EOS of Spin-polarized Matter and a Possible New “Skin Thickness” in High-spin Isomers

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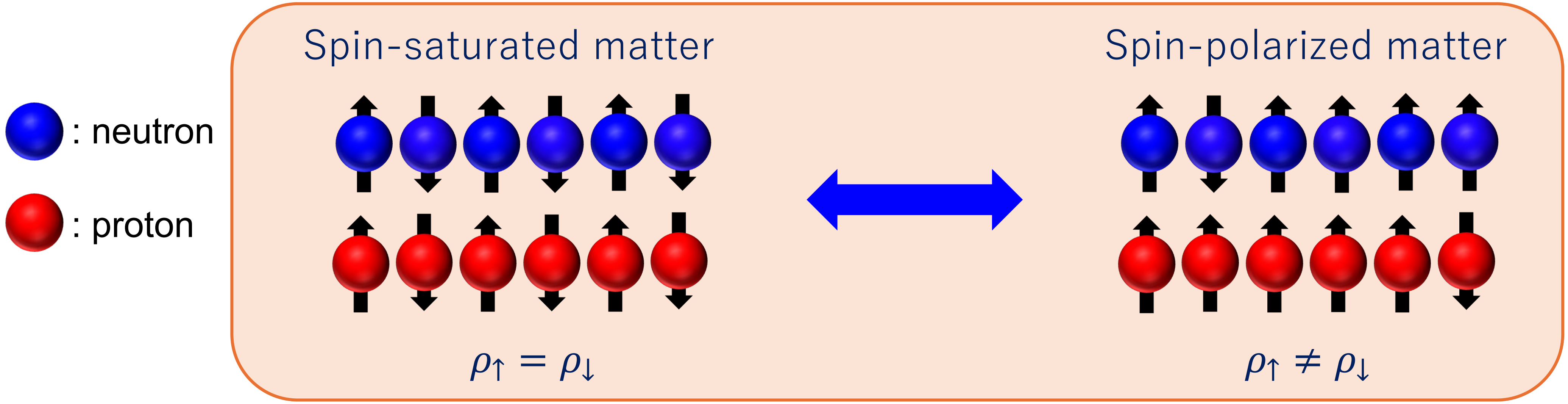
2025/12/17-20 @Workshop on recent developments from QCD to nuclear matter

Outline

1. **Motivation – Spin-polarized matter –**
2. The relativistic point-coupling model
3. EOS of spin polarized matter
4. New “skin thickness” in high-spin isomers
5. Summary

Spin-polarized Nuclear Matter



- The ferromagnetic phase in high-density region ▶ Origin of the magnetic field of magnetar

  I. Vidana, I. Bombaci, Phys. Rev. C 66, 045801 (2002)

  T. Maruyama, T. Tatsumi, Nucl. Phys. A 693 (2001) 710-730

- Strong magnetic field in the remnant of neutron star merger ▶ Possible formation of spin-polarized matter

  B. D. Metzger, et al., Astrophys. J. Lett. 856, 101 (2018)

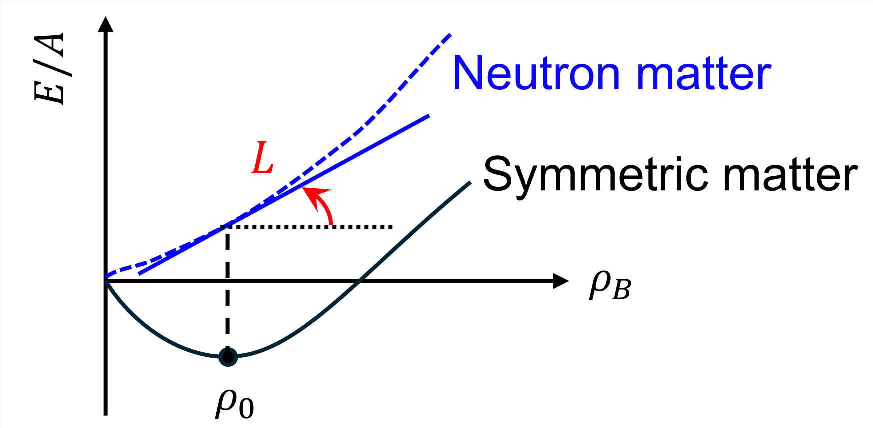
- Changes in particle fraction due to spin polarization ▶ Affecting neutron star cooling.

  N. H. Khoa, N. H. Tan, D. T. Khoa, Phys. Rev. C 105, 065802 (2022)

▶ EOS for spin-polarized matter has been increasingly important.

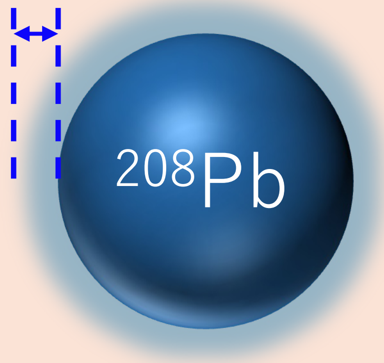
Neutron skin thickness and Slope parameter

Slope parameter L



Linear correlation

Neutron skin thickness Δr_{np}



$Z = 82$

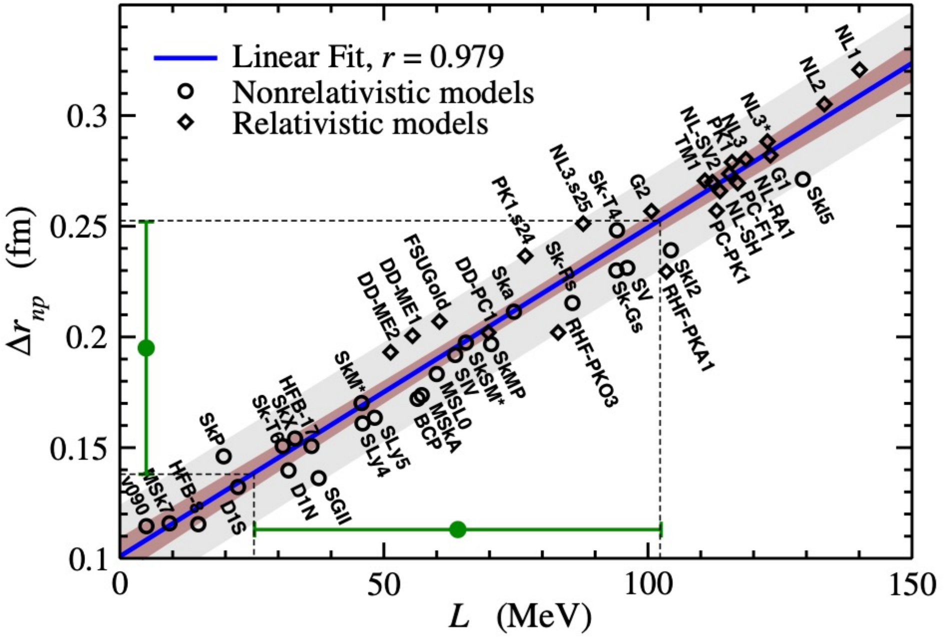
$N = 126$

PREX

D. Adhikari, et al., Phys. Rev. Lett. 126, 172502 (2021)

p elastic scattering

J. Zenihiro, et al., Phys. Rev. C 82, 044611 (2010)



X. Roca-Maza, et al., Phys. Rev. Lett. 106, 252501 (2011)

Current State

Neutron skin thickness is <u>an experimental probe</u> to constrain the EOS for asymmetric nuclear matter.

There are very few analogous probes for constraining the EOS for spin-polarized matter.

Short Summary

Our goals

- Proposing a method to constrain the EOS for spin-polarized matter by terrestrial nuclear experiments.

This Study

- Calculating the EOS for spin-polarized matter in the relativistic model, which can naturally treat the spin d.o.f. of nucleons
- Exploring a new skin "thickness" correlated with the spin slope parameter (details to follow)
- Calculating high-spin isomer state of ^{52}Fe

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Relativistic Point-coupling Model

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

$$\mathcal{L}_{\rm int} = -\frac{1}{2}\alpha_S(\bar\psi\psi)^2 - \frac{1}{2}\alpha_{tS}(\bar\psi\vec\tau\psi)^2 - \frac{1}{2}\alpha_V(\bar\psi\gamma^\mu\psi)^2 - \frac{1}{2}\alpha_{tV}(\bar\psi\gamma^\mu\vec\tau\psi)^2 - \frac{1}{2}\alpha_{PS}(\bar\psi\gamma_5\psi)^2 - \frac{1}{2}\alpha_{tPS}(\bar\psi\gamma_5\vec\tau\psi)^2$$
$$-\frac{1}{2}\alpha_{PV}(\bar\psi\gamma_5\gamma^\mu\psi)^2 - \frac{1}{2}\alpha_{tPV}(\bar\psi\gamma_5\gamma^\mu\vec\tau\psi)^2 - \frac{1}{2}\alpha_T(\bar\psi\sigma^{\mu\nu}\psi)^2 - \frac{1}{2}\alpha_{tT}(\bar\psi\sigma^{\mu\nu}\vec\tau\psi)^2$$

Mean Field approx.

$$\psi = \sum_{\boldsymbol{p},s,t}\left[u(p,s,t)a_{\boldsymbol{p},s,t}e^{-ip\cdot x} + \cancel{v(p,s,t)b^\dagger_{\boldsymbol{p},s,t}e^{ip\cdot x}}\right]$$

(no sea approximation)

$$\langle[\bar\psi(\mathcal{O}\Gamma)_i\psi]^2\rangle = \sum_{\alpha,\beta}\int_{V_{p_F}} dp^3dq^3[\bar u(p,\alpha)(\mathcal{O}\Gamma)_i u(p,\alpha)\times\bar u(q,\beta)(\mathcal{O}\Gamma)_i u(q,\beta)$$
$$-\bar u(p,\alpha)(\mathcal{O}\Gamma)_i u(q,\beta)\times\bar u(q,\beta)(\mathcal{O}\Gamma)_i u(p,\alpha)]$$

$\mathcal{O}\in\{1,\tau_3\}$

$\Gamma\in\{1,\gamma_5,\gamma^\mu,\gamma_5\gamma^\mu,\sigma^{\mu\nu}\}$

Fierz transf.

▶ $\Lambda_{ij}\,\bar u(p,\alpha)(\mathcal{O}\Gamma)_j u(p,\alpha)\times\bar u(q,\beta)(\mathcal{O}\Gamma)_j u(q,\beta)$

$$= (\delta_{ij}-\Lambda_{ij})\left[\sum_\alpha\int_{V_{p_F}} dp^3\bar u(p,\alpha)(\mathcal{O}\Gamma)_j u(p,\alpha)\right]^2$$

Relativistic Point-coupling Model

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

Total Energy Density

$$\mathcal{E} = \langle \mathcal{H} \rangle = \mathcal{E}_{\text{kin}} + \frac{1}{2}\alpha_S\rho_S^2 + \frac{1}{2}\alpha_{tS}\rho_{tS}^2 + \frac{1}{2}\alpha_V\rho_V^2 + \frac{1}{2}\alpha_{tV}\rho_{tV}^2 - \frac{1}{2}\alpha_{PV}\rho_{PV}^2 - \frac{1}{2}\alpha_{tPV}\rho_{tPV}^2 + \alpha_T\rho_T^2 + \alpha_{tT}\rho_{tT}^2$$

$$\rho_S = \sum_{s,t}\int \frac{d^3p}{(2\pi)^3}\bar{u}(p,s,t)\,u(p,s,t)\ , \qquad \rho_V = \sum_{s,t}\int \frac{d^3p}{(2\pi)^3}u^\dagger(p,s,t)\,u(p,s,t)$$

$$\rho_{PV} = \sum_{s,t}\int \frac{d^3p}{(2\pi)^3}u^\dagger(p,s,t)\,\gamma^0\gamma^3\gamma_5\,u(p,s,t) = \sum_{s,t}\int \frac{d^3p}{(2\pi)^3}u^\dagger(p,s,t)\,\Sigma_3\,u(p,s,t)\,,$$

$$\Sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$\rho_T = \sum_{s,t}\int \frac{d^3p}{(2\pi)^3}u^\dagger(p,s,t)\,\gamma_0\sigma^{12}\,u(p,s,t) = \sum_{s,t}\int \frac{d^3p}{(2\pi)^3}u^\dagger(p,s,t)\,\gamma_0\Sigma_3\,u(p,s,t)\,,$$

$u(p,s,t)$: positive-energy plane wave spinor

Relativistic Point-coupling Model

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

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Independent parameters are only $\alpha_S, \alpha_V, \alpha_{tS}, \alpha_{tV}, \alpha_T,$ (because $\mathrm{rank}(1-\Lambda) = 5$)

and other coupling constants are determined by coefficients of Fierz transf. :

$$\alpha_{tT} = \frac{1}{18}(-\alpha_S + 3\alpha_{tS} + 2\alpha_V - 6\alpha_{tV} + 6\alpha_T),$$
$$\alpha_{PV} = \frac{1}{3}(2\alpha_S + 3\alpha_{tS} + 2\alpha_V + 3\alpha_{tV} + 6\alpha_T),$$
$$\alpha_{tPV} = \frac{1}{9}(2\alpha_S + 3\alpha_{tS} + 5\alpha_V - 6\alpha_{tV} + 6\alpha_T).$$

The four coupling constants $\alpha_S, \alpha_V, \alpha_{tS}$ and α_{tV} are density-dependent :

$$\alpha_i(\rho_B) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \times \alpha_i(\rho_0) \qquad \left(x = \frac{\rho_B}{\rho_0} \right) \qquad i = S, V, tS, tV$$

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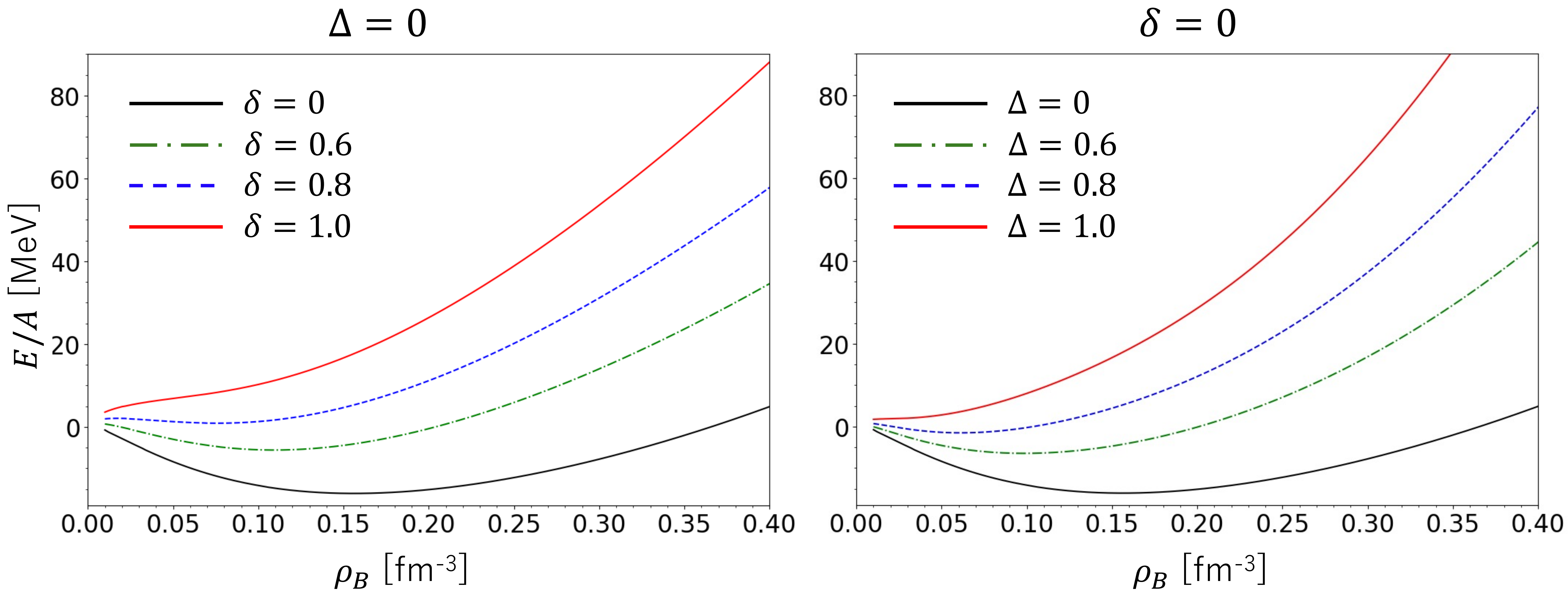
Numerical Results (EOS)

EOS for spin-unpolarized asymmetric matter and spin-polarized symmetric matter (using PCF-PK1)

$\delta \equiv (\rho_n - \rho_p)/\rho_B,\ \Delta_t \simeq (\rho_{\uparrow,t} - \rho_{\downarrow,t})/\rho_t$

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

For simplicity, we assume $\Delta_n = \Delta_p = \Delta$ ▶ $E(\rho_{\uparrow,n}, \rho_{\downarrow,n}, \rho_{\uparrow,p}, \rho_{\downarrow,p}) = E(\rho_B, \delta, \Delta)$



Spin Slope parameter

Slope parameter

$$\frac{E}{A}(\rho,\delta) \cong \frac{E}{A}(\rho,\delta=0) + \underline{S(\rho)}\delta^2$$

Symmetry energy

$$L = 3\rho_0 \frac{\partial S(\rho)}{\partial \rho}\bigg|_{\rho=\rho_0}$$

Spin slope parameter

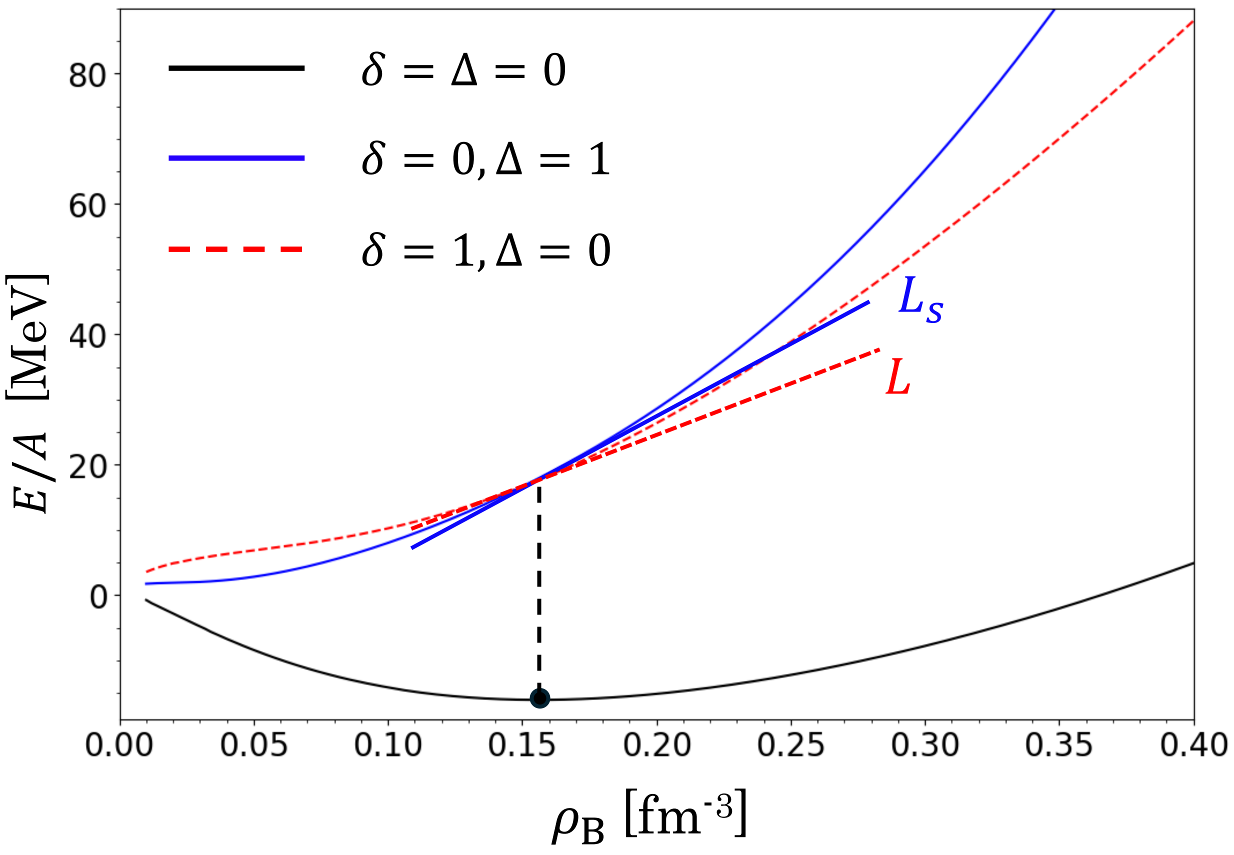
$$\frac{E}{A}(\rho,\langle\Sigma_3\rangle,\delta) \cong \frac{E}{A}(\rho,\Delta=0,\delta) + \underline{W(\rho,\delta)}\Delta^2$$

Spin symmetry energy

$$L_s = 3\rho_0 \frac{\partial W(\rho)}{\partial \rho}\bigg|_{\rho=\rho_0}$$

$$\delta \equiv (\rho_n - \rho_p)/\rho_B, \quad \Delta_\tau \simeq (\rho_{\uparrow,\tau} - \rho_{\downarrow,\tau})/\rho_\tau$$

$$\Delta \equiv |\Delta_n| = |\Delta_p| \; (\neq \langle\Sigma_3\rangle)$$



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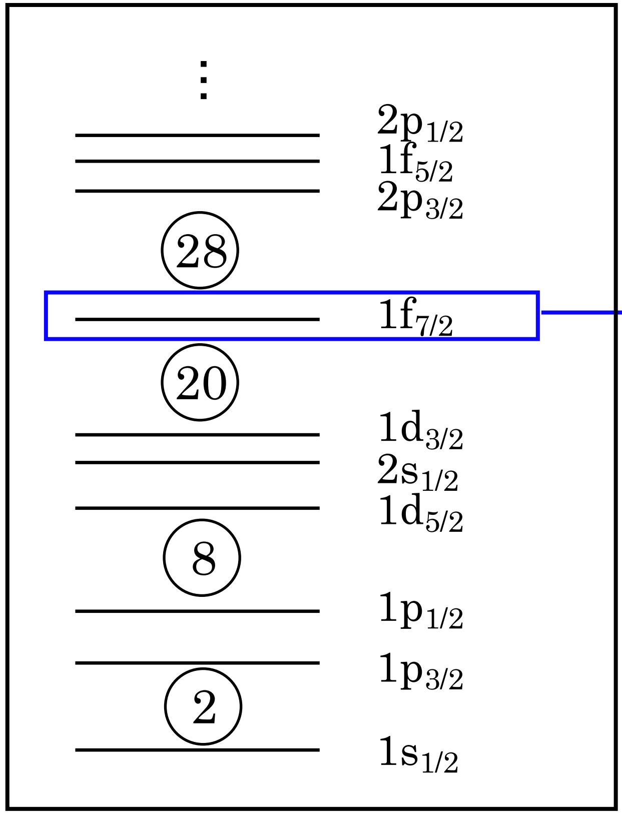
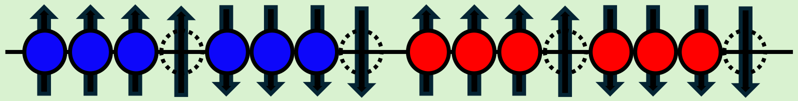
Nuclear Isomer and 12^+ Excited State of ^{52}Fe ($N = Z = 26$)

Nuclear isomer : excited state with a long lifetime

Lifetime of 12^+ excited state of ^{52}Fe is 45.9s. ▶ Isomer with high spin

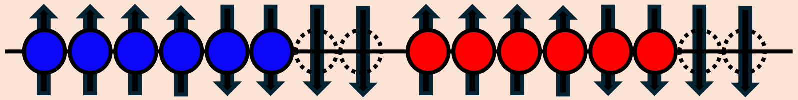


If 6 nucleons occupy f7/2 orbit,

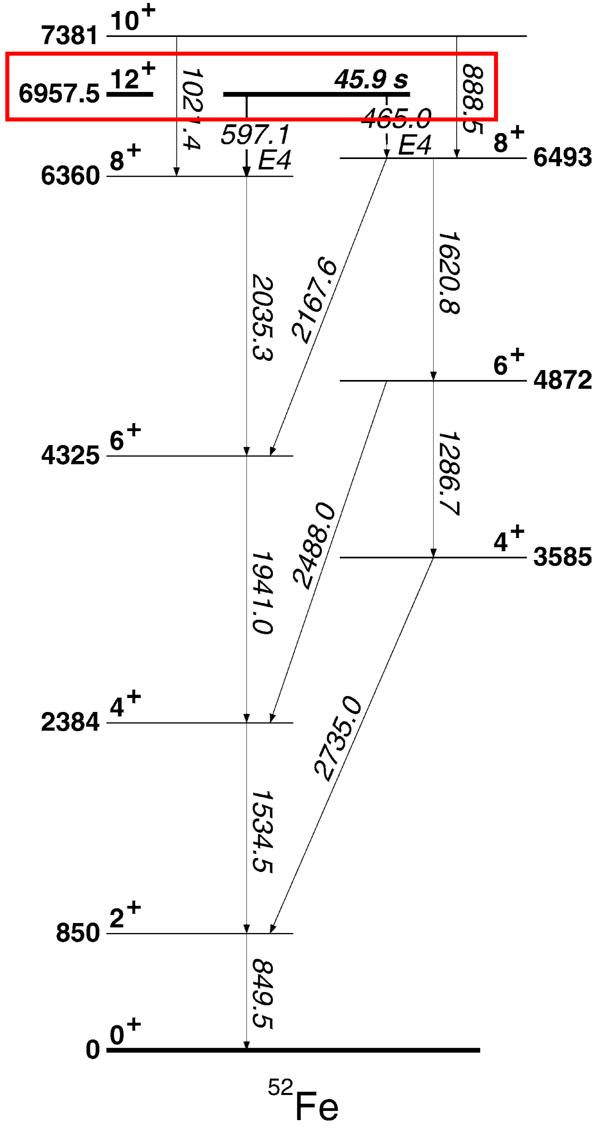


▶ ^{52}Fe(0^+)

In ^{52}Fe(12^+), j_z is fully aligned.



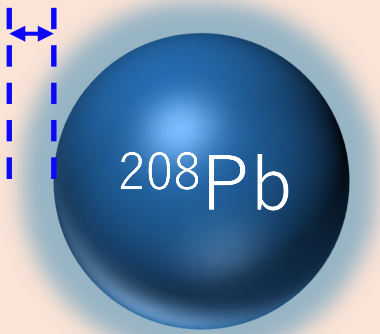
$\frac{5}{2} + \frac{7}{2} = +6$ $\frac{5}{2} + \frac{7}{2} = +6$

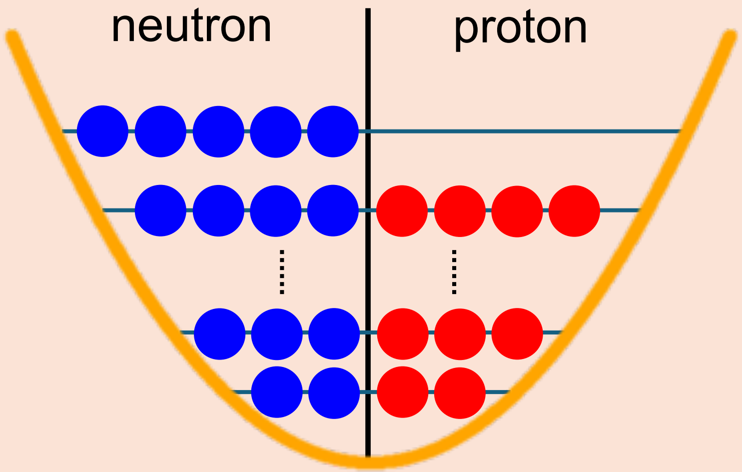


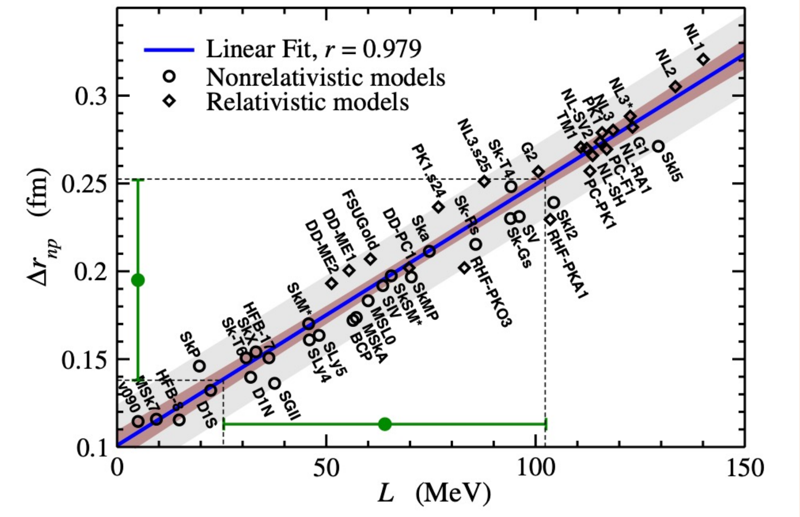
A. Gadea, et al., Phys. Lett. B, 619 (2005) 88

Why High-spin Isomers?

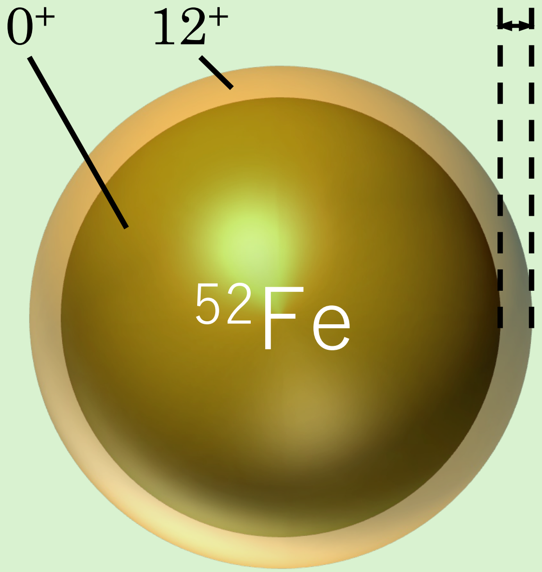
Neutron skin thickness

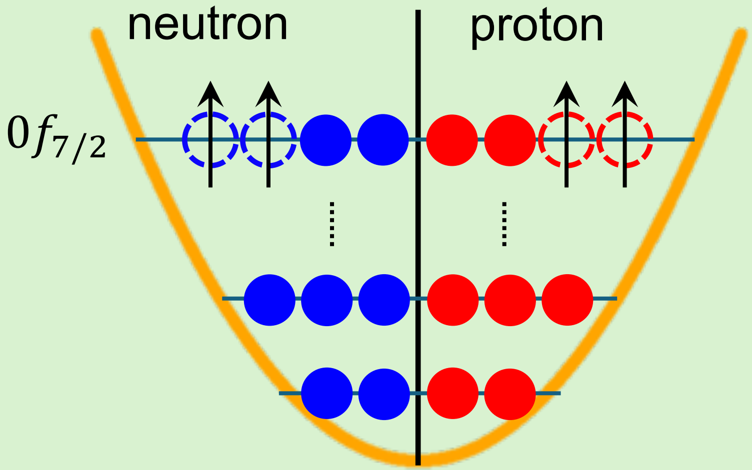




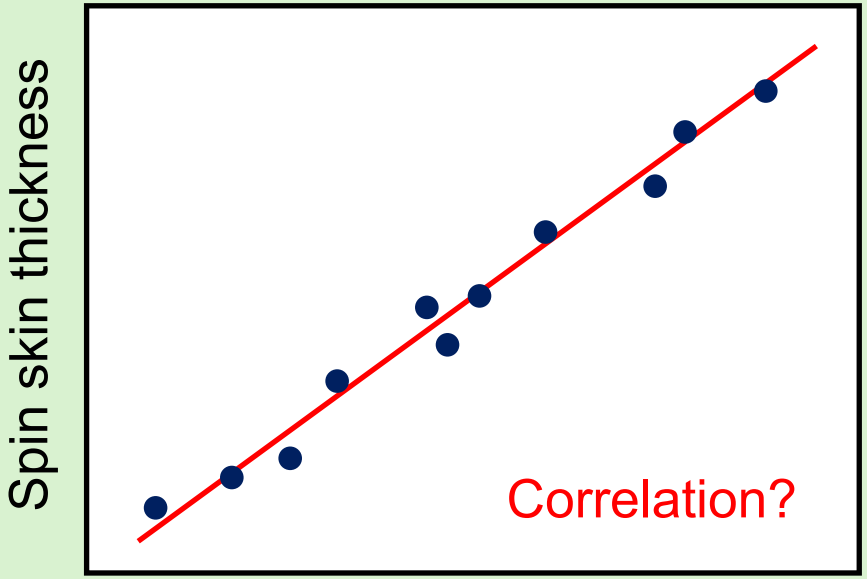


Spin skin thickness?





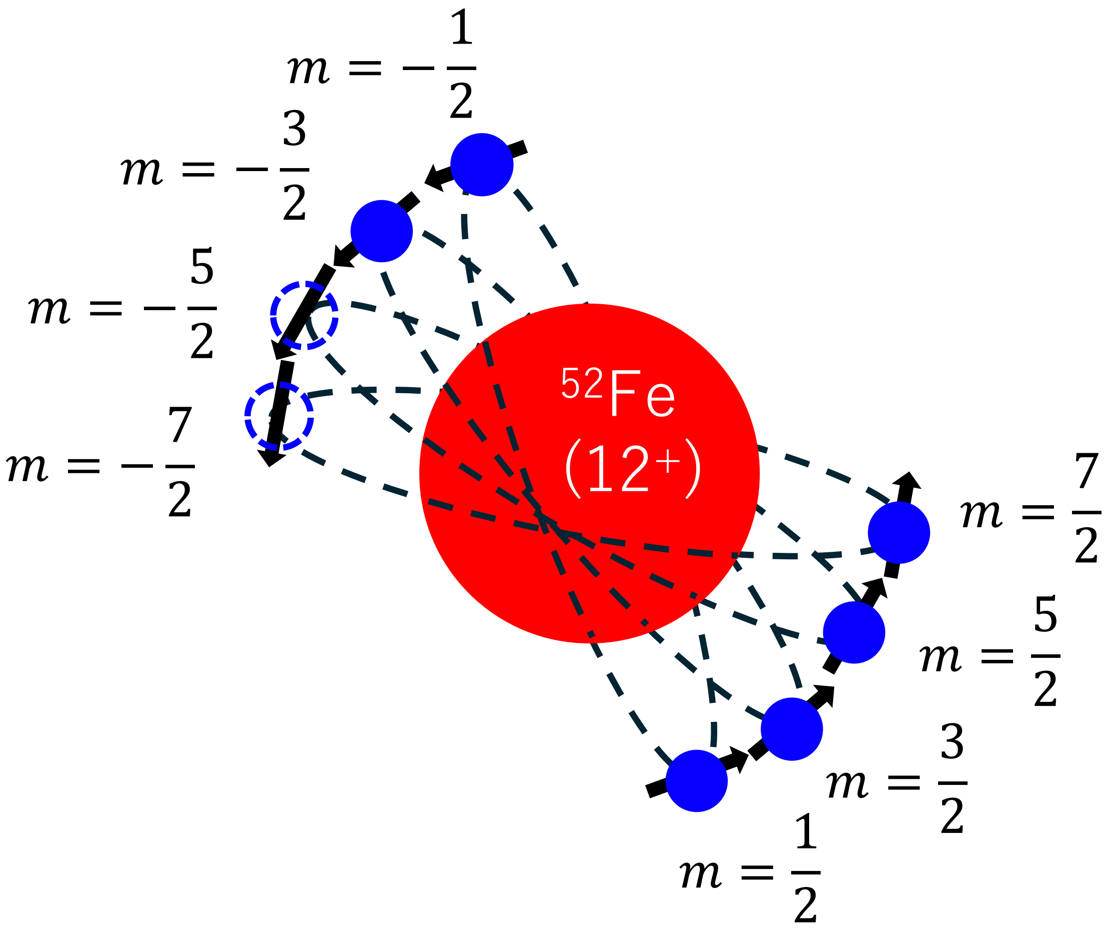
Spin slope parameter



Calculation of 0⁺ g.s. and 12⁺ isomer of ^{52}Fe

0⁺ ground state : spherical HFB calculation with separable paring interaction

12⁺ isomer state : HF calculation for ^{52}Fe with the fixed configuration



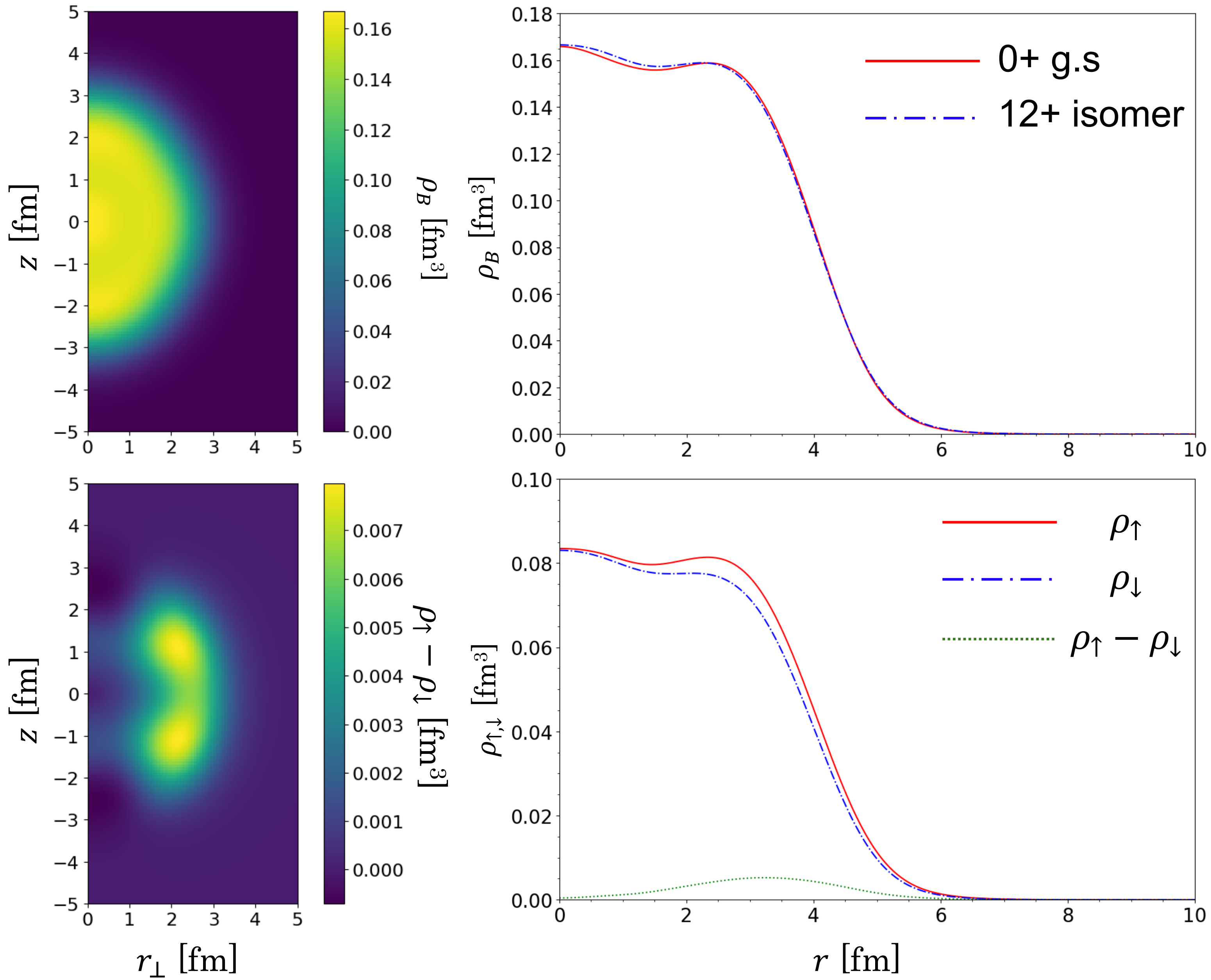
- The configurations in $f7/2$ orbit are fixed by blocking $m = -5/2$ and $m = -7/2$ orbits.

▼

The total angular momentum is 6 for both neutrons and protons.

- Pairing correlations are neglected in calculation of 12⁺ isomer state.

Nucleon and Spin Density of $^{52}Fe(12+)$ State



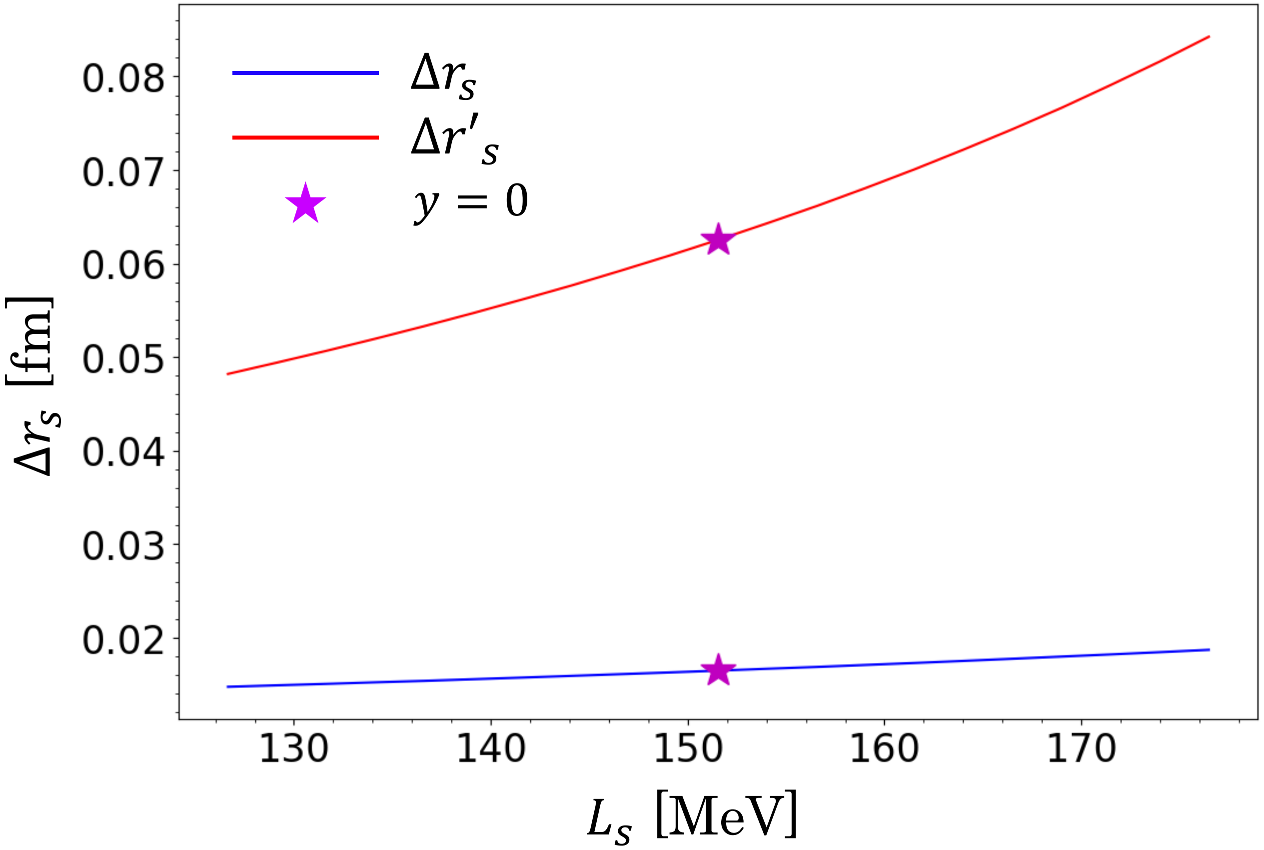
Spin densities are localized near the surface!

Correlation between spin skin thickness and spin slope parameter

Introducing a new variable y:

$$\alpha_{tV}(\rho) \rightarrow (1-y)\alpha_{tV}(\rho) + y\alpha_{tV}(\rho_0)$$

cf. T. Inakura, H. Nakada, Phys. Rev. C 92, 064302 (2015)



Two definitions of the "spin skin thickness"

1. $\Delta r_s = (\text{rad. of } 12^+ \text{ isomer}) - (\text{rad. of } 0^+ \text{ g.s.})$

2. $\Delta r'_s = r_+ - r_-$

   $r_\pm$: rms radius of $\rho_\pm = \langle\psi^\dagger(1 \pm \Sigma_3)\psi\rangle/2$,
   the spin-up (-down) nucleon density.

$\Delta r'_s$ is determined only by the properties of 12^+ state of ^{52}Fe.

Δr_s : positive but weak correlation

cf. $\Delta R_{\text{exp}} = 0.003$ fm

A. R. Vernon, et al., PRL 134 252501 (2025)

$\Delta r'_s$: positive and stronger correlation

However, <u>how to observe</u>?

Summary

- Our goal : to propose an experimental probes for constraining the EOS of spin-polarized nuclear matter.

- Correlation between spin skin thickness Δr_s ($\Delta r'_s$) and spin slope parameter L_s
  - Δr_s :the difference in radii between 0^+ ground state and 12^+ isomer state
  - $\Delta r'_s$:the difference in radii between spin-up and spin-down nucleon in 12^+ state
  - a positive but weak correlation between Δr_s and L_s
  - a positive and stronger correlation between $\Delta r'_s$ and L_s

- Future work
  - Calculating high-spin isomer state of other nuclei
  - Including the paring correlation in the calculation of high-spin isomers