Threshold effect: σ =0 when E_f (**p**$_f$=0) > E_i. This threshold condition depends on the potentials.
- More generally, we need to know how the cross section σ depends on the potentials.

Momentum dependence of the nucleon potentials



- Skyrme interaction (SLy4, m^*/m=0.70) (not used)

U(p) at p>500 MeV/c is important for the Δ, π productions
=> p^2 dependence needs modification in the high-momentum region

- Λ_{md}=5.0 fm^{-1}: Used in AMD

- **rel** (relativistic form): Used in sJAM

Momentum-dependent potential (in AMD): $U_\alpha(\boldsymbol{r},\boldsymbol{p}) = A_\alpha(\boldsymbol{r})\dfrac{[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2}{1+[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2/\Lambda_{\mathrm{md}}^2} + \tilde{C}_\alpha(\boldsymbol{r})$,

Nucleon single-particle energy $E_a(\boldsymbol{r},\boldsymbol{p}) = \sqrt{(m_N+\Sigma_a^s(\boldsymbol{r}))^2 + (\boldsymbol{p}-\boldsymbol{\Sigma}_a(\boldsymbol{r}))^2} + \Sigma_a^0(\boldsymbol{r})$.

$$U_{\mathrm{rel}}(p) = \sqrt{(m_N+\Sigma^s)^2+p^2} + \Sigma^0 - \sqrt{m_N^2+p^2}$$

$$m^* = m_N + \Sigma^s,$$
$$E^* = \sqrt{m_N^{*2} + \boldsymbol{p}^{*2}},$$
$$\boldsymbol{p}^* = \boldsymbol{p} - \boldsymbol{\Sigma}$$

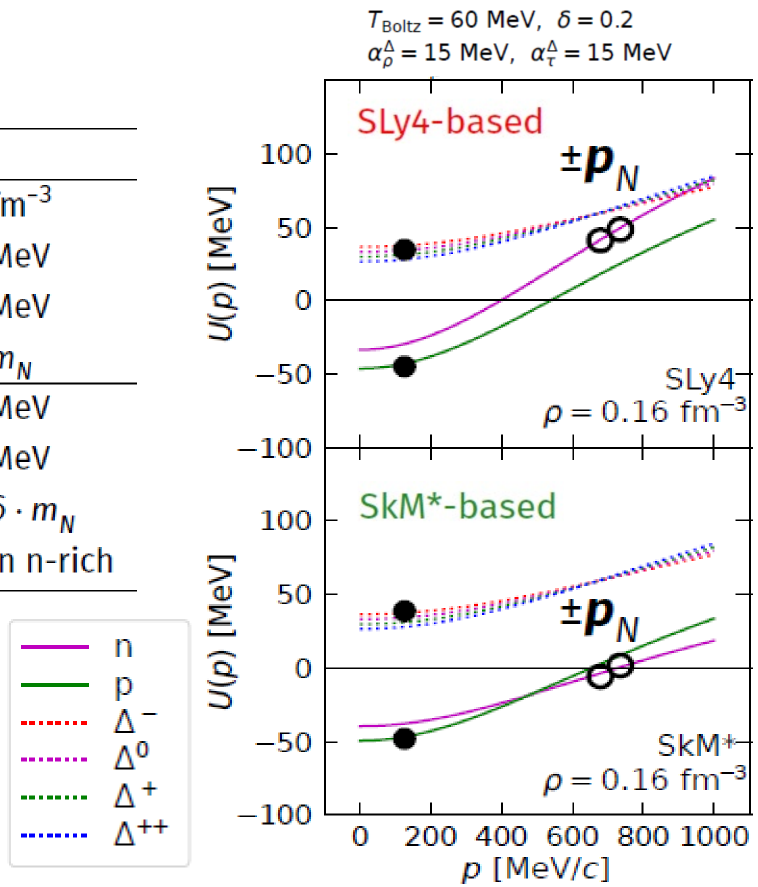
NN→NΔ cross sections under potentials

N. Ikeno and A. Ono, PRC108, 044601 (2023)

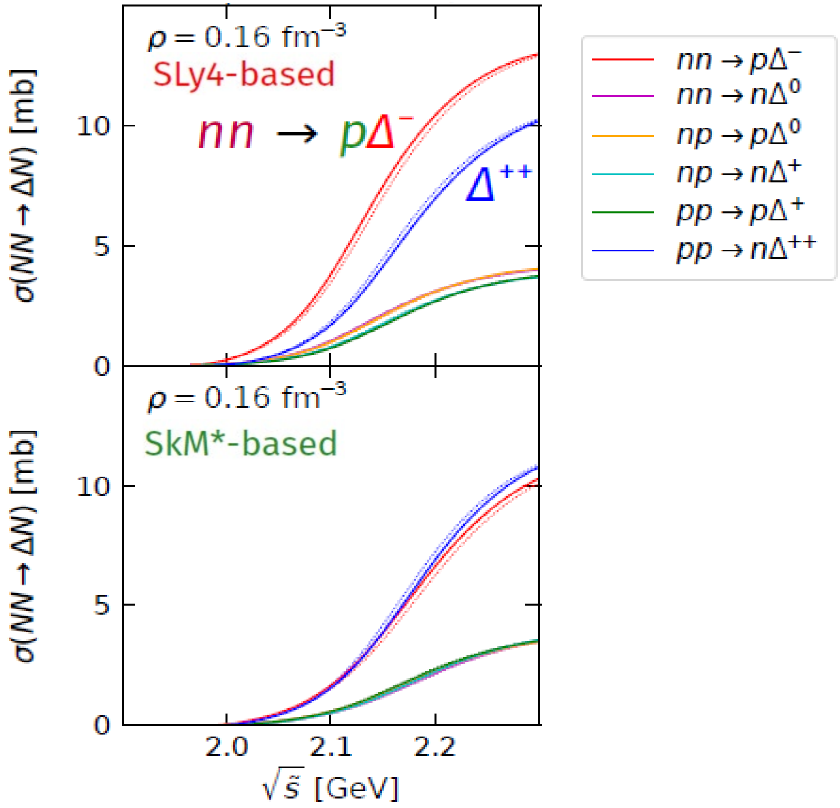
Two different momentum dependences of neutrons $U_n(p)$ and protons $U_p(p)$

Similar EOS for SLy4 and SkM*

	SLy4	SkM*	
ρ_0	0.16	0.16	fm^{-3}
E_0	-15.97	-15.77	MeV
K	230	217	MeV
m^*	0.69	0.79	m_N
S_0	32.0	30.0	MeV
L	46	46	MeV
$m_n^* - m_p^*$	-0.18	+0.33	$\delta \cdot m_N$
	$m_n^* < m_p^*$	$m_n^* > m_p^*$	in n-rich



$\sigma(NN \to N\Delta)$ with the initial nucleon momenta $\pm\boldsymbol{p}_N$ in nuclear matter, as a function of $\sqrt{\tilde{s}} = 2\sqrt{m_N^2 + \boldsymbol{p}_N^2}$



$$\sigma(\pm\boldsymbol{p}_N;\text{environment}) \sim p_f \sim \sqrt{\epsilon^*} \quad \text{with} \quad \epsilon^* = \underbrace{2\sqrt{m_N^2 + (\pm\boldsymbol{p}_N)^2} - m_N - m_\Delta}_{\text{same as in vaccum}} + \underbrace{U_1(+\boldsymbol{p}_N) + U_2(-\boldsymbol{p}_N) - U_3(0) - U_\Delta(0)}_{\text{effect of potentials}}$$

Strong impact on the isospin dependence of Δ production by e.g. $2U_n(p_N) - U_p(0) - U_\Delta(0)$

Modified Model: AMD+sJAM

N. Ikeno and A. Ono, PRC108, 044601 (2023)

sJAM: newly developed JAM

AMD wave function

$$|\Phi_{\rm AMD}\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\boldsymbol{r}_j - \frac{\boldsymbol{Z}_i(t)}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right].$$

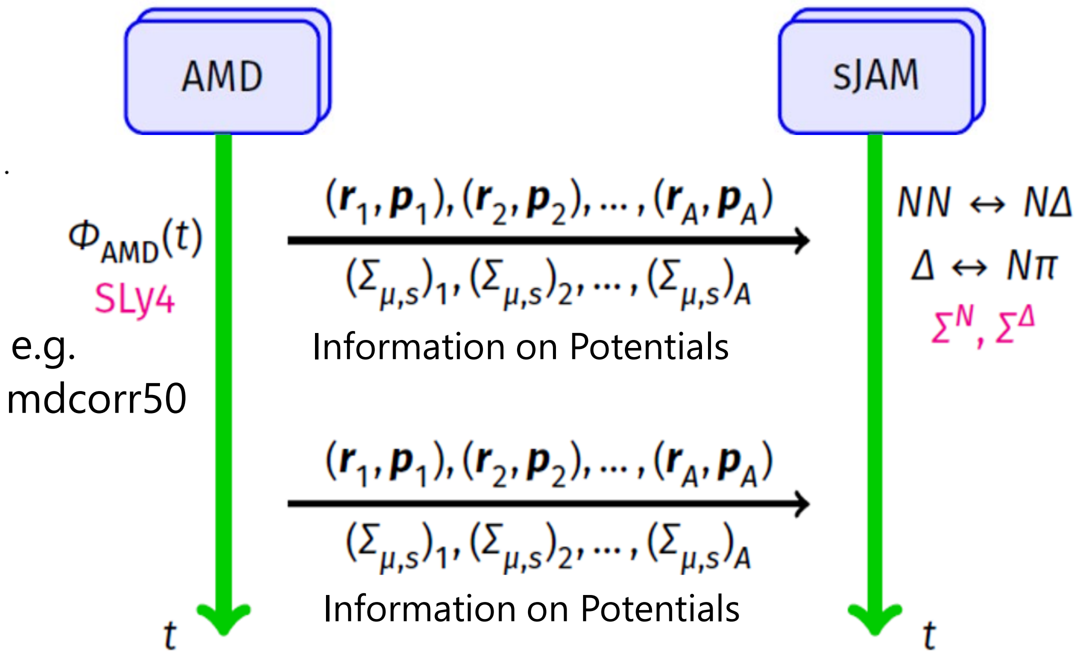
Effective interaction:
SLy4, SkM* etc.

Collison term with cluster:

$$N_1 + N_2 + B_1 + B_2 \rightarrow C_1 + C_2$$

$$\frac{d\sigma}{d\Omega} = P(C_1, C_2)\left(\frac{p_i}{v_i}\frac{p_f}{v_f}\right)|M|^2\frac{p_f}{p_i}$$

- Energy is conserved precisely
- Cross section naturally depends on potentials



Collisions including ...

$NN \leftrightarrow NN$
$NN \leftrightarrow N\Delta$
$\Delta \leftrightarrow N\pi$

e.g. $NN \rightarrow N\Delta$

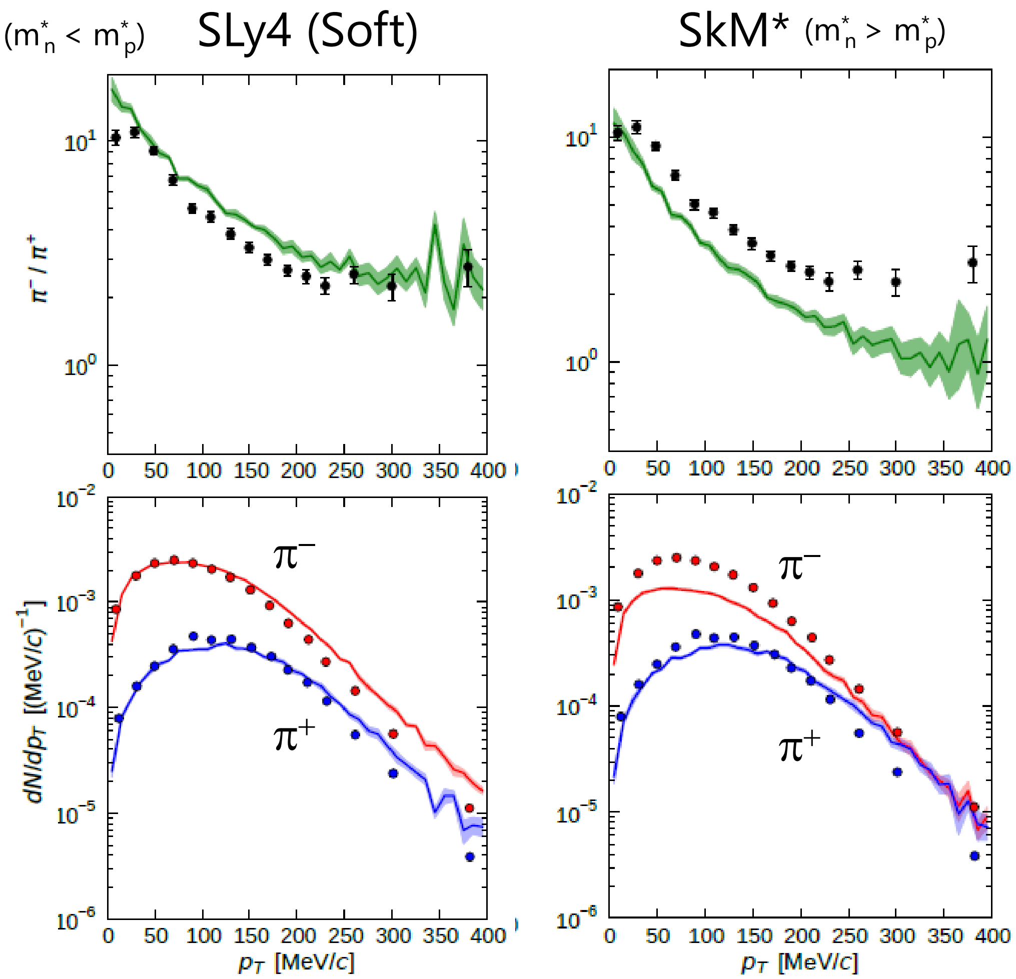
$$\frac{d\sigma_{NN\to N\Delta}}{dm_\Delta} \propto f_i f_f \frac{|M|^2}{16\pi s}\frac{p_f}{p_i}A_\Delta(m_\Delta)$$

is calculated as a function of ($\mathbf{p_1}$, $\mathbf{p_2}$: environment) for every possible collision.

The JAM code (without pot.) in the AMD+JAM model has been replaced by the new sJAM code (with pot.).

Effect of nucleon potential on pion production

N. Ikeno and A. Ono, PRC108, 044601 (2023)



Data: J. Estee et al. [SπRIT], PRL26,162701(2021).

^{132}Sn+^{124}Sn, E/A=270 MeV

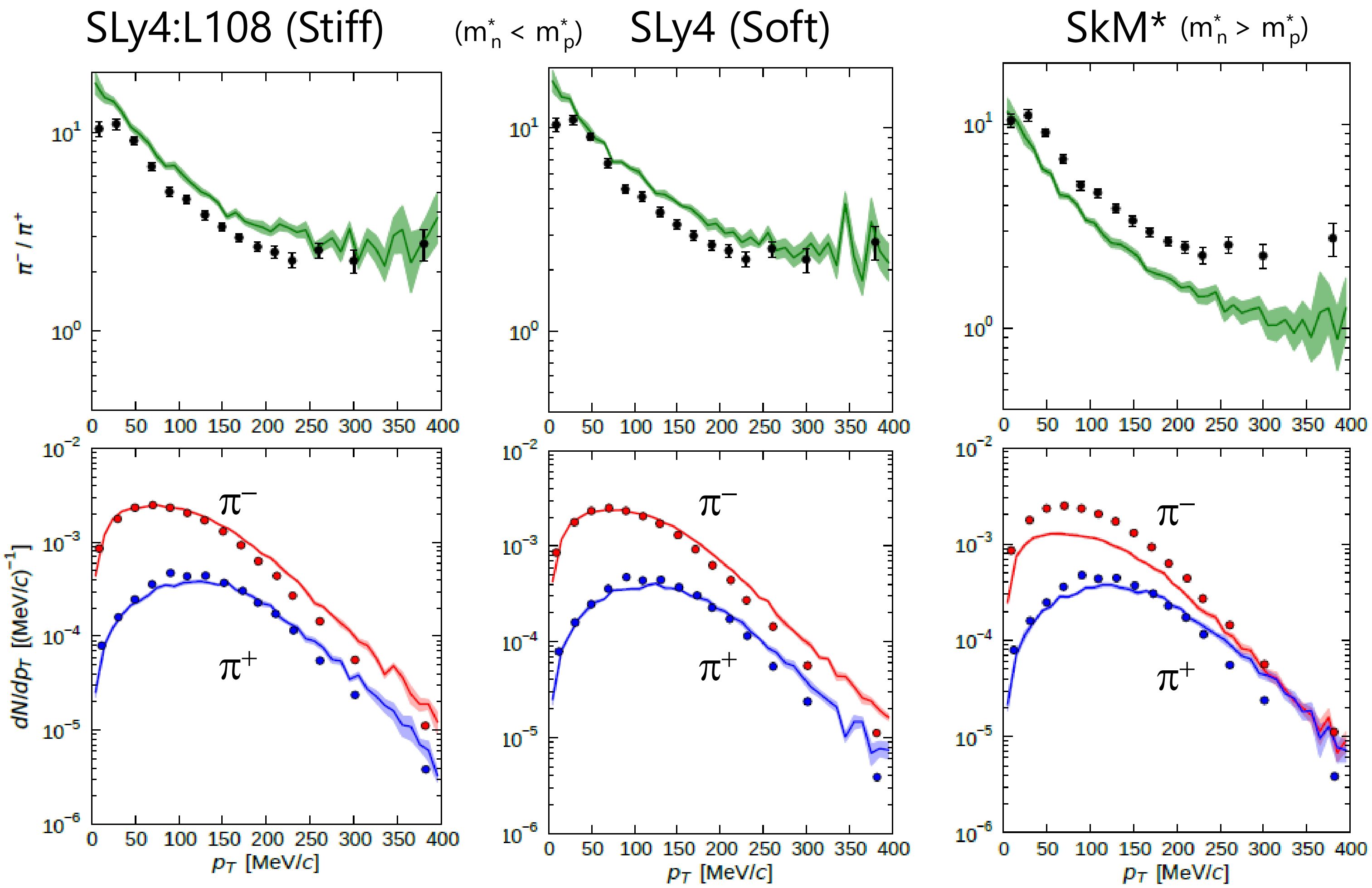
Impact parameter: 0< b <3 fm

✓ SLy4 vs. SkM*: Momentum dependence of U_n and U_p has a strong effect on pion production

Nuclear dynamics are discussed SπRIT data within AMD:
M. Kurata-Nishimura et al. [SπRIT], PLB871, 139970 (2025)

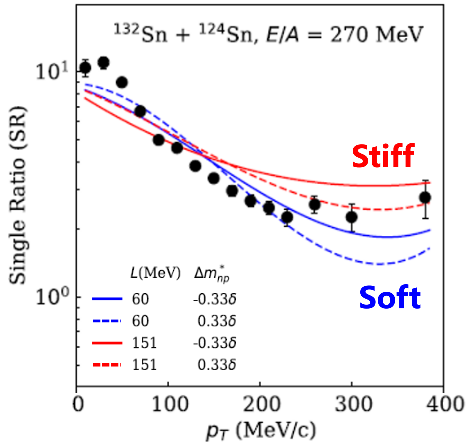
Effect of nucleon potential on pion production

N. Ikeno and A. Ono, PRC108, 044601 (2023)



J. Estee et al. [SπRIT], PRL26,162701(2021).

Data: SπRIT, Cal: dcQMD



^{132}Sn+^{124}Sn, E/A=270 MeV

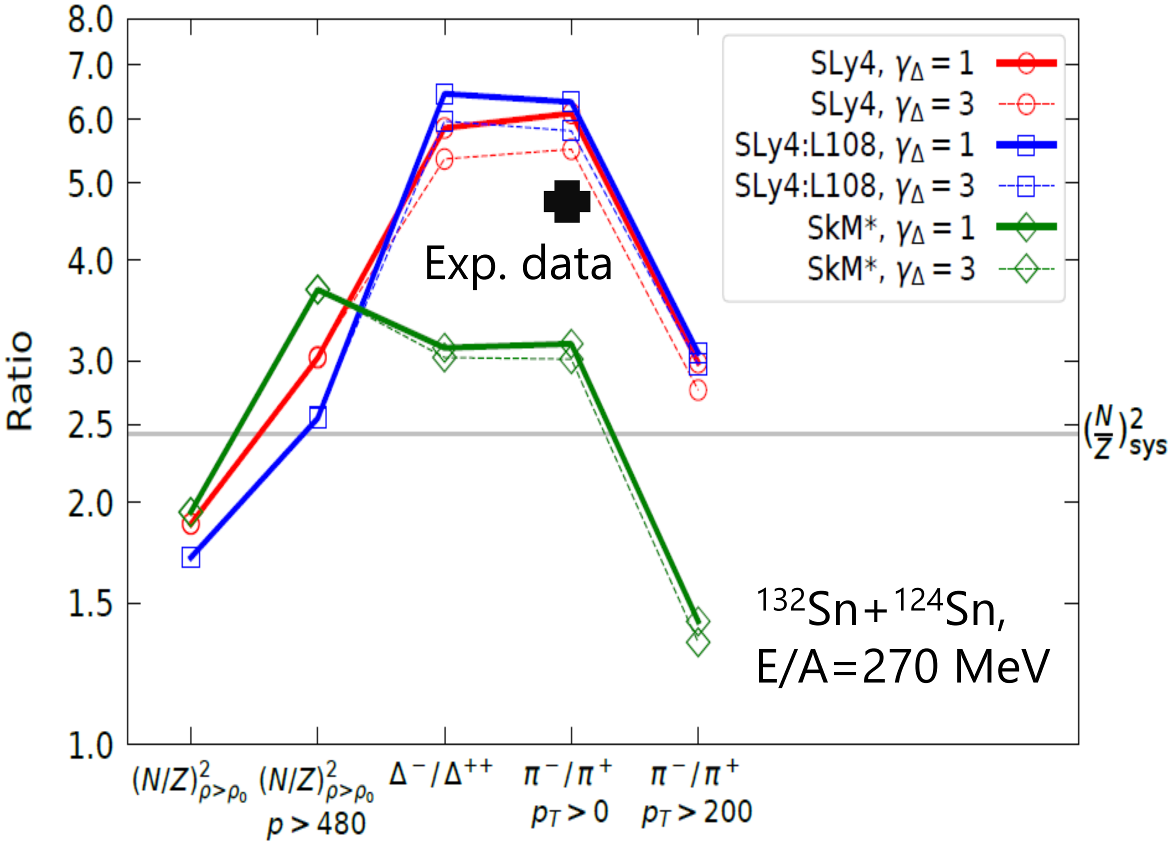
Impact parameter: 0< b <3 fm

- ✓ SLy4 vs. SkM*: Momentum dependence of U_n and U_p has a strong effect on pion production
- ✓ SLy4 vs. SLy4:L108: Relatively small dependence of symmetry energy (L) on pion production

Nuclear dynamics are discussed SπRIT data within AMD:
M. Kurata-Nishimura et al. [SπRIT], PLB871, 139970 (2025)

From nucleons to pion ratios

N. Ikeno and A. Ono, PRC108, 044601 (2023)



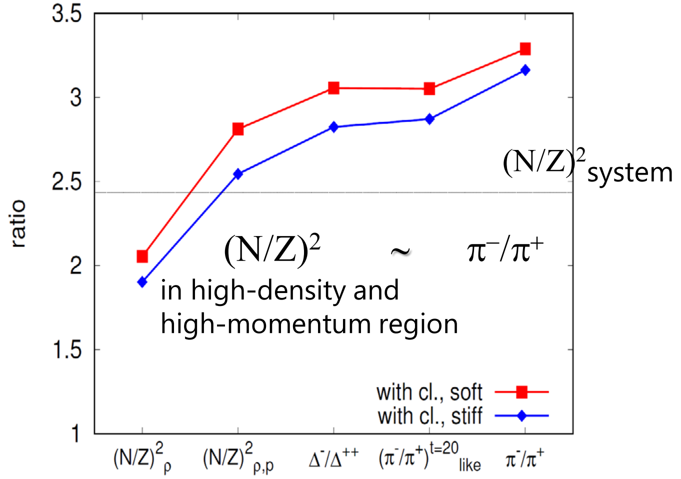
Representative ratios:

$$\left(\frac{N}{Z}\right)^2 = \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt} \qquad \frac{\Delta^-}{\Delta^{++}} = \frac{\int_0^\infty (nn \to p\Delta^-) dt}{\int_0^\infty (pp \to n\Delta^{++}) dt}$$

N(t), Z(t) : Numbers of nucleon which satisfy the conditions

➢ Without consideration of potential effects

N. Ikeno, A. Ono, Y. Nara, A. Ohnishi, PRC93 (2016) 044612; PRC97(2018) 069902(E)



✓ L dependence (SLy4 vs SLy4:L108) in N/Z is inverted in the Δ production.
✓ Effect of the symmetry energy L (SLy4 vs SLy4:L108) : Relatively small on pion production

✓ Effect of the momentum dependence of U_n and U_p (SLy4 vs SkM*): Strong
✓ π^-/π^+ carries strong information on the momentum-dependence of U_n and U_p

• Pion potential is NOT included here

Pion potential effect on pion production

- Low-density and low-momentum region: s-wave potential is well-known

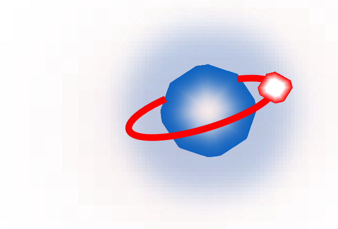
  Exp. data and chiral perturbation theory

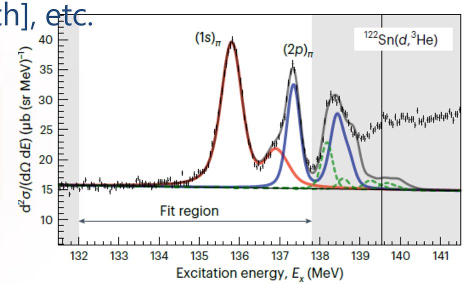
  E.E. Kolomeitsev, N. Kaiser, W. Weise, PRL90(03)092501: D. Jido, T. Hatsuda, T. Kunihiro, PLB670(08)109, K. Kwon, D. Jido et al, arXiv:2507.01398 [nucl-th], etc.

  ex) Deeply bound pionic atom, Recent Experimental Results @ RIKEN/RIBF

  T. Nishi, K. Itahashi, .., N. Ikeno, et al. [piAF], Nature Phys. 19 (2023) 788.

  S. Hirenzaki and N. Ikeno., `Handbook of Nuclear Physics', Springer (2022), ``Theoretical study of Deeply Bound Pionic Atoms with an Introduction to Mesonic Nuclei''.





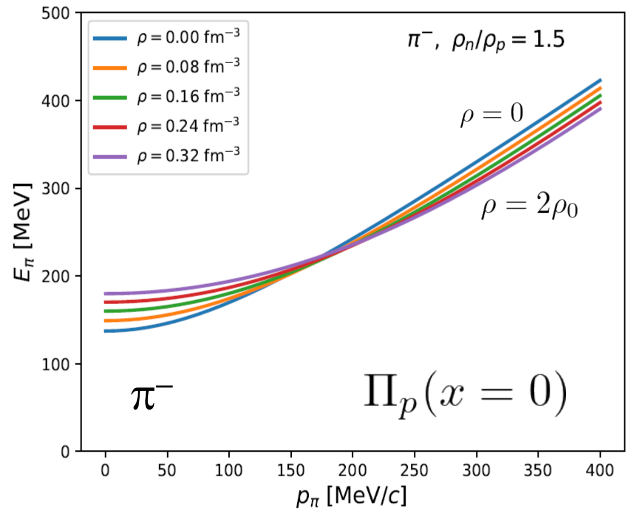
- High-density ($>2\rho_0$) and high-momentum region: p-wave potential includes large uncertainties

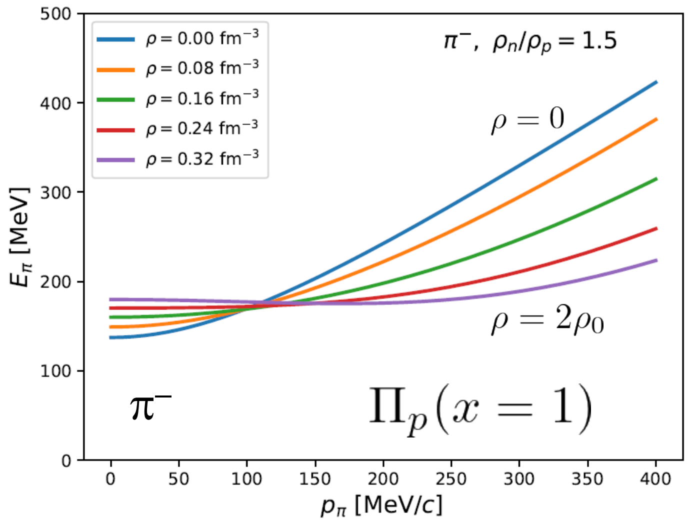
  [an example of theory]: Δ-hole model in HIC, p-wave interaction: attractive

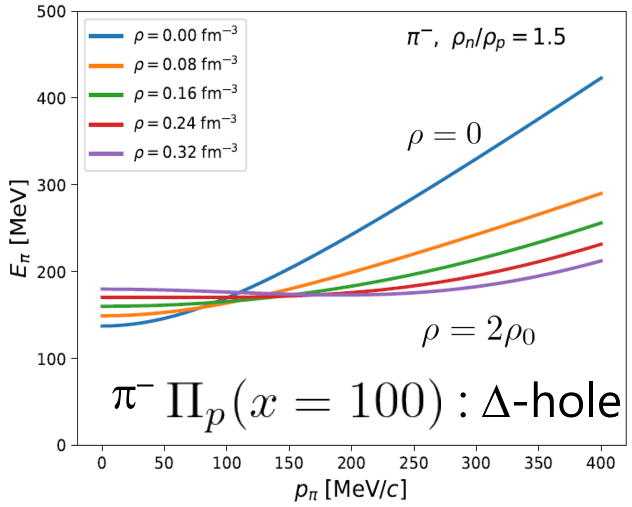
  C.M. Ko, L. Xiong, V. Koch. PRC.47.788(1993): Z. Zheng, C.M. Ko, PRC 95, 064604 (2017) …

- Pion energy: $E_\pi^2 = m_\pi^2 + \boldsymbol{p}^2 + \Pi_s + \Pi_p(E_\pi, \boldsymbol{p})$

Several choices of p-wave potential:

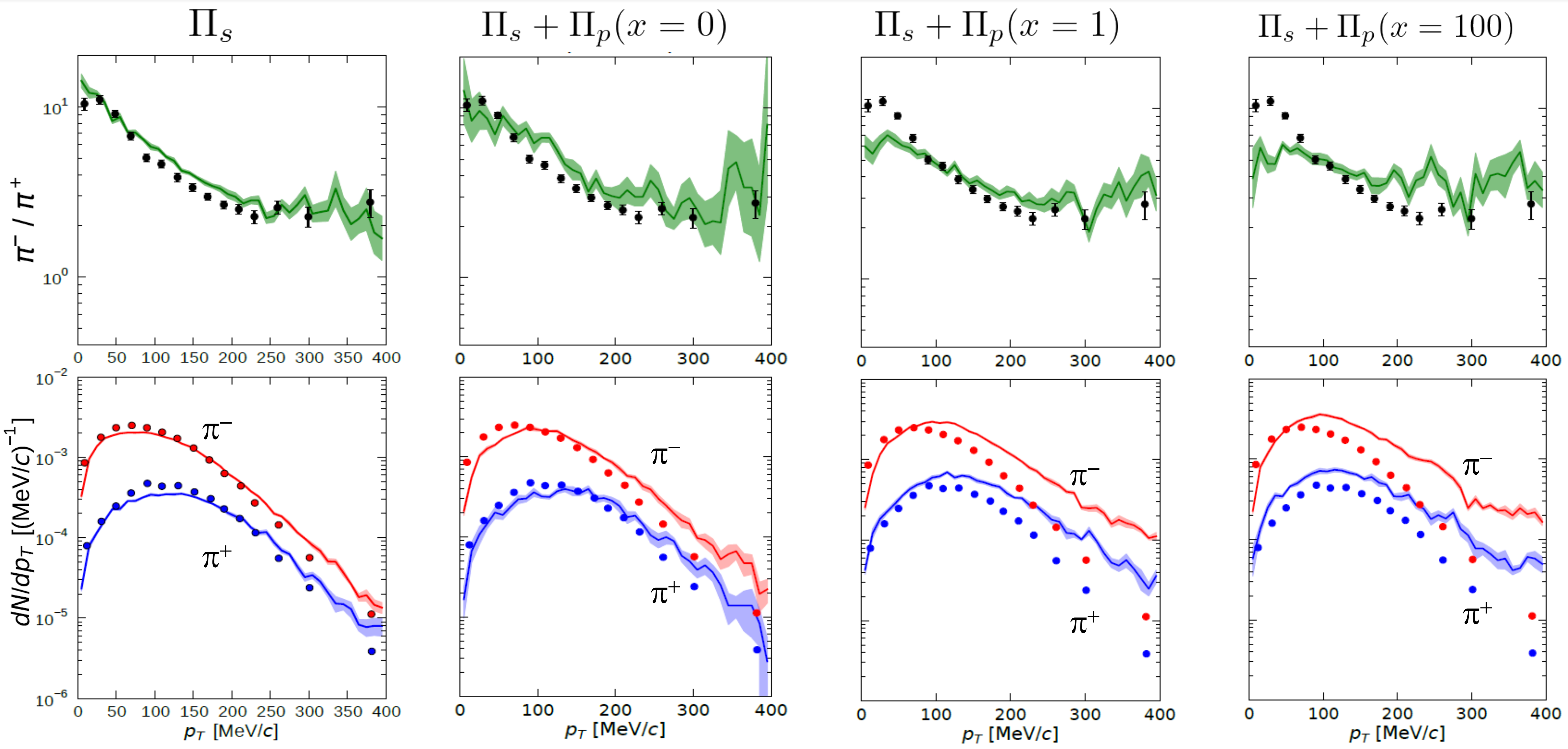






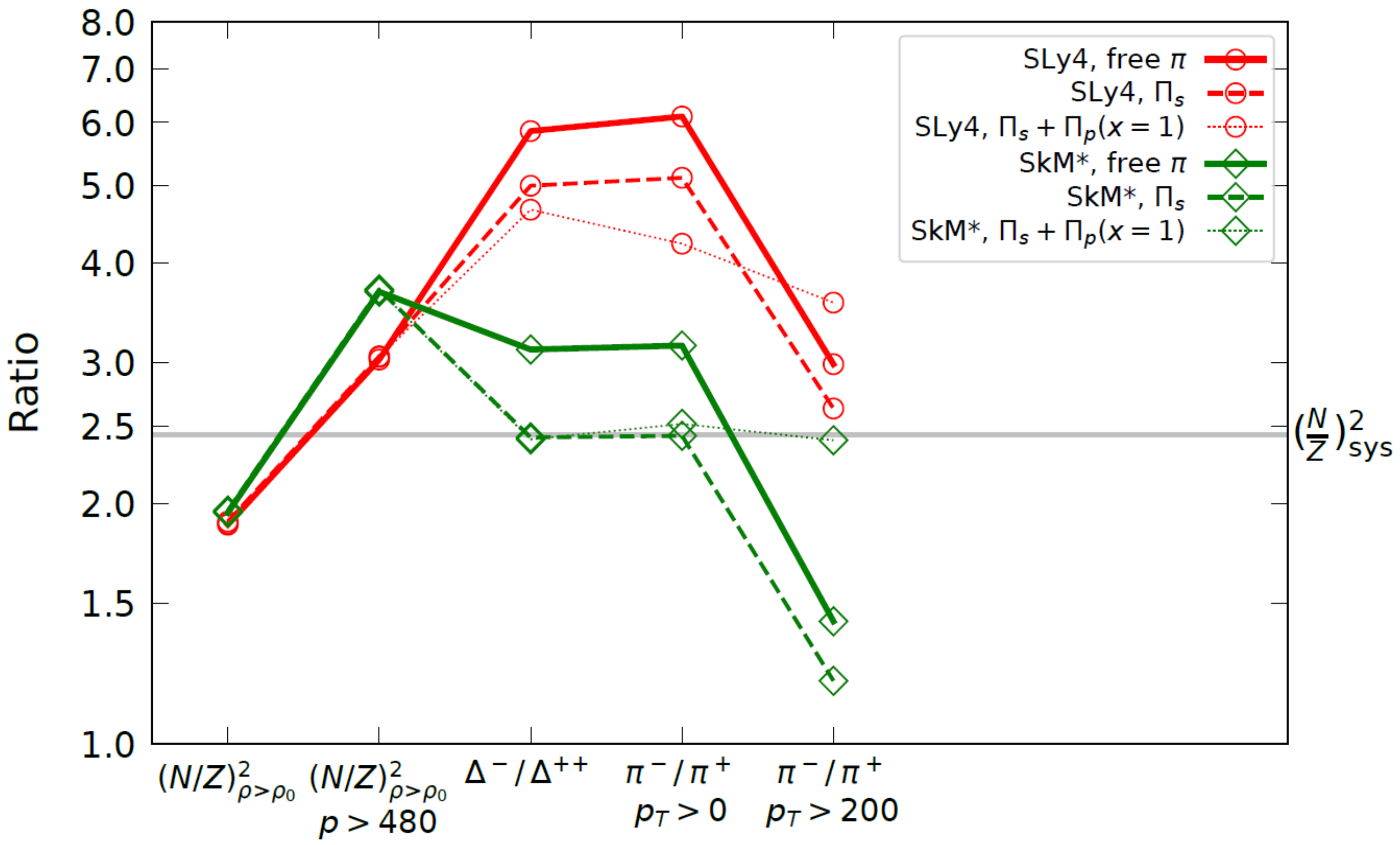
Pion spectra for different p-wave pion potentials

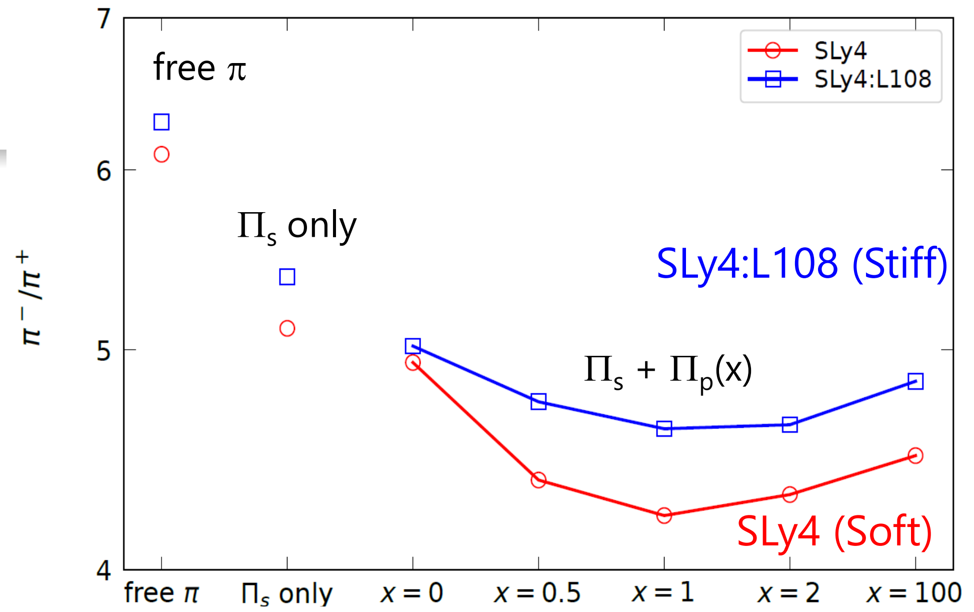
SLy4 case

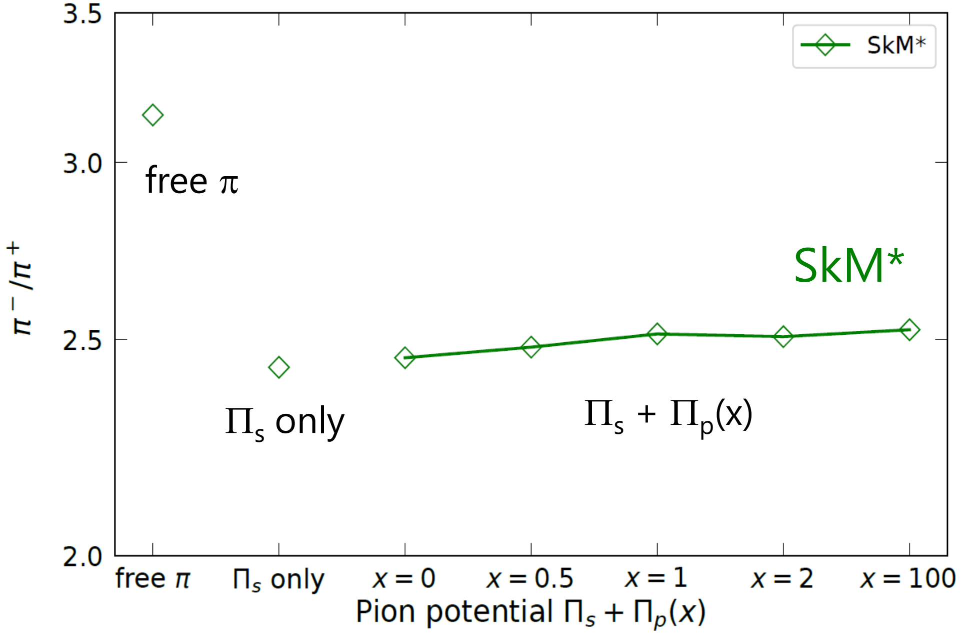


Effect of p-wave potentials (high-p and high-ρ) on high–momentum pion yield
-> Does this uncertainty affect the final result?

Pion potential effect on pion production





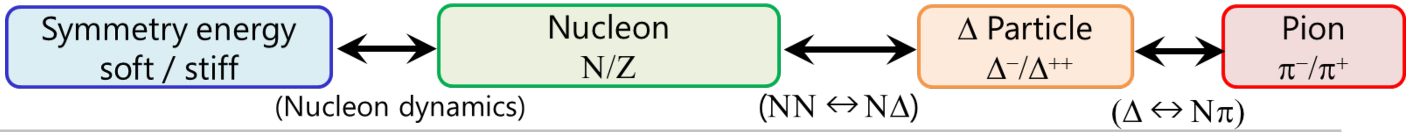


$\Delta^-/\Delta^{++} = (nn \to p\Delta^-)/(pp \to n\Delta^{++})$

- Δ production ratio (Δ^-/Δ^{++}) is reduced by mainly Π_s, not by Π_p
- This Δ^-/Δ^{++} ratio is almost directly reflected in the π^-/π^+ ratio
- At high momentum, the π^-/π^+ (p_T>200) increases due to Π_p

=>The π^-/π^+ ratio with some dependence on the p-wave potential

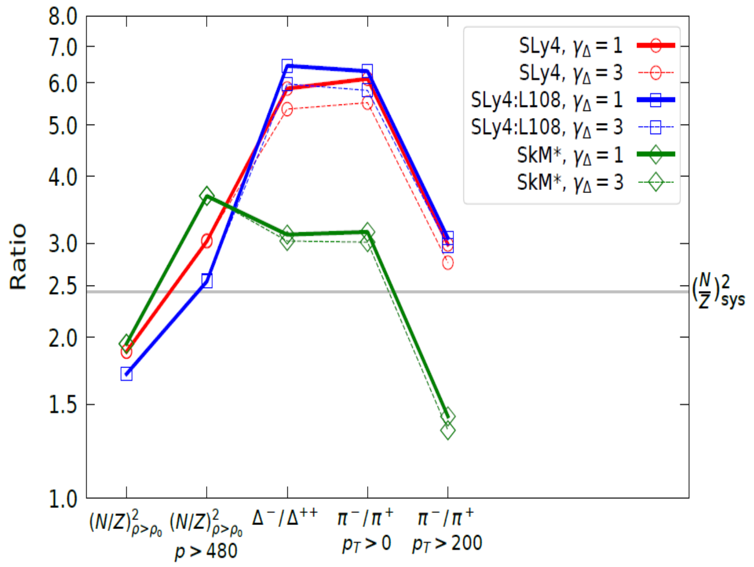
Summary



- Pion production in HICs and High-density symmetry energy

- Transport models: AMD+sJAM
  - ✓ Cluster correlation
  - ✓ Collision term under potentials
    - Strong influence on the NN ↔ NΔ process (SLy4 vs. SkM*)

- Pion ratios are more sensitive to the momentum dependence of U_n and U_p than other factors



- Better observables and ways to determine the symmetry energy?

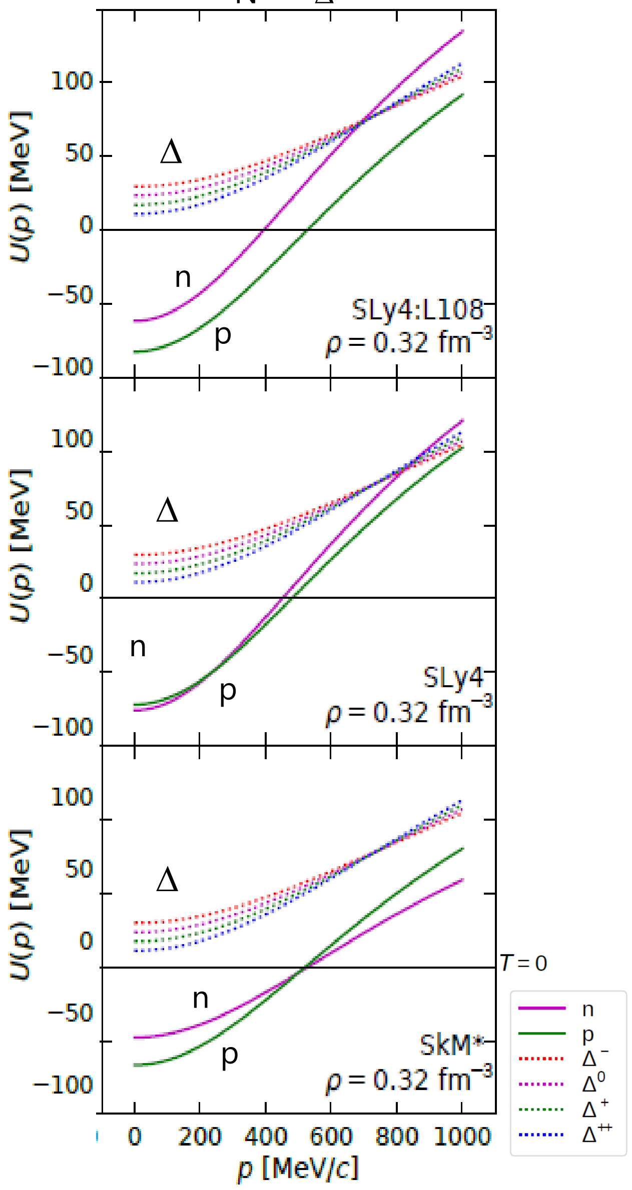
=> Pions combined with nucleon fragments and other observables

New experimental data: Different systems and energies

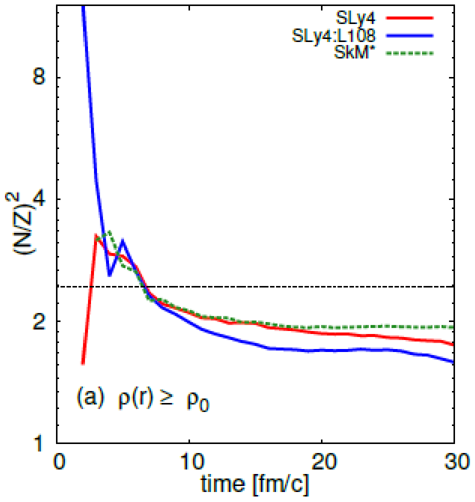
=> Comprehensive studies are expected

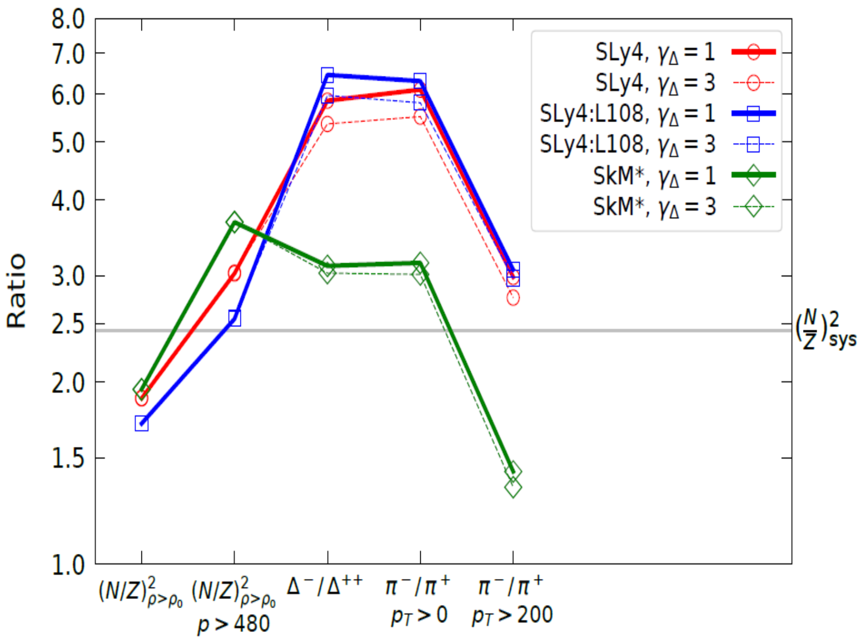
How to understand the effects in Nucleon dynamics

U_N, U_Δ



High density region
$(N/Z)^2$





Representative ratios: $\left(\frac{N}{Z}\right)^2 = \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt}$

N(t), Z(t) : Numbers of nucleon which satisfy the conditions

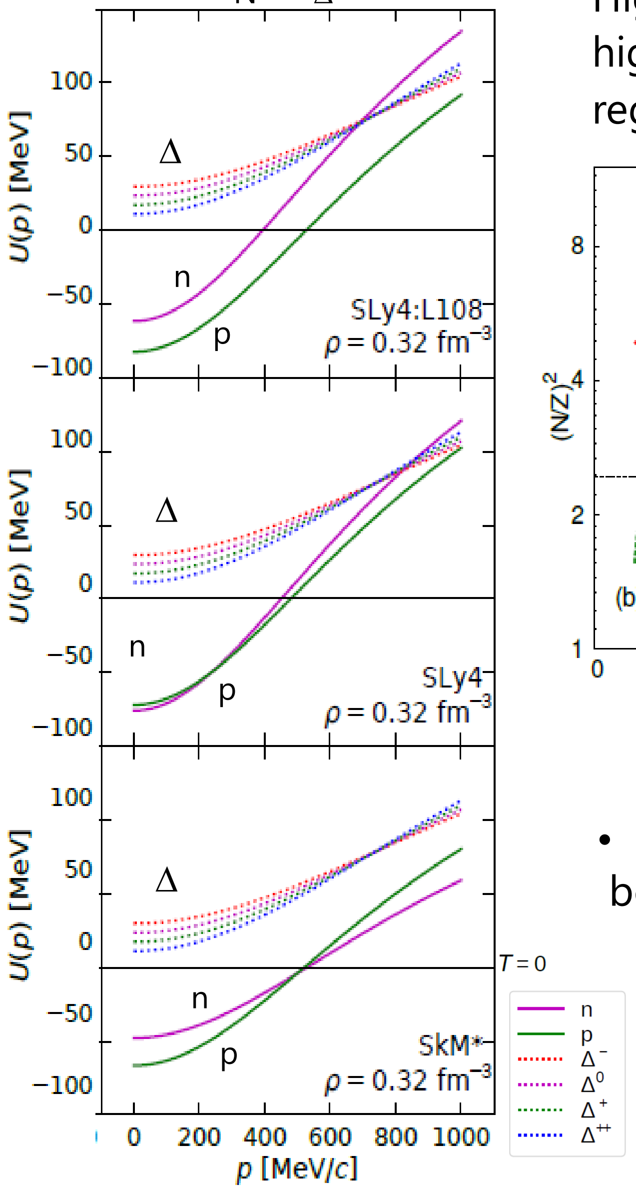
From Ratio's Fig, we can see

- Effect of the symmetry energy L (SLy4 vs SLy4:L108)
- Effect of the momentum dependence of U_n and U_p (SLy4 vs SkM*)

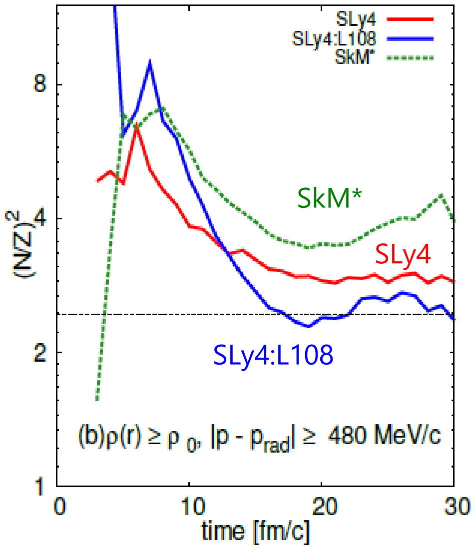
=> There is clearly the effect of the symmetry energy L between SLy4 and SLy4:L108

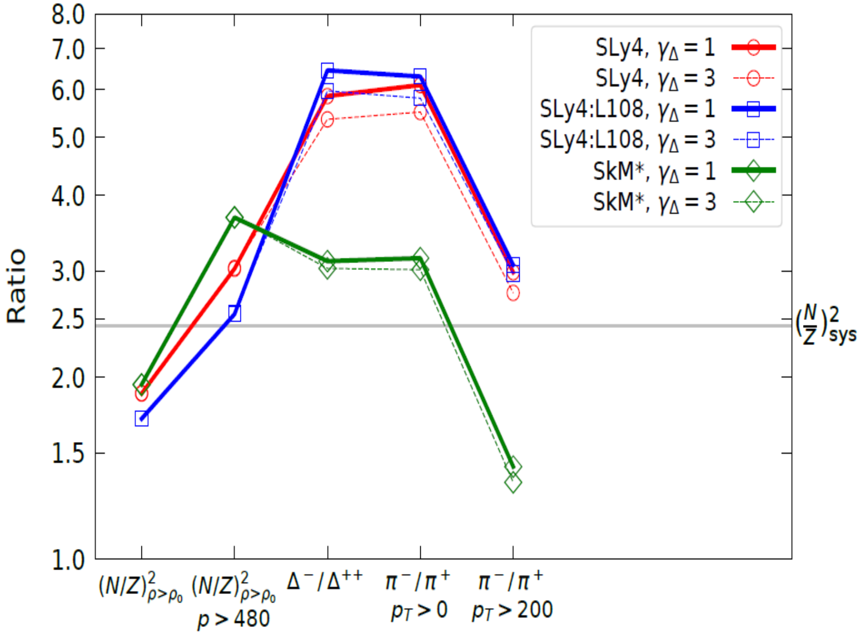
How to understand the effects in Nucleon dynamics

U_N, U_Δ



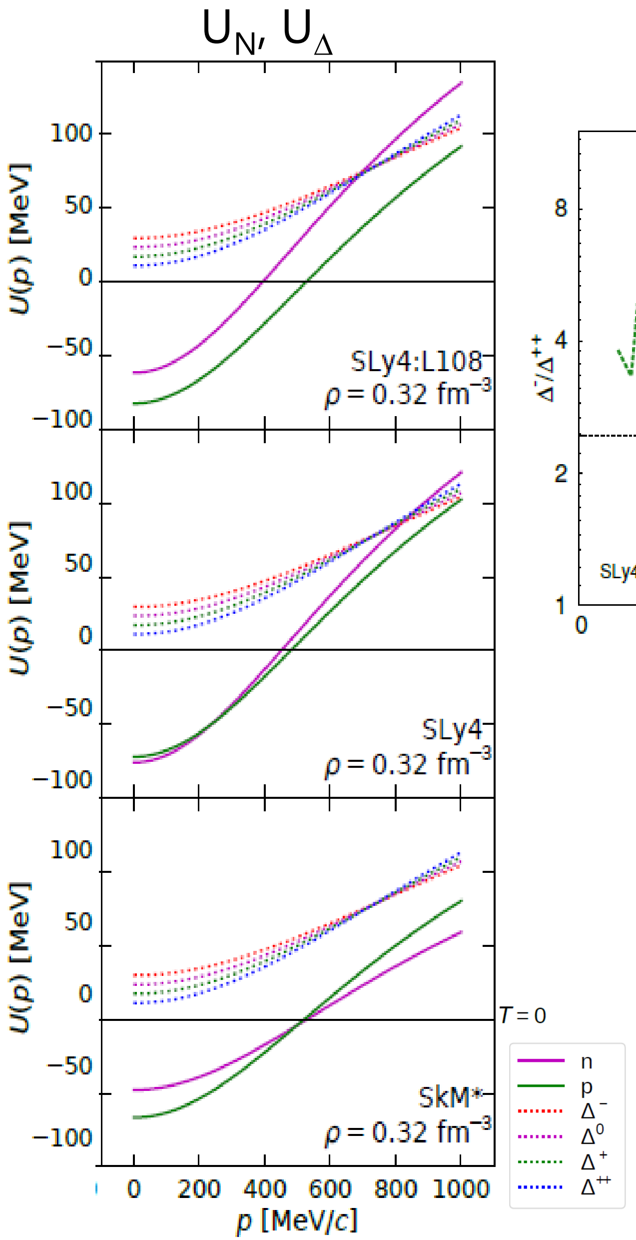
High-density and high-momentum region $(N/Z)^2$

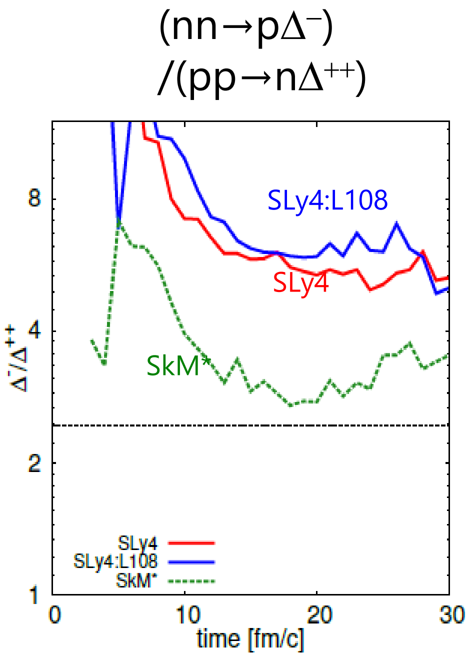


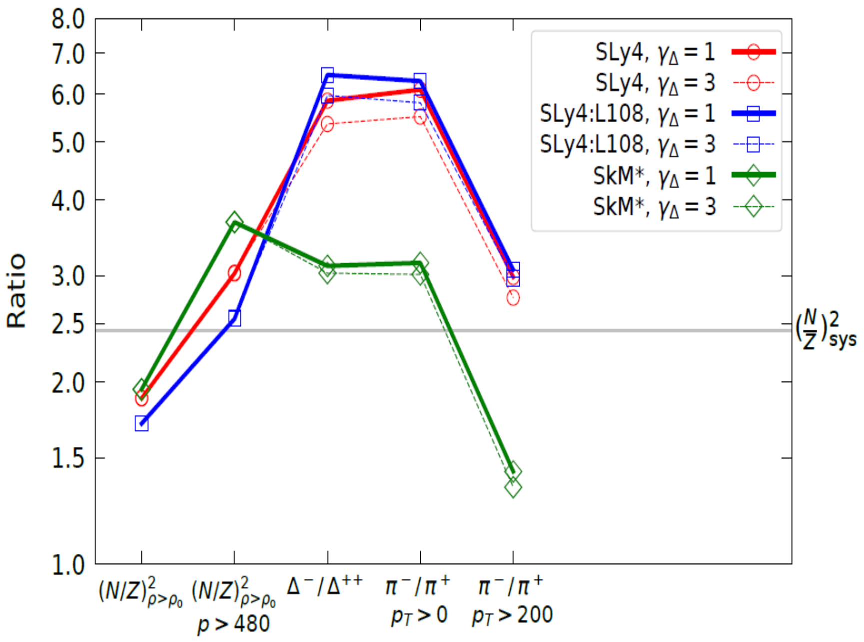


- In the high ρ and p region, $(N/Z)^2$ of SkM* drastically increases because U_n has weaker momentum dependence than that in SLy4 (m^*_n(SkM*) > m^*_n(SLy4))

How to understand the effects in Delta and pion





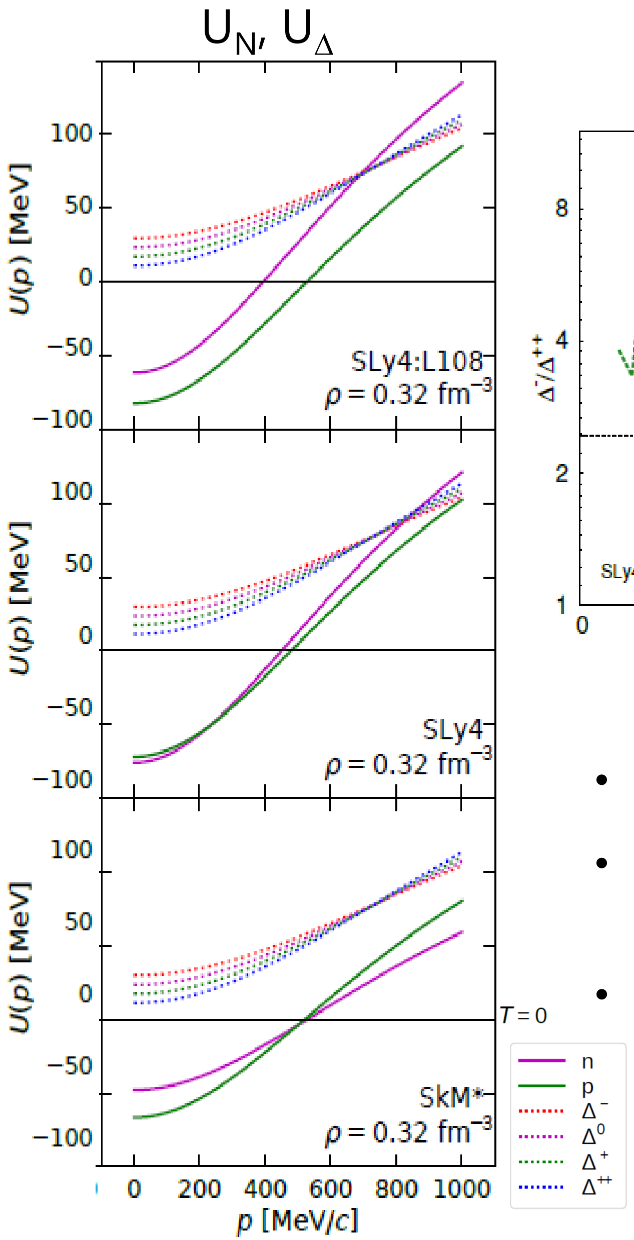


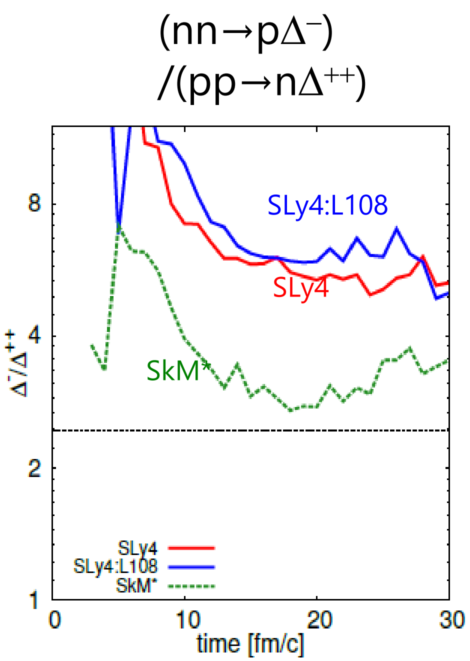
- In the Δ production, Δ^-/Δ^{++} in SLy4:L108 increases drastically.

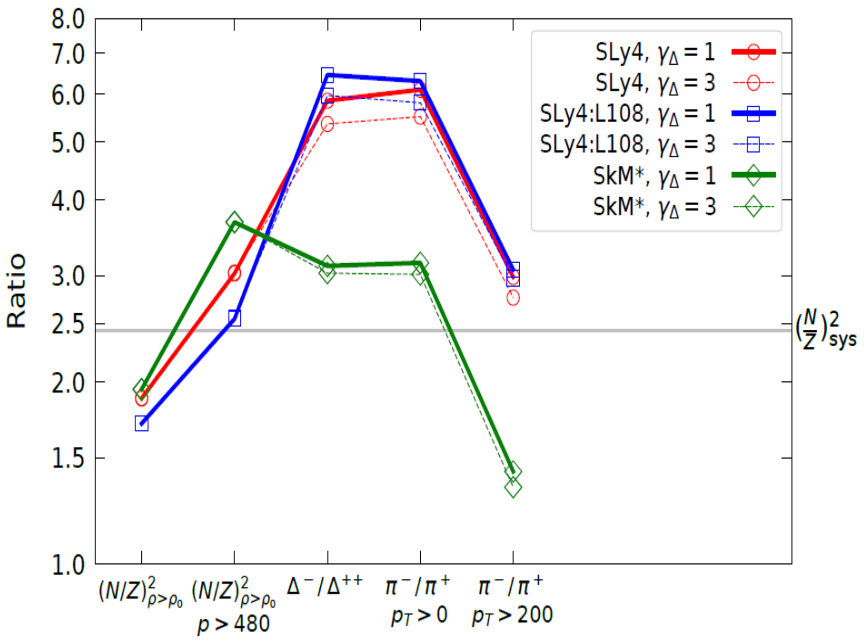
nn→pΔ^- is favored, due to the momentum dependence of U_n and U_p.

- L dependence (SLy4 vs SLy4:L108) in N/Z is inverted in the Δ production.

How to understand the effects in Delta and pion

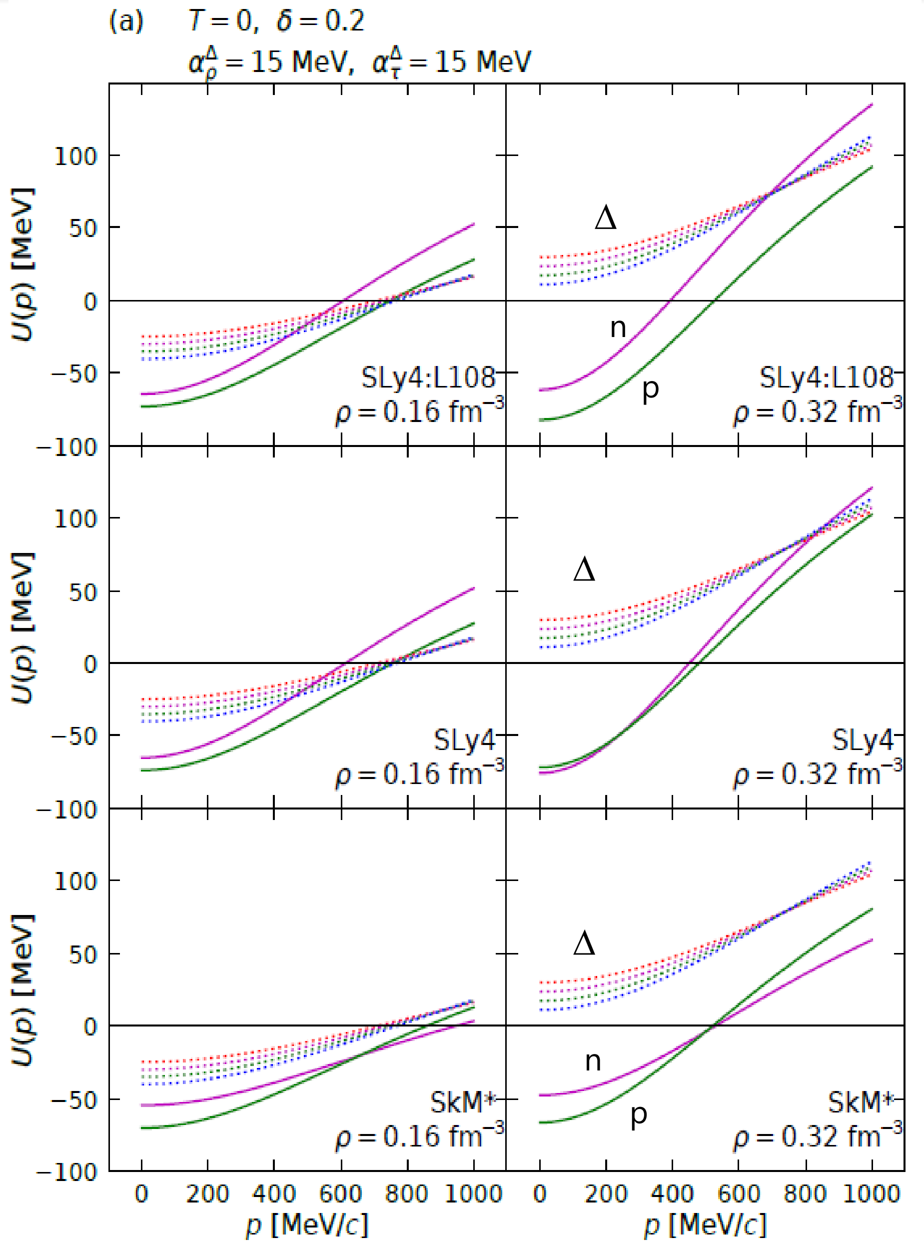






- Then π^-/π^+ seems to reflect to Δ^-/Δ^{++}
- Effect of the symmetry energy L (SLy4 vs SLy4:L108) :
  Relatively small on pion production
- Effect of the momentum dependence of U_n and U_p (SLy4 vs SkM*):
  Strong effect on pion production

Nucleon and Δ potentials



- Nucleon potential

  SLy4:L108 (Stiff), SLy4 (Soft), SkM* in the relativistic form

- Δ potentials:

  Consist of isosc$\Sigma_\Delta = (\Sigma^s_\Delta, \Sigma^0_\Delta, \boldsymbol{\Sigma}_\Delta)$or part

$$\Sigma_{\Delta^-} = \Sigma_{\rm is} + \frac{3}{2}\Sigma_{\rm iv}$$
$$\Sigma_{\Delta^0} = \Sigma_{\rm is} + \frac{1}{2}\Sigma_{\rm iv}$$
$$\Sigma_{\Delta^+} = \Sigma_{\rm is} - \frac{1}{2}\Sigma_{\rm iv}$$
$$\Sigma_{\Delta^{++}} = \Sigma_{\rm is} - \frac{3}{2}\Sigma_{\rm iv}$$

isoscalar part:

$$\Sigma^s_{\rm is} = \frac{1}{2}(\Sigma^s_n + \Sigma^s_p)_{\rm SkM*},$$
$$\Sigma^0_{\rm is} = \frac{1}{2}(\Sigma^0_n + \Sigma^0_p)_{\rm SkM*} + \alpha^\Delta_\rho \frac{\rho}{\rho_0} + \alpha^\Delta_\tau \frac{\tau}{\tau_0},$$
$$\boldsymbol{\Sigma}_{\rm is} = \alpha^\Delta_\rho \frac{\boldsymbol{J}}{\rho_0},$$

isovector part:

$$\Sigma^s_{\rm iv} = \frac{\gamma^\Delta}{3}(\Sigma^s_n - \Sigma^s_n)_{\rm SkM*},$$
$$\Sigma^0_{\rm iv} = \frac{\gamma^\Delta}{3}(\Sigma^0_n - \Sigma^0_p)_{\rm SkM*},$$
$$\boldsymbol{\Sigma}_{\rm iv} = \boldsymbol{0}.$$

based on the nucleon potential in the SkM* parametrization

Free parameters: $\alpha^\Delta_\rho, \alpha^\Delta_\tau, \gamma^\Delta$

- No Pion potential

Delta potential (isoscalar and isovector)

- Effects of the **isovector part** of U_Δ

- Δ potentials: $\Sigma_\Delta = (\Sigma^s_\Delta, \Sigma^0_\Delta, \boldsymbol{\Sigma}_\Delta)$

Consist of isoscalar and isovector part

$$\Sigma_{\Delta^-} = \Sigma_{\rm is} + \frac{3}{2}\Sigma_{\rm iv}$$

$$\Sigma_{\Delta^0} = \Sigma_{\rm is} + \frac{1}{2}\Sigma_{\rm iv}$$

$$\Sigma_{\Delta^+} = \Sigma_{\rm is} - \frac{1}{2}\Sigma_{\rm iv}$$

$$\Sigma_{\Delta^{++}} = \Sigma_{\rm is} - \frac{3}{2}\Sigma_{\rm iv}$$

isoscalar part:

$$\Sigma^s_{\rm is} = \frac{1}{2}(\Sigma^s_n + \Sigma^s_p)_{\rm SkM^*},$$

$$\Sigma^0_{\rm is} = \frac{1}{2}(\Sigma^0_n + \Sigma^0_p)_{\rm SkM^*} + \alpha^\Delta_\rho \frac{\rho}{\rho_0} + \alpha^\Delta_\tau \frac{\tau}{\tau_0},$$

$$\boldsymbol{\Sigma}_{\rm is} = \alpha^\Delta_\rho \frac{\boldsymbol{J}}{\rho_0},$$

isovector part:

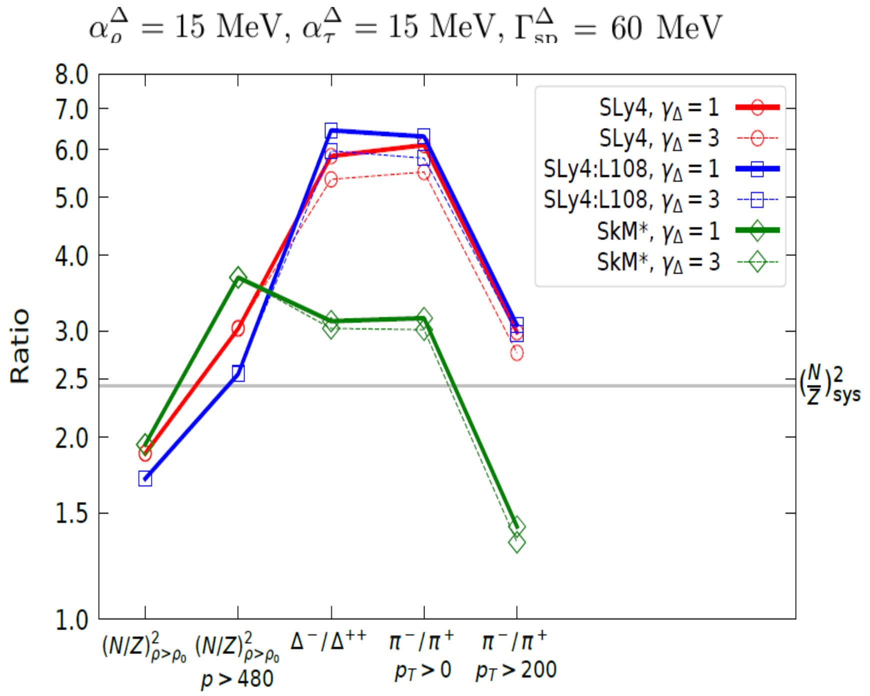
$$\Sigma^s_{\rm iv} = \frac{\gamma^\Delta}{3}(\Sigma^s_n - \Sigma^s_n)_{\rm SkM^*},$$

$$\Sigma^0_{\rm iv} = \frac{\gamma^\Delta}{3}(\Sigma^0_n - \Sigma^0_p)_{\rm SkM^*},$$

$$\boldsymbol{\Sigma}_{\rm iv} = \boldsymbol{0}.$$

based on the nucleon potential in the SkM* parametrization

Free parameters: $\alpha^\Delta_\rho, \alpha^\Delta_\tau, \gamma^\Delta$



Solid line $\gamma_\Delta = 1$
Dashed line $\gamma_\Delta = 3$

$\gamma_\Delta=1 \Rightarrow \Sigma_{\Delta^-} - \Sigma_{\Delta^{++}} = \Sigma_n - \Sigma_p$
$\gamma_\Delta=3 \Rightarrow \Sigma_{\Delta^0} - \Sigma_{\Delta^+} = \Sigma_n - \Sigma_p$

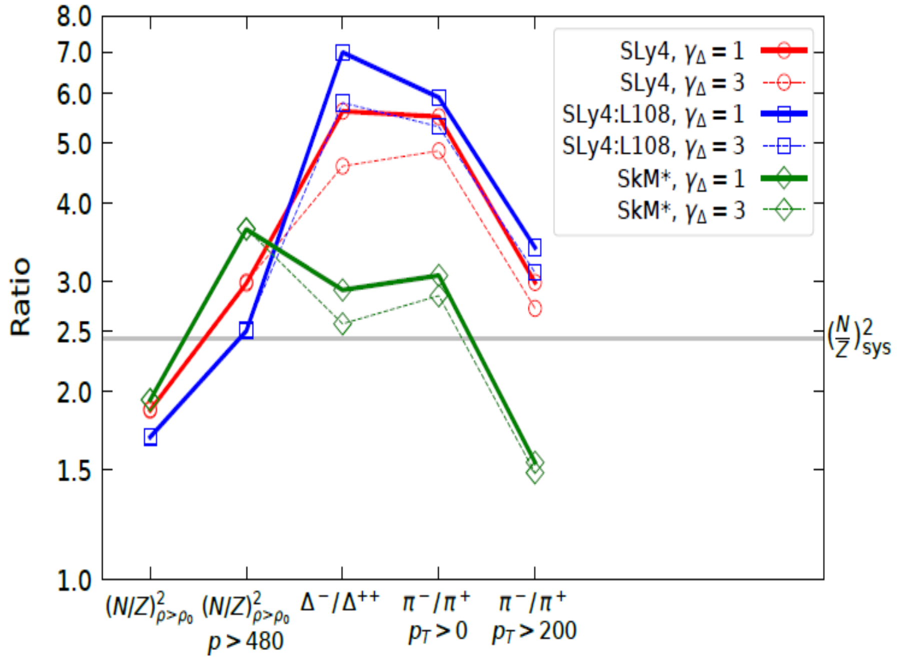
✓ Effect of the isospin splitting of the Δ potential ($\gamma_\Delta=1$ vs. $\gamma_\Delta=3$) is of the same order as that of the nuclear symmetry energy (SLy4 vs SLy4:L108).

Delta potential (isoscalar and isovector)

- Effects of the **isoscalar part** of U_Δ and spreading width Γ^Δ
- Δ potentials: $\Sigma_\Delta = (\Sigma^s_\Delta, \Sigma^0_\Delta, \boldsymbol{\Sigma}_\Delta)$ $\alpha^\Delta_\rho = 0,\ \alpha^\Delta_\tau = 0\ \ \Gamma^\Delta_{sp} = 0.$ (No repulsive terms)

Consist of isoscalar and isovector part

$$\Sigma_{\Delta^-} = \Sigma_{is} + \frac{3}{2}\Sigma_{iv}$$
$$\Sigma_{\Delta^0} = \Sigma_{is} + \frac{1}{2}\Sigma_{iv}$$
$$\Sigma_{\Delta^+} = \Sigma_{is} - \frac{1}{2}\Sigma_{iv}$$
$$\Sigma_{\Delta^{++}} = \Sigma_{is} - \frac{3}{2}\Sigma_{iv}$$



isoscalar part:

$$\Sigma^s_{is} = \frac{1}{2}(\Sigma^s_n + \Sigma^s_p)_{SkM*},$$
$$\Sigma^0_{is} = \frac{1}{2}(\Sigma^0_n + \Sigma^0_p)_{SkM*} + \alpha^\Delta_\rho \frac{\rho}{\rho_0} + \alpha^\Delta_\tau \frac{\tau}{\tau_0},$$
$$\boldsymbol{\Sigma}_{is} = \alpha^\Delta_\rho \frac{\boldsymbol{J}}{\rho_0},$$

isovector part:

$$\Sigma^s_{iv} = \frac{\gamma^\Delta}{3}(\Sigma^s_n - \Sigma^s_n)_{SkM*},$$
$$\Sigma^0_{iv} = \frac{\gamma^\Delta}{3}(\Sigma^0_n - \Sigma^0_p)_{SkM*},$$
$$\boldsymbol{\Sigma}_{iv} = \boldsymbol{0}.$$

- spreading width Γ^Δ $\quad \Gamma_\Delta(m) = \Gamma_{sp}\frac{\rho}{\rho_0} + \sum \Gamma_{\Delta\to N\pi}(m)$

Δ spectral function A(m): $\quad A(m) = \dfrac{4m^2\Gamma_\Delta(m)}{(m^2 - M_\Delta^2)^2 + m^2\Gamma_\Delta(m)^2}$

Solid line $\gamma_\Delta = 1$
Dashed line $\gamma_\Delta = 3$

✓ Results are similar qualitatively
✓ Effect of the symmetry energy (SLy4 vs SLy4:L108) is now stronger
✓ Effect of the difference in the momentum dependence of U_n and U_p (SLy4 vs SkM*) is always the most significant

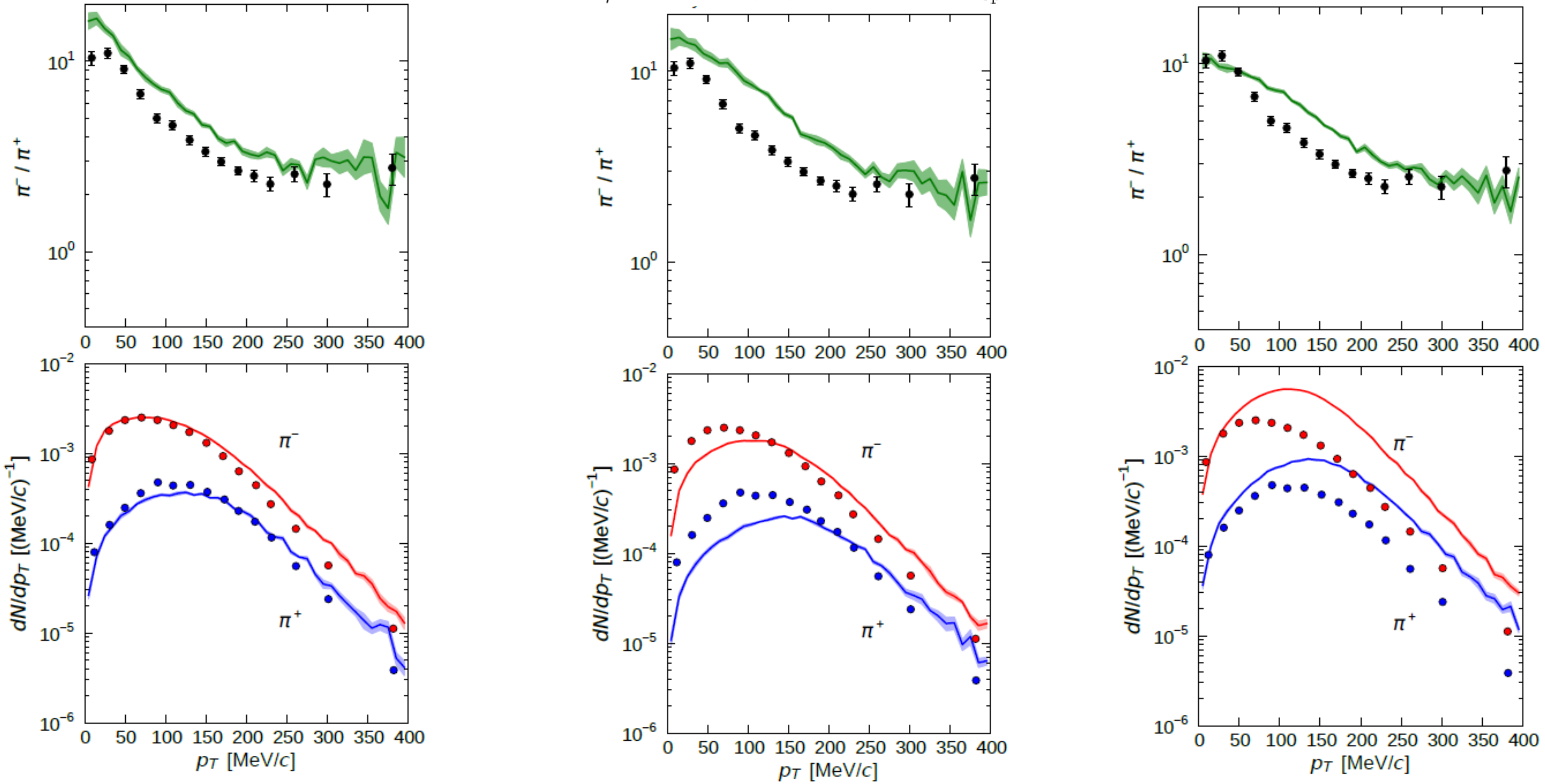
Delta potential (isoscalar and isovector)

- Effects of the isoscalar part of U_Δ and spreading width Γ^Δ [No repulsive terms]

$\alpha_\rho^\Delta = 15$ MeV, $\alpha_\tau^\Delta = 15$ MeV, $\Gamma_{\rm sp}^\Delta = 60$ MeV

$\alpha_\rho^\Delta = 15$ MeV, $\alpha_\tau^\Delta = 15$ MeV, $\Gamma_{\rm sp}^\Delta = 0.$

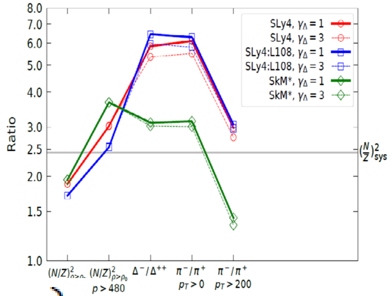
$\alpha_\rho^\Delta = 0,\ \alpha_\tau^\Delta = 0\ \ \Gamma_{\rm sp}^\Delta = 0.$



π^-/π^+ ratio of the spectra is not affected much

- Low momentum region of the spectra is significantly affected by Γ^Δ
- Pion yield is overestimated due to the lack of the repulsive terms in U_Δ

Interactions: SLy4, SLy4:L108, SkM*



- Energy density:

$$\mathcal{E}_{\rm int}(\boldsymbol{r}) = \sum_{\alpha\beta}\Big\{U^{t_0}_{\alpha\beta}\rho_\alpha(\boldsymbol{r})\rho_\beta(\boldsymbol{r}) + U^{t_3}_{\alpha\beta}\rho_\alpha(\boldsymbol{r})\rho_\beta(\boldsymbol{r})[\rho(\mathbf{r})]^\gamma \ + U^{\tau}_{\alpha\beta}\tilde{\tau}_\alpha(\boldsymbol{r})\rho_\beta(\boldsymbol{r}) + U^{\nabla}_{\alpha\beta}\nabla\rho_\alpha(\boldsymbol{r})\nabla\rho_\beta(\boldsymbol{r})\Big\},$$

Densities: $\rho_\alpha(\boldsymbol{r}) = \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3} f_\alpha(\boldsymbol{r},\boldsymbol{p}),\quad \tilde{\tau}_\alpha(\boldsymbol{r}) = \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3} \frac{[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2}{1+[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2/\Lambda^2_{\rm md}} f_\alpha(\boldsymbol{r},\boldsymbol{p}),$

with $\bar{\boldsymbol{p}}(\boldsymbol{r}) = \frac{1}{\sum_\alpha \rho_\alpha(\boldsymbol{r})} \sum_\alpha \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3} \boldsymbol{p} f_\alpha(\boldsymbol{r},\boldsymbol{p}).$

The coefficients are related to the Skyrme parameters

$$U^{t_0}_{\alpha\beta} = \langle\alpha\beta|\frac{1}{2}t_0(1+x_0P_\sigma)|\alpha\beta-\beta\alpha\rangle,$$

$$U^{t_3}_{\alpha\beta} = \langle\alpha\beta|\frac{1}{12}t_3(1+x_3P_\sigma)|\alpha\beta-\beta\alpha\rangle,$$

$$U^{\tau}_{\alpha\beta} = \langle\alpha\beta|\frac{1}{4}t_1(1+x_1P_\sigma)|\alpha\beta-\beta\alpha\rangle + \langle\alpha\beta|\frac{1}{4}t_2(1+x_2P_\sigma)\ |\alpha\beta+\beta\alpha\rangle,$$

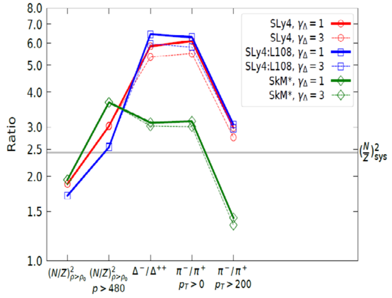
$$U^{\nabla}_{\alpha\beta} = \langle\alpha\beta|\frac{3}{16}t_1(1+x_1P_\sigma)|\alpha\beta-\beta\alpha\rangle - \langle\alpha\beta|\frac{1}{16}t_2(1+x_2P_\sigma)\ |\alpha\beta+\beta\alpha\rangle,$$

In the case of cut-off parameter $\Lambda_{\rm md} = \infty$, interaction is equivalent to the Skyrme type interaction

$$\begin{aligned} v_{ij} &= t_0(1+x_0P_\sigma)\delta(\mathbf{r}) \\ &+ \tfrac{1}{2}t_1(1+x_1P_\sigma)[\delta(\mathbf{r})\mathbf{k}^2 + \mathbf{k}^2\delta(\mathbf{r})] \\ &+ t_2(1+x_2P_\sigma)\mathbf{k}\cdot\delta(\mathbf{r})\mathbf{k} \\ &+ \tfrac{1}{6}t_3(1+x_3P_\sigma)[\rho(\mathbf{r}_i)]^\gamma\delta(\mathbf{r}), \end{aligned}$$

the spin–isospin label α (or β) $= p\uparrow, p\downarrow, n\uparrow$ and $n\downarrow$

Interactions: SLy4, SLy4:L108, SkM*



- Momentum-dependent potential **(in AMD):**

$$U_\alpha(\boldsymbol{r},\boldsymbol{p}) = (2\pi\hbar)^3 \frac{\delta}{\delta f_\alpha(\boldsymbol{r},\boldsymbol{p})} \int \mathcal{E}_{\rm int}(\boldsymbol{r}) d\boldsymbol{r} = A_\alpha(\boldsymbol{r}) \frac{[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2}{1+[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2/\Lambda_{\rm md}^2} + \tilde{C}_\alpha(\boldsymbol{r}),$$

with $A_\alpha(\boldsymbol{r}) = \sum_\beta U^\tau_{\alpha\beta}\rho_\beta(\boldsymbol{r})$

$$\tilde{C}_\alpha(\boldsymbol{r}) = \sum_\beta \Big\{ 2U^{t_0}_{\alpha\beta}\rho_\beta(\boldsymbol{r}) + 2U^{t_3}_{\alpha\beta}\rho_\beta(\boldsymbol{r})[\rho(\mathbf{r})]^\gamma + U^\tau_{\alpha\beta}\tilde{\tau}_\beta(\boldsymbol{r}) - 2U^\nabla_{\alpha\beta}\nabla^2\rho_\beta(\boldsymbol{r}) \Big\} + \Big(\sum_{\alpha'\beta'} U^{t_3}_{\alpha'\beta'}\rho_{\alpha'}(\boldsymbol{r})\rho_{\beta'}(\boldsymbol{r}) \Big) \gamma[\rho(\mathbf{r})]^{\gamma-1}.$$

- Relativistic version **(in sJAM):**

Nucleon single-particle energy $E_a(\boldsymbol{r},\boldsymbol{p}) = \sqrt{(m_N+\Sigma^s_a(\boldsymbol{r}))^2 + (\boldsymbol{p}-\boldsymbol{\Sigma}_a(\boldsymbol{r}))^2} + \Sigma^0_a(\boldsymbol{r}).$

Parametrization from Skyrme interaction: equivalent up to $O(p^2)$:

$$\frac{\boldsymbol{p}^2}{2m_N} + A_a(\boldsymbol{p}-\bar{\boldsymbol{p}})^2 + \tilde{C}_a + m_N \approx \sqrt{(m_N+\Sigma^s_a)^2 + (\boldsymbol{p}-\boldsymbol{\Sigma}_a)^2} + \Sigma^0_a$$

c.f. Zhen Zhang and Che Ming Ko, PRC 98 (2018) 054614.

$\Sigma^s_a = m^*_a - m_N$ with the nucleon effective mass $m^*_a = (m_N^{-1} + 2A_a)^{-1}$

$$\boldsymbol{\Sigma}_a = 4A_a m^*_a \bar{\boldsymbol{p}} = 2m^*_a \sum_b U^\tau_{ab}\boldsymbol{J}_b$$

$$\Sigma^0_a = \tilde{C}_a - \Sigma^s_a + A_a\bar{\boldsymbol{p}}^2 - 8m^*_a A^2_a \bar{\boldsymbol{p}}^2 = C_a - \Sigma^s_a - \frac{\boldsymbol{\Sigma}^2_a}{2m^*_a}$$

$$\boldsymbol{J}_b(\boldsymbol{r}) = \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3}\boldsymbol{p} f_b(\boldsymbol{r},\boldsymbol{p})$$

$$\tau_b(\boldsymbol{r}) = \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3}\boldsymbol{p}^2 f_b(\boldsymbol{r},\boldsymbol{p}).$$

$$U_{\rm rel}(p) = \sqrt{(m_N+\Sigma^s)^2 + p^2} + \Sigma^0 - \sqrt{m_N^2 + p^2}$$