Evgeny Epelbaum, Ruhr University Bochum

Workshop on recent developments from QCD to nuclear matter,
Institute of Physics, Academia Sinica, 17-20 Dec. 2025

Chiral EFT for Nuclear Interactions

Review & some new developments

- Which methods are used in *ab initio* calculations (and what are their limitations)?
- Where do we stand with understanding nuclear interactions from chiral EFT?
- What is the role of chiral symmetry?

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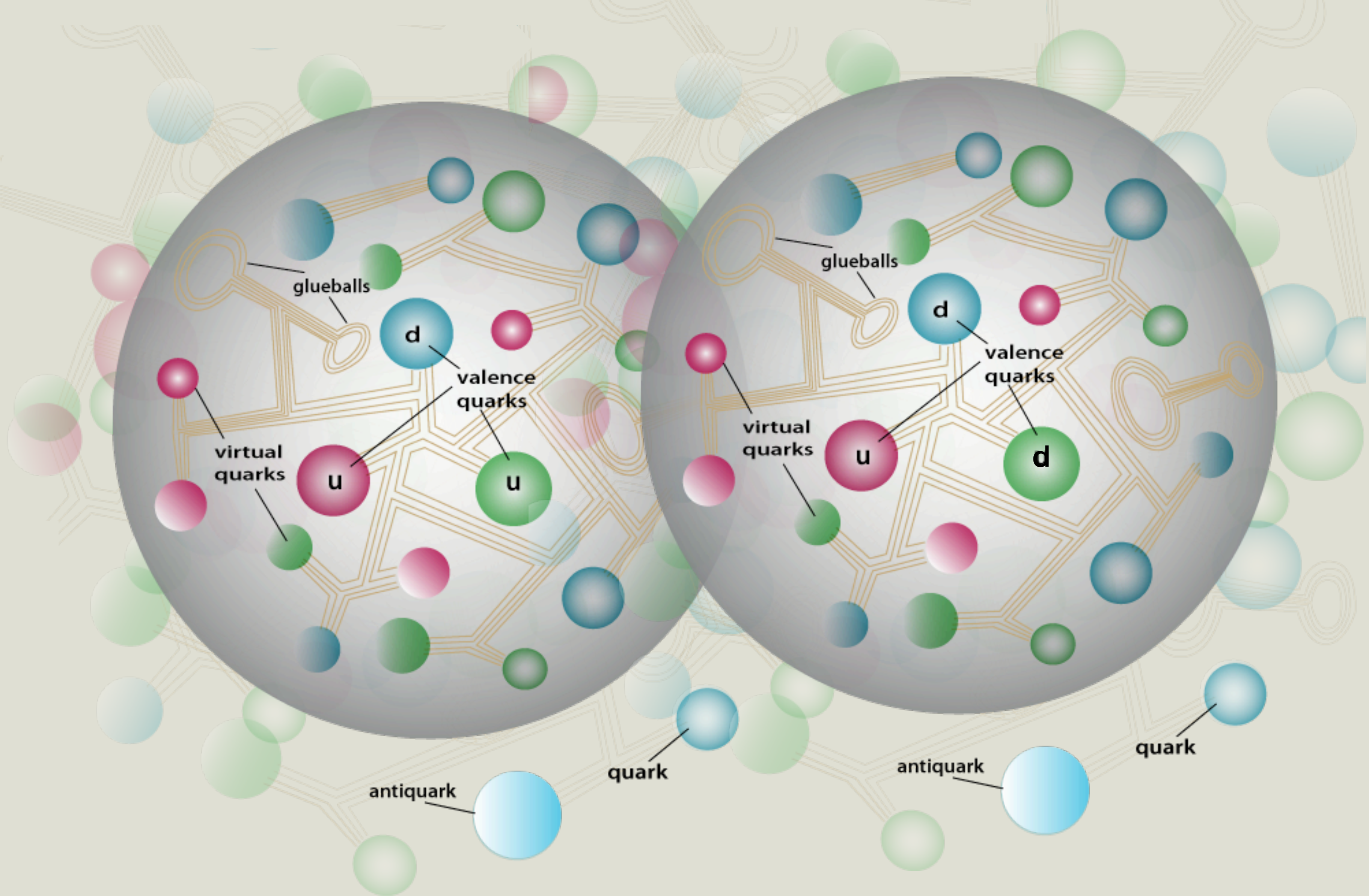
TOMOE

Deuteron as a bound state of quarks and gluons



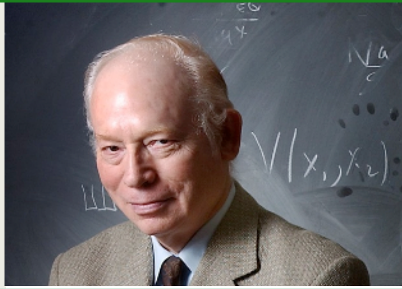
Is there a way to simplify the picture (without losing connection to QCD)?

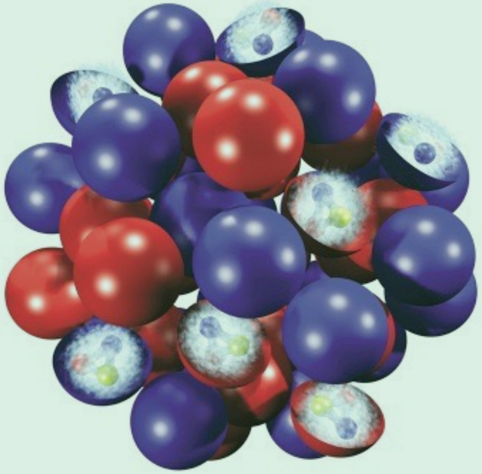
Degrees of freedom

Weinberg's 3rd law of progress in theoretical physics:

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry…

in Asymptotic Realms of Physics, MIT Press, Cambridge, 1983





Typical momenta of nucleons in nuclei:

$$\langle \Psi | \hat{p} | \Psi \rangle \sim 50 - 300 \text{ MeV}$$

Fermi-momentum at the saturation density:

$$p_F = (3/2\pi^2 \rho)^{1/3} \sim 270 \text{ MeV}$$

⇒ non-relativistic description in the framework of the A-body Schrödinger equation:

$$\left[\left(\sum_{i=1}^{N} \frac{-\vec{\nabla}^2}{2m} + \mathcal{O}(m^{-3}) \right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived in ChPT}} \right] |\Psi\rangle = E|\Psi\rangle$$

Two main challenges:

- **solution of the quantum mechanical A-body problem** (*ab-initio* methods)
- **derivation/construction of nuclear interactions** (chiral EFT)

⇒ non-relativistic description in the framework of the A-body Schrödinger equation:

$$\left[\left(\sum_{i=1}^{N} \frac{-\vec{\nabla}^2}{2m} + \mathcal{O}(m^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \dots}_{\text{derived in ChPT}}\right] |\Psi\rangle = E|\Psi\rangle$$

The QM A-body problem

2N: Rewrite to the integral **Lippmann-Schwinger eq.**: $t = V_{2N} + V_{2N}G_0 t$ — easy to solve in p-space.

3N: **Faddeev equations,** e.g. for elastic Nd scattering:

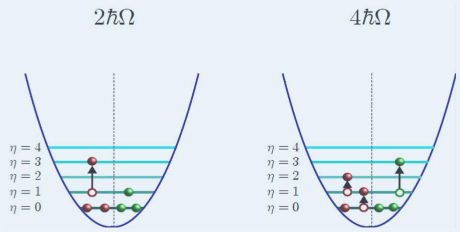
$$\underbrace{T\phi}_{\text{asymptotic Nd state}} = tP\phi + (1+tG_0)\underbrace{V_{3N}^{(1)}}_{\text{symmetric under exchange of nucleons 2,3}}(1+P)\phi + tPG_0T\phi + (1+tG_0)V^{(1)}(1+\underbrace{P}_{P_{12}P_{23}+P_{13}P_{23}})G_0T\phi$$

Solved iteratively in partial-waves (for fixed J, $T \sim 10^5 \times 10^5$), few minutes on 1 CPU.

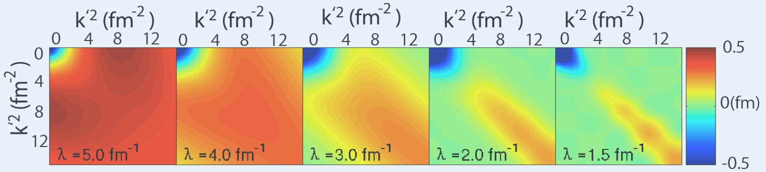
4N: **Yakubovsky equations.** Take several hours on JURECA@FZJ to solve for bound state; only very few groups can do scattering (with large restrictions)

>4N: So far, mainly for bound states. E.g., **the No-Core-Shell-Model**:

$$H|\Psi_i\rangle = E_i|\Psi_i\rangle, \quad |\Psi_i\rangle = \sum_{n=0}^{\infty} A_n^i|\Phi_n\rangle, \quad |\Phi_n\rangle = \underbrace{[a_\alpha^\dagger \ldots a_\zeta^\dagger]_n|0\rangle}_{n\ =\ 1,2,\ldots,10^{12}\text{ or more}}$$



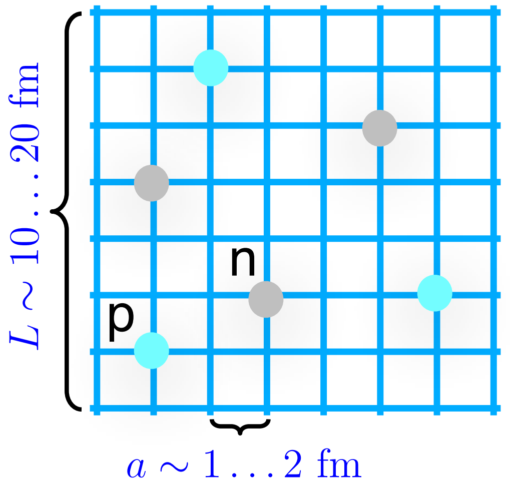
⇒ sparse matrix $H_{mn} = \langle \Phi_m | H | \Phi_n \rangle$ diagonalization (Lánczos), extrapolation in N$_{\text{max}}$, pre-diagonalization of H (SRG):

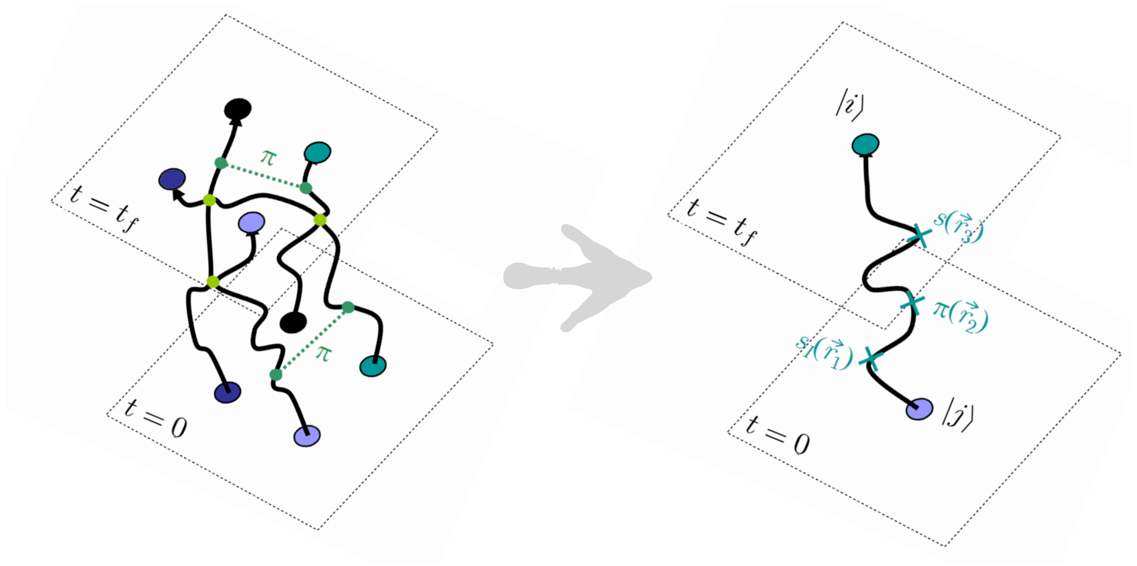


$$H(s) = U(s)HU^\dagger(s) \Rightarrow \frac{dH_s}{ds} = \underbrace{[\eta_s, H_s]}_{\text{e.g., } \eta_s = [T_{\text{kin}}, H_s]}, \quad H_{s\to 0} = H$$

Other ab-initio methods: **QMC, CC expansion, IM-SRG, Lorentz IT, Green's functions, NL-EFT, …**

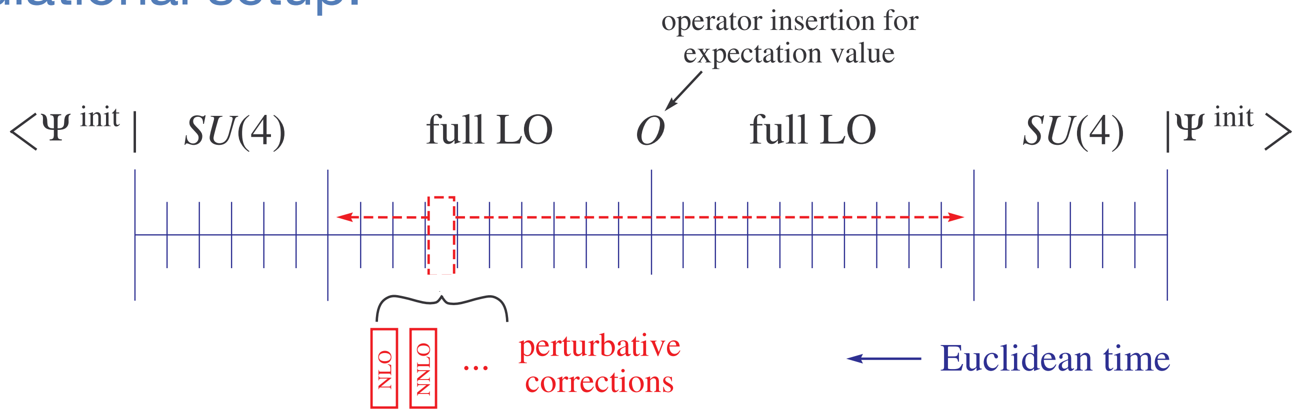
Nuclear lattice simulations with Dean Lee, Ulf-G. Meißner, et al.

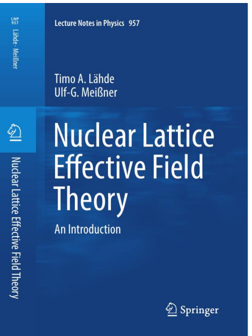




Auxiliary-field Quantum Monte-Carlo calculations

Calculational setup:




Lecture Notes in Physics 957

Timo A. Lähde
Ulf-G. Meißner

Nuclear Lattice
Effective Field
Theory

An Introduction

Springer

nature International weekly journal of science

Wavefunction matching for solving quantum many-body problems

https://doi.org/10.1038/s41586-024-07422-z

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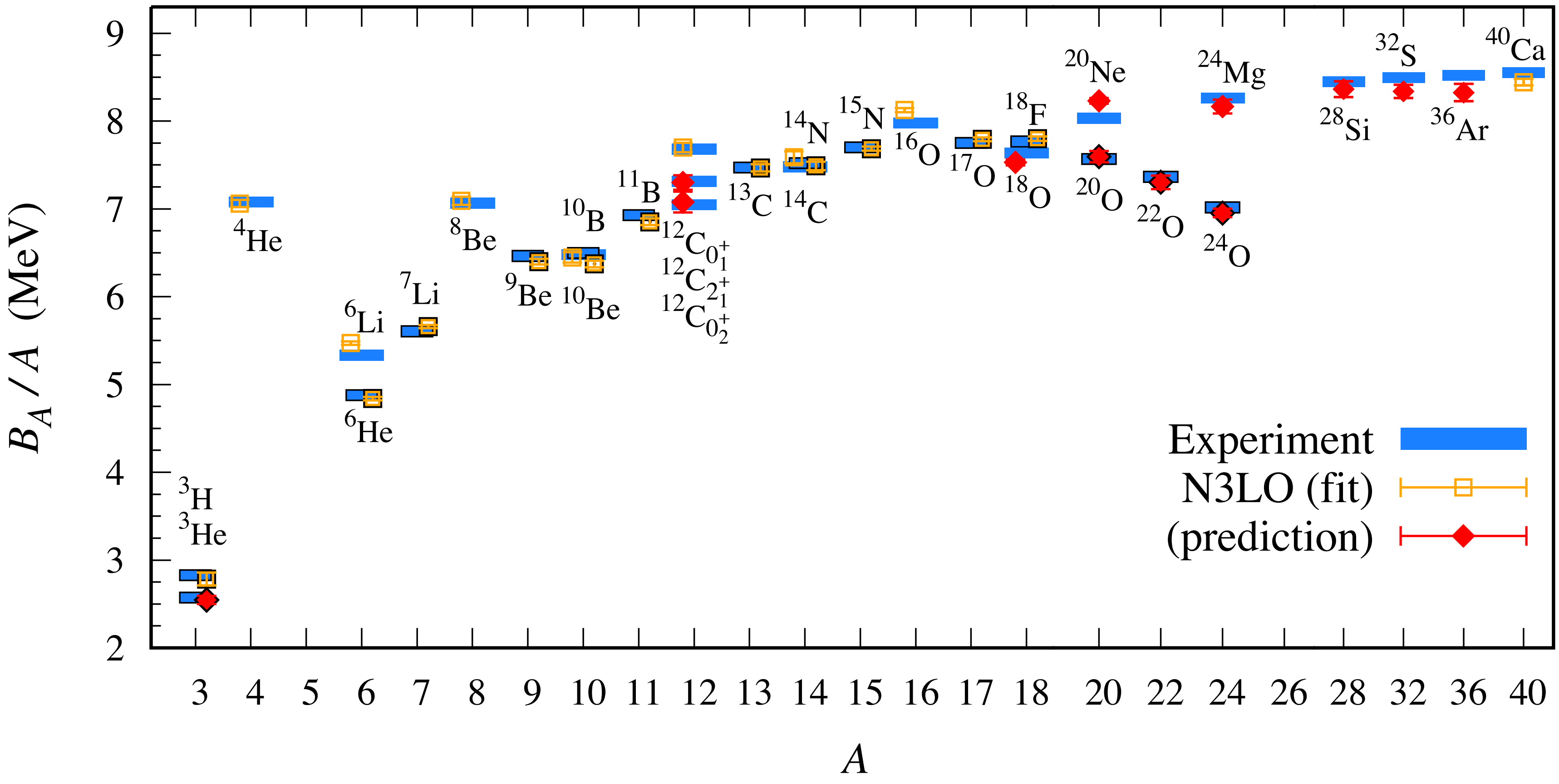
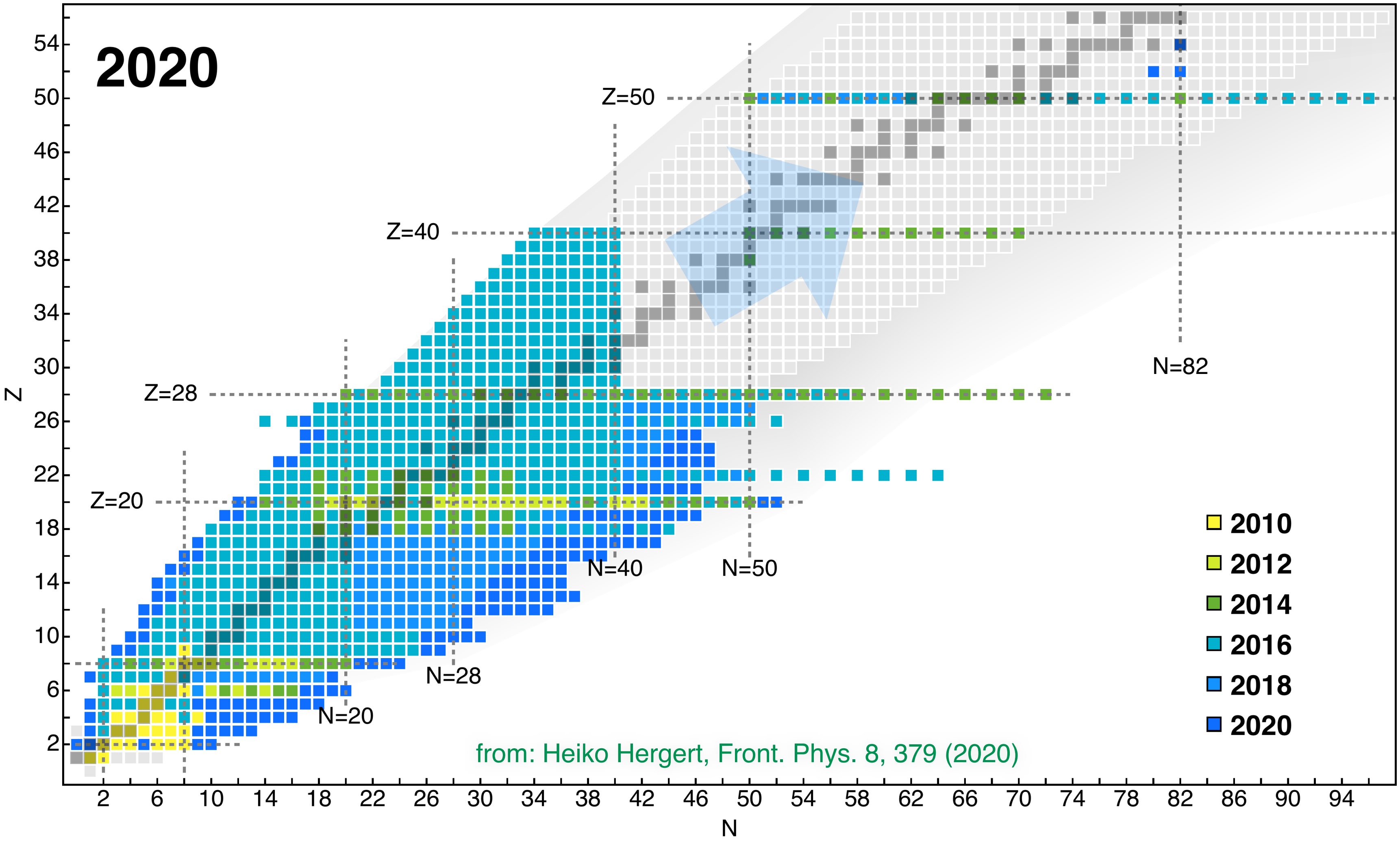
Serdar Elhatisari[1,2], Lukas Bovermann[3], Yuan-Zhuo Ma[4,5], Evgeny Epelbaum[3], Dillon Frame[6,7], Fabian Hildenbrand[6,7], Myungkuk Kim[8], Youngman Kim[8], Hermann Krebs[3], Timo A. Lähde[6,7], Dean Lee[4]✉, Ning Li[9], Bing-Nan Lu[10], Ulf-G. Meißner[2,6,7,11], Gautam Rupak[12], Shihang Shen[6,7], Young-Ho Song[13] & Gianluca Stellin[14]



Ab initio many-body calculations



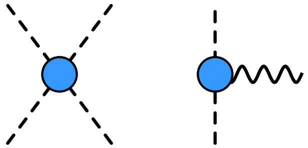
from: Heiko Hergert, Front. Phys. 8, 379 (2020)

The main bottleneck in developing nuclear physics into precision and predictive science is the accuracy of the interaction (especially of 3N forces).

Chiral EFT in a nutshell

GB dynamics

Weinberg, Gasser, Leutwyler, …

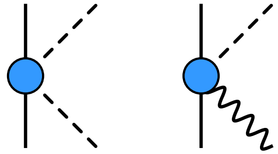


Chiral Perturbation Theory

$$Q = \frac{\text{momenta of particles or } M_\pi}{\text{breakdown scale } \Lambda_b} \sim \frac{1}{4} \dots \frac{1}{3}$$

πN dynamics

Bernard-Kaiser-Meißner et al.

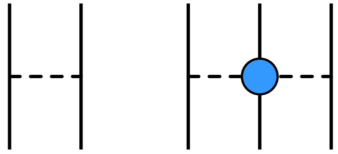


Effective Lagrangian (HB):

$$\begin{aligned}
\mathcal{L}_\pi &= \frac{F^2}{4}\text{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) \;+\; \dots\,, \\
\mathcal{L}_{\pi N} &= \bar{N}(iv\cdot D + g_A\, u\cdot S)N \;+\; \dots\,, \\
\mathcal{L}_{NN} &= -\frac{1}{2}C_S(\bar{N}N)^2 + 2C_T(\bar{N}SN)^2 \;+\; \dots
\end{aligned}$$

Nuclear forces

Weinberg, van Kolck, Kaiser, EGM, …



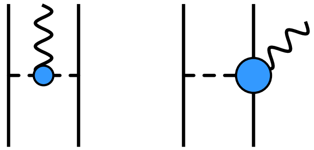
Enhanced ladder graphs are re-summed by solving the many-body Schrödinger equation

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+V_{2N}+V_{3N}+V_{4N}+\dots\right]|\Psi\rangle = E|\Psi\rangle$$

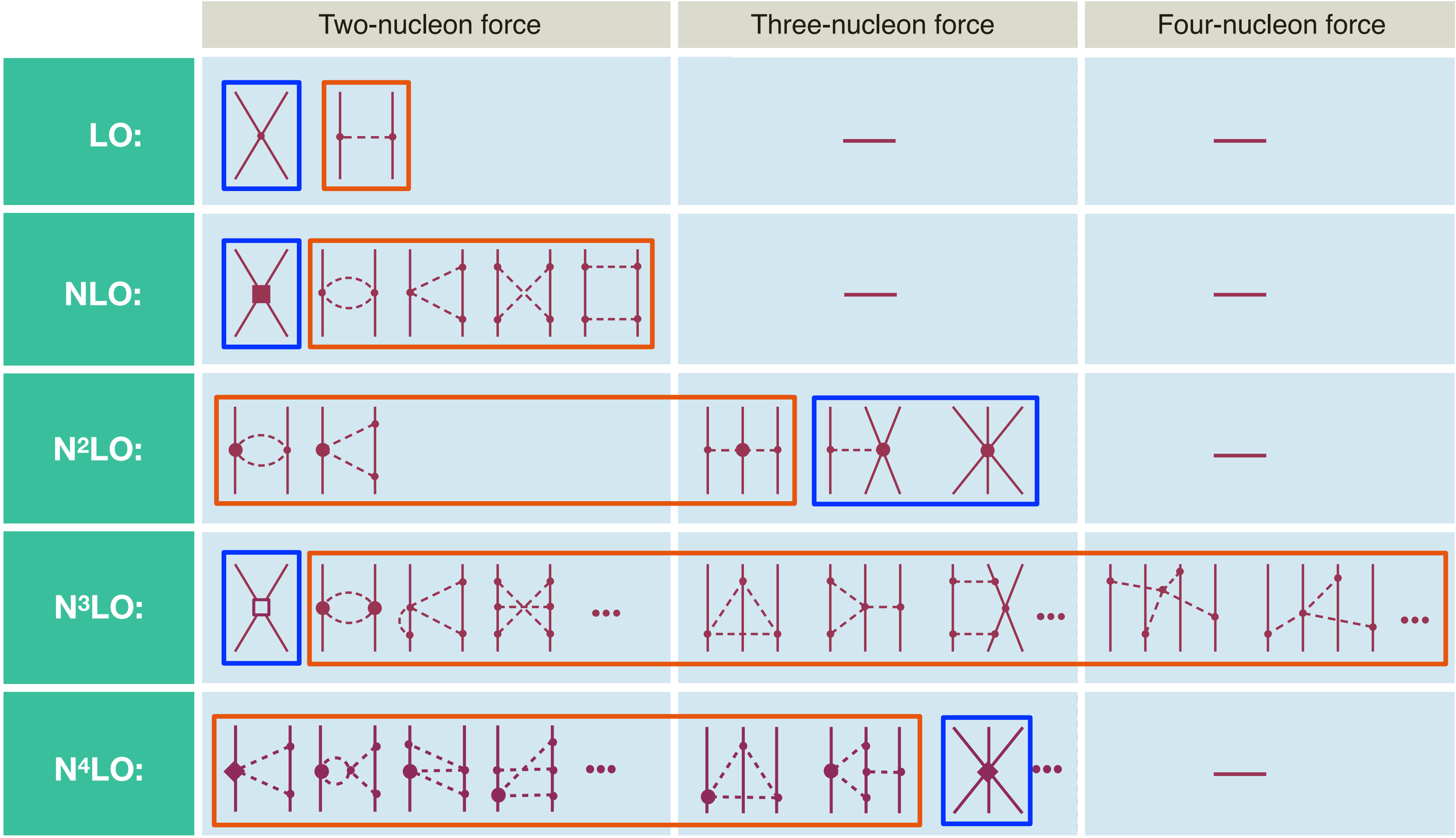
Need to introduce a (finite) cutoff

Nuclear currents

Park et al, Bochum-Bonn, JLab-Pisa



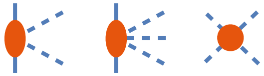
Chiral expansion of nuclear forces



	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO:		—	—
NLO:		—	—
N²LO:			—
N³LO:	•••	•••	•••
N⁴LO:	•••	•••	—

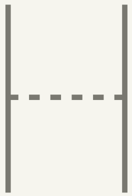
Short-range few-N interactions are tuned to experimental data

Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes



Predictive power

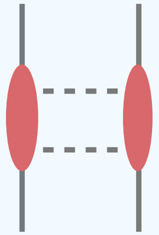
Chiral symmetry + πN data = predictions for the large-distance behavior of the nuclear forces.



$$\left.\begin{aligned} \mathscr{L}_{pv} &= -\frac{g}{2m_N}\bar{N}\gamma^5\gamma^\mu\tau N\cdot\partial_\mu\pi \\ \cancel{\mathscr{L}_{ps}} & \cancel{= -ig\bar{N}\gamma^5\tau N\cdot\pi} \end{aligned}\right\} \Rightarrow$$

the same OPEP (on-shell)

i.e., ***not*** constrained by χ symmetry…



⟵ strongly constrained by χ symmetry:

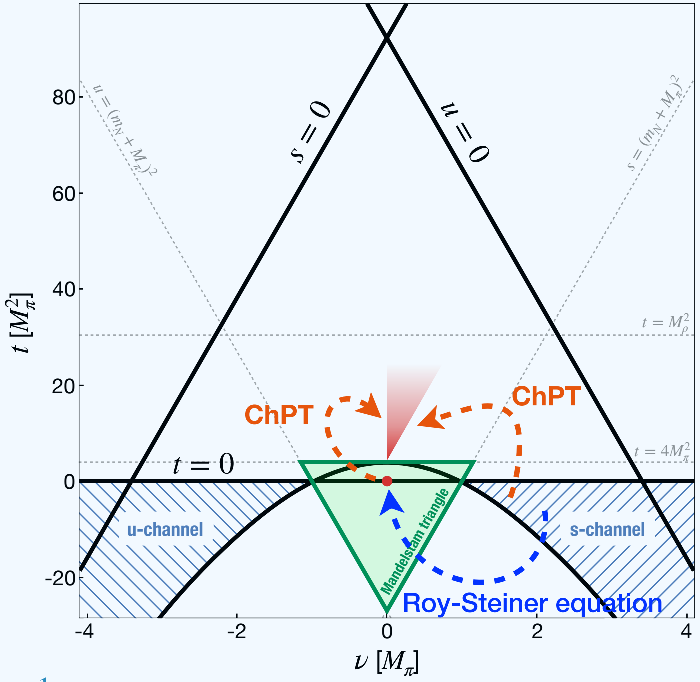
$\mathscr{L}_{ps}$ vs. $\mathscr{L}_{pv}$ matters, also $\pi\pi$, $\pi\pi N$, etc. interactions play a role…

Dispersive representation:

$$V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty}\mu d\mu\,\frac{\rho(\mu)}{q^2+\mu^2} + \ \ldots$$

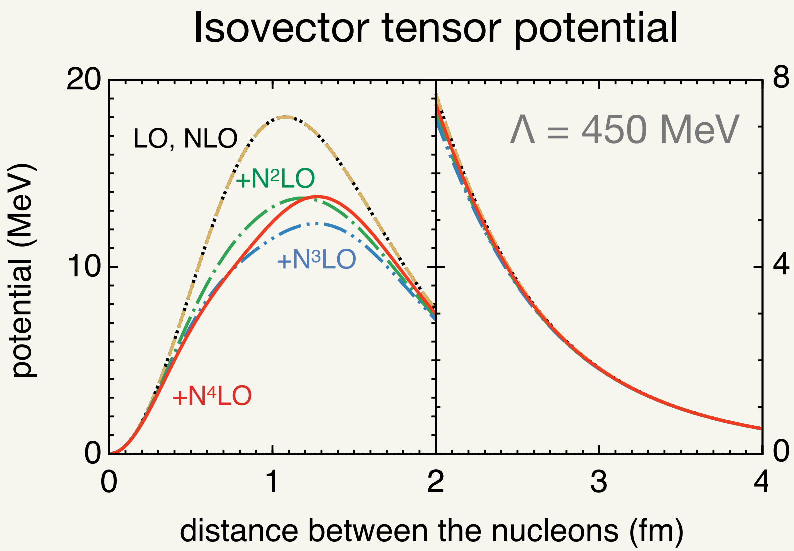
$\rho(\mu)$ can be extracted from (analytically continued) $T_{\pi N}(s,t)$ obtained in ChPT (need data to fix LECs)

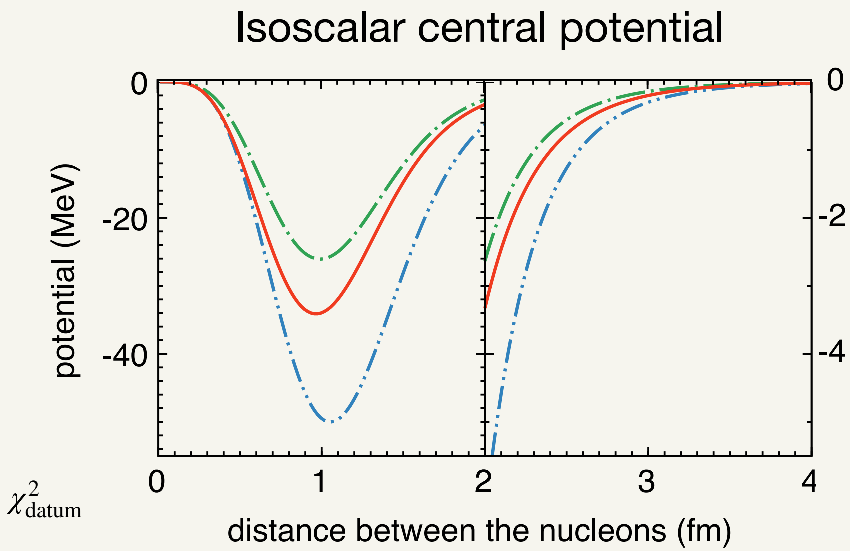
Mandelstam plane for πN scattering



⇒ parameter-free predictions for $V_{2\pi}(r)$ at $r \gtrsim M_\pi^{-1}$

Chiral expansion of the long-range NN force



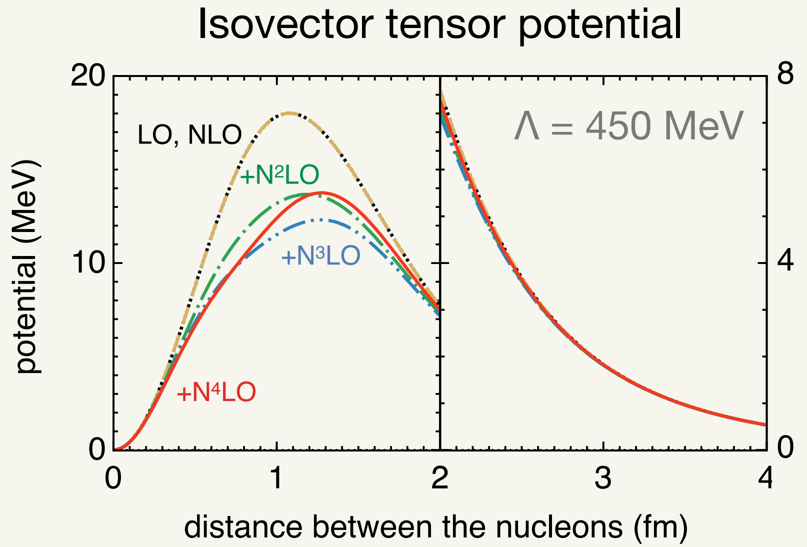


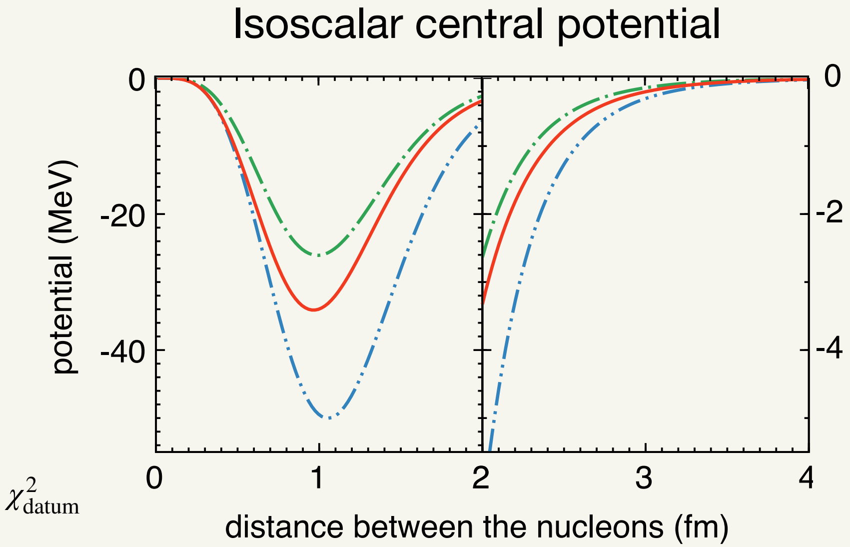
SMS NN potentials Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2} \; e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}} + \text{subtraction}, \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\,\frac{\rho(\mu)}{\vec{q}^{\,2} + \mu^2} \; e^{-\frac{\vec{q}^{\,2} + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts

Chiral expansion of the long-range NN force





Can these predictions be tested in NN scattering?

$\chi^2_{\rm datum}$ for the description of neutron-proton and proton-proton scattering data

$E_{\rm lab}$ bin	CD Bonn	Nijm I	Nijm II	Reid 93	Bochum N^{4}LO$^+$
0-300 MeV	1.042	1.061	1.070	1.078	1.013

Switching off the 2π-exchange leads to $\chi^2_{\rm datum} \sim 1.99$ (i.e., 50σ away!), consistent fits only for $E_{\rm lab} \leq 135$ MeV…

The landscape of chiral NN interactions

Different regularizations (cutoff choices)

- — fully nonlocal Entem, Machleidt, Nosyk 2017 **(Idaho)**; Ekström et al. 2013-18 **(GO)**: $\mathrm{NNLO_{opt}}$, $\mathrm{NNLO_{sat}}$, NNLO-Δ
- — semi-local EE, Krebs, Meißner 2015; Reinert, EE, Krebs 2018 **(LENPIC)**
- — local Gezerlis et al. 2013; Piarulli et al., 2016 **(Norfolk models)**; Saha, Entem, Machleidt, Nosyk 2023
- — local, nonlocal + lattice Lee, Elhatisari, EE, Lähde, Meißner, Krebs et al. **(Nuclear Lattice EFT)**

Highest available EFT order

- — $\mathrm{N^4LO^+}$: Low-Energy Nuclear Physics International Collaboration (LENPIC), Idaho
- — $\mathrm{N^3LO}$: Norfolk, NLEFT
- — $\mathrm{N^2LO}$: Gothenburg-Oak Ridge (GO)

the only chiral EFT interactions that provide a statistically perfect description of NN data below π-production threshold

Degrees of freedom in the effective Lagrangian

- — π, N: LENPIC, Idaho, GO, NLEFT
- — π, N, Δ: Norfolk, GO

Strategy in the determination of LECs

- — πN from the Roy-Steiner analysis (not fitted), NN LECs from two-nucleon data
  LENPIC, Idaho, Norfolk, NLEFT
- — LECs determined from a global fit to πN, NN, nuclei, EoS GO

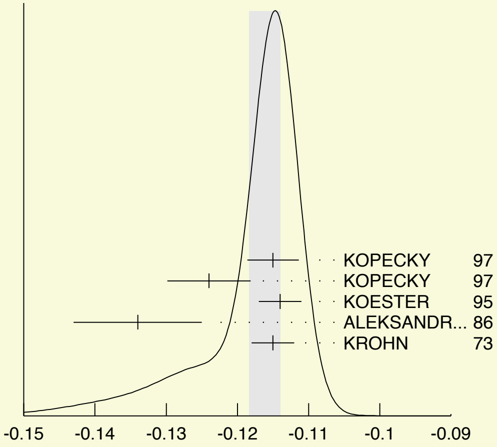
Application: How big is a neutron?

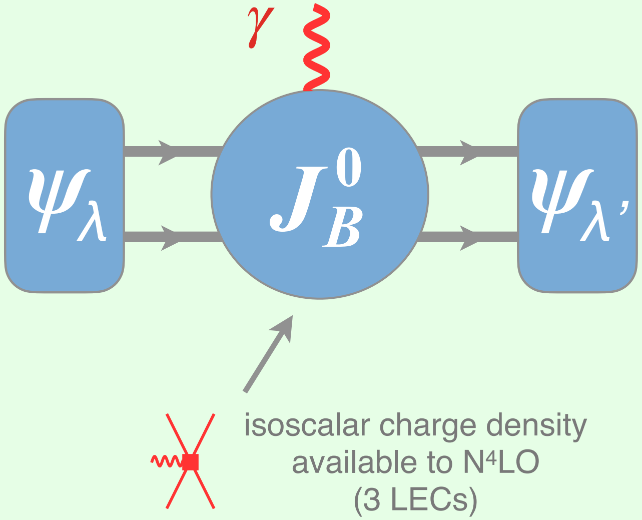
The proton radius puzzle settled. What about the neutron radius?

— no neutron targets; extrapolations of $G_C^n(Q^2)$ extracted from ^{2}H not reliable…

— the only information comes from (fairly old) n-scattering experiments on Pb, Bi, …

⟶ PDG value: $r_n^2 = -0.1161 \pm 0.0022$ fm^2





isoscalar charge density available to N^{4}LO (3 LECs)

Idea: accurate calculation of the ^{2}H structure radius, which incorporates all nuclear effects

$$r_d^2 = r_{str}^2 + \left(r_p^2 + \frac{3}{4m_p^2}\right) + r_n^2$$

combined with ^{1}H-^{2}H isotope shifts data

$$r_d^2 - r_p^2 = 3.82070(31)\ \text{fm}^2$$

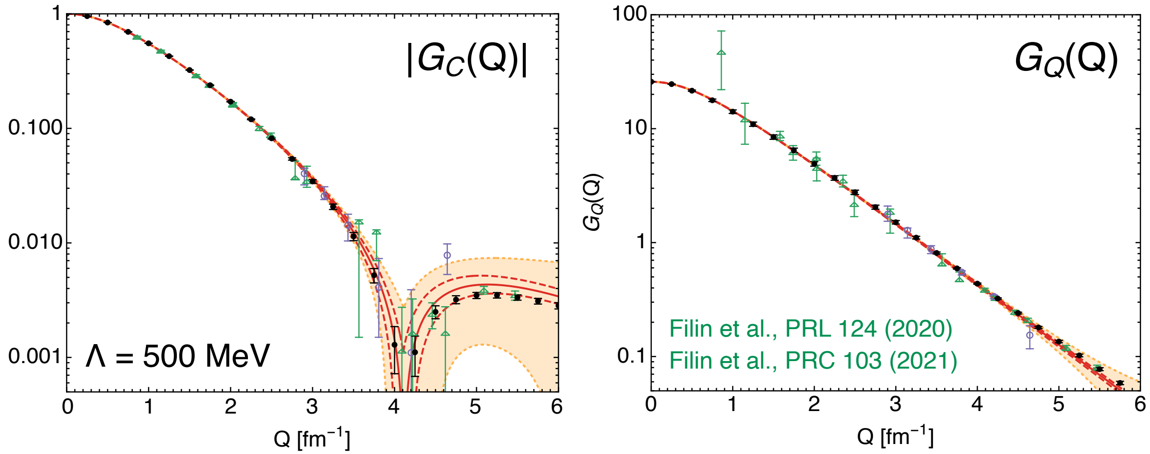
Jentschura et al. ’11; Pachucki et al. ’18

can be used to extract r_n^2 !

Deuteron charge and quadrupole FFs

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

The charge and quadrupole form factors of the deuteron at N^{4}LO



The extracted structure radius and quadrupole moment:

$$r_{\rm str} = 1.9729^{+0.0015}_{-0.0012}\ \text{fm}$$

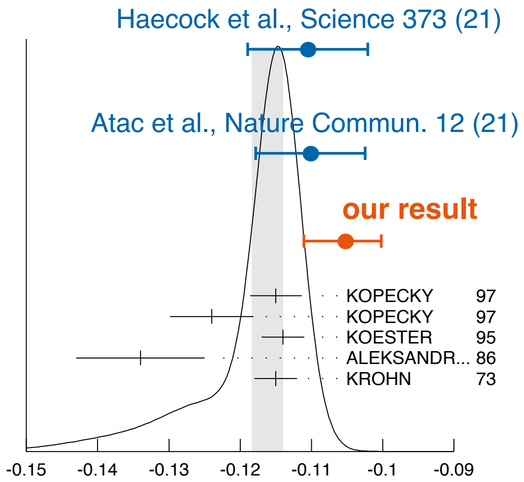
$$Q_{\rm d} = 0.2854^{+0.0038}_{-0.0017}\ \text{fm}^2$$

statistical and systematic errors due to the EFT truncation, choice of fitting range and πN LECs

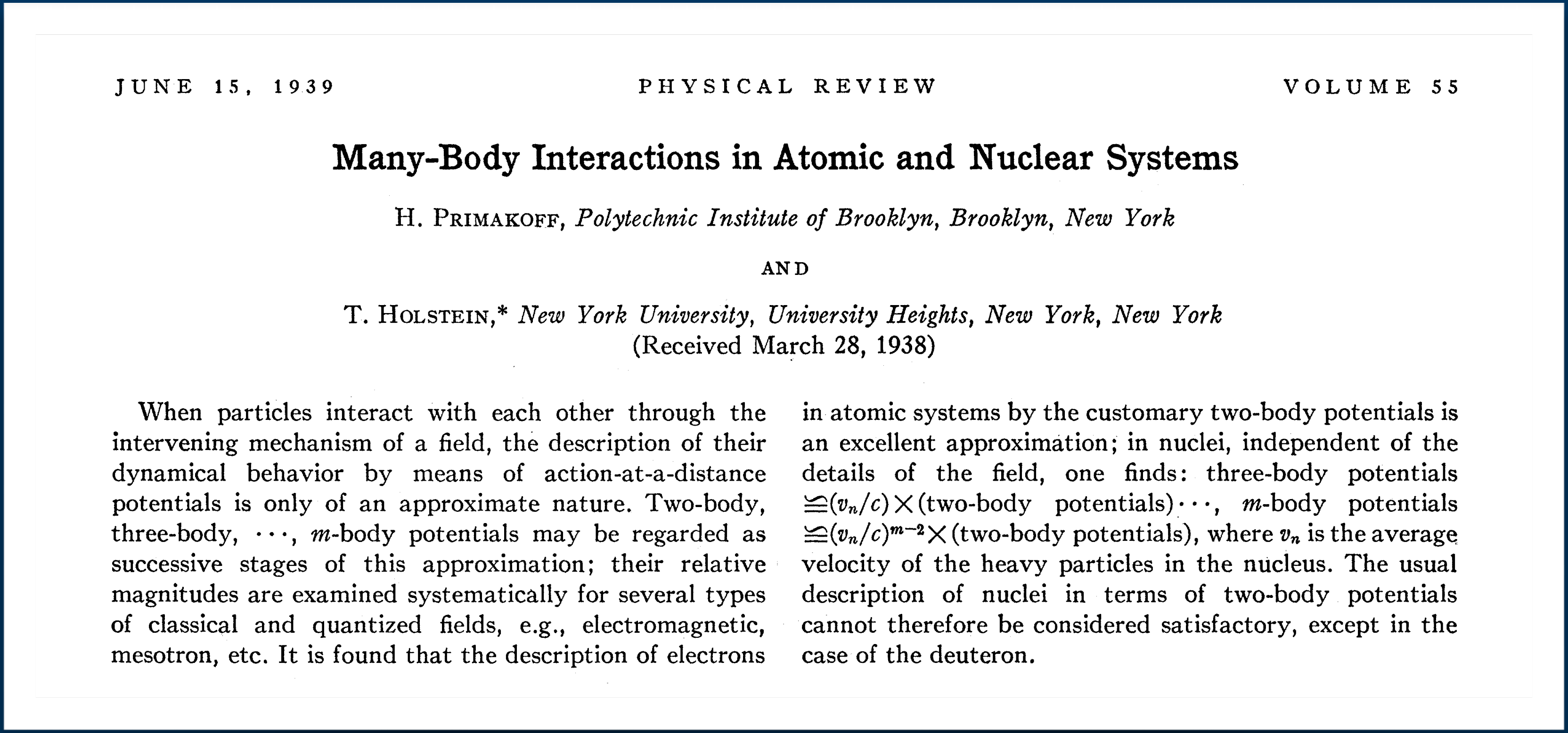
The value of $Q_{\rm d}$ is to be compared with $Q_{\rm d}^{\rm exp} = 0.285\,699(15)(18)\ \text{fm}^2$ Puchalski et al., PRL 125 (2020)

Combining our result for $r_{\rm str}^2 = r_d^2 - r_p^2 - r_n^2 - \dfrac{3}{4m_p^2}$ with the ^{1}H-^{2}H isotope shift datum $r_d^2 - r_p^2 = 3.82070(31)\ \text{fm}^2$ leads to the prediction for the neutron radius:

$$\boxed{r_n^2 = -\,0.105^{+0.005}_{-0.006}\ \text{fm}^2}$$



Three-body forces



JUNE 15, 1939 PHYSICAL REVIEW VOLUME 55

Many-Body Interactions in Atomic and Nuclear Systems

H. Primakoff, *Polytechnic Institute of Brooklyn, Brooklyn, New York*

AND

T. Holstein,* *New York University, University Heights, New York, New York*

(Received March 28, 1938)

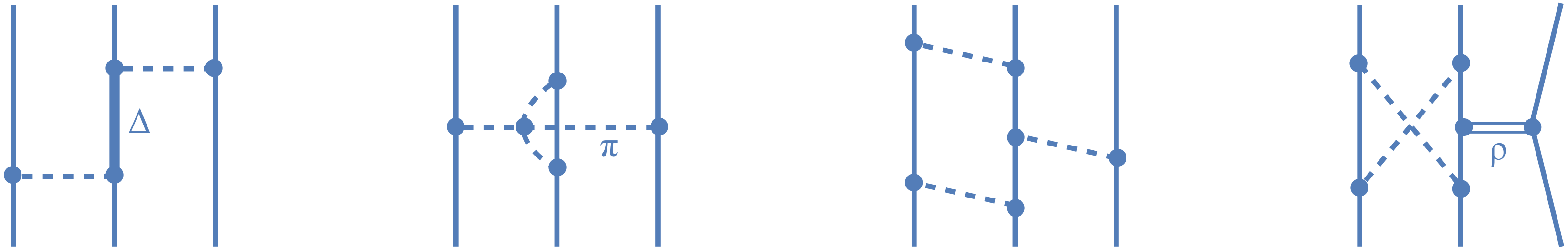
When particles interact with each other through the intervening mechanism of a field, the description of their dynamical behavior by means of action-at-a-distance potentials is only of an approximate nature. Two-body, three-body, $\cdots$, m-body potentials may be regarded as successive stages of this approximation; their relative magnitudes are examined systematically for several types of classical and quantized fields, e.g., electromagnetic, mesotron, etc. It is found that the description of electrons in atomic systems by the customary two-body potentials is an excellent approximation; in nuclei, independent of the details of the field, one finds: three-body potentials $\cong (v_n/c) \times$ (two-body potentials) $\cdots$, m-body potentials $\cong (v_n/c)^{m-2} \times$ (two-body potentials), where v_n is the average velocity of the heavy particles in the nucleus. The usual description of nuclei in terms of two-body potentials cannot therefore be considered satisfactory, except in the case of the deuteron.

3-body force: A frontier in nuclear & atomic physics

Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, EPJA 61 (2025) 9

- Three-nucleon forces (3NF) are small but important corrections to the dominant NN forces

- 3NF mechanisms:



intermediate Δ-excitation

Fujita, Miyazawa ’57

multi-pion interactions

off-shell behavior of the V_{NN}

$V_{\text{ring}} = \mathcal{A}_{3\pi} - V_\pi G_0 V_\pi G_0 V_\pi$

short-range

⇒ 3NF are not directly measurable and depend on the scheme (DoF, off-shell V_{NN}, …)

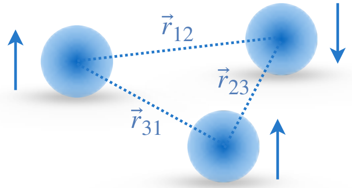
- Still poorly understood (in spite of extensive research) Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rept. Prog. Phys. 75 (12)

- 3NF have extremely rich and complex structure

  — most general ***local*** 3NF: $V_{3N} = \sum_{i=1}^{20} O_i \, f_i(r_{12}, r_{23}, r_{31}) + \text{permutations}$

  EE, Gasparyan, Krebs, Schat ’15

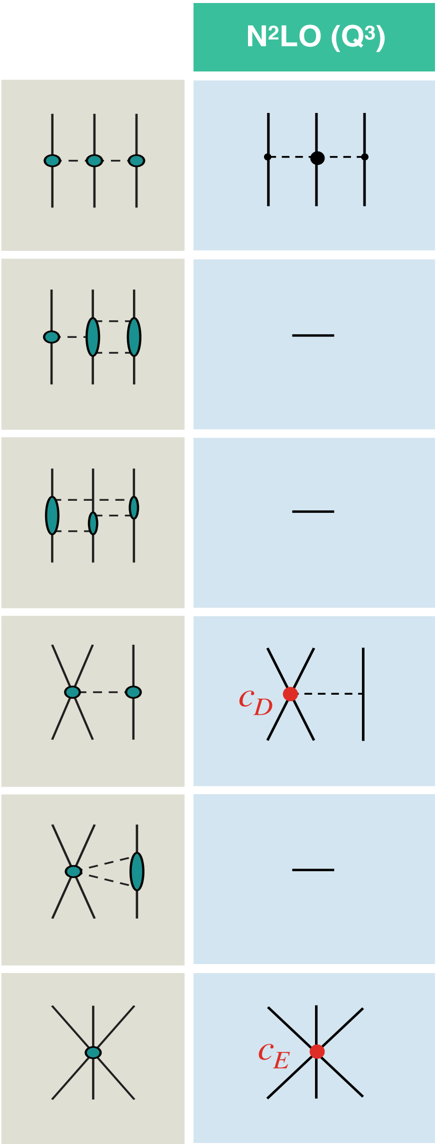
  — most general ***nonlocal*** 3NF: **320 (!)** operators Topolnicki ’17



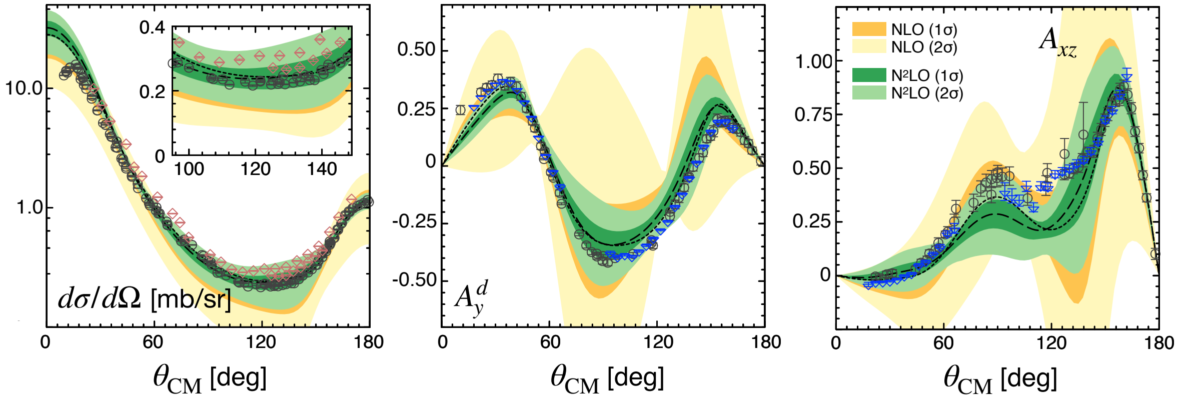
⇒ Guidance from theory indispensable — an opportunity for χEFT!

Three-body force: A frontier in nuclear physics

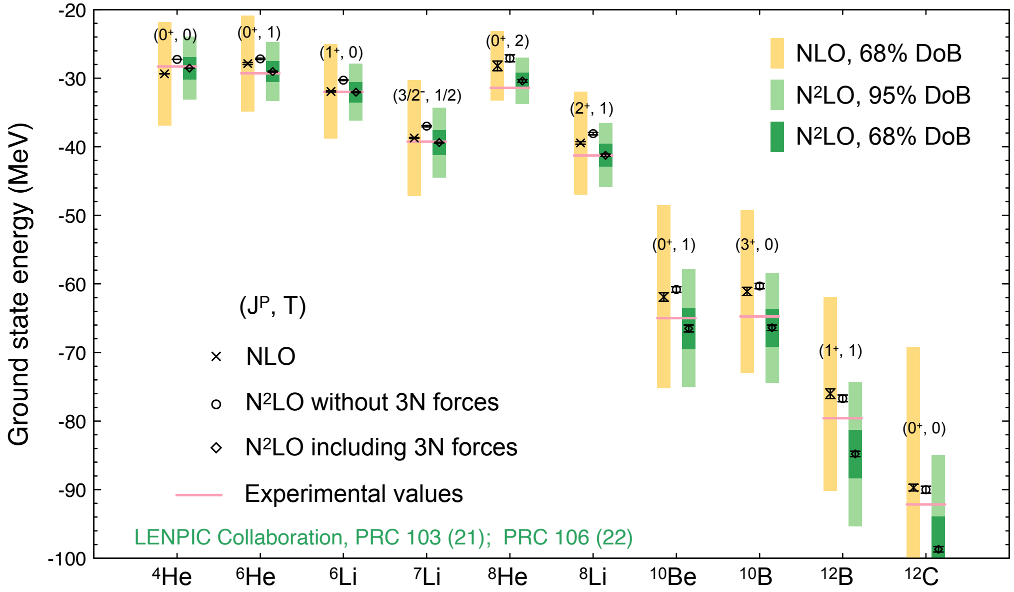
Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, EPJA 61 (2025) 9



Elastic Nd scattering at 135 MeV



Predictions for light p-shell nuclei

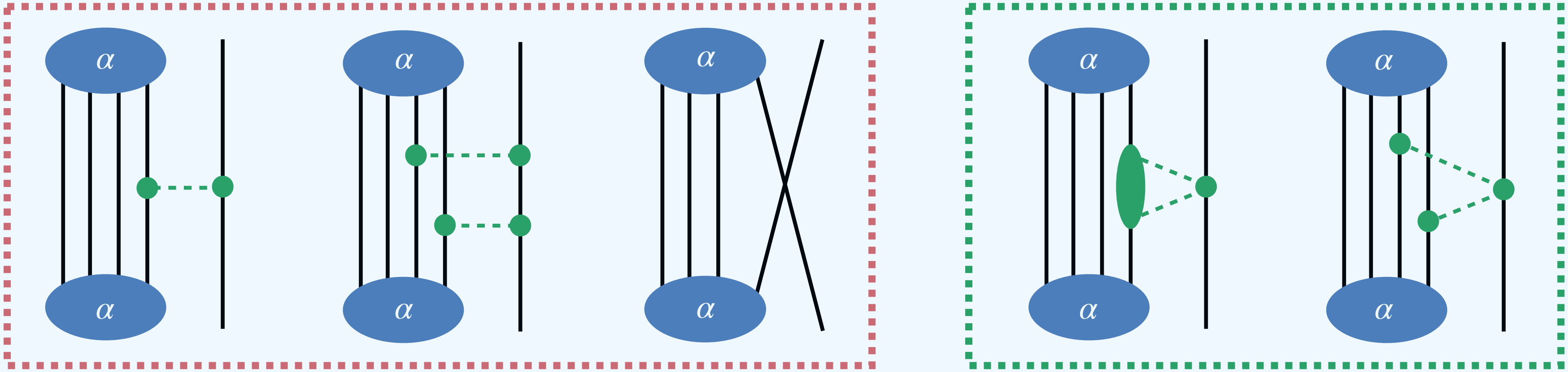


LENPIC Collaboration, PRC 103 (21); PRC 106 (22)

LENPIC

Probing long-range 3NF in peripheral n-α scattering

Yilong Yang, EE, Jie Meng, Lu Meng, Pengwei Zhao, PRL 135 (25) 172502

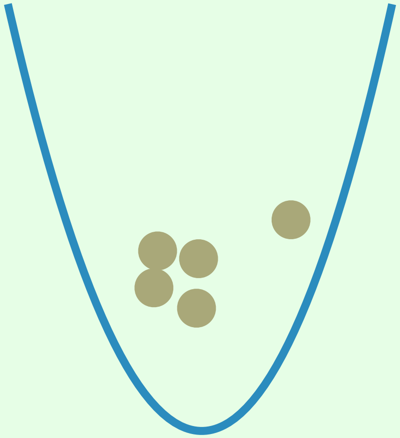


suppressed

allowed

Suggested long ago by Manoel Robilotta,

but so far only S-, P-waves considered Carlson, Nollett, Lynn, Lazauskas, Kravvaris, Shirokov, Zhang, Mercenne, Bagnarol, …



Calculate the $^5\mathrm{He}_l$ GS energy ϵ_l in a harmonic trap (for $l = 2$) using VMC + neural network and apply the Busch formula:

$$k^{2l+1}\cot\delta_l(\epsilon_l) \;=\; (-1)^{l+1}(4\mu\omega)^{l+1/2}\frac{\Gamma\big[(3+2l)/4-\epsilon_l/(2\omega)\big]}{\Gamma\big[(1-2l)/4-\epsilon_l/(2\omega)\big]}$$

Busch et al., Found. Physical. 28 (2008) 549;
Suzuki et al., PRA 80 (2009) 033601

Probing χ symmetry in peripheral n-α scattering

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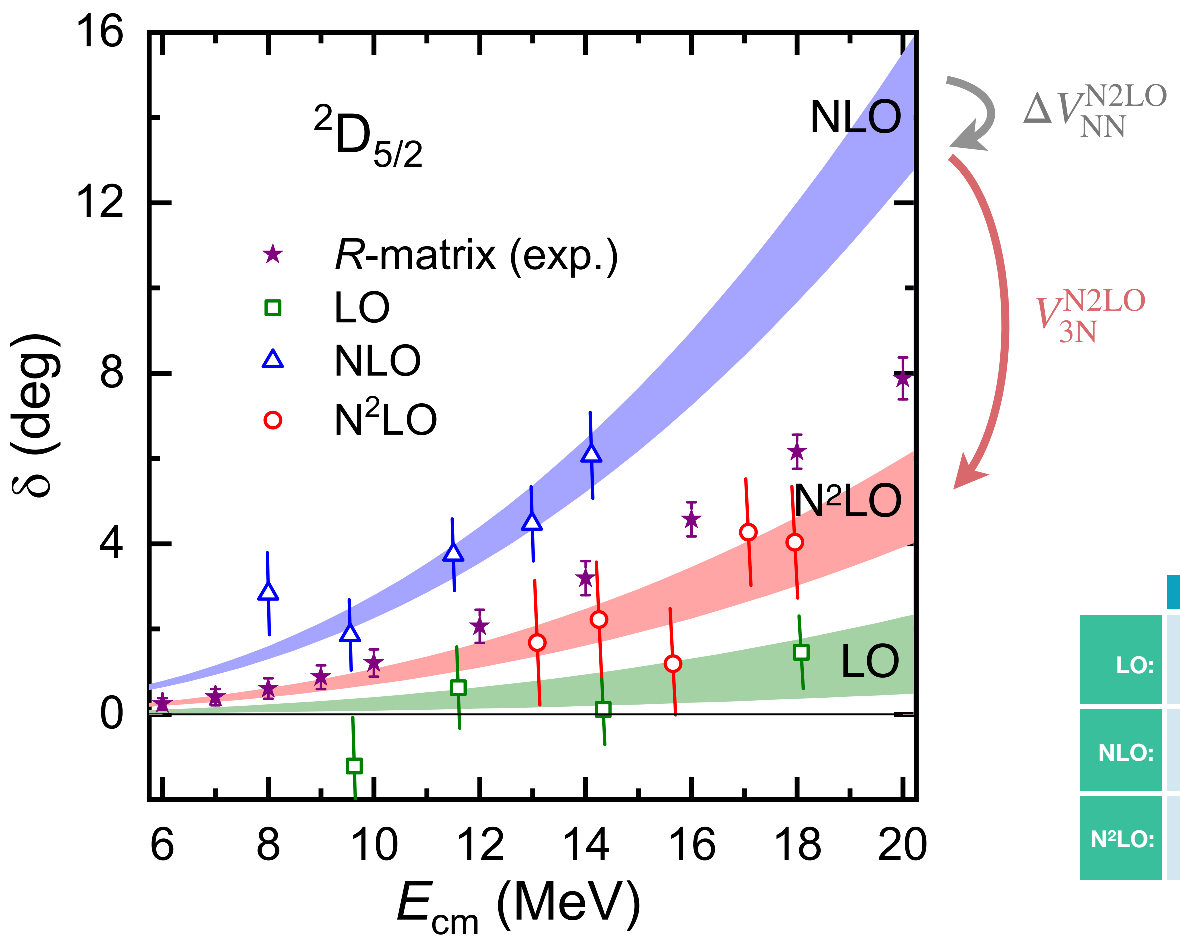
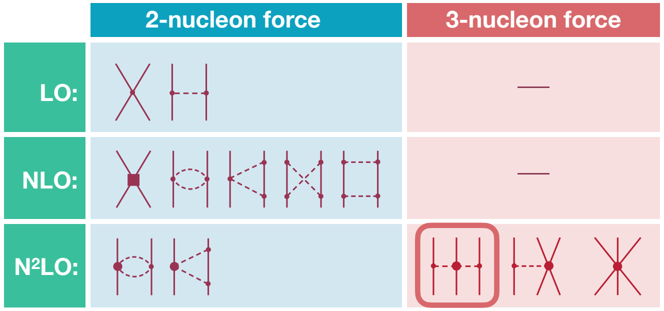


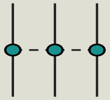
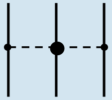
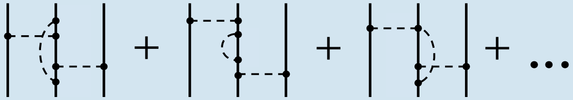
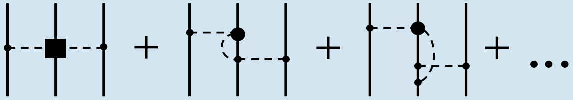
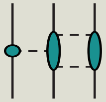
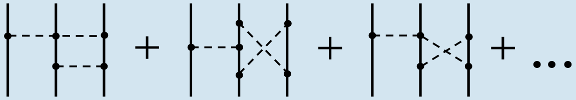
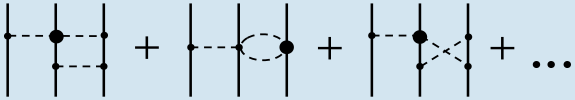
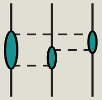
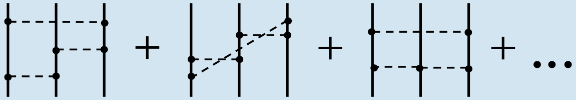
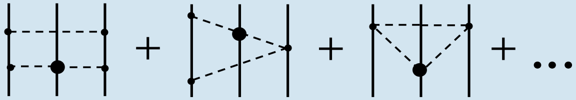
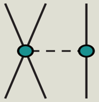
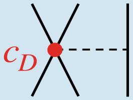
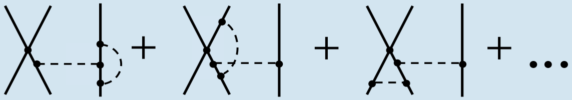
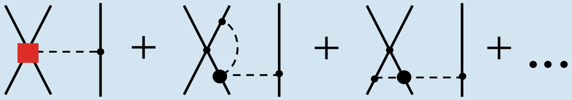
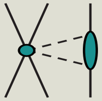
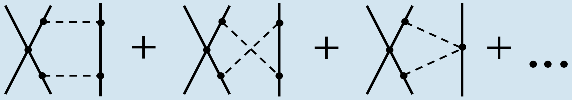
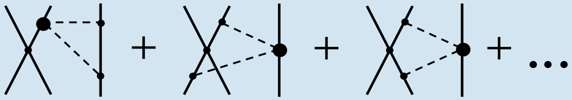
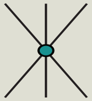
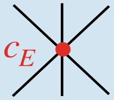
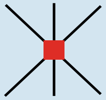


	2-nucleon force	3-nucleon force
LO:		—
NLO:		—
N²LO:		

Peripheral $n\alpha$-scattering provides a sensitive probe of the long-range 3NF (governed by the chiral symmetry)

Three-body force: A frontier in nuclear physics

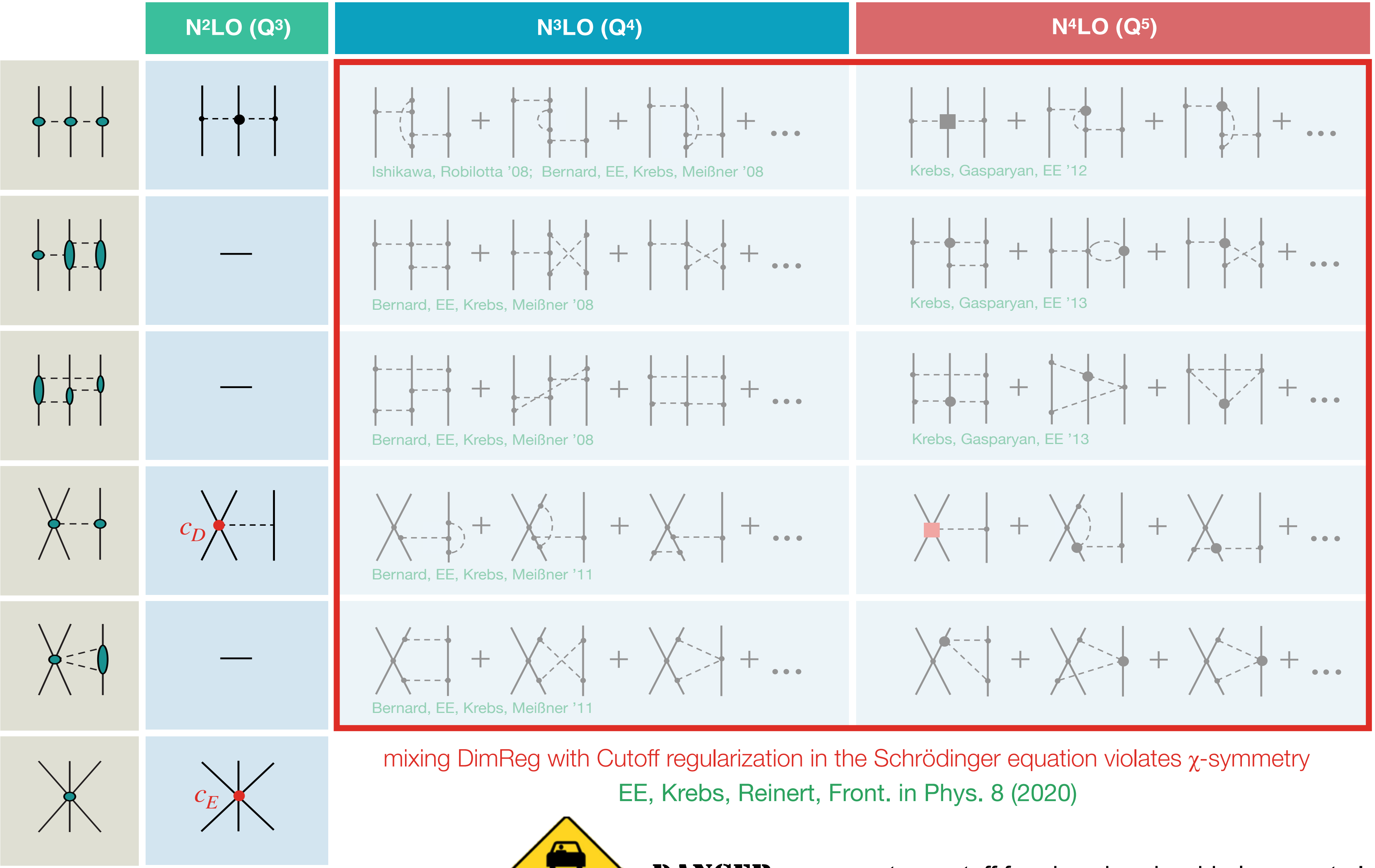
Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, EPJA 61 (2025) 9

	N²LO (Q³)	N³LO (Q⁴)	N⁴LO (Q⁵)
		 Ishikawa, Robilotta ’08; Bernard, EE, Krebs, Meißner ’08	 Krebs, Gasparyan, EE ’12
	—	 Bernard, EE, Krebs, Meißner ’08	 Krebs, Gasparyan, EE ’13
	—	 Bernard, EE, Krebs, Meißner ’08	 Krebs, Gasparyan, EE ’13
		 Bernard, EE, Krebs, Meißner ’11	
	—	 Bernard, EE, Krebs, Meißner ’11	
		—	 **13 LECs** Girlanda, Kievski, Viviani ’11

New experiment @RIKEN will measure spin-transfer coefficients in elastic d-p scattering Sekiguchi et al.

Three-body force: A frontier in nuclear physics

Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, EPJA 61 (2025) 9



Ishikawa, Robilotta ’08; Bernard, EE, Krebs, Meißner ’08

Krebs, Gasparyan, EE ’12

Bernard, EE, Krebs, Meißner ’08

Krebs, Gasparyan, EE ’13

Bernard, EE, Krebs, Meißner ’08

Krebs, Gasparyan, EE ’13

Bernard, EE, Krebs, Meißner ’11

Bernard, EE, Krebs, Meißner ’11

mixing DimReg with Cutoff regularization in the Schrödinger equation violates χ-symmetry

EE, Krebs, Reinert, Front. in Phys. 8 (2020)

DANGER: momentum cutoff for pions breaks chiral symmetry!

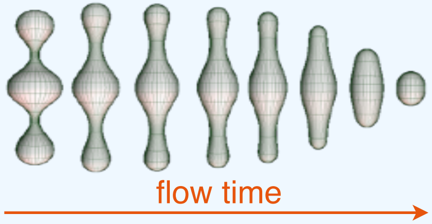
D. B. Kaplan ~ INT ~ 4/19/16

Chiral gradient flow

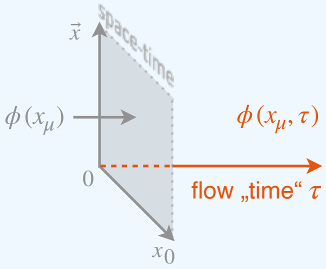
Hermann Krebs, EE, PRC 110 (2024) 044003; PRC110 (2024) 044003

Gradient flows: methods for smoothing manifolds

(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory

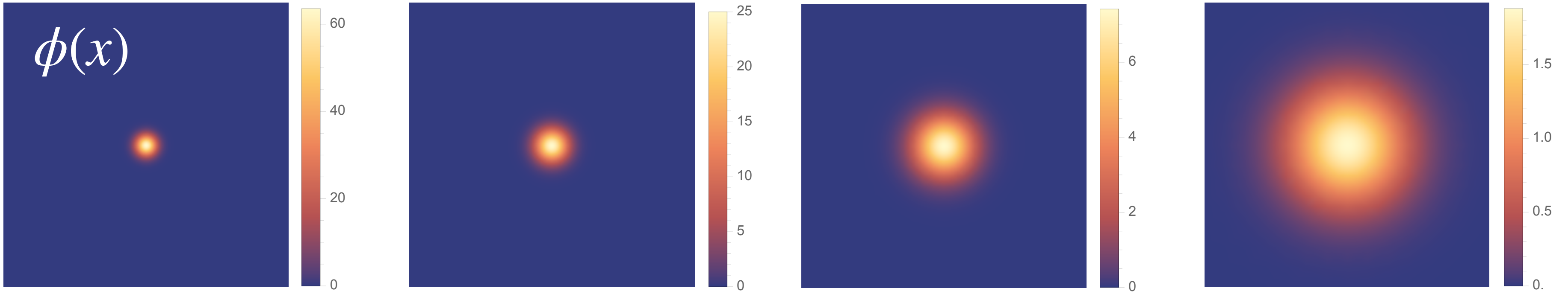


Flow equation: $$\frac{\partial}{\partial\tau}\phi(x,\tau)=-\frac{\delta S[\phi]}{\delta\phi(x)}\bigg|_{\phi(x)\to\phi(x,\tau)}$$

subject to the boundary condition $\phi(x,0)=\phi(x)$

Free scalar field:

$$\left[\partial_\tau-(\partial_\mu^x\partial_\mu^x-M^2)\right]\phi(x,\tau)=0\quad\Rightarrow\quad\phi(x,\tau)=\int d^4y\,\overbrace{\underbrace{G(x-y,\tau)}_{\textit{heat kernel}}}^{G(x,\tau)=\frac{\theta(\tau)}{16\pi^2\tau^2}e^{-\frac{x^2+4M^2\tau^2}{4\tau}}}\phi(y)\quad\Rightarrow\quad\tilde{\phi}(q,\tau)=e^{-\tau(q^2+M^2)}\tilde{\phi}(q)$$



$\tau=0$ $\tau=1$ $\tau=2$ $\tau=4$

Chiral gradient flow

Hermann Krebs, EE, PRC 110 (2024) 044003; PRC110 (2024) 044003

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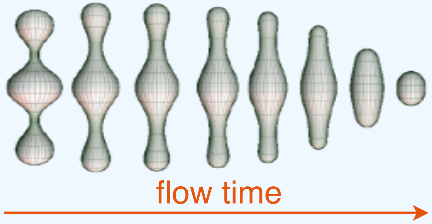

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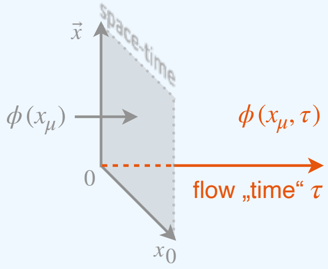

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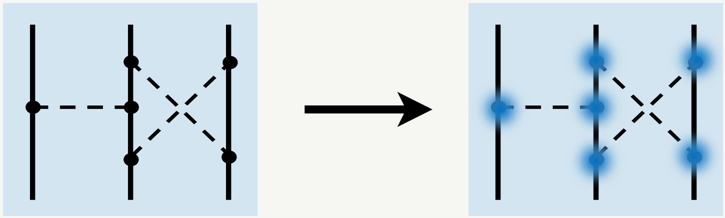
YM gradient flow Narayanan, Neuberger ’06, Lüscher, Weisz ’11: $\partial_\tau A_\mu(x,\tau) = D_\nu G_{\nu\mu}(x,\tau)$ ← extensively used in LQCD

Chiral gradient flow **Hermann Krebs**, EE, PRC 110 (2024) 044003

PRC 110 (2024) 044004

Generalize $U(x)$, $U(x)\to RU(x)L^\dagger$ to $W(x,\tau)$:

$$\partial_\tau W = -i\underbrace{w}_{\sqrt{W}}\,\mathrm{EOM}(\tau)\,w, \quad W(x,0)=U(x)$$



Summary and outlook

- The chiral symmetry of QCD and its breaking pattern play the key role for understanding low-energy nuclear physics

- Chiral EFT has already been developed into a precision tool in the NN sector!

Frontiers and challenges:

- Precision physics beyond the 2N system (the 3NF challenge)
  - — high-precision 3NFs (gradient flow method) and 3N scattering
  - — precision test of chiral EFT for nuclear forces & electroweak currents in nuclei
  - — ab-initio theory for heavier nuclei and reactions
- Chiral EFT as a tool to deal with nuclear effects (SM and BSM): PV, EDM, 0νββ,…
- EFT and lattice-QCD

Thank you for your attention