Anomalous Dynamical Screening of Relativistic Plasma in a Magnetic Field

Sota Hanai

Sota Hanai, arXiv:2509.16652

$$c = \hbar = k_{\mathrm{B}} = \varepsilon_0 = \mu_0 = 1$$

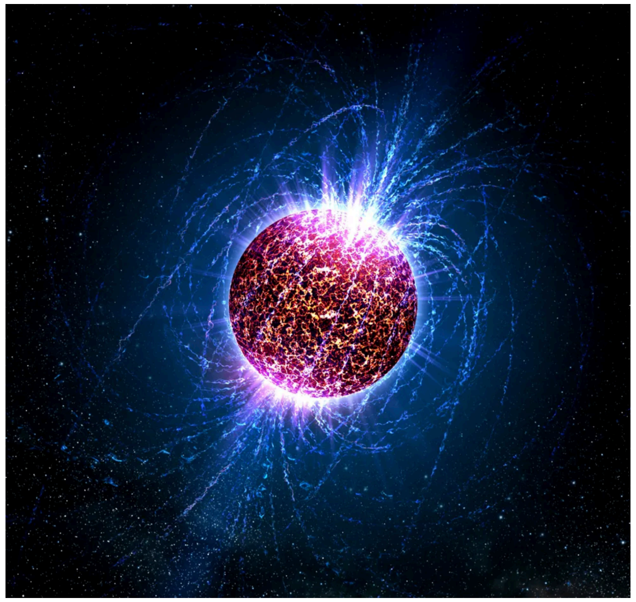
Neutron stars

- Densest matter in the Universe：Neutron star

- Typical quantities
  - mass ~ solar mass
  - radius ~ $10\,\mathrm{km}$

  density ~ $10^{14}\,\mathrm{g/cm^3}$

  - magnetic field (surface) ~ 10^{12}-$10^{15}\,\mathrm{G}$
  - temperature ~ 10^{6}-$10^{9}\,\mathrm{K}$



Credit: Casey Reed/
Penn State University

- Many-body system of relativistic fermions
  → electrons, quarks

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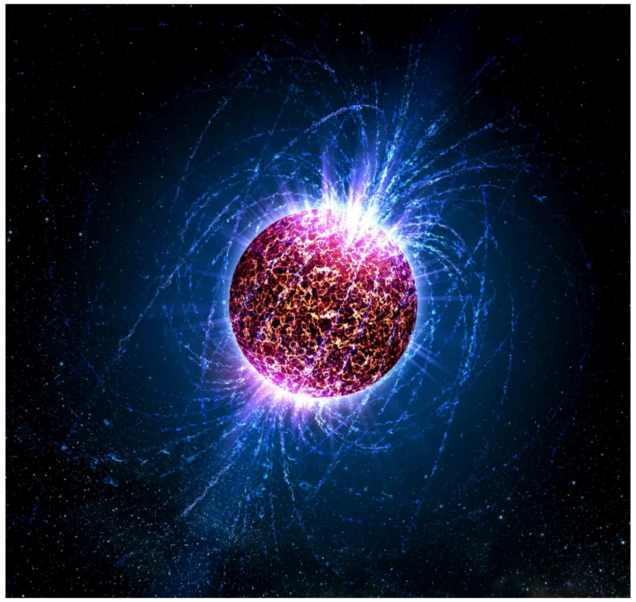

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- Many-body system of relativistic fermions
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Chirality ➡ Collective modes in magnetized dense matter

Outline

- Introduction
- Electromagnetic waves in a background magnetic field
- Anomalous dynamical screening Hanai (2025)
- Conclusion and Outlook

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- **Electromagnetic waves in a background magnetic field**
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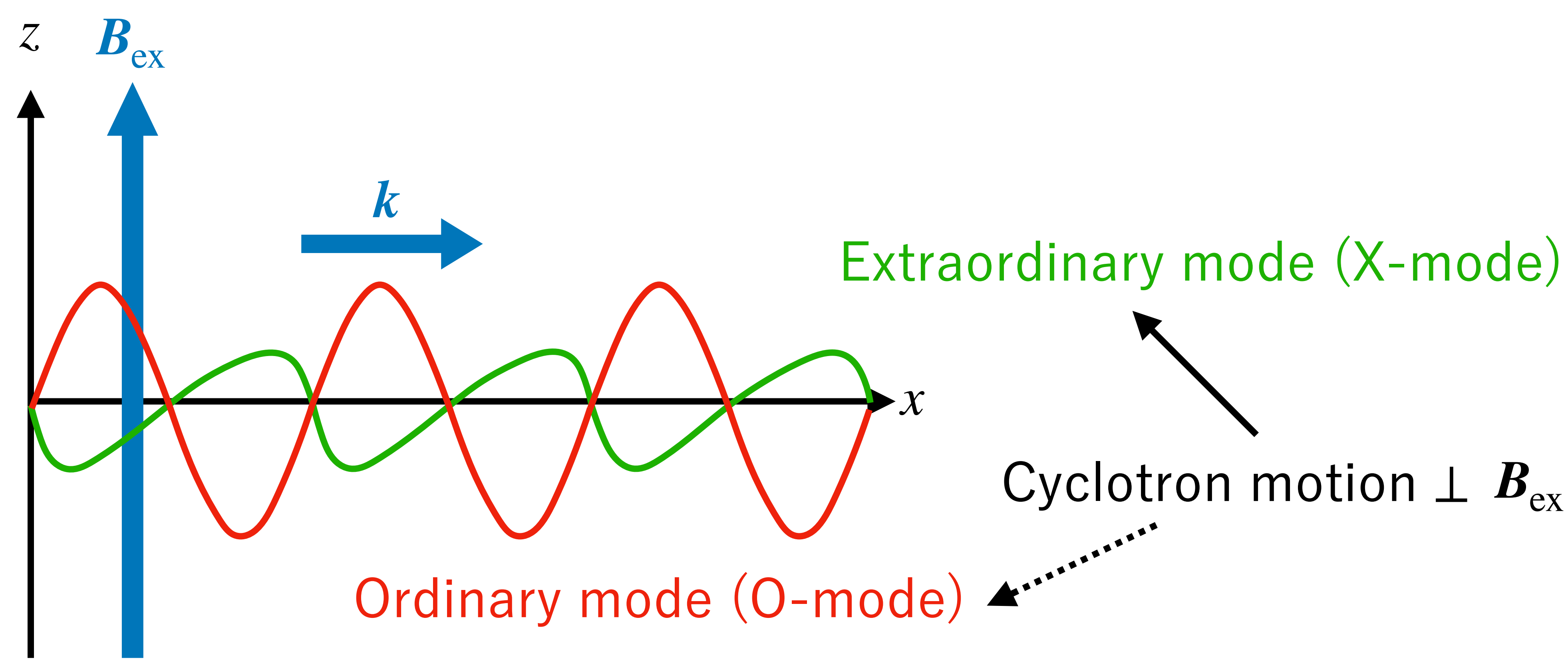
Electromagnetism in magnetic fields

See, e.g., Landau, Lifshitz vol.8

- Collective exciations in a magnetized collisionless plasma

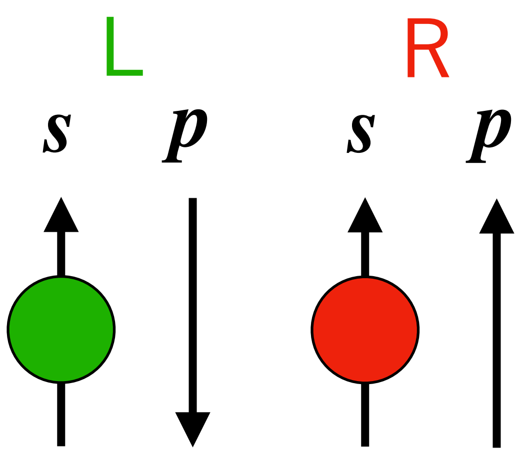
$$\boldsymbol{E} = \delta\boldsymbol{E}\,, \qquad \boldsymbol{B} = \boldsymbol{B}_{\rm ex} + \delta\boldsymbol{B}$$

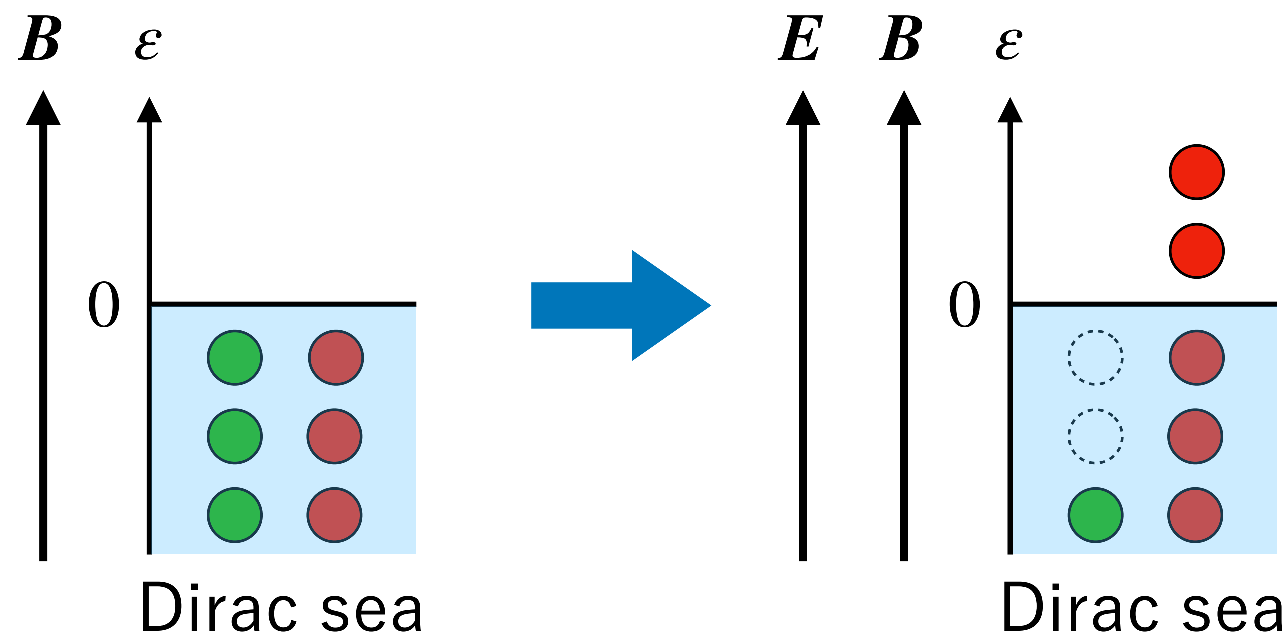


- O-mode in relativisitc plasma is modifed by quantum effect

Chiral anomaly

- Massless fermions have **chirality**





$$\partial_\mu j_5^\mu = \frac{e^2}{2\pi^2}\boldsymbol{E}\cdot\boldsymbol{B}\,, \qquad j_5^\mu \equiv j_{\mathrm{R}}^\mu - j_{\mathrm{L}}^\mu$$

Adler (1969); Bell, Jackiw (1969)

- Dispersion relation of O-mode

$$\left(\frac{|k_x|}{\omega}\right)^2 = \varepsilon_{zz}\,, \qquad e\delta j_z = -\,\mathrm{i}\omega(\varepsilon_{zz}-1)\delta E_z$$

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- **Anomalous dynamical screening** Hanai (2025)
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Chiral kinetic theory

Son, Yamamoto (2012); Stephanov, Yin (2012)

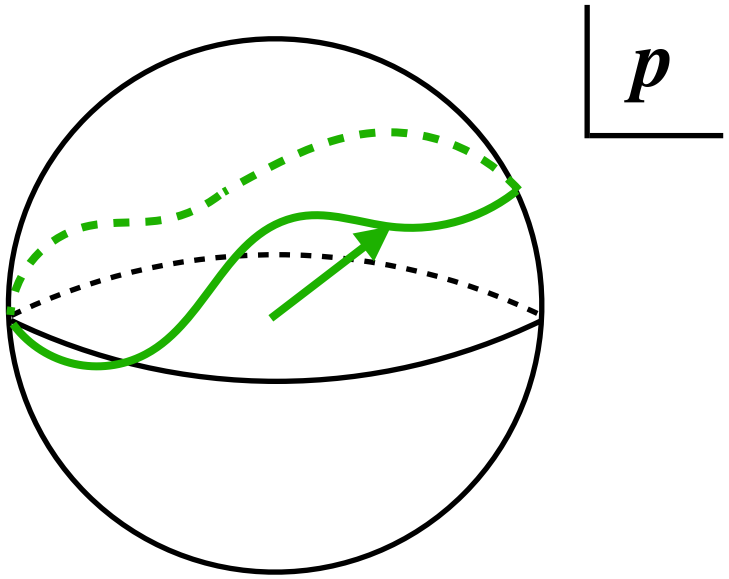
- Boltzmann equation in collisionless regime

$$\frac{\partial f_\lambda}{\partial t} + \dot{\boldsymbol{x}} \cdot \frac{\partial f_\lambda}{\partial \boldsymbol{x}} + \dot{\boldsymbol{p}} \cdot \frac{\partial f_\lambda}{\partial \boldsymbol{p}} = 0 \qquad (\lambda = \pm \text{ : chirality})$$

- Topology of chiral fermions

$$H = \lambda \boldsymbol{\sigma} \cdot \boldsymbol{p} \Longrightarrow \boldsymbol{\Omega} = \lambda \frac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}$$

Berry curvature



- Equations of motion

$$\dot{\boldsymbol{x}} = \boldsymbol{v} + \dot{\boldsymbol{p}} \times \underline{\boldsymbol{\Omega}}, \qquad \dot{\boldsymbol{p}} = e\boldsymbol{E} + e\dot{\boldsymbol{x}} \times \boldsymbol{B}$$

- Currents can be obtained by solving the Boltzmann equation

Anomalous correction to O-mode

Hanai (2025)

- Dispersion relation

$$\omega^2 = |k_x|^2 + m_\mathrm{D}^2 F(\xi) + \underbrace{m_\mathrm{a}^2 G(\xi)}_{\text{Anomalous correction}} \qquad \left(\xi \equiv \frac{\omega}{|k_x|}, \qquad m_\mathrm{a}^2 \equiv \frac{3e^4 B_\mathrm{ex}^2}{4\pi^4 \chi} \right)$$

- Long wavelength limit：Correction to plasma oscillation
  $(\xi \gg 1)$

- Quasi-static limit
  $(\xi \ll 1)$

Landau damping

$$\omega \simeq -\mathrm{i}\frac{4|k_x|^3}{\pi m_\mathrm{D}^2}\left(1 + \frac{m_\mathrm{a}^2}{|k_x|^2}\right)$$

$|k_x| \gtrsim m_\mathrm{a}$ ➡ **Anomalous dynamical screening**

Relaxation time in dense matter

- Electron-electron scattering

$$\mathscr{M} = \frac{C_1}{\omega^2 - |\boldsymbol{k}|^2 + \Pi_\mathrm{L}} + \frac{C_2}{\omega^2 - |\boldsymbol{k}|^2 + \Pi_\mathrm{T}}$$

Landau damping

- Temperature dependence

$$\frac{1}{\tau_\mathrm{T}} = \frac{1}{\tau_\mathrm{c}} + \frac{1}{\tau_\mathrm{a}} \qquad \left(\frac{1}{\tau_\mathrm{c}} \sim \alpha^2 \frac{T^{5/3}}{m_\mathrm{D}^{2/3}} \qquad \frac{1}{\tau_\mathrm{a}} \sim \alpha^2 \zeta^2 T \right)$$

Landau damping

Heiselberg, Pethick (1993)

Anomalous dynamical screening

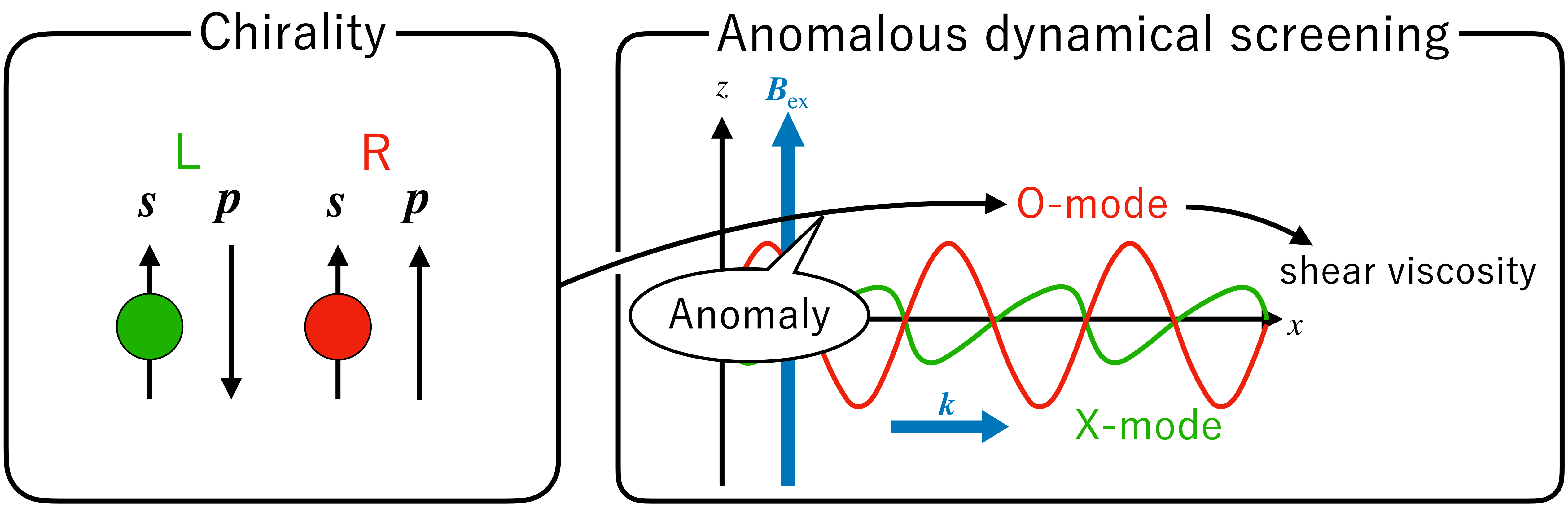
Hanai (2025)

- Shear viscosity : $\eta \propto \tau_\mathrm{T}$

$$\frac{\tau_\mathrm{c}}{\tau_\mathrm{a}} \sim 1 \left(\frac{T}{10^6\ \mathrm{K}} \right)^{-2/3} \left(\frac{B}{10^{17}\ \mathrm{G}} \right)^{2} \left(\frac{\mu_\mathrm{e}}{10^2\ \mathrm{MeV}} \right)^{-10/3}$$

→ would enhance the r-mode instability

Conclusion & outlook

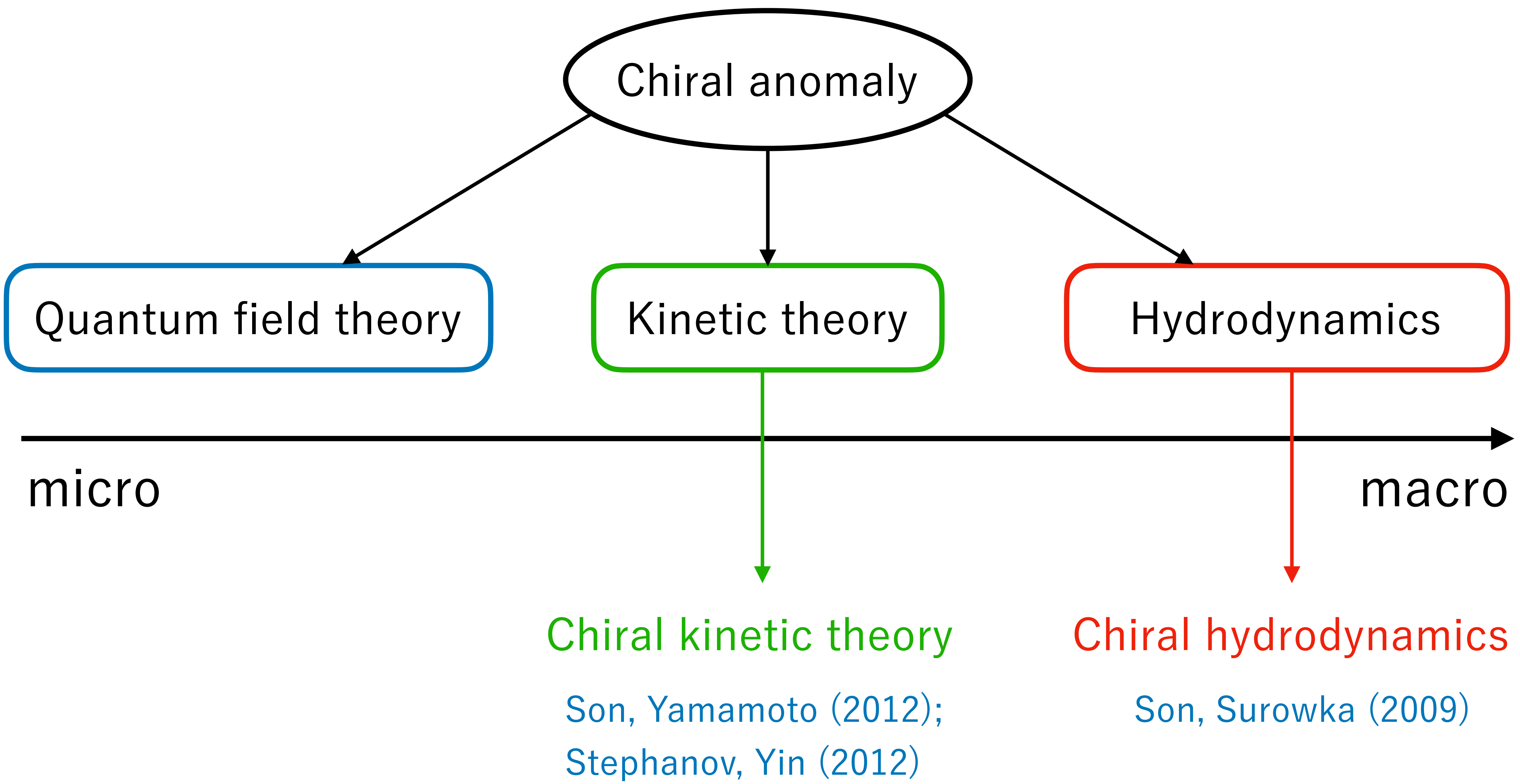


- Anomalous effect may be dominant in strong magnetic fields
- Application to quark matter?

Appendix

Chiral effects in effective theory

- Chiral anomaly is independent of scales 't Hooft (1980)



Parity of transport

- Electric fields cause electric currents （Ohm's law）

$$e\boldsymbol{j} = \sigma \boldsymbol{E} \xrightarrow{\text{parity transformation}} e(-\boldsymbol{j}) = \sigma(-\boldsymbol{E})$$

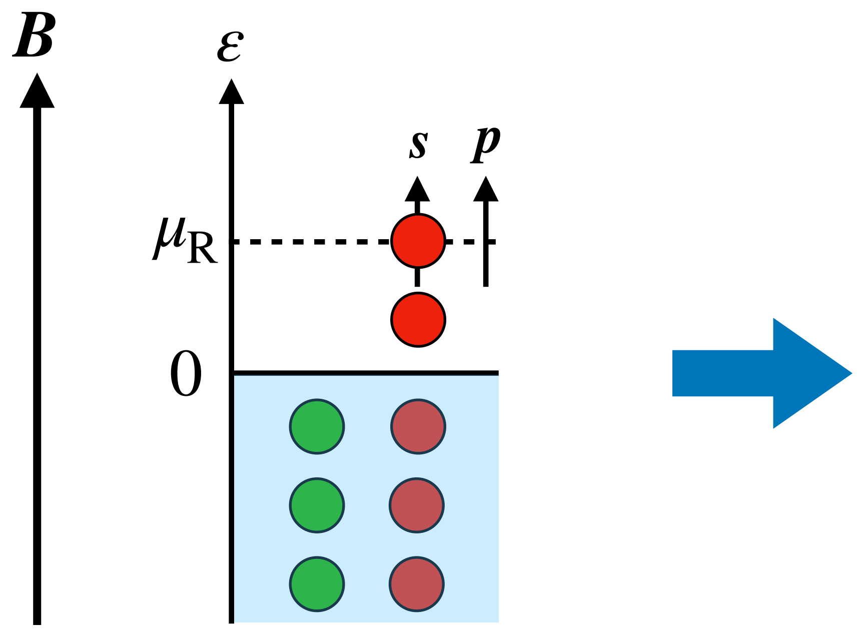
- Do magnetic fields cause electric currents ?

$$e\boldsymbol{j} = \sigma_{\mathrm{m}} \boldsymbol{B} \xrightarrow{\text{parity transformation}} e(-\boldsymbol{j}) = \sigma_{\mathrm{m}} \boldsymbol{B}$$

If $\sigma_{\mathrm{m}} \to -\sigma_{\mathrm{m}}$ is satisfied, the current can be generated.

Chiral magnetic/separation effects

- At finite density system



$$\boldsymbol{j}_{\mathrm{R}} = \frac{e\mu_{\mathrm{R}}}{4\pi^2}\boldsymbol{B}\,, \qquad \boldsymbol{j}_{\mathrm{L}} = -\frac{e\mu_{\mathrm{L}}}{4\pi^2}\boldsymbol{B}$$

- For Dirac fermions

CME $$\boldsymbol{j} = \boldsymbol{j}_{\mathrm{R}} + \boldsymbol{j}_{\mathrm{L}} = \frac{e\mu_5}{2\pi^2}\boldsymbol{B},$$

Vilenkin (1980); Nilsen, Ninomiya (1983);
Fukushima, Kharzeev, Warringa (2008); ⋯

CSE $$\boldsymbol{j}_5 = \boldsymbol{j}_{\mathrm{R}} - \boldsymbol{j}_{\mathrm{L}} = \frac{e\mu}{2\pi^2}\boldsymbol{B}$$

Son, Zhitnitsky (2004);
Metlitsky, Zhitnitsky (2005); ⋯

$$\left(\mu = \frac{\mu_{\mathrm{R}} + \mu_{\mathrm{L}}}{2}, \qquad \mu_5 = \frac{\mu_{\mathrm{R}} - \mu_{\mathrm{L}}}{2}\right)$$

chiral chemical potential

CME & chiral anomaly

Nielsen, Ninomiya (1983)

- Energy necessary for the chirality imbalance

$$\mu_5 \frac{\mathrm{d}Q_5}{\mathrm{d}t} = \mu_5 \int \mathrm{d}^3\boldsymbol{x} \underline{\frac{e^2}{2\pi^2}} \boldsymbol{E} \cdot \boldsymbol{B}$$ chiral anomaly

- Energy is suppled by electric currents

$$\mu_5 \frac{\mathrm{d}Q_5}{\mathrm{d}t} = \int \mathrm{d}^3 e\boldsymbol{j} \cdot \boldsymbol{E}$$

- For arbitrary electric fields,

$$\Rightarrow \quad \boldsymbol{j} = \underline{\frac{e\mu_5}{2\pi^2}} \boldsymbol{B}$$

Relaxation time of quark matter

Heiselberg, Pethick (1993)

- Medium effects
  - longitudinal: Debye screening (usual Fermi liquid like)
  - transverse: Landau damping

$$\frac{1}{\tau} \sim \#\alpha_s^2 \underline{\frac{T^2}{q_D}} + \#\alpha_s^2 \underline{\frac{T^{5/3}}{q_D^{2/3}}}$$

usual Fermi liquid

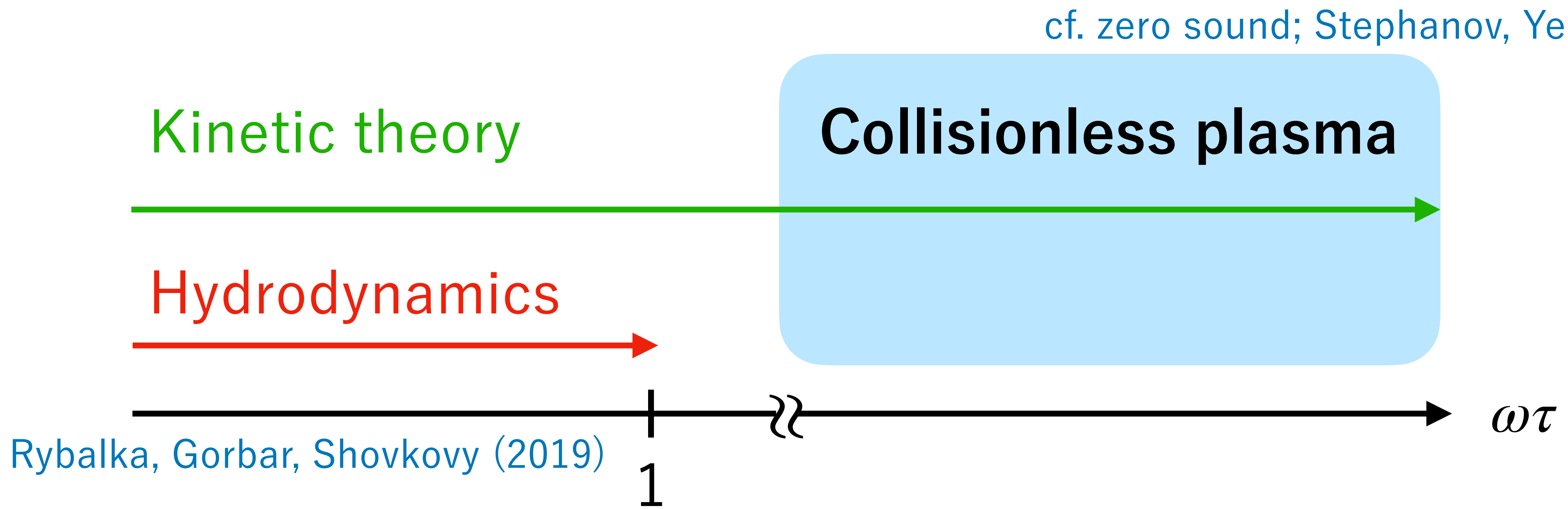
$(q_{IR} \sim q_D)$

Landau damping

$(q_{IR} \sim (q_D^2 T)^{1/3})$

Relativistic collisionless plasma

- Collective modes also appear in the collisionless regime



- Focus on the fluctuations

$$\boldsymbol{E} = \delta\boldsymbol{E}\,, \quad \boldsymbol{B} = \boldsymbol{B}_{\rm ex} + \delta\boldsymbol{B}\,, \quad n_5 = \delta n_5$$

Electromagnetism in magnetic fields

See, e.g., Landau, Lifshitz vol.8

- Maxwell equations in media

$$\boldsymbol{k} \times \delta \boldsymbol{E} = \omega \delta \boldsymbol{B}\,, \qquad \boldsymbol{k} \times \delta \boldsymbol{B} = -\,\omega \delta \boldsymbol{E} - \mathrm{i} e \delta \boldsymbol{j}$$

- Linear response

microscopic details

$$e \delta \boldsymbol{j} = -\,\mathrm{i}\omega \delta \boldsymbol{P} = -\,\mathrm{i}\omega(\varepsilon - 1)\delta \boldsymbol{E}$$

- Permittivity tensor in an external magnetic field

$$\varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ -\varepsilon_{xy} & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

Onsager's theorem: $\varepsilon^{ij}(B_{\mathrm{ex}}) = \varepsilon^{ji}(-B_{\mathrm{ex}})$

・Diagonal：even in B_{ex}
・Off-diagonal：odd in B_{ex}

$$\boldsymbol{r}^2 \delta \boldsymbol{E} - (\boldsymbol{r} \cdot \delta \boldsymbol{E})\boldsymbol{r} - \varepsilon \delta \boldsymbol{E} = 0$$

$$\left(\boldsymbol{r} \equiv \frac{\boldsymbol{k}}{\omega} \right)$$

$\Rightarrow$

$$r_x^2 = \varepsilon_{zz}$$

O-mode

Dynamics of electric field and chiral charge

- Maxwell eq. + linear response

$$(r_x^2 - 1)\delta E_z = \mathrm{i}\frac{1}{\omega} e\delta j_z$$

- Continuity eq. of chiral charge

$$\omega\delta n_5 - k_x \delta j_{5x} = \underbrace{\mathrm{i}\frac{e^2 B_{\mathrm{ex}}}{2\pi^2}\delta E_z}_{\text{chiral anomaly}}$$

- Electric and axial currents

$$e\delta j_z = e\sum_{\lambda=\pm}\delta j_{\lambda z} = \mathrm{i}\frac{m_{\mathrm{D}}^2}{\omega}F(\xi)\delta E_z + \frac{e^2 B_{\mathrm{ex}}}{2\pi^2\chi}G(\xi)\delta n_5\,,$$

$$\delta j_{5x} = \sum_{\lambda=\pm}\lambda\delta j_{\lambda x} = 0$$

$$\left(\xi \equiv \frac{\omega}{|k_x|}\,,\quad L(\xi) \equiv \frac{\xi}{2}\ln\left|\frac{1+\xi}{1-\xi}\right| - \mathrm{i}\frac{\pi\xi}{2}\theta(1-\xi)\,,\quad F(\xi) \equiv \frac{1}{2}[\xi^2 + (1-\xi^2)L(\xi)]\,,\quad G(\xi) \equiv (1-\xi^2)[1-L(\xi)]\right)$$

Transport coefficients

- Counting: $A^\mu = \mathcal{O}(\epsilon)\,,\ \partial_\mu = \mathcal{O}(\delta)\,,\ e = \mathcal{O}(\delta)$

- Solve Boltzmann equation by expansion

$$f_\lambda(t, \boldsymbol{x}, \boldsymbol{p}) = f_\lambda^{(0)} + f_\lambda^{(\delta\epsilon)} + f_\lambda^{(\delta^2\epsilon)}$$

- Currents at each order

$$\boldsymbol{j}_\lambda^{(\delta\epsilon)}(\omega, \boldsymbol{k}) = \mathrm{i}\frac{m_{\mathrm{D}\lambda}^2}{2e\omega}[\xi^2 + (1-\xi^2)L(\xi)]\boldsymbol{E}\,, \quad \text{Ohmic-like}$$

$$\boldsymbol{j}_\lambda^{(\delta^2\epsilon)}(\omega, \boldsymbol{k}) = \lambda\frac{e\mu_\lambda}{4\pi^2}(1-\xi^2)[1-L(\xi)]\boldsymbol{B} \quad \text{CME}$$

$$\left(L(\xi) \equiv \frac{\xi}{2}\ln\left|\frac{1+\xi}{1-\xi}\right| - \mathrm{i}\frac{\pi\xi}{2}\theta(1-\xi)\,, \qquad \xi \equiv \frac{\omega}{|\boldsymbol{k}|} \right)$$

Relaxation time in dense matter

- Relaxation time of degenerate matter

$$\frac{1}{\tau} \sim \alpha^2 \frac{T^2}{k_{\mathrm{IR}}}$$

Pauli blocking

- Electron-electron scattering

$$\mathscr{M} = \frac{C_1}{\omega^2 - |\boldsymbol{k}|^2 + \Pi_{\mathrm{L}}} + \frac{C_2}{\omega^2 - |\boldsymbol{k}|^2 + \Pi_{\mathrm{T}}}$$

Landau damping

$$\Rightarrow \quad k_{\mathrm{IR}}^2 \sim \Pi_{\mathrm{T}}$$

- Temperature dependence

$$\frac{1}{\tau_{\mathrm{T}}} = \frac{1}{\tau_{\mathrm{c}}} + \frac{1}{\tau_{\mathrm{a}}} \qquad \left(\frac{1}{\tau_{\mathrm{c}}} \sim \alpha^2 \frac{T^{5/3}}{m_{\mathrm{D}}^{2/3}} \qquad \frac{1}{\tau_{\mathrm{a}}} \sim \alpha^2 \zeta^2 T \right)$$

Landau damping
Heiselberg, Pethick (1993)

Anomalous dynamical screening
Hanai (2025)

Strong magnetic field limit

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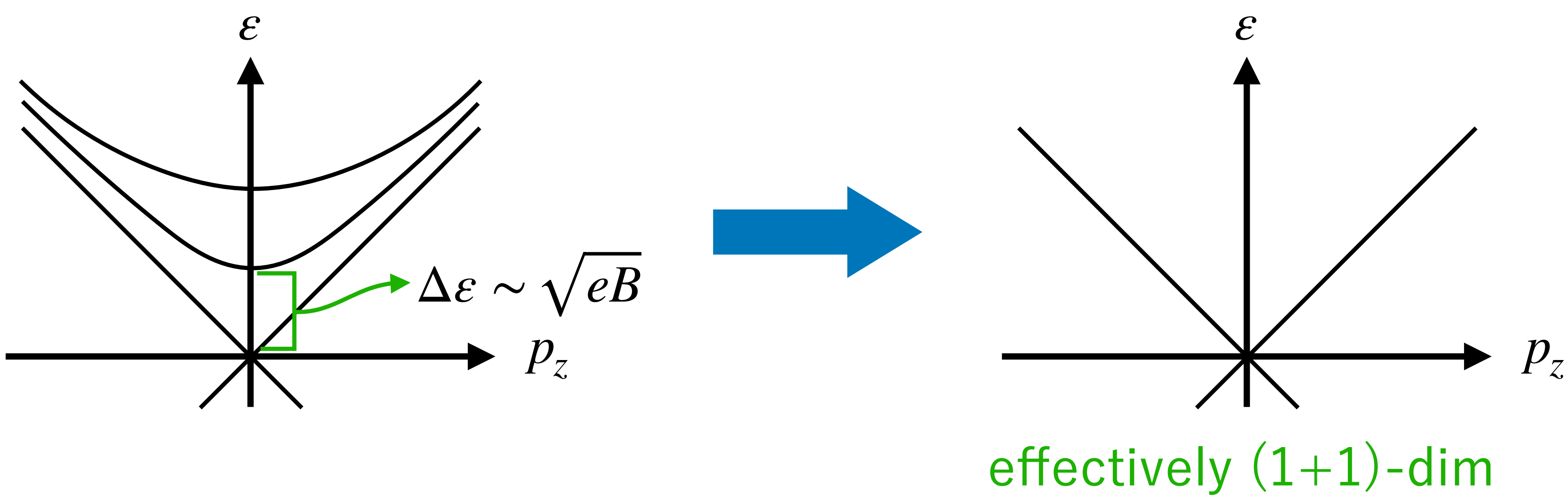
- Chiral kinetic theory is applicable for weak field $(\zeta \ll 1)$

- Lowest Landau level approximation



- Dispersion relation

$$\omega^2 = m_{\mathrm{a}}^2$$

Viscosity and r-mode instability

- Shear viscosity : $\eta = \frac{1}{5} n_e \mu_e \tau$

$$\frac{\tau_c}{\tau_a} \sim 1 \left(\frac{T}{10^6\ \mathrm{K}}\right)^{-2/3} \left(\frac{B}{10^{17}\ \mathrm{G}}\right)^{2} \left(\frac{\mu_e}{10^2\ \mathrm{MeV}}\right)^{-10/3}$$

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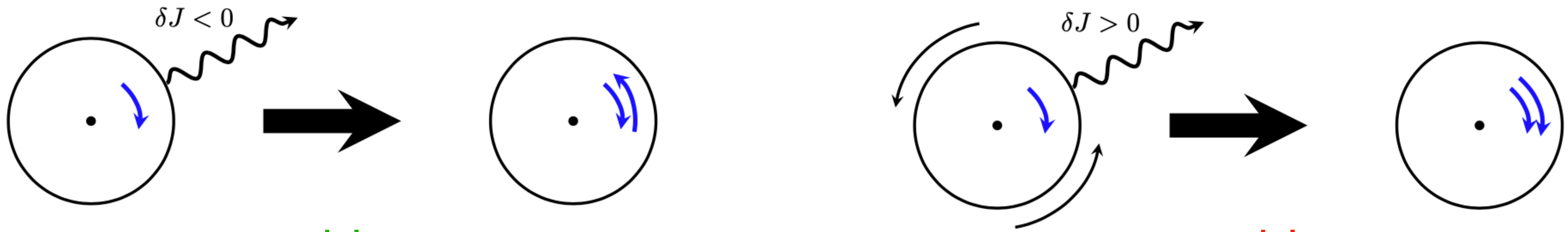
- Chandrasekhar-Friedman-Schutz (CFS) instability

Chandrasekhar (1970); Friedman, Schutz (1978)



stable unstable

- r-mode is unstable due to CFS instability (r-mode instability)

Andersson (1998)

Anomalous dynamical screening would enhance the r-mode instability