Nuclear equation of state and neutrino reaction rates in dense astrophysical environments

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Outline

1 : Introduction

2 : Nuclear EOS with the cluster variational method

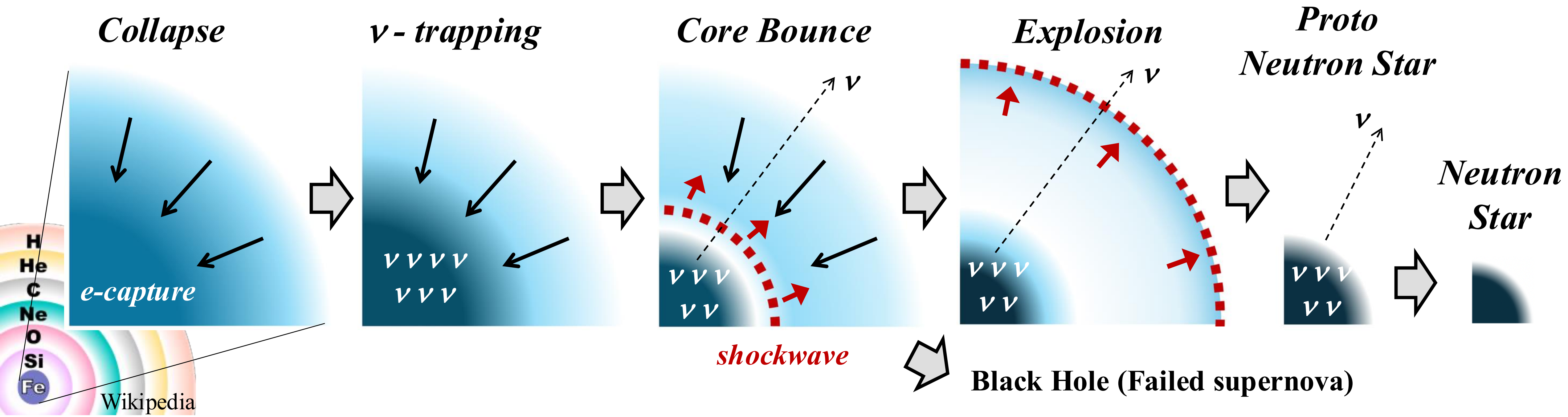
3: Supernova simulation with the variational EOS

4 : Neutrino-nucleon reaction rates consistent with the EOS

1. Introduction

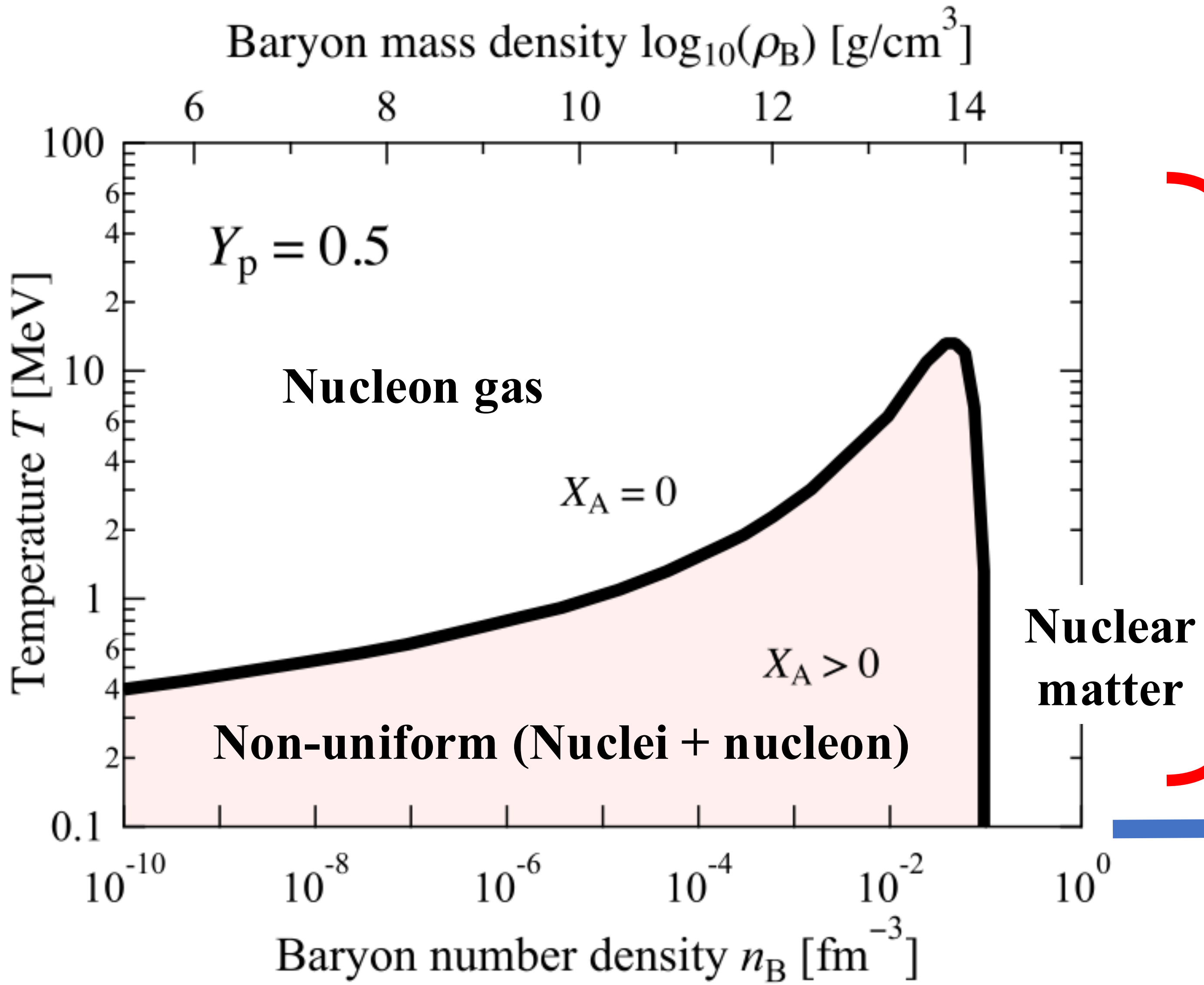
- **Nuclear equation of state (EOS) for dense matter**
- **Nuclear weak reactions with neutrino**

Core-collapse mechanism of massive stars



- **Gravitational energy released by core-collapse: $\sim 10^{53}$ erg**
- **Explosion energy (Kinetic energy of mass ejecta): $\sim 10^{51}$ erg**

Nuclear EOS for core-collapse simulations



Phase diagram of nuclear matter [based on NPA 961 (2017) 78]

Supernova & PNS ($T > 0$)

- Temperature T : $0 \leq T \leq 400$ MeV
- Density ρ : $10^{5.1} \leq \rho_B \leq 10^{16.0}$g/cm^3
- Proton fraction Y_p : $0 \leq Y_p \leq 0.65$

Neutron Star ($T = 0$)

Representative EOS for core-collapse simulations

(Rev. Mod. Phys. 89 (2017) 015007)

Model	Nuclear Interaction	Degrees of Freedom	$M_{\max}$ ($M_\odot$)	$R_{1.4M_\odot}$ (km)	Ξ	publ. avail.	References
H&W	SKa	$n, p, \alpha, \{(A_i, Z_i)\}$	2.21^a	13.9^a		n	El Eid and Hillebrandt (1980); Hillebrandt *et al.* (1984)
LS180	LS180	$n, p, \alpha, (A, Z)$	1.84	12.2	0.27	y	Lattimer and Swesty (1991)
LS220	LS220	$n, p, \alpha, (A, Z)$	2.06	12.7	0.28	y	Lattimer and Swesty (1991)
LS375	LS375	$n, p, \alpha, (A, Z)$	2.72	14.5	0.32	y	Lattimer and Swesty (1991)
STOS	TM1	$n, p, \alpha, (A, Z)$	2.23	14.5	0.26	y	Shen *et al.* (1998); Shen *et al.* (1998, 2011)
FYSS	TM1	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.22	14.4	0.26	n	Furusawa *et al.* (2013b)
HS(TM1)	TM1*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.21	14.5	0.26	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(TMA)	TMA*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.02	13.9	0.25	y	Hempel and Schaffner-Bielich (2010)
HS(FSU)	FSUgold*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.74	12.6	0.23	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(NL3)	NL3*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.79	14.8	0.31	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(DD2)	DD2	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.42	13.2	0.30	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(IUFSU)	IUFSU*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.95	12.7	0.25	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
SFHo	SFHo	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.06	11.9	0.30	y	Steiner *et al.* (2013a)
SFHx	SFHx	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.13	12.0	0.29	y	Steiner *et al.* (2013a)
SHT(NL3)	NL3	$n, p, \alpha, \{(A_i, Z_i)\}$	2.78	14.9	0.31	y	Shen *et al.* (2011b)
SHO(FSU)	FSUgold	$n, p, \alpha, \{(A_i, Z_i)\}$	1.75	12.8	0.23	y	Shen *et al.* (2011a)
SHO(FSU2.1)	FSUgold2.1	$n, p, \alpha, \{(A_i, Z_i)\}$	2.12	13.6	0.26	y	Shen *et al.* (2011a)

+ Nuclear EOS tables based on the Liquid drop model with **Skyrme interaction** by A. S. Schneider (2017)

Microscopic EOS with bare nuclear potentials

- **TNTYST EOS: Variational (AV18 +UIX) + Thomas-Fermi method** (NPA 961 (2017) 78)

Current Status of Supernova Simulations

EOS dependence in core-collapse supernova simulations is gradually being investigated.

- More than 20 EOSs applicable to the simulations
(only two EOSs were available about 10 years ago)

TODAY'S TALK

- **Overview of our supernova EOS with the variational many-body theory**
- **Its application to neutron star and core-collapse supernova simulations**
- **Recent improvement in neutrino reaction rates**

2. Nuclear EOS with the cluster variational method

Nuclear Hamiltonian

$$H = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i<j}^{N} V_{ij} + \sum_{i<j<k}^{N} V_{ijk}$$

Argonne v18 (AV18) pot.

Urbana IX (UIX) pot.

- **Argonne v18 potential:**
  (PRC 51 (1995) 38)

$$V_{ij} = \sum_{t=0}^{1}\sum_{s=0}^{1}[V_{\mathrm{C}ts}(r_{ij}) + sV_{\mathrm{T}t}(r_{ij})S_{\mathrm{T}ij} + sV_{\mathrm{SO}t}(r_{ij})(\boldsymbol{L}_{ij}\cdot\boldsymbol{s}) + V_{\mathrm{qL}ts}(r_{ij})\left|\boldsymbol{L}_{ij}\right|^2 + sV_{\mathrm{qSO}t}(r_{ij})(\boldsymbol{L}_{ij}\cdot\boldsymbol{s})^2]P_{tsij}$$

- **Urbana IX potential:**
  (PRL 74 (1995) 4396)

$$V_{ijk} = U\sum_{\mathrm{cyc}}[T(r_{ij})]^2[T(r_{ik})]^2 + A\sum_{\mathrm{cyc}}\left[\{x_{ij},x_{ik}\}\{\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k\} + \frac{1}{4}[x_{ij},x_{ik}][\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k]\right]$$

$$x_{ij} = Y(r_{ij})\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + T(r_{ij})S_{\mathrm{T}ij}$$

Energy per nucleon for uniform matter

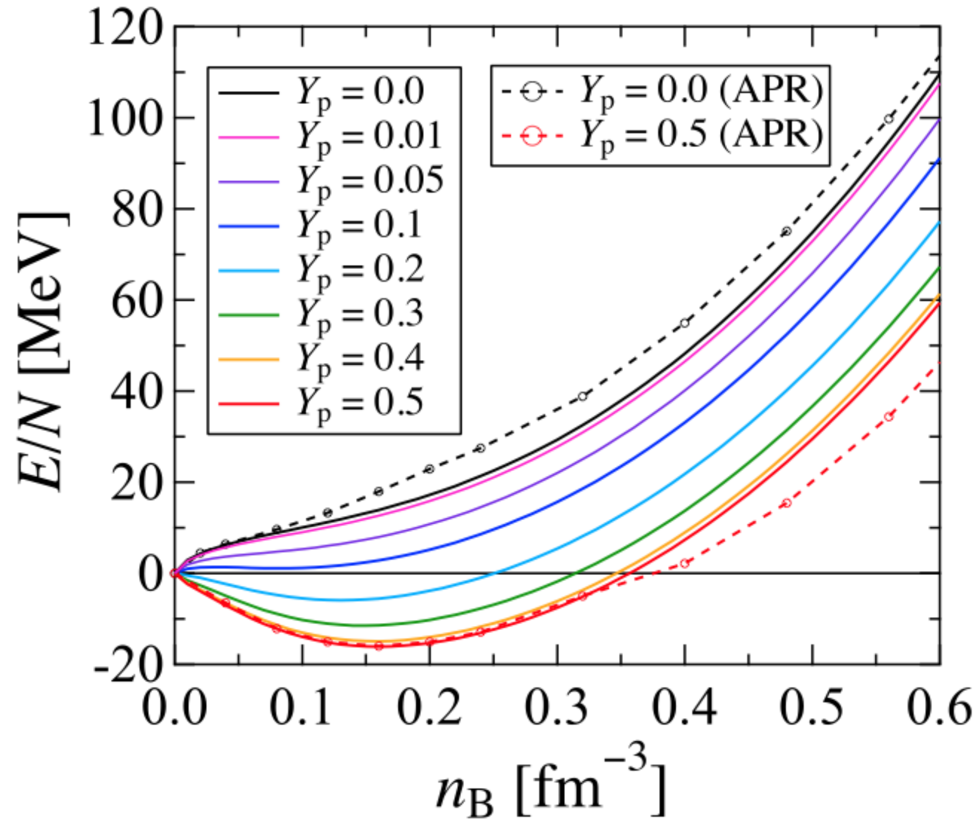
Jastrow wave function

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right] \Phi_F$$

Φ_F: The Fermi-gas wave function

Two-body correlation function: $$f_{ij} = \sum_{t=0}^{1} \sum_{\mu} \sum_{s=0}^{1} \left[f^{\mu}_{Cts}(r_{ij}) + s f^{\mu}_{Tt}(r_{ij}) S_{Tij} + s f^{\mu}_{SOt}(r_{ij}) (\boldsymbol{L}_{ij} \cdot \mathbf{s}) \right] P^{\mu}_{tsij}$$

t : Total isospin μ: 3rd component of t s : Total spin



The two-body cluster approximation is adopted to obtain the energy per nucleon.

n_0[fm^{-3}]	E_0[MeV]	K [MeV]	S [MeV]	L [MeV]
0.16	–16.1	245	30.0	38.7

Our EOS : HT and M. Takano, NPA 902 (2013) 53

APR : A. **A**kmal, V. R. **P**andharipande, D. G. **R**avenhall, PRC 58 (1998) 1804

Nuclear EOS at finite T and low density

- **Finite temperature**

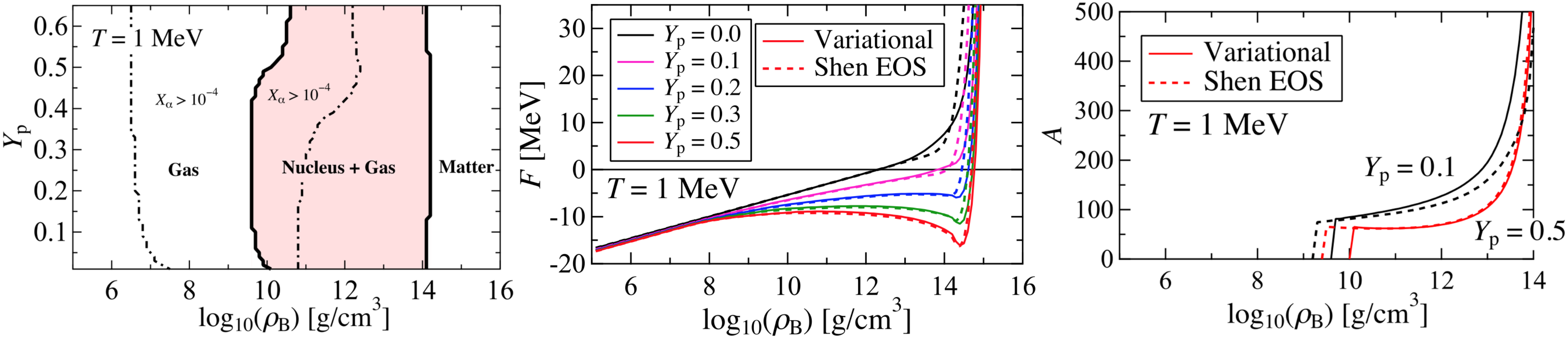
Free energy at finite temperature *F/N* is calculated with the variational method proposed by Schmidt and Pandharipande.

(Phys. Lett. 87B(1979) 11, PRC 75(2007) 035802))

- **Non-uniform phase**

Thomas-Fermi approximation is adopted to calculate the heavy nuclei in low-density nuclear matter.

NPA 961 (2017) 78



Home Page of Variational EOS Table

http://www.np.phys.waseda.ac.jp/EOS/

Equation of state for nuclear matter w

Equation of state (EOS) based on the variational man
matter, the EOS is constructed with the cluster variat
potential and the Urbana IX three-body nuclear poten
approximation. Alpha particle mixing is also taken int
This EOS table is open for general use in any studies
referred to in your publication.

User's Guide (read m...

guide.pdf

User's Guide

User Note for the Variational EOS Table

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Abstract

This is a guide for users of the nuclear equation of state (EOS) table based on the Argonne v18 two-body and Urbana IX three-body potentials. We construct the nuclear EOS using the cluster variational method for uniform matter and the Thomas-Fermi calculation for non-uniform matter.

EOS tables

eoszip

Numerical Data

```
cccccccccccc
 Log10(Temp)    Temp
-1.000000E+00  1.000000E-01
```

5.100E+00	7.581427E-11	1.000E-02	-1.516998E+00	7.968970E+00	1.427632E+01	1.004453E+02
5.200E+00	9.544451E-11	1.000E-02	-1.494684E+00	7.968916E+00	1.405264E+01	1.005867E+02
5.300E+00	1.201575E-10	1.000E-02	-1.472371E+00	7.968862E+00	1.382897E+01	1.007294E+02
5.400E+00	1.512693E-10	1.000E-02	-1.450059E+00	7.968809E+00	1.360532E+01	1.008713E+02
5.500E+00	1.904368E-10	1.000E-02	-1.427748E+00	7.968757E+00	1.338169E+01	1.010151E+02
5.600E+00	2.397458E-10	1.000E-02	-1.405439E+00	7.968705E+00	1.315807E+01	1.011550E+02
5.700E+00	3.018220E-10	1.000E-02	-1.383130E+00	7.968653E+00	1.293447E+01	1.013009E+02

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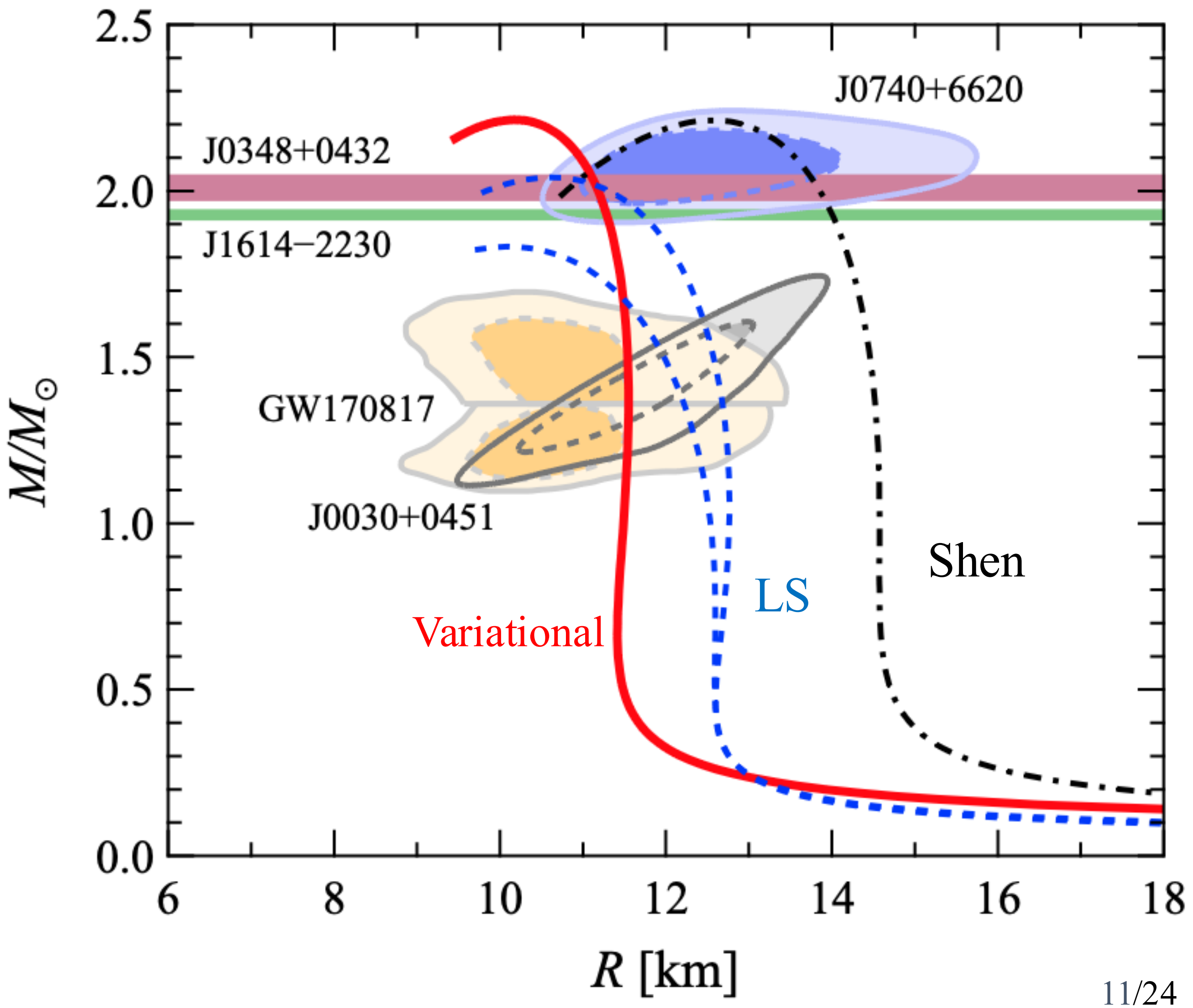
3. Supernova simulation with standard reaction rates

Core-collapse supernova simulations are performed with the following three nuclear EOSs.

- **Variational:** Variational method (AV18+UIX) + Thomas-Fermi calculation
- **LS EOS:** Skyrme Hartree-Fock + Compressible Liquid Drop model
- **Shen EOS:** Relativistic Mean Field (TM1) + Thomas-Fermi calculation



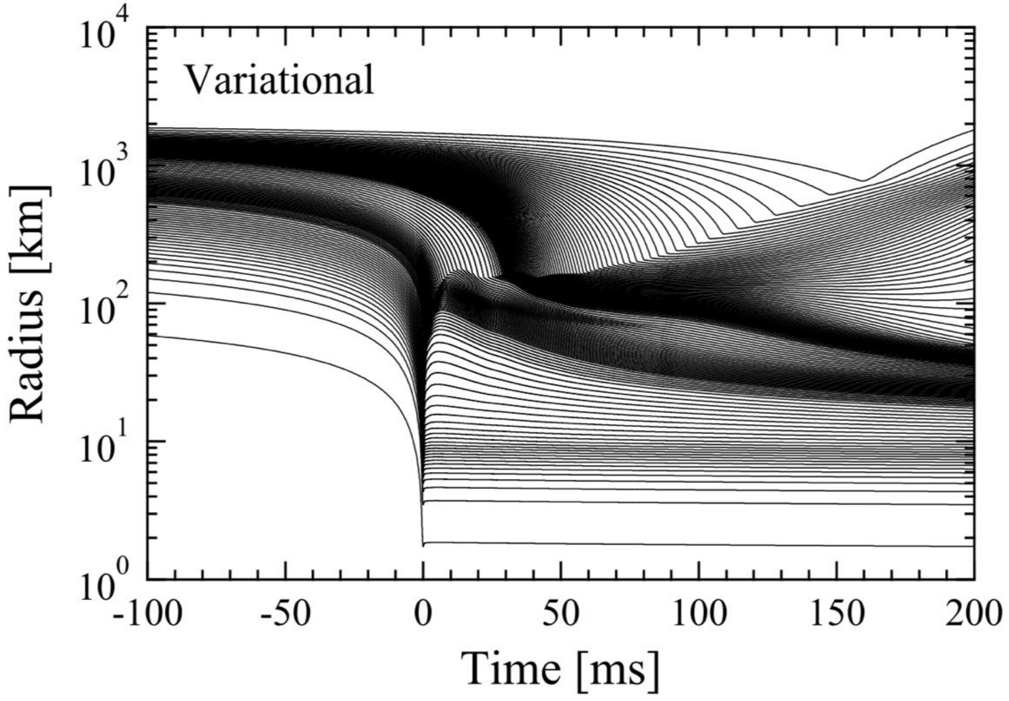
Application to core-collapse simulations

1D neutrino-radiation hydrodynamics simulations

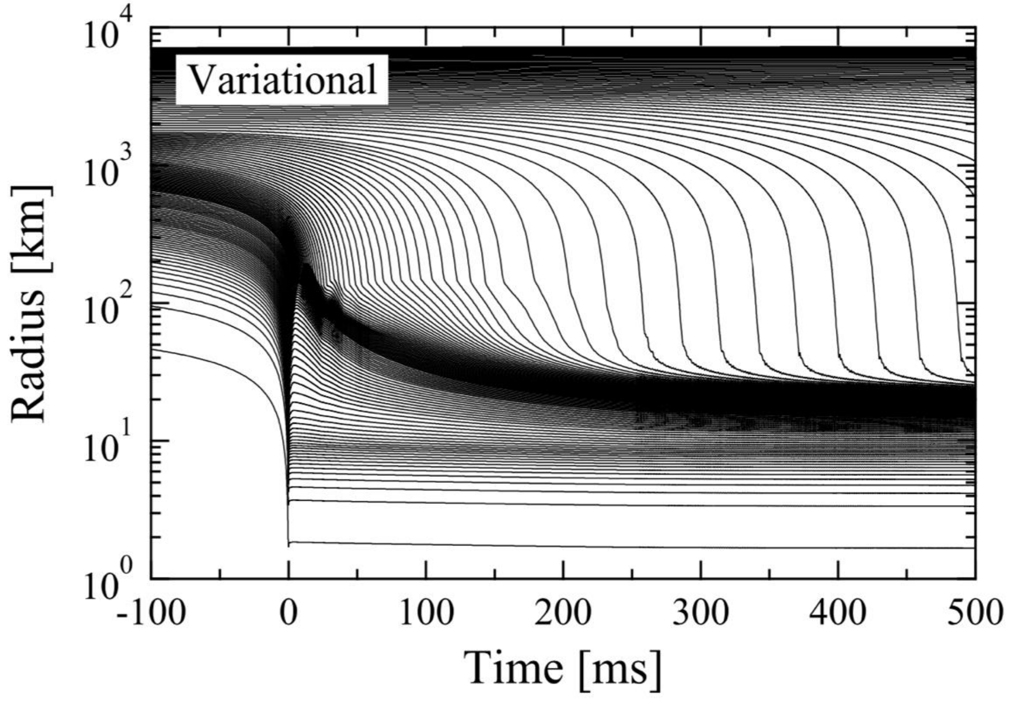
(HT et al., PTEP 023D05 (2014))
(Nakazato, Sumiyoshi & HT, PASJ 73 (2021) 639)

- **EOS: Togashi / Shen / LS220 / LS180**
- Progenitor model : 9.6 $M_\odot$ / 15 $M_\odot$ / 30 $M_\odot$
- Neutrino Transport: Directly solve the Boltzmann equation

Progenitor model : 9.6 $M_\odot$
(provided by A. Heger)



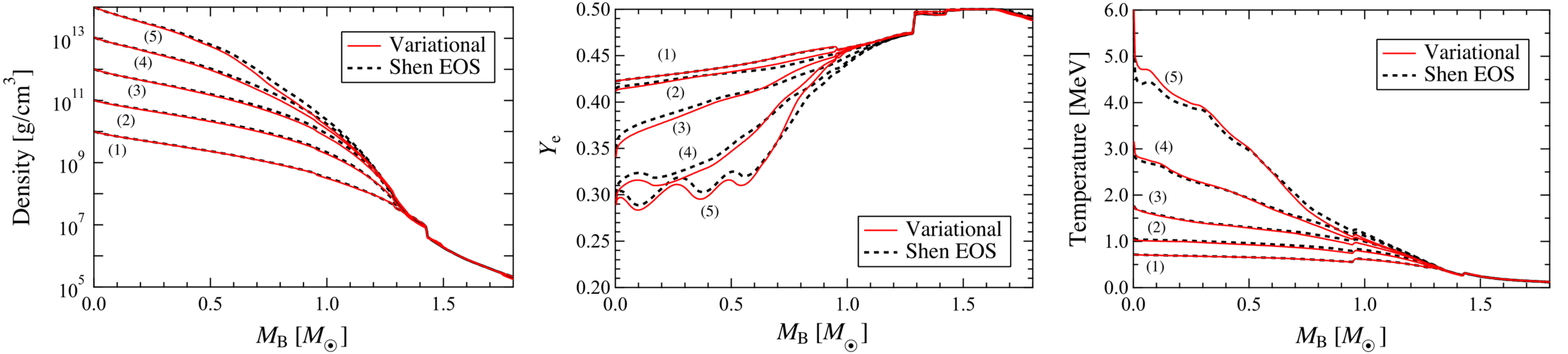
Progenitor model: WW 15$M_\odot$
(Astrophys. J. Suppl. 101 (1995) 181)

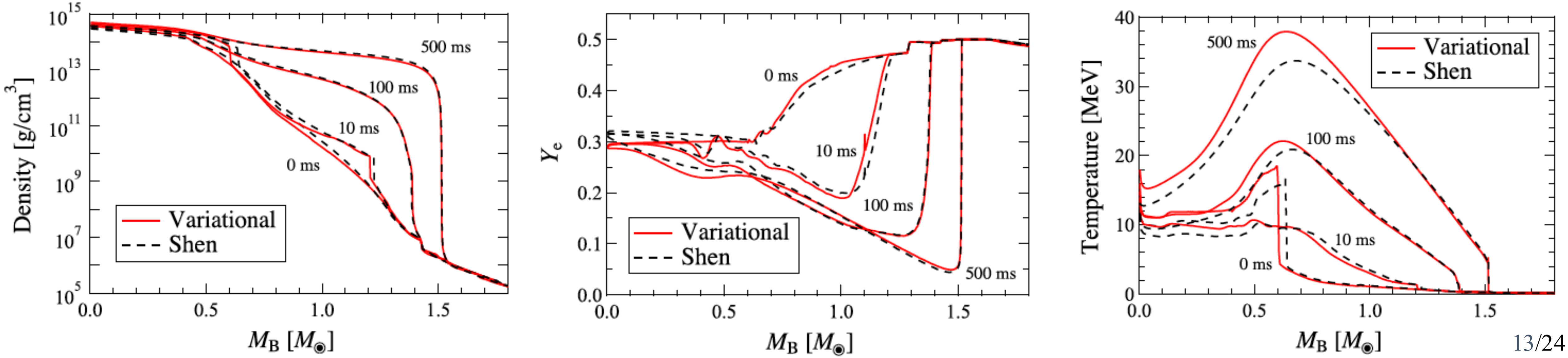


Radial trajectories of mass elements

Thermodynamic profiles in SN simulations

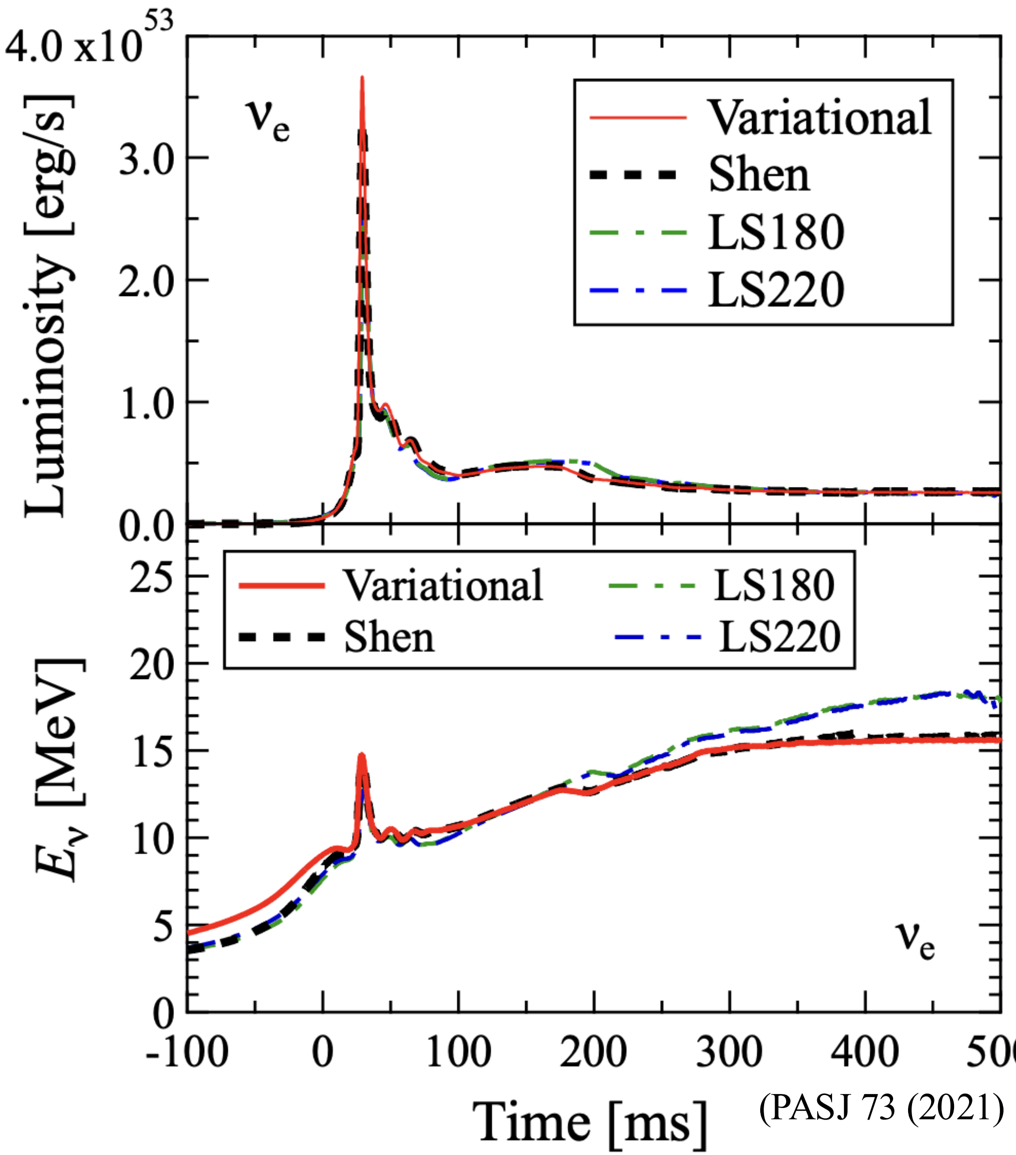
The numbers (1)–(5) :the times when the central density reaches 10^{10}, 10^{11}, 10^{12}, 10^{13}, 10^{14} g/cm^3





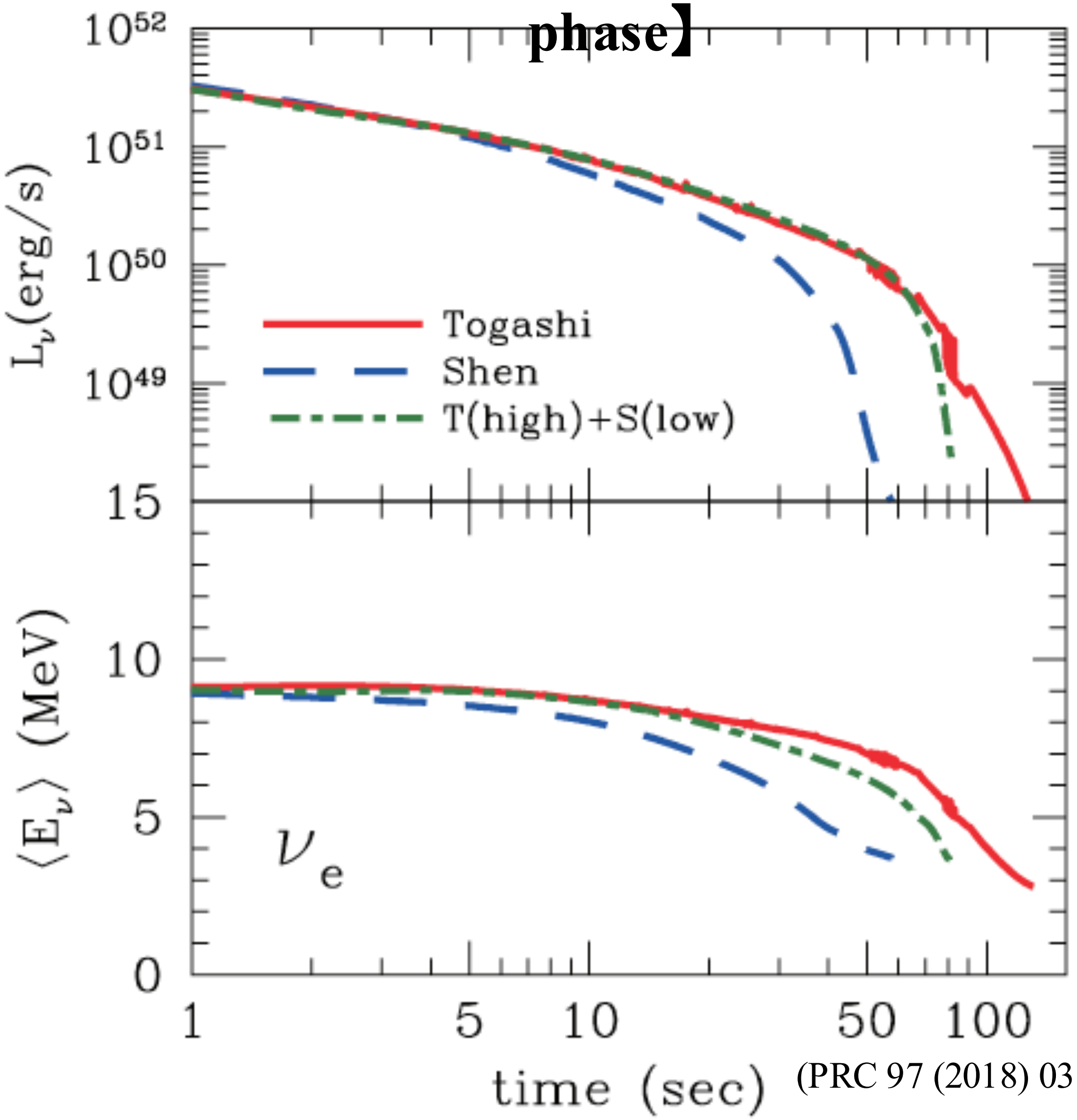
Neutrino Luminosity and Average Energy

【Post-bounce phase】



(PASJ 73 (2021) 639)

【Proto-neutron star cooling phase】



(PRC 97 (2018) 035804)

4. Neutrino-nucleon reaction rates consistent with the EOS

Effect of the nuclear EOS is not so large in emitted neutrino from 1D core-collapse supernova simulation. (~1s)

For a more sophisticated simulations to understand the supernova mechanism, we aim to construct nuclear reaction rates with neutrino in a self-consistent manner.

Difficulties in applying to astrophysical simulations

- Wide range of T, Y_p, n_B and neutrino energy E_ν
- Interaction rates need to be given in analytical forms!

(Computer memory is already fully occupied by the EOS table)

Neutrino reaction rates adopted in the simulations

1. Electron-type neutrino absorption on neutrons

$$\nu_e + n \leftrightarrow e^- + p$$

2. Electron-type antineutrino absorption on protons

$$\bar{\nu}_e + p \leftrightarrow e^+ + n$$

3. Neutrino bremsstrahlung in NN collisions

$$N + N' \leftrightarrow N + N' + \nu + \bar{\nu}$$

4. Neutrino scattering on nucleons

$$\nu + N \leftrightarrow \nu + N$$

5. Electron-type neutrino absorption on nuclei

$$\nu_e + A \leftrightarrow A + e^-$$

6. Neutrino coherent scattering on nuclei

$$\nu + A \leftrightarrow \nu + A$$

7. Neutrino scattering on electrons

$$\nu + e^- \leftrightarrow \nu + e^-$$

8. Electron-positron pair annihilation and creation

$$e^- + e^+ \leftrightarrow \nu + \bar{\nu}$$

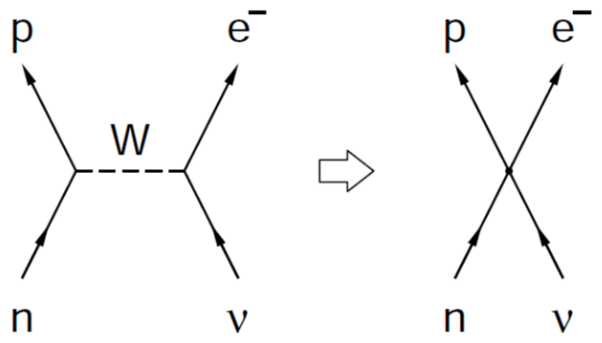
9. Plasmon decay and creation

We aim to calculate the neutrino reaction rates in a nuclear medium by using the cluster variational method.

I. Neutrino Charged Current Reactions on Nucleons

1. Electron-type neutrino absorption on neutrons:

$$\nu_e + n \leftrightarrow e^- + p$$



Emissivity [s^{-1}]:

$$\epsilon(n_B, Y_p, T, E_\nu) = 2\hbar^2 G_F^2 \cos^2\theta_c (1+3g_A^2)(2\pi)^4 \int \frac{d\boldsymbol{p}_n}{(2\pi\hbar)^3} \int \frac{d\boldsymbol{p}_p}{(2\pi\hbar)^3} \int \frac{d\boldsymbol{p}_e}{(2\pi\hbar)^3}$$
$$\times\ \delta(E_p + E_e - E_n - E_\nu)\delta^3(\boldsymbol{p}_p + \boldsymbol{p}_e - \boldsymbol{p}_n - \boldsymbol{p}_\nu) f_n(E_n)\,[1 - f_p(E_p))]\,[1 - f_e(E_e))]$$

Occupation probabilities obtained from the variational method are applied.

$$f_i(k) = \left\{1 + \exp\left[\frac{\varepsilon_i(k) - \mu_{0i}}{k_B T}\right]\right\}^{-1} \qquad \varepsilon_i(k) = \frac{\hbar^2 k^2}{2m_i^*}$$

μ_{0i} : Determined by the normalization condition

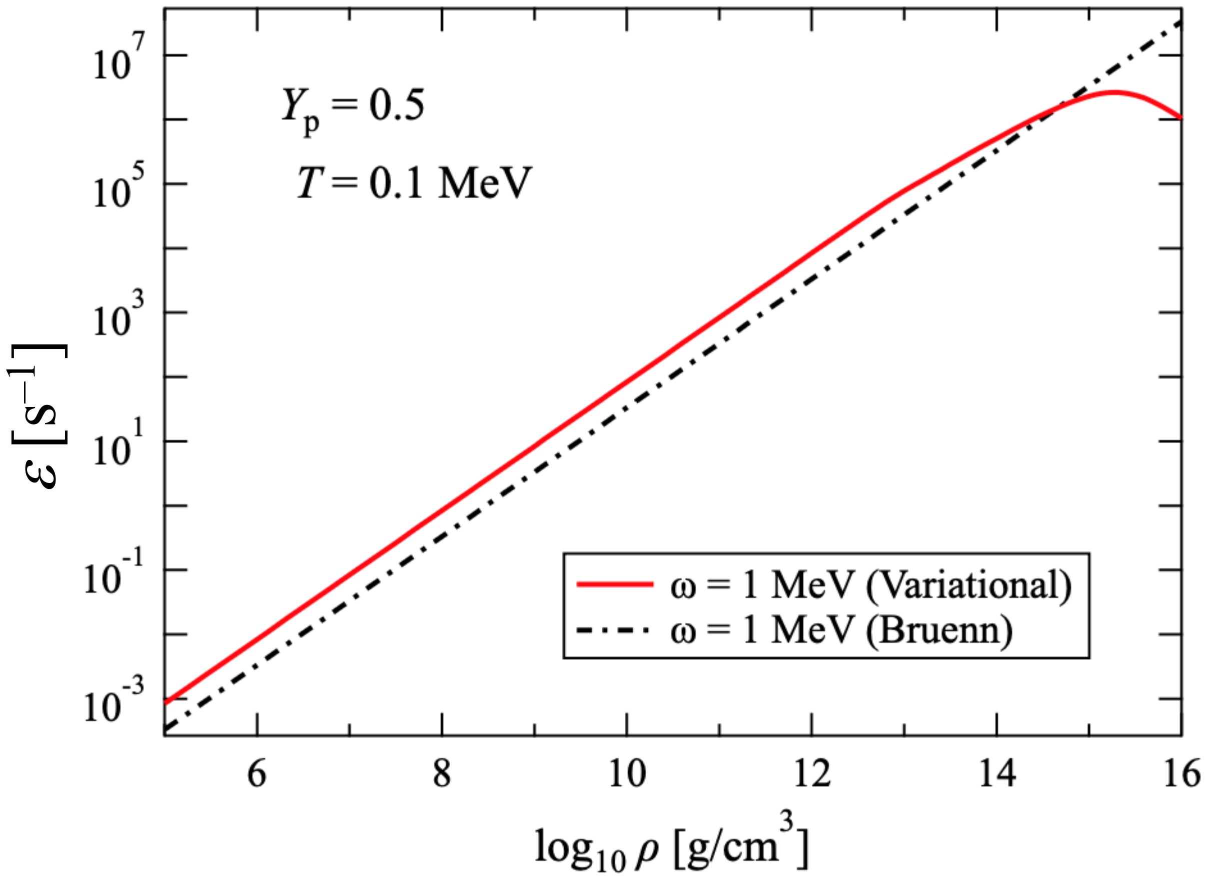
m_i* : Variational parameter

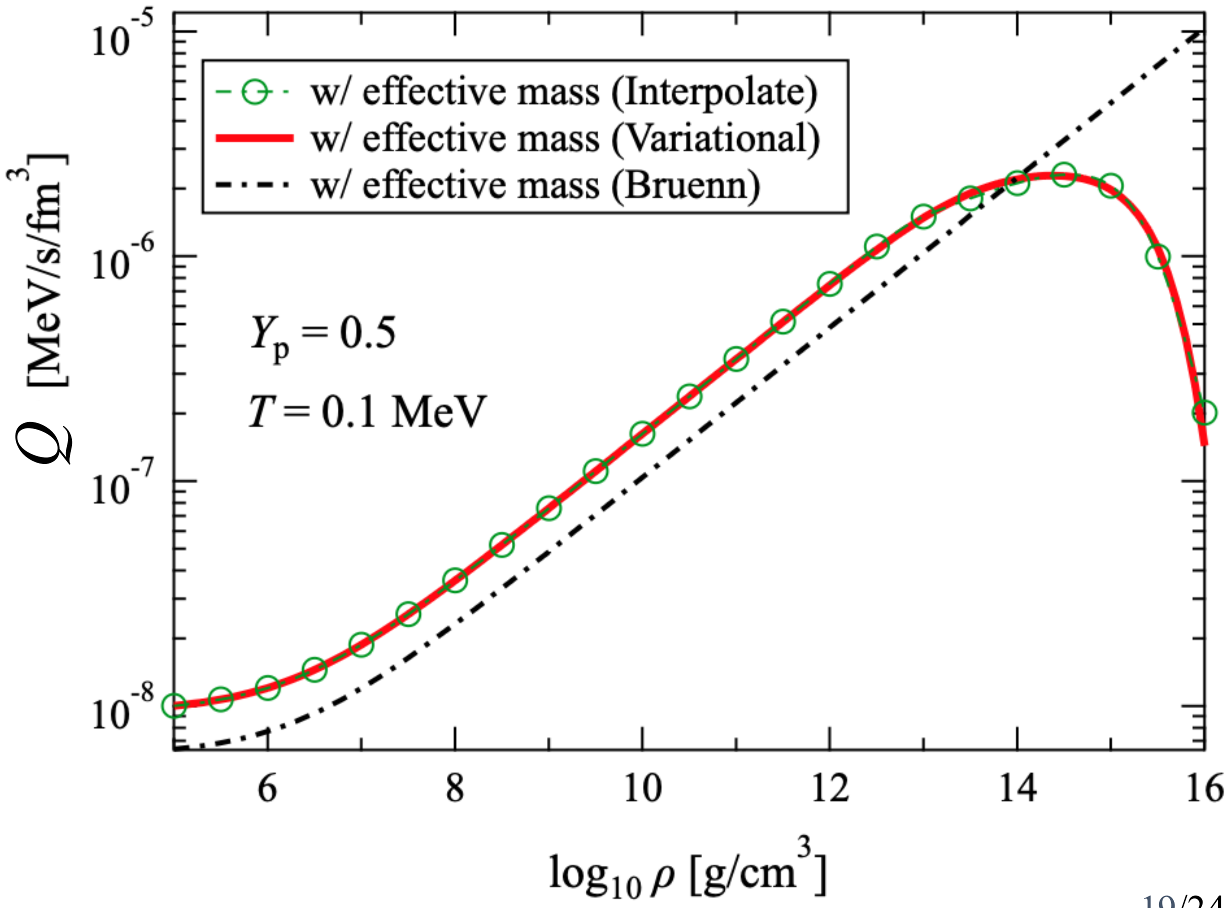
Neutrino Emissivity with the Variational Method

Procedure by Bruenn (APJS 58 (1985) 771)

- $\delta^3(\boldsymbol{p}_\mathrm{p} + \boldsymbol{p}_\mathrm{e} - \boldsymbol{p}_\mathrm{n} - \boldsymbol{p}_\nu) \sim \delta^3(\boldsymbol{p}_\mathrm{p} - \boldsymbol{p}_\mathrm{n})$
- $m_\mathrm{p}^* \sim m_\mathrm{n}^*$

$$Q(n_\mathrm{B}, Y_\mathrm{p}, T) = \int \frac{d\boldsymbol{p}_\nu}{(2\pi\hbar)^3} \epsilon(n_\mathrm{B}, Y_\mathrm{p}, T, E_\nu) E_\nu f_\nu(E_\nu)$$





Summary

Nuclear EOS and **Neutrino Reaction Rates** are consistently developed based on the cluster variational method for core-collapse supernova simulations.

- The EOS dependence in core-collapse supernova simulations is gradually being investigated.
- Uncertainty in neutrino reaction rates in nuclear matter is still large.
- Nuclear medium effects on neutrino emissivity becomes relatively large at high-density region.

Future Plans

- Analytical expression of the interaction rates
- Supernova simulations with the obtained neutrino reaction rates