How do hadrons connect to quark–gluon matter?

Yoshimasa Hidaka
(YITP, Kyoto University, iTHEMS, RIKEN)

based on: Fujimoto, Fukushima, YH, McLerran, Phys. Rev. D 112, 074006 (2025)
Hayata, YH, Nishimura, JHEP 07, 106 (2024)
Fujikura YH, in progress
Hayata, YH, Kondo, JHEP 03 (2025) 006

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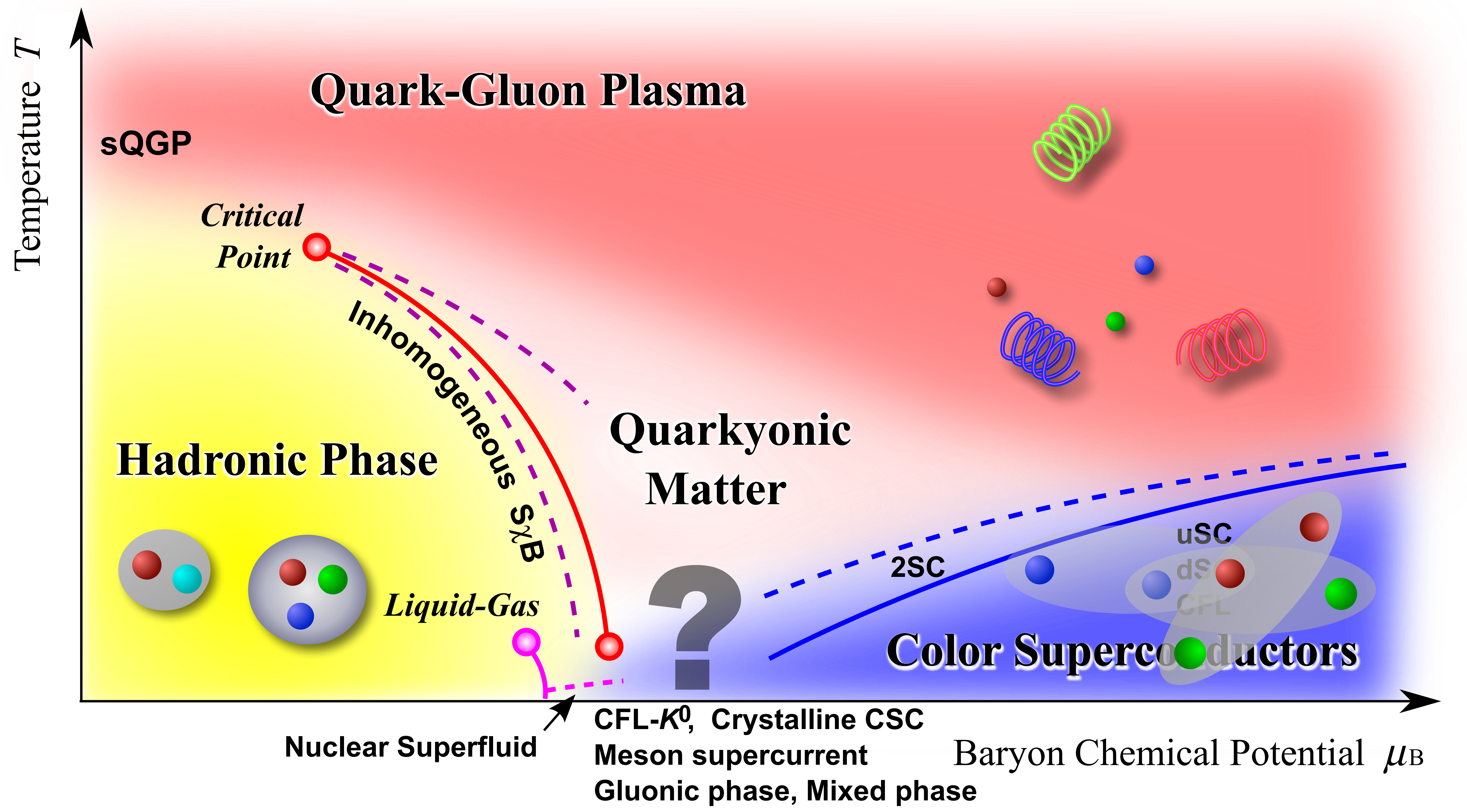
QCD phase diagram

Fukushima, Hatsuda, Rept. Prog. Phys. 74 (2011) 014001

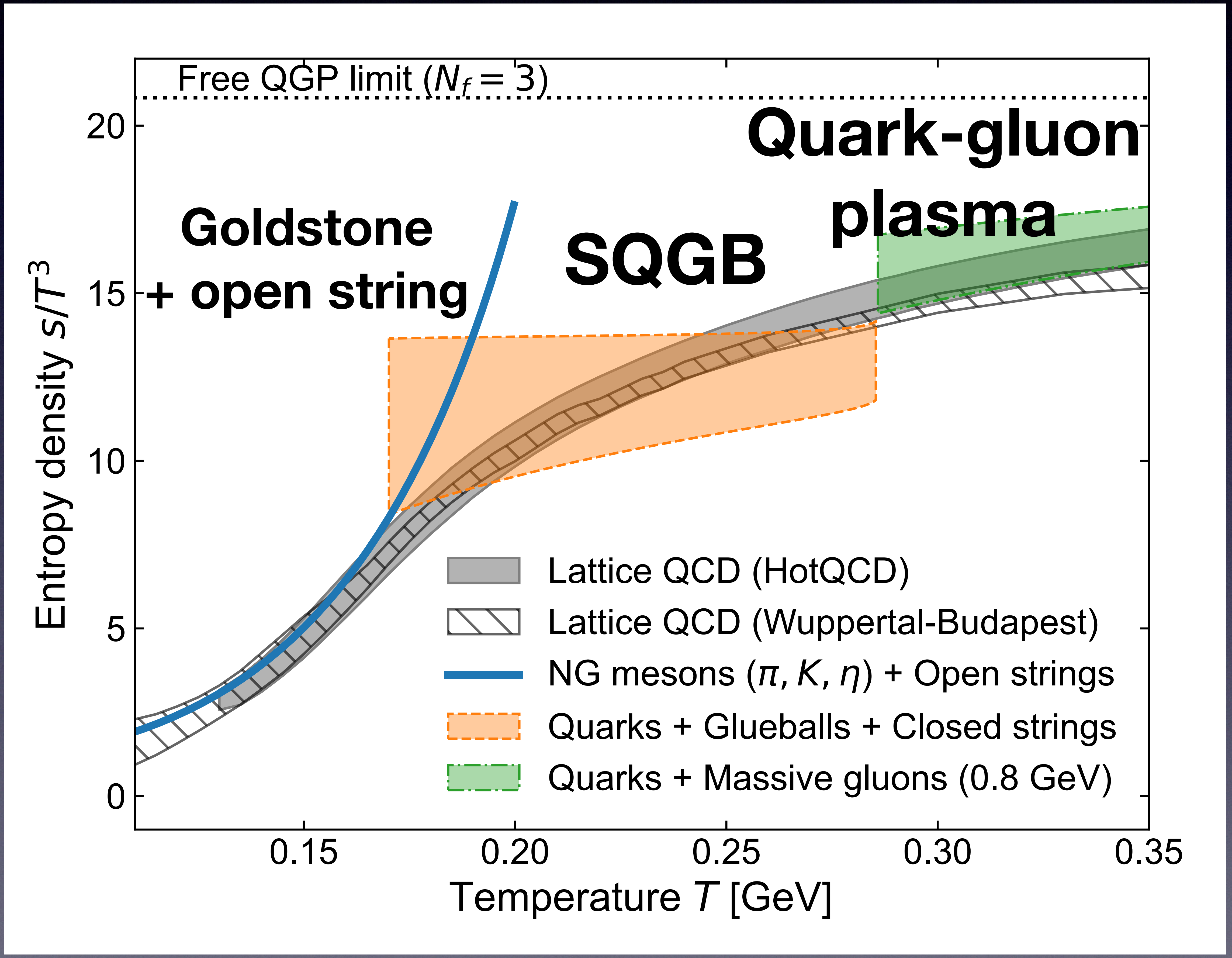


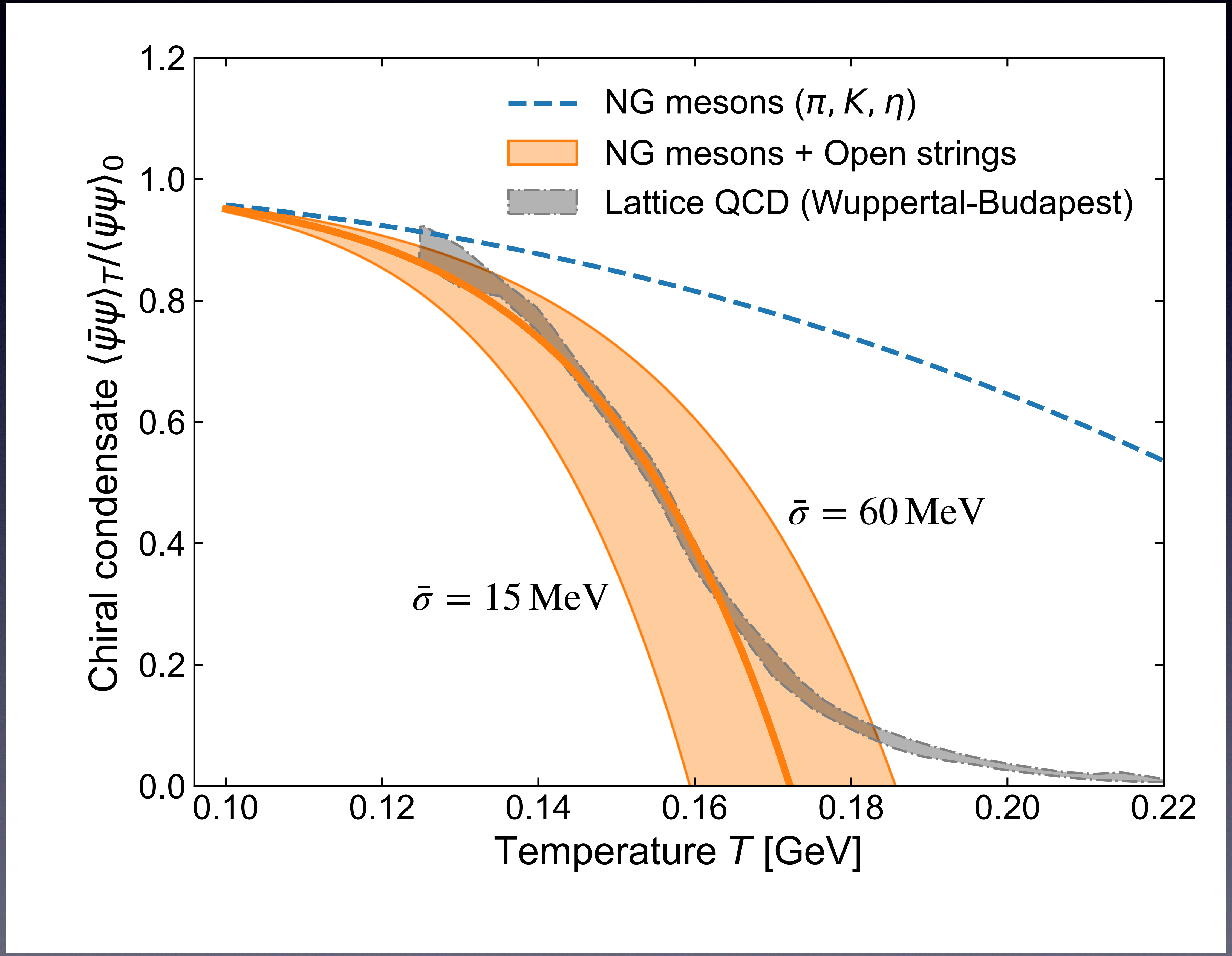
Finite-Temperature

we proposed Spaghetti of quarks with glueballs (SQGB)

There are three regimes

See Yuki's talk

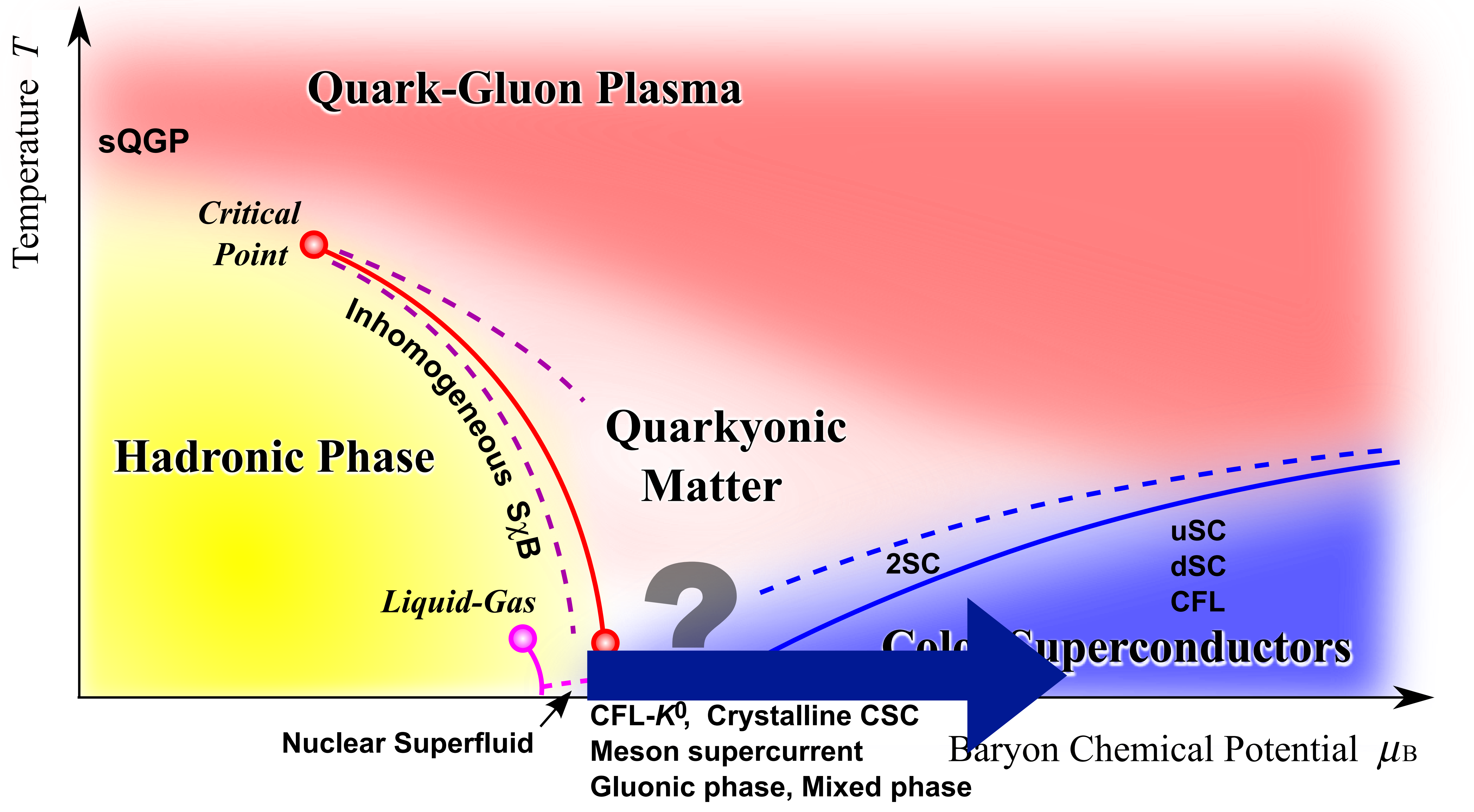




$$\langle\bar{\psi}\psi\rangle_T \approx \langle\bar{\psi}\psi\rangle_0 - \frac{\partial p(M)}{\partial M}\frac{\sigma_M}{m_q}$$

In this talk, we focus on Dense QCD matter

Fukushima, Hatsuda, Rept. Prog. Phys. 74 (2011) 014001



What we know

For 3-flavor QCD : $G = SU(3)_f \times U(1)_B$

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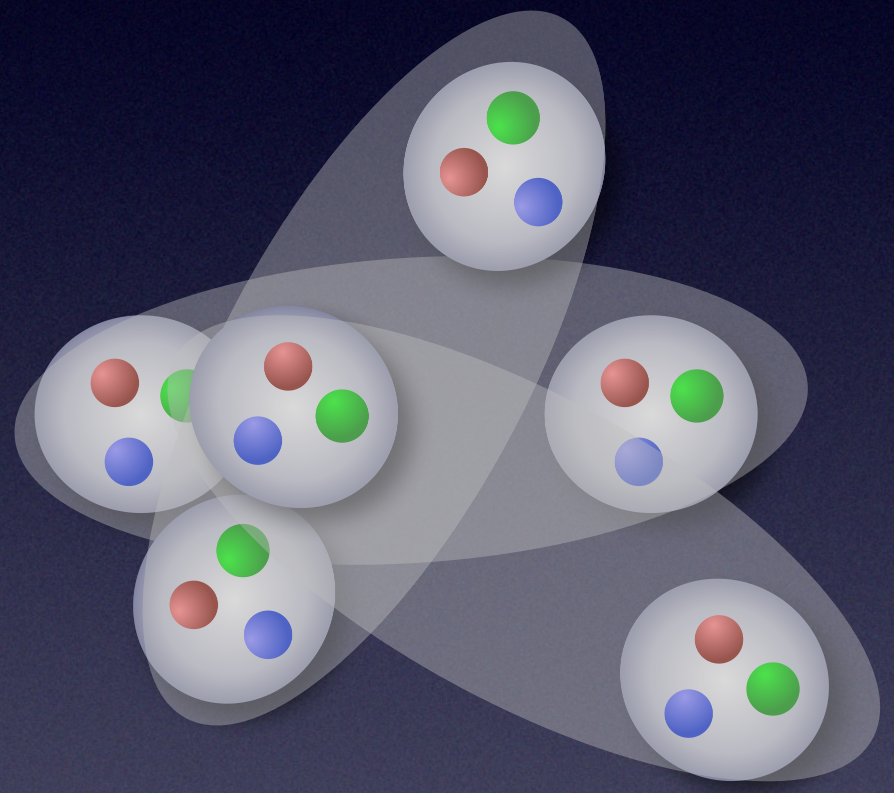
- **Superfluid(dilute phase)**



Baryon pair condensation

$$\Delta = \langle \Lambda\Lambda \rangle \neq 0 \qquad \Lambda \sim uds$$

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

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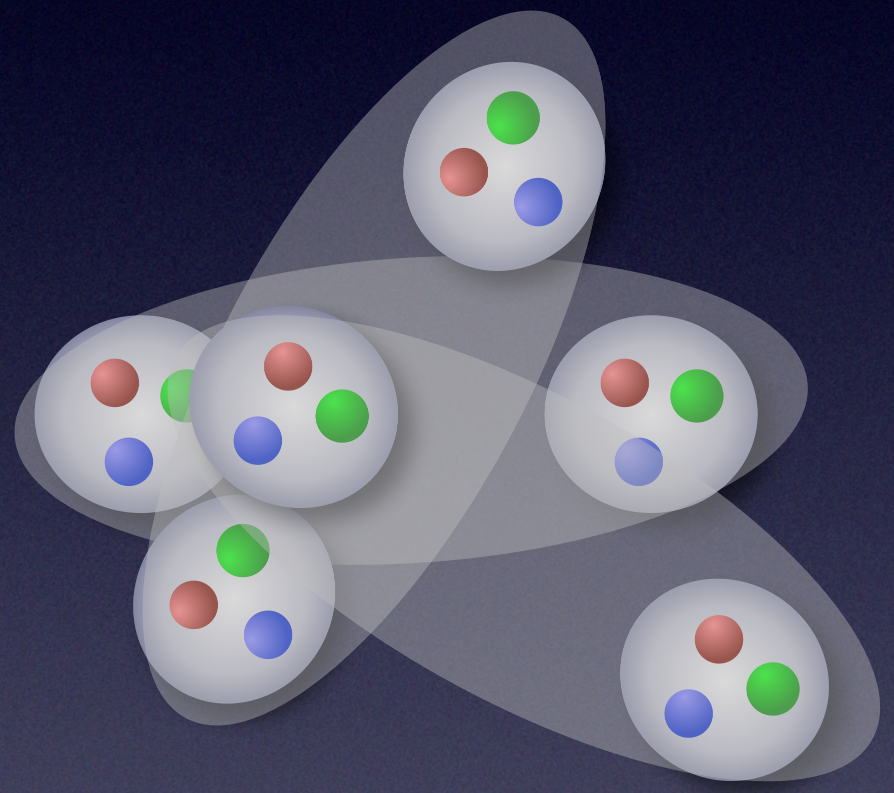

Baryon pair condensation

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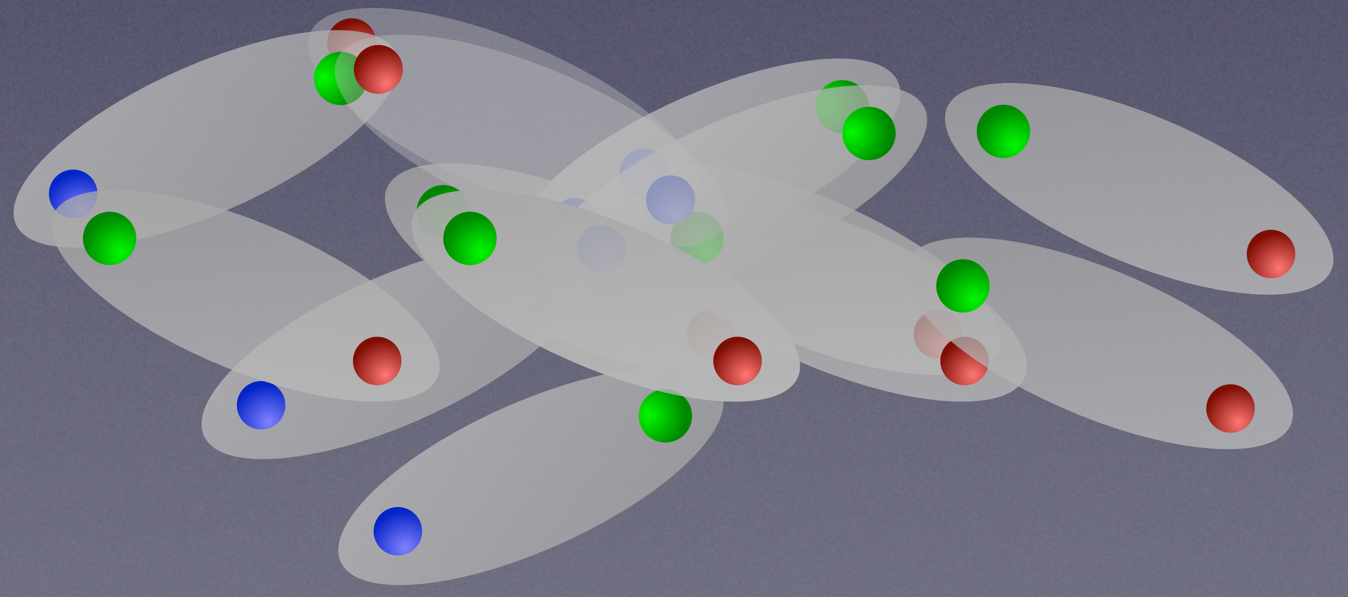
$$SU(3)_f \times U(1)_B \to SU(3)_f$$

- **Color super conductor (dense phase)**



“quark pair condensate”

$$(\Phi_L)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_L)^b_j (Cq_L)^c_k \rangle = -\epsilon^{ijk}\epsilon_{abc}\langle (q_R)^b_j (Cq_R)^c_k \rangle$$

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

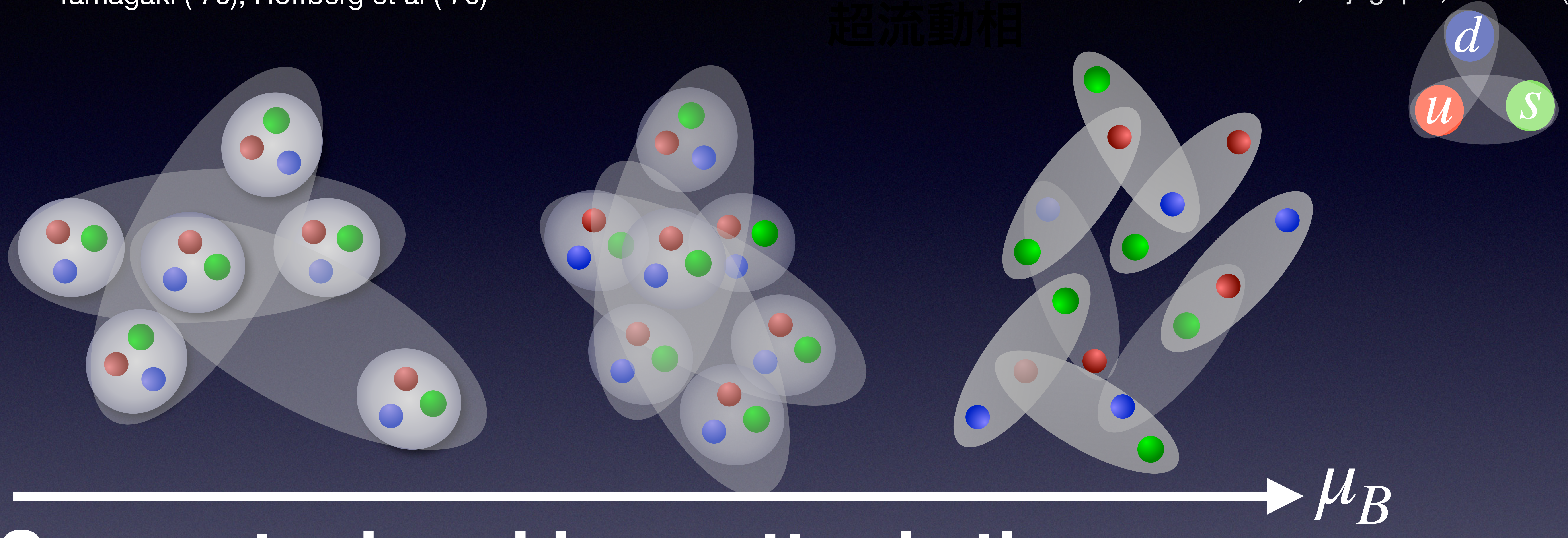
Quark hadron continuity

Hadronic superfluid

Tamagaki ('70), Hoffberg et al ('70)

Color flavor locked phase (CFL phase)

Alford, Rajagopal, Wilczek ('99)



Symmetry breaking patter is the same

⇒Quark hadron continuity

Excitations

Baryons ⇒ Quarks

Vector meson ⇒ Gluons

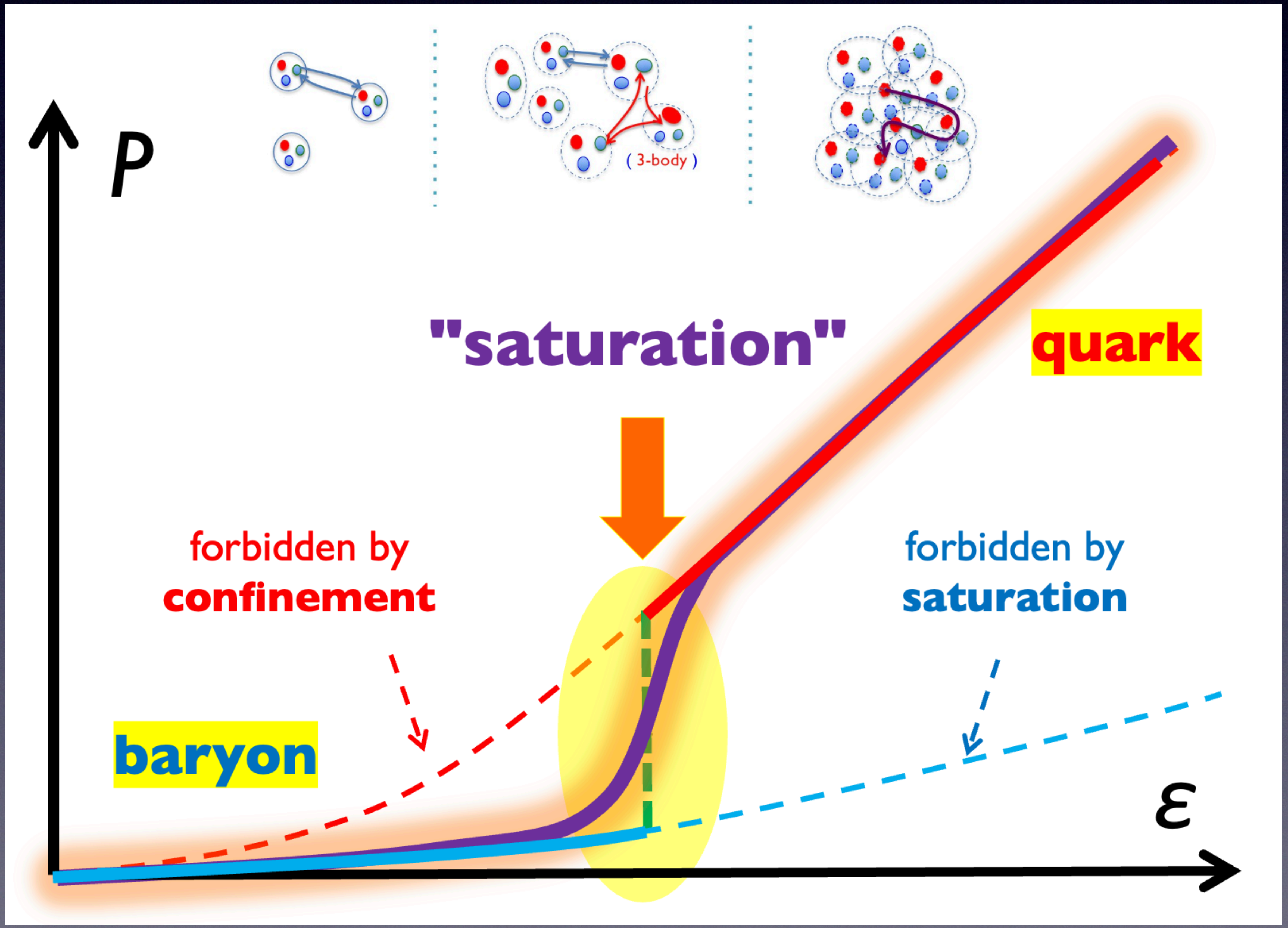
cf. Hatsuda, Tachibana, Yamamoto, Baym ('06)

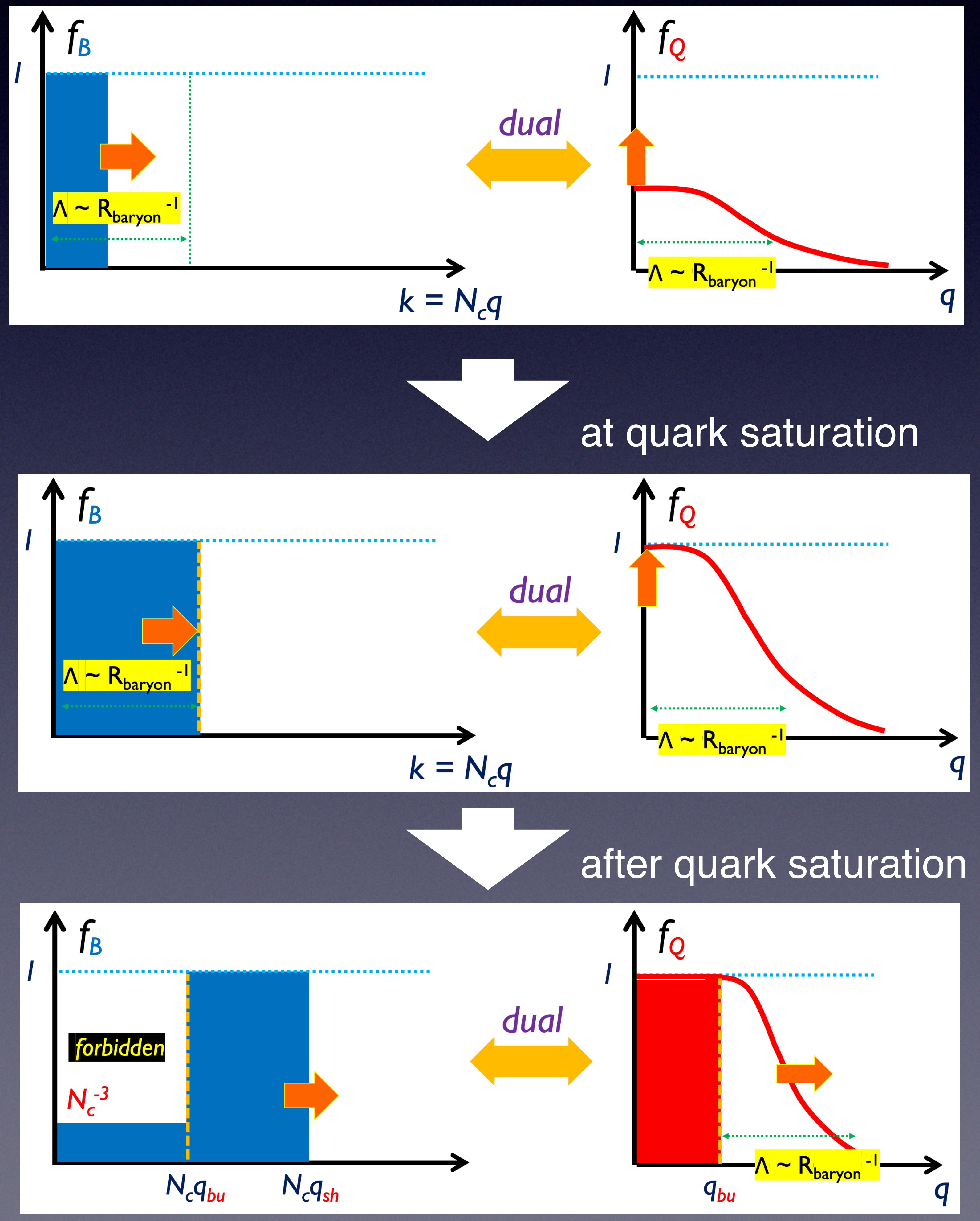
Ideal dual Quarkyonic (IdylliQ) model

Kojo ('21), McLerran, Reddy ('19), Fujimoto, Kojo, McLerran ('24)

$$f_Q(\boldsymbol{q}, n_B) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B; n_B)\varphi_Q^B(\boldsymbol{q}_B; \boldsymbol{P}_B)$$

Figure is taken from Kojo, 2412.20442





Study of QCD_2

Numerical calculations
can be done by using tensor network technique

(Bosonization tequnique also useful.)

Kojo ('11), Lajer, Konik, Pisarski, Tsvelik ('21)

- **Finite box simulation**

Hayata, YH, Nishimura, JHEP 07 (2024) 106

- **Infinite Volume simulation**

work in progress with Yohei Fujikura

(dimensionless) QCD_2 Hamiltonian

$$J = \frac{ag_0}{2}, w = \frac{1}{2g_0 a}, m = m_0/g_0$$

We use $g_0 = 1$ unit

$$H/g_0 = J\sum_{n=1}^{N-1} E_i^2(n)$$ **Electric field term**

$$+w\sum_{n=1}^{N-1}\left(\chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1)\right)$$

Hopping term

$$+m\sum_{n=1}^{N}(-1)^n\chi^\dagger(n)\chi(n)$$ **Mass term**

As a variational ansatz of wave function

- We employ a matrix product state

$$|\psi\rangle = \sum_{\{n_i\}} |n_1\rangle \cdots |n_N\rangle \mathrm{tr} M_1^{n_1} \cdots M_N^{n_N}$$

$[M_i^{n_i}]_{ij}$: $D \times D$ matrix

- Optimize the wave function by density matrix renormalization group technique

$$E = \min_{\psi} \langle \psi | H | \psi \rangle$$

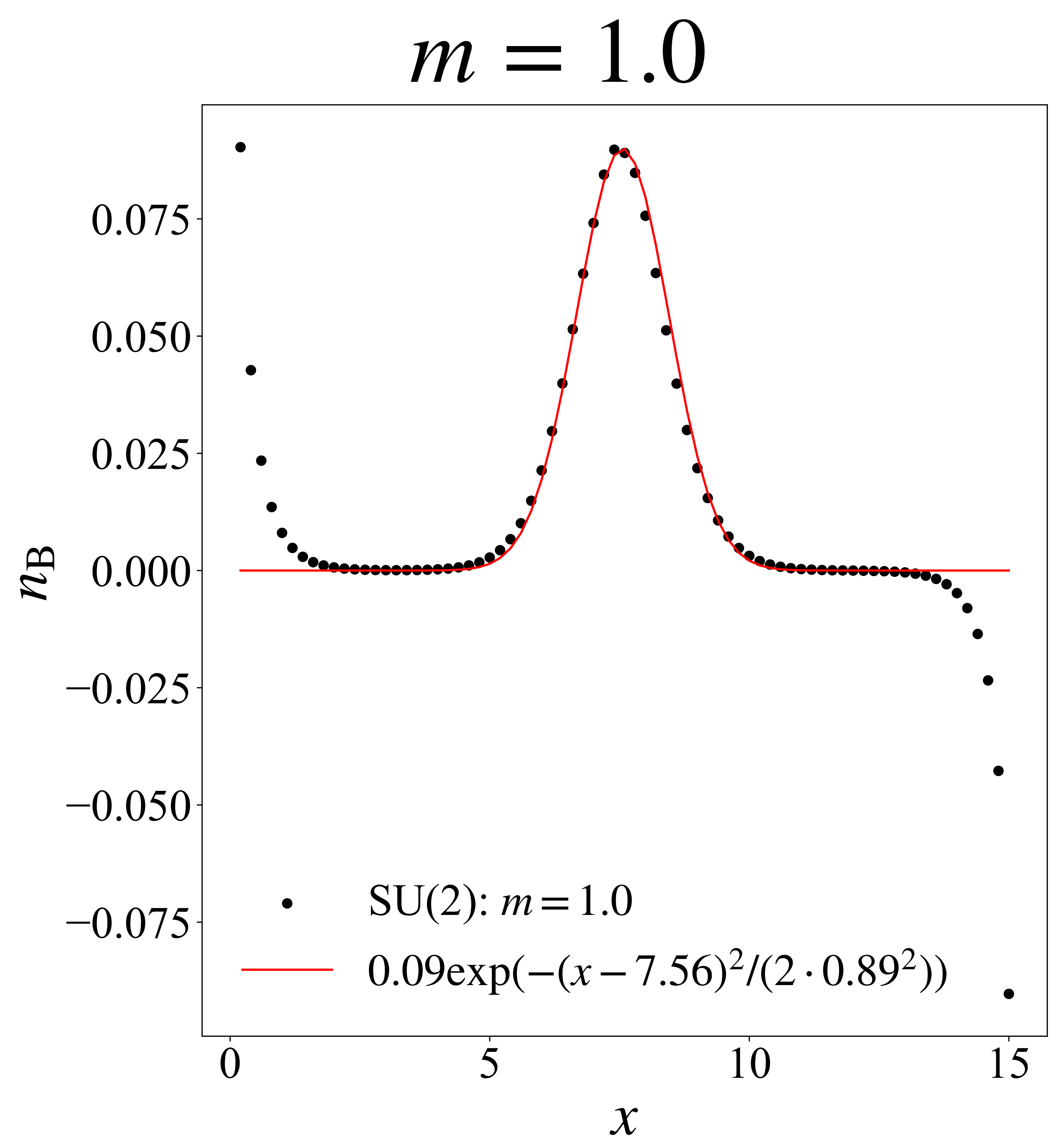
We employ iTensor library for a finite box system

Numerical results

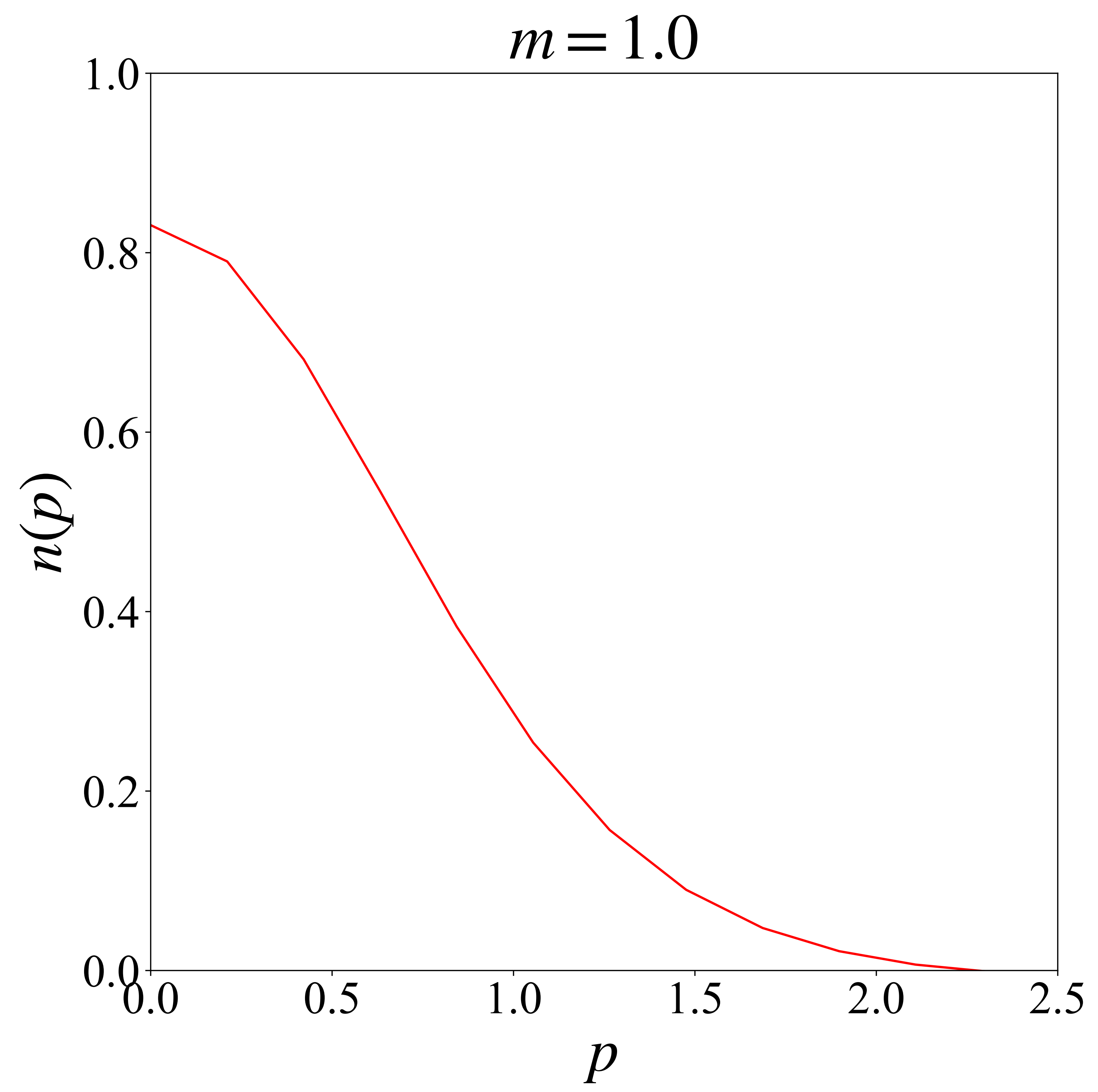
Color SU(2), 1 flavor, vacuum

single baryon state $\dim\mathscr{H} = 2^{300}$ $J = 1/20$ $w = 5$ volume $V = 15$

Baryon number density



Quark distribution function



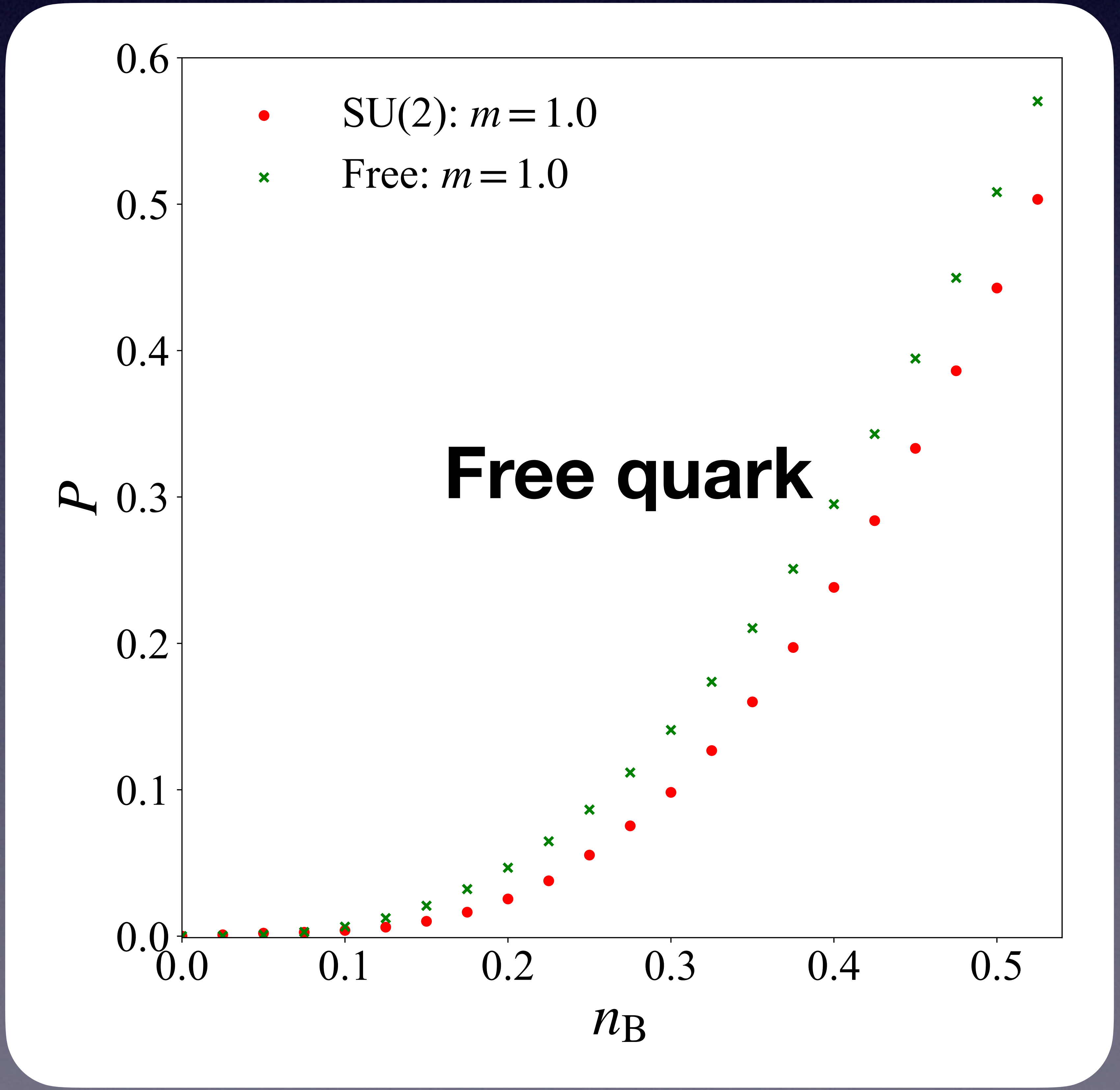
Baryon size ~ 1

QCD_2 Hamiltonian Lattice simulation
with density matrix renormalization technique

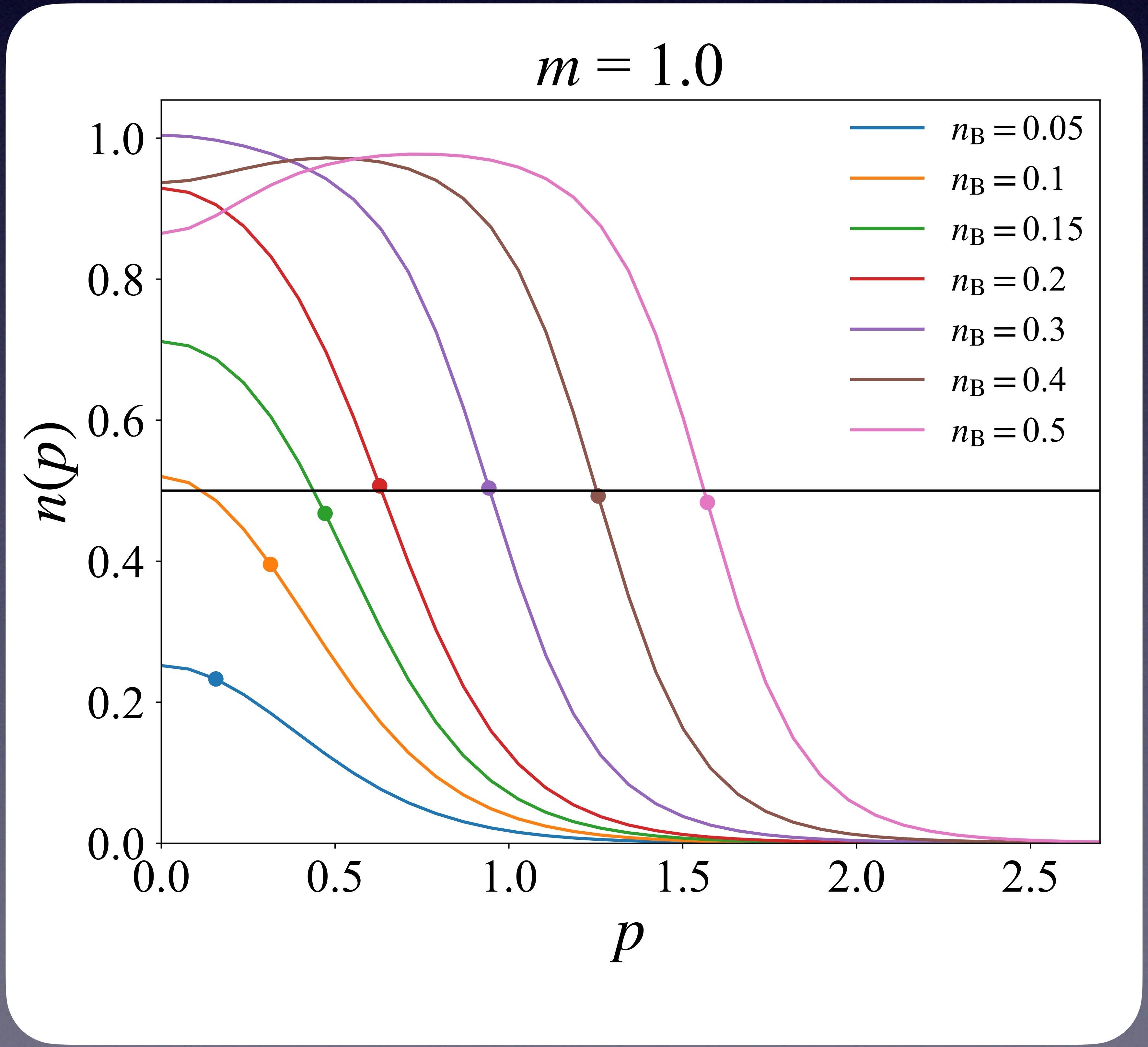
Tomoya Hayata, YH, Nishimura ('23)

Two color QCD $N_f = 1$

$\dim\mathscr{H} = 2^{320}$



$\dim\mathscr{H} = 2^{480}$



QCD_2 Hamiltonian Lattice simulation
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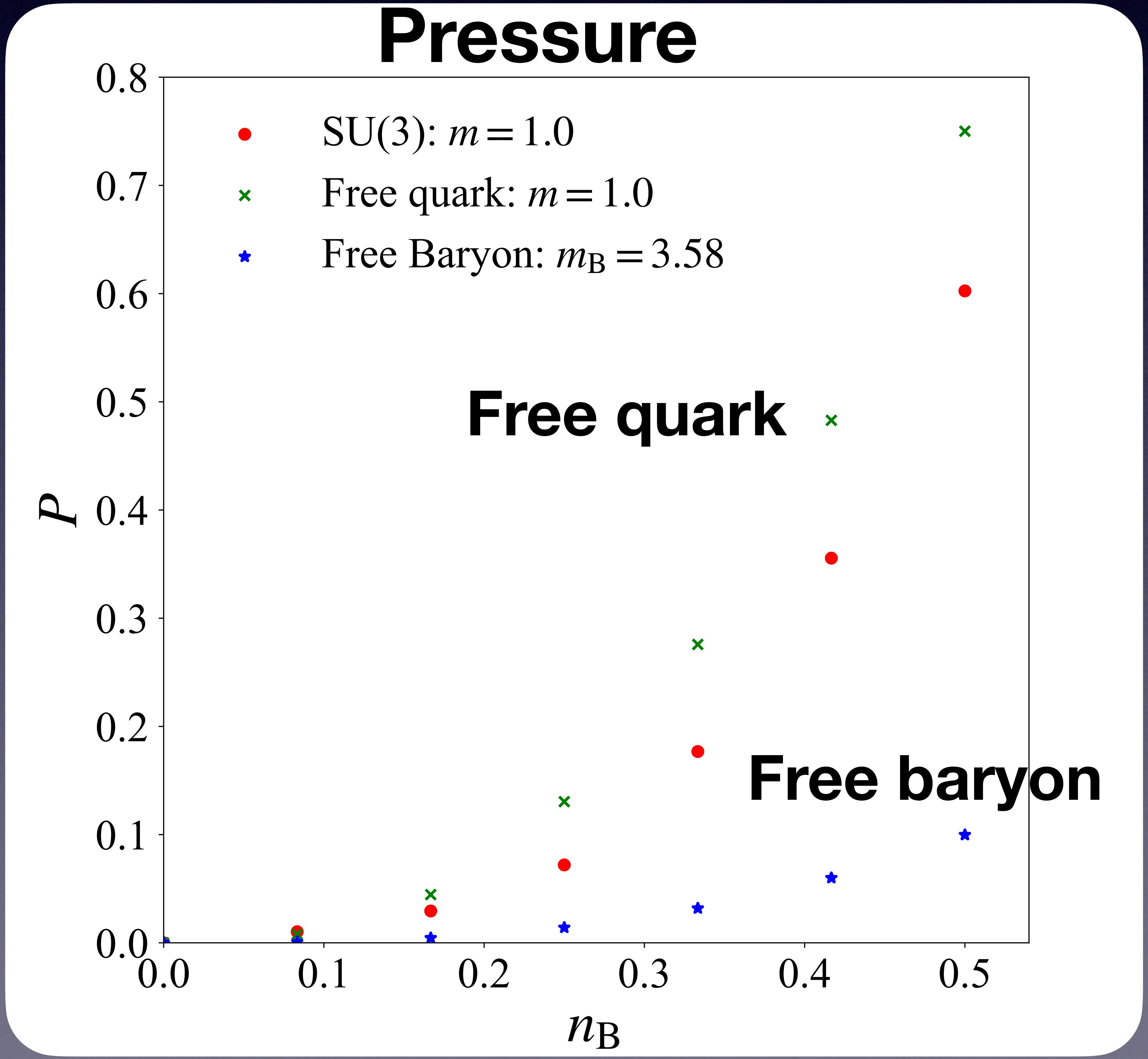
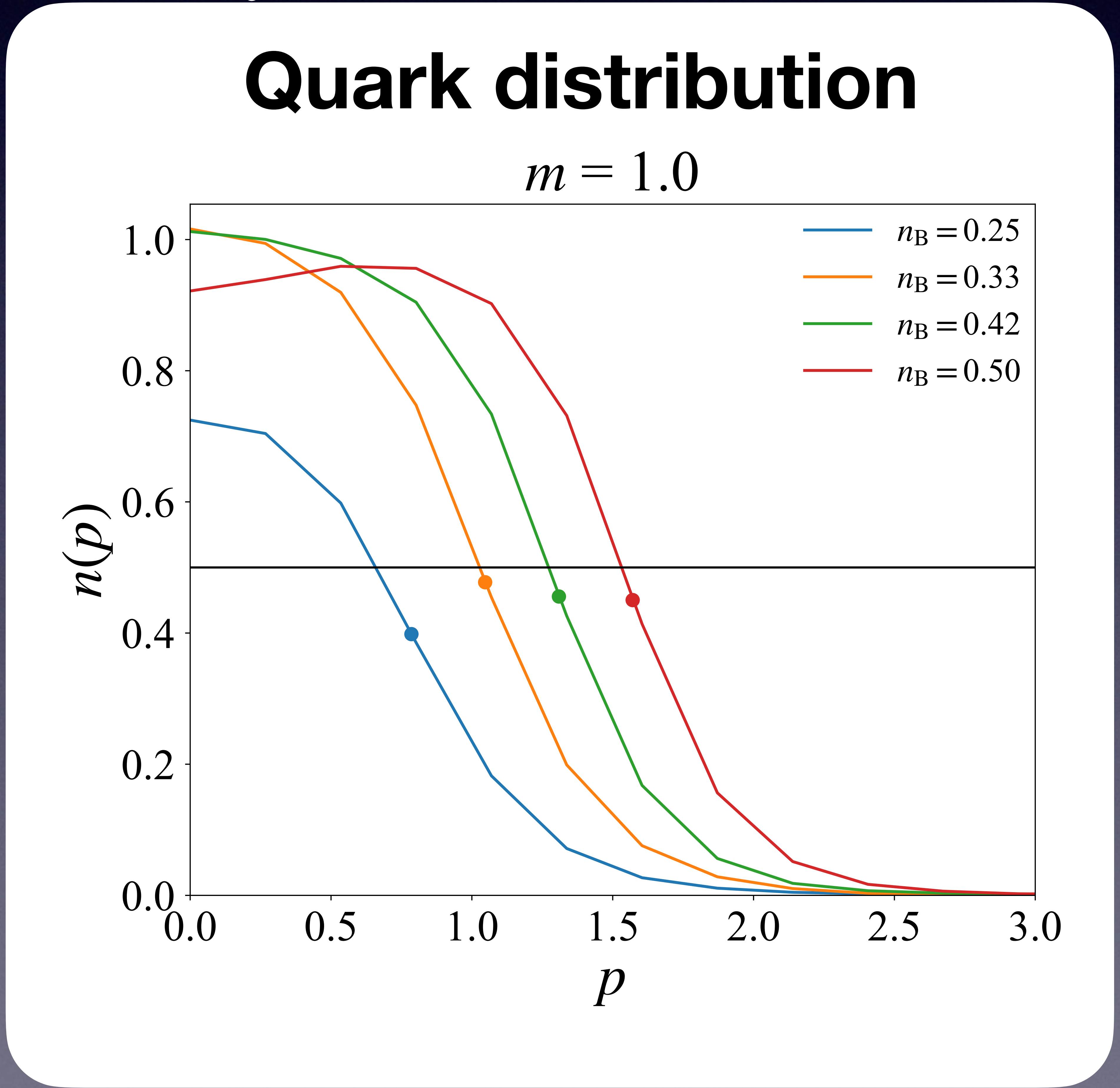
Tomoya Hayata, YH, Nishimura ('23)

Three color QCD, $N_f = 1 \quad \dim\mathscr{H} = 2^{144}$



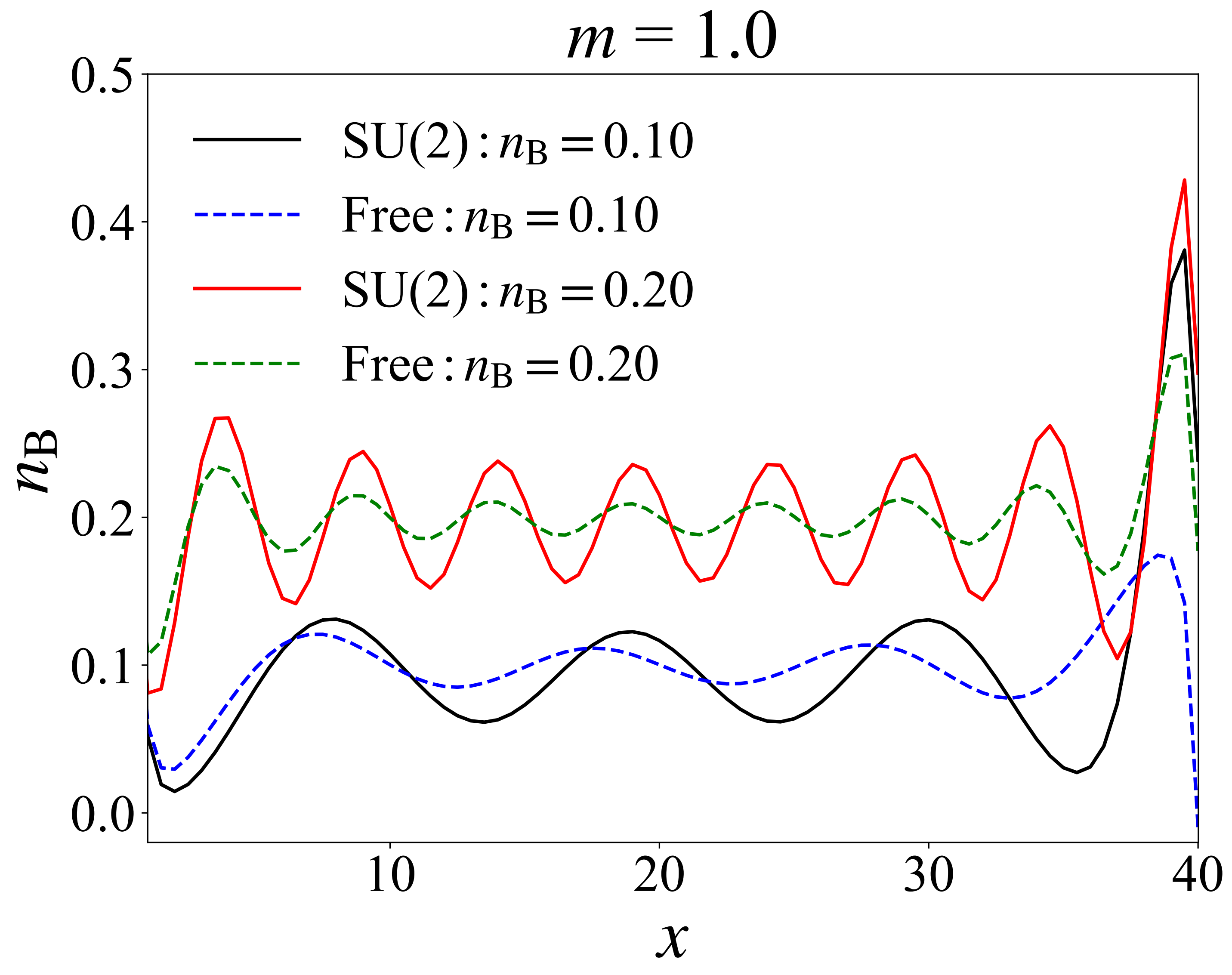


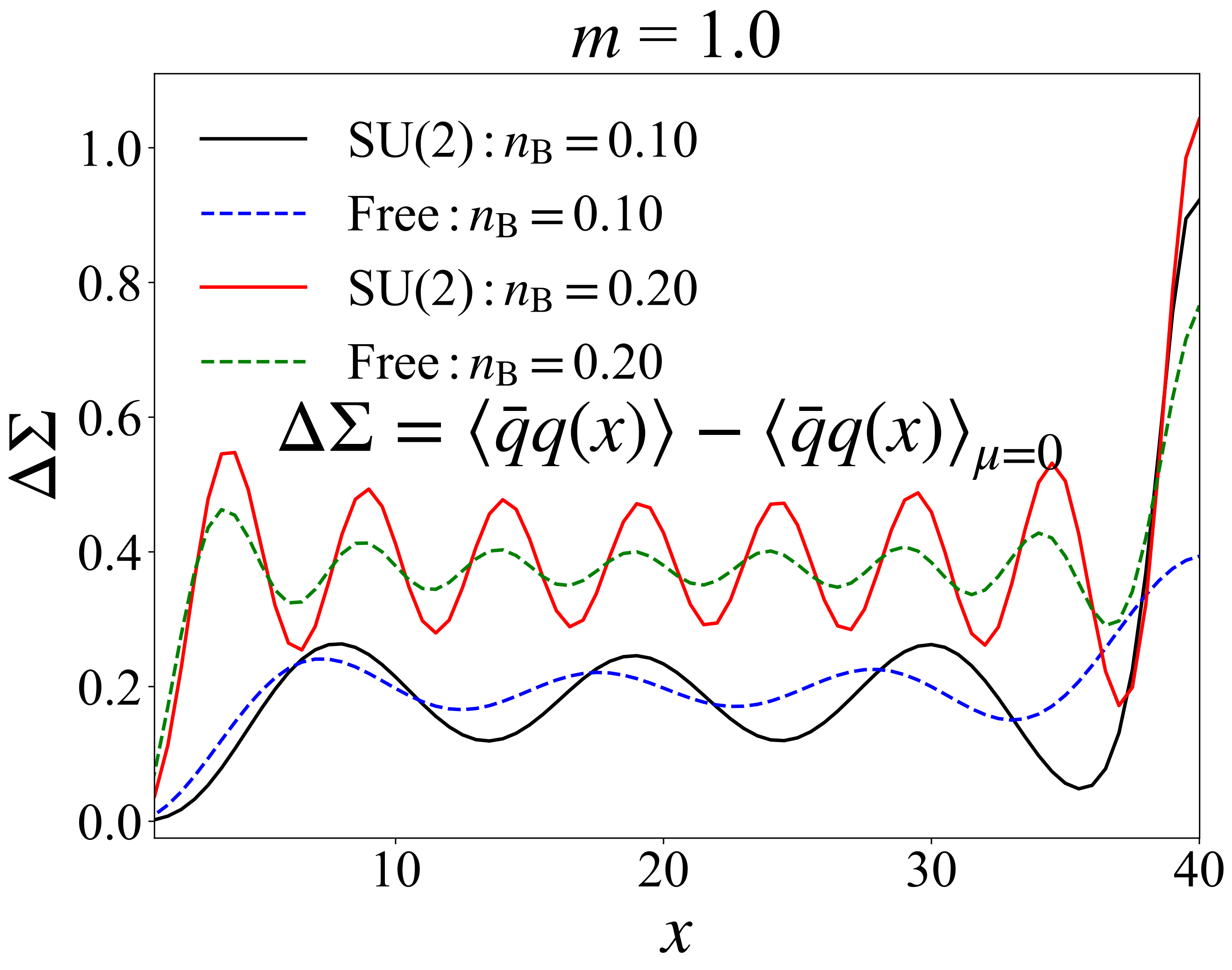
Inhomogeneous phase in QCD_2

corresponding to 'quarkyonic chiral spirals' Kojo, Hidaka, McLerran, Pisarski (2010)

SU(2)

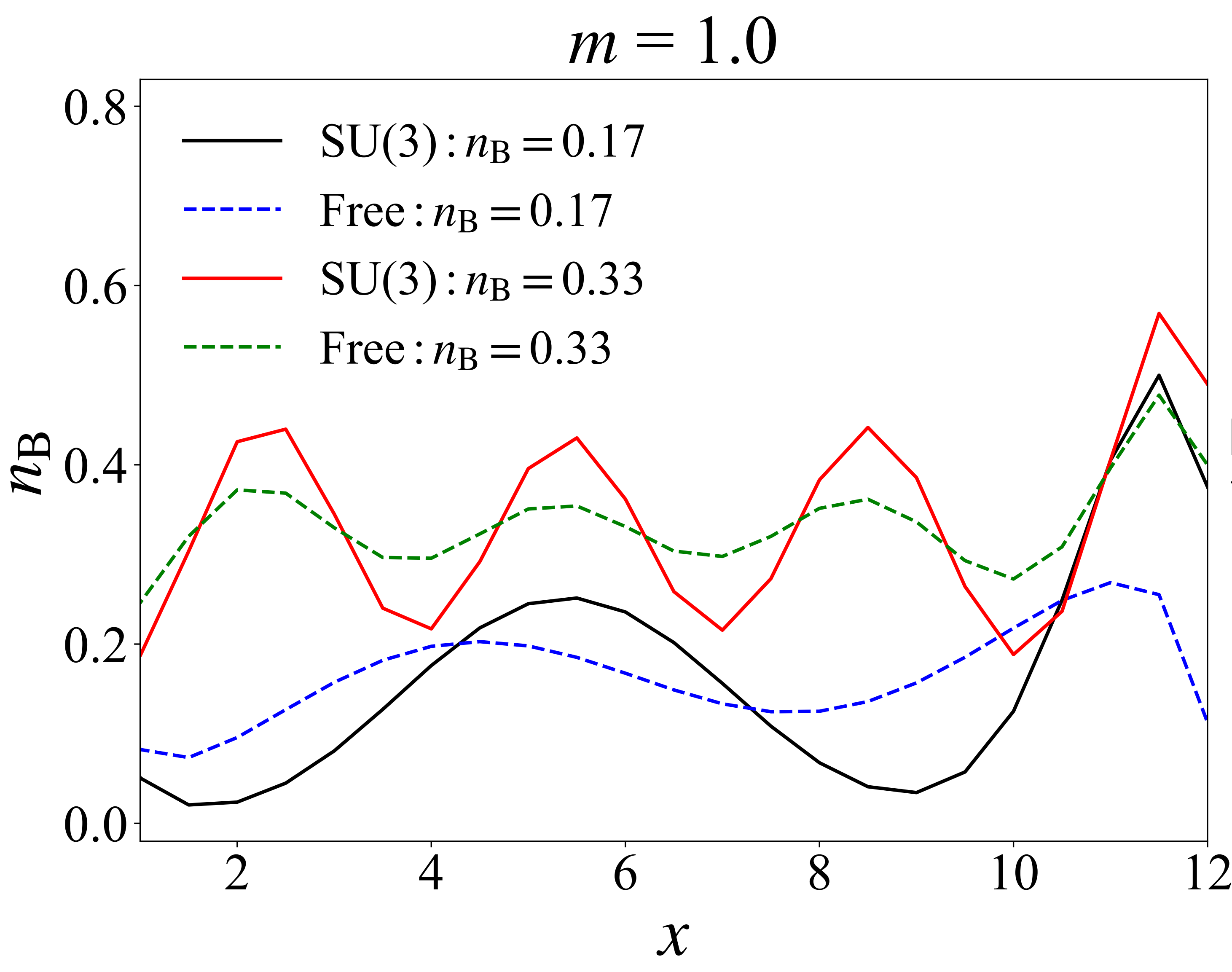
$\dim\mathscr{H} = 2^{320}$

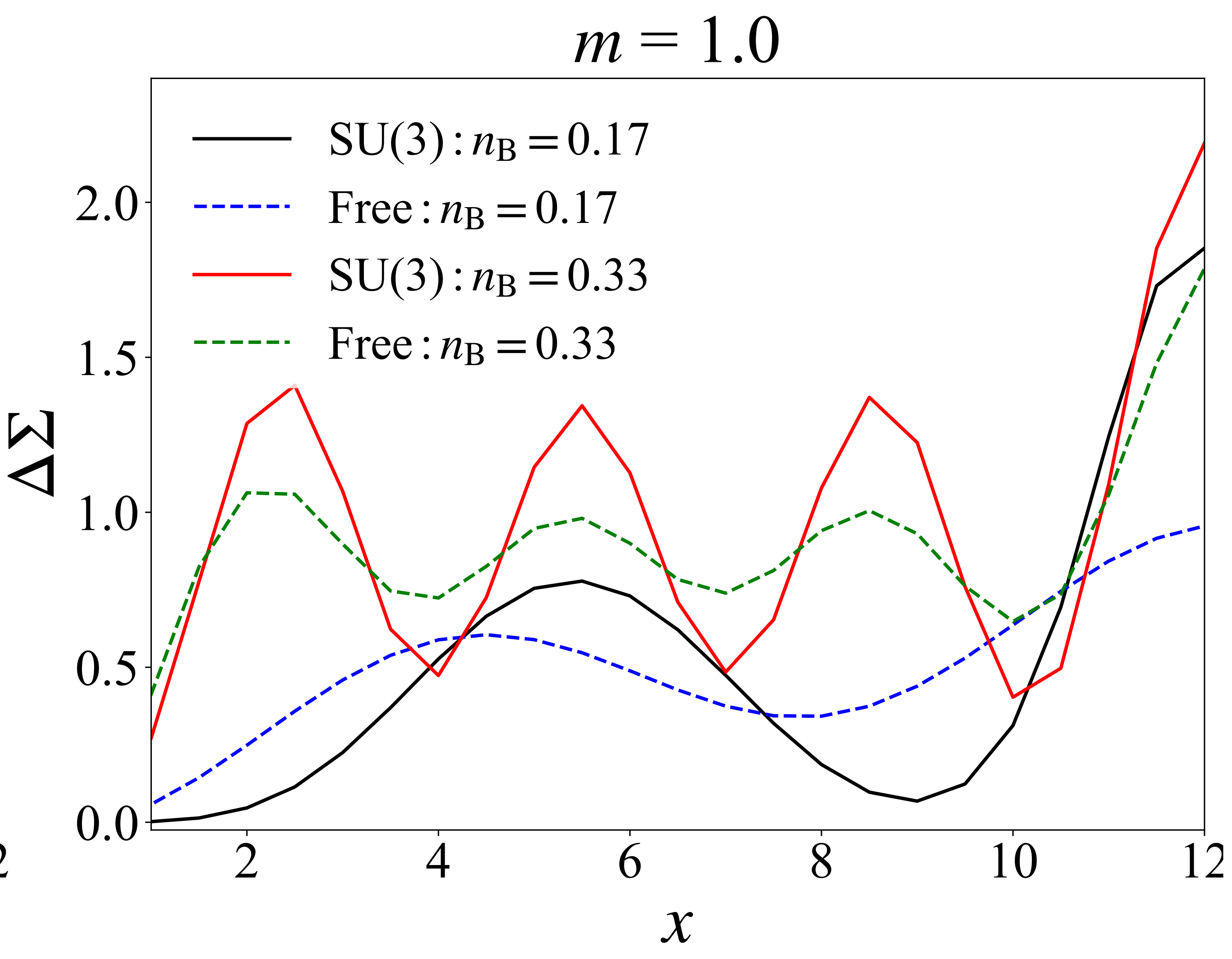




SU(3)

$\dim\mathscr{H} = 2^{320}$

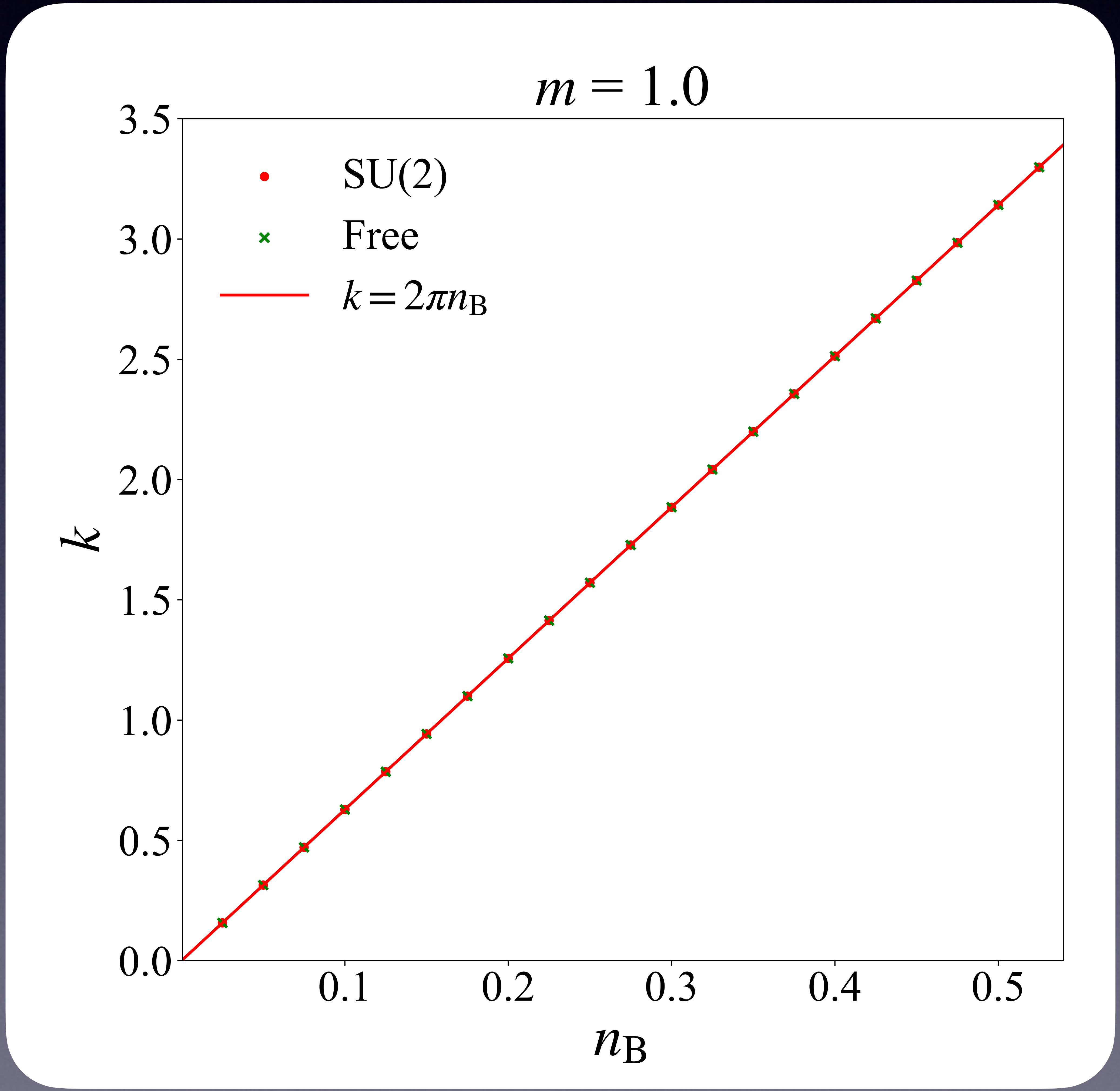




Wave number dependence

$$J = 1/8 \quad w = 2 \quad V = 40 \quad \dim\mathscr{H} = 2^{320}$$

Wave number dependence



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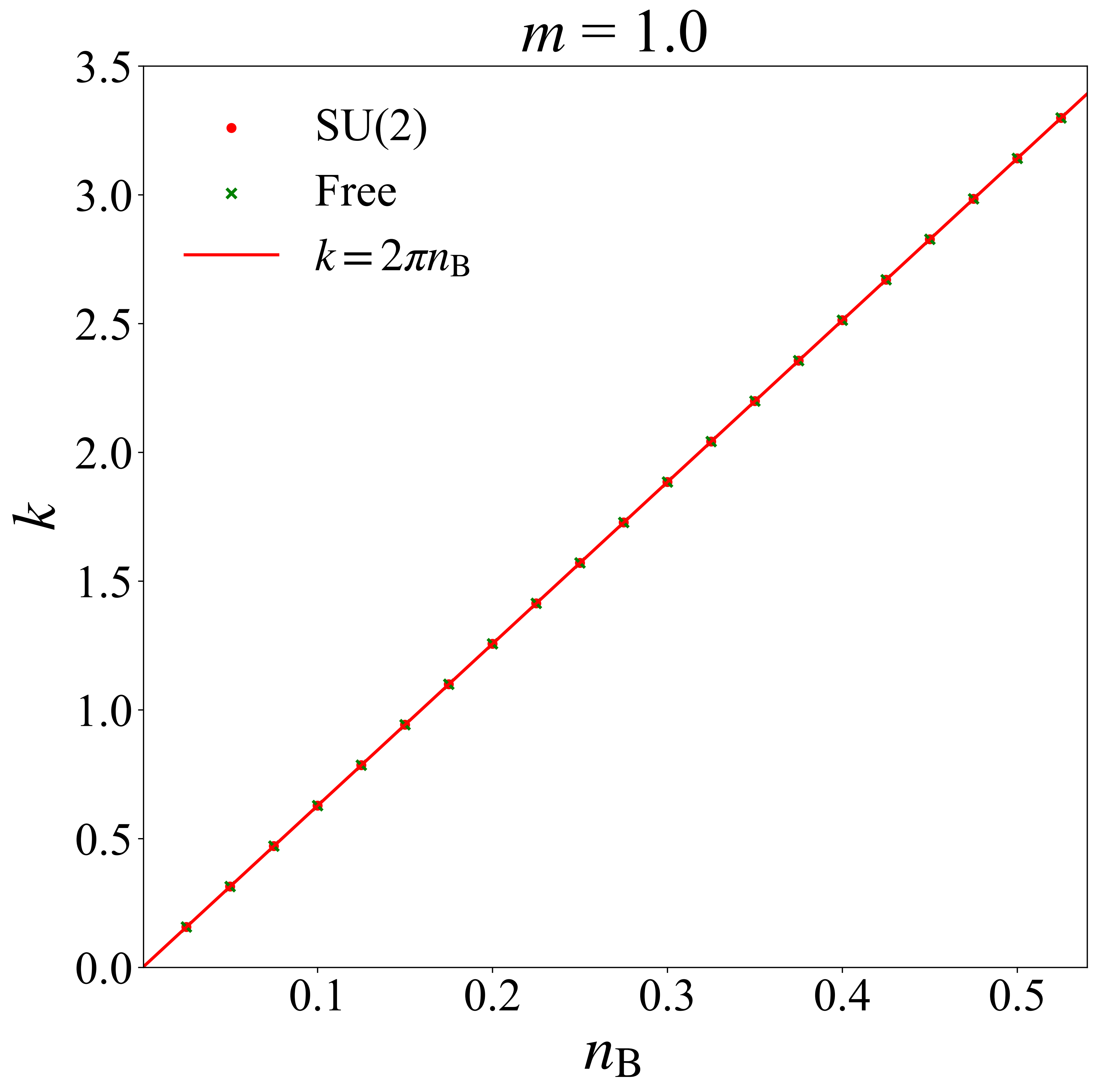
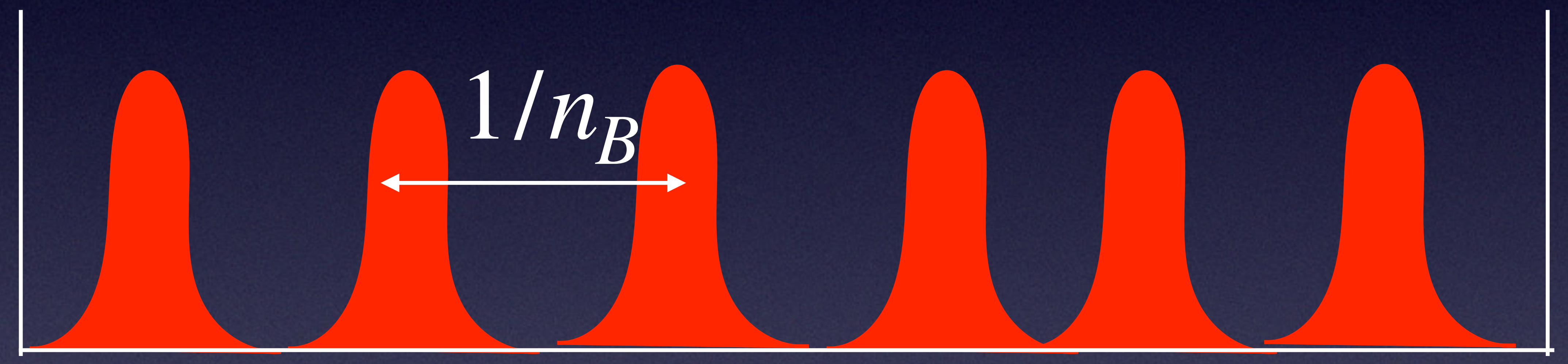


Hadronic picture

If hadron interactions are repulsive



distance $1/n_B \Rightarrow k = 2\pi n_B$

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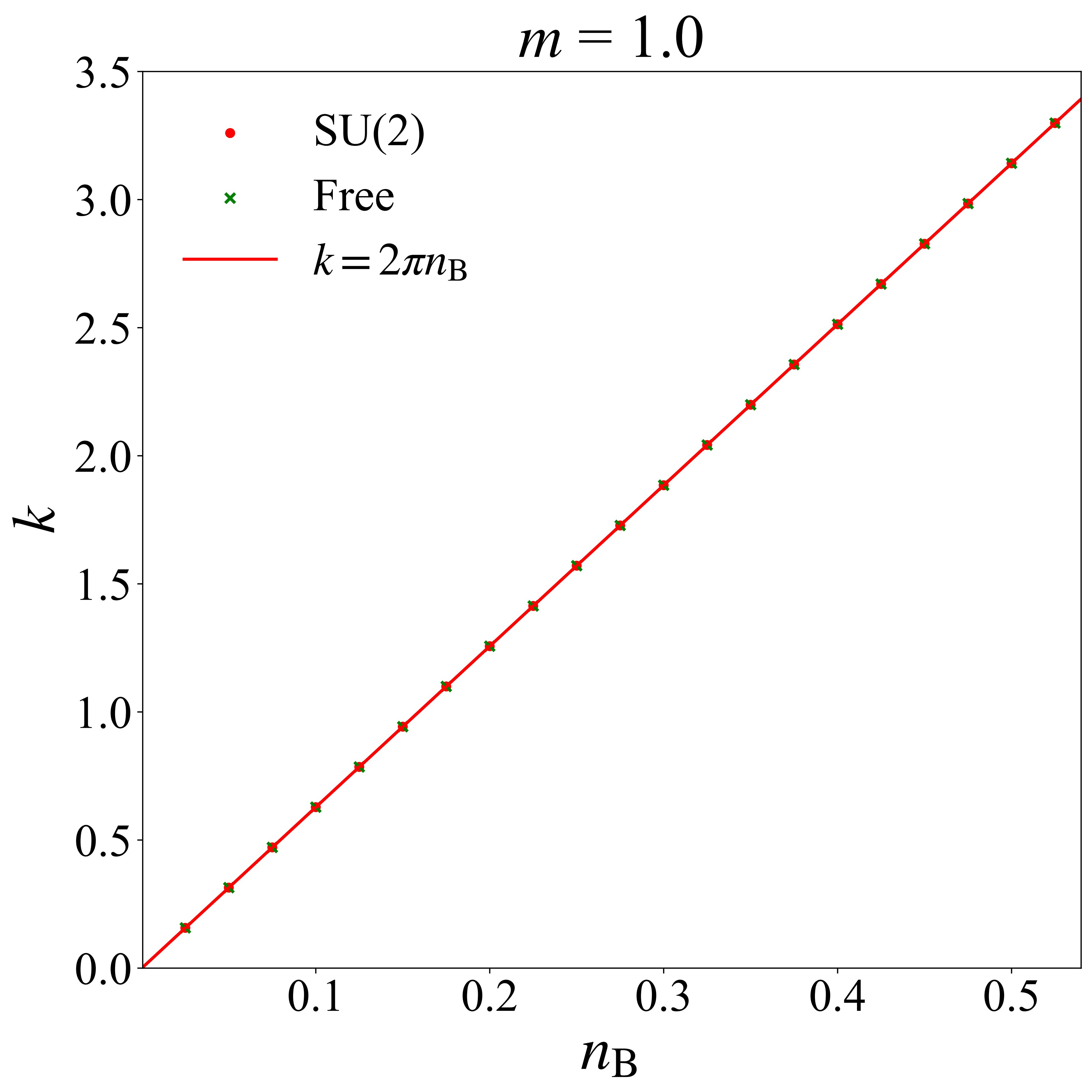

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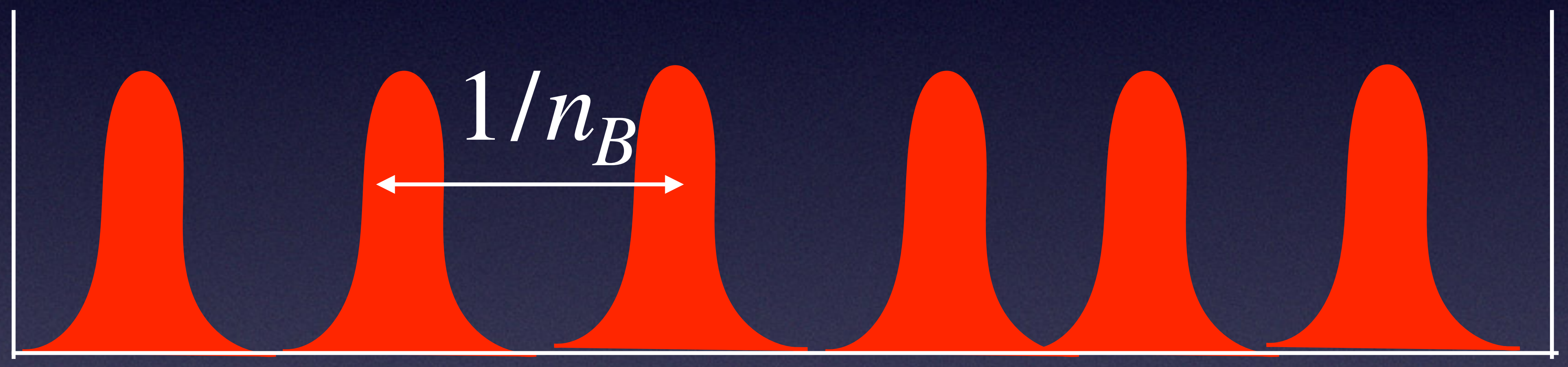


distance $1/n_B \Rightarrow k = 2\pi n_B$

Quark picture

If interactions between quarks
Fermi surface is unstable

$\Rightarrow$density wave $k = 2p_F = 2\pi n_B$

Infinite volume

Using Variational Uniform Matrix Product State,   Fujikura, YH (work in progress)

Zauner-Stauber, Vanderstraeten, Fishman, Verstraete, Haegeman ('18)

No continuous symmetry breaking occurs in (1+1)d

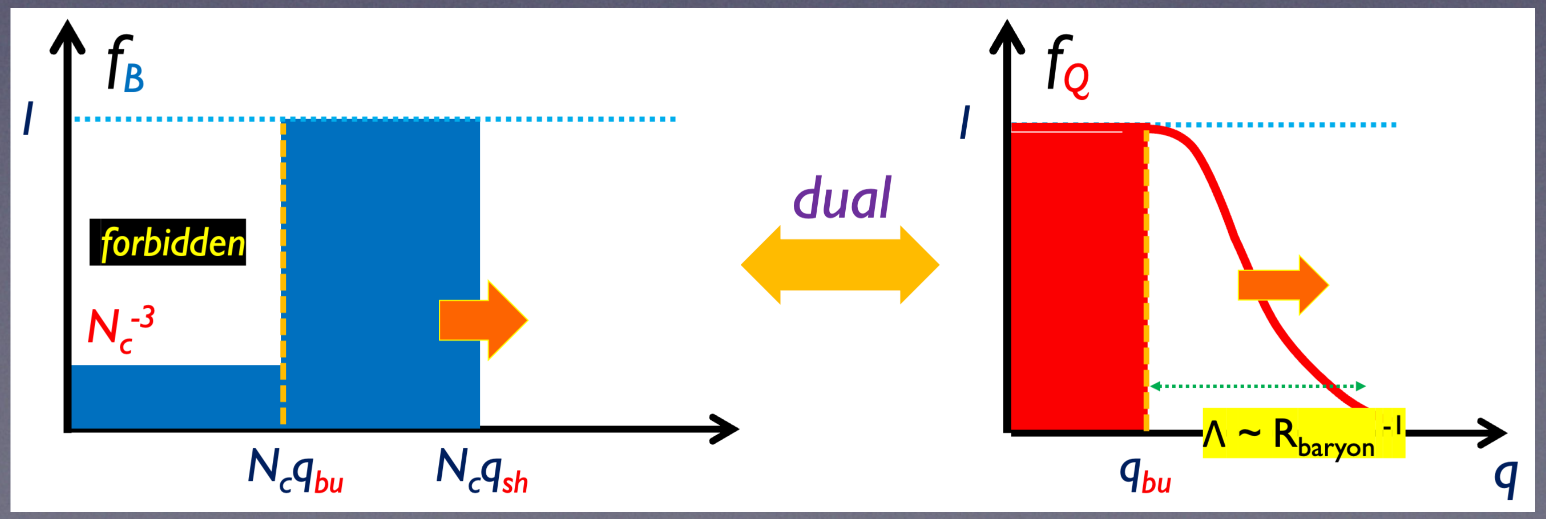
Translation symmetry is restored

⇒ Tomonaga-Luttinger liquid

Hadronic to quark Luttinger liquid cross over

Correlation function of baryons has oscillation

⇒consistent with Quarkyonic picture



Thought experiment : rotating neutron stars

Vortices as phase probes

Quantum vortex

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Quantum vortex

Vortices in CFL phase has nontrivial magnetic flux

Is it topological phase ? Cherman, Sen, Yaffe ('19)

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Is there a possibility that something happens on vortices?

Our answer is YES! (at least in QCD like theory)

Phase transition on the vortex

$\mathrm{U(1)_{gauge}} \times \mathrm{U(1)_{global}}$ **lattice model**

cf. Motrunich, Senthil ('05)

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Field strength | **Scalar field** (phase dof) | **Gauge field**

$$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$$

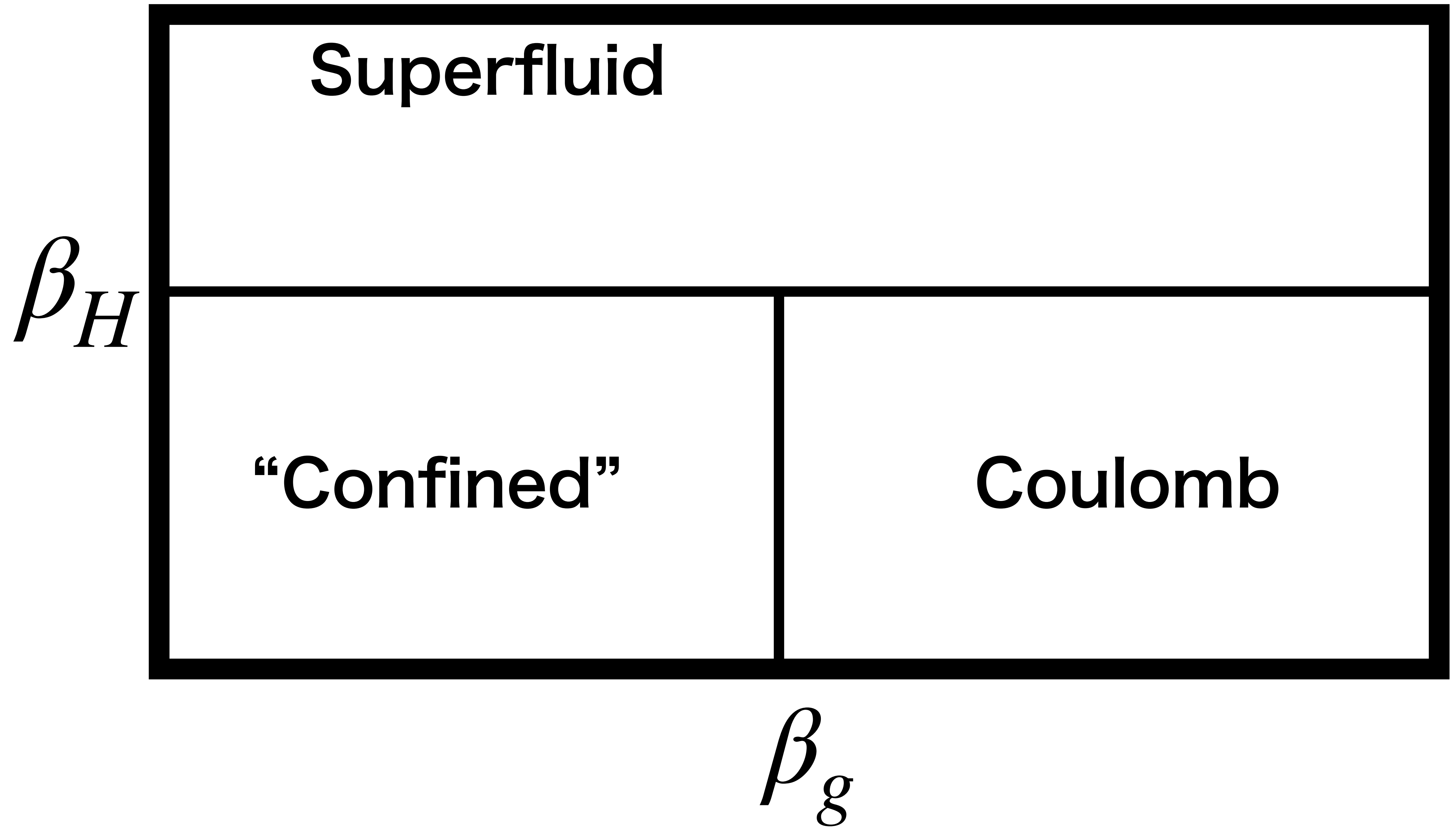
Symmetry

$$\mathrm{U(1)_{gauge}} : \begin{array}{l} \varphi_1 \to \varphi_1 - \lambda \\ \varphi_2 \to \varphi_2 - \lambda \\ A_\mu \to A_\mu + \Delta_\mu \lambda \end{array}$$

$$\mathrm{U(1)_{global}} : \begin{array}{l} \varphi_1 \to \varphi_1 + \theta \\ \varphi_2 \to \varphi_2 - \theta \end{array}$$

$$\mathbb{Z}_{2F} : \begin{array}{l} \varphi_1 \to \varphi_2 \\ \varphi_2 \to \varphi_1 \end{array}$$

Phase diagram



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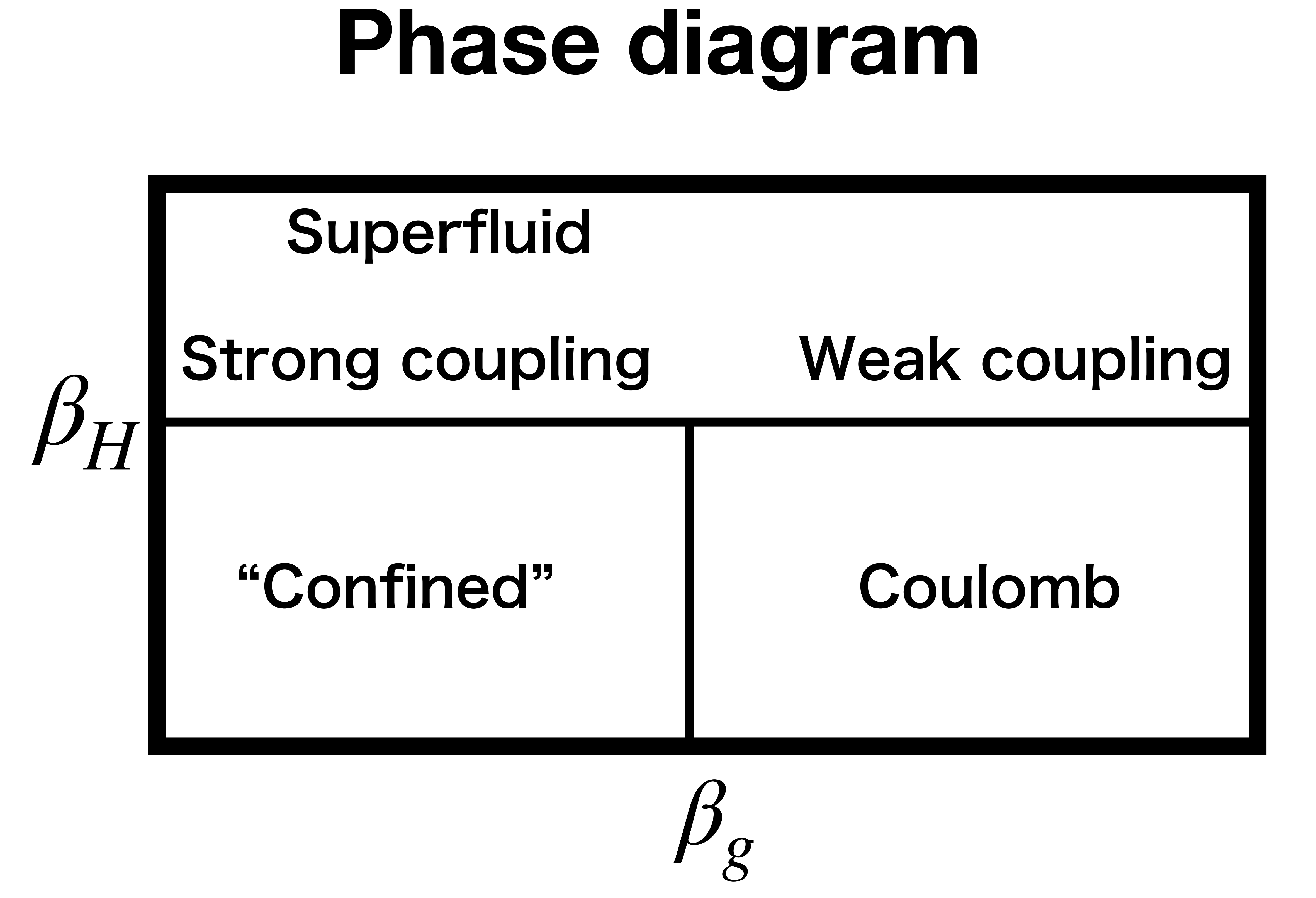

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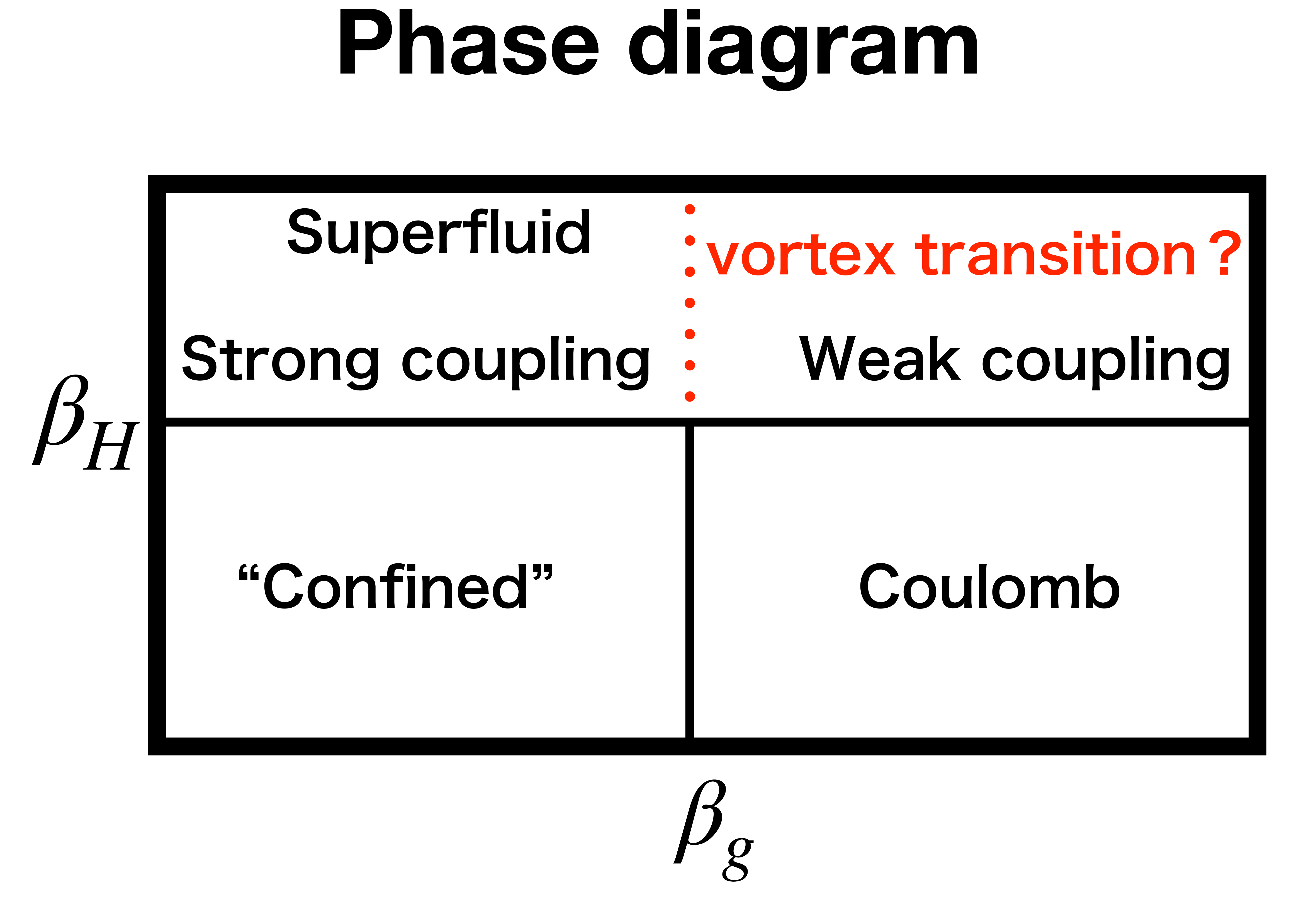
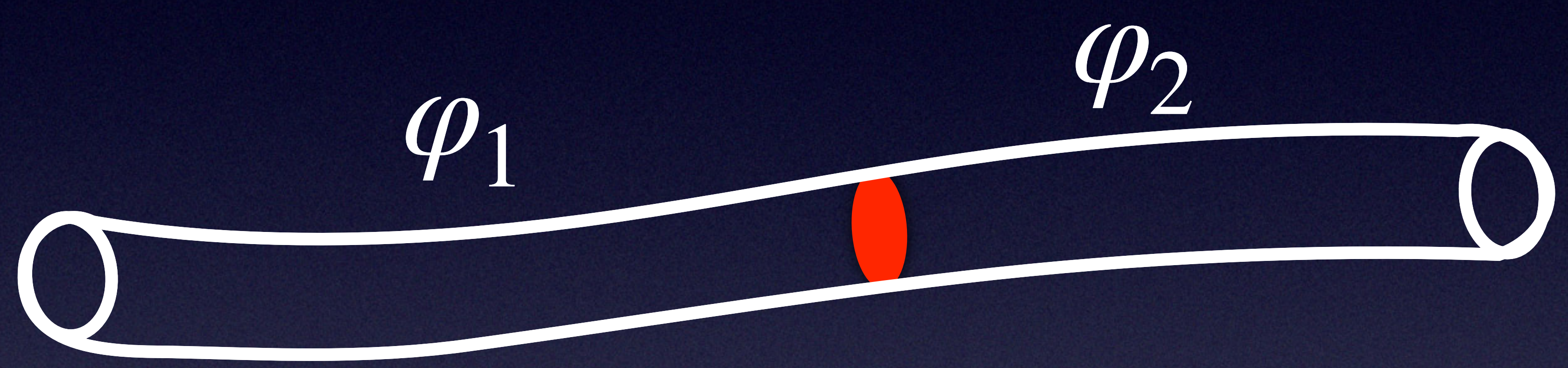


There are two ‘phases’ on a vortex

$\mathbb{Z}_{2F}$ broken phase

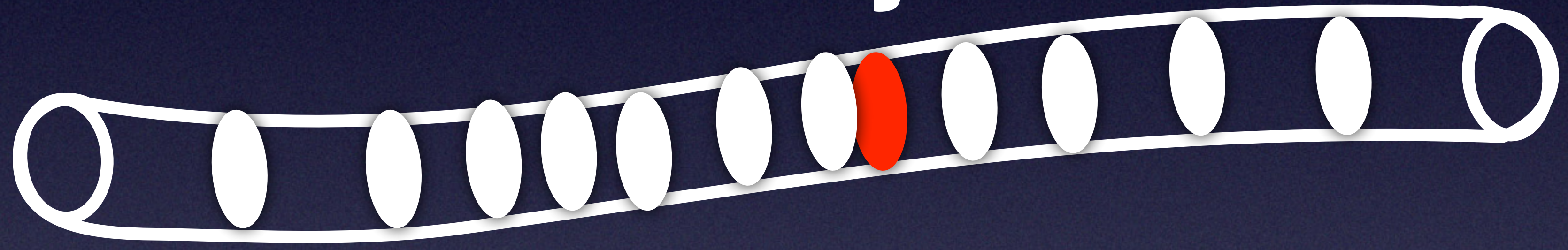
Weak coupling



$\mathbb{Z}_{2F}$ unbroken

Strong coupling

randomized junctions



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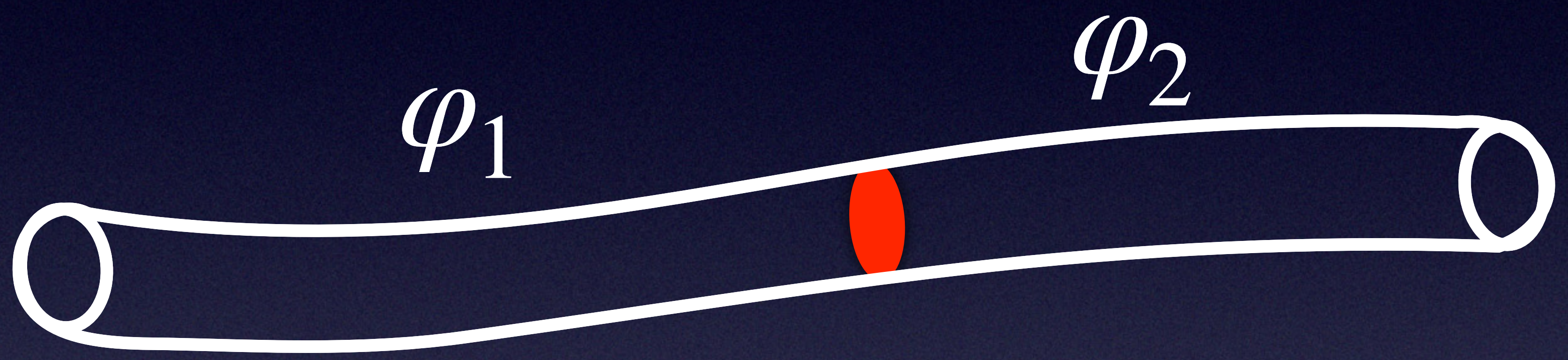

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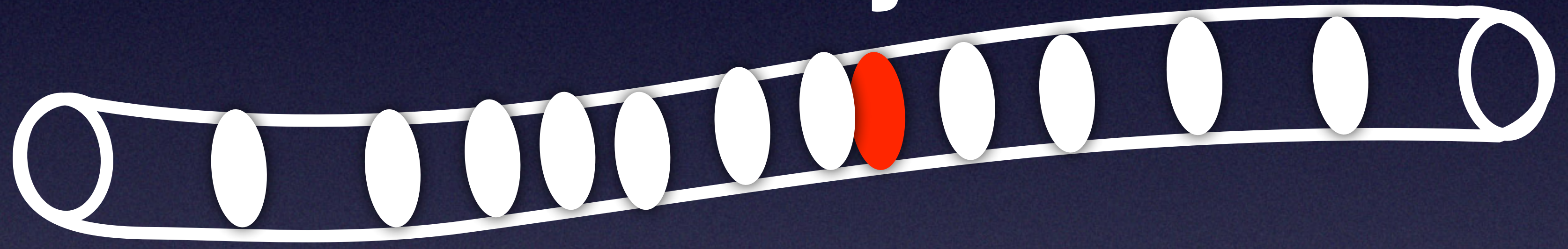


This is analogous to the Ising model

$\mathbb{Z}_2$ broken phase

↑↑↑↑↑↑|↓↓↓↓↓↓

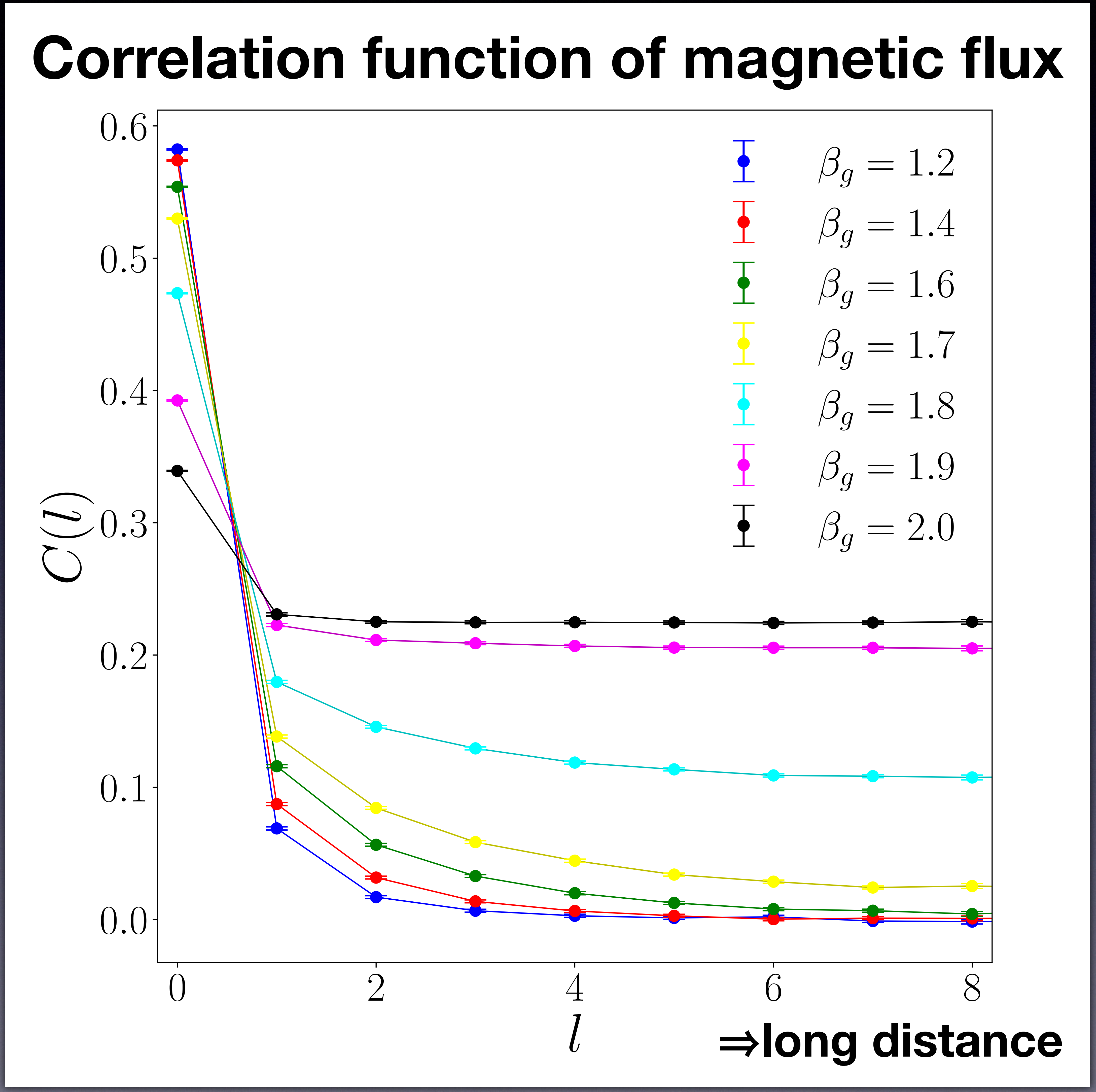
domain wall

$\mathbb{Z}_2$ unbroken phase

random configuration

↑↓↑↑↓↑|↑↓↑↓↓↓

Numerical simulation



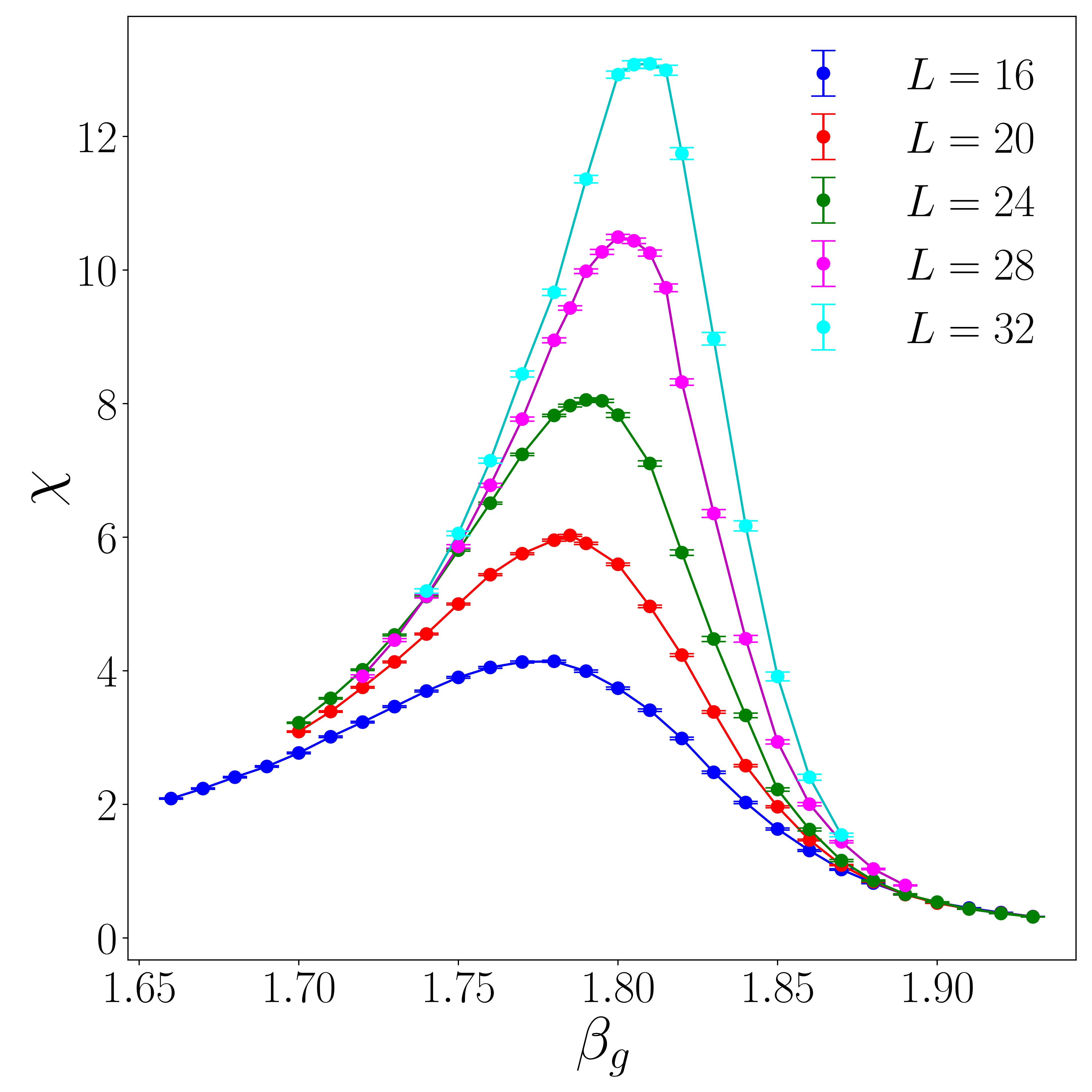
vortex

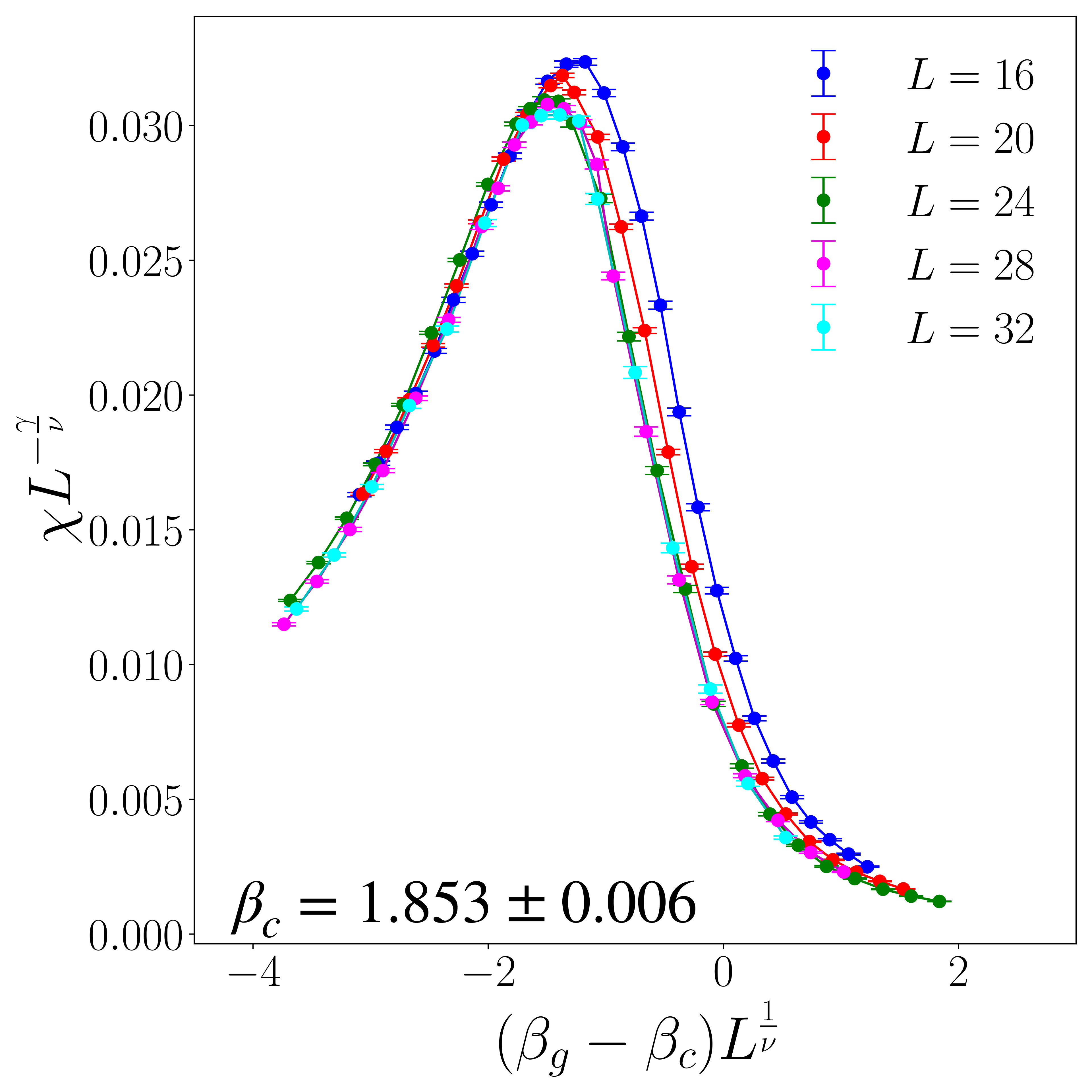
At weak coupling long-range correlation

Spontaneous symmetry breaking

⇒ **Phase transition on a vortex**

Critical point





Ising universality class $\nu = 1, \gamma = 7/4$

predicted in Motrunich, Senthil ('05)

Implications from Vortex phase transition

Single vortex transition does not mean bulk phase transition

$$f_{\text{total}} = \frac{F_{\text{total}}}{V} \sim f_{\text{bulk}} + \frac{L}{V} f_{\text{vortex}}$$

No sigular behavior at $V \to \infty$

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No sigular behavior at $V \to \infty$

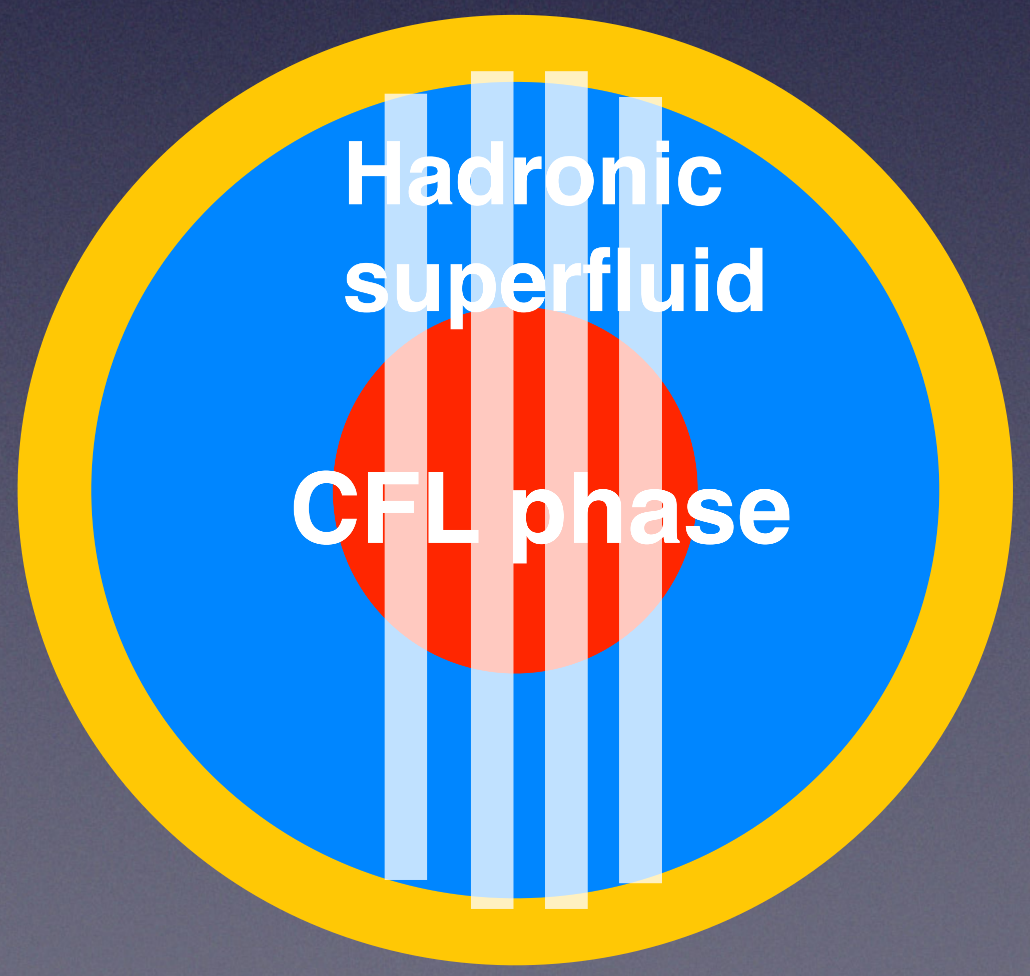
In the case of high rotation where

$$n_{\text{vrotex}} = N_{\text{vortex}} / V_{\text{perp}} \neq 0$$

$$f_{\text{total}} = \frac{F_{\text{total}}}{V} \sim f_{\text{bulk}} + n_{\text{vortex}} f_{\text{vortex}}$$

can be singular

at the vortex phase transition point



vortex

Summary

Finite-temperature QCD:

We have proposed Quark Spaghetti with Glue Balls

Finite-density QCD:

We discussed crossover between Hadronic and quark matter

We examined QCD in (1+1)d using tensor network technique.

⇒Hadronic to quark Luttinger liquid cross over

We found the phase transition on a vortex in $U(1) \times U(1)$ model

⇒ some phase transition might occur in highly rotating QCD matter

How about real QCD?