Dense QCD and neutron stars:
quark deconfinement
at high temperature and baryon density

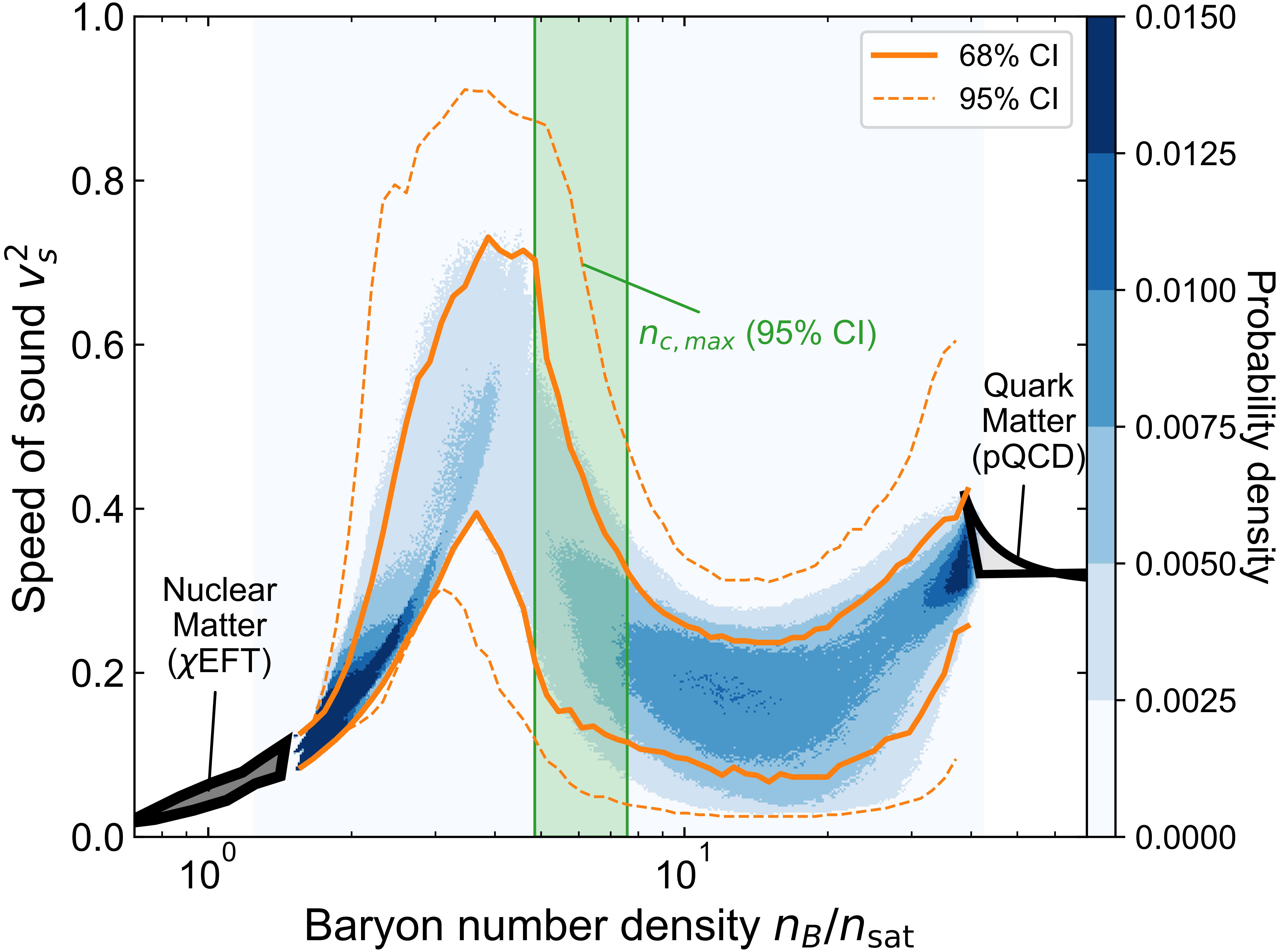
Yuki Fujimoto
(Niigata U)

References:
[1] Y. Fujimoto, K. Fukushima, Y. Hidaka, L. McLerran, PRD112 (2025)
[2] Y. Fujimoto, T. Kojo, L. McLerran, PRL132 (2024); arXiv:2410.22758.

December 18, 2025 - Workshop on recent developments from QCD to nuclear matter, Academia Sinica

Emerging picture of neutron star EoS

Figure based on method by Altiparmak,Ecker,Rezzolla (2022)

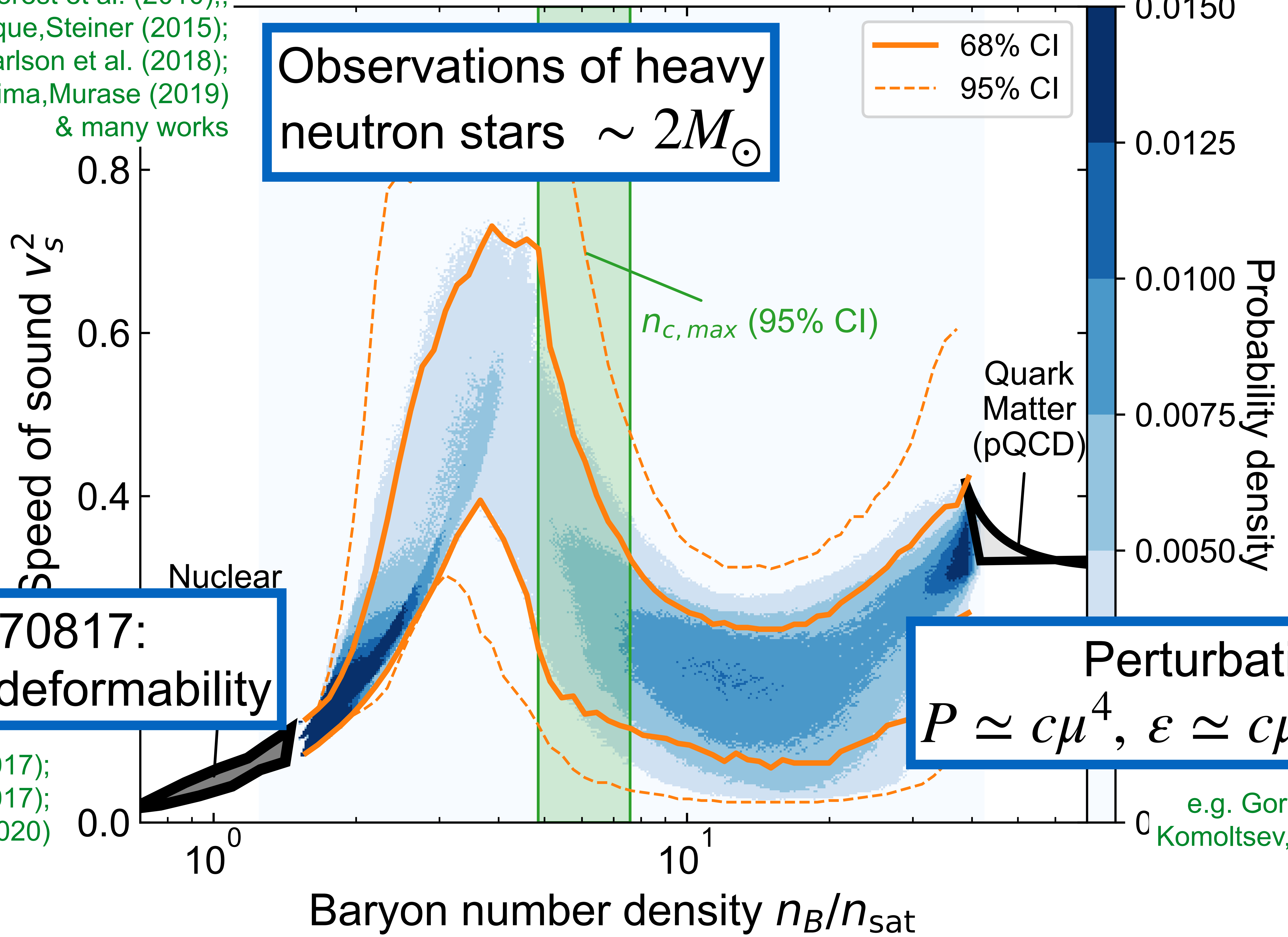


Emerging picture of neutron star EoS

Figure based on method by Altiparmak,Ecker,Rezzolla (2022)

Demorest et al. (2010);;
Bedaque,Steiner (2015);
Tews, Carlson et al. (2018);
Fujimoto,Fukushima,Murase (2019)
& many works

Observations of heavy neutron stars $\sim 2M_\odot$



GW170817: small tidal deformability

LIGO-Virgo (2017);
e.g. Annala et al. (2017);
Drischler, Han et al. (2020)

Perturbative QCD

$$P \simeq c\mu^4,\ \varepsilon \simeq c\mu^4/3,\ n \simeq 4c\mu^3$$

e.g. Gorda,Komoltsev,Kurkela (2022);
Komoltsev, Somasundaram et al. (2023)

Emerging picture of neutron star EoS

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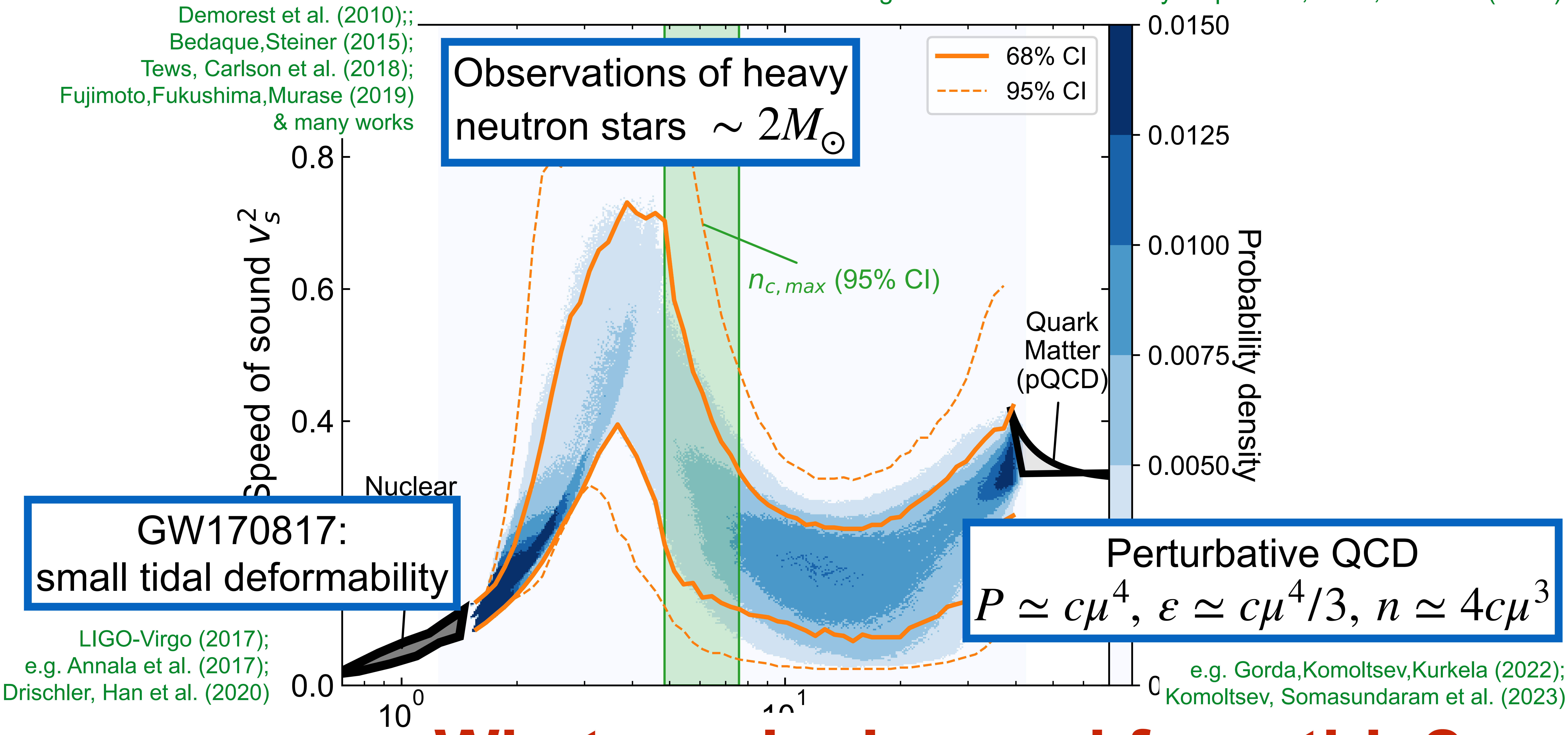

What can be learned from this?

Outline

1. Revised view on quark deconfinement in crossover at high T
2. Quark (de)confinement at large density and quarkyonic matter

1. Revised view on quark deconfinement in crossover at high *T*

Quark deconfinement is hard to capture as there is
no well-defined order parameter for deconfinement with dynamical quarks present

Deconfinement in EoS of hadrons & resonances

Schematic behavior of high-temperature equation of state:

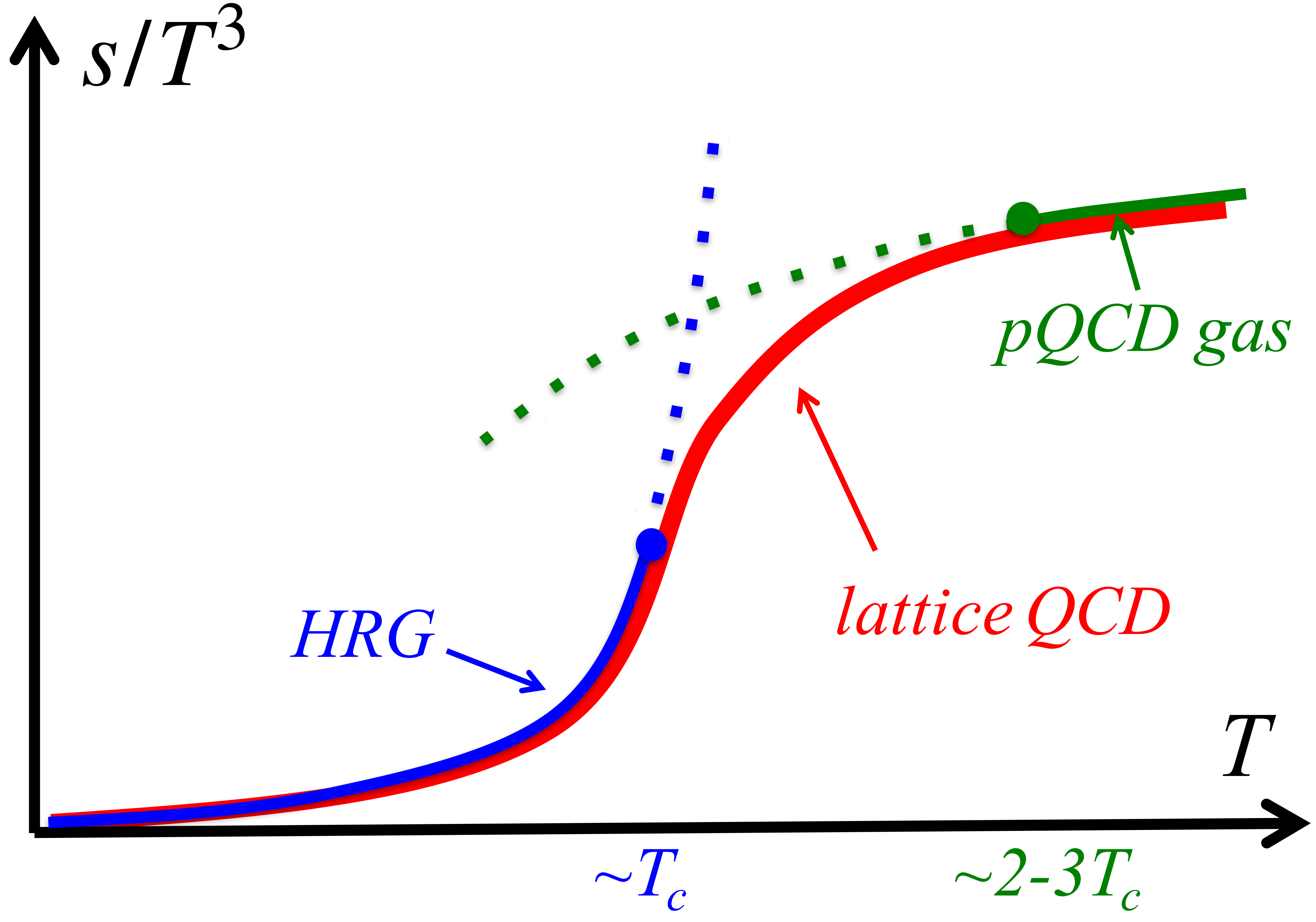

Figure adapted from: Baym,Hatsuda,Kojo,Powell,Song,Takatsuka (2017)

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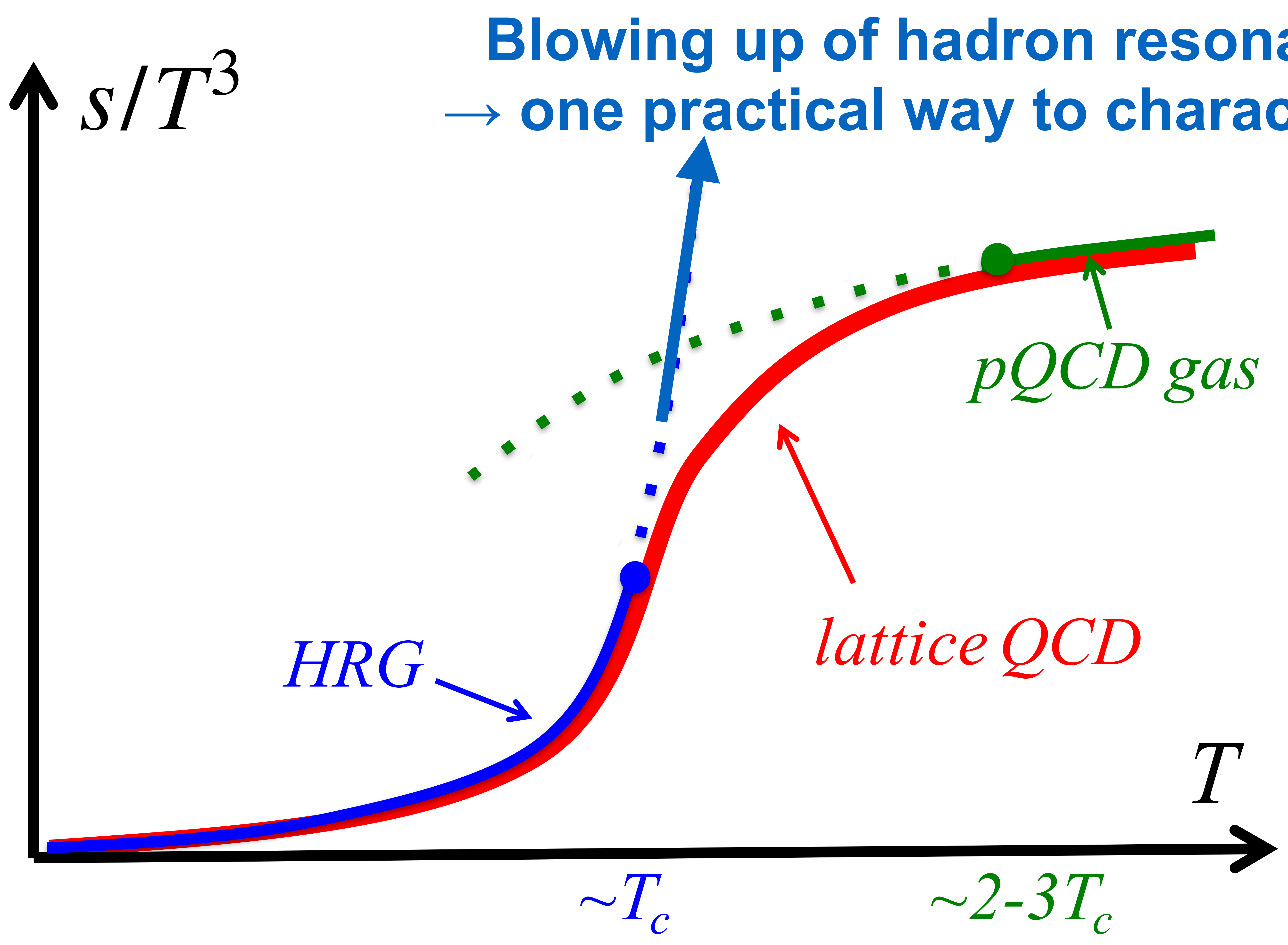

Figure adapted from: Baym,Hatsuda,Kojo,Powell,Song,Takatsuka (2017)

Hagedorn temperature

Partition function: $Z \propto \int_{m_0}^{\infty} dm\, \rho(m) e^{-m/T}$,

$$\rho(m) \propto m^a \exp(m/T_\mathrm{H})$$

T_H: Hagedorn's limiting temperature

Hagedorn (1965)

T_H … Later reinterpreted as temperature of quark-gluon liberation

Cabibbo & Parisi (1975)

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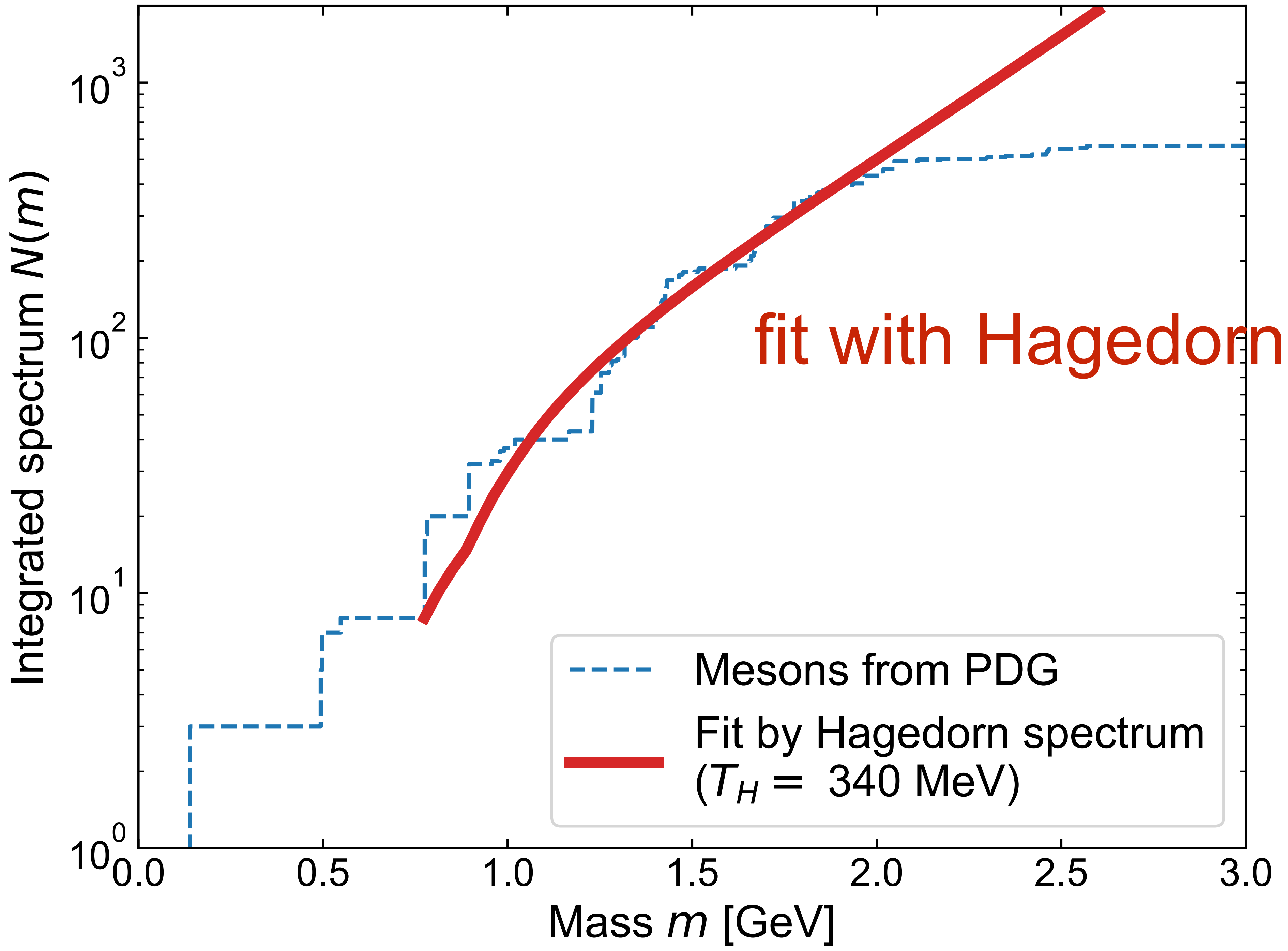
T_H: Hagedorn's limiting temperature

Hagedorn (1965)

Integrated mass spectrum
of hadrons & resonances in PDG:



fit with Hagedorn spectrum

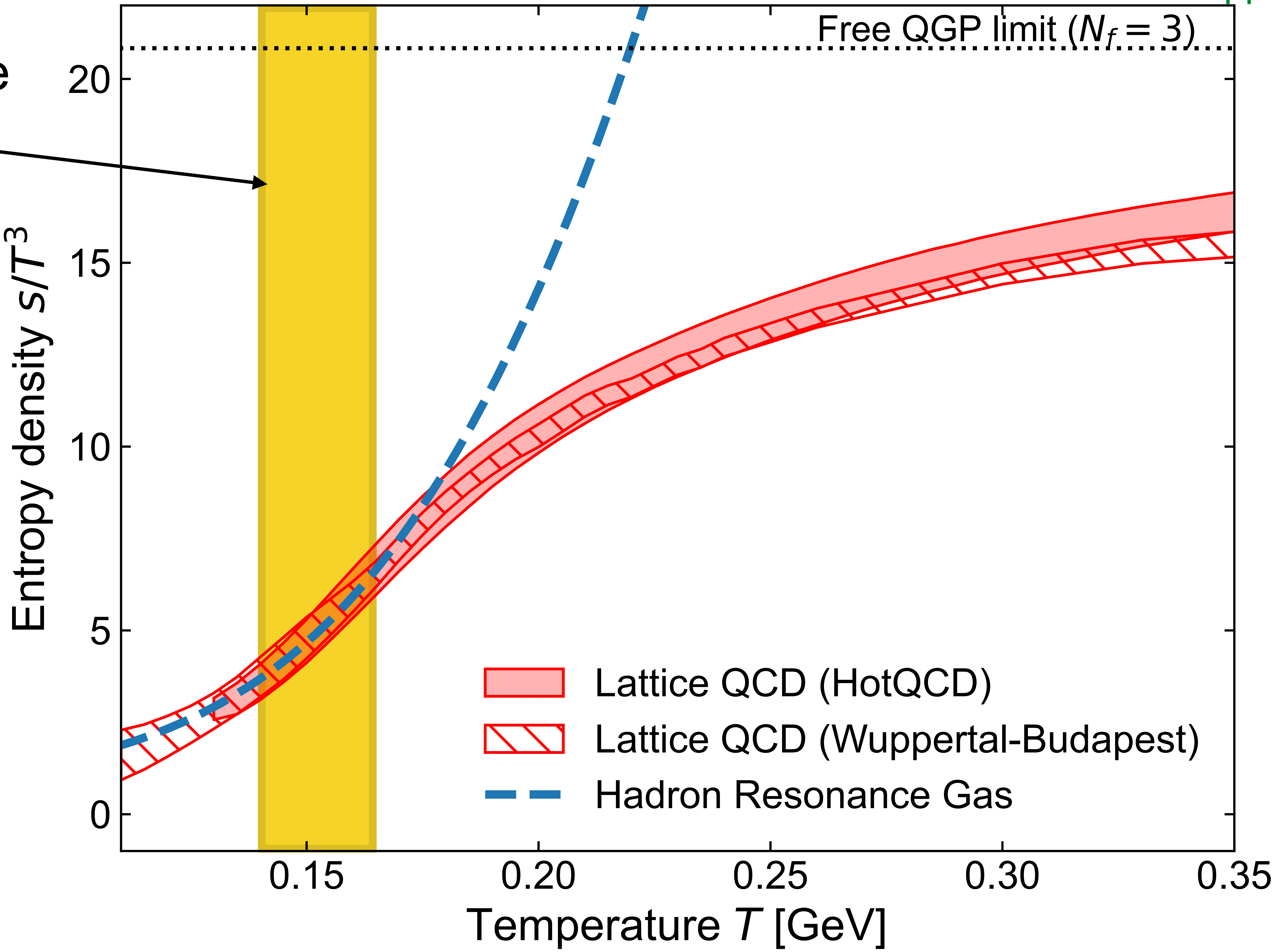
T_H … Later reinterpreted as
temperature of quark-gluon liberation

Cabibbo & Parisi (1975)

Lattice QCD equation of state

Wuppertal-Budapest (2014);
HotQCD (2014)

Critical temperature
for chiral transition
& deconfinement



Lattice QCD equation of state

Wuppertal-Budapest (2014);
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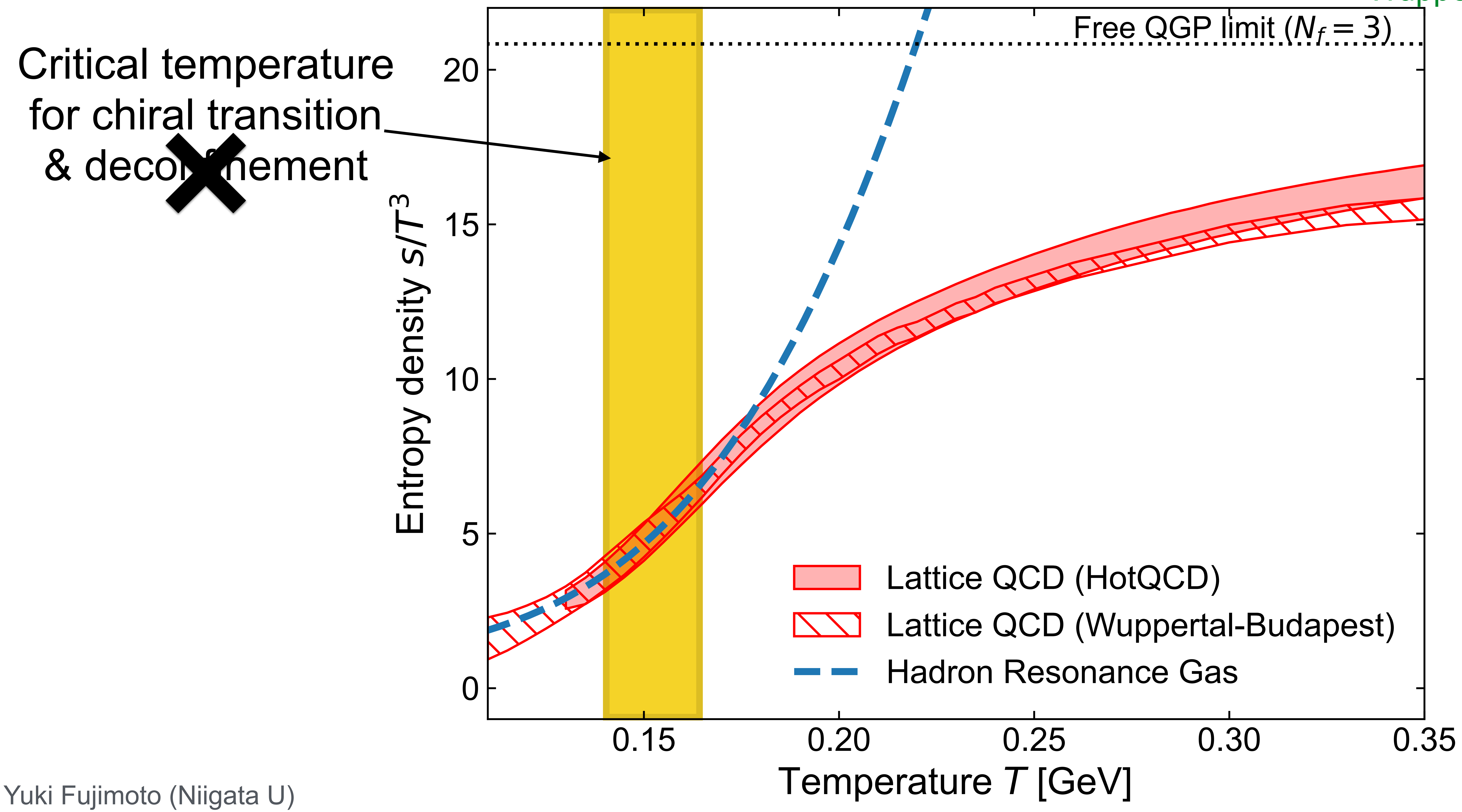


Hagedorn spectrum from string theory

Hagedorn spectrum of open strings:

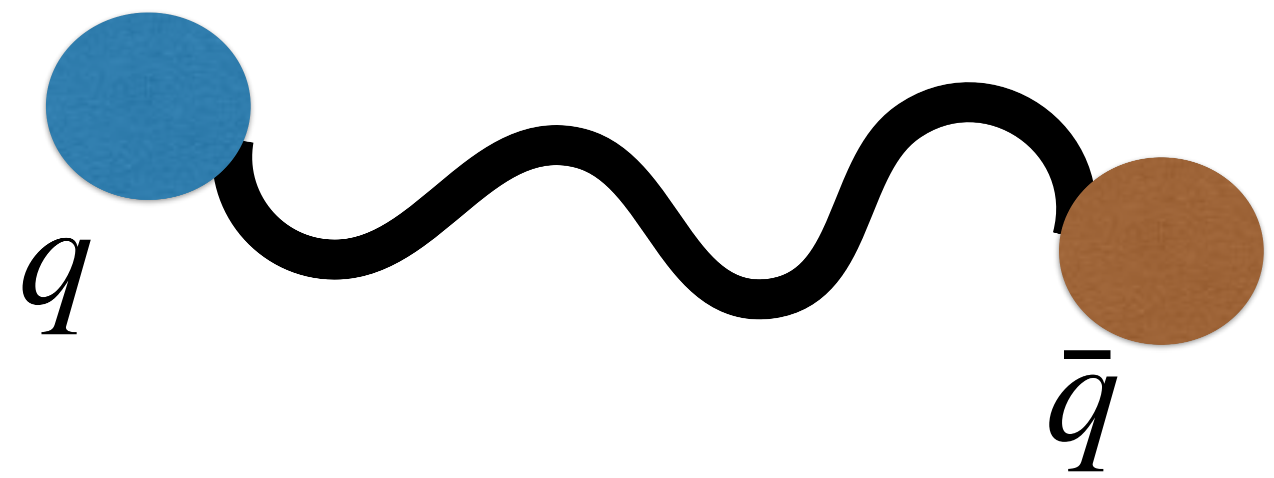
$$\rho(m) = \frac{\sqrt{2\pi}}{6T_H}\left(\frac{T_H}{m}\right)^{3/2} e^{m/T_H}$$

$$T_H = \sqrt{\frac{2\sigma}{3\pi}}$$

Hagedorn temp. ↔ string tension

See, e.g., Green,Schwarz,Witten

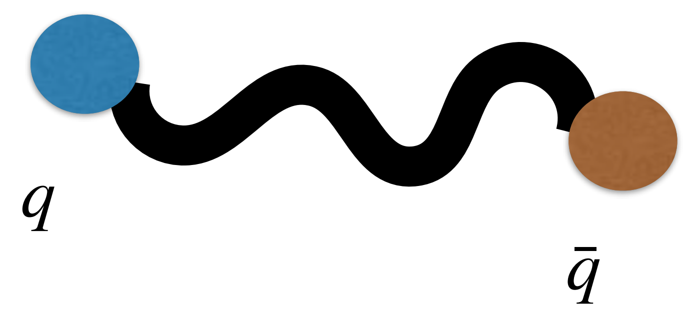
… Mesons can be approximately regarded as an **open string** (w/ quarks attached at the end)



This stringy picture is implied from the Regge trajectory: Chew-Frautchi plot

Fitting EoS with open string formula

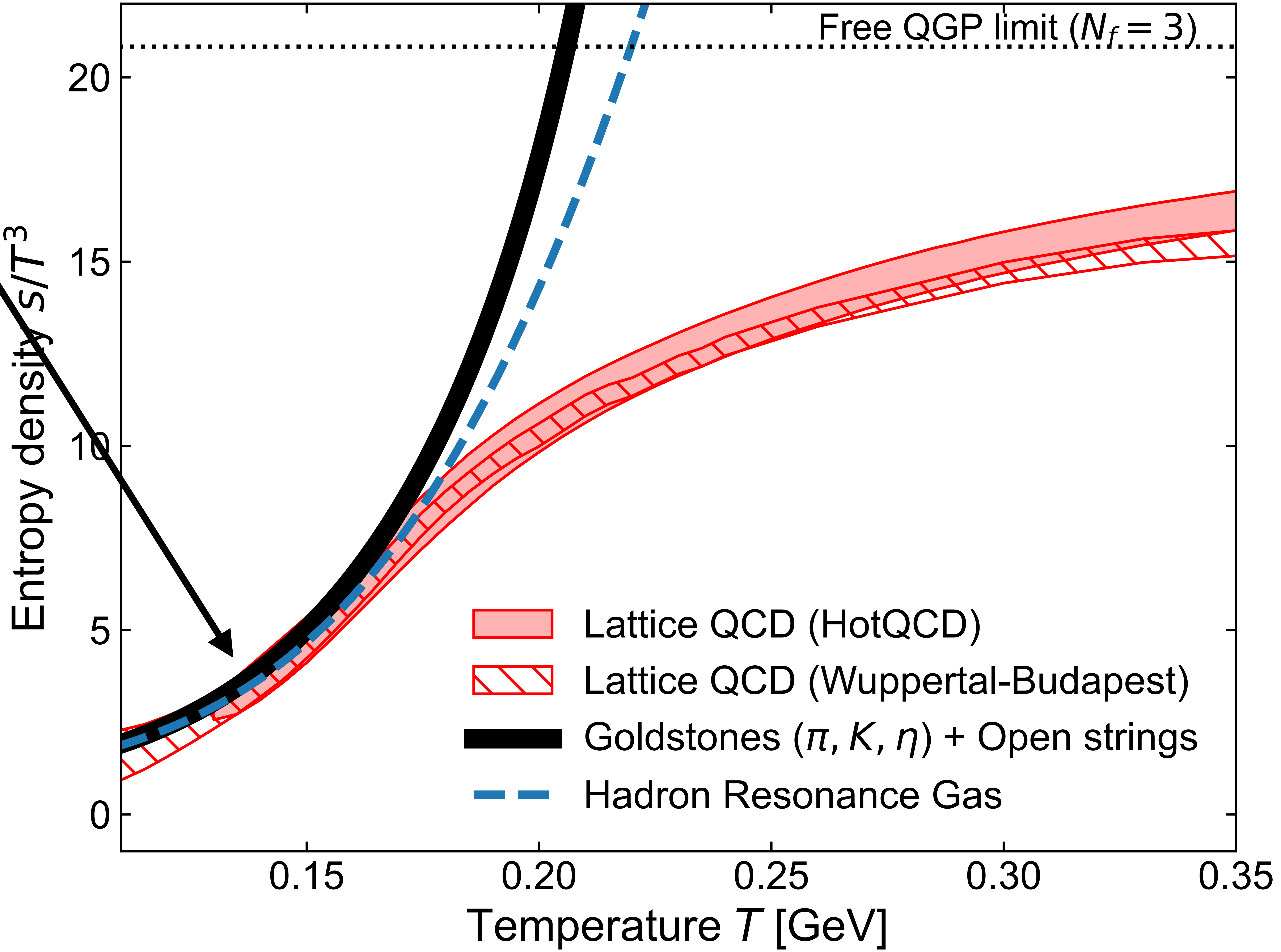
Fujimoto,Fukushima,Hidaka,McLerran (2025)



Fit with open string

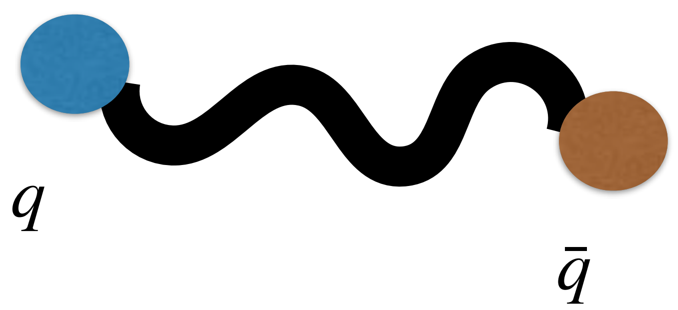
w/ $T_H \simeq 300$ MeV

… the value is common for quarkless theory (pure glue theory)



Deconfinement may take place at higher T

Fujimoto, Fukushima, Hidaka, McLerran (2025)



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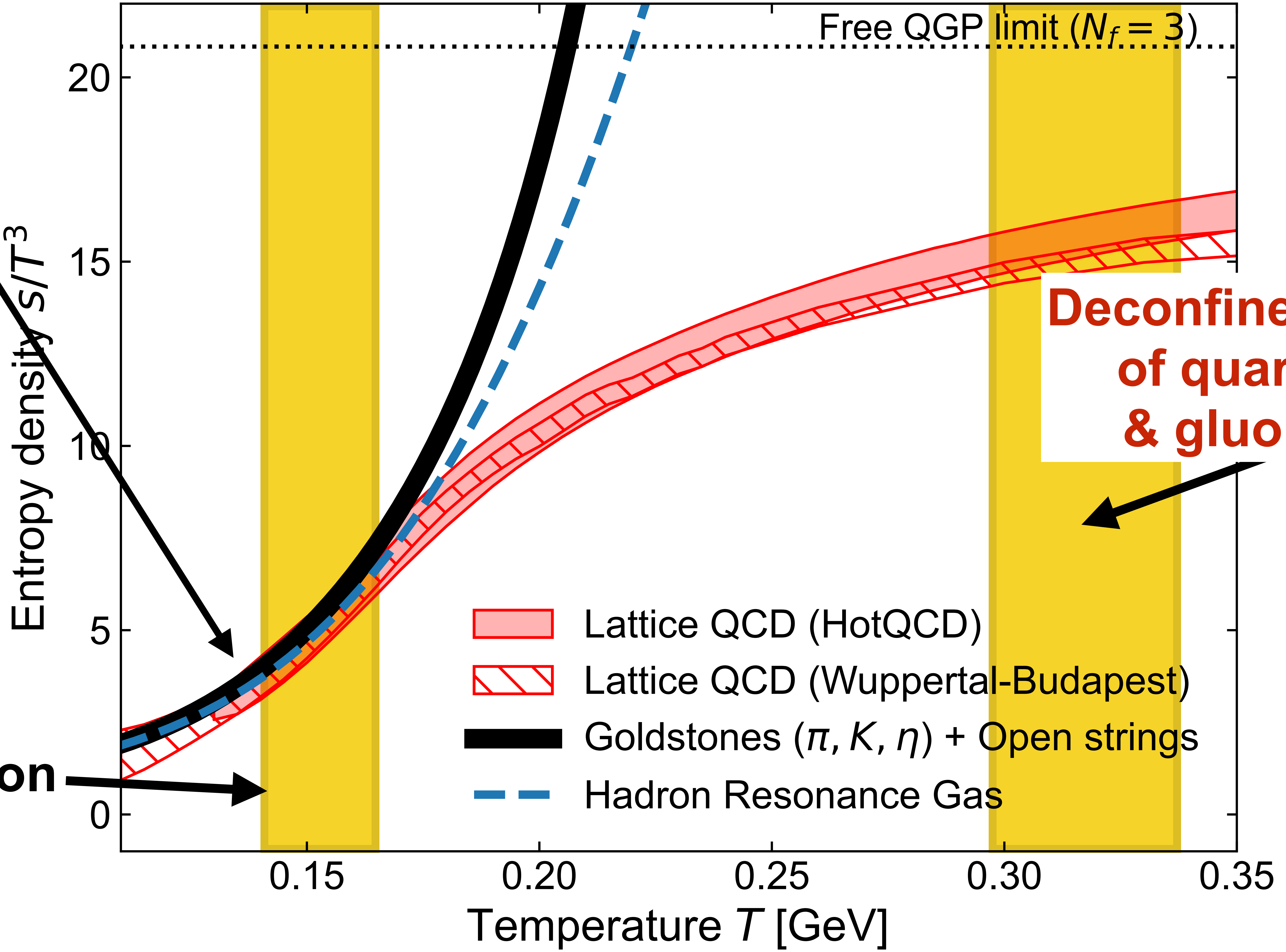
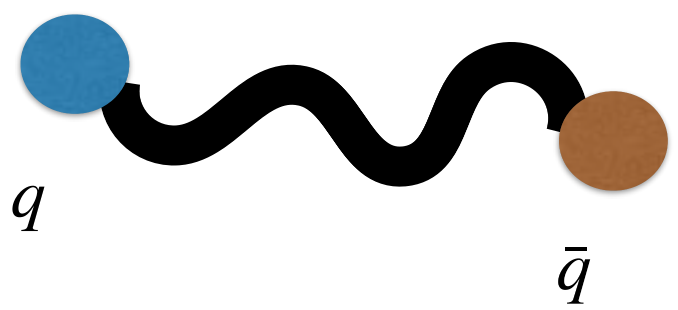
Chiral transition



Deconfinement of quarks & gluons

Deconfinement may take place at higher T

Fujimoto,Fukushima,Hidaka,McLerran (2025)



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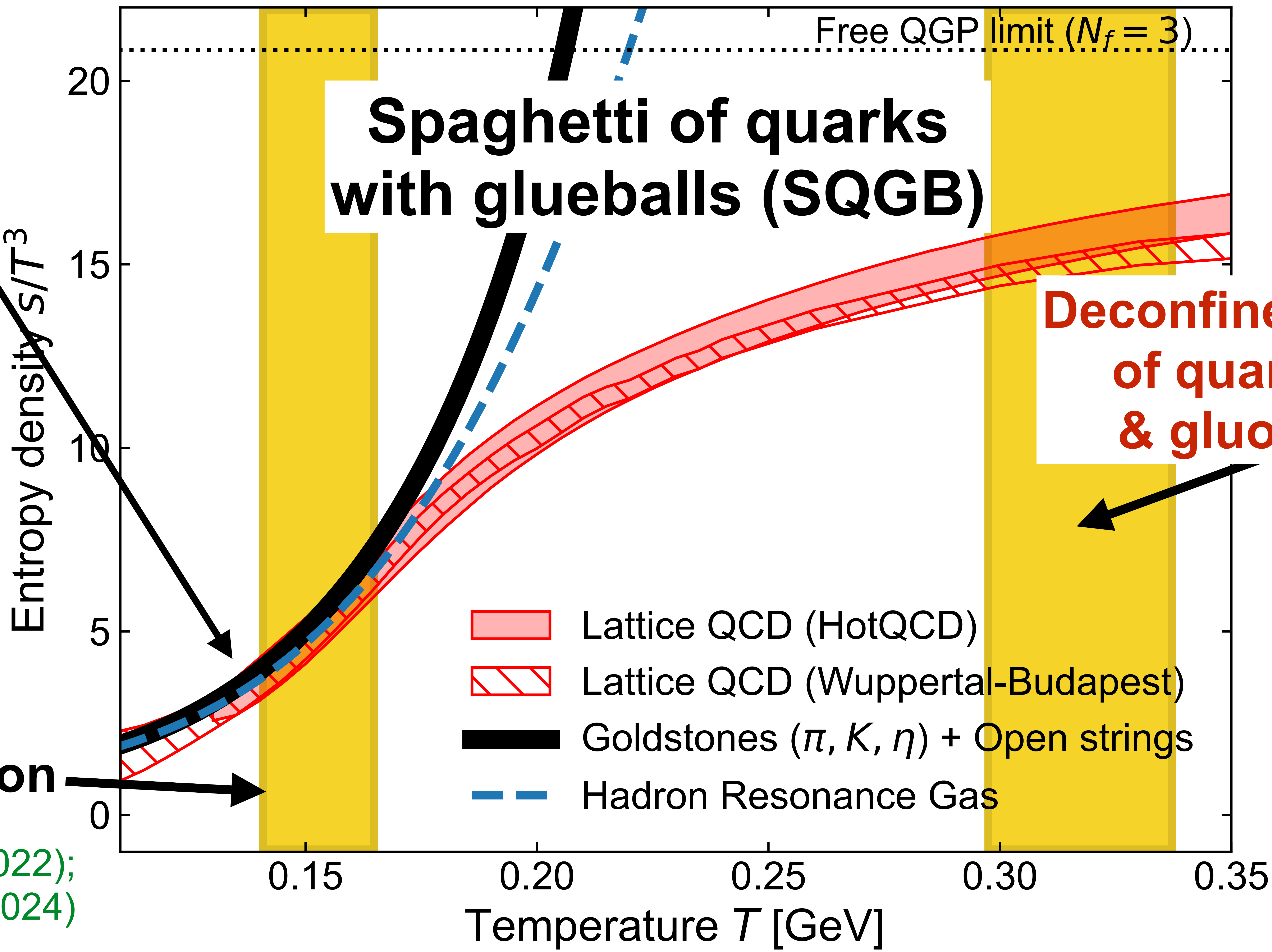
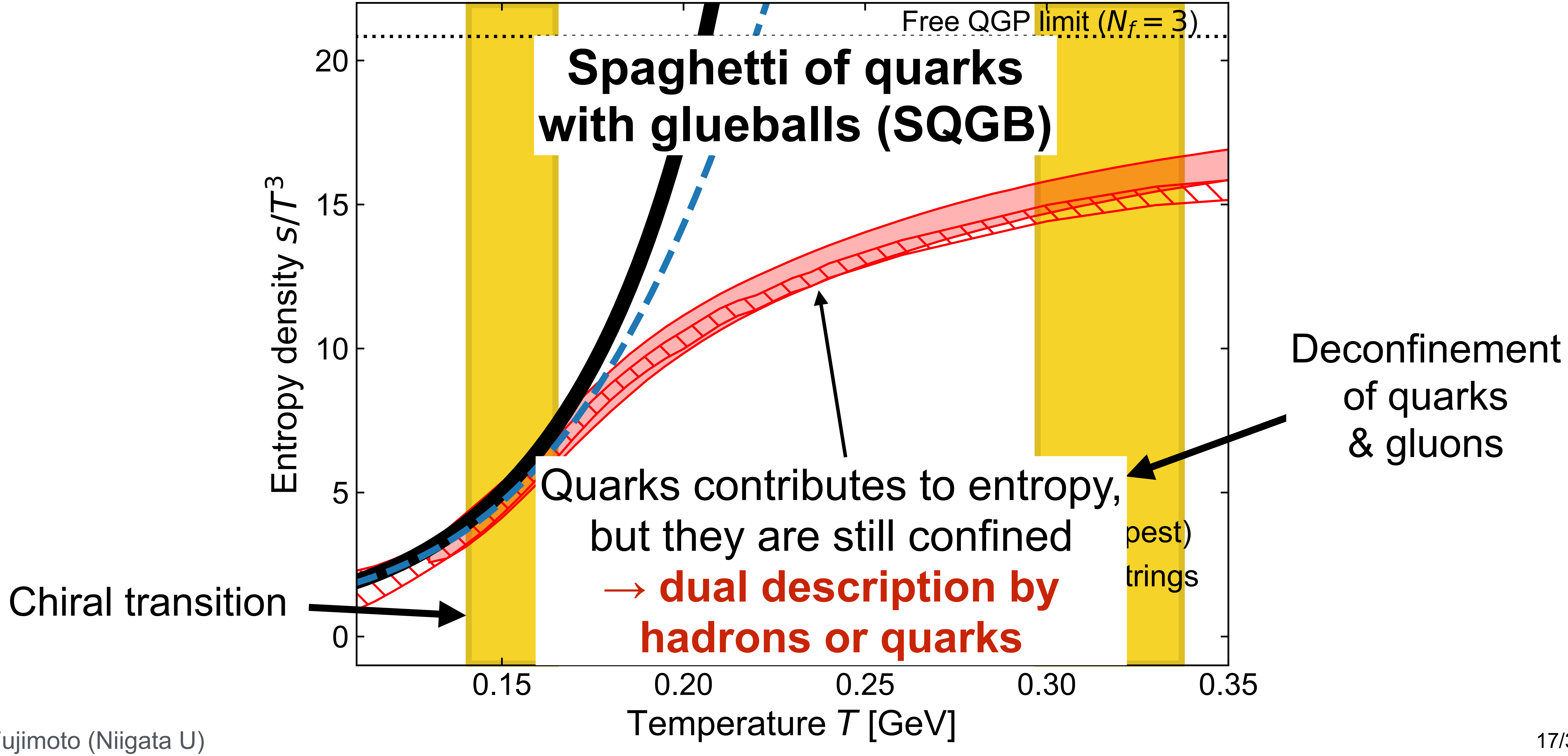


See also: Glozman,Philipsen,Pisarski (2022); Cohen,Glozman (2024)

Dual description at high T

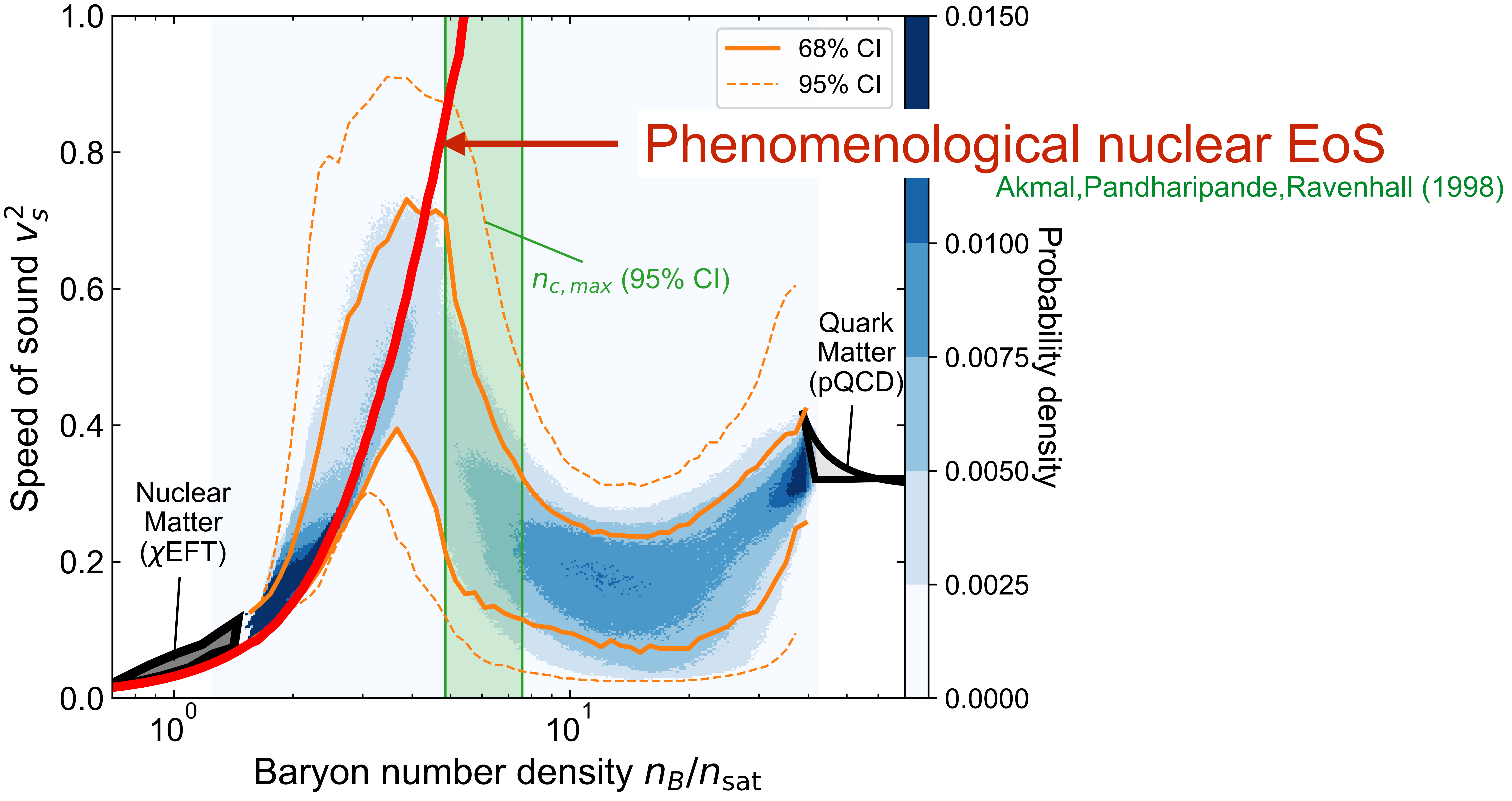
Fujimoto, Fukushima, Hidaka, McLerran (2025)



2. Quark (de)confinement at large μ_B and quarkyonic matter

Analogy with high-T QCD

Phenomenological nuclear EoS describes the behavior well up to certain density…



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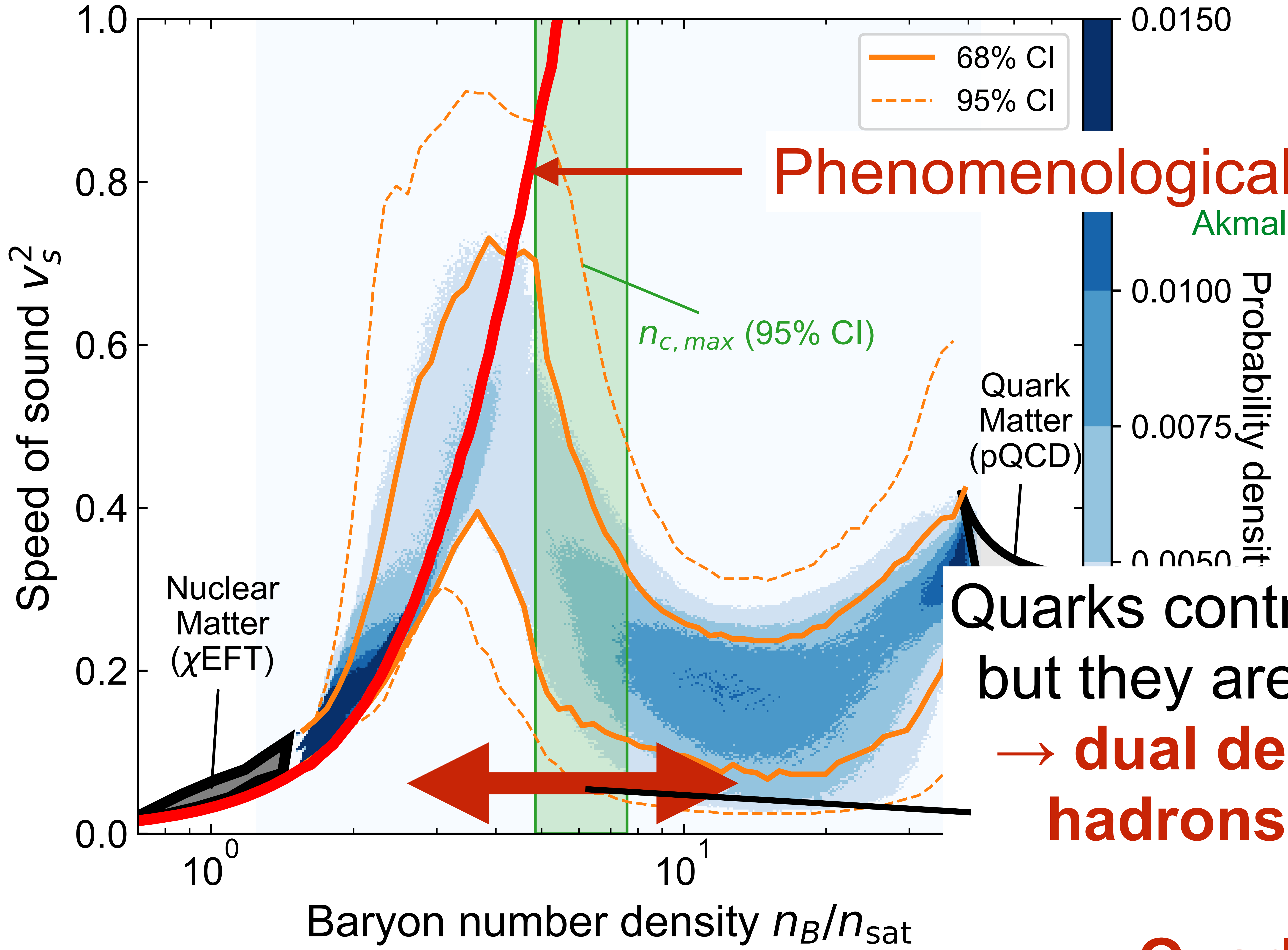


Quarks contributes to EoS, but they are still confined

→ dual description by hadrons or quarks

Quark yonic

Quarkyonic duality in Fermi gas: IdylliQ model

Kojo (2021); Fujimoto,Kojo,McLerran, PRL 132 (2023)

Implement Quarkyonic duality in Fermi gas model
(= simultaneous description in terms of baryons & quarks)

Fermi gas model w/ an explicit duality:

$$\varepsilon = \int_{\boldsymbol{k}} E_{\mathrm{B}}(k) f_{\mathrm{B}}(k) = \int_{\boldsymbol{q}} E_{\mathrm{Q}}(q) f_{\mathrm{Q}}(q)$$

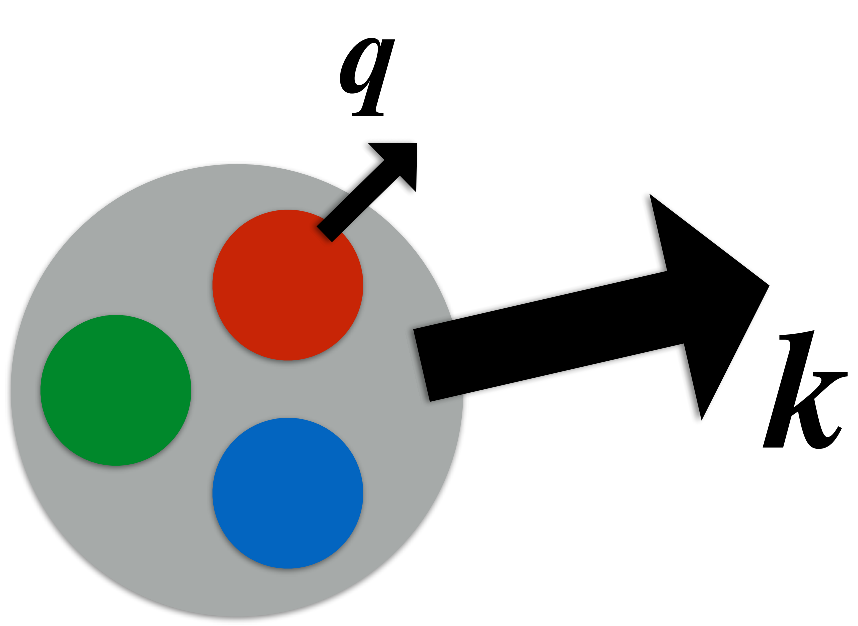
$$n_{\mathrm{B}} = \int_{\boldsymbol{k}} f_{\mathrm{B}}(k) = \int_{\boldsymbol{q}} f_{\mathrm{Q}}(q)$$

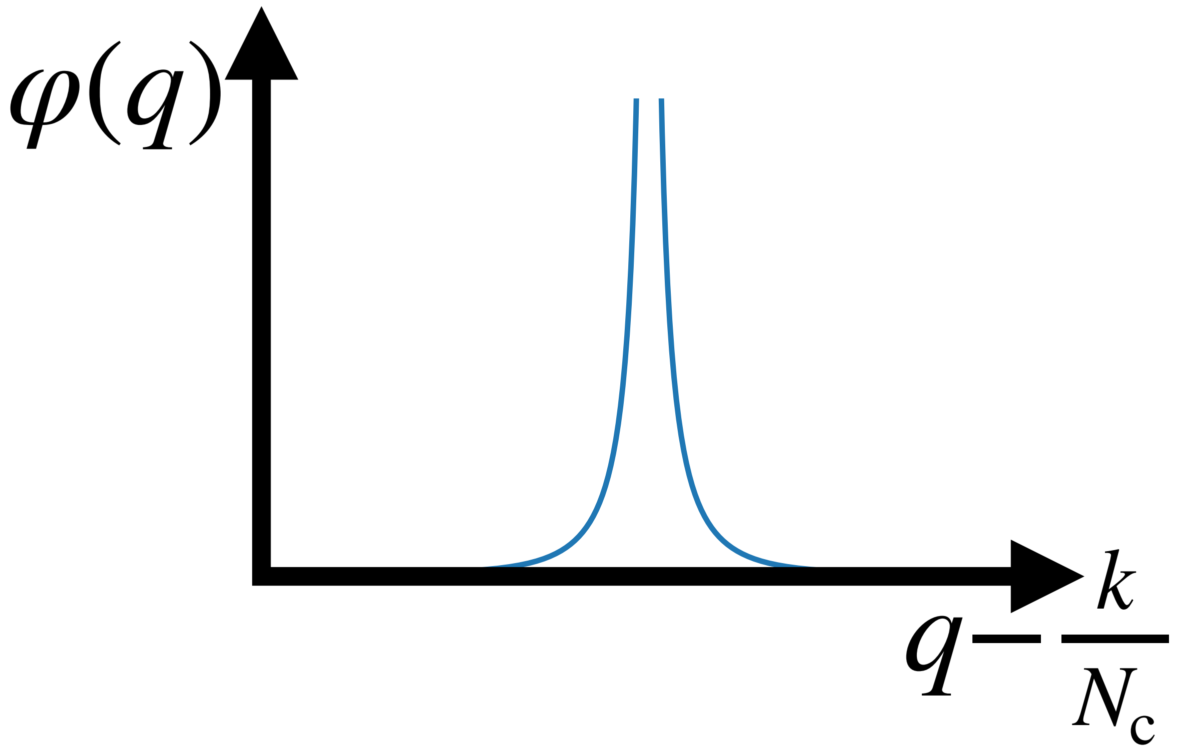
$0 \leq f_{\mathrm{B,Q}} \leq 1$: Pauli exclusion

$E_{\mathrm{B}}(k) = \sqrt{k^2 + M_N^2}$: ideal baryon dispersion relation

Modeling of confinement:

$$f_{\mathrm{Q}}(q) = \int_{\boldsymbol{k}} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right) f_{\mathrm{B}}(k)$$





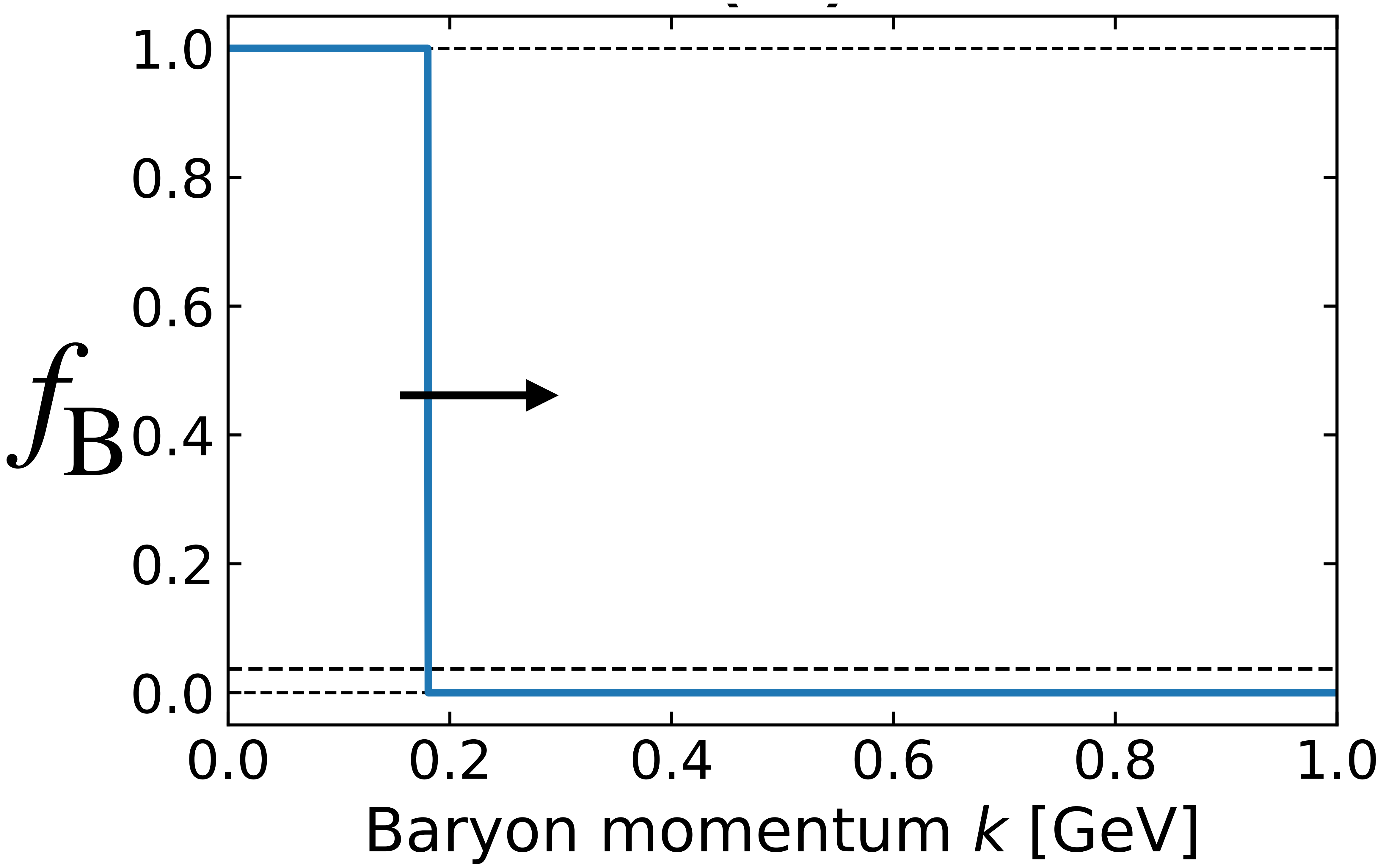
Ideal dual Quarkyonic model
→ **Find a solution for f_{B} and f_{Q} with minimum ε at a given n_B**

Solution of IdylliQ model

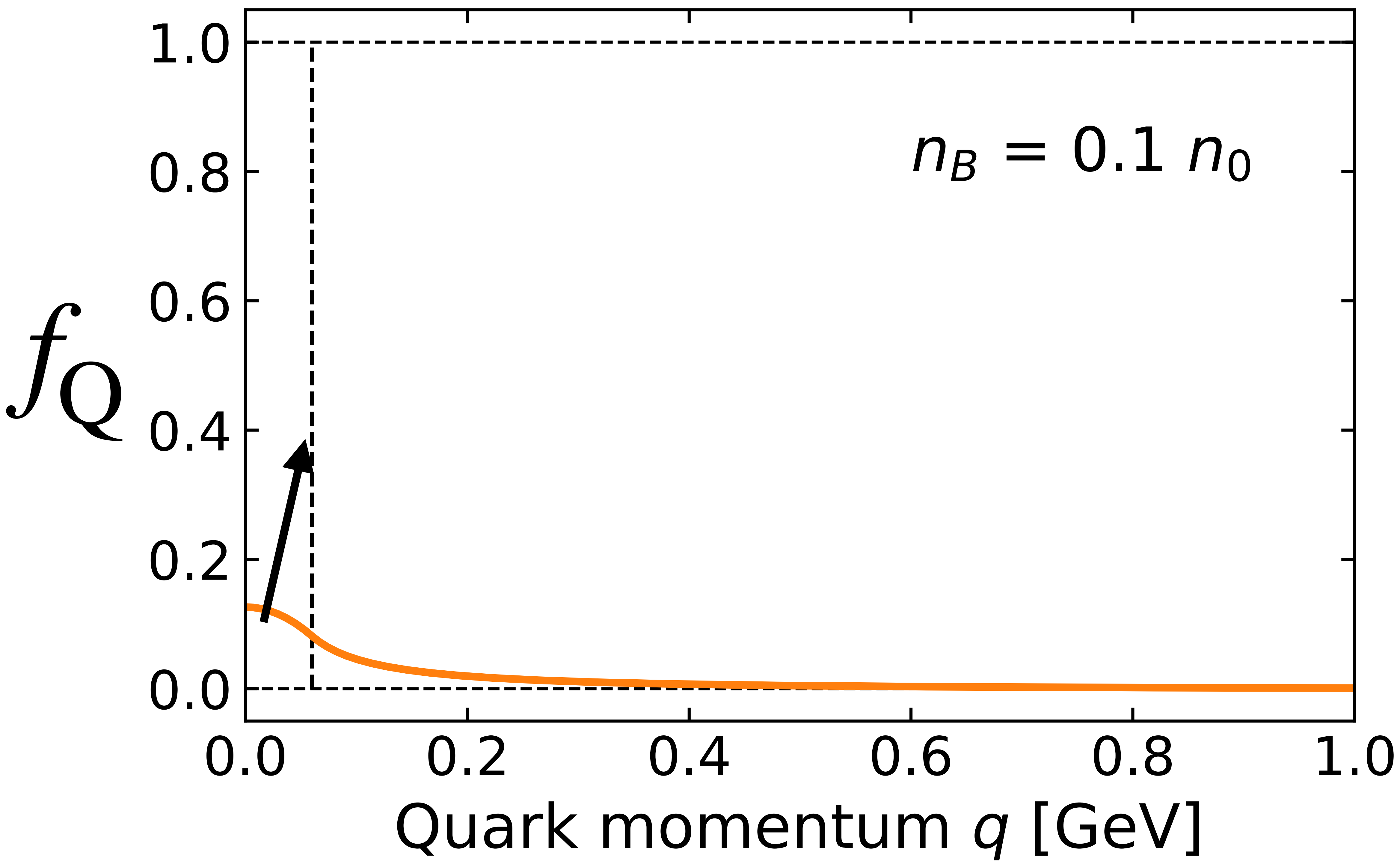
Kojo, PRD 104 (2021); Fujimoto,Kojo,McLerran, PRL 132 (2023)

At low density…

Fermi-Dirac distribution
for baryons



Quarks do not fill up
the Fermi sea yet

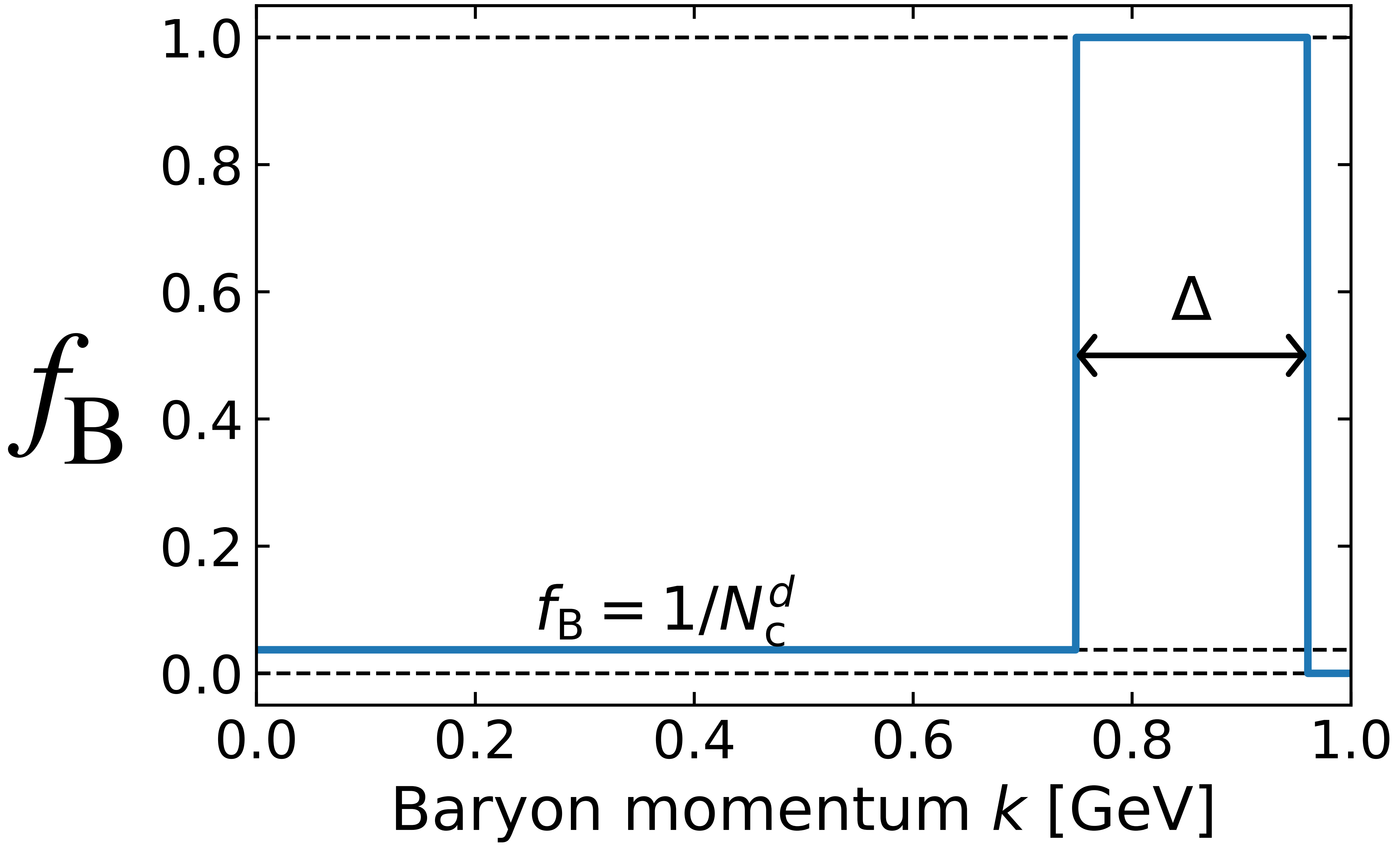


Solution of IdylliQ model

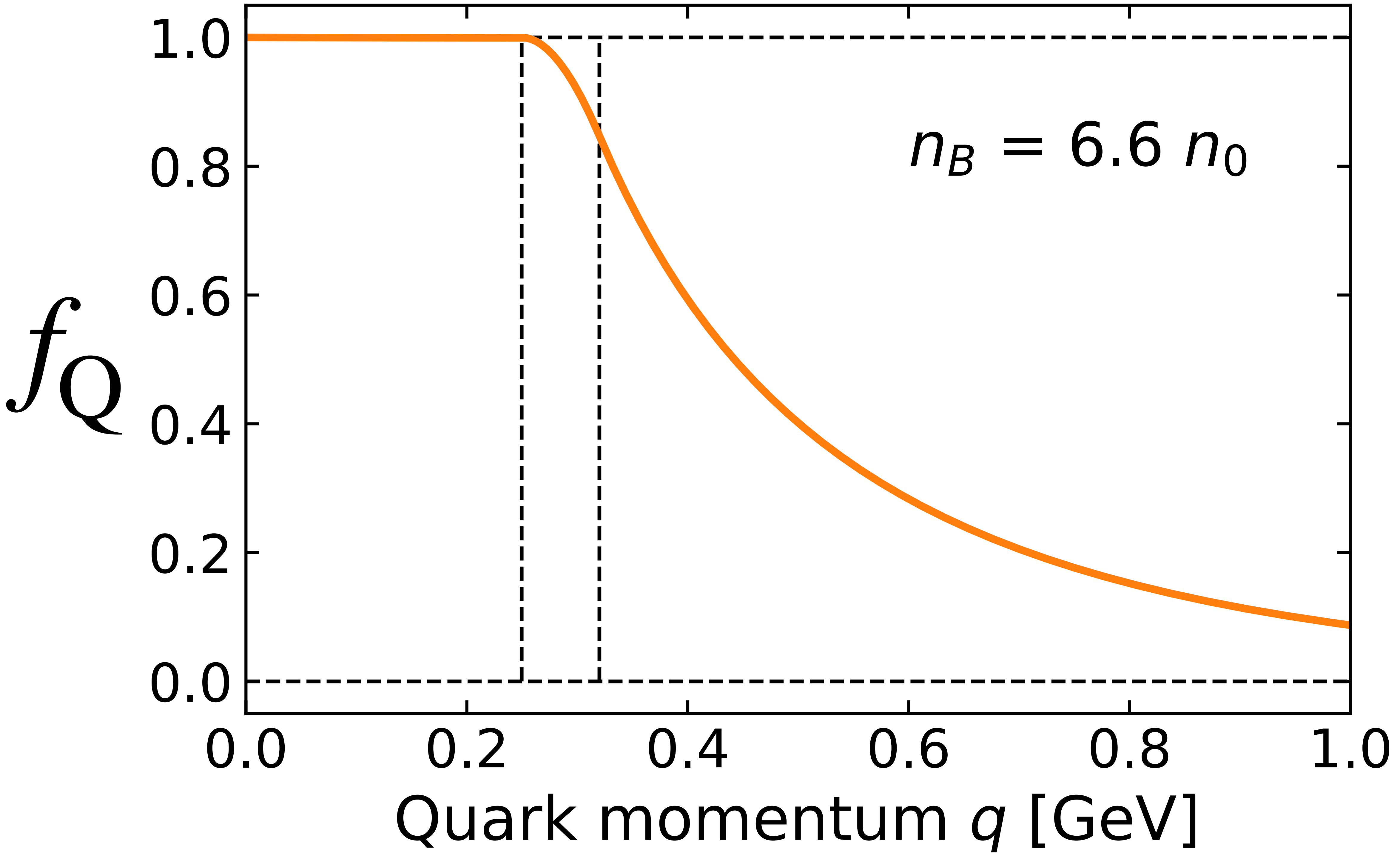
Fujimoto,Kojo,McLerran, PRL 132 (2023)

At sufficiently high density…

Fermi-Dirac distribution for baryons is modified



Quark obeys the FD distribution (with a tail from confinement)



… characteristic feature of Quarkyonic matter

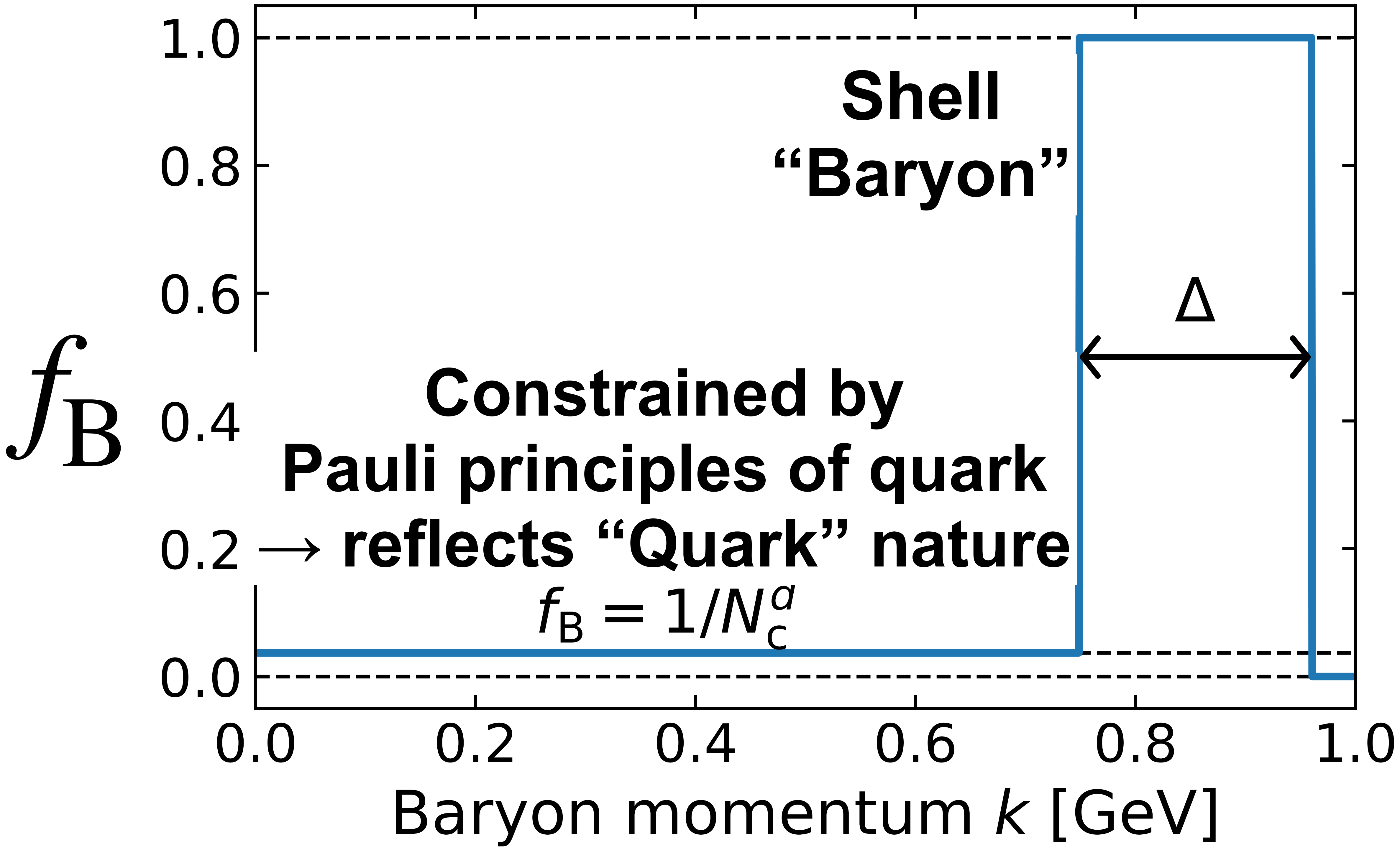
Solution of IdylliQ model

Fujimoto,Kojo,McLerran, PRL 132 (2023)

McLerran,Pisarski (2007); McLerran,Reddy (2018);
Jeong,McLerran,Sen (2019); many other works

At sufficiently high density…

Fermi-Dirac distribution for baryons is modified



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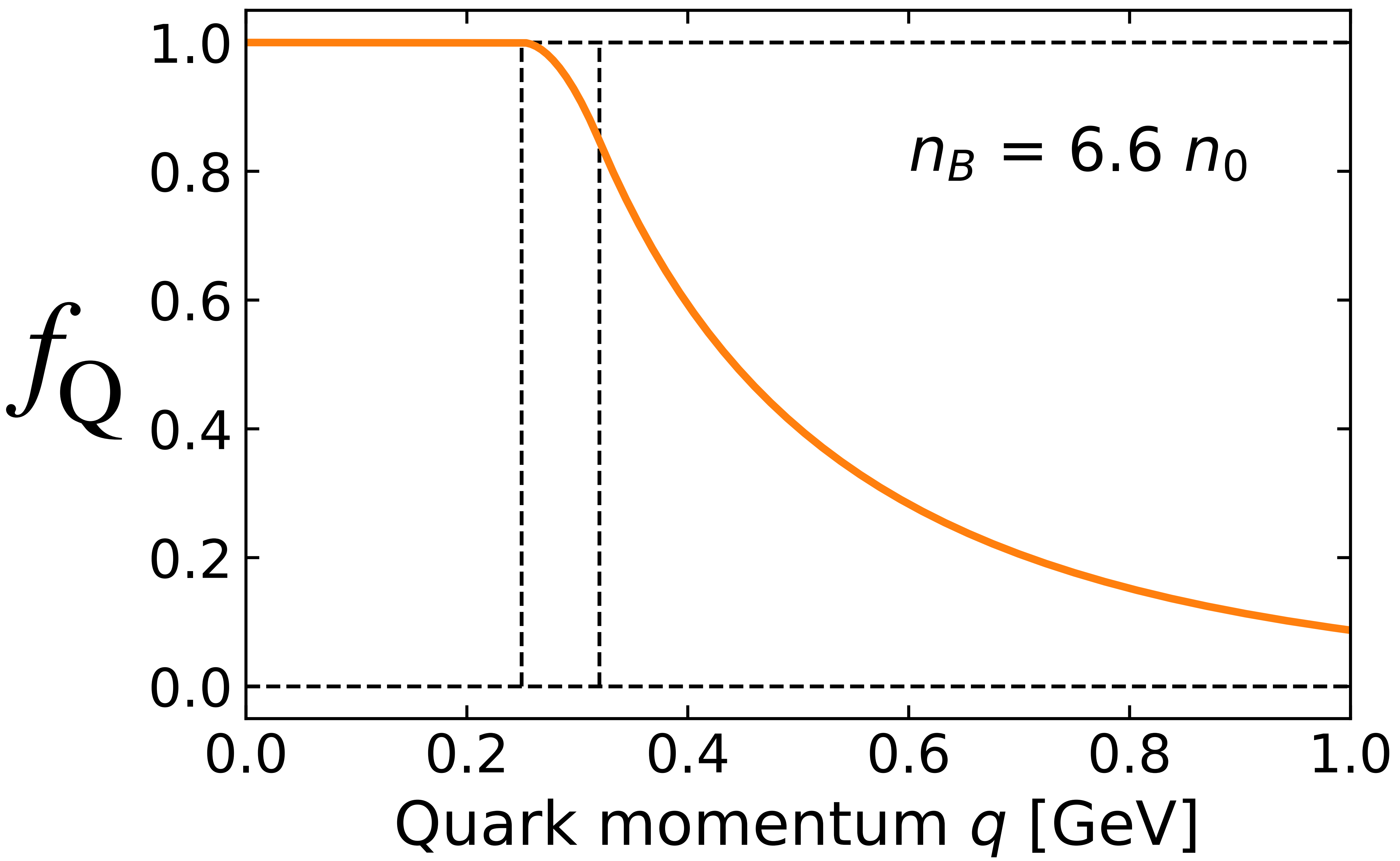
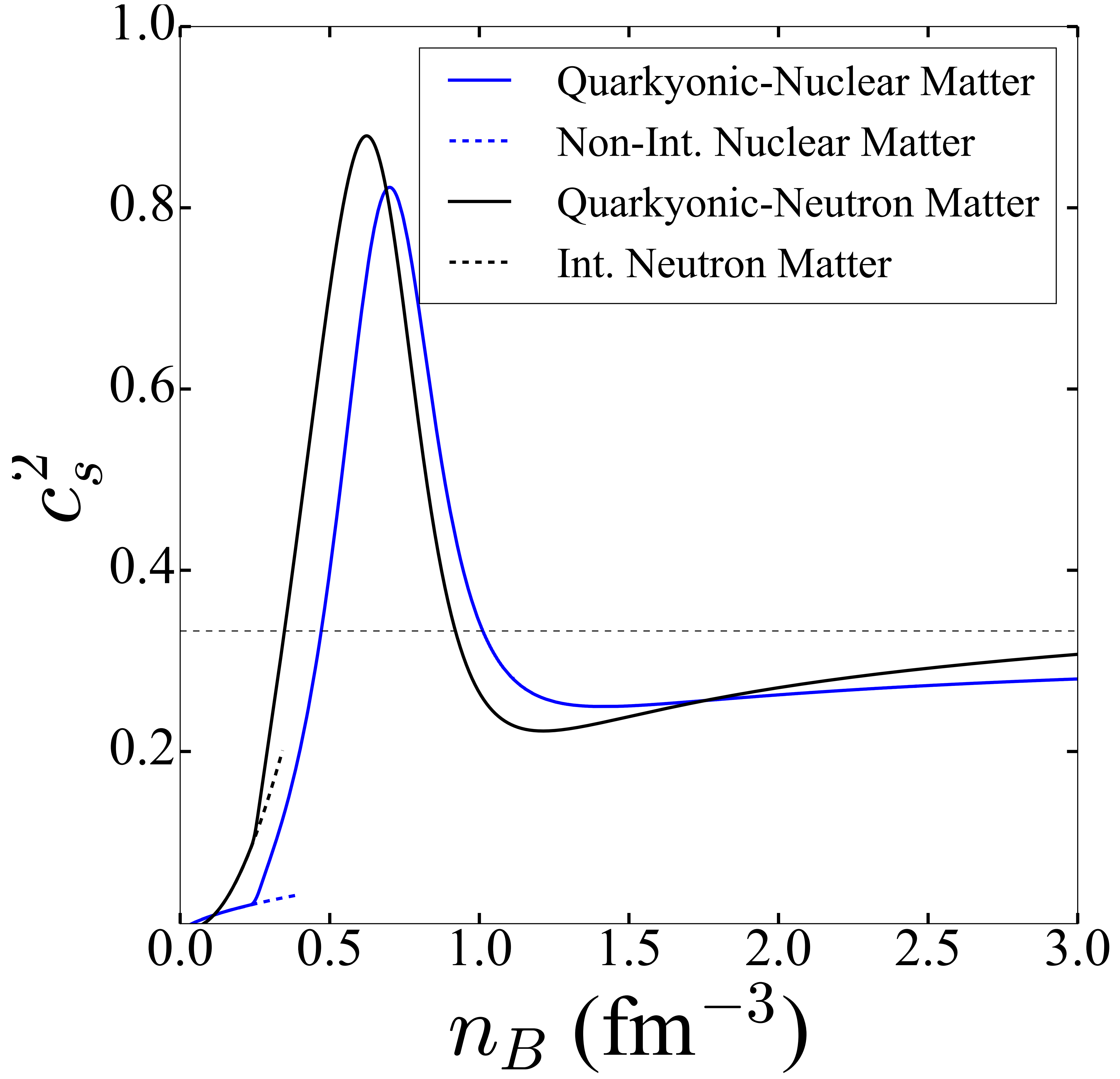


… characteristic feature of Quarkyonic matter

EoS comparison: Quarkyonic model & Bayesian

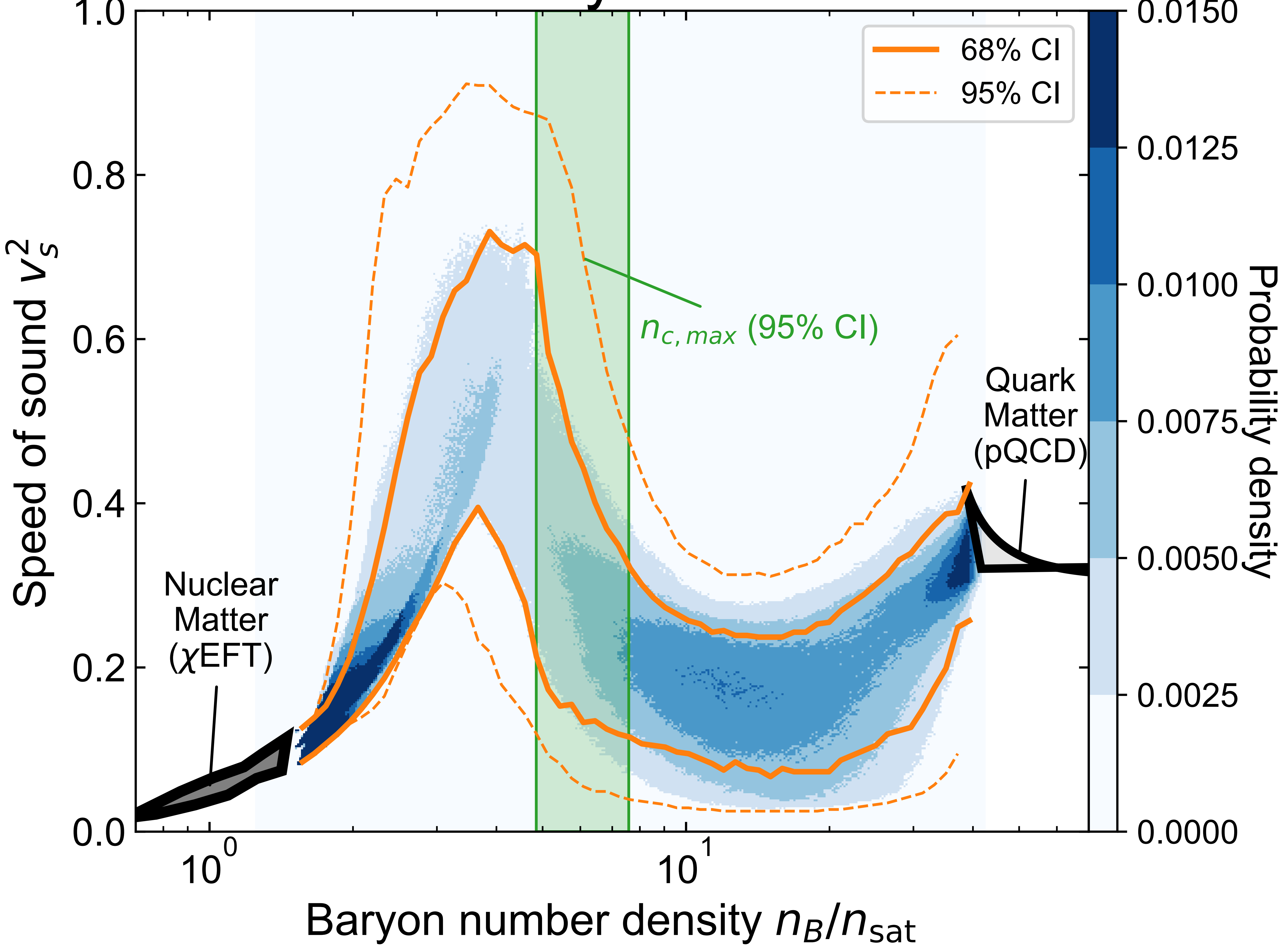
Looks very similar…

Model EoS of Quarkyonic matter

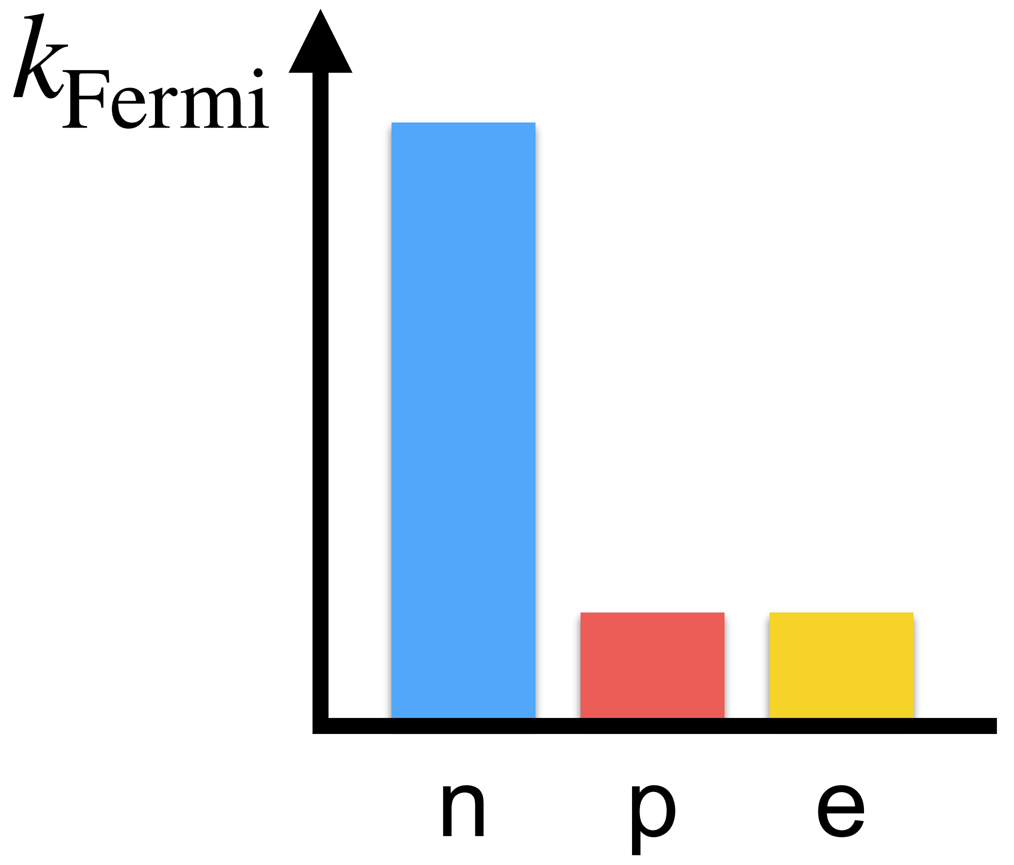


McLerran,Reddy (2018)

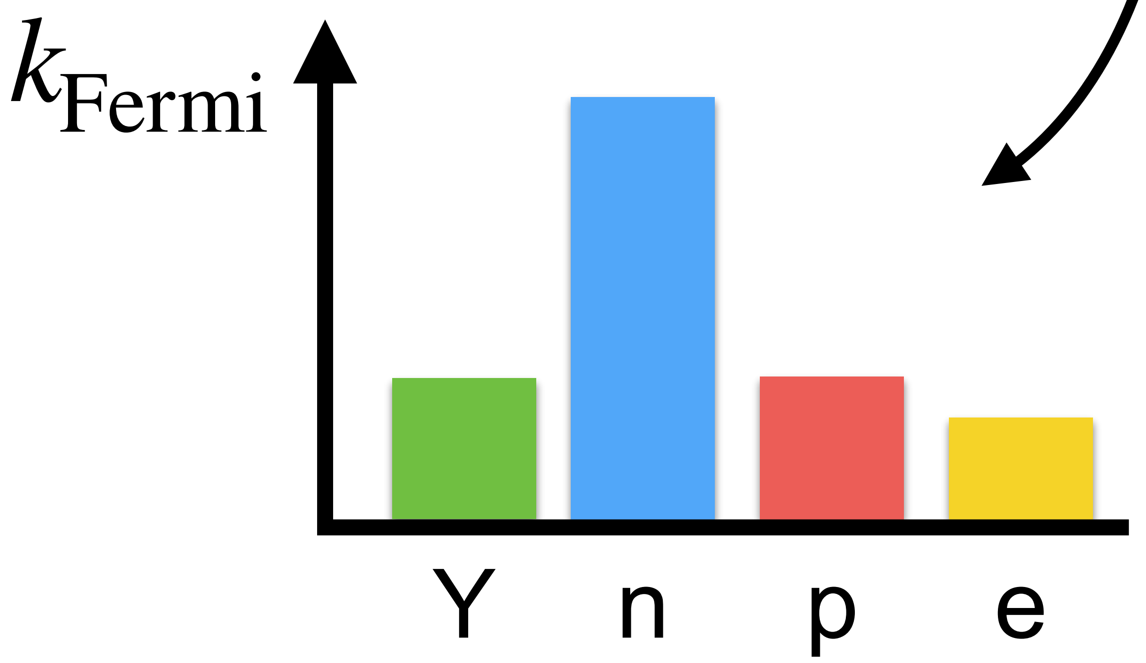
Bayesian



Strangeness in neutron stars

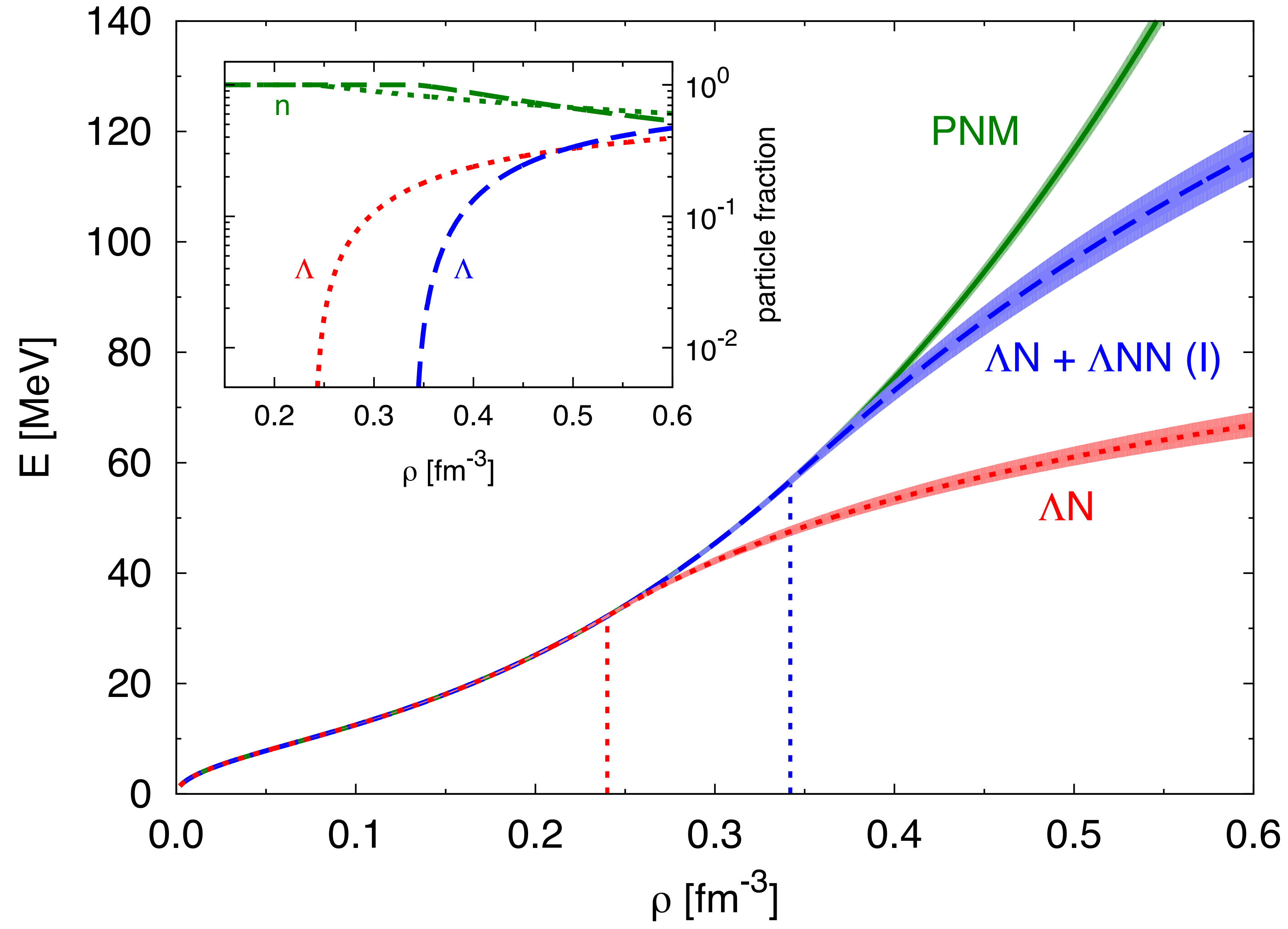


Inclusion of hyperons



Hyperons (Y) lower the energy density at a given baryon density

Hyperons soften the EoS drastically …



Cannot support heavy neutron stars

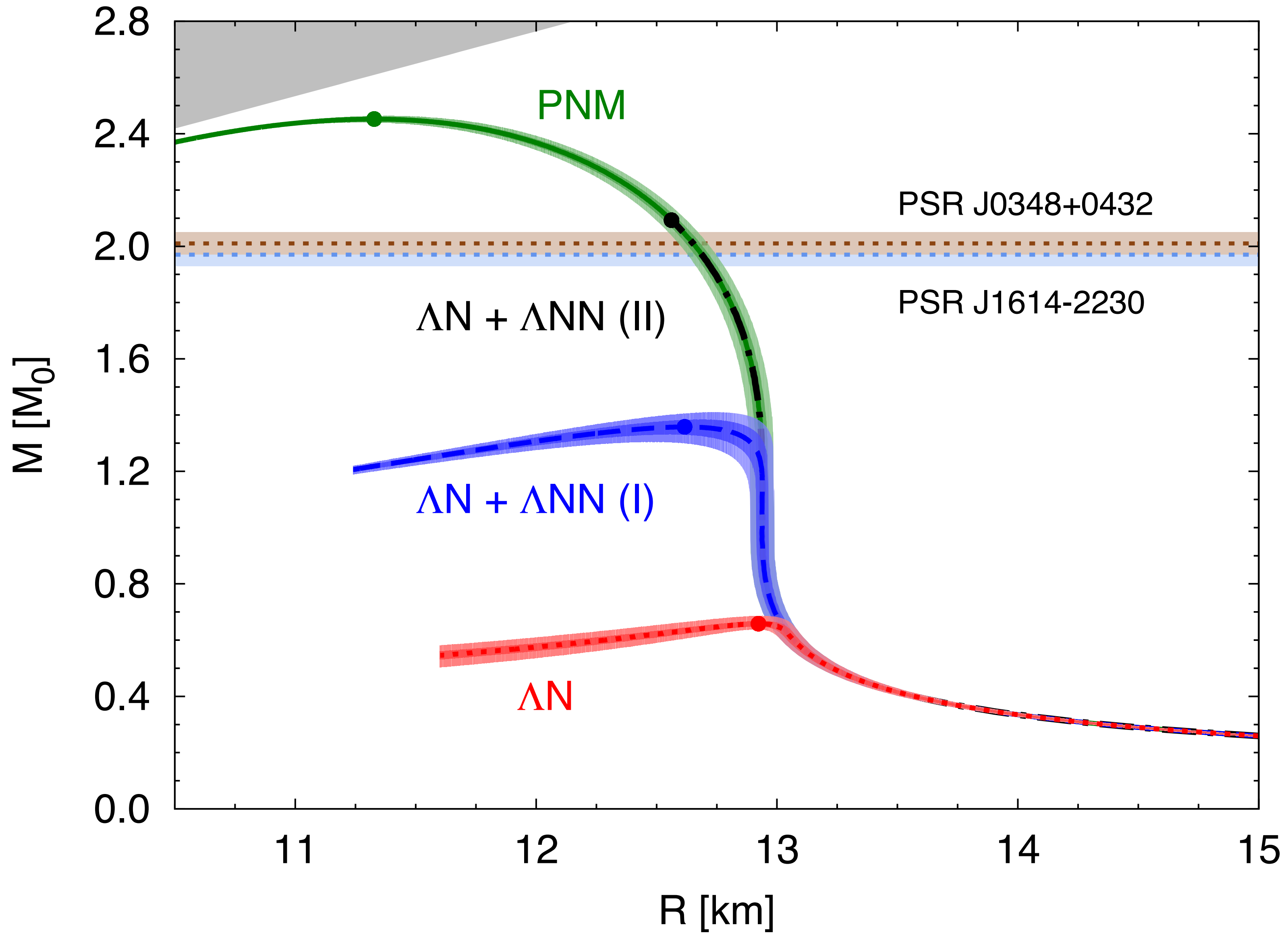


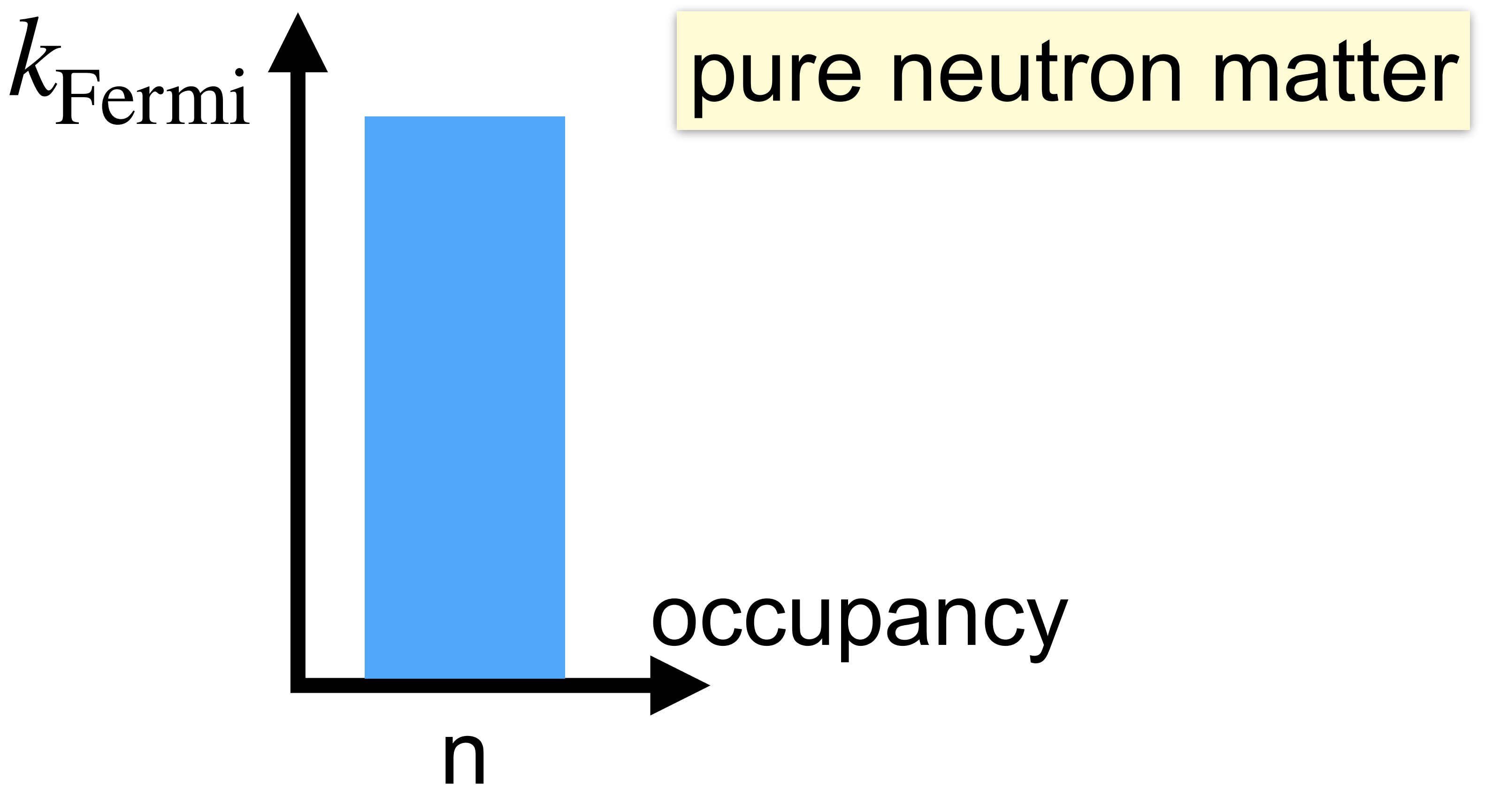
figure from Lonardoni et al. (2014)

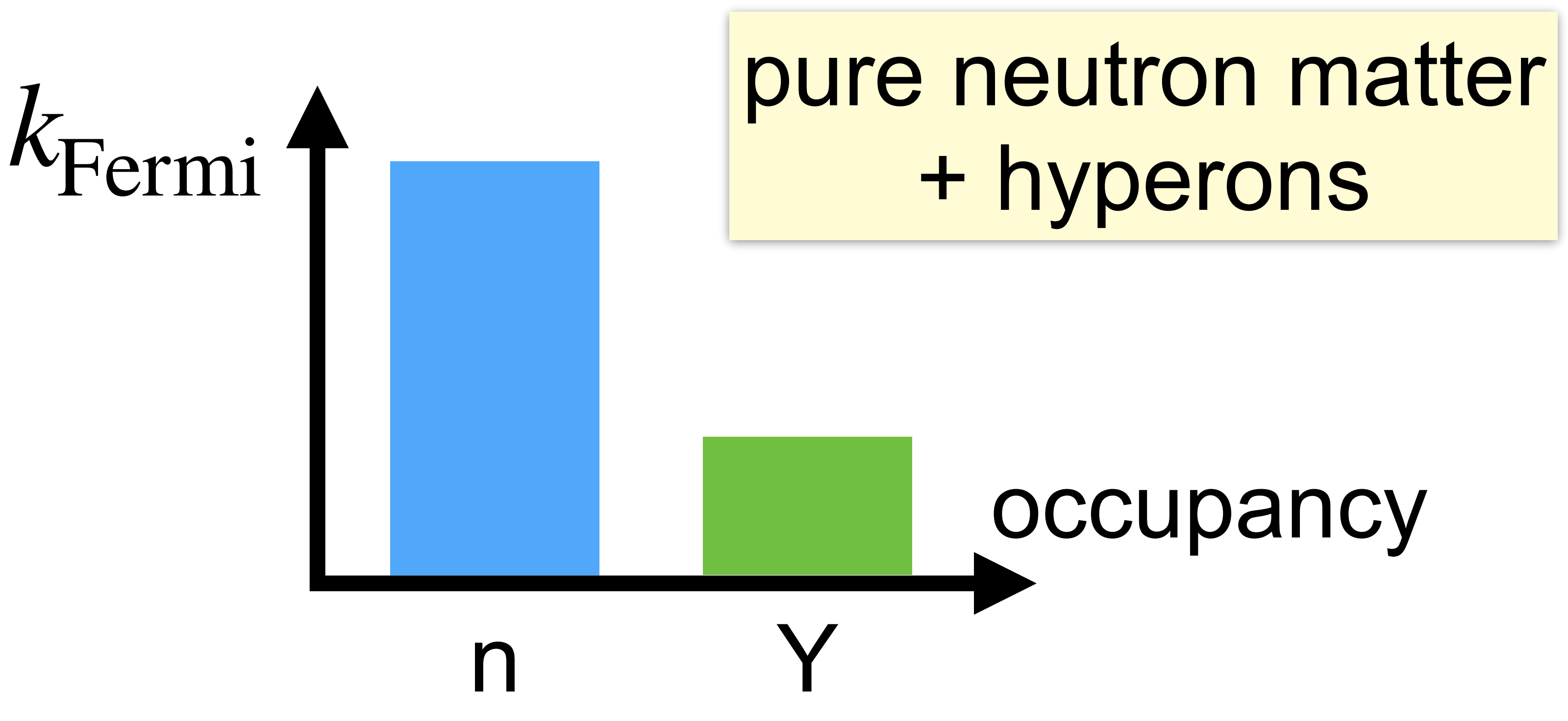
Hyperon “puzzle”

Quarkyonic solution to the hyperon puzzle

Fujimoto,Kojo,McLerran, 2410.22758 (2024)

Conventional picture:

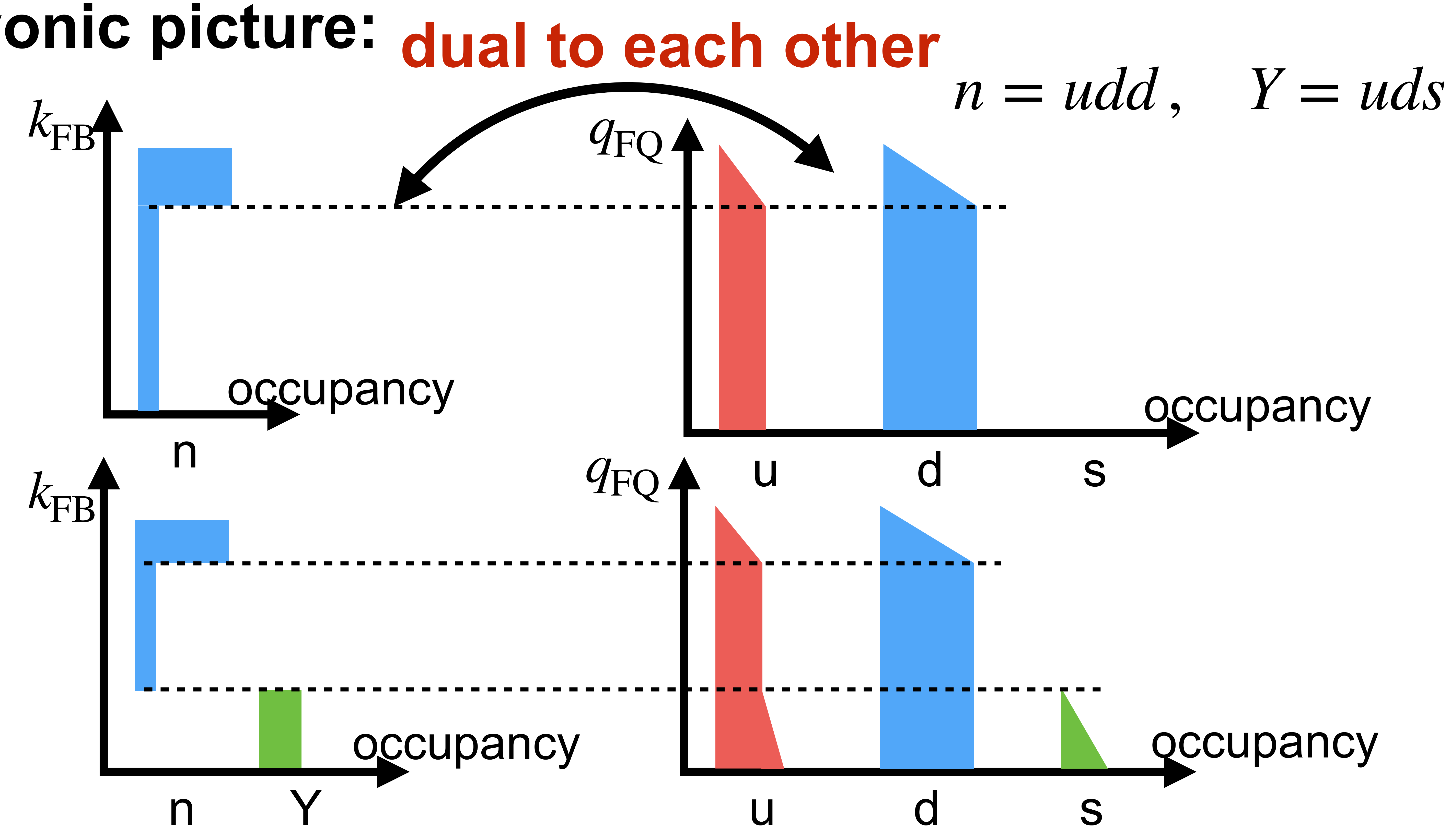




Hyperons (Y) lower the energy density at a given baryon density

Threshold: $\mu_B = M_Y$

Quarkyonic picture:



Y has to appear so that d-quark states are kept saturated:

$\frac{1}{2}$n = $\frac{1}{2}$ud → Y = uds $\rightarrow \mu_Y = E_Y(k_Y) - \frac{1}{2}E_n(k_Y) + \frac{1}{2}\mu_n$

Threshold ($\mu_n = \mu_Y = \mu_B$ & $k_Y = 0$)

is shifted to: $\mu_B = 2M_Y - M_n = M_Y + (M_Y - M_n)$

Quarkyonic solution to the hyperon puzzle

Fujimoto,Kojo,McLerran, 2410.22758 (2024)

Equation-wise, one can understand the threshold shift as follows:

Hyperon chemical potential:

$$\mu_Y = \left(\frac{\partial \varepsilon}{\partial n_Y}\right)_{n_n} = E_Y(k_{\mathrm{F},Y}) - \frac{1}{2}E_N(k_{\mathrm{F},Y}) + \frac{1}{2}\mu_n$$

Beta equilibrium condition:

$$\mu_S = 0 \Rightarrow \mu_i = B_i\mu_B + Q_i\mu_Q$$

i.e. $\mu_n = \mu_B\,,\ \mu_Y = \mu_B$ (now we limit ourselves to $\mu_Q = 0$)

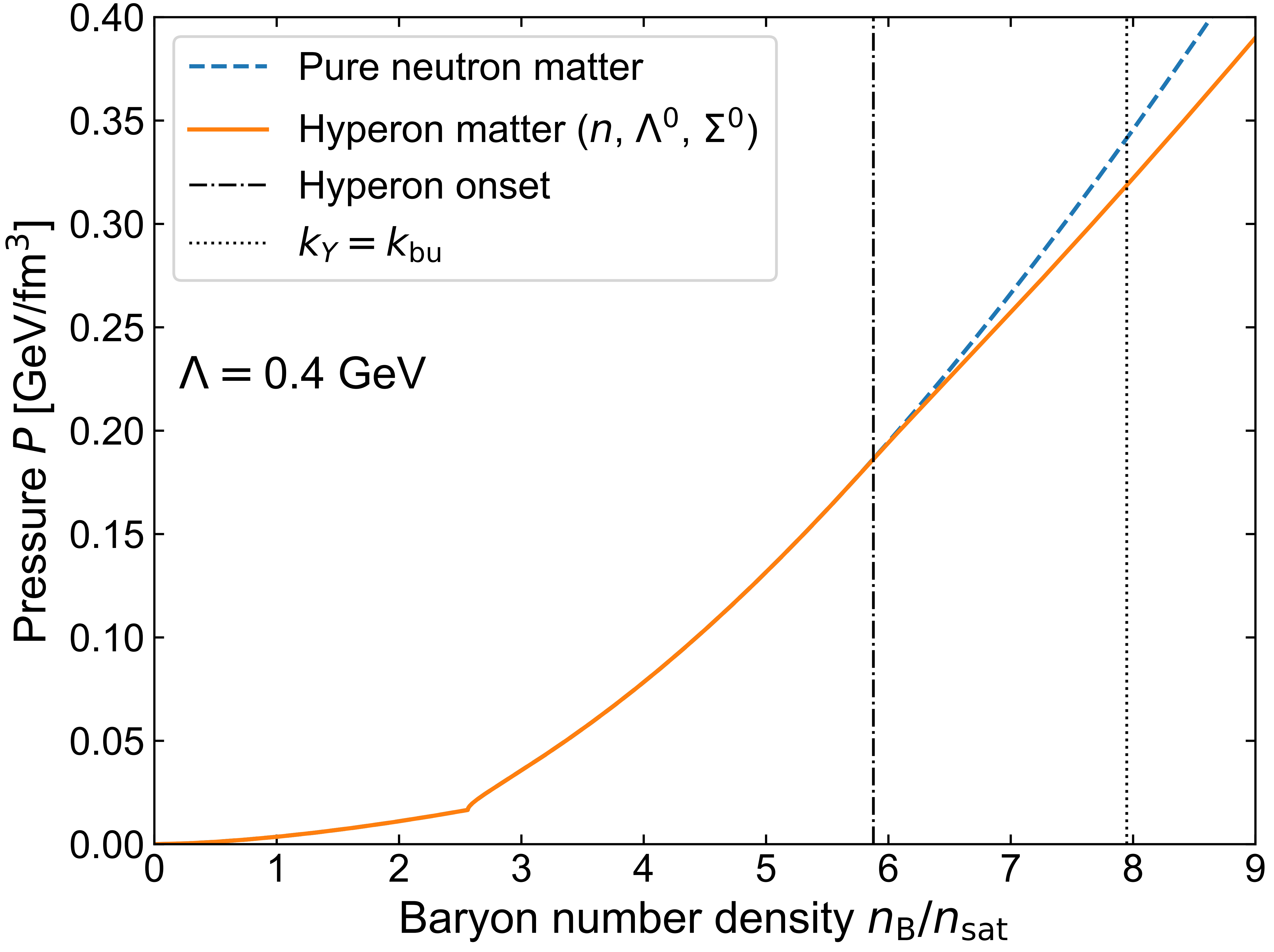
Hyperon threshold:

when the Fermi momentum of hyperons is $k_{\mathrm{F},Y} = 0$

$$\mu_B^{\mathrm{thres}} = M_Y + (M_Y - M_N)$$

Quarkyonic solution to the hyperon puzzle

Fujimoto,Kojo,McLerran, 2410.22758 (2024)



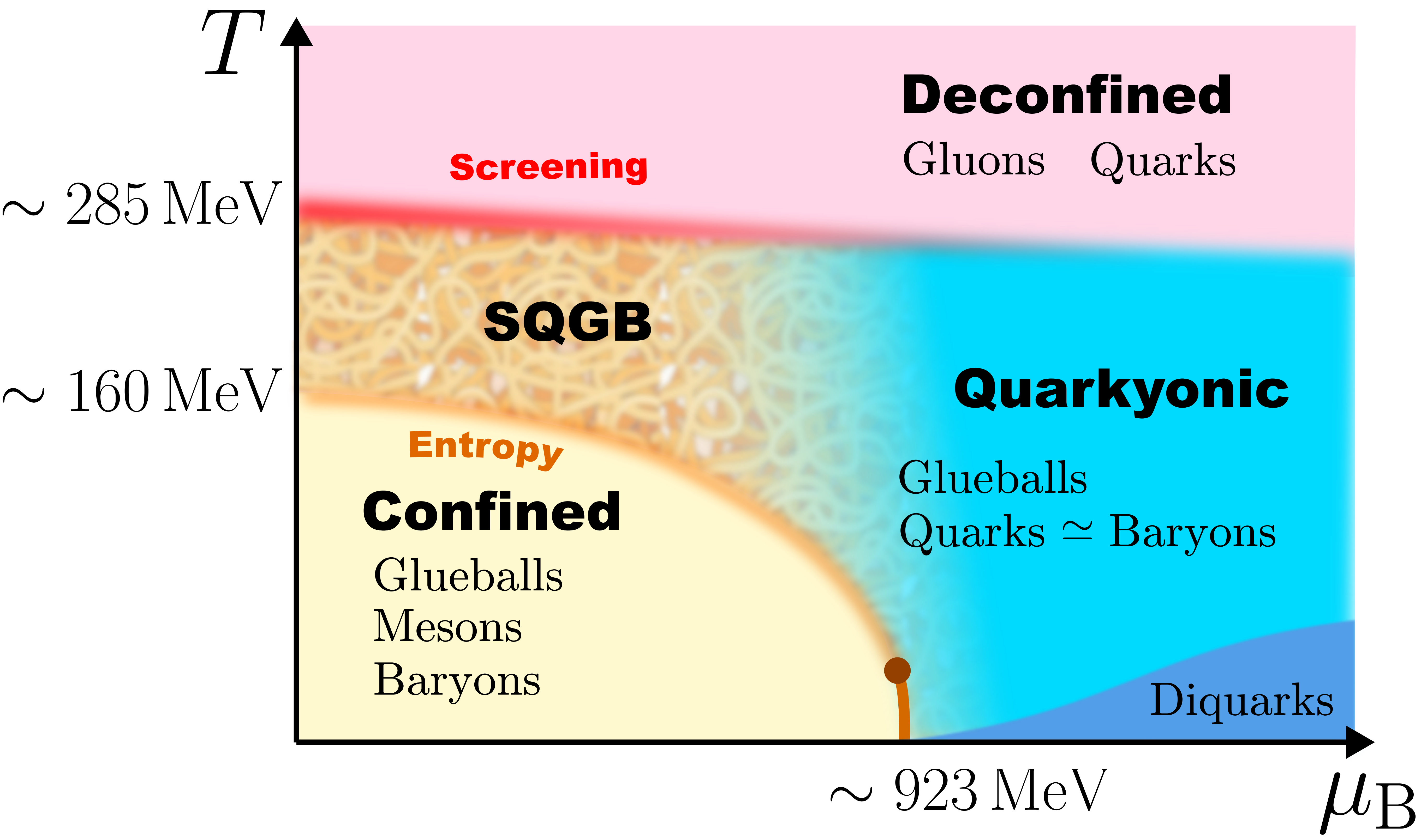
Due to the saturation of d-quark states, softening in the hyperon EoS is mild

This is purely the effect of FD statistics! No interaction except for the one implicitly in the confining relation.

Usual solutions of the hyperon puzzle requires very strong repulsive interaction

Summary

Fujimoto,Fukushima,Hidaka,McLerran (2025)



- Deconfinement is difficult to capture
- Hagedorn temperature (≈ deconfinement) may be T_H ~ 300–340 MeV … common for quarks & gluons
- Confinement may persist at large density … Quarkyonic regime.
- Including hyperons in Quarkyonic matter … Hyperon threshold lifted. Missing piece in hyperon "puzzle"

Bonus materials

Difference in Hagedorn spectrum

Conventionally used form (from bootstrap condition):

Hagedorn (1965)?

$$\rho(m) = \frac{a}{(m^2 + m_0^2)^{5/4}} e^{m/T_H}$$

… ambiguity in the power-law term in the coefficient

Hagedorn spectrum from string theory:

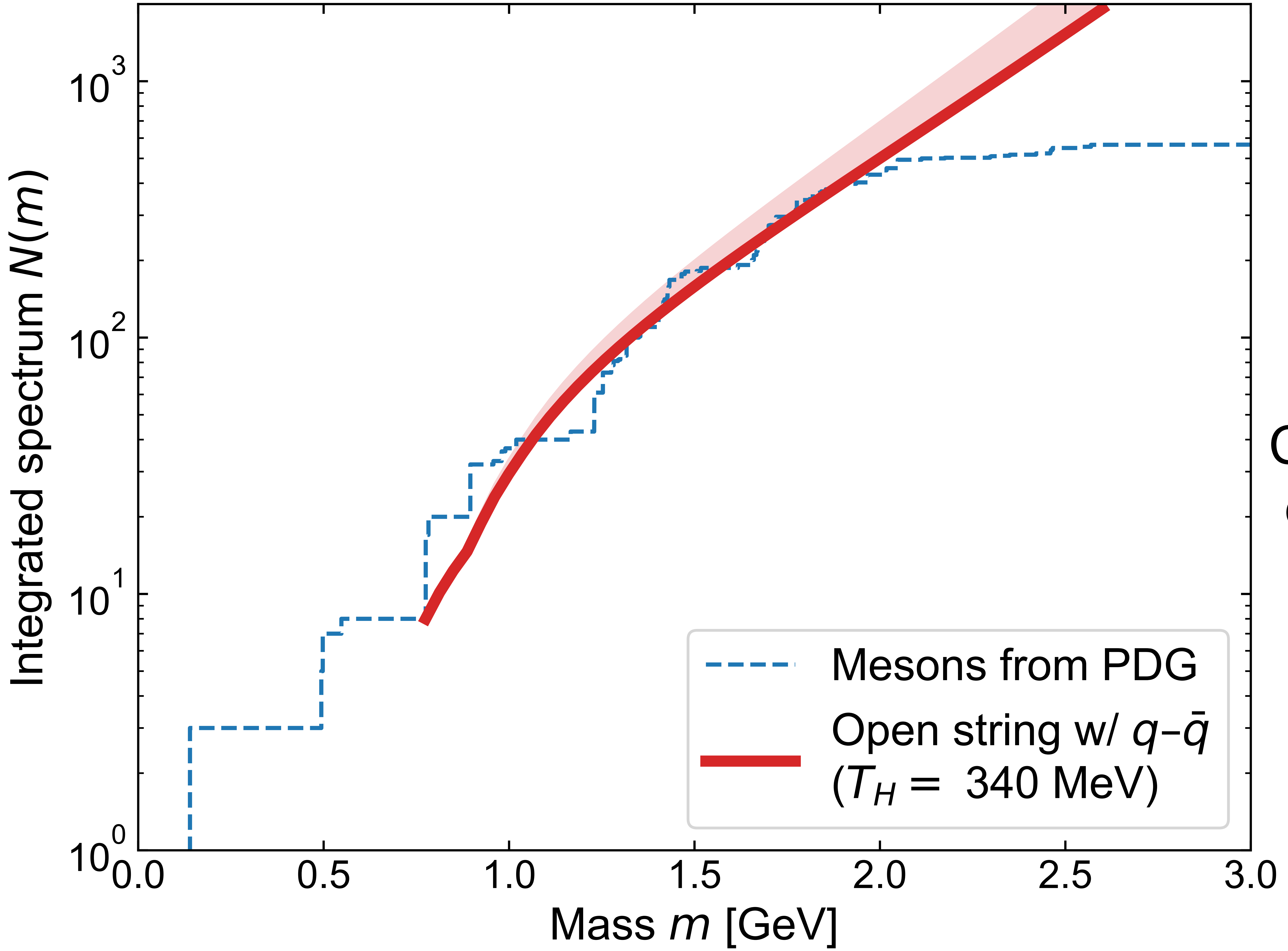
$$\rho(m) = \frac{\sqrt{2\pi}}{6T_H} \left(\frac{T_H}{m}\right)^{3/2} e^{m/T_H}$$

… no ambiguity in the coefficient

Huang,Weinberg (1970)

Fitting PDG with open string Hagedorn spectrum

Marczenko,Kovacs,McLerran,Redlich (2025)
Fujimoto, to appear (2025)



Fit gives $T_H \simeq 340$ MeV

Consistent with the string tension determined on the lattice at T=0

$\sqrt{\sigma}$ = 481.7 ± 9.7 MeV

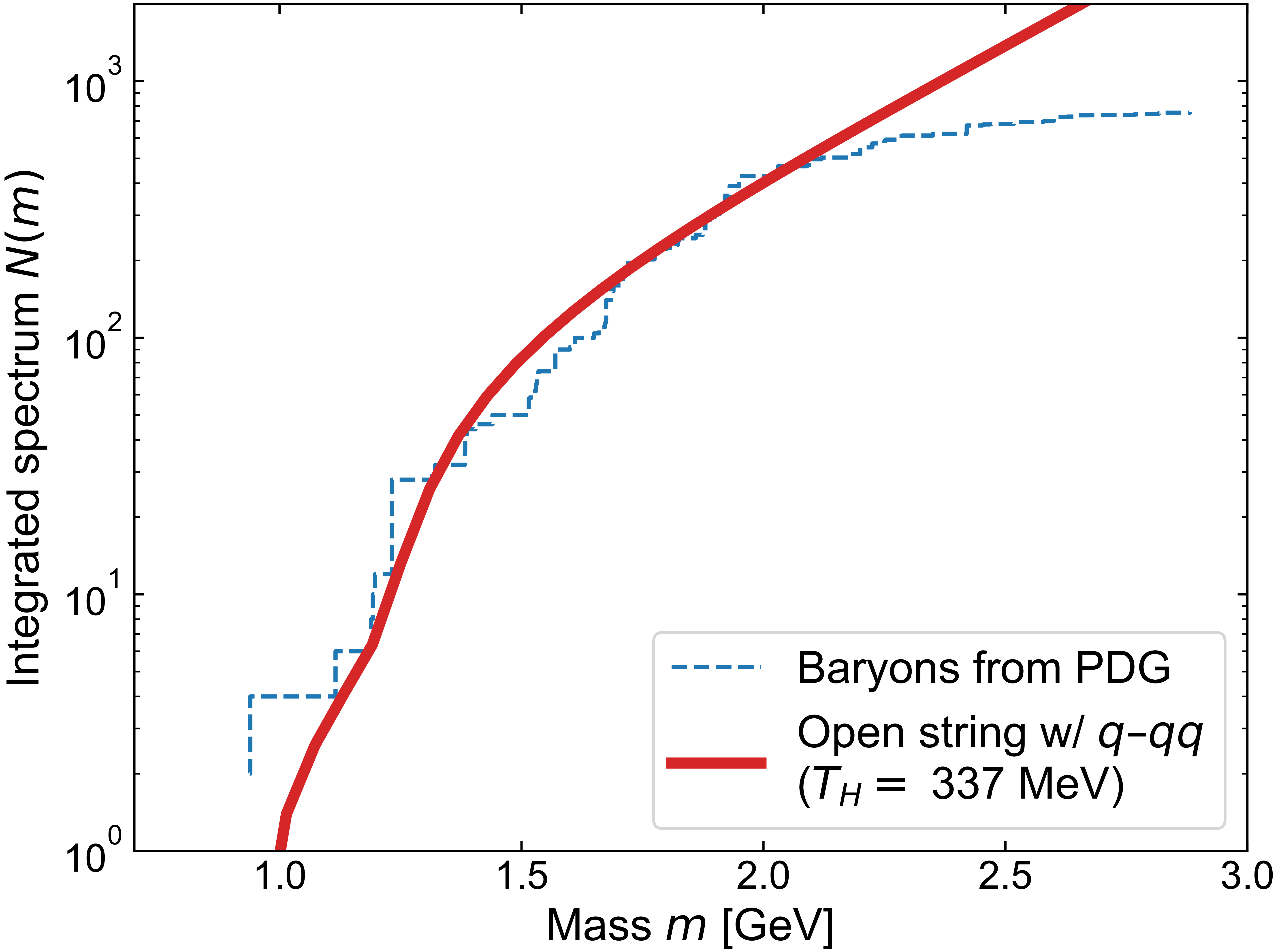
$\to T_H =$ 332.9 ± 6.7 MeV

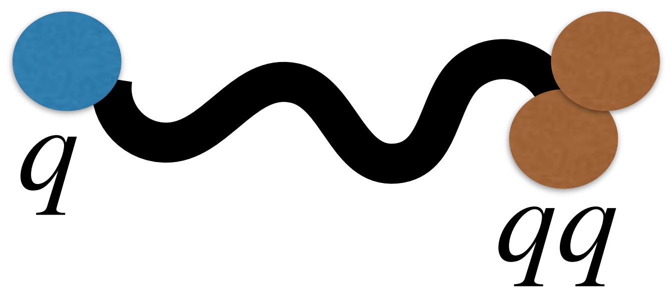
Brambilla,Delgado,Kronfeld,Leino,
Petreczky,Steinbeißer,Vairo,Weber (2022)

Fitting PDG with open string Hagedorn spectrum

Fujimoto, to appear (2025)

Also, baryons can be described by open string as a quark-diquark system! Selem,Wilczek (2006)





Fit to baryons gives $T_H \simeq 337$ MeV

Consistent with the string tension determined on the lattice at T=0

$\sqrt{\sigma}$ = 481.7 ± 9.7 MeV

$\rightarrow T_H =$ 332.9 ± 6.7 MeV

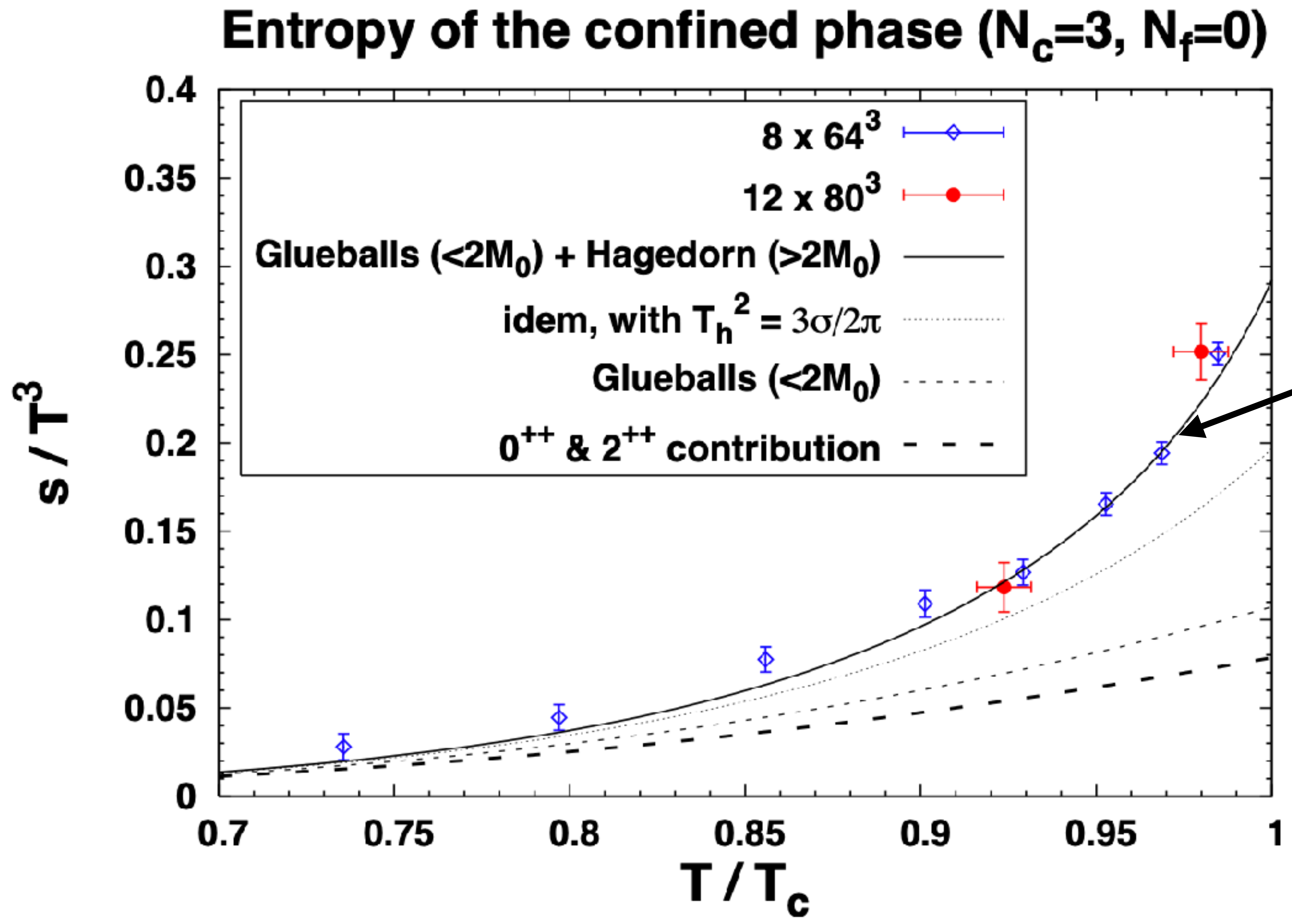
Brambilla,Delgado,Kronfeld,Leino, Petreczky,Steinbeißer,Vairo,Weber (2022)

Thermal pure glue QCD and Hagedorn spectrum

Pure SU(3) Yang-Mills → Deconfinement takes place at $T_d \simeq 285\,\mathrm{MeV}$

→ Below T_d is explained by glueballs

Borsanyi et al., PRD 105 (2022)

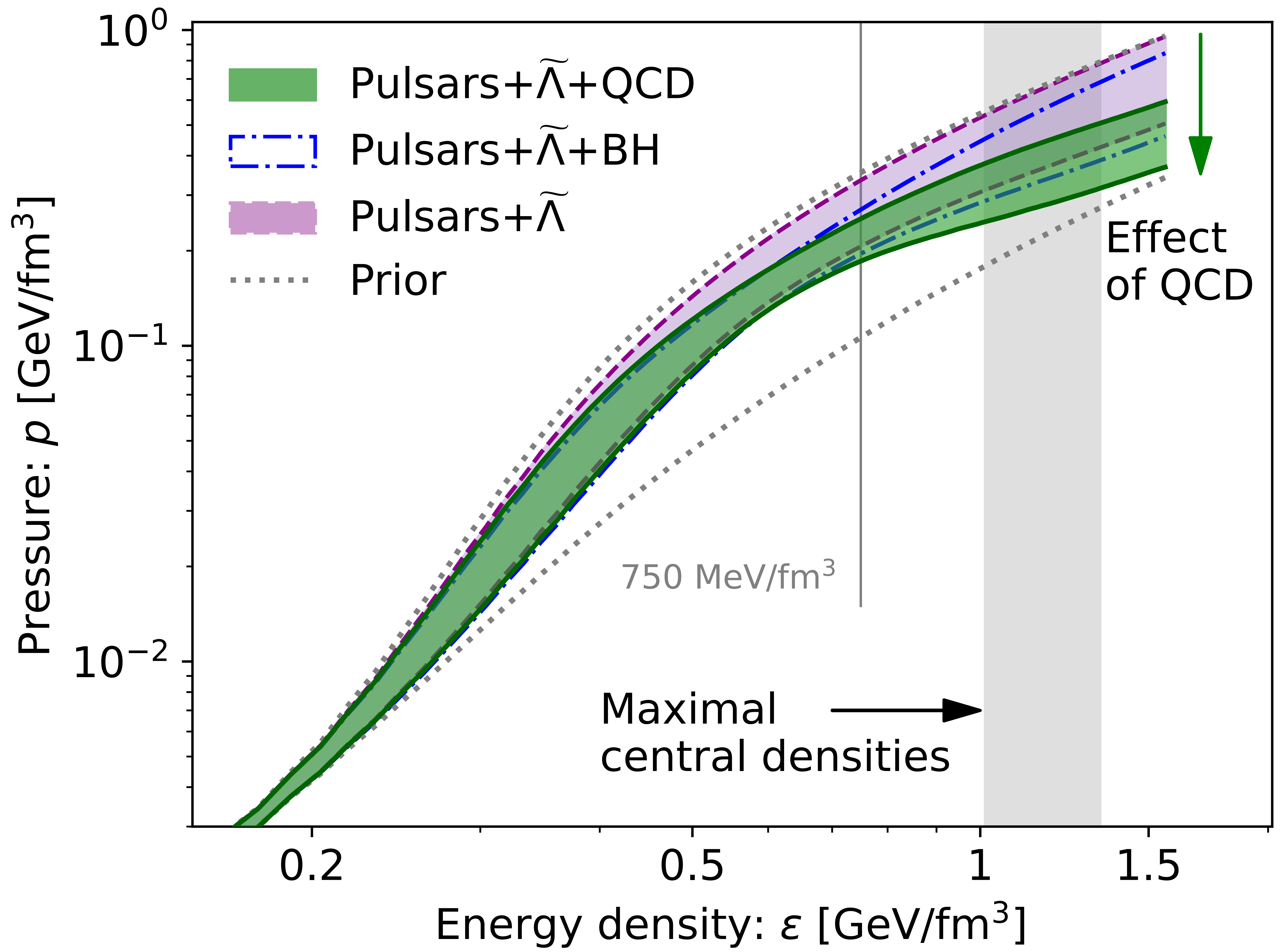


Solid line is fit w/ Hagedorn spectrum

Meyer, PRD 80 (2009)

Role of weak-coupling QCD in constraining EoS

Gorda,Komoltsev,Kurkela (2022);
Komoltsev,Somasundaram, et al. (2023)



- QCD effect significantly softens the equation of state at high density

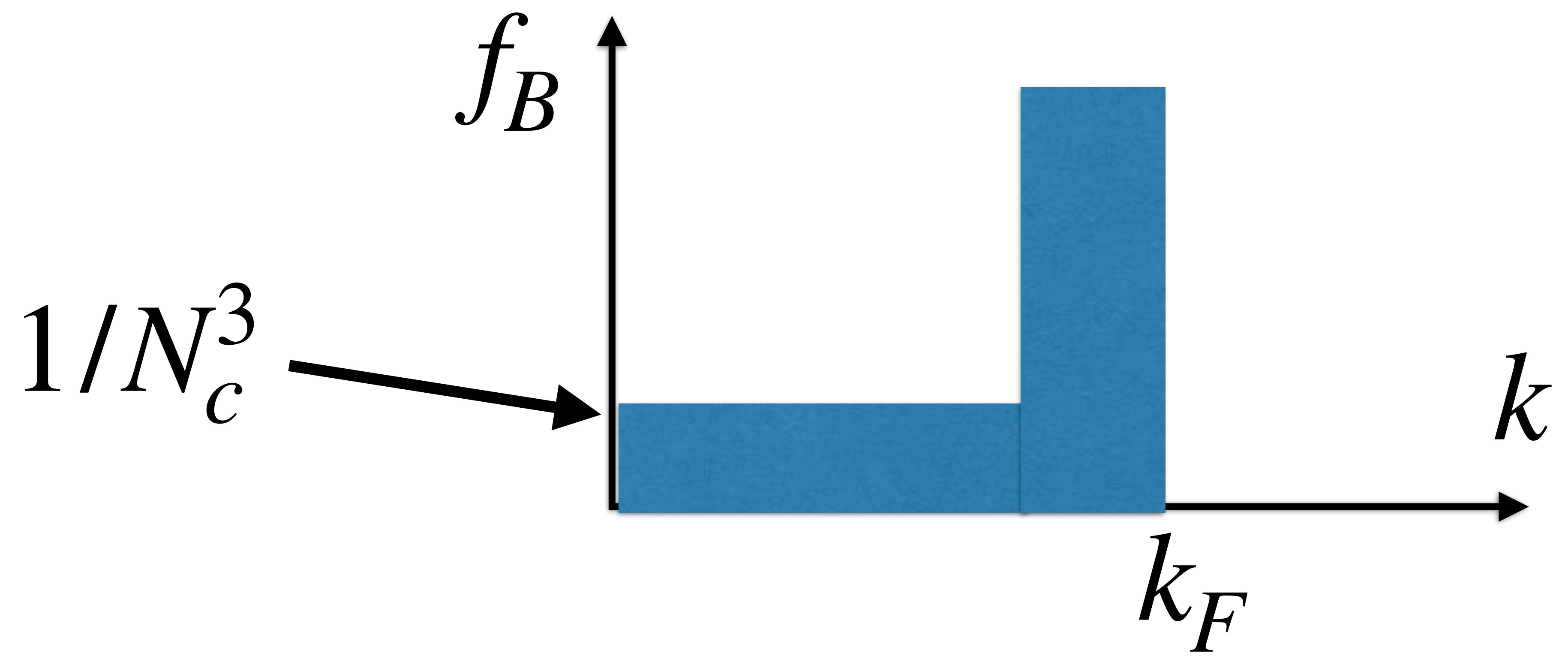
Quarkyonic matter favors large sound speed

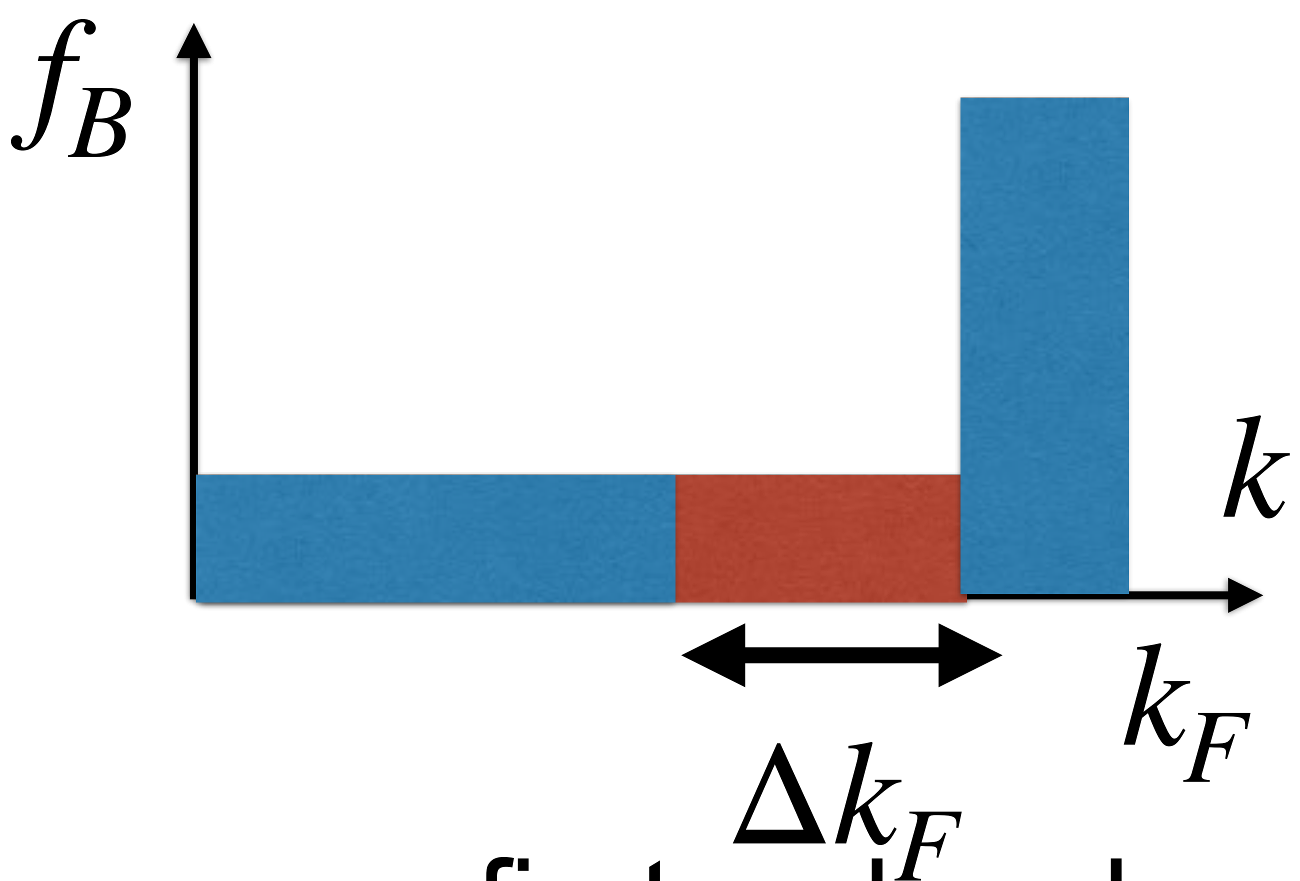
Fujimoto,Kojo,McLerran, PRL 132 (2023)

A partial occupation of available baryon phase space leads to **large sound speed:**

$$v_s^2 = \frac{n_{\mathrm{B}}}{\mu_{\mathrm{B}} dn_{\mathrm{B}}/d\mu_{\mathrm{B}}} \to \frac{\delta\mu_{\mathrm{B}}}{\mu_{\mathrm{B}}} \sim v_s^2 \frac{\delta n_{\mathrm{B}}}{n_{\mathrm{B}}}$$

If baryons have underoccupied state, the change in density is small while the change in chemical potential is large

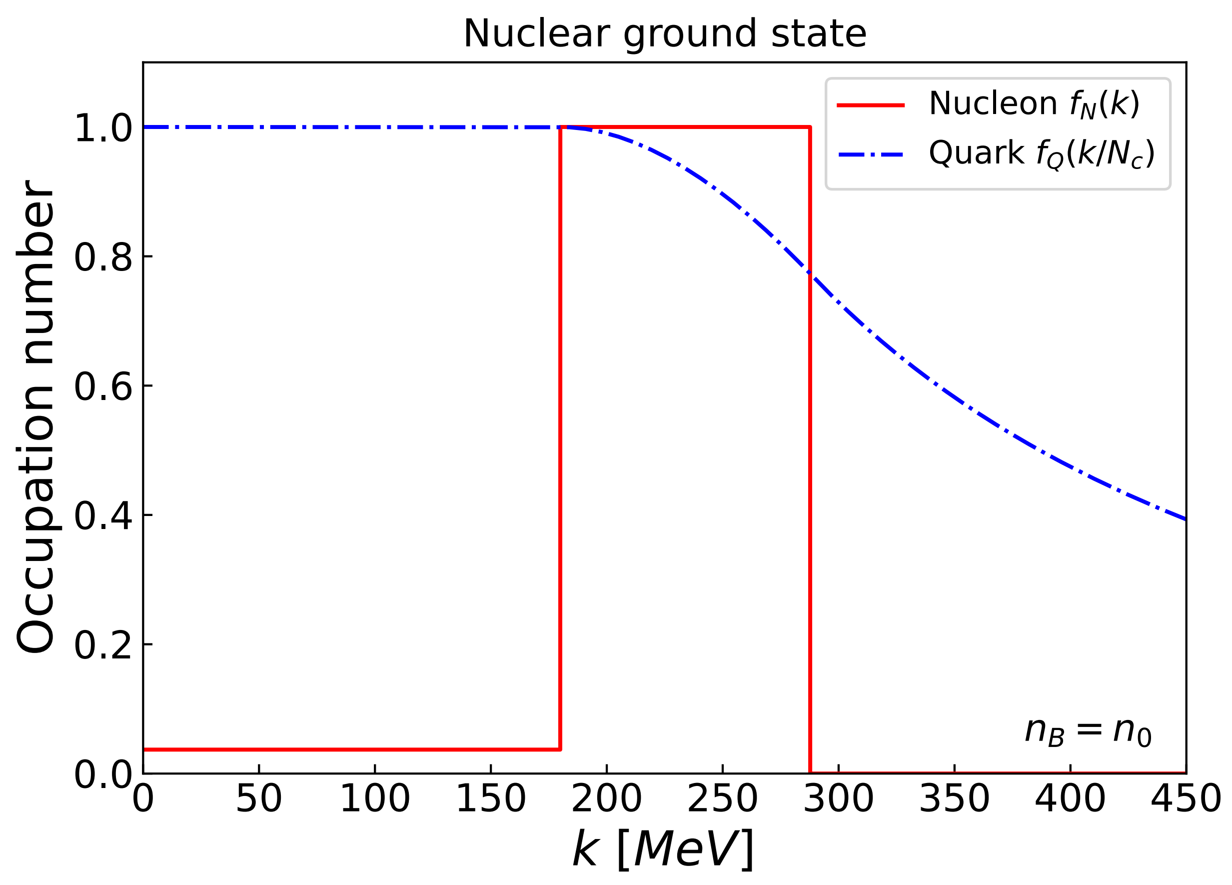


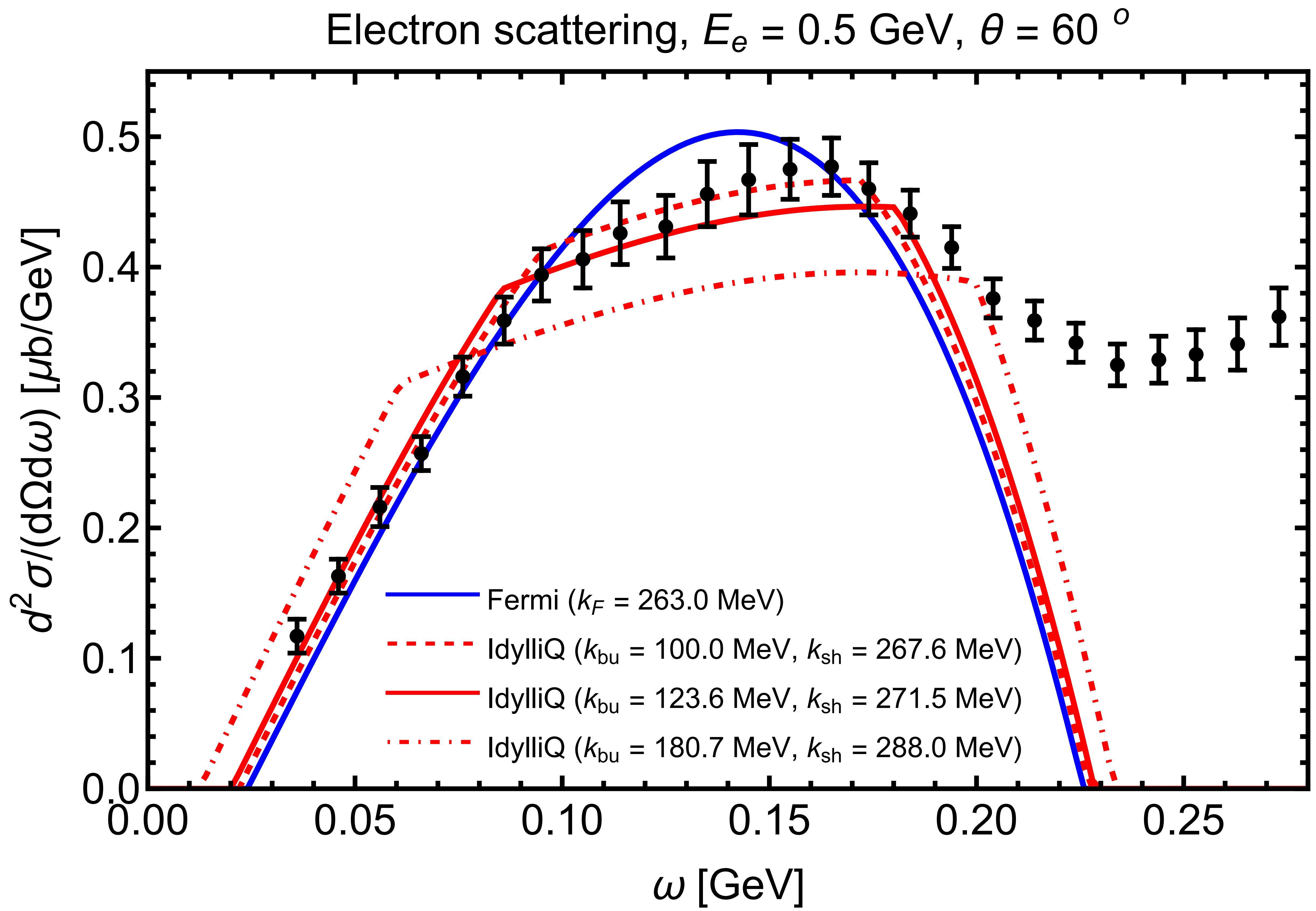


→ Favor the crossover over first-order phase transition ($v_s^2 = 0$)

Possible signature of Quarkyonic matter in experiment

Koch, McLerran, Miller, Vovchenko, PRC 110 (2024)





Suppression in low-momentum part of baryon distribution explains the data well