

Description of high-density QCD matter in heavy ion collisions by transport models: A new relativistic quantum molecular dynamics (RQMD2)

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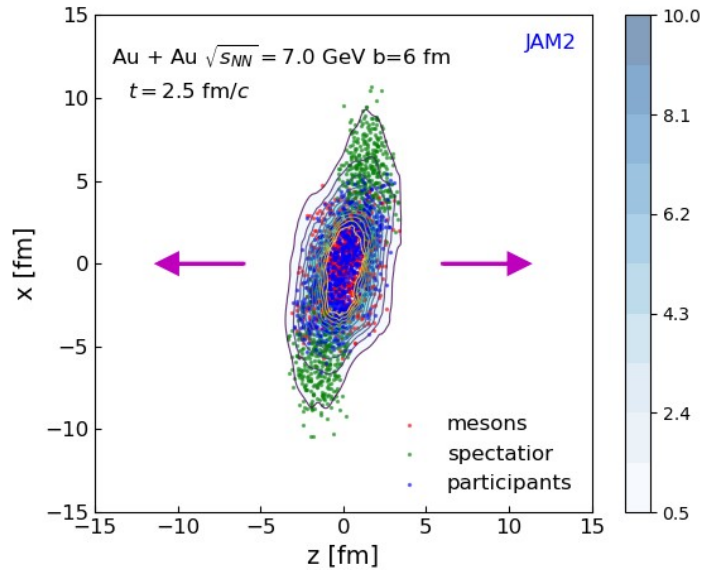
Collaboration with A. Jinnō (Kyoto Univ.), K. Murase (RCNP)

[1] Y. Nara, A. Jinnō, T. Maruyama, K. Murase, and A. Ohnishi, Phys. Rev. C 108, 2 (2023).

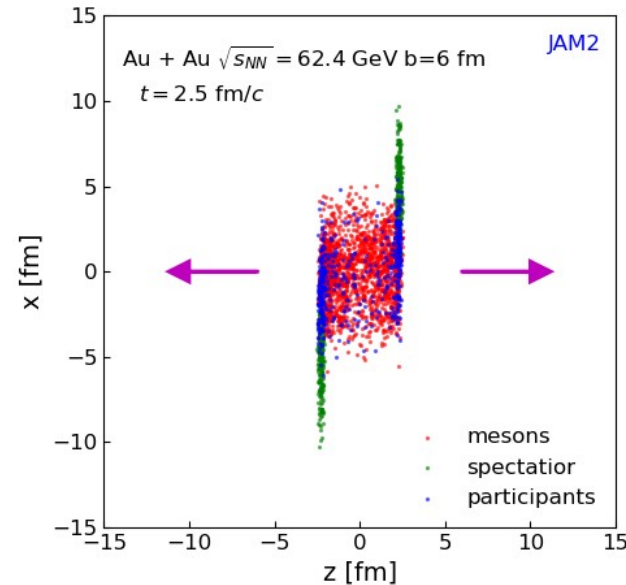
[2] Y. Nara, A. Jinnō, and K. Murase, [arXiv:2507.23294 [hep-ph]]

Relativistic heavy-ion collisions

$$t_{\text{pass}} = 2R/\gamma \approx 1 \text{ fm}/c \text{ at } \sqrt{s_{NN}} = 30 \text{ GeV}$$



Nuclei are compressed and create baryon rich matter.

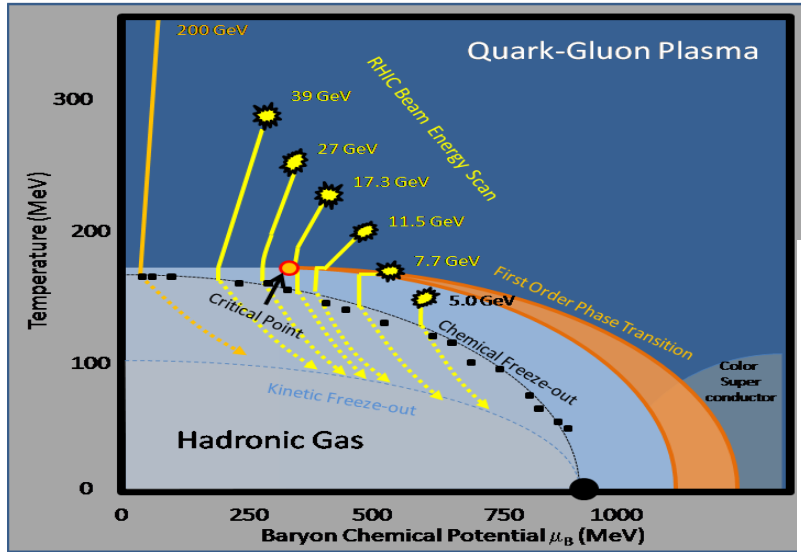


More than 30GeV:
Colliding nuclei become transparent, and baryon free plasma is created at mid-rapidity.

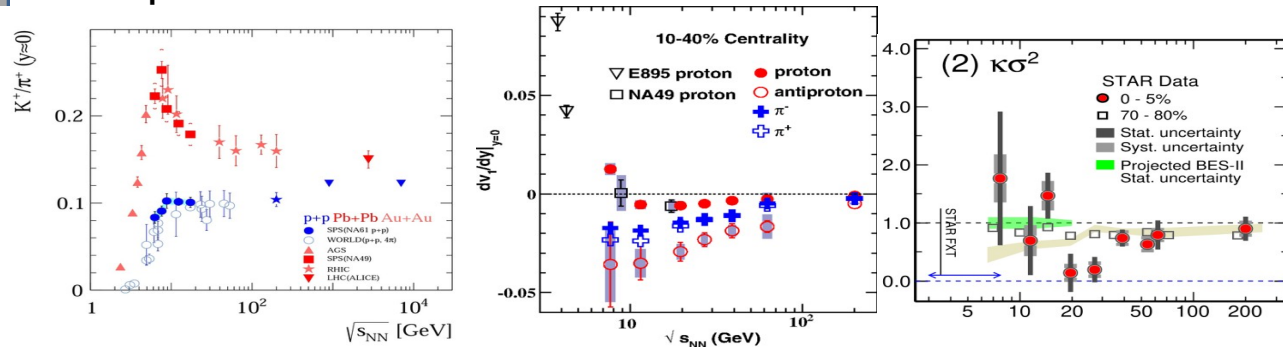
Factorized model possible:

Initial condition
Hydro
Hadoron cascade

Search for the QCD equation of state (EoS) by the beam energy scan



On going RHIC BES II, FXT and NA61/SHINE explore the phase structure of QCD matter. New experiments FAIR, NICA, J-PARC-HI, HIAF are planned.



Discovery of **non-monotonic behavior** of beam energy dependence in $K^+/\pi^+, v_1, \text{kurtosis}, N_t N_p / N_d^2$.
Are they signals of a phase transition?

To extract the information on the EoS, transport model is needed.

Microscopic transport models

Cascade model: simulate Boltzmann type collision term:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_r \cdot f = I_{\text{coll}}(\text{elastic, inelastic collisions}) \quad \text{EoS: ideal gas}$$

Boltzmann-Uehling-Uhlenbeck (BUU): follow the time evolution of one-particle distribution function $f(\mathbf{x}, \mathbf{p})$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_r \cdot f - \nabla_r U \cdot \nabla_p f = I_{\text{coll}} \quad U(n(\mathbf{x}_i)): \text{mean-field simulating EoS}$$

Quantum molecular dynamics (QMD): N-body theory of interacting Gaussian system

$$\frac{d\mathbf{x}_i}{dt} = \frac{\partial H}{\partial \mathbf{p}_i}, \quad \frac{d\mathbf{p}_i}{dt} = -\frac{\partial H}{\partial \mathbf{x}_i} + I_{\text{coll}}$$

Motivation and purpose

- In QMD, potentials are the function of ‘interaction density’ which is different from real density → density dependence of the potentials is underestimated.
- Relativistic version of QMD uses the mass shell condition using one-particle potential, not single particle potential.

single-particle potential: $U(n) = \alpha n + \beta n^\gamma$

$$m^* = m + U$$

one-particle potential: $V(n) = \frac{1}{n} \int U(n) dn$

$$m^* = m + V$$

- Develop a new method to correctly estimate a density dependence of potential.
- Use single-particle potential to define mass shell condition

A system interacting with a single-particle potential U

$$H = \int d^3x d^3p \frac{\mathbf{p}^2}{2m} f(x, p) + \int d^3x \int U(n) dn$$

The distribution function is represented by particles with Gaussian profile G,

$$f(x, p) = \frac{1}{N_{\text{test}}} \sum_{i=1}^{N N_{\text{test}}} G(\mathbf{x} - \mathbf{x}_i(t)) \delta(\mathbf{p} - \mathbf{p}_i(t)) \quad N_{\text{test}} = 1 \text{ for QMD}$$

$$\frac{d\mathbf{x}_i}{dt} = \frac{\mathbf{p}_i}{m}, \quad \frac{d\mathbf{p}_i}{dt} = - \int d^3x U(n) \frac{\partial G(\mathbf{x} - \mathbf{x}_i)}{\partial \mathbf{x}_i}$$

Numerical integration:
lattice Hamiltonian method in BUU
MC-integration in AMD,
Gauss integration in ImQMD

Approximations are used to save CPU time.

EOM for new QMD (QMD2)

$$\frac{d\mathbf{p}_i}{dt} = - \int d^3x U(n) \frac{\partial G(x - x_i)}{\partial \mathbf{x}_i} \quad \leftarrow \text{exact EoM}$$

Rewrite EOM with the one-particle potential: $U(n) = V(n) + n \frac{dV(n)}{dn}$

$$\frac{d\mathbf{p}_i}{dt} = - \sum_{j=1}^N \int d^3x \left[\frac{dV(n)}{dn} \frac{\partial G(x - x_j)}{\partial \mathbf{x}} G(x - x_i) + \frac{dV(n)}{dn} \frac{\partial G(x - x_i)}{\partial \mathbf{x}_i} G(x - x_j) \right]$$

We approximate the integral as

$$\frac{d\mathbf{p}_i}{dt} = - \sum_{j=1}^N \left[\frac{dV(n(x_i))}{dn} + \frac{dV(n(x_j))}{dn} \right] \frac{\partial n_{ij}}{\partial \mathbf{x}_i} \quad n_{ij} = \int d^3x G(x - x_i) G(x - x_j)$$

QMD v.s. new QMD (QMD2)

Exact: $\frac{d\mathbf{p}_i}{dt} = - \int d^3x U(n) \frac{\partial G(x - x_i)}{\partial \mathbf{x}_i}$

Rewrite EOM with the one-particle potential:

$$U(n) = V(n) + n \frac{dV(n)}{dn}$$

QMD: $\frac{d\mathbf{p}_i}{dt} = - \sum_{j=1}^N \left[\frac{dV(\langle n_i \rangle)}{dn} + \frac{dV(\langle n_j \rangle)}{dn} \right] \frac{\partial n_{ij}}{\partial \mathbf{x}_i}$

Exact momentum conservation

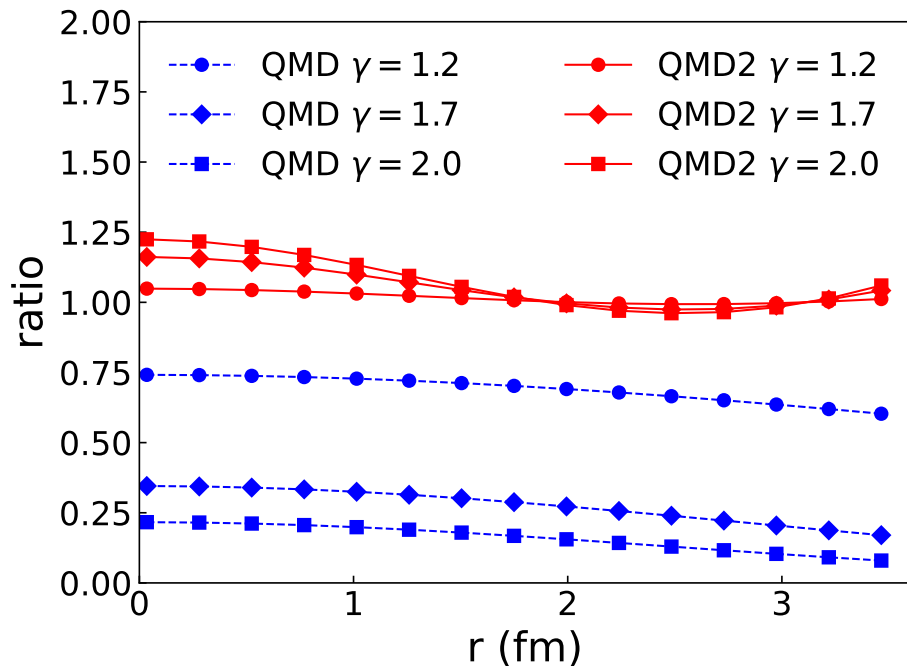
QMD2: $\frac{d\mathbf{p}_i}{dt} = - \sum_{j=1}^N \left[\frac{dV(n(\mathbf{x}_i))}{dn} + \frac{dV(n(\mathbf{x}_j))}{dn} \right] \frac{\partial n_{ij}}{\partial \mathbf{x}_i}$

QMD: $\langle n_i \rangle = \sum_{j \neq i}^N \int d^3x G_i G_j \equiv \sum_{j \neq i}^N n_{ij}$

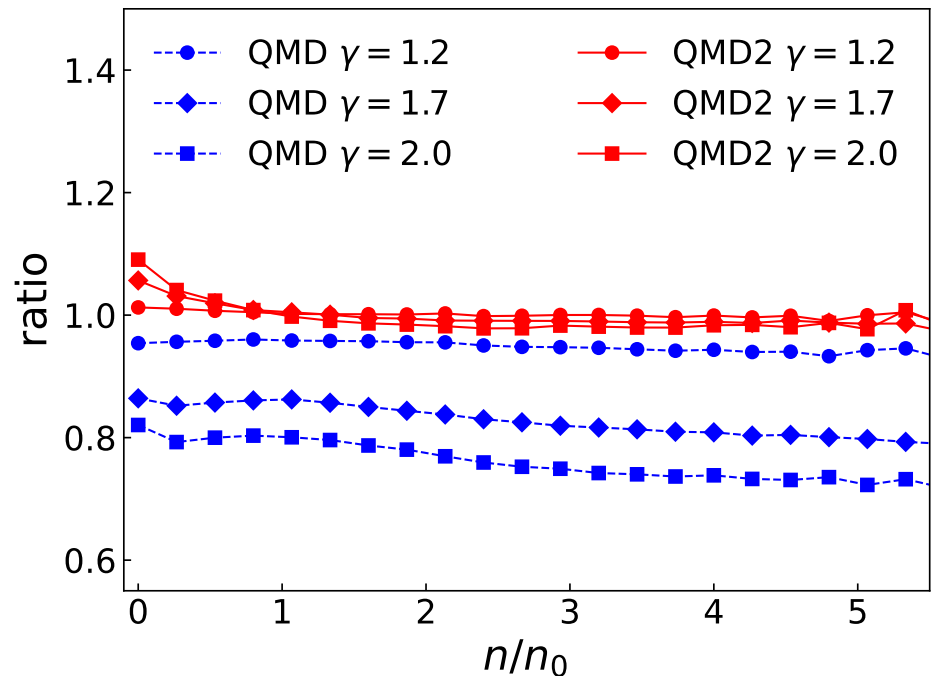
QMD2: real density $n = \sum_{i=1}^N \int d^3x G_i$

New QMD results for $U=n^\gamma$

Ratio = QMD/(numerical integration)



Two particles



400 particles

Covariant cascade method Phys. Rev. C 108, 2 (2023)

We consider 8N-dimensional phase space: $\{x_i^\mu(s), p_i^\mu(s)\}, i = 1, \dots, N$

Hamiltonian is given by the sum of mass-shell constraints

$$H = \sum_{i=1}^N \lambda_i \frac{p_i^2 - m_i^2}{2}, \quad \lambda_i : \text{Lagrange multiplier}$$

Equation of motion:

$$\frac{dx_i}{ds} = \lambda_i p_i$$

which is equivalent to $\frac{dx_i}{dx_i^0} = \frac{p_i}{p_i^0}$, or $\frac{dx_i}{d\tau_i} = \frac{p_i}{m_i}$

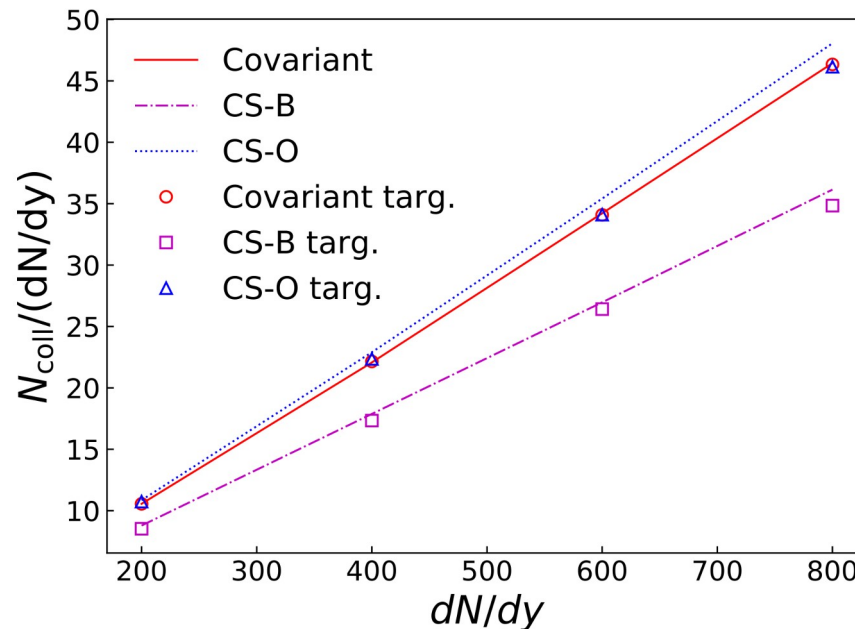
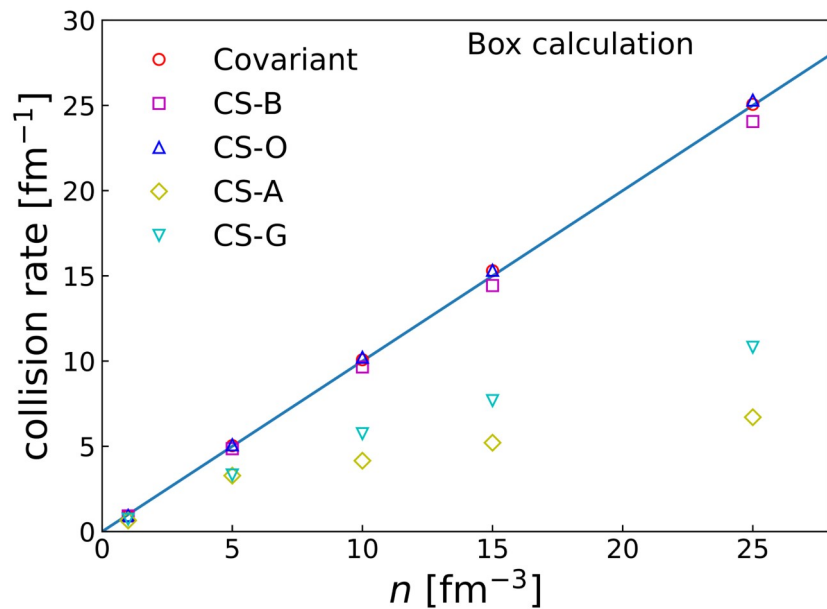
If we assume that time of all particle is the same at a Lorentz frame \hat{a}

$$\hat{a} \cdot x_i = s, \quad i = 1, \dots, N \quad \rightarrow \quad \hat{a} \cdot \frac{dx_i}{ds} = 1 \quad \rightarrow \quad \lambda_i = \frac{1}{\hat{a} \cdot p_i}$$

$$\text{EoM: } \frac{dx_i}{ds} = \frac{p_i}{\hat{a} \cdot p_i}$$

Based on this EoM, covariant cascade method can be obtained (MC simulation of Boltzmann type collision term.)

Static box and one-dimensionally expanding system

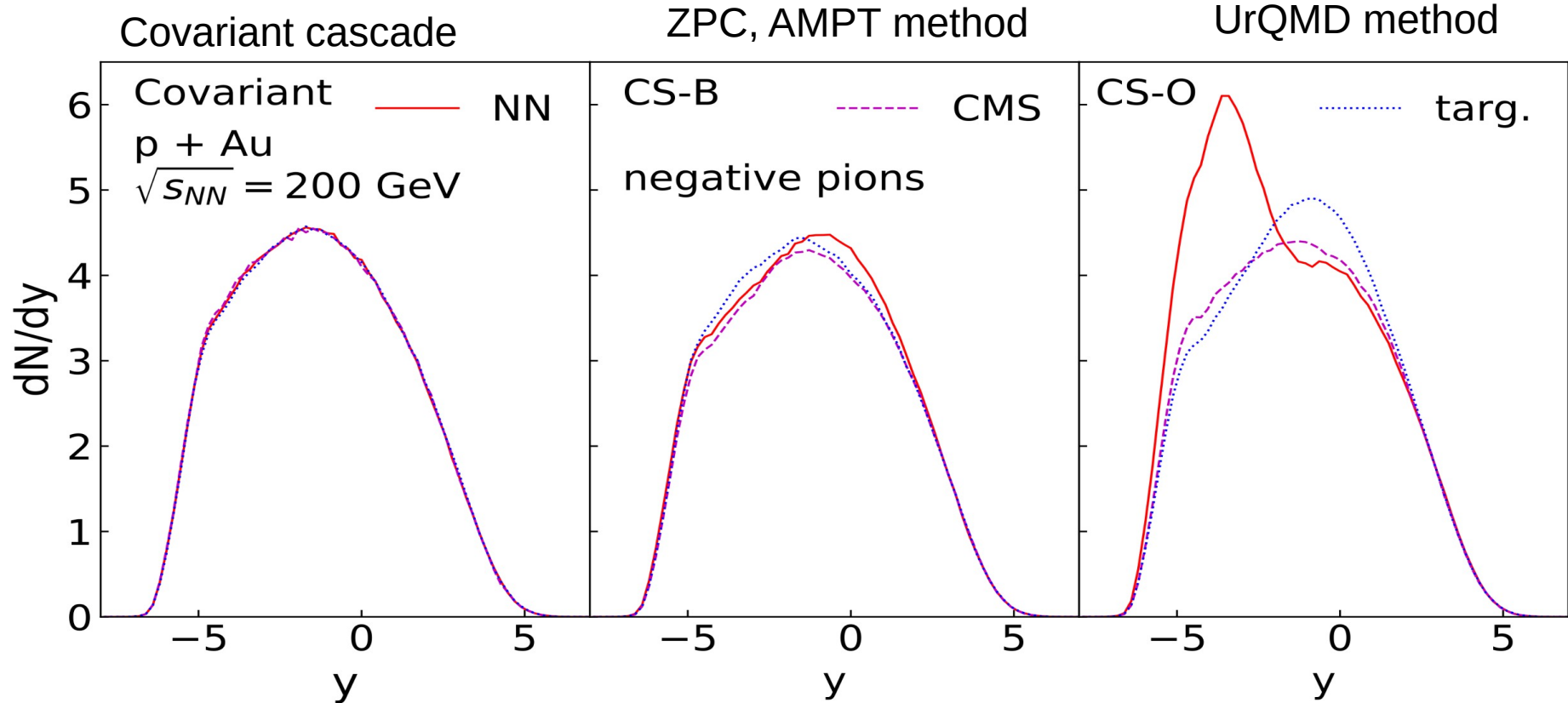


- CS-B: ZPC new X.-L. Zhao, et.al, PRC102, 024904 (2020)
- CS-G: ZPC old B. Zhang
- CS-O: UrQMD, SMASH

CS-B predicts lower collision rate for expanding system.

pA collisions at 200 GeV

Validation of frame independence



All methods are agree with each other at CM frame.

Action for RQMD2

$$S = S_{\text{part}} + S_{\text{field}}$$

$$S_{\text{part}} = \sum_{i=1}^N \int p_i \cdot \frac{dx_i(s)}{ds} ds - \int d^4x d^4p W(x, p) f(x, p) \quad S_{\text{field}} = \int d^4x \mathcal{L}_{\text{field}}$$

$$\text{Generalized potential: } W(x, p) = \frac{p^{*2}(x, p) - m^{*2}(x, p)}{2}$$

$$\text{Potential contribution: } p^{*\mu} = p^\mu - U^\mu, \quad m^* = m + U_s$$

$$f(x, p) = \sum_{i=1}^N \int ds \lambda_i(s) \delta(\hat{a} \cdot (x - x_i(s))) G(x - x_i(s)) \delta(p - p_i(s))$$

$\hat{a} = (1, 0, 0, 0)$ at the CM frame: synchronization of particle clocks in CM.

EoM for RQMD2

$$\frac{dx_i^\mu}{ds} = \lambda_i \bar{\Pi}_i^\mu, \quad \frac{dp_i^\mu}{ds} = -\lambda_i \bar{Q}_i^\mu$$

$$\lambda_i = \frac{1}{\hat{a} \cdot \bar{\Pi}_i}$$

$$\bar{\Pi}_i^\mu = \int d^4x \Pi^\mu(x, p_i) G(x - x_i(s))$$

$$\bar{Q}_i^\mu = \int d^4x Q^\mu(x, p_i) G(x - x_i(s))$$

$$\Pi_\mu(x, p) = \partial_\mu^p W(x, p) = p^* \cdot \partial_\mu^p p^* - m^* \partial_\mu^p m^*,$$

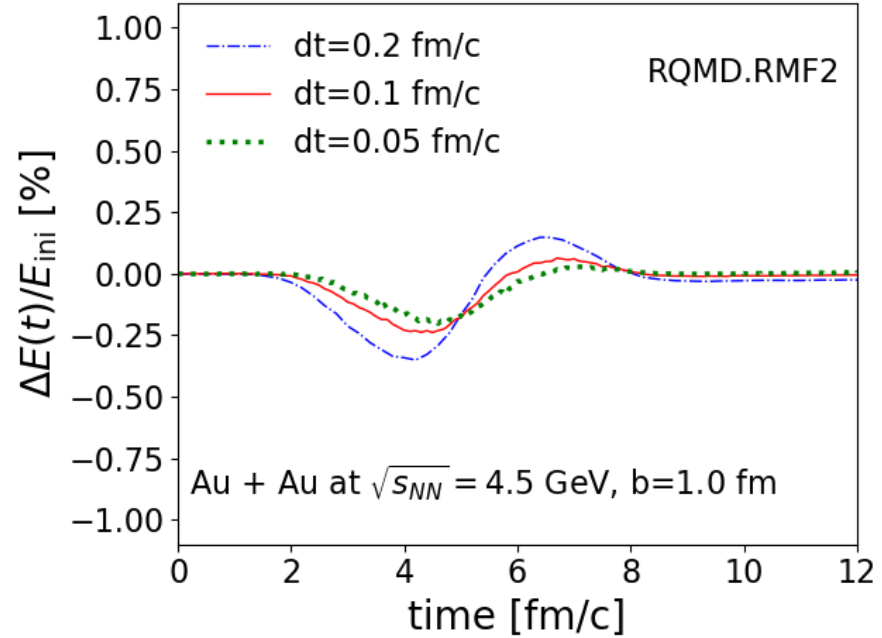
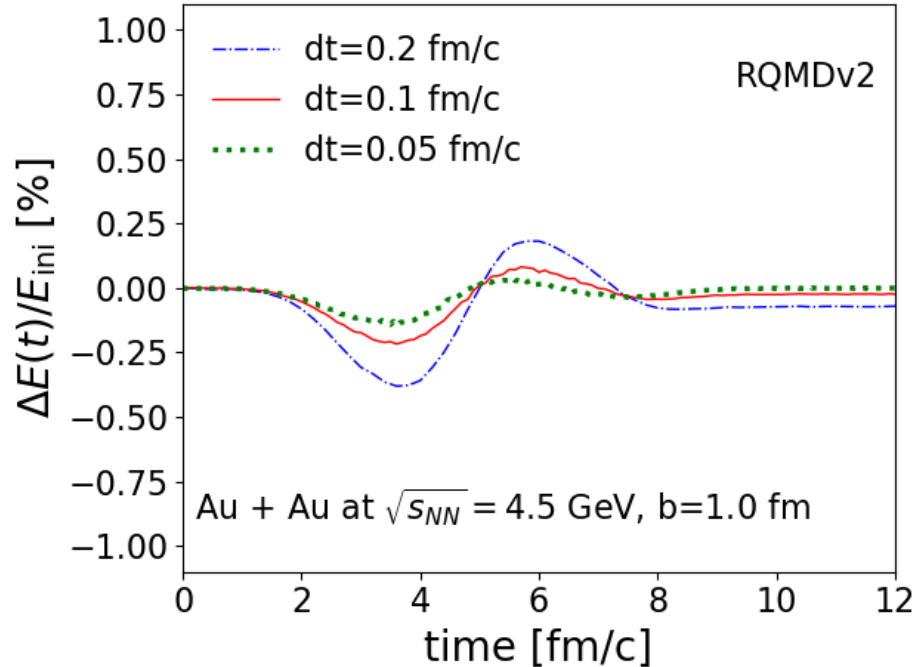
$$Q_\mu(x, p) = \partial_\mu^x W(x, p) = p^* \cdot \partial_\mu^x p^* - m^* \partial_\mu^x m^*,$$

$$G(x - x_i) = \delta((x - x_i) \cdot \hat{a}) \frac{\hat{a} \cdot u_i}{(2\pi L)^{3/2}} e^{\frac{(x-x_i)^2 - [(x-x_i) \cdot u_i]^2}{2L}}. \quad \text{Lorentz contracted Gaussian}$$

We apply the same idea as QMD to approximate the spatial integral to get the efficient RQMD EoM.

Validation of the code

Energy conservation

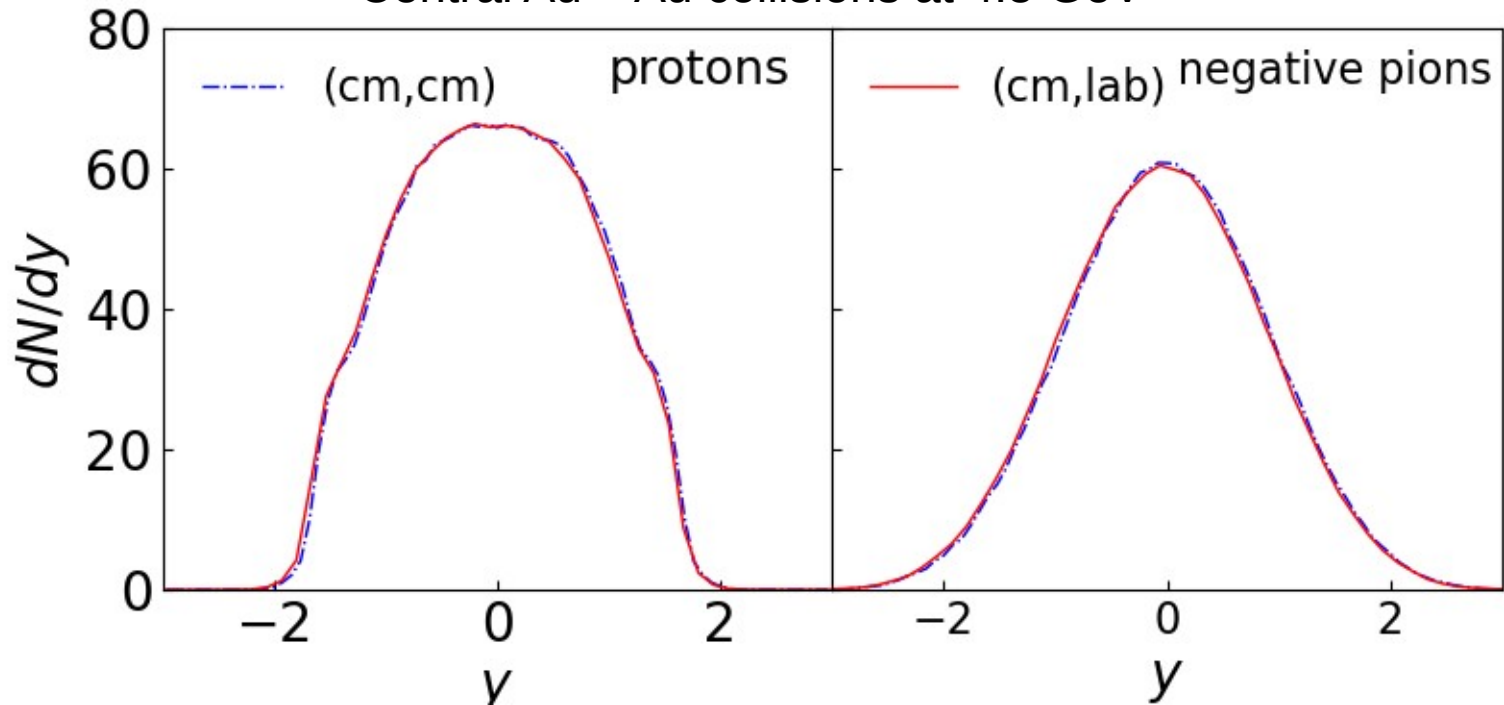


RQMDv2: Lorentz vector Skyrme force

RQMD.RMF: relativistic mean-field

Lorentz covariance

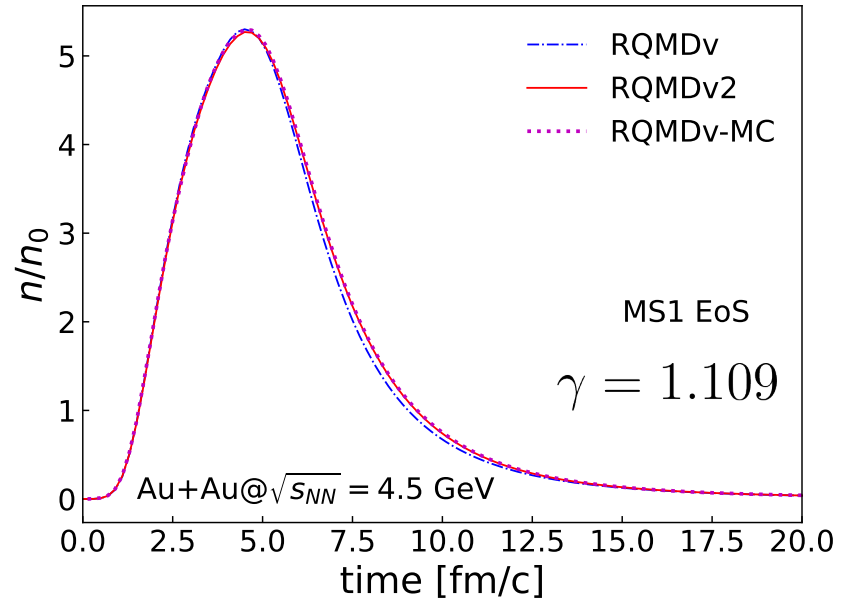
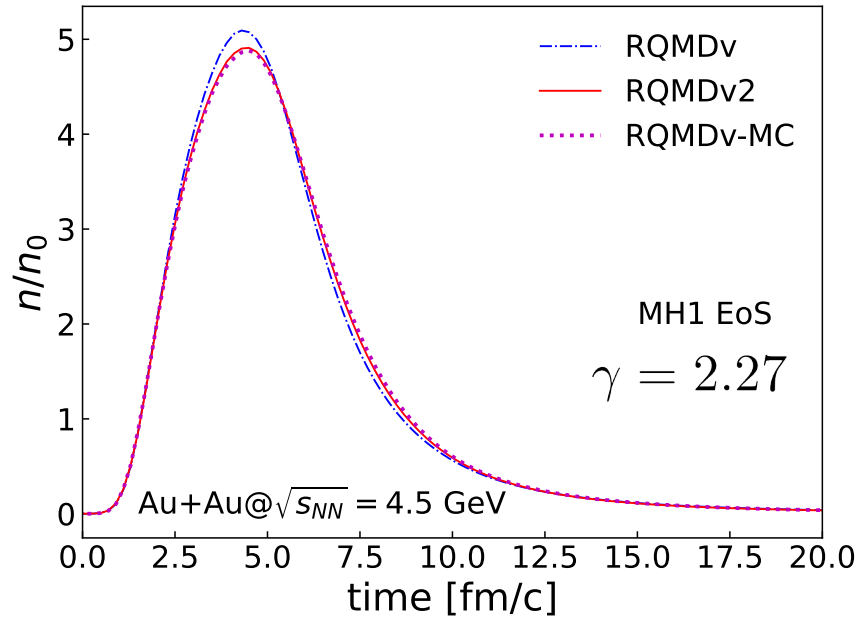
Central Au + Au collisions at 4.5 GeV



(cm, cm) = ($\hat{a} = (1, 0, 0, 0)$ at CM, simulation at CM frame)

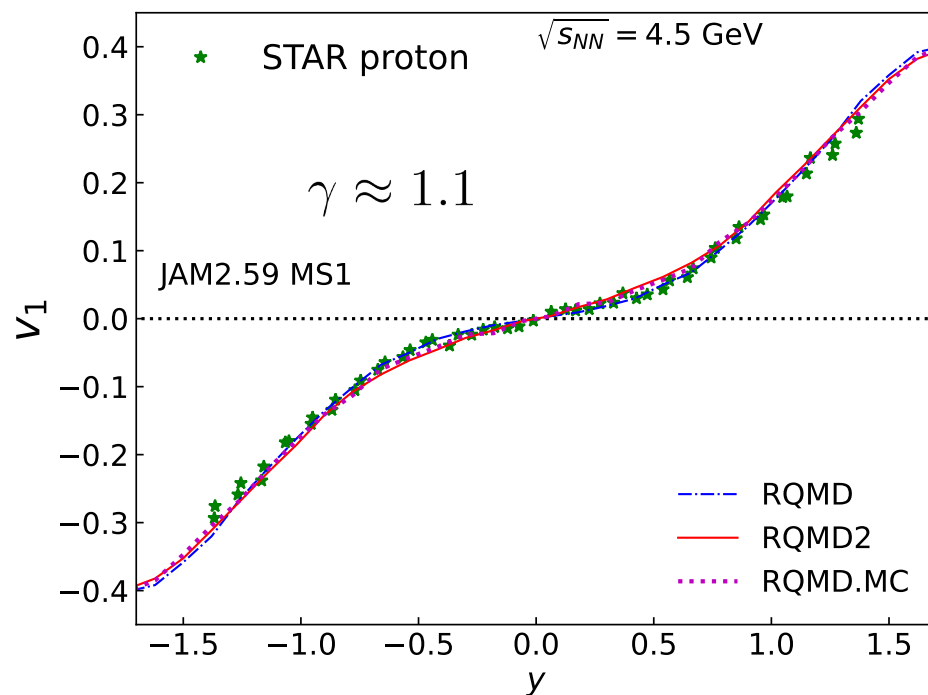
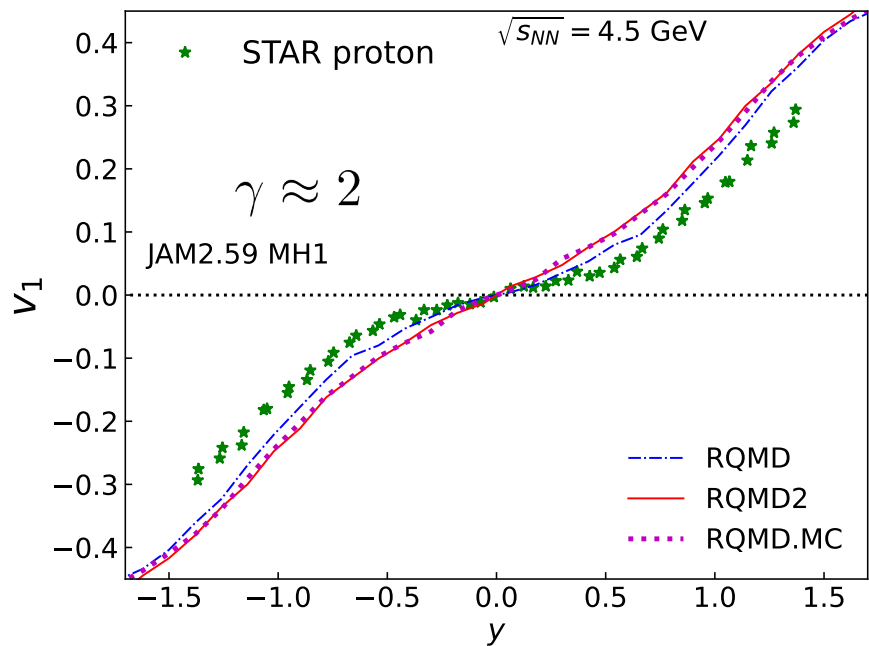
Frame-independence: dN/dy at CM frame = dN/dy at Laboratory frame

Time evolution of density at 4.5GeV



RQMDv (Lorentz vector Skyrme potential)
RQMDv2: new RQMDv
RQMDv-MC: exact integration

Directed flow in Au + Au collisions



RQMDv (Lorentz vector Skyrme potential)

RQMDv2: new RQMDv

RQMDv-MC: exact integration

$$v_1 = \langle \cos \phi \rangle = \left\langle \frac{p_x}{p_T} \right\rangle$$

Summary

- A new RQMD formulation (RQMD2) is presented based on a covariant cascade method and the variational principle.
- RQMD2 is implemented in the JAM2 code, which is publicly available <https://gitlab.com/transportmodel/jam2>.
- RQMD2 accurately simulates the equation of state (EoS).
 - ✓ Propose a good approximation to estimate non-linear density dependence.
 - ✓ Consistent on-mass shell condition in RQMD2

Back up

Equations of motion for QMD

One-particle potential: $V(n) = \frac{1}{n} \int U(n) dn$ single-particle potential: $U(n) \sim n^\gamma$

$$\text{density } n = \sum_{i=1}^N \int d^3x G(\mathbf{x} - \mathbf{x}_i) = \sum_{i=1}^N \int d^3x G_i$$

QMD uses the approximation for the density dependence of potential. $\langle n^\gamma \rangle \approx \langle n \rangle^\gamma$

$$\int d^3x \int U(n) dn = \int d^3x n V(n) = \sum_{i=1}^N \int d^3x G_i V(n) \approx \sum_{i=1}^N V(\langle n_i \rangle)$$

$$\text{interaction density } \langle n_i \rangle = \sum_{j \neq i}^N \int d^3x G_i G_j \equiv \sum_{j \neq i}^N n_{ij}$$

Equations of motion for QMD

One-particle potential: $V(n) = \frac{1}{n} \int U(n) dn$ single-particle potential: $U(n) \sim n^\gamma$

QMD uses the approximation for the density dependence of potential. $\langle n^\gamma \rangle \approx \langle n \rangle^\gamma$

$$H = \sum_{i=1}^N \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i=1}^N V(\langle n_i \rangle) \quad \text{Potential is not a function of real density.}$$

$$\text{Interaction density } \langle n_i \rangle = \sum_{j \neq i}^N \int d^3x G_i G_j \equiv \sum_{j \neq i}^N n_{ij}$$

$$\text{QMD: } \frac{d\mathbf{p}_i}{dt} = - \sum_{i=1}^N \left[\frac{dV(\langle n_i \rangle)}{dn} + \frac{dV(\langle n_j \rangle)}{dn} \right] \frac{\partial n_{ij}}{\partial \mathbf{x}_i}$$

Covariant cascade method Phys. Rev. C 108, 2 (2023)

We consider 8N-dimensional phase space: $\{x_i^\mu(s), p_i^\mu(s)\}, i = 1, \dots, N$

Hamiltonian is given by the sum of mass-shell constraints

$$H = \sum_{i=1}^N \lambda_i \frac{p_i^2 - m_i^2}{2}$$

Equation of motion:

$$\frac{dx_i}{ds} = \lambda_i p_i \quad \lambda_i : \text{Lagrange multiplier}$$

EoM with respect to the particle time: $\frac{dx_i}{dx_i^0} = \frac{dx_i}{ds} \frac{ds}{dx_i^0} = \frac{\lambda_i p_i}{\lambda_i p_i^0} = \frac{p_i}{p_i^0}$

If we take s to be a proper time of a particle, $\lambda = \frac{1}{m_i}$, EoM: $\frac{dx_i}{d\tau_i} = \frac{p_i}{m_i}$.

Covariant cascade method Phys. Rev. C 108, 2 (2023)

$$\frac{dx_i}{ds} = \lambda_i p_i \quad \lambda_i : \text{Lagrange multiplier}$$

If we assume that time of all particle is the same at a Lorentz frame \hat{a}

$$\hat{a} \cdot x_i = s, \quad i = 1, \dots, N \quad \rightarrow \hat{a} \cdot \frac{dx_i}{ds} = 1 \quad \rightarrow \lambda_i = \frac{1}{\hat{a} \cdot p_i}$$

$$\text{EoM: } \frac{dx_i}{ds} = \frac{p_i}{\hat{a} \cdot p_i}$$

2N constraints reduce the 8N phase space to physical 6N dimensions.

$$\text{In the } \hat{a} \text{ frame (CM): } \frac{dx_i}{ds} = \frac{p_i}{p_i^0}, \quad s = x_i^0, \quad (i = 1, \dots, N)$$

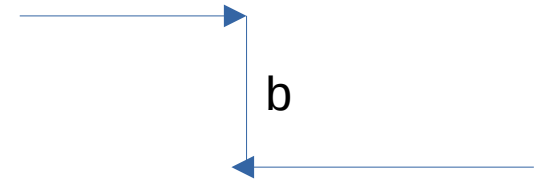
Covariant cascade method cont.

Two particles scatter when the impact parameter becomes less than $b \leq \sqrt{\frac{\sigma}{\pi}}$

Impact parameter is defined in the two-body c.m. frame:

$$b^2 = x_{cm}^2 - \frac{(x_{cm} \cdot v_{cm})^2}{v_{cm}^2} = -x_T^2 + \frac{(x \cdot p_T)^2}{p_T^2}$$

$$x_{cm} = x_{cm,1} - x_{cm,2}, \quad v_{cm} = v_{cm,1} - v_{cm,2}$$



Particles collide at the point of closest approach.

The transverse distances are defined as

$$x_T = x - (x \cdot u)u, \quad p_T = p - (p \cdot u)u, \quad x = x_1 - x_2, \quad p = p_1 - p_2, \quad u = \frac{p_1 + p_2}{\sqrt{(p_1 + p_2)^2}}$$

$$\text{time of the closet approach } s_{\text{coll}} = s_1 - \frac{[x_1(s_1) - x_2(s_1)] \cdot v_T}{v_T^2}$$

Cascade method in 6N dimensions

The time of closest approach is computed in the two-body c.m.s.

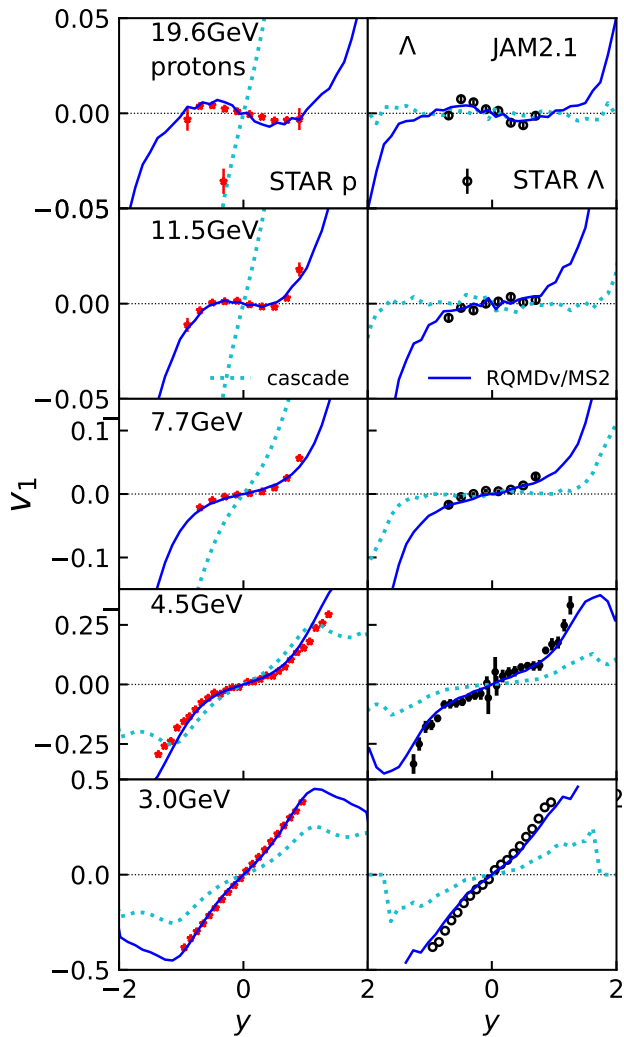
$$t_{cm} = t_{cm,1} - \frac{[\mathbf{x}_{cm,1}(t_{cm,1}) - \mathbf{x}_{cm,2}(t_{cm,1})] \cdot (\mathbf{v}_{cm,1} - \mathbf{v}_{cm,2})}{(\mathbf{v}_{cm,1} - \mathbf{v}_{cm,2})^2}$$

then transform t_{cm} to the computational frame to get the collision times t_{c1} and t_{c2} . but in general, t_{c1} is different from t_{c2} . To determine the collision time

- ZPC: $t_{coll} = \frac{t_{c1} + t_{c2}}{2}$ B. Zhang and Y. Pang, PRC56, 2185 (1997).
- new ZPC: $t_{coll} = \min(t_{c1}, t_{c2})$ X.-L. Zhao, et.al, PRC102, 024904 (2020)
- UrQMD computes the time of closest approach in the computational frame:

$$t_{coll} = t_1 - \frac{[\mathbf{x}_1(t_1) - \mathbf{x}_2(t_1)] \cdot (\mathbf{v}_1 - \mathbf{v}_2)}{(\mathbf{v}_1 - \mathbf{v}_2)^2}$$

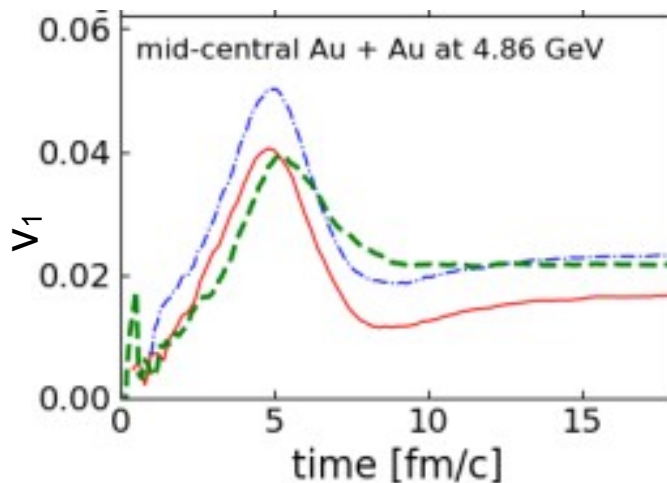
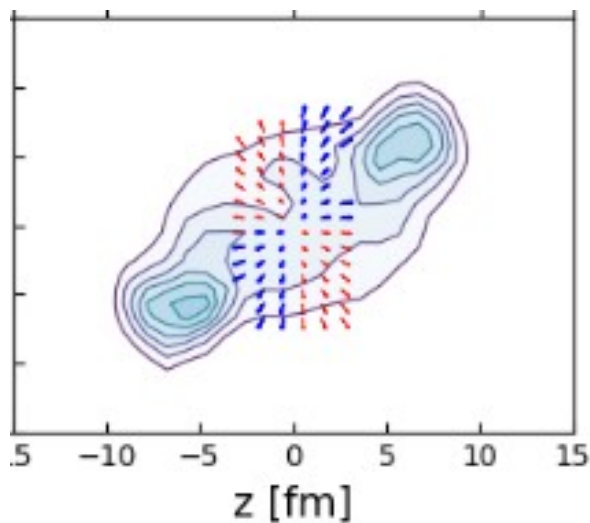
Beam energy dependence of v_1 from RQMDv



Y.N. and A. Ohnishi, PRC(2022)

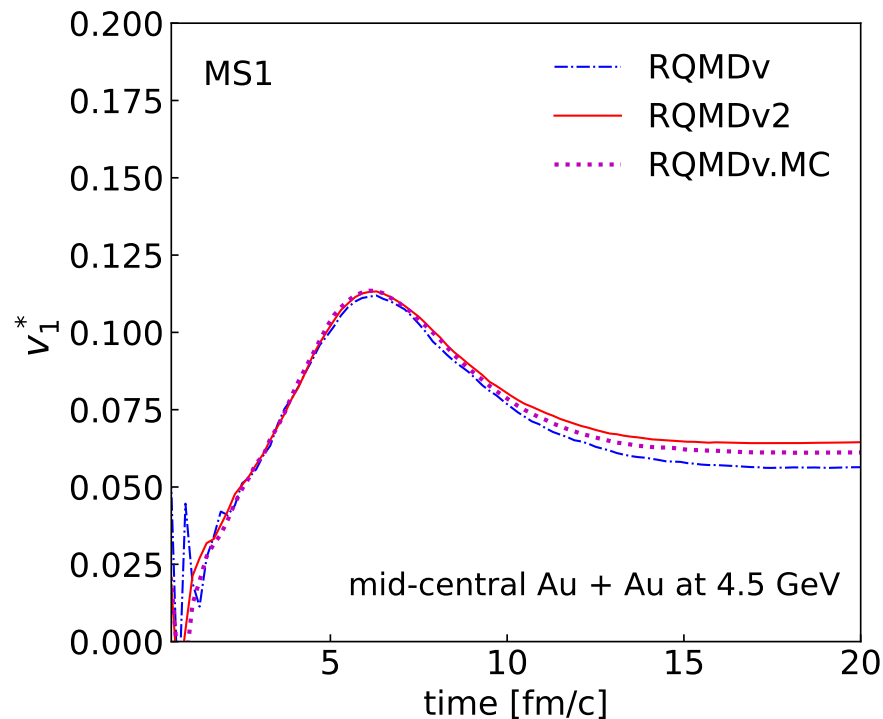
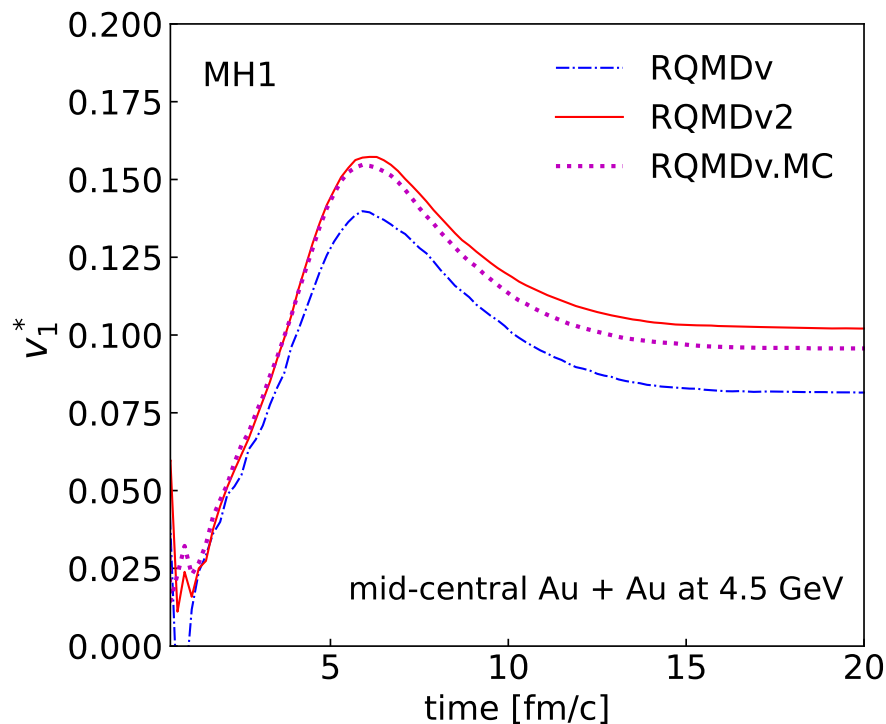
Y.N, A. Jinno, K. Murase, A. Ohnishi, PRC106 (2022)

Skyrme type Lorentz vector potential



v_1 is determined by the cancellation of positive and negative components

Time evolution of directed flow v_1



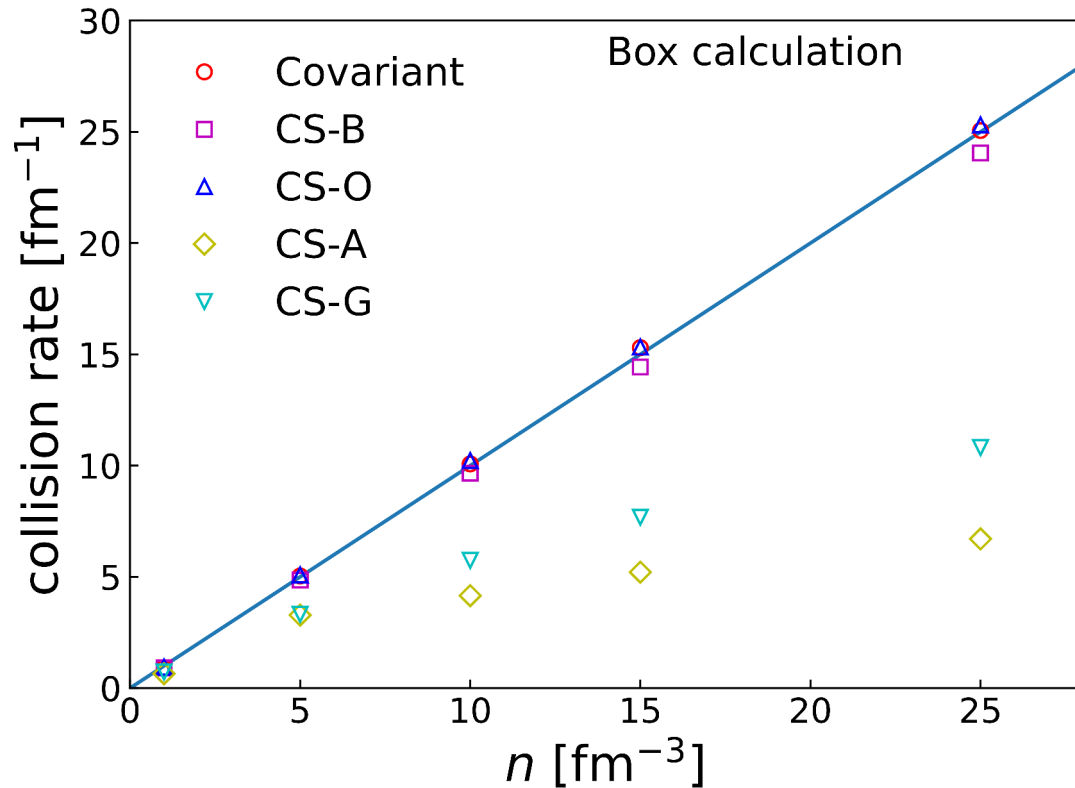
RQMDv (Lorentz vector Skyrme potential)

RQMDv2: new RQMDv

RQMDv-MC: exact integration

$$v_1 = \langle \cos \phi \rangle = \left\langle \frac{p_x}{p_T} \right\rangle \quad v_1^* = \int_{-1}^1 dy v_1(y) \text{sgn}(y) \quad 28$$

Collision rate in a box simulation



All models define impact parameter in the two-body c.m. frame, which is Lorentz invariant. Collision ordering time is frame dependent in 6N cascade method.

- CS-B: ZPC new X.-L. Zhao, et.al

$$t_{\text{coll}} = \min(t_{c1} + t_{c2})$$

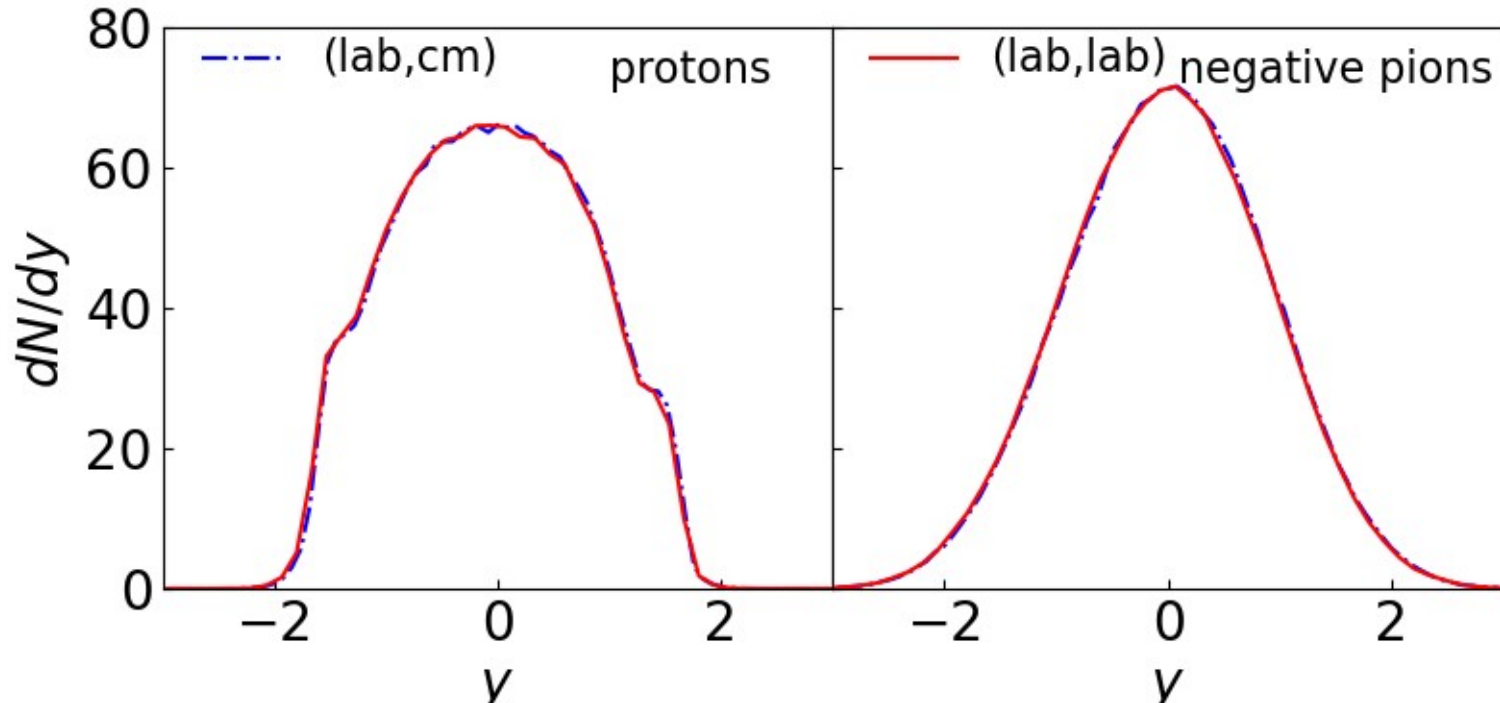
- CS-G: ZPC old B. Zhang

$$t_{\text{coll}} = \frac{t_{c1} + t_{c2}}{2}$$

- CS-O: UrQMD, SMASH
collision time is computed in the computational frame

Foliation dependence

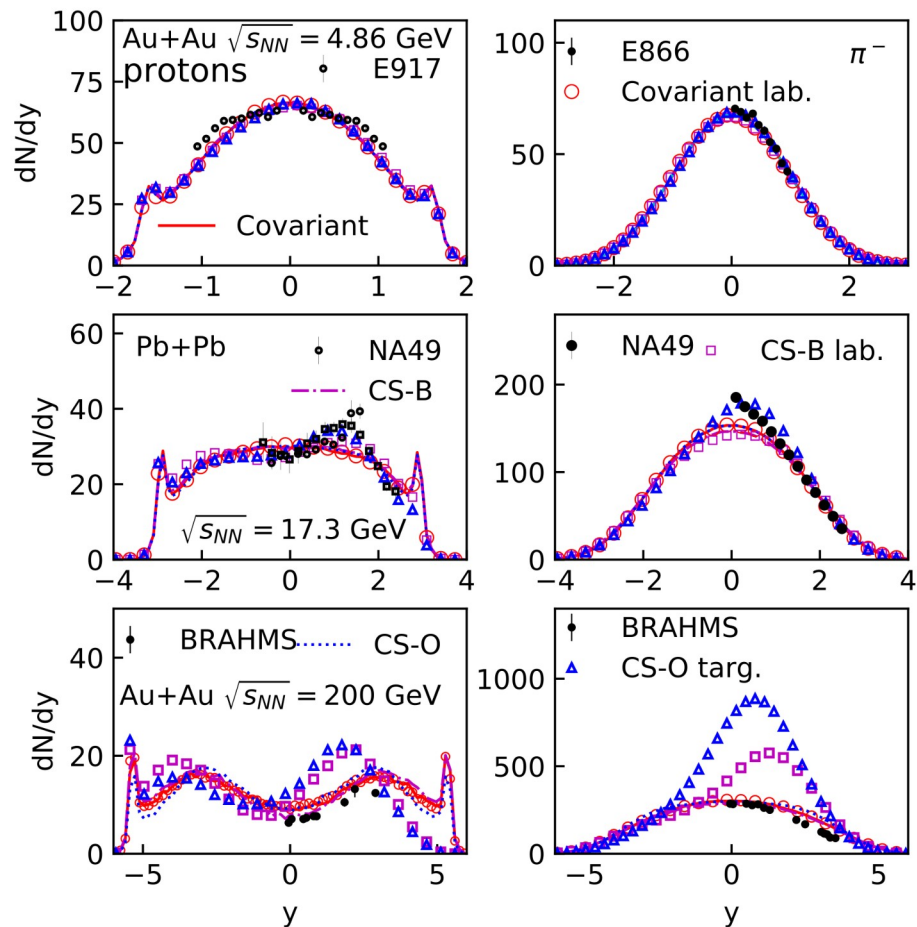
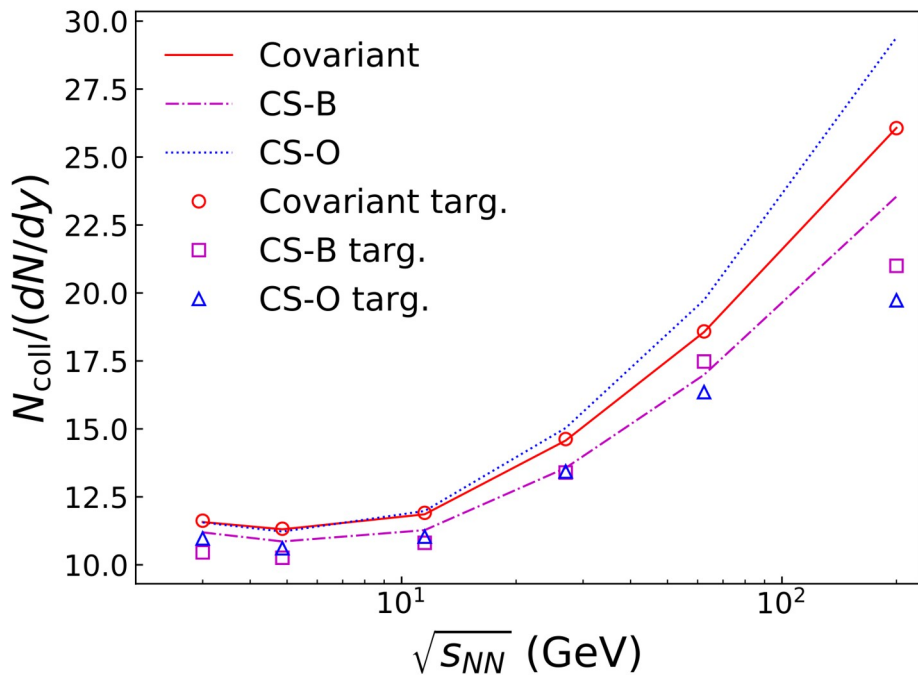
time constraints $\hat{a} \cdot x_i - s = 0$ foliation $\hat{a} = (1, 0, 0, .0)$ in the laboratory frame



dN/dy remain similar, but distribution is no longer perfectly symmetric.
However, dN/dy is Lorentz invariant between different frames.

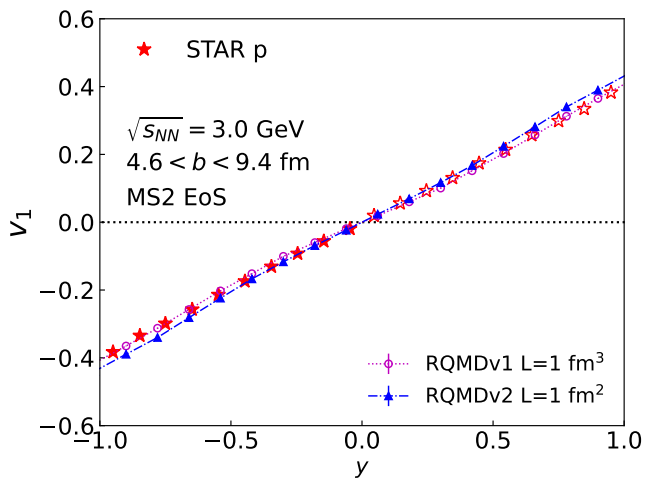
Au+Au collisions at 4.86, 17.3 and 200 GeV

Validation of frame independence

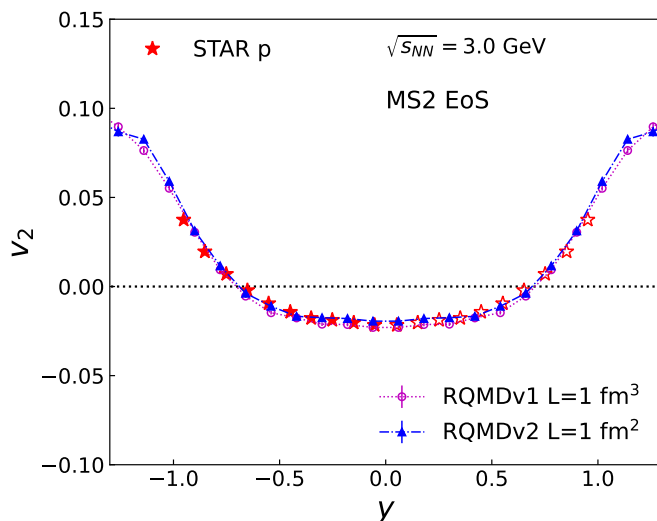


flows in Au + Au collisions at 3GeV

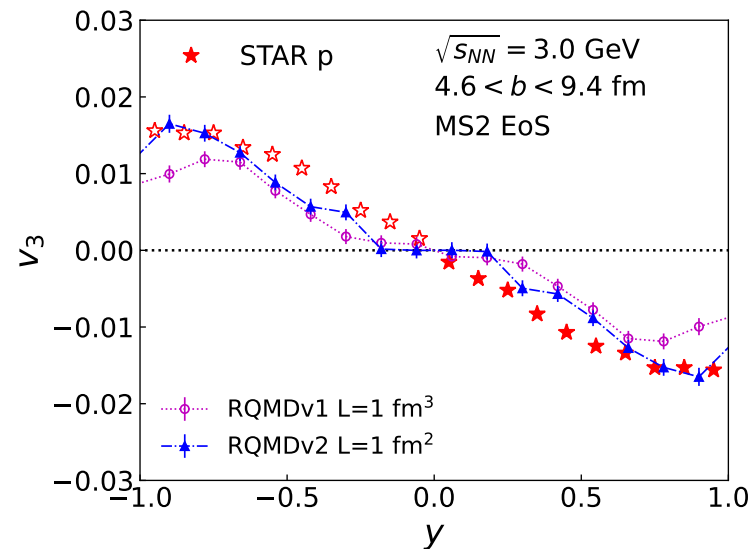
$$v_1 = \langle \cos \phi \rangle$$



$$v_2 = \langle \cos 2\phi \rangle$$



$$v_3 = \langle \cos 3\phi \rangle$$



Even for the soft momentum-dependent EoS, RQMDv2 predicts larger v_3 .