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Magnetic field effects on neutrino emission in dense quark matter

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Workshop on recent developments from QCD to nuclear matter

Neutron Stars and Cooling

-Extremely dense remnants of massive stars after supernova explosions.

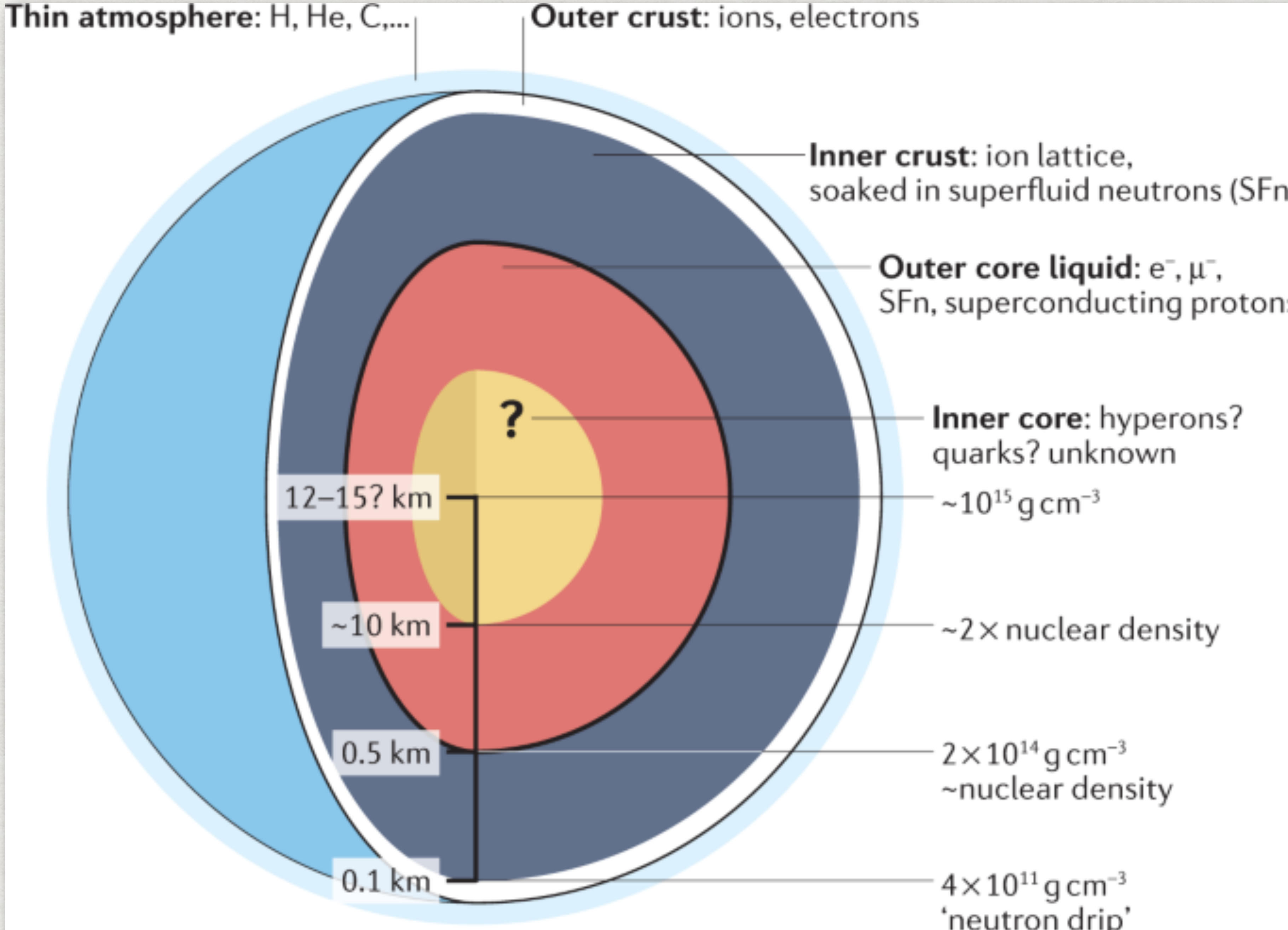
- **Key Properties:**
- Radius: ~10 km
- Mass: 1.4–2.2 solar masses
- Density ~ $10^{17}\ \mathrm{kg/m^3}$
- Strong magnetic fields in core

$(\sim 10^{14}\mathrm{G} - 10^{17}\mathrm{G})$

- **Significance:**

Laboratories for testing physics under extreme conditions (e.g., strong gravity, high densities).

[G. Baym, et. al,1707.04966 [astro-ph.HE]]
[N. Iwamoto, PRL. 44, 1637 (1980)]
[M. Alford et. al, 2409.09423 [nucl-th]]



Nature Reviews Physics volume **4**, 237–246 (2022)

QUARKS (?)

E. Annala et al, Nature Commun. 14 (2023) 8451

Cooling of compact stars

•**Energy Loss Mechanisms in quark matter:**
•**Neutrino Emission:**

direct URCA processes $\propto R^3T^6$

Modified URCA processes $\propto R^3T^8$

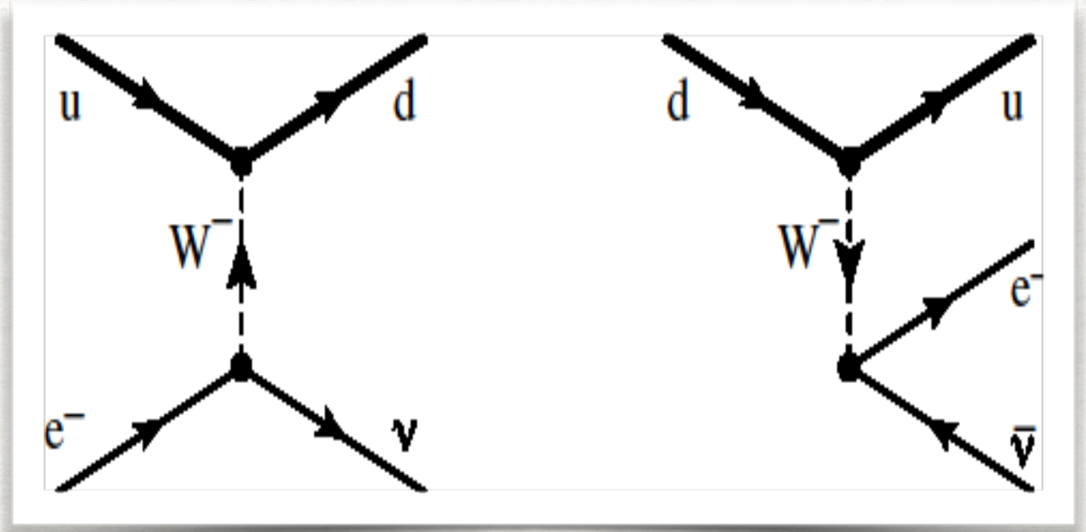
•**Photon Emission:** Come only from the star's surface. $\propto R^2T^4$

How do magnetic fields influence cooling?

[Ghosh, Shovkovy, *JHEP* 04 (2025) 110]

$\nu(\bar{\nu})$ emission

direct Urca processes



Fermi theory of weak interactions

- electron capture by an up quark, producing down quark and an electron neutrino

$$u + e^- \rightarrow d + \nu_e$$

- decay of a down quark

$$d \rightarrow u + e^- + \bar{\nu}_e$$

β-equilibrium

$$\mu_d = \mu_u + \mu_e$$

- Assumption: quark matter temperature is below the threshold for neutrino trapping

$$\mu_\nu = 0$$

Neutrino emission From Star

Use Kadanoff-Baym transport equation for neutrinos

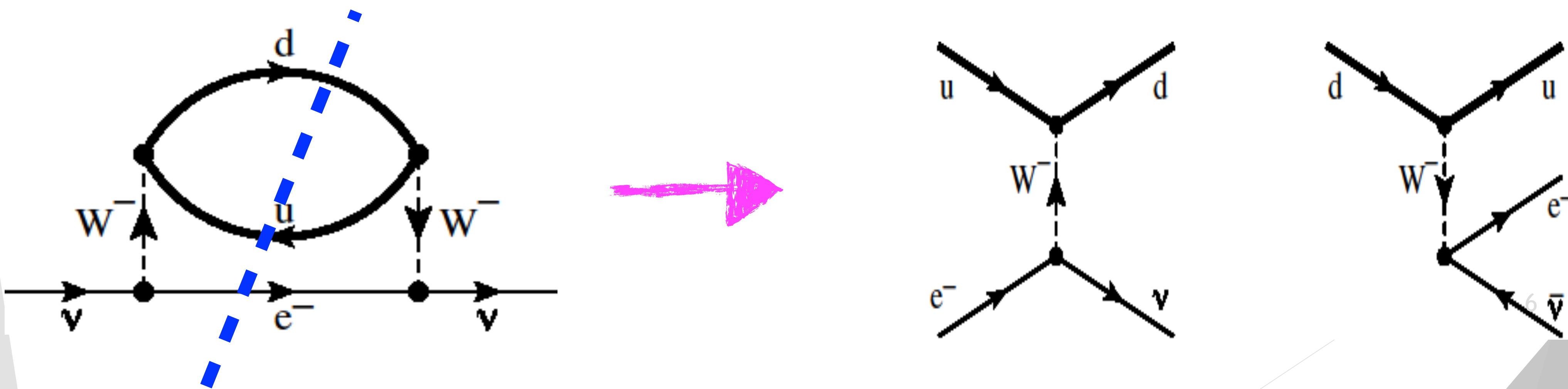
$$i\partial_t \mathrm{Tr}[\gamma^0 G_\nu^<(t,P_\nu)] = -\mathrm{Tr}[G_\nu^>(t,P_\nu)\Sigma_\nu^<(t,P_\nu) - \Sigma_\nu^>(t,P_\nu)G_\nu^<(t,P_\nu)].$$

Neutrino-number production rate:

$$\frac{\partial f_\nu(t,\boldsymbol{p}_\nu)}{\partial t} = \frac{G_F^2\cos^2\theta_C}{2}\sum_{\lambda=\pm}\sum_{n=0}^{\infty}(-1)^n\int\frac{d^3\boldsymbol{p}_e e^{-p_{e,\perp}^2\ell^2}}{(2\pi)^3 p_\nu E_{e,n}} n_F(E_{e,n}-\mu_e)n_B(p_\nu+\mu_e-E_{e,n})L_{n,\lambda}^{\delta\sigma}(\boldsymbol{p}_e,\boldsymbol{p}_\nu)\Im\left[\Pi_{\delta\sigma}^R(Q)\right]$$

Fermi coupling

imaginary (absorptive) part of W^- self-energy



[Ghosh, Shovkovy, J. High Energy Phys. 04 (2025) 110]

Assumptions:

- Chemical potentials:

  $\mu_f(\text{where} f = u, d) \sim 300\ \text{MeV},\ \mu_e \sim 50\ \text{MeV}$

  -Landau quantization of electron state
- Temperature: $T \lesssim 5\ \text{MeV}$
- Magnetic field : $|eB| < 10^{17}\,\text{G}\ (\sqrt{|\text{eB}|} < 25\,\text{MeV})$

Energy emission rate:

$$\dot{\mathcal{E}}_\nu = 2\int \frac{d^3\boldsymbol{p}_\nu}{(2\pi)^3} p_{\nu,0} \frac{\partial f_\nu(t,\boldsymbol{p}_\nu)}{\partial t},$$

Momentum emission rate:

$$\dot{\mathcal{P}}_{\nu,z} = 2\int \frac{d^3\boldsymbol{p}_\nu}{(2\pi)^3} p_{\nu,z} \frac{\partial f_\nu(t,\boldsymbol{p}_\nu)}{\partial t}.$$

→ indicates that neutrinos are emitted asymmetrically relative to the magnetic field

$$B = 0$$

$$\dot{\mathcal{E}}_\nu^{(\text{Iwamoto})} \simeq \frac{457}{630}\alpha_s G_F^2 \cos^2\theta_C \mu_u \mu_d \mu_e T^6 + O\left(\alpha_s^2, \frac{\mu_e}{\mu_u}\right)$$

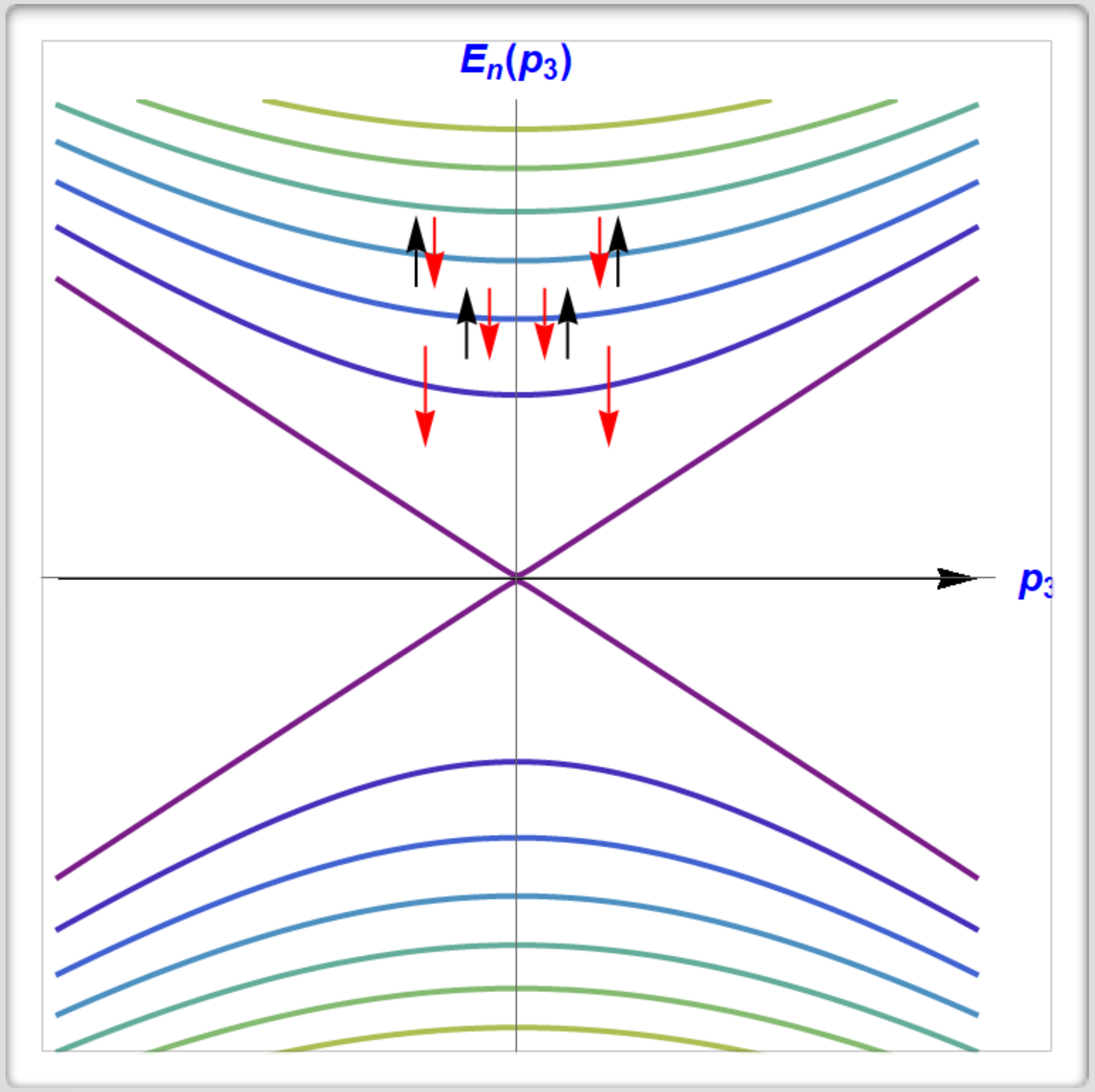
[Iwamoto, Phys. Rev. Lett. 44 (1980) 1637], [Iwamoto, Annals Phys. 141 (1982) 1]

Landau Levels

Relativistic:

$E_{kin} \geq E_{rest}$

$v \sim c$



Dirac Equation

$$\left\{ i\gamma^0 \partial_0 - i\vec{\gamma} \cdot (\nabla - iq\vec{A}) \right\} \Psi = 0$$

Energy spectrum:

$$E_n(p_3) = \pm\sqrt{p_3^2 + 2n|eB| + m^2}$$

$$n = s + k + \frac{1}{2}$$

Spin quantum number: s=± 1/2

Orbital: k=0,1,2,...

[Miranski & Shovkovy, Phys.Rept. 576 (2015) 1-209]

Expression for energy emission $@B \neq 0$

$$\dot{\mathcal{E}}_\nu = \frac{12 N_c G_F^2 \cos^2\theta_C T^5}{\pi^5} v_F \mu_u \mu_d \sum_{n=0}^{\infty} \frac{(-1)^n}{\ell^2} \int_0^\infty \int_0^\infty \frac{\Theta(u,v) du dv}{\sqrt{u}\sqrt{u+v}} \frac{e^{-v}}{e^{\epsilon_n^u}+1} \left(\mathrm{Li}_5\left(e^{\epsilon_n^u}\right) - \frac{\epsilon_n^u}{4}\mathrm{Li}_4\left(e^{\epsilon_n^u}\right)\right)$$
$$\times \left[L_n(2v) - L_{n-1}(2v)\right] \left[1 + \frac{\sqrt{2n+u}}{2\ell\mu_u}\left(1 - \frac{v_F^2 \mu_e (\mu_d+\mu_u)\ell^2}{u+v}\right)\right]$$

$\ell = 1/\sqrt{|eB|}$

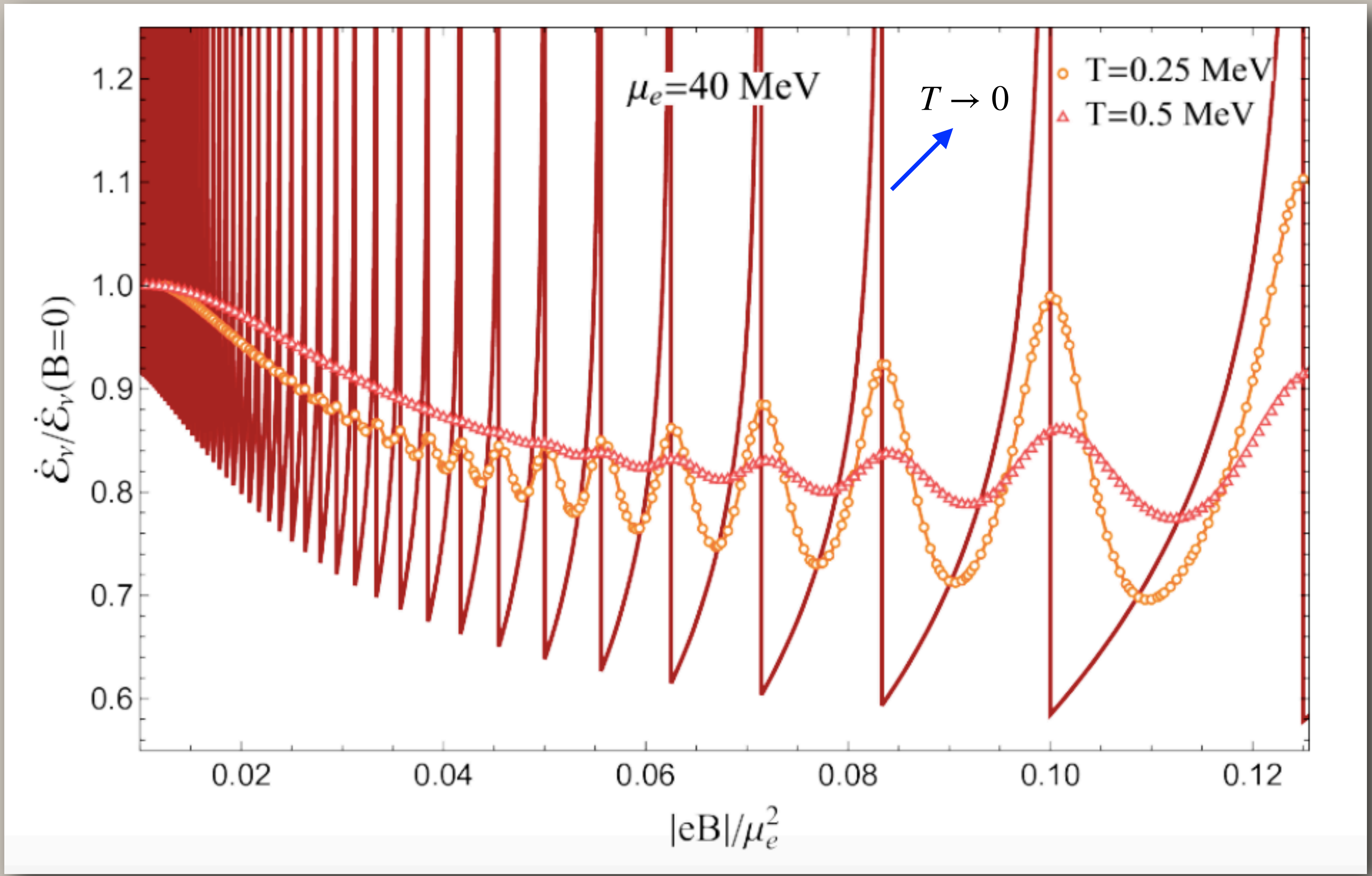
$T \ll |eB|/\mu_e$ (Low temperature approximation) $\quad u_n \equiv \mu_e^2 \ell^2 - 2n$

$$\dot{\mathcal{E}}_\nu^{(0)} = \frac{457\pi N_c G_F^2 \cos^2\theta_C}{5040} v_F \mu_u \mu_d \mu_e T^6 \left(1 + \frac{\mu_e}{2\mu_u}\right)$$
$$\times \sum_{n=0}^{n_{\max}} \frac{(-1)^n}{\sqrt{u_n}} \int \frac{\Theta(u_n, v) e^{-v} dv}{\sqrt{u_n + v}} \left(1 - \frac{v_F^2 \mu_e^2 \ell^2}{u_n + v}\right) \left[L_n(2v) - L_{n-1}(2v)\right]$$

LLL approximation

$$\dot{\mathcal{E}}_\nu^{(\mathrm{LLL})} \simeq \frac{457\pi^{3/2} N_c G_F^2 \cos^2\theta_C}{5040\ell} v_F \mu_u \mu_d T^6 \left(1 + \frac{\mu_e}{2\mu_u}\right)\left(\left(1 + 2v_F^2\mu_e^2\ell^2\right) e^{\mu_e^2\ell^2} \mathrm{erfc}(\mu_e \ell) - \frac{2v_F^2 \mu_e \ell}{\sqrt{\pi}}\right)$$

Energy emission $T \ll |eB|/\mu_e$



$$B = 1.69 \times 10^{14} \frac{|eB|}{\text{MeV}^2} \text{ G.}$$

- Divergences as $T \to 0$

$$|eB|/\mu_e^2 = \frac{1}{2n}$$

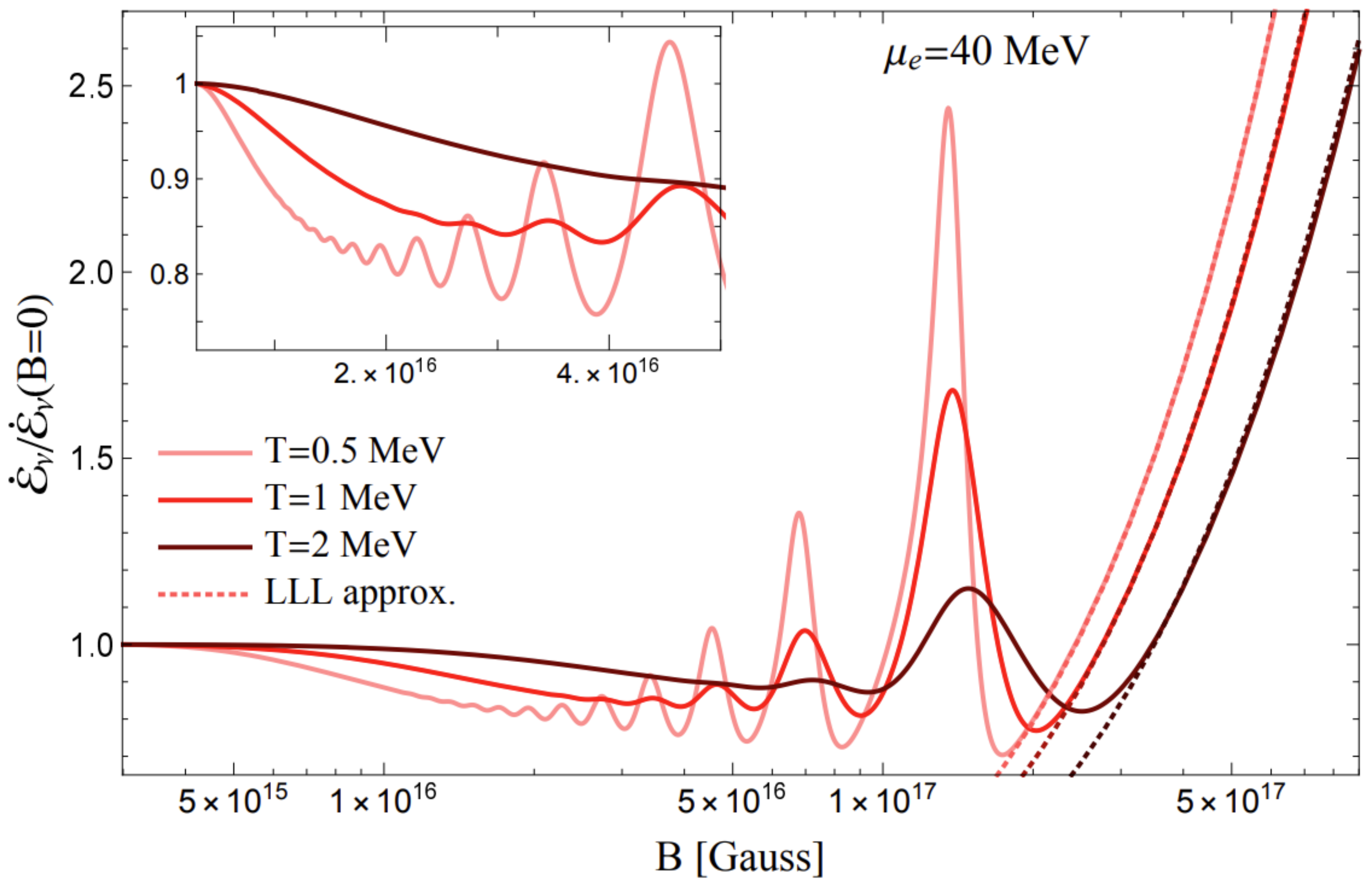
- As T→0, the energy emission rate exhibits a characteristic sawtooth behavior as a function of |eB|/μ , with sharp features appearing at Landau-level thresholds.

- Thermal effects wash out the oscillations, leading to a smoother dependence on the magnetic field

Energy emission

[Ghosh, Shovkovy, J. High Energy Phys. 04 (2025) 110]

$$\dot{\mathcal{E}}_\nu = 2\int \frac{d^3\boldsymbol{p}_\nu}{(2\pi)^3} p_{\nu,0} \frac{\partial f_\nu(t,\boldsymbol{p}_\nu)}{\partial t}$$

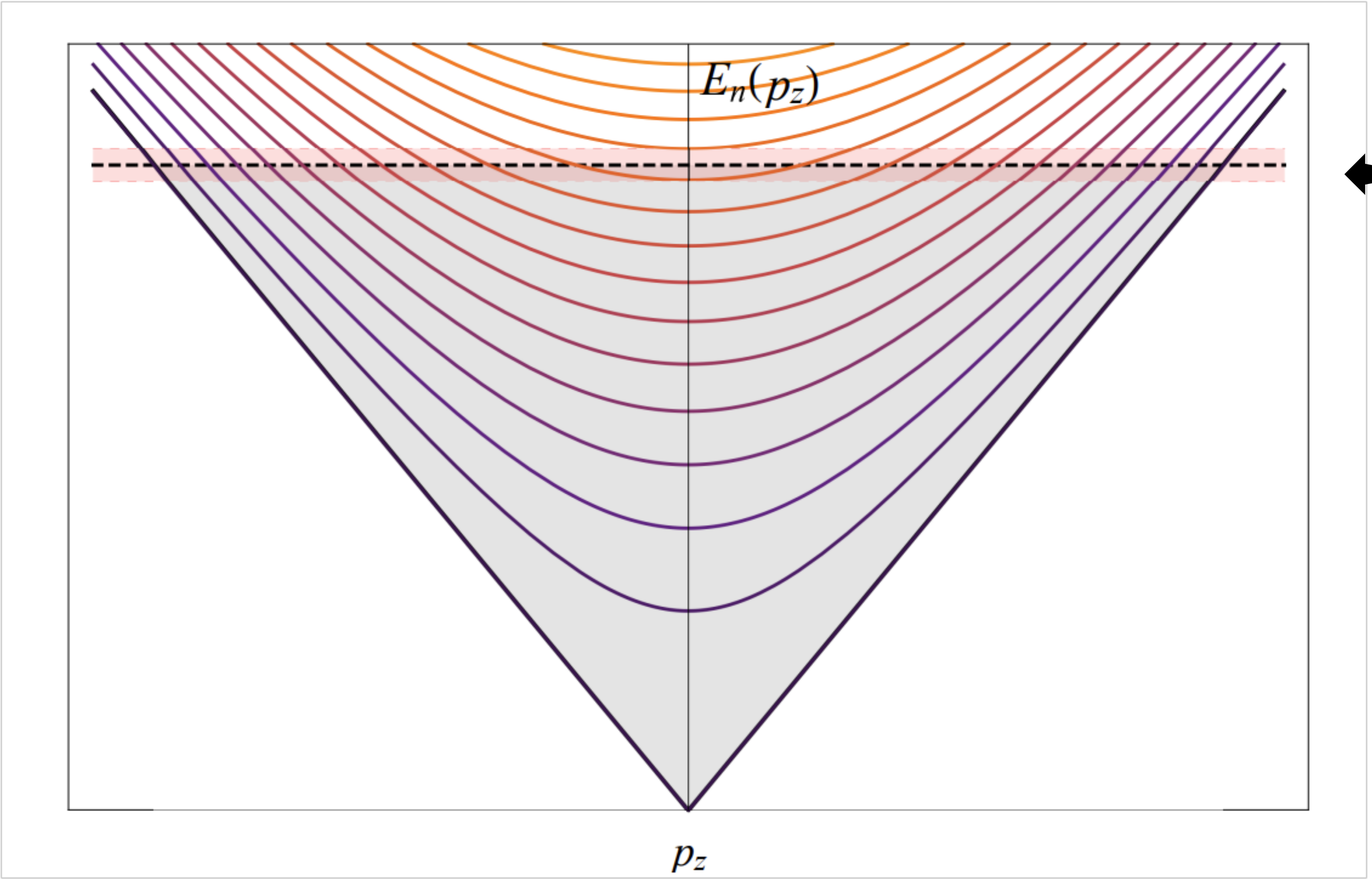


local peaks occur around the threshold values:

$$|eB| = \mu_e^2/(2n)$$

Electron's Landau-level spectrum:

- Fermi surface states associated with the LLL (n = 0) → electron longitudinal momenta are largest in magnitude $|p_{e,z}| \simeq \mu_e$
  - LL index increases → Fermi surface states move towards $p_{e,z} \simeq 0$



Fermi level

$$|eB| = \mu_e^2/(2n)$$

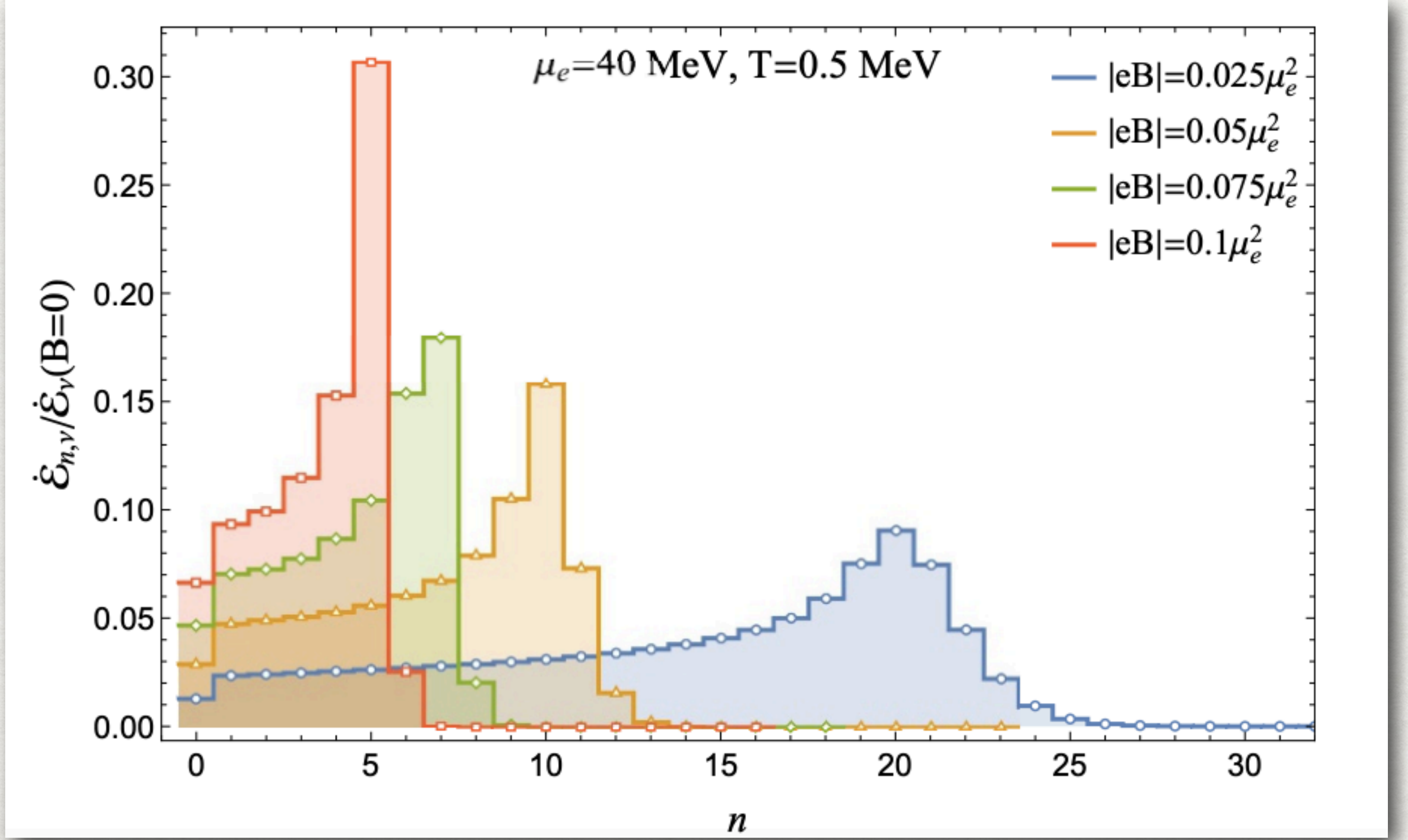
$$E_{e,n} = \sqrt{2n|eB| + p_{e,z}^2 + m_e^2}$$

[Ghosh, Shovkovy, *JHEP* 04 (2025) 110]

LANDAU LEVEL CONTRIBUTIONS TO $\dot{\mathcal{E}}_\nu$

- largest partial contributions originate from electron states at the Fermi surface with the smallest values of $|p_F|$.

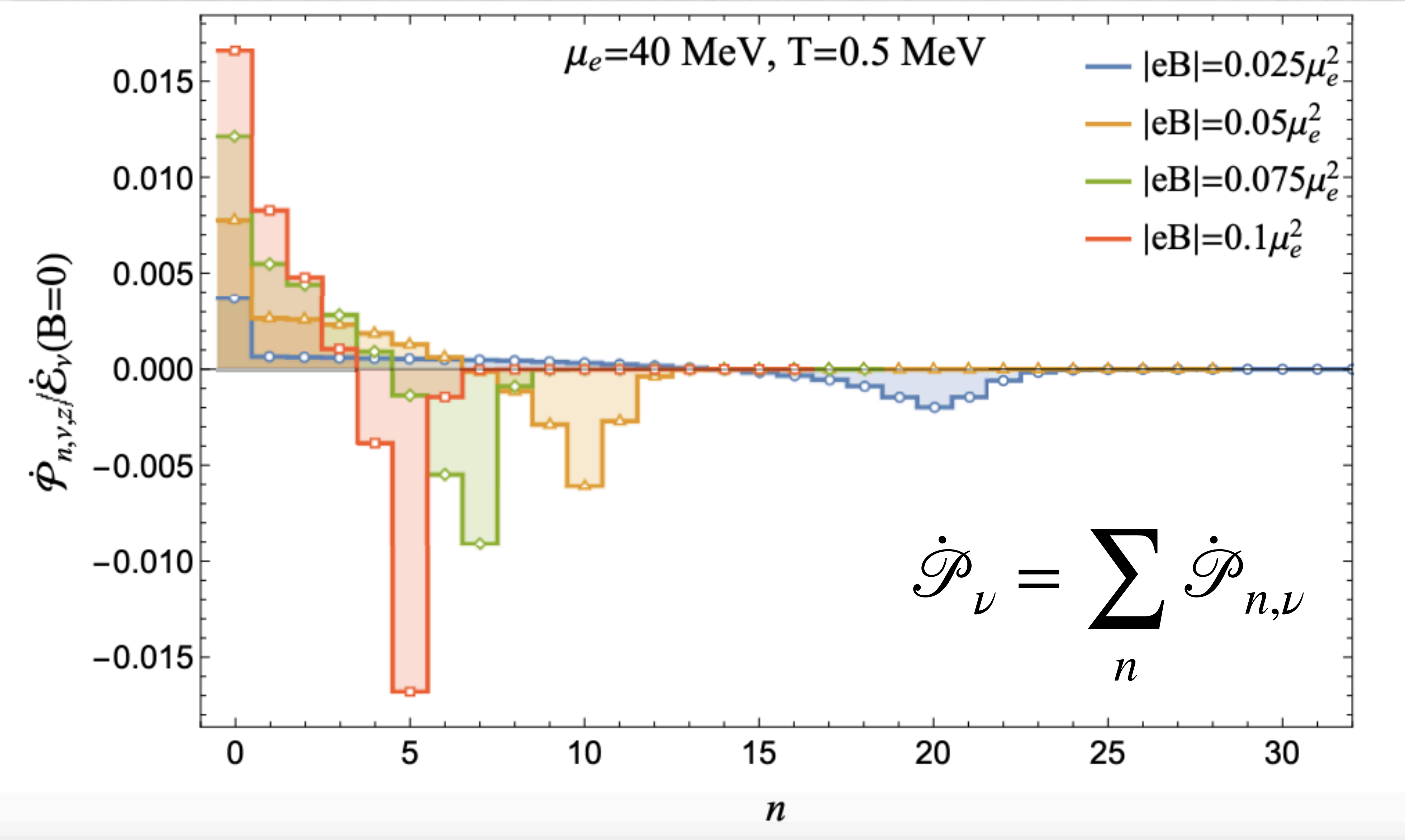
  → states are associated with the Landau level whose energy minimum lies closest to the chemical potential.



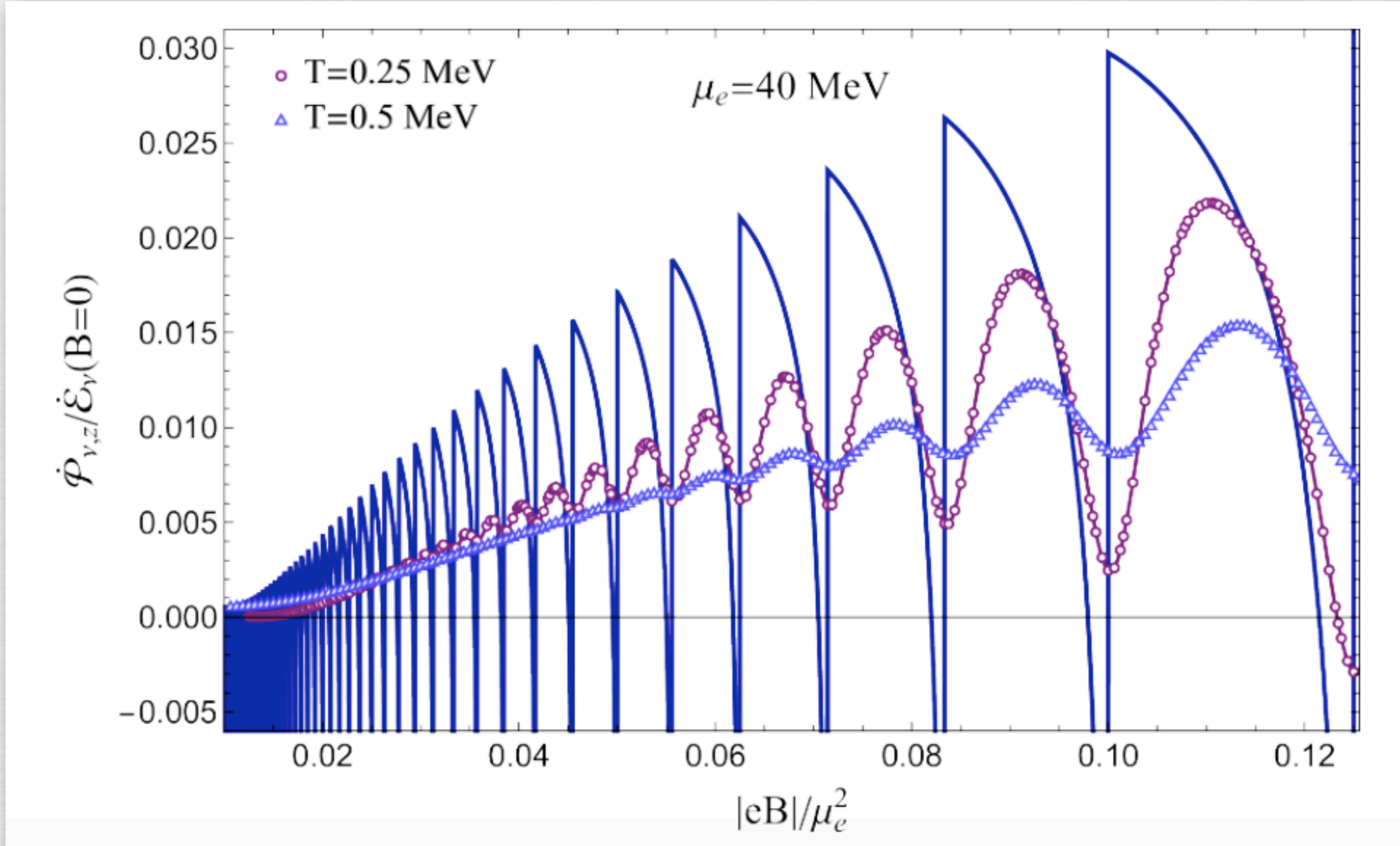
$$\dot{\mathcal{E}}_\nu = \sum_n \dot{\mathcal{E}}_{n,\nu}$$

LANDAU LEVEL CONTRIBUTIONS TO $\dot{\mathscr{P}}_\nu$

- Spin-down polarization of n=0 states → correlates with the emission of nonzero net momentum in B-direction
- higher Landau levels, states with opposite spins drive neutrino emissions with opposing net momenta



Momentum emission $T \ll |eB|/\mu_e$



- Divergences as $T \to 0$

$$|eB|/\mu_e^2 = \frac{1}{2n}$$

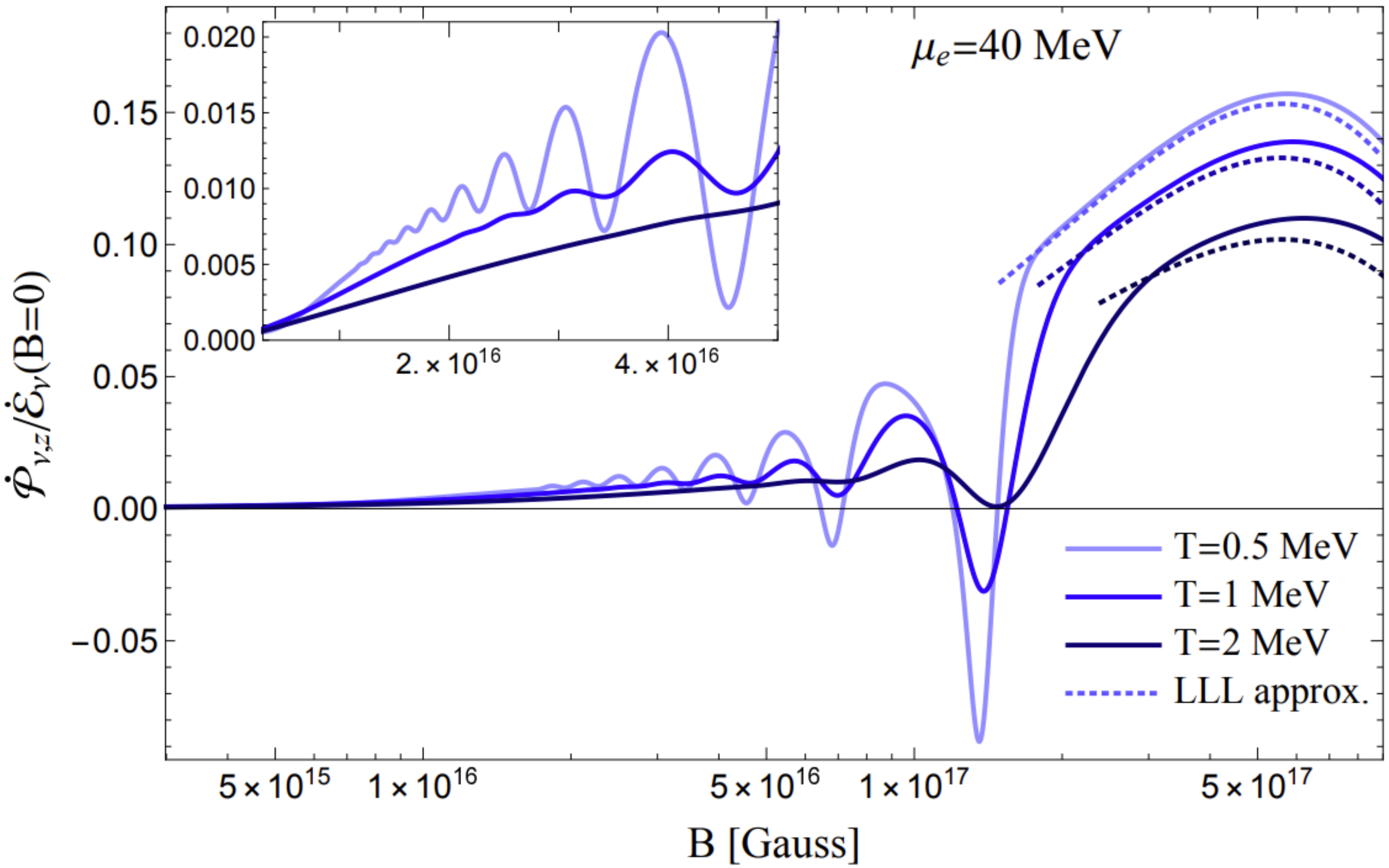
- momentum emission rate does not have a definite sign

$$\dot{\mathcal{P}}_{\nu,z} = 2\int \frac{d^3\boldsymbol{p}_\nu}{(2\pi)^3} p_{\nu,z} \frac{\partial f_\nu(t,\boldsymbol{p}_\nu)}{\partial t}$$

[Ghosh, Shovkovy, J. High Energy Phys. 04 (2025) 110]

Momentum Rate

- indicates that neutrinos are emitted asymmetrically relative to the magnetic field



Can be negative

- the thermal effects tend to completely wash away oscillations

[Ghosh, Shovkovy, *JHEP* 04 (2025) 110]

FERMI-LIQUID CORRECTION

Iwamoto (1980)

$$B = 0$$

Energy–momentum conservation:

$$\vec{k}_d = \vec{k}_u + \vec{k}_e + \vec{k}_{\bar{\nu}}$$

$$k_d \approx v_F \mu_d \qquad k_u \approx v_F \mu_u \qquad k_e \approx \mu_e \qquad k_{\bar{\nu}} \approx T$$

$$T \ll \mu_e \qquad \mu_e \ll \mu_u \lesssim \mu_d$$

- Three momenta $\vec{k}_u, \vec{k}_d, \vec{k}_e$ will be collinear without Fermi-liquid correction

FERMI-LIQUID CORRECTION

$$E_f = \mu_f + v_F(p - p_F)$$

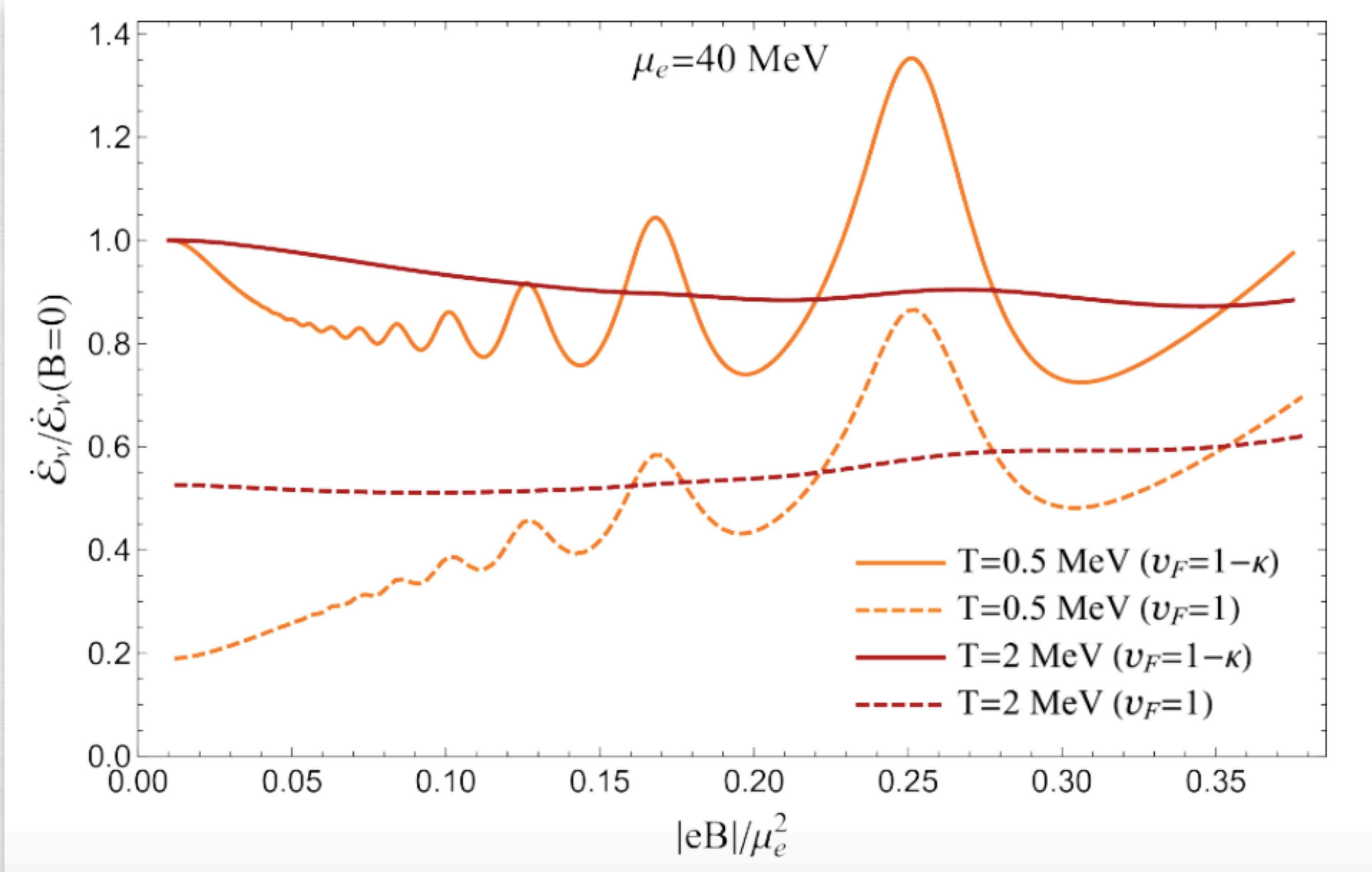
With Fermi velocity $v_F = 1 - 2\alpha_s/(3\pi)$

[Iwamoto, Phys. Rev. Lett. 44 (1980) 1637], [Iwamoto, Annals Phys. 141 (1982) 1]

Interplay of B vs Fermi liquid corrections to $\dot{\mathcal{E}}_\nu$

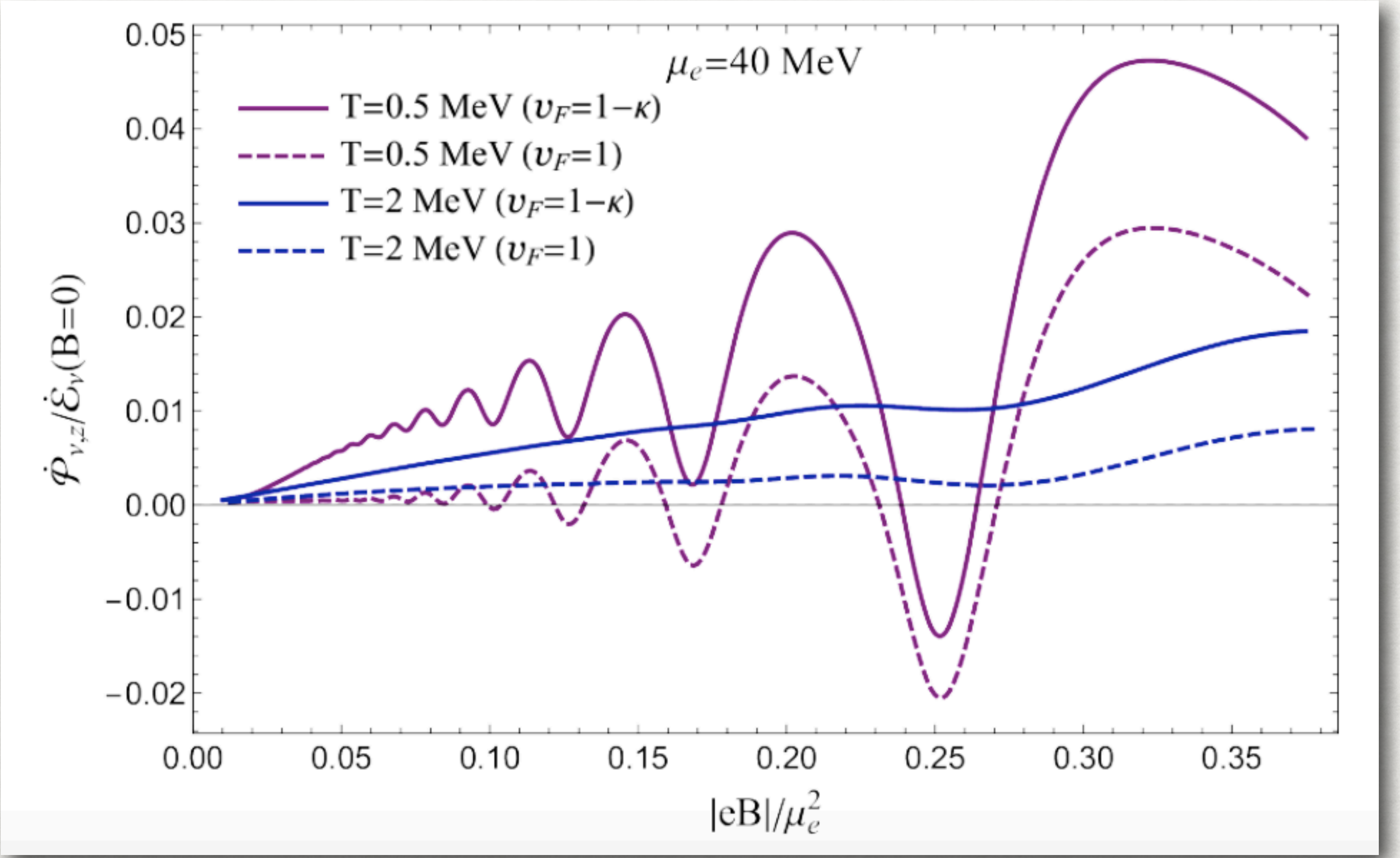
- Weak field → $\dot{\mathcal{E}}_\nu$ smaller when Fermi-liquid corrections are omitted
- magnetic field naturally resolves the phase space restriction even without the inclusion of the Fermi-liquid corrections.

the transverse components of the electron momentum are not conserved quantum numbers any longer



Interplay of B vs Fermi liquid corrections to $\dot{\mathscr{P}}_\nu$

- When the quark Fermi-liquid effects are excluded $\dot{\mathscr{P}}_\nu \rightarrow 0$



PULSAR KICK

- Net non-zero longitudinal momentum emission leads to pulsar kick

- **Estimated pulsar kick velocity:**

$$v_k = \frac{4\pi R_c^3}{3M} \int \dot{\mathcal{P}}_{\nu,z} dt = \frac{4\pi R_c^3}{3M} \int \eta \, \dot{\mathcal{E}}_\nu dt,$$

asymmetry in momentum emission

$$\eta \equiv \frac{\dot{\mathcal{P}}_{\nu,z}}{\dot{\mathcal{E}}_\nu} \sim 2 \times 10^{-3} \frac{|eB|}{\mu_e T}.$$

- **This results in**

$$v_k \simeq 1.9 \text{ km/s} \left(\frac{B}{10^{16} \text{ G}}\right) \left(\frac{M_\odot}{M}\right) \left(\frac{R_c}{10 \text{ km}}\right)^3 \left(\frac{40 \text{ MeV}}{\mu_e}\right) \left(\frac{\mu_f}{300 \text{ MeV}}\right)^2 \left(\frac{\Delta T}{10 \text{ MeV}}\right)$$

SUMMARY AND OUTLOOK

- Neutrino emission from dense quark matter is calculated for a plasma in a strong magnetic field from first principles field theoretical method.

- The Landau-level discretization of electron states at the Fermi surface leads to an oscillatory dependence of the neutrino emission rate on the magnetic field strength.

- Pulsar kick velocity is estimated.

- Future direction:
  -color superconducting phases or other phase?
  - other processes?

THANK YOU