Vorticity induced-effects from WZW terms

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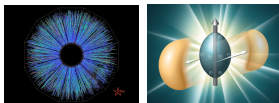
Wess-Zumino-Witten terms

> **Quantum Anomaly**: when a conserved current of the classical theory is not conserved in the quantum theory, or
>
> $$\partial_\mu j^\mu_{\text{classical}} = 0 \quad \Rightarrow \quad \partial_\mu j^\mu_{\text{quantum}} \neq 0 \,.$$

▶ **'t Hooft anomaly matching condition**: anomaly should manifest in the effective field theory [G. 't Hooft, NATO SS B 59, (1980)]

▶ **Chiral anomaly** in QCD $\Rightarrow$ anomalous hadronic interactions described by **Wess-Zumino-Witten (WZW) effective action** [J. Wess and B. Zumino, PLB 37 (1971); E. Witten, NPB 223 (1983)]

▶ Anomalous effects, e.g., chiral magnetic and chiral vortical [K. Fukushima et al., PRD 78, (2008); D. Kharzeev and A. Zhitnitsky, NPA 797, (2007)]

Images: Brookhaven National Laboratory

Vorticity as an axial-vector field

From the two facts:

1. Four-vorticity $\omega^\mu = \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta / 2$ can be regarded as a fictitious spacetime torsion (u^ν is the four-velocity) [A. Shitade and T. Kimura, PRB 90 (2014); Z. V. Khaidukov and M. A. Zubkov, JETP Lett. 108 (2018)]

2. Massless Dirac fermions in a spacetime with torsion ⇔ massless Dirac fermions coupled to an axial-vector field in flat spacetime [I. L. Shapiro, P. Rept. 357 (2002)]

one concludes that *vorticity can be introduced as an emergent axial gauge field*
[N. Yamamoto and D. L. Yang, PRD 103 (2021)]

$$A^\mu \equiv \frac{1}{2}\omega^\mu \, .$$

We may also introduce a chemical potential μ_f for flavour f via

$$\bar{\psi}_f \gamma^0 \mu_f \psi_f \quad \Rightarrow \quad \bar{\psi}_f \slashed{V}_f \psi_f \, ,$$

with fermions ψ_f and vector field V_f^μ [D. T. Son and Ariel R. Zhitnitsky, PRD 70 (2004)].

Goal

Re-derive the WZW terms in external vector, axial-vector, and pseudoscalar fields to determine the anomalous effects induced by vorticity.

How?

Using a derivative expansion of the fermion determinant starting from a linear sigma model [I. J. R. Aitchison and C. M. Fraser, PRD 31 (1985)].

Linear sigma model

We start from

$$\mathcal{L} = \bar{\psi} \left(i\slashed{\partial} - \slashed{V} + \gamma^5 \slashed{A} - m e^{i\theta\gamma^5} \right) \psi \,,$$

where m is the fermion mass and θ is a pseudoscalar field which we decompose as

$$\theta = \Pi + \phi \,.$$

For Abelian A^μ, it has the *local* axial symmetry

$$\psi \to e^{-i\lambda\gamma^5}\psi \,, \qquad A_\mu \to A_\mu + \partial_\mu\lambda \,, \qquad \phi \to \phi + 2\lambda \,,$$

and a vector-gauge symmetry under which ψ, V^μ and Π transform.

Derivative expansion of fermion determinants (1/2)

▶ Integrate out the fermions for the effective action Γ

$$e^{i\Gamma} = \int D\psi D\bar{\psi} \, \exp\left(i \int d^4x \mathcal{L} \right),$$

$$\Rightarrow \Gamma = -i \, \text{tr} \ln \left[\slashed{p} - \slashed{V} + \gamma^5 \slashed{A} - m e^{i\theta\gamma^5} \right],$$

where p^μ is the momentum operator

▶ Expand around $V_\mu = A_\mu = \theta = 0$

$$\Gamma = -i \, \text{tr} \ln \left(\slashed{p} - m \right) + i \, \text{tr} \frac{1}{\slashed{p} - m} \tilde{M} + \frac{i}{2} \, \text{tr} \frac{1}{\slashed{p} - m} \tilde{M} \frac{1}{\slashed{p} - m} \tilde{M} + \dots,$$

where $\tilde{M} = \slashed{V} - \gamma^5 \slashed{A} + m(e^{i\theta\gamma^5} - 1)$

Derivative expansion of fermion determinants (2/2)

- We want to extract the effective Lagrangian $\mathcal{L}_{\text{eff}}$ via

$$\Gamma = \int \mathrm{d}^4 x \, \mathcal{L}_{\text{eff}}$$

- Write functional trace as momentum and position space integral

- Use commutation relations $[p^\mu, \phi] = \mathrm{i}\partial^\mu \phi$ and

$$\left[\phi, \frac{1}{p^2 - m^2}\right] = \frac{1}{(p^2 - m^2)^2} \left[p^2, \phi\right] + \frac{1}{(p^2 - m^2)^3} \left[p^2, \left[p^2, \phi\right]\right] + \ldots,$$

where $\phi(x)$ is some field operator, to shift all momentum operators to one side and integrate in the momentum

What do we get?

To fourth order, we get

$$
\begin{aligned}
\Gamma_{\text{WZW}} = & \frac{\mathrm{i}N_c}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \int \mathrm{d}^4x \, \mathrm{Tr}_f \Big(-2\mathrm{i}\partial_\mu\theta \left(3\partial_\nu V_\alpha V_\beta + \partial_\nu A_\alpha A_\beta\right) \\
& - \partial_\mu\theta\partial_\nu\theta\partial_\alpha\theta V_\beta + 6(\partial_\mu\theta V_\nu V_\alpha V_\beta + \theta V_\nu \partial_\mu V_\alpha V_\beta) \\
& + 2\theta\partial_\mu V_\nu A_\alpha A_\beta - 2\theta V_\nu A_\alpha \partial_\mu A_\beta + 2\partial_\mu\theta A_\nu V_\alpha A_\beta \\
& - 6\theta A_\nu \partial_\mu V_\alpha A_\beta - 2\theta\partial_\mu A_\nu A_\alpha V_\beta + 2\theta A_\nu A_\alpha \partial_\mu V_\beta \\
& + 2[\partial_\mu\theta, \theta]\{\partial_\nu V_\alpha, A_\beta\} + 2(\partial_\mu\theta\partial_\nu V_\alpha\theta - \theta\partial_\nu V_\alpha\partial_\mu\theta)A_\beta \\
& + \theta\theta[\partial_\nu V_\alpha, \partial_\mu A_\beta] \Big) + \dots
\end{aligned}
$$

which agrees with previous results, where N_c is the number of colours, Tr_f is the trace over flavour space [Ö . Kaymakcalan et al., PRD 30,(1984); J. L. Mañes, Nucl. Phys. B 250, 369 (1985); N. K. Pak and P. Rossi, Nucl. Phys. B 250 (1985) and many others . . .].

Abelian case and the chiral anomaly

▶ Specialising to $V_\mu = eQV_\mu^Q$, with V_μ^Q and A_μ Abelian,

$$\Gamma = \frac{N_c}{8\pi^2} \epsilon^{\mu\nu\alpha\beta} \int \mathrm{d}^4 x \, \mathrm{Tr}_f \left\{ \partial_\mu (\Pi + \phi)(e^2 Q^2 \partial_\nu V_\alpha^Q V_\beta^Q + \frac{1}{3} \partial_\nu A_\alpha A_\beta) \right.$$
$$\left. - \frac{\mathrm{i}e}{3} Q \partial_\mu (\Pi + \phi) \, \partial_\nu \Pi \partial_\alpha \Pi V_\beta^Q + \frac{\mathrm{i}e}{3} Q[\partial_\mu \Pi, \Pi] \partial_\nu V_\alpha^Q A_\beta \right\} ,$$

where we've decomposed $\theta = \Pi + \phi$ and Q is a charge matrix

▶ We reproduce the anomaly;

$$\partial_\mu j_V^\mu = 0 ,$$
$$\partial_\mu j_A^\mu = \frac{N_c}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} \left(\frac{e^2}{4} F_{\mu\nu}^V F_{\alpha\beta}^V \, \mathrm{Tr}_f \, Q^2 + \frac{N_f}{12} F_{\mu\nu}^A F_{\alpha\beta}^A \right) ,$$

where $j_{V/A}^\mu$ is the vector/axial current with field strength tensor $F_{\mu\nu}^{V/A}$ and N_f is the number of flavours [W. A. Bardeen, Phys. Rev. 184 (1969)]
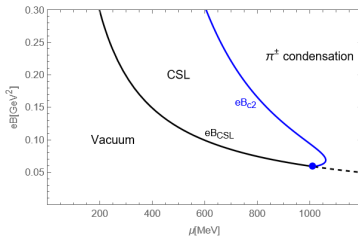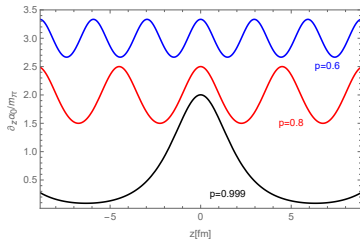
Neutral pions at large magnetic field

▶ Focus on pions → $N_f = 2$, $N_c = 3$

▶ When $A_\mu = 0$, we can extract

$$\mathcal{L}_{\text{baryon}} = \frac{e}{4\pi^2 f_\pi} \mu_B \nabla \pi_0 \cdot \boldsymbol{B} ,$$

$$\mathcal{L}_{\text{isospin}} = \frac{e}{8\pi^2 f_\pi} \mu_I \nabla \pi_0 \cdot \boldsymbol{B} ,$$

where $\mu_{B/I}$ is the baryon/isospin chemical potential and f_π is the pion decay constant

▶ Responsible for Chiral Soliton Lattice (CSL) [T. Brauner and N. Yamamoto, JHEP 4 (2017); M. S. Grønli and T. Brauner, Eur. Phys. J. C 82 (2022)]

Anomalous quantities for charged pions
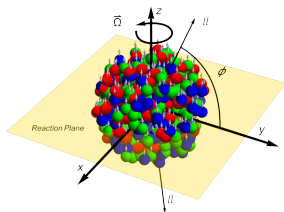
When $A_\mu = \omega_\mu/2$, we can extract

$$\mathcal{L}_{\pi\pi V\omega} = -\frac{\mathrm{i}e}{16\pi^2 f_\pi^2}\epsilon^{\mu\nu\alpha\beta}(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+)F^V_{\nu\alpha}\omega_\beta \, .$$

Assuming constant vorticity, we obtain the anomalous charge and angular momentum density;

$$n = -\frac{i}{4\pi^2 f_\pi^2}(\nabla\pi^+ \times \nabla\pi^-)\cdot\boldsymbol{\omega} \, ,$$

$$\boldsymbol{J} = -\frac{en_I}{8\pi^2 f_\pi^2}\boldsymbol{B} \, ,$$

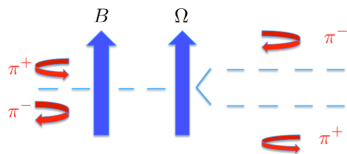respectively, where $n_I = j_I^0$ and j_I^μ is the non-anomalous isospin current.

Charged pions in electromagnetic fields and vorticity

[M. Wei et al., PRD 105, (2022)]

▶ Vorticity-modified photon-pion coupling

▶ Could modify the current-current correlators associated with photon and dilepton production

▶ Rotation causes split in charged pion Landau Levels $\Rightarrow$ charged pion condensation

▶ Anomalous magneto-vortical contribution?

[Y. Liu and I. Zahed, PRL 120 (2018)]

Summary and Outlook

▶ Re-derived the WZW effective action from a linear sigma model using a derivative expansion of the fermion determinant

▶ New anomalous terms involving vorticity from axial-vector field correspondence $\Rightarrow$ anomalous charge and angular momentum density

▶ Phenomenological implications e.g. modifying the photon and dilepton production in a pion gas at finite temperature and/or density [B. Singh et al., PRD 100 (2019); M. Wei et al., PRD 105 (2022); L. Dong and S. Lin, EPJ. A 58 (2022)]

▶ No explicit chiral vortical or helical magnetic effect [X. G. Huang et al. JHEP 02 (2018), N. Yamamoto and D. L. Yang, PRD 103 (2021)]

▶ Anomalous effects in superfluid nuclear/quark matter? [D. T. Son and A. R. Zhitnitsky, PRD **70** (2004)]