QCD-based approach on isospin symmetry breaking in nuclear density functional theory

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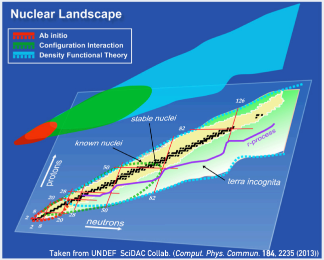
Workshop on recent developments from QCD to nuclear matter

Institute of Physics, Academia Sinica, Taipei, TAIWAN

MLPhYs
Foundation of "Machine Learning Physics"
学習物理学の創成

The first accelerator experiment in east Asia (1934, NTU)

Nuclear structure calculation



- ***Ab initio* calculation** (see: Evgeny's talk)
  - Starting from nuclear interaction in vacuum
  - Sophisticated ground-state wave function
  - Difficult to approach all the nuclei
- **Large-scale shell model calculation**
- **Density functional theory** (see: Nakatsukasa-san's talk)
  - Starting from nuclear interaction in medium
  - Simple ground-state wave function (Slater determinant)
  - Can approach for (almost) all the nuclei

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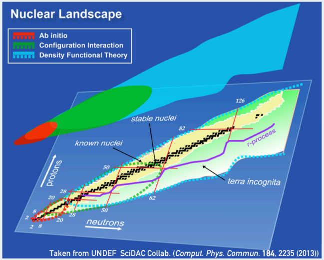

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Introduction

Density functional theory (DFT)

- DFT is one of the strong methods to approach nuclear structure
- In DFT, an energy density functional (EDF) $E[\rho]$ is minimized to obtain the ground-state energy E_{gs} and density ρ_{gs}: $E_{\mathrm{gs}} = E\left[\rho_{\mathrm{gs}}\right]$
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- How to pin down an EDF theoretically?
- EoS is one of the reference of an EDF
  $\rightarrow$ EoS should be determined (see: Isobe-san's talk)
- However, different data estimates different properties of EoS, especially for the isovector properties (see: Ikeno-san's talk)
  $\rightarrow$ Are there any missing physics not considered?

Isospin symmetry breaking of nuclear interaction

- Nuclear interaction: *almost* isospin symmetric $v_{pp}^{T=1} \simeq v_{pn}^{T=1} \simeq v_{nn}^{T=1}$

Miller, Opper, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

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- Nuclear interaction: *almost* isospin symmetric
- **Charge symmetry breaking (CSB)**
  - Difference between p-p int. and n-n int.

$$v_{\mathrm{CSB}} \equiv v_{nn}^{T=1} - v_{pp}^{T=1} \sim \tau_{zi} + \tau_{zj}$$

  - Originates from mass difference of nucleons ($m_p \neq m_n$) and π^0-η & ρ^0-ω mixings in meson-exchange process
  - Contribute to β term (β^{2n+1} terms) in nuclear EoS
- **Charge independence breaking (CIB)**
  - Difference between like-particle int. and diff.-particle int.

$$v_{\mathrm{CIB}} \equiv \frac{v_{nn}^{T=1} + v_{pp}^{T=1}}{2} - v_{np}^{T=1} \sim \tau_{zi}\tau_{zj}$$

  - Originates from mass difference of pions ($m_{\pi^0} \neq m_{\pi^\pm}$)
  - Contribute to SNM and β^2 term (β^{2n} terms) in nuclear EoS

Miller, Opper, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

Introduction

Isospin symmetry breaking of atomic nuclei

- Mirror nuclei: A pair of nuclei of (Z, N) and (N, Z)
- Properties of mirror nuclei are identical if no Coulomb nor nuclear ISB
- Coulomb int. is not enough to describe the mass diff. of mirror nuclei "Okamoto-Nolen-Schiffer (ONS) anomaly"
- Other different properties of mirror nuclei
  - Ground-state spin ($^{73}_{38}\mathrm{Sr}$ and $^{73}_{35}\mathrm{Br}$ at NSCL, $^{71}_{36}\mathrm{Kr}$ and $^{71}_{35}\mathrm{Br}$ at RIBF)
  - Shape ($^{70}_{36}\mathrm{Kr}$ and $^{70}_{34}\mathrm{Se}$ at RIBF)
- Finite (negative) neutron-skin thickness $\Delta R_{np} = R_n - R_p$ of $N = Z$ nuclei

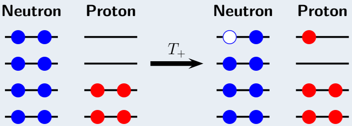
Okamoto. *Phys. Lett.* **11**, 150 (1964)
Nolen and Schiffer. *Annu. Rev. Nucl. Sci.* **19**, 471 (1969)
Hoff *et al. Nature* **580**, 52 (2020)
Wimmer *et al. Phys. Rev. Lett.* **126**, 072501 (2021)

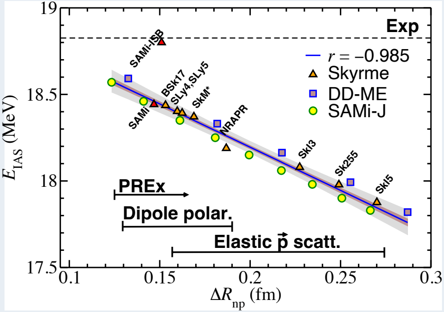
Algora *et al.* arXiv:2411.00509 [nucl-ex]

Isobaric analog energy (IAE) and neutron-skin thickness

Neutron-skin thickness

$$\Delta R_{np} = R_n - R_p$$

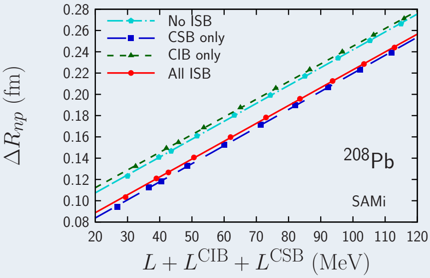




- Isobaric analog energy: Energy difference between $|\Psi\rangle$ and $T_{\pm}\,|\Psi\rangle$
  - E.g. ${}^{208}_{82}\mathrm{Pb}$ and ${}^{208}_{83}\mathrm{Bi}^{*}$
  - Spatial-spin wave function is almost the same
  - Energy difference originates from Coulomb interaction
- There is a correlation between E_{IAS} and ΔR_{np} of ${}^{208}\mathrm{Pb}$
- Exp. values of E_{IAS} and ΔR_{np} cannot be described at the same time

Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

Neutron-skin thickness of ^{208}Pb



- Neutron-skin thickness
  $\Delta R_{np} = R_n - R_p$
  ↑
  Correlation with L
- ΔR_{np} obtained by Skyrme DFT
- On top of it,
  ISB terms are considered

- If we assume the same ΔR_{np},
  difference between estimated L_{full} without & that with ISB is $11.1\,\mathrm{MeV}$
  CSB contribution $13.9\,\mathrm{MeV}$
  CIB contribution $-2.7\,\mathrm{MeV}$
- $L_{\mathrm{CIB}} = 2.3\,\mathrm{MeV}$ and $L_{\mathrm{CSB}} = -3.2\,\mathrm{MeV}$ → Change of L is $12\,\mathrm{MeV}$

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **107**, 064302 (2023)

Mysterious of CSB strength (leading-order strength s_0)

$$v_{\mathrm{CSB}}(\boldsymbol{r}) = [s_0 \delta(\boldsymbol{r}) + \dots] (\tau_1 + \tau_2)$$

- ISB effects on nuclear properties depends on ISB strengths
- **Phenomenological determination** (Referring experimental data)
- **CSB strength s_0 extracted from *ab initio* calculation**

Method: Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)
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  - $s_0 \simeq -10\,\text{MeV}\,\text{fm}^3$ (MDE and TDE)

$O(10)\ \text{MeV}\,\text{fm}^3$

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- **We need to determine ISB strength microscopically!**

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Simplest model systems for ONS anomaly

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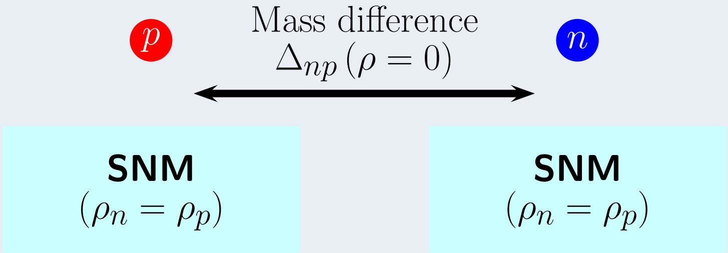

Hatsuda, Høgaasen, and Prakash. *Phys. Rev. Lett.* **66**, 2851 (1991)

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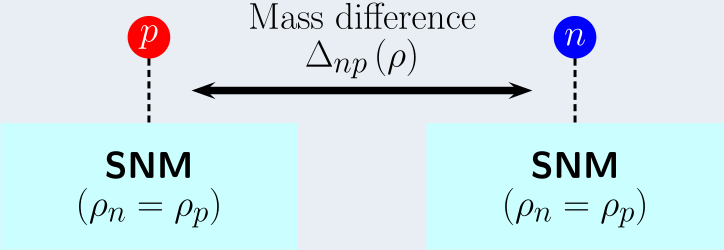

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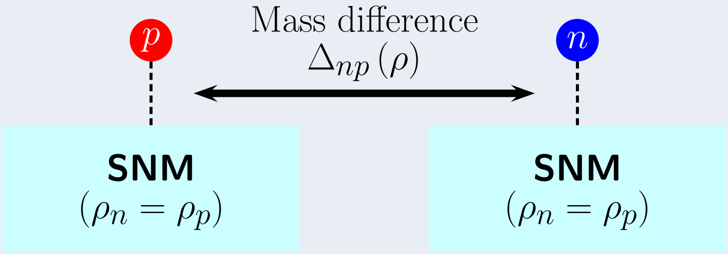

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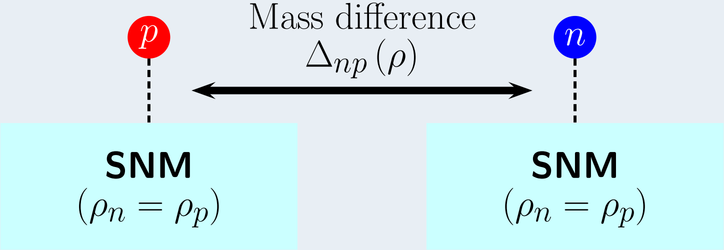
- In this model, we need $\Delta_{np}(\rho)$ from QCD knowledge



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QSR approach for Δ_{np} (considering only quark condensation)

- In-medium self-energy Σ_τ can be calculated by QSR

$$\Delta_{np}(\rho) = \omega_n - \omega_p \simeq \Sigma_n^{\mathrm{S}} - \Sigma_p^{\mathrm{S}}$$

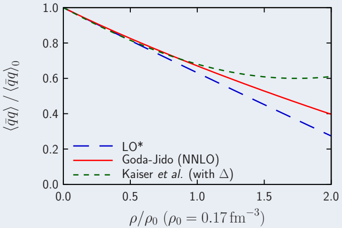
- Using the Borel sum and QSR,

$$\Delta_{np}(\rho) = C_1 \left(\frac{\langle \bar{q} q \rangle_\rho}{\langle \bar{q} q \rangle_0} \right)^{1/3} - C_2 \qquad C_1 = -a\gamma \qquad \gamma = \frac{\langle \bar{d} d \rangle_0}{\langle \bar{u} u \rangle_0} - 1$$

(ω_τ: Time-component of 4-momentum, Σ_τ^{S}: Scalar self-energy, $C_1 = 5.24^{+2.48}_{-1.21}\,\mathrm{MeV}$)

Hatsuda, Høgaasen, and Prakash. *Phys. Rev. Lett.* **66**, 2851 (1991)

Estimation of in-medium chiral condensation



$$\frac{\langle \bar{q} q \rangle_\rho}{\langle \bar{q} q \rangle_0} \simeq 1 + k_1 \frac{\rho}{\rho_0} + k_2 \left(\frac{\rho}{\rho_0} \right)^{5/3}$$

$$k_1 = -\frac{\sigma_{\pi N} \rho_0}{f_\pi^2 m_\pi^2} \qquad k_2 = -k_1 \frac{3 k_{\mathrm{F}0}^2}{10 m_N^2}$$

$\sigma_{\pi N}$: π-N sigma term, f_π: pion decay const.

Goda and Jido. *Phys. Rev. C* **88**, 065204 (2013)

QCD sum rule approach for CSB EDF

Estimation of Δ_{np}

- Eventually, the ONS anomaly $\delta_{\mathrm{QSR}} = \Delta_{np}(\rho = 0) - \Delta_{np}(\rho)$ is

$$\begin{aligned}\delta_{\mathrm{QSR}} &= C_1 \left[1 - \left(\frac{\langle \overline{q} q \rangle_\rho}{\langle \overline{q} q \rangle_0} \right)^{1/3} \right] \\ &= C_1 \left[\frac{\sigma_{\pi N}}{3 f_\pi^2 m_\pi^2} \rho - \left(\frac{3\pi^2}{2} \right)^{2/3} \frac{\sigma_{\pi N}}{10 f_\pi^2 m_N^2 m_\pi^2} \rho^{5/3} + \dots \right]\end{aligned}$$

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

QCD Sum Rule Approach for CSB EDF

Effective interaction approach

- We introduce Skyrme-type CSB interaction

$$v_{\mathrm{Sky}}^{\mathrm{CSB}}(\boldsymbol{r}) = \left\{ s_0 (1 + y_0 P_\sigma) \delta(\boldsymbol{r}) + \frac{s_1}{2} (1 + y_1 P_\sigma) \left[\boldsymbol{p}^{\dagger 2} \delta(\boldsymbol{r}) + \delta(\boldsymbol{r}) \boldsymbol{p}^2 \right] + s_2 (1 + y_2 P_\sigma) \boldsymbol{p}^\dagger \cdot \delta(\boldsymbol{r}) \boldsymbol{p} \right\} \frac{\tau_{1z} + \tau_{2z}}{4}$$

- Contribution of $v_{\mathrm{Sky}}^{\mathrm{CSB}}$ to nuclear EoS is

$$\frac{E^{\mathrm{CSB}}}{A} = \left[\frac{\tilde{s}_0}{8} \rho + \frac{1}{20} \left(\frac{3\pi^2}{2} \right)^{2/3} (\tilde{s}_1 + 3 \tilde{s}_2) \rho^{5/3} \right] \frac{\rho_n - \rho_p}{\rho}$$

$\tilde{s}_0 = s_0 (1 - y_0)$, $\tilde{s}_1 = s_1 (1 - y_1)$, $\tilde{s}_2 = s_1 (1 + y_2)$

- Therefore,

$$\delta_{\mathrm{Skyrme}} \simeq -\frac{\tilde{s}_0}{4} \rho - \frac{1}{10} \left(\frac{3\pi^2}{2} \right)^{2/3} (\tilde{s}_1 + 3 \tilde{s}_2) \rho^{5/3}$$

since $\left(\rho_n - \rho_p\right) / \rho \simeq (N - Z) / A$

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Comparison of δ obtained by two methods

$$\delta_{\mathrm{QSR}} \simeq C_1 \left[\frac{\sigma_{\pi N}}{3 f_\pi^2 m_\pi^2} \rho - \left(\frac{3\pi^2}{2} \right)^{2/3} \frac{\sigma_{\pi N}}{10 f_\pi^2 m_N^2 m_\pi^2} \rho^{5/3} \right]$$

$$\delta_{\mathrm{Skyrme}} \simeq -\frac{\tilde{s}_0}{4} \rho - \frac{1}{10} \left(\frac{3\pi^2}{2} \right)^{2/3} (\tilde{s}_1 + 3\tilde{s}_2) \rho^{5/3}$$

These two results should be identical; therefore

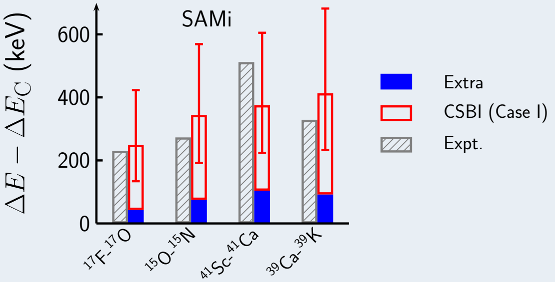
$$\tilde{s}_0 \simeq -\frac{4}{3} \frac{C_1 \sigma_{\pi N}}{f_\pi^2 m_\pi^2} = -15.5^{+8.8}_{-12.5} \, \mathrm{MeV\,fm^3}$$

$$\tilde{s}_1 + 3\tilde{s}_2 \simeq \frac{1}{m_N^2} \frac{C_1 \sigma_{\pi N}}{f_\pi^2 m_\pi^2} = 0.52^{+0.42}_{-0.29} \, \mathrm{MeV\,fm^5}$$

$\sigma_{\pi N}$ has the large uncertainty $\sigma_{\pi N} = 45 \pm 15\,\mathrm{MeV}$ (conservative estimation)

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Application for actual Skyrme Hartree-Fock calculation



- "Extra" contribution is not enough to describe ΔE
  - Higher-order correction for the Coulomb interaction
  - Change of kinetic energy due to $m_p \neq m_n$
- Uncertainty comes from the $\sigma_{\pi N}$ term only
- QCD-CSB interaction describe ΔE quite nicely
  $\rightarrow$ ONS anomaly may be solved?

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Mirror-Skin Thickness

How do the CSB EDF determined empirically?

- So far, CSB EDF strength has been determined by masses
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- Can we find observable sensitive only to CSB?
- Can we pin down the density dependence of CSB EDF?
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  ← Density depepdence of CSB EDF is almost unknown

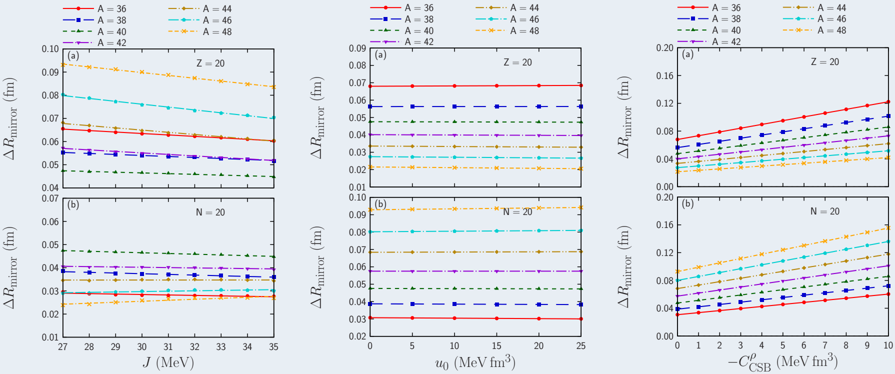
Our proposal: **Mirror-skin thickness**

$$\Delta R_{\mathrm{mirror}}(Z,N) = R_p(Z,N) - R_n(N,Z)$$

- $\Delta R_{\mathrm{mirror}}$ is zero if there is not Coulomb nor ISB
- $\Delta R_{\mathrm{mirror}}$ for $N = Z$ nuclei is a proton-skin thickness

Naito, Hijikata, Zenihiro, Colò, and Sagawa. *Eur. Phys. J. A* **61**, 177 (2025)

Sym. energy, CIB, and CSB dependence of mirror-skin thickness

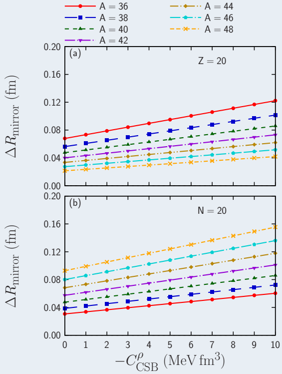


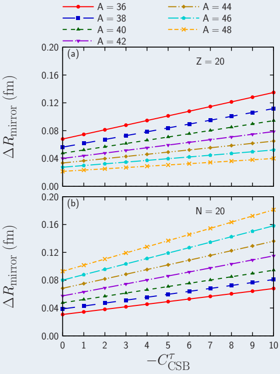
- $\Delta R_{\mathrm{mirror}}$ is not sensitive to symmetry energy or CIB
- $\Delta R_{\mathrm{mirror}}$ is sensitive to CSB strength ($\gtrsim 0.05\,\mathrm{fm}$ for $^{36}\mathrm{Ca}$)

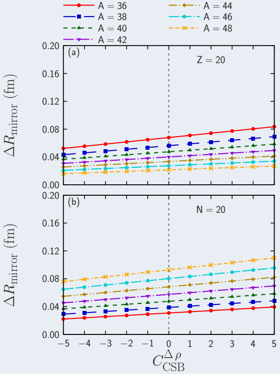
Naito, Hijikata, Zenihiro, Colò, and Sagawa. *Eur. Phys. J. A* **61**, 177 (2025)

Various density dependence of mirror-skin thickness

$$\mathcal{E}_{\text{CSB}} = C_{\text{CSB}}^{\rho}\left(\rho_n^2 - \rho_p^2\right) + C_{\text{CSB}}^{\tau}\left(\rho_n \tau_n - \rho_p \tau_p\right) + C_{\text{CSB}}^{\Delta\rho}\left(\rho_n \Delta\rho_n - \rho_p \Delta\rho_p\right)$$







- ΔR_{mirror} is not sensitive to $C_{\text{CSB}}^{\Delta\rho}$
- C_{CSB}^{ρ} and C_{CSB}^{τ} show almost the same dependences

Naito, Hijikata, Zenihiro, Colò, and Sagawa. *Eur. Phys. J. A* **61**, 177 (2025)

Conclusion

- CSB and CIB terms contribute to
  ΔR_{np} of $^{208}\mathrm{Pb}$ in $-0.02\,\mathrm{fm}$ ($12\,\mathrm{MeV}$ in L value)
- CSB is related to chiral symmetry breaking and its restoration
- QCD-based CSB EDF gives reasonable description of ONS anomaly
- Mirror-skin thickness can be an alternative way
  to pin down CSB EDF phenomenologically
- Combining phenomenological CSB EDF & QCD-based knowledge,
  we may determine $\langle \overline{q} q \rangle$ as done in a pionic atom
- Comparison among the models of nucleon effective mass?
- **Next step**: QCD-based CIB EDF
  $\leftarrow$ One-pion exchange interaction dominates for bare CIB int.
  $\uparrow$ Evgeny *et al.* derived pion-exchange CIB int. in χEFT
- **Future**: Can we connect QCD knowledge and EDF
  as done in χEFT + *ab initio* calculation?

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Thank you for attention!!

Appendix Slides

Method in this work

- **Nuclear density functional theory (DFT)** is used
  because only this can be applied to whole nuclear chart at this moment
  $$E\left[\rho_p, \rho_n\right] = T_{\text{KS}}\left[\rho_p, \rho_n\right] + E_{\text{nucl}}\left[\rho_p, \rho_n\right] + E_{\text{CH}}\left[\rho_{\text{ch}}\right] + E_{\text{Cx}}\left[\rho_{\text{ch}}\right]$$
- **Skyrme-type effective interaction** is used for E_{nucl} (non-rel. calc.)

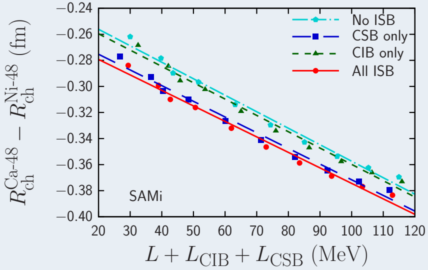
$$v_{\text{Sky}}^{\text{IS}}(\boldsymbol{r}) = t_0 (1 + x_0 P_\sigma) \delta(\boldsymbol{r}) + \frac{t_1}{2} (1 + x_1 P_\sigma) \left[\boldsymbol{p}^{\dagger 2} \delta(\boldsymbol{r}) + \delta(\boldsymbol{r}) \boldsymbol{p}^2\right] + t_2 (1 + x_2 P_\sigma) \boldsymbol{p}^\dagger \cdot \delta(\boldsymbol{r}) \boldsymbol{p}$$
$$+ \frac{t_3}{6} (1 + x_3 P_\sigma) \delta(\boldsymbol{r}) \left[\rho(\boldsymbol{R})\right]^\alpha + i W_0 \boldsymbol{\sigma} \cdot \boldsymbol{p}^\dagger \times \delta(\boldsymbol{r}) \boldsymbol{p}$$

  SAMi and its family are used for E_{nucl}
- Doubly-magic nuclei are focused on
  → Spherical symmetry is assumed and pairing is neglected

Isospin Symmetry Breaking in Nuclear DFT

Charge-radii difference of mirror nuclei ^{48}Ca-^{48}Ni

$$
\begin{aligned}
\Delta R_{\text{ch}}^{(Z,N)} &= R_{\text{ch}}^{(Z,N)} - R_{\text{ch}}^{(N,Z)} \\
&\simeq R_{p}^{(Z,N)} - R_{p}^{(N,Z)} \\
&\simeq R_{p}^{(Z,N)} - R_{n}^{(Z,n)} \\
&\simeq -\Delta R_{np}^{(Z,N)}
\end{aligned}
$$



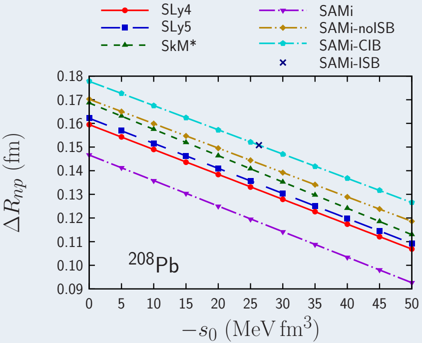
- ΔR_{ch} may also correlate to L (Still under discussion)
- ISB terms change R_{ch}, and thus ΔR_{ch}
- CSB & CIB effects on L-ΔR_{ch} correlation are same direction in contrast to L-ΔR_{np} correlation

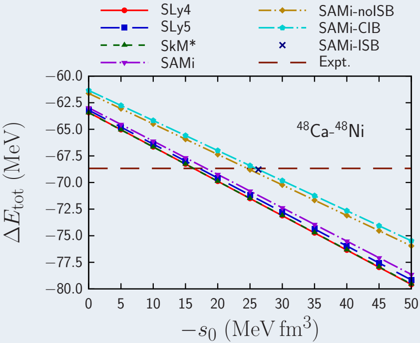
Naito, Roca-Maza, Colò, Liang, and Sagawa. *Phys. Rev. C* **106**, L061306 (2022)

Neutron-Skin Thickness and Charge Radii Difference

Isospin-Symmetric Terms Dependence

- If parameters of $v_{\mathrm{Sky}}^{\mathrm{IS}}$ is changed, how s_0- or u_0-dependence on ΔR_{np} and ΔE_{tot} changes??

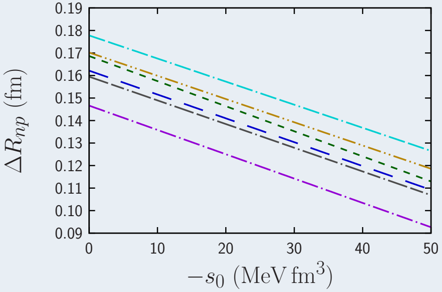




- Isospin symmetic part E_{IS} hardly affects slope of s_0 dep. of ΔR_{np} or ΔE_{tot}

A Method to Determine s_0

If *ab initio* calculation provides ΔR_{np} (or ΔE_{tot}) without and with CSB term, CSB strength s_0 can be determined by using this property



0. Calculate ΔR_{np} for various s_0 and fit to $\Delta R_{np} = a - b s_0$
1. Derive avaraged value $\overline{b}$

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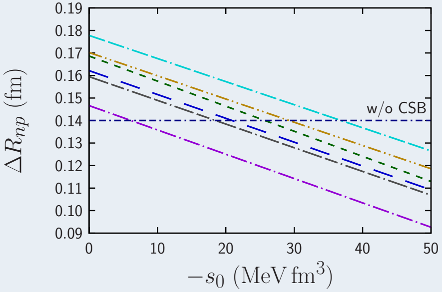
If *ab initio* calculation provides ΔR_{np} (or ΔE_{tot}) without and with CSB term, CSB strength s_0 can be determined by using this property



0. Calculate ΔR_{np} for various s_0 and fit to $\Delta R_{np} = a - b s_0$
1. Derive avaraged value $\overline{b}$
2. Calculate ΔR_{np} w/o CSB terms: $\Delta R_{np}^{\text{w/o CSB}}$

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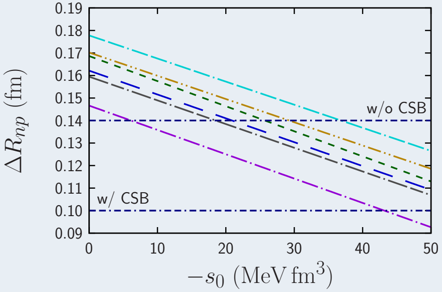
If *ab initio* calculation provides ΔR_{np} (or ΔE_{tot}) without and with CSB term, CSB strength s_0 can be determined by using this property



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3. Calculate ΔR_{np} w/ CSB terms: $\Delta R_{np}^{\text{w/ CSB}}$

Neutron-Skin Thickness and Charge Radii Difference

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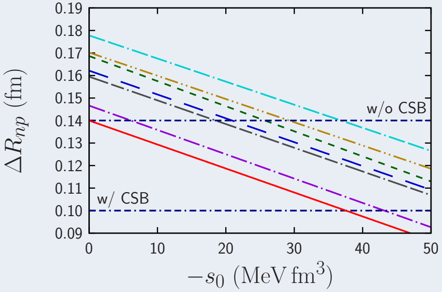
If *ab initio* calculation provides ΔR_{np} (or ΔE_{tot}) without and with CSB term, CSB strength s_0 can be determined by using this property



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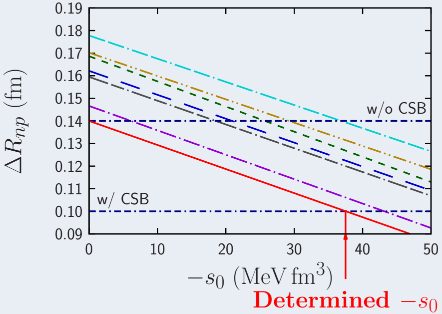


0. Calculate ΔR_{np} for various s_0 and fit to $\Delta R_{np} = a - bs_0$
1. Derive avaraged value $\overline{b}$
2. Calculate ΔR_{np} w/o CSB terms: $\Delta R_{np}^{\text{w/o CSB}}$
3. Calculate ΔR_{np} w/ CSB terms: $\Delta R_{np}^{\text{w/ CSB}}$
4. s_0 can be determined by

$$s_0 = -\frac{\Delta R_{np}^{\text{w/ CSB}} - \Delta R_{np}^{\text{w/o CSB}}}{\overline{b}}$$

New Observable to Pin Down CSB Strength

Motivation

- Most experimental data are sensitive not only CSB but also others
- Can we find observable sensitive only to CSB?

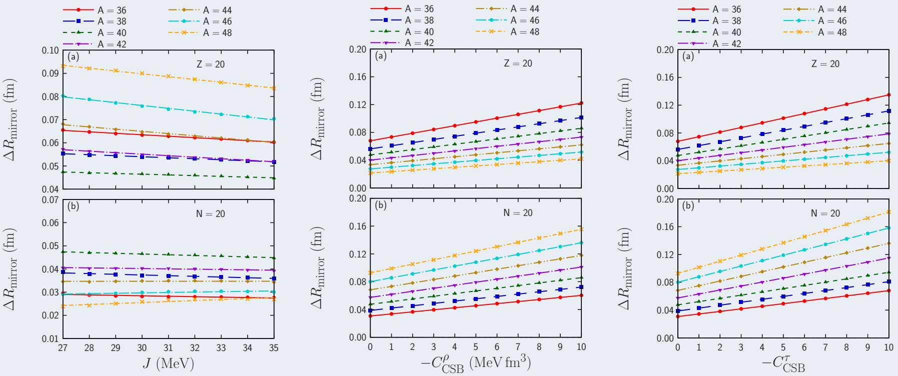
Our proposal: **Mirror skin**

$$\Delta R_{\text{mirror}}(Z, N) = R_p(Z, N) - R_n(N, Z)$$

- ΔR_{mirror} is zero if there is not Coulomb nor ISB
- ΔR_{mirror} for $N = Z$ nuclei is a proton-skin thickness

Naito, Colò, Hijikata, Sagawa, and Zenihiro. *To be submitted*

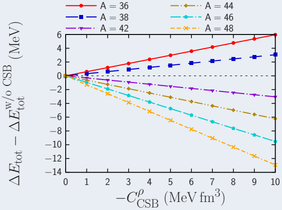
Mirror-skin thickness

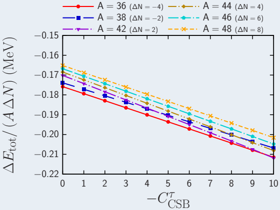


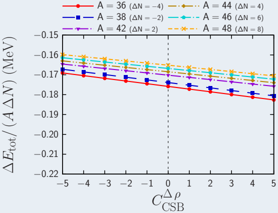
- Mirror-skin thickness is not sensitive to symmetry energy ($\lesssim 0.01\,\mathrm{fm}$)
- Mirror-skin thickness is sensitive to CSB strength ($\gtrsim 0.05\,\mathrm{fm}$ for $^{36}\mathrm{Ca}$)

Can we determine the form of CSB EDF in mass difference of mirror nucl?

$$\mathcal{E}_{\mathrm{CSB}} = C_{\mathrm{CSB}}^{\rho}\left(\rho_n^2 - \rho_p^2\right) + C_{\mathrm{CSB}}^{\tau}\left(\rho_n\tau_n - \rho_p\tau_p\right) + C_{\mathrm{CSB}}^{\Delta\rho}\left(\rho_n\,\Delta\rho_n - \rho_p\,\Delta\rho_p\right)$$





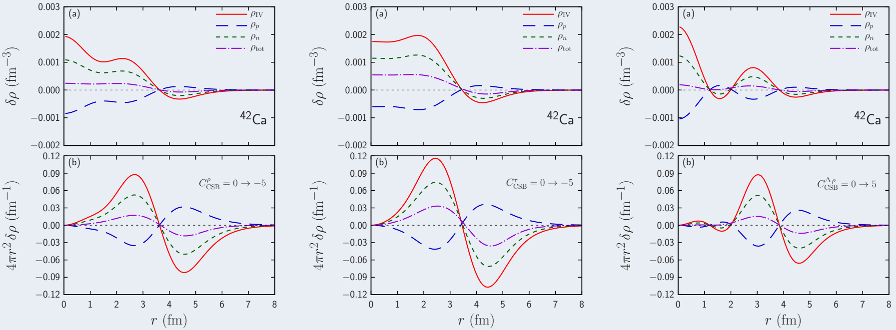


- Mass difference of mirror nuclei is not sensitive to $C_{\mathrm{CSB}}^{\Delta\rho}$
- C_{CSB}^{ρ} dependence is quite similar to C_{CSB}^{τ}

New Observable to Pin Down CSB Strength

Can we determine the form of CSB EDF in density?

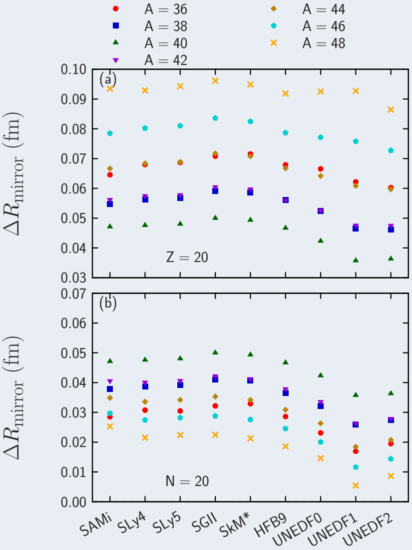
$$\mathcal{E}_{\mathrm{CSB}} = C_{\mathrm{CSB}}^{\rho}\left(\rho_n^2 - \rho_p^2\right) + C_{\mathrm{CSB}}^{\tau}\left(\rho_n \tau_n - \rho_p \tau_p\right) + C_{\mathrm{CSB}}^{\Delta\rho}\left(\rho_n \Delta\rho_n - \rho_p \Delta\rho_p\right)$$

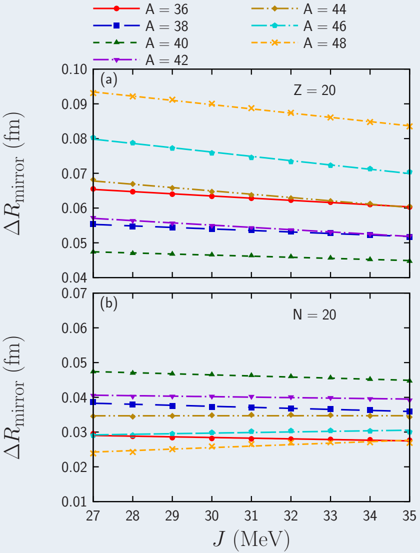


- Different term changes densities in different way
- In reallity, "experimental data without CSB term" does not exist
  → Change of CSB for the observable is a future perspective

New Observable to Pin Down CSB Strength

Mirror-skin thickness





QCD Sum Rule Approach for CSB EDF

Motivation

Determine strengths of CSB effective interaction by a fundamental theory

QCD Sum Rule (QSR)

QCD in short-range region	Perturbative approach is available (Asymptotic freedom)
QCD in long-range region	Non-perturbative effect is non-negligible

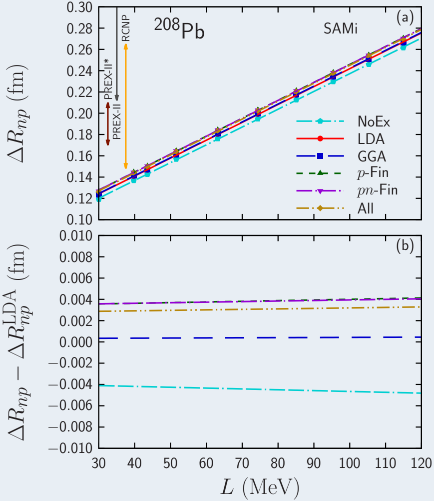
- In QSR, non-perturbative effect is considered starting from
  - perturbative calculation in short-range region
  - operator product expansion in long-range region
- QSR has been extended to the finite ρ and finite T

Shifman, Vainshtein, and Zakharov. *Nucl. Phys. B* **147**, 385 (1979)

Review: Dey and Dey. *Nuclear and Particle Physics* Chap. 9 (Springer 1993)

Neutron-Skin Thickness and Charge Radii Difference

Coulomb Effect on Neutron-Skin Thickness



- ISB terms are not considered
- Treatment of Coulomb int. does not change the slope of L-ΔR_{np} dependence
- Absolute value of ΔR_{np} changes, but its value is tiny $\rightarrow$ negligible

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **107**, 064302 (2023)