Neutron and Quark Star Matter

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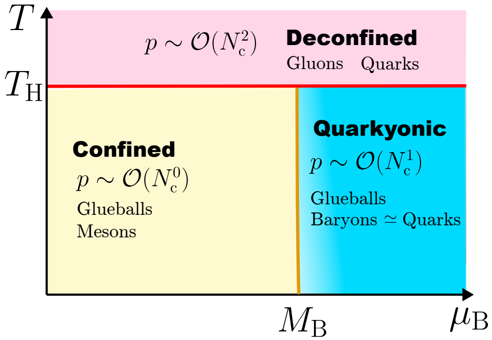
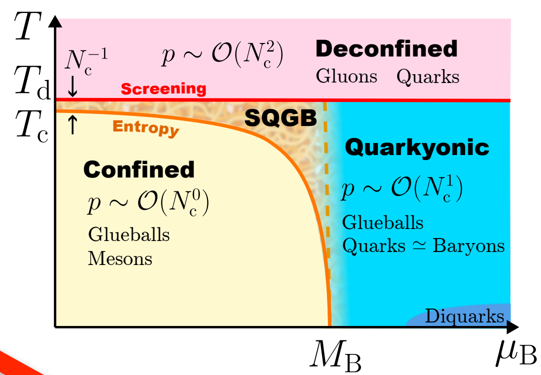
Based on a work with Josuke Minamiguchi, Tomoya Uji

— Workshop on recent developments from QCD to nuclear matter —

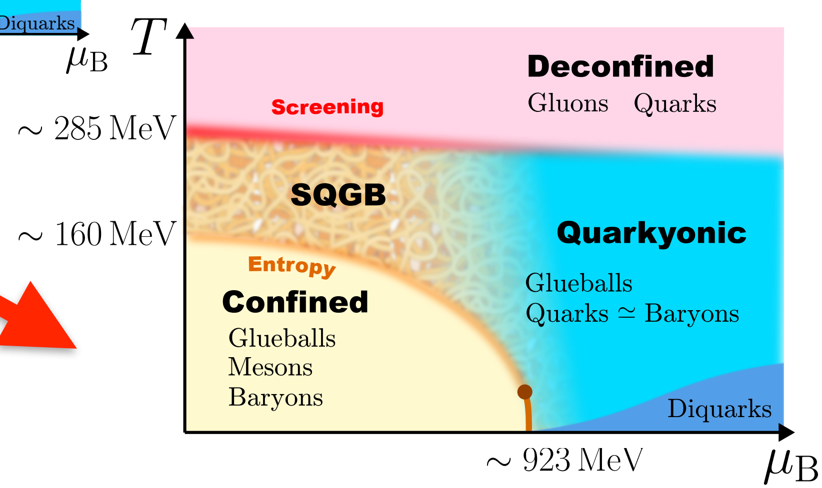
QCD Phase Diagram



Spaghetti of Quarks with Glueballs



From $N_c = \infty$ to $N_c = 3$



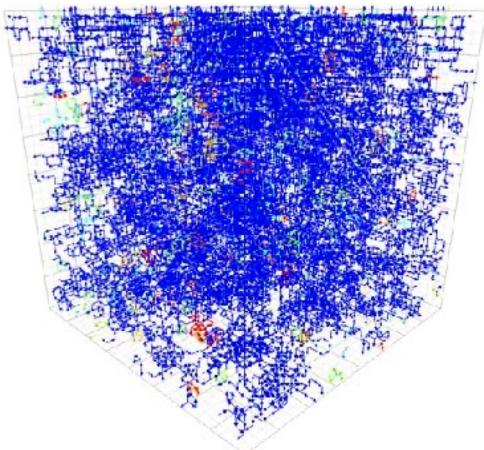
Fujimoto-Fukushima-
-Hidaka-McLerran (2025)

QCD Phase Diagram

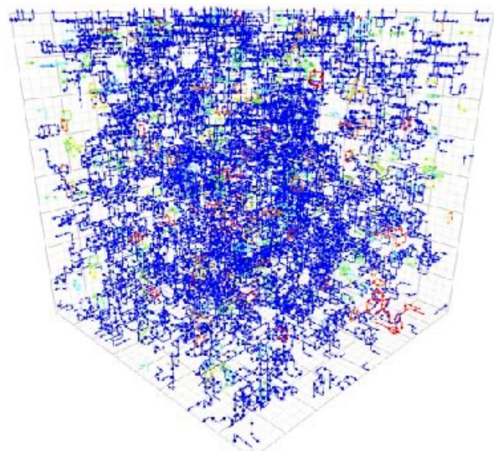
arXiv:2411.19446

Centre vortex evidence for a second finite-temperature QCD transition

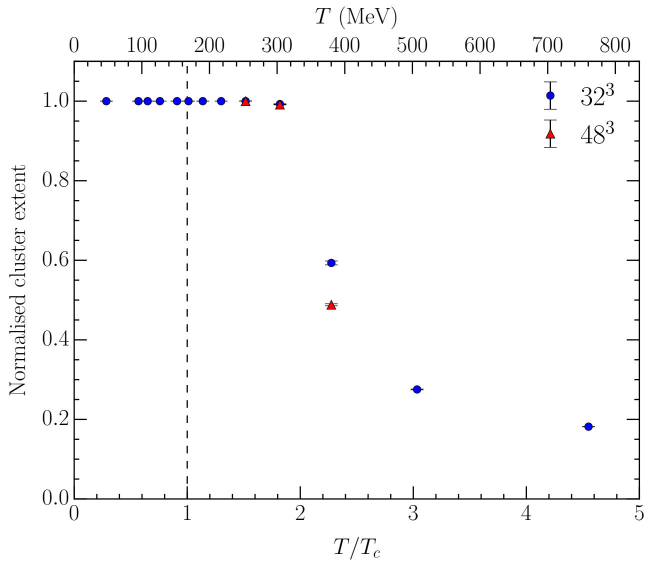
Jackson A. Mickley[1] Chris Allton[2,3] Ryan Bignell[4] and Derek Leinweber[1]



$T/T_c = 0.28$



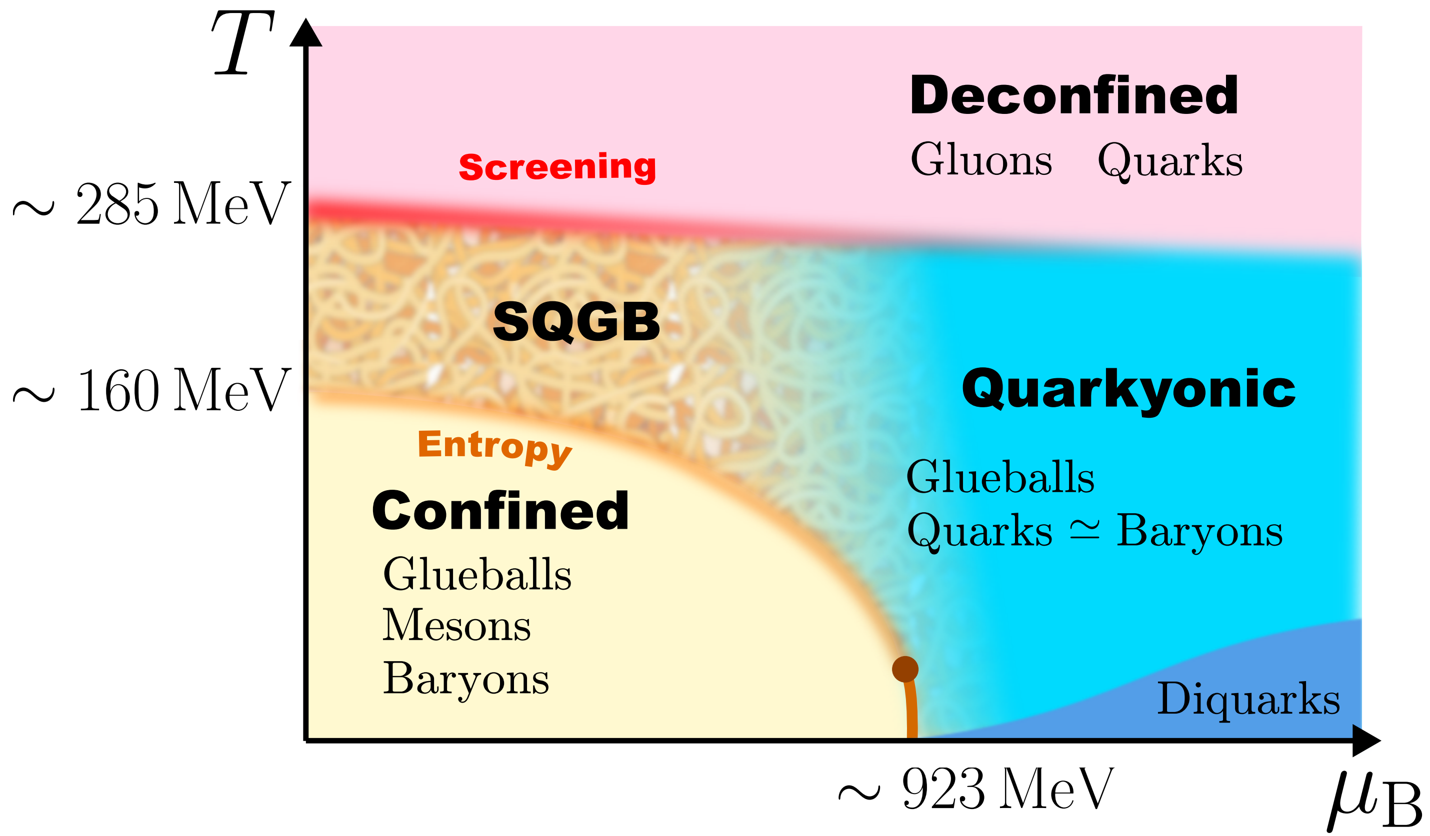
$T/T_c = 2.28$



Even above T_c, non-perturbative gluons still remain!

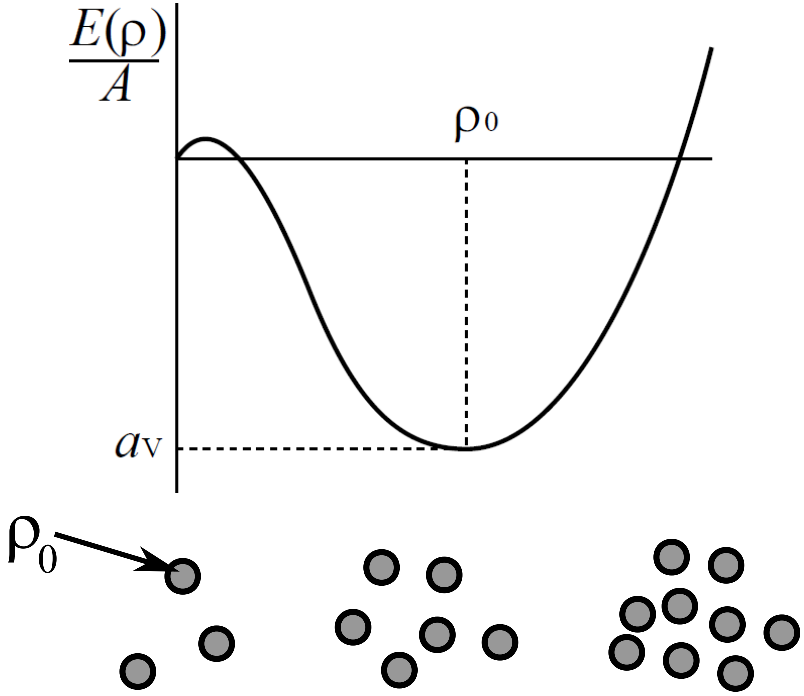
QCD Phase Diagram

How much do we know about the phase diagram?
Nuclear Liquid/Gas Critical Point? QCD Critical Point??



Stability of Nuclear/Quark Matter

Nuclear Saturation



Self-bound fermionic systems have a preferred density.
Diluteness is realized as a "mixed phase" of nuclei.

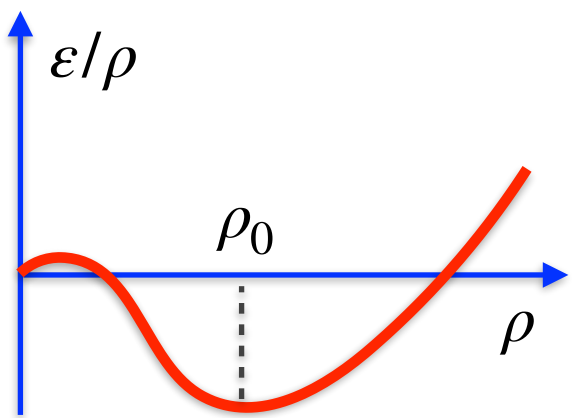
This is how this world is like what we know.

Stability of Nuclear/Quark Matter

Extremal point → 1st-order PT

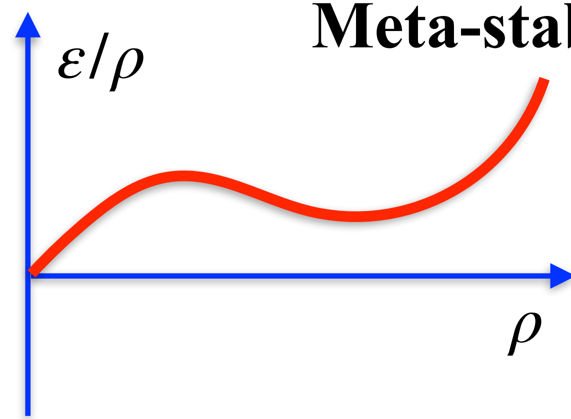
$$\frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\bigg|_{\text{gas}} - \frac{\varepsilon}{\rho}\bigg|_{\text{liquid}}\right) = \frac{p_{\text{gas}} - p_{\text{liquid}}}{\rho^2} = 0$$

Stationary Cond. **Phase Transition Cond.**



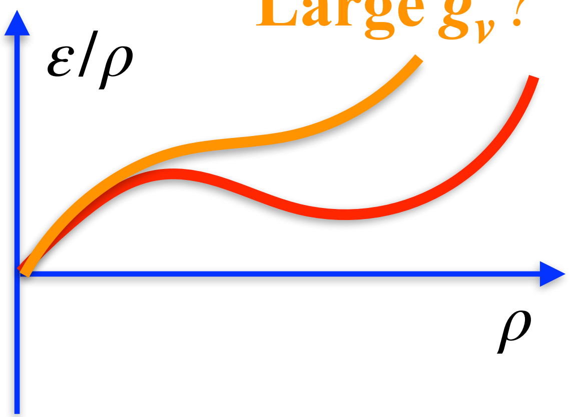
Nuclear Matter

Meta-stable QM?

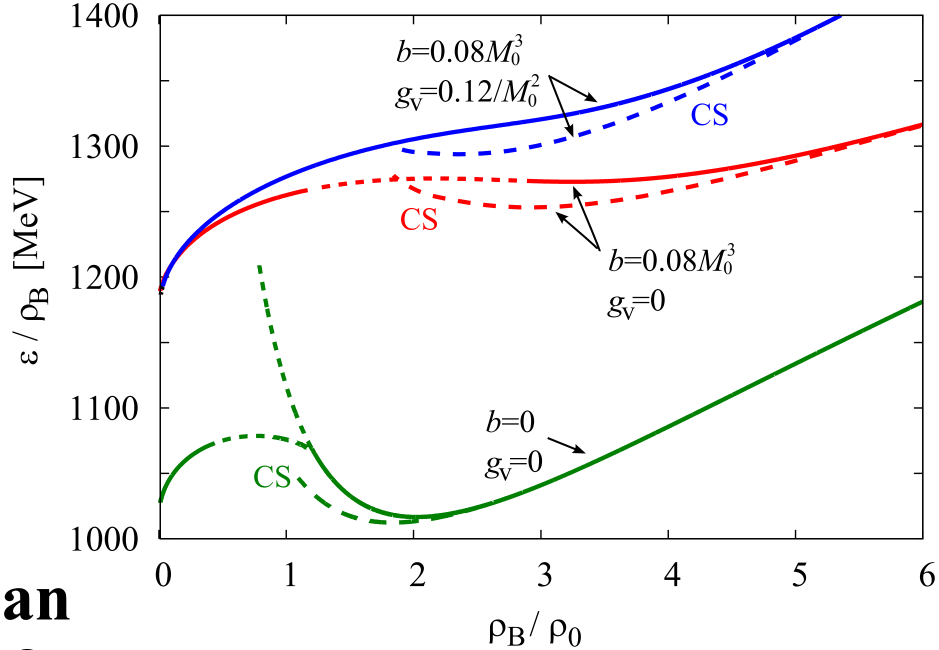


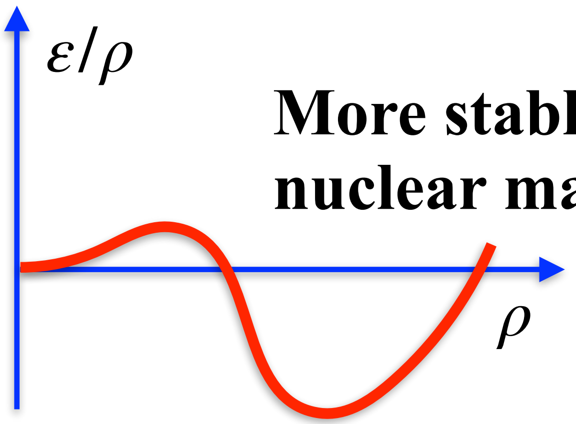
Many model studies

Stability of Nuclear/Quark Matter



Fukushima (2012)

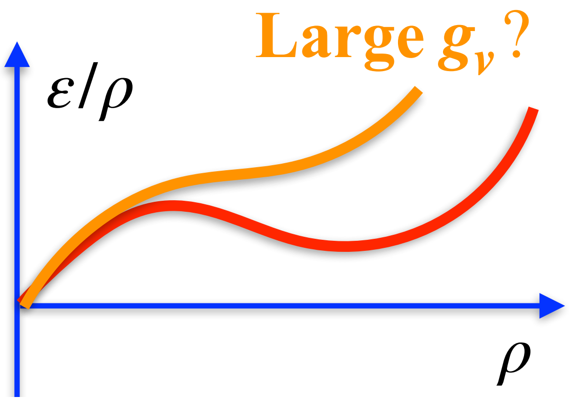




More stable than nuclear matter?

1D chiral spirals (CS) decrease the energy per particle, inducing another 1st-order PT.

Stability of Nuclear/Quark Matter



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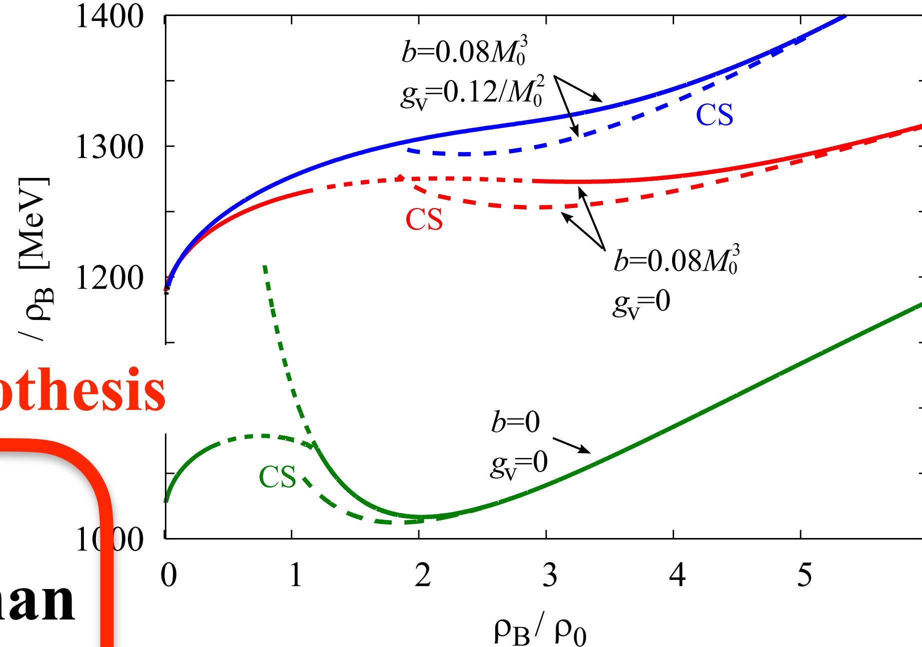
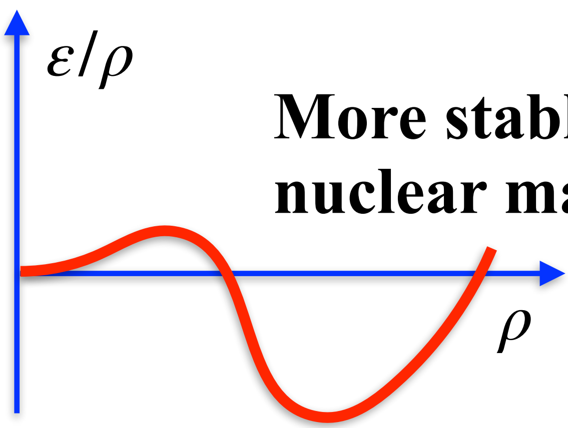


Strange Quark Matter Hypothesis



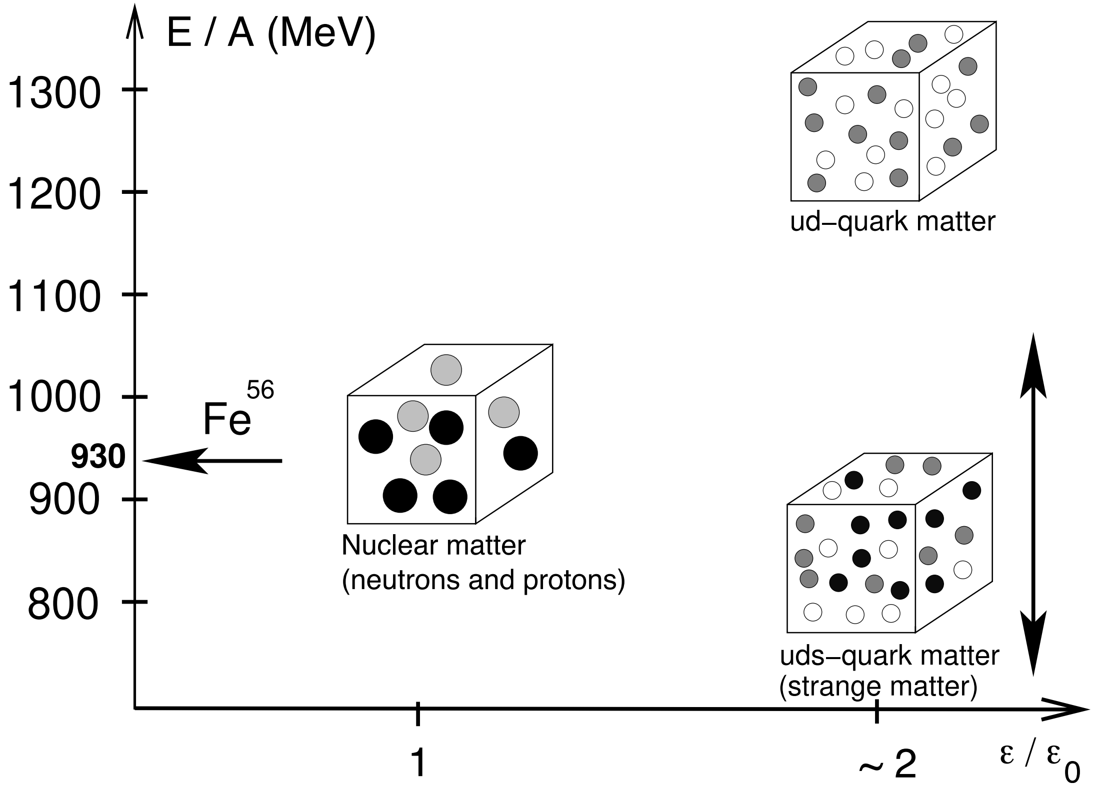
More stable than nuclear matter?

1D chiral spirals (CS) decrease the energy per particle, inducing another 1st-order PT.

Strange Quark Matter Hypothesis

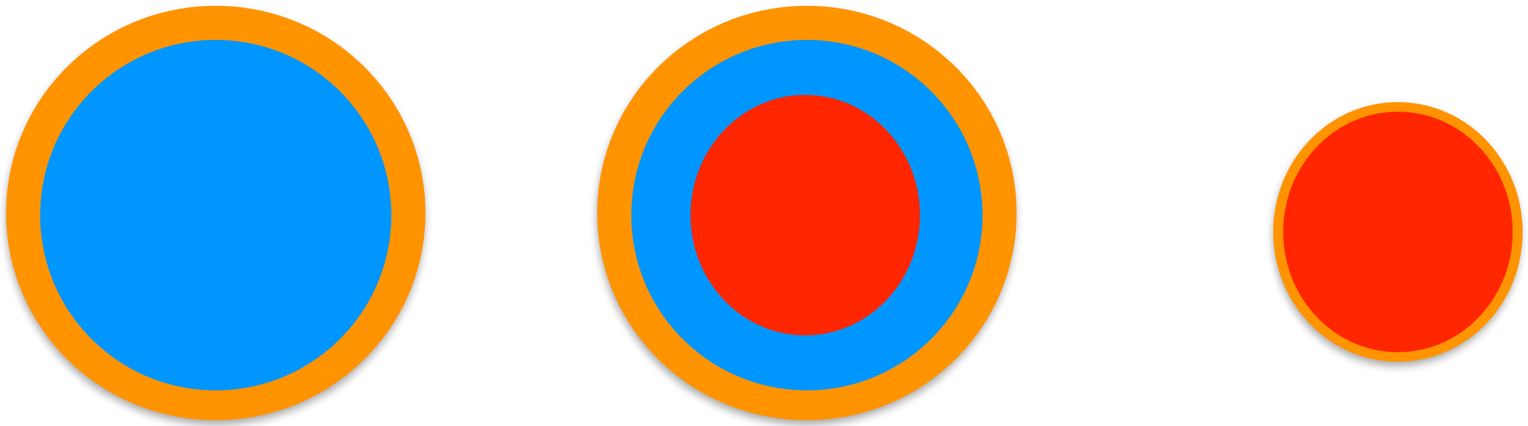
Bodmer (1971) / Terazawa (1979) / Witten (1984)

Schematic illustration by Weber (2004)



Differences

Neutron Stars | **Hybrid Stars** | **Quark Stars**



Softening in NS mergers

$R \sim$ 10-12km / $M \sim 1.4\text{-}2.1 M_\odot$

Crust ~ 1km

Atmosphere ~ ions + electrons

Signature???

$R \sim <$ 12km / $M \sim < 2M_\odot$

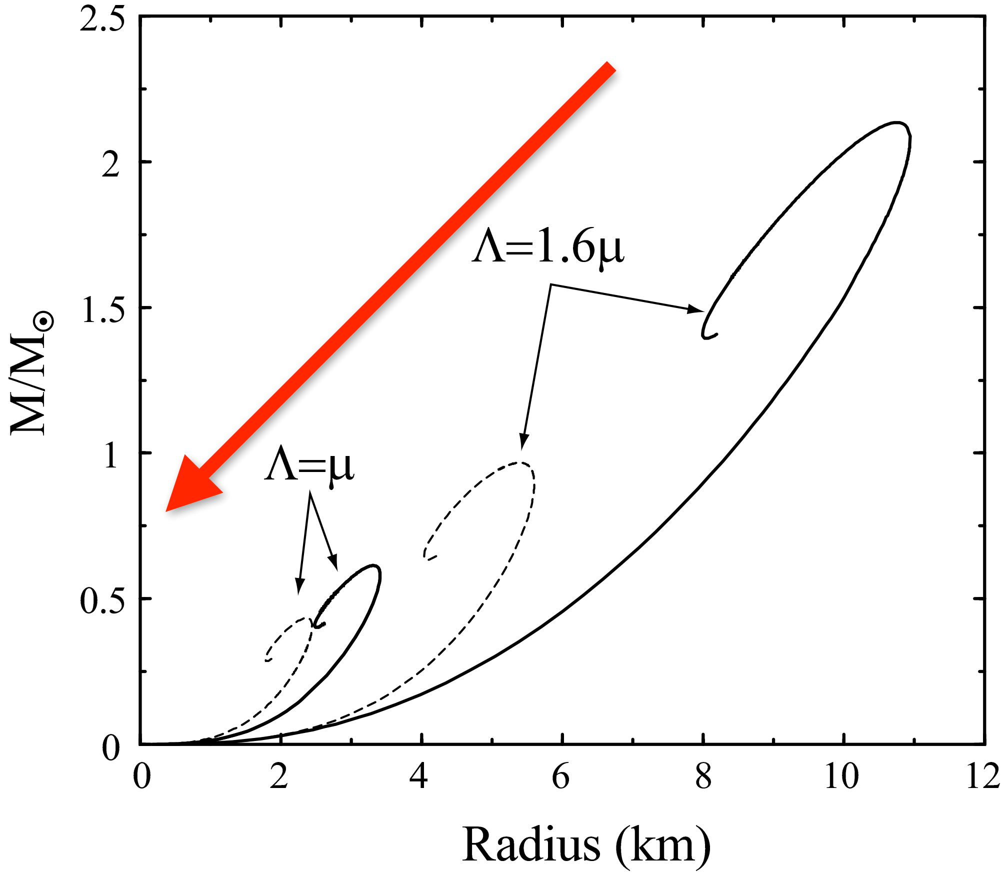
Thin crust

No atmosphere

Differences

Mass-Radius Relation of QS

Andersen-Strickland (2002)



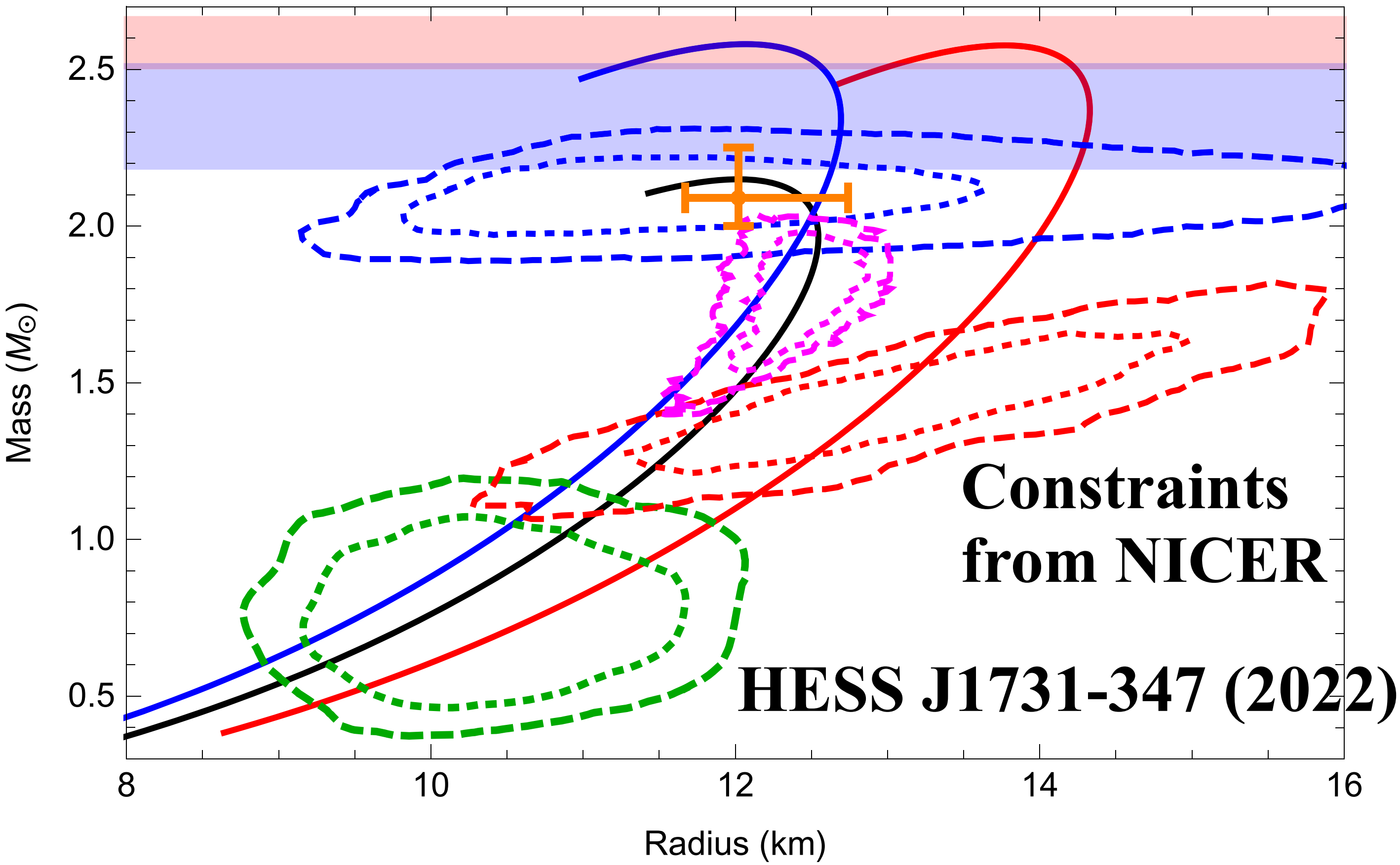
Resummed PT in QCD

This is a bit misleading since the EoS is very sensitive for such *too small* scales Λ.

Quark stars may not be light nor small…

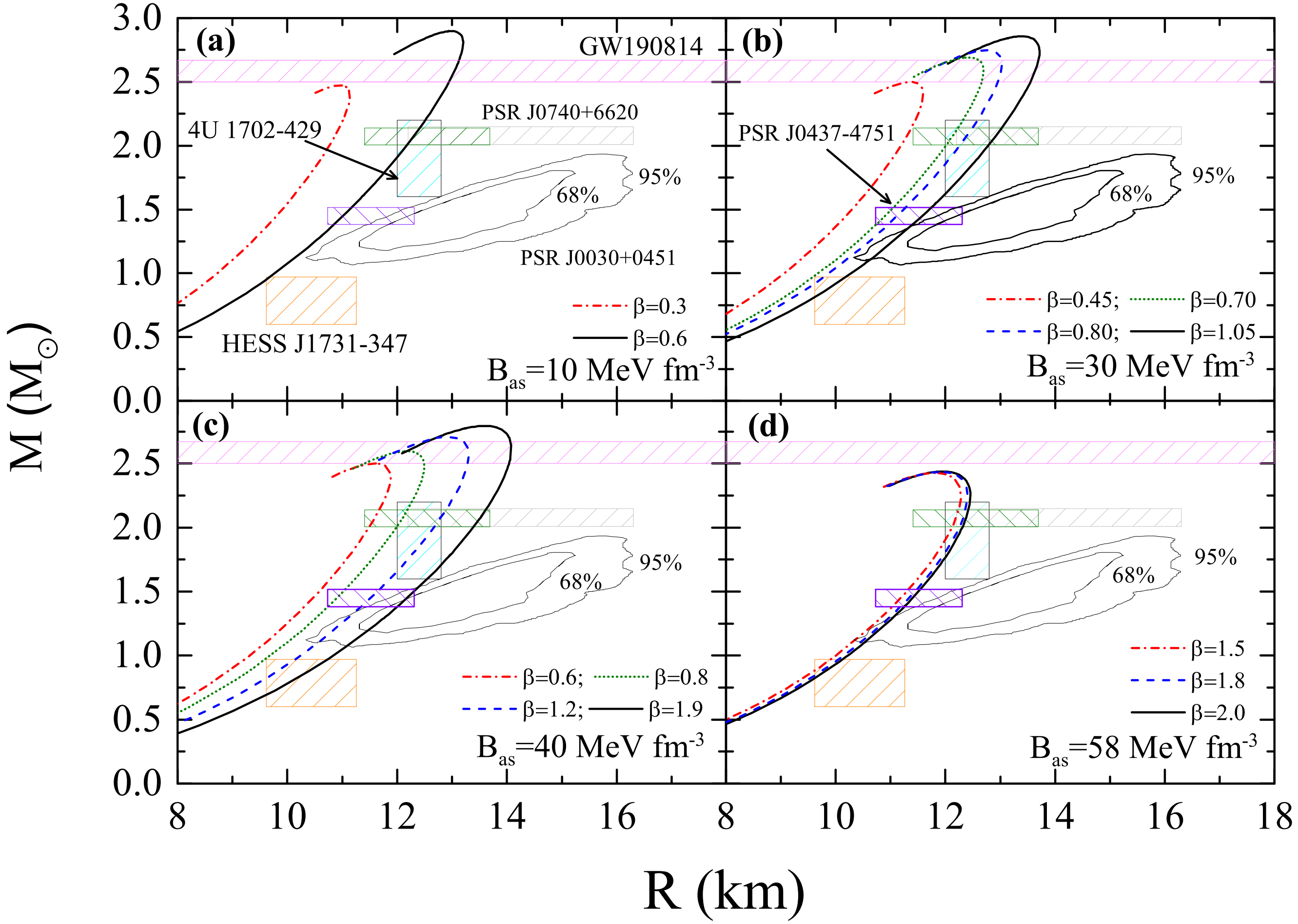
Candidate?

Clemente-Drago-Pagliara (2024) **Strange Dwarfs?**



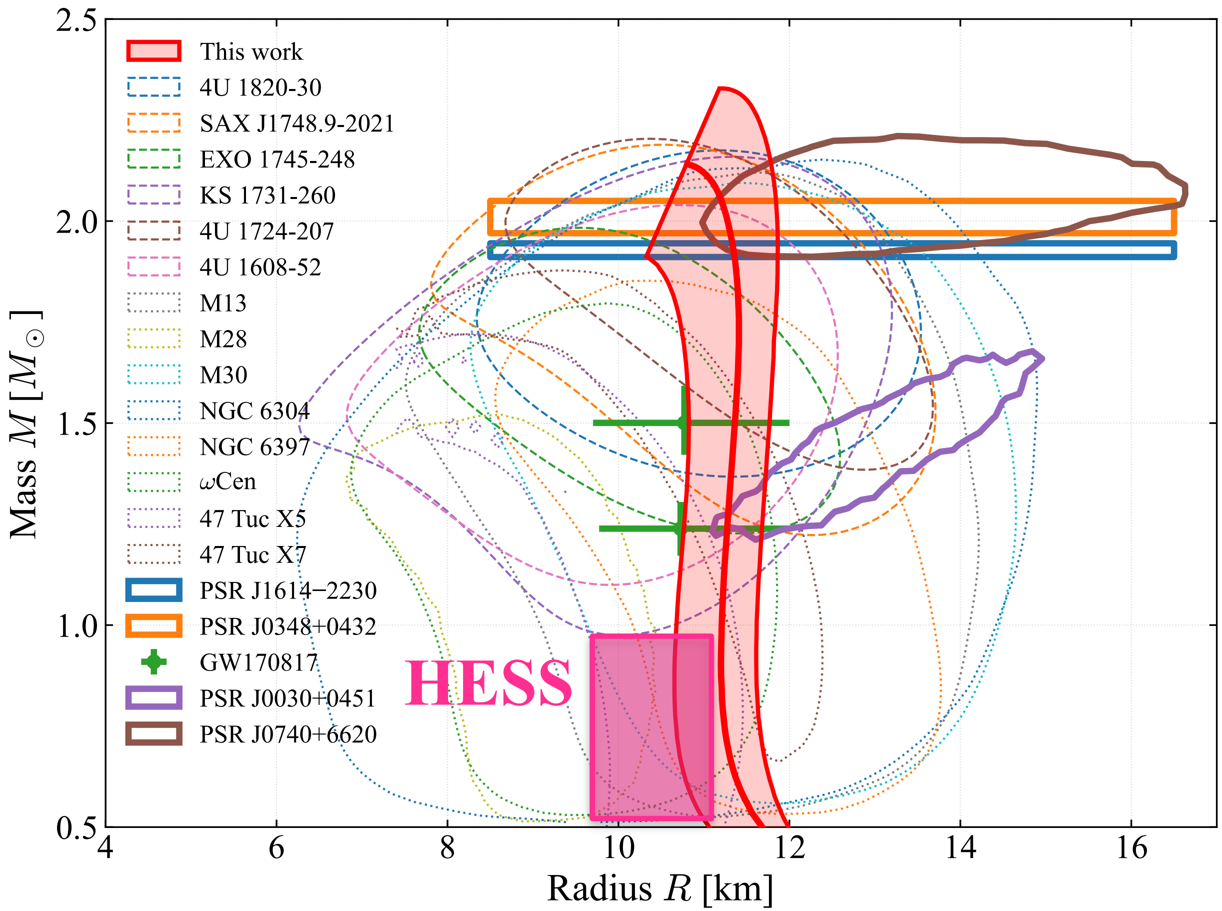
Candidate?

Ju-Chu-Wu-Liu (2024)



Candidate?

Fujimoto-Fukushima-Kamata-Murase (2018-2024)



Candidate?

There is no stability bound for light NSs ?

Supernovae simulations favor NSs with $M \gtrsim M_\odot$.

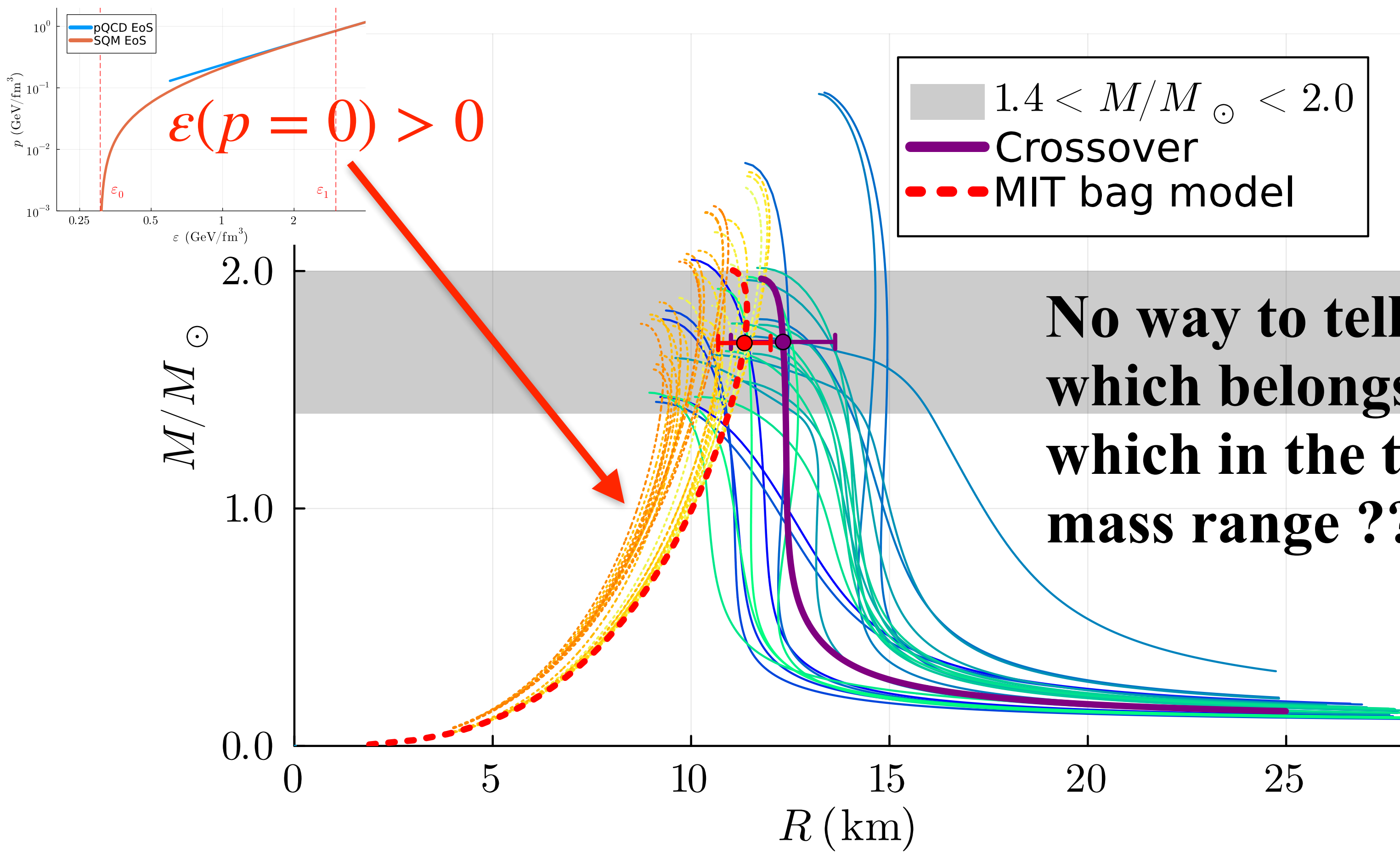
The supernova remnant of HESS may be an exceptionally light NS ?

Only from the *M*-*R* data, one can only claim possible consistency with the QS scenario…

Any more decisive observable (from GW signals)?

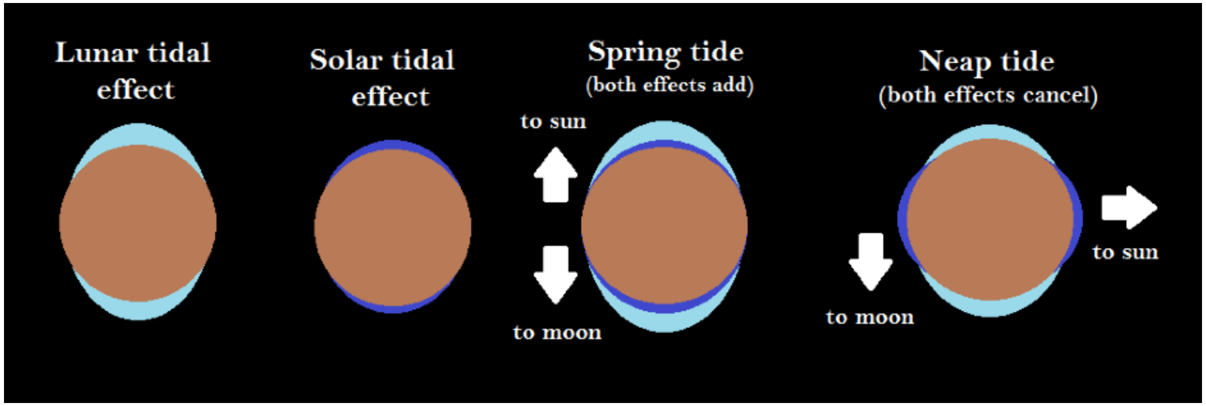
M-R Indistinguishability

Fukushima-Minamiguchi-Uji

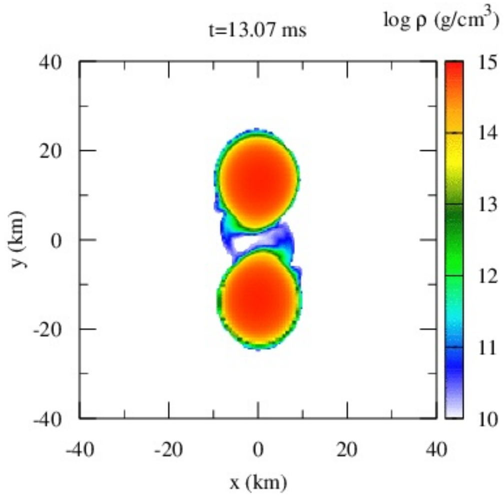


No way to tell which belongs to which in the typical mass range ??

Tidal Responses



from energyeducation



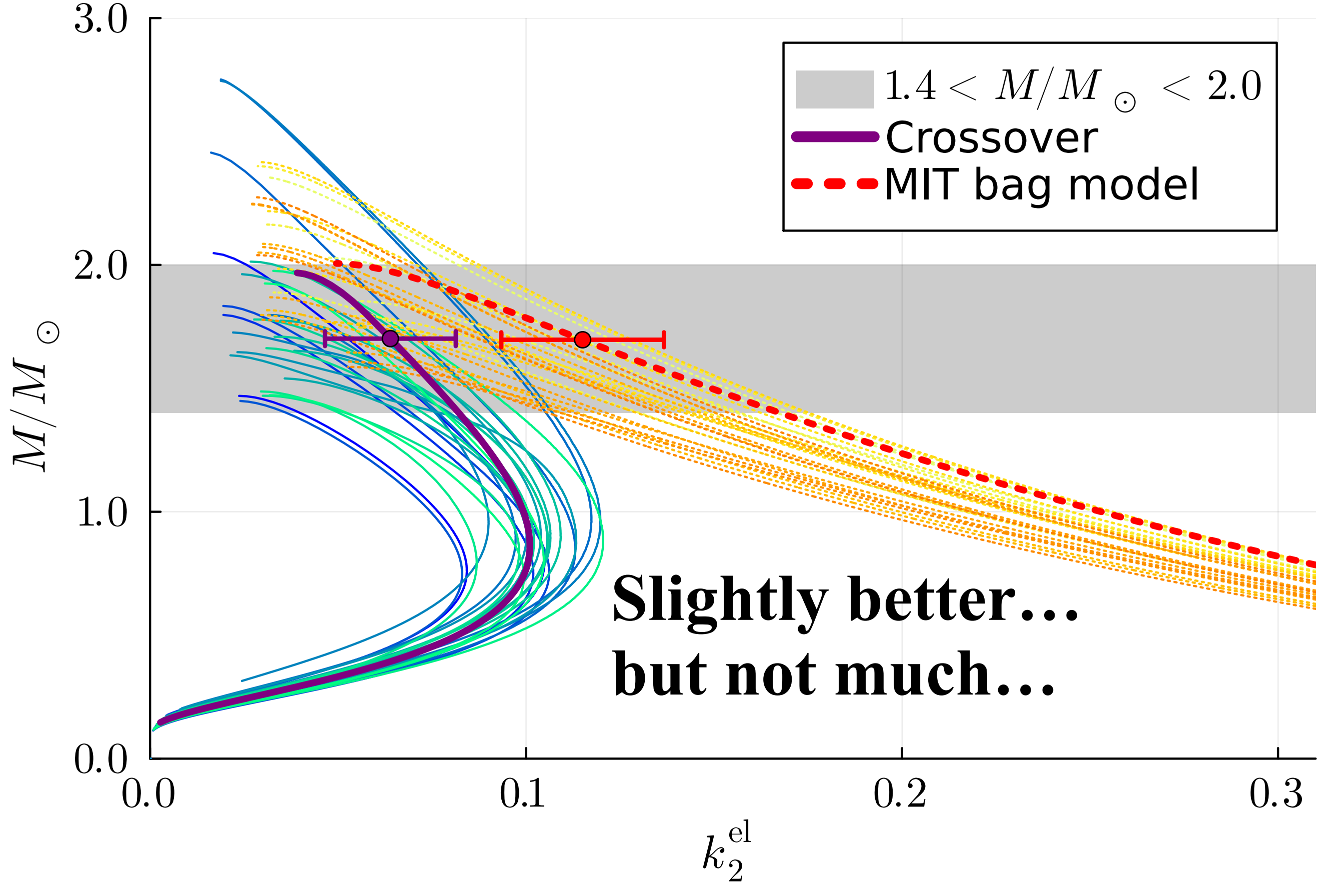
Kyutoku+

Distortion in response to the gravitational force from the binary companion.
Affects the GW waveforms.

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

Tidal Responses

Fukushima-Minamiguchi-Uji



$$\Lambda := \frac{2k_2^{\rm el}}{3C^5}$$

$$C := \frac{M}{R}$$

Slightly better…
but not much…

Tidal Responses

Damour-Nagar (2009) / Binnington-Poisson (2009)

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

$$h_{\mu\nu} = h_{\mu\nu}^{\text{even}} + h_{\mu\nu}^{\text{odd}}$$

Parity-even = Electric $\leftarrow \mathscr{E}_{ij}$

Parity-odd = Magnetic $\leftarrow \mathscr{B}_{ij}$

$\boldsymbol{\ell = 2}$ $$h^{\text{even}} \sim a_1 P_2^{(2)}(r/M - 1) + a_2 Q_2^{(2)}(r/M - 1)$$

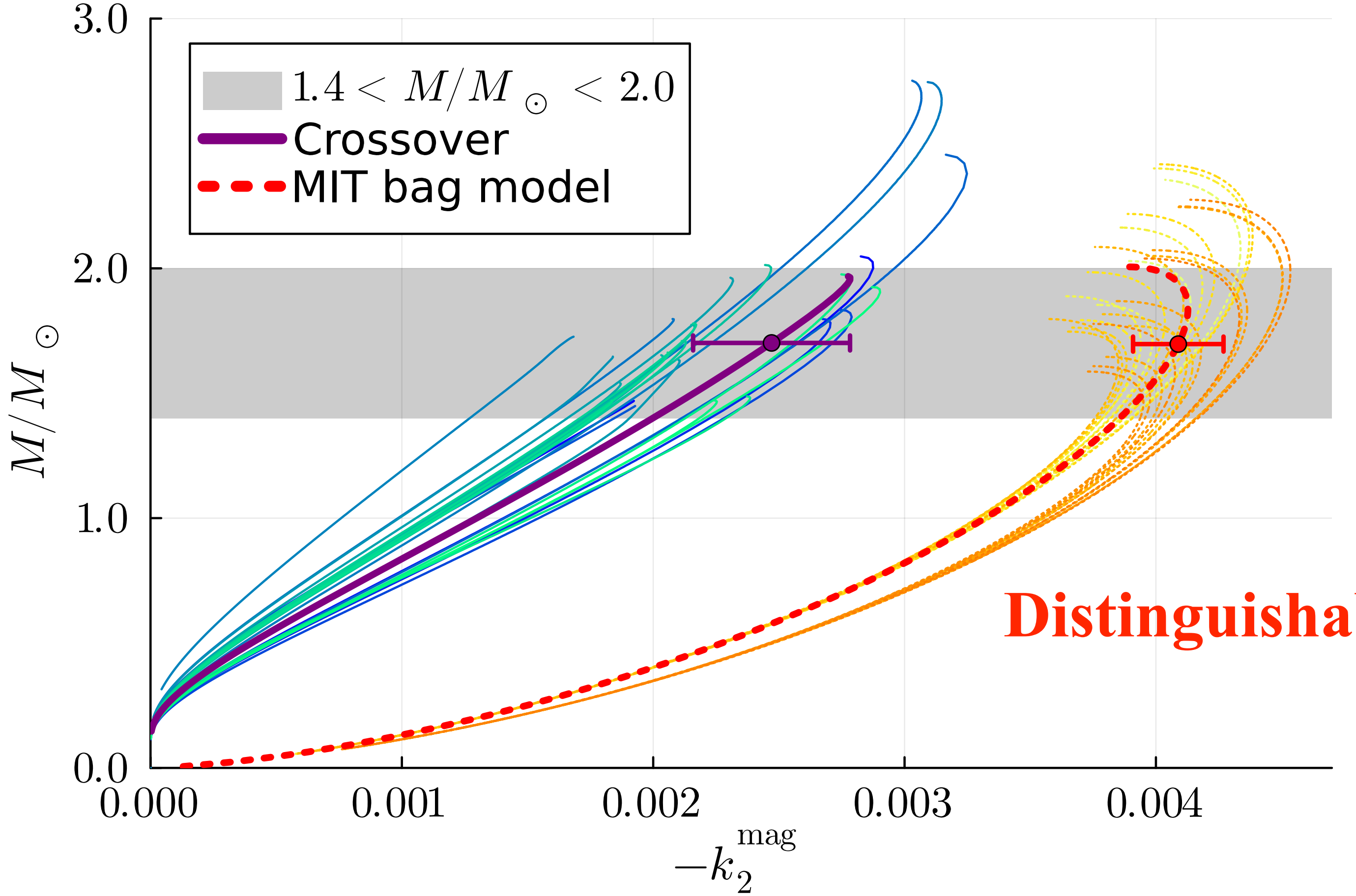
$$k_2^{\text{el}} \propto a_2/a_1$$

h^{odd} **involves complicated special functions…**

But the strategy to read k_2^{mag} **is the same.**

Tidal Responses

Fukushima-Minamiguchi-Uji



Distinguishable!!!

Detectability via Gravitational Waves

Fukushima-Minamiguchi-Uji

GW Phase: $\Psi(f) = \Psi(f) + \Psi^{\rm el}_{\rm tidal}(f) + \Psi^{\rm mag}_{\rm tidal}(f)$

5PN (for $\Psi^{\rm el}_{\rm tidal}$) **6PN** (for $\Psi^{\rm mag}_{\rm tidal}$)

***n*-Post-Newtonian approx. = Expansion up to $\mathcal{O}\left((v/c)^{2n}\right)$**

NS [Crossover EoS] vs. QS [MIT bag model] distinguishable?

$$\langle h_1 | h_2 \rangle = 4\mathrm{Re}\int_{f_{\rm min}}^{f_{\rm max}} \frac{h_1(f)h_2^*(f)}{S_n(f)}df \;\to\; F(h_1,h_2) = \max\frac{\langle h_1 | h_2\rangle}{\sqrt{\langle h_1|h_1\rangle\langle h_2|h_2\rangle}}$$

$$\boxed{2\rho^2[1-F] \geq \chi^2_k(1-p)}$$ (k : parameter num.)

k	ρ (68%)	ρ (90%)	
4	63.9	82.2	$(k_2^{\rm el}, k_2^{\rm mag}) \times 2$
6	78.0	96.2	$(M, k_2^{\rm el}, k_2^{\rm mag}) \times 2$

SNR $\rho \sim$ 32.4
for GW170817…

Not easy, but
not impossible!

Summary

Strange quark matter may be self-bound?

- QCD Critical Point and Quark Star both discuss the stability / meta-stability of quark matter.
- QCD Critical Point is a necessary condition for the existence of Quark Stars and vice versa.

Quark Star Hunting

- Small-M and small-R objects are likely candidates but more candidates in wider regions overlapping NSs…
- Tidal responses: the magnetic Love number can resolve the degeneracy very clearly!