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Diagnosing electron-neutrino lepton number crossings and their sensitivity to supernova core physics

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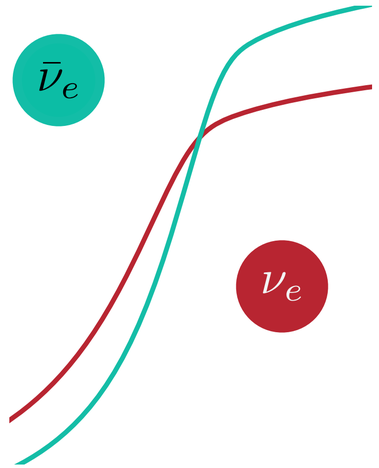
Collective Neutrino Oscillations
in Supernovae and Neutron Star Mergers
March 23 2026



Outline

- Introduction
- Part I: How ELN crossings are affected by the supernova core physics
- Part II: Comparison of methods that diagnose ELN crossings

Based on the papers:

Cornelius et al., *Phys. Rev. D* **112** 063006 (2025)
Cornelius et al., *Phys. Rev. D* **112** 063004 (2025)

Collaborators:

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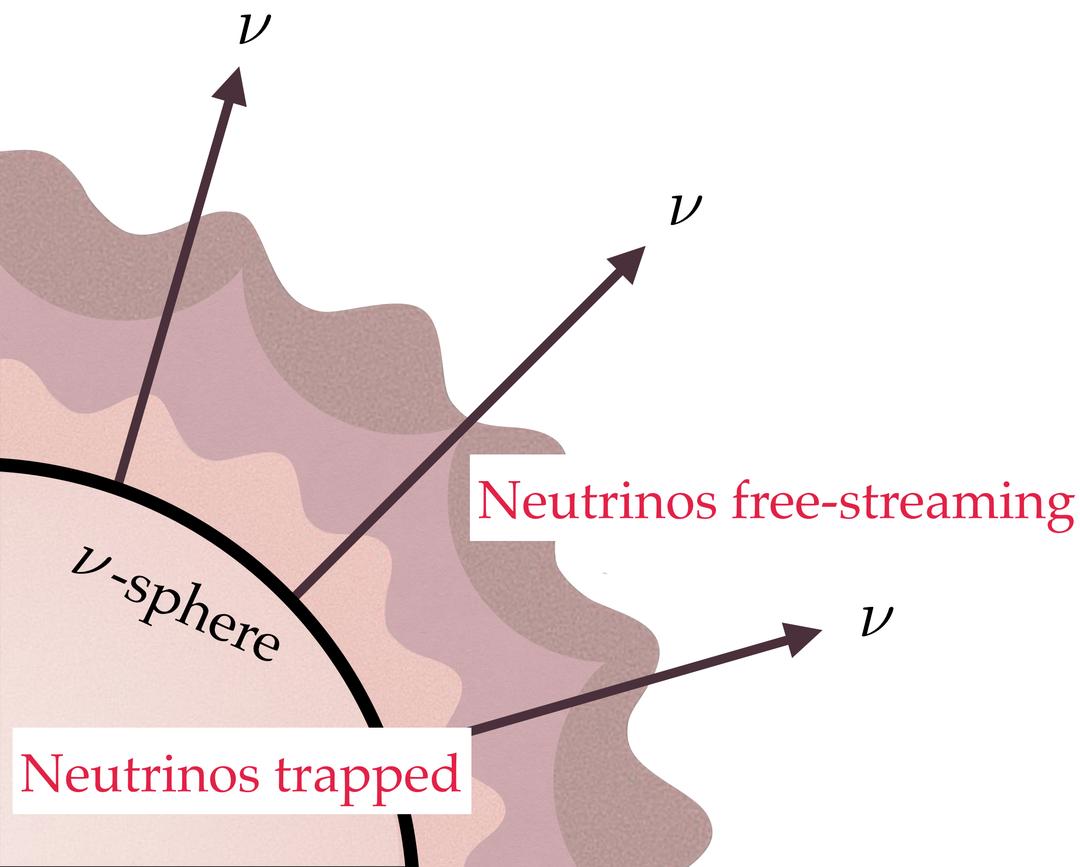
Shashank Shalgar (NBI)
Hans-Thomas Janka (MPA Garching)

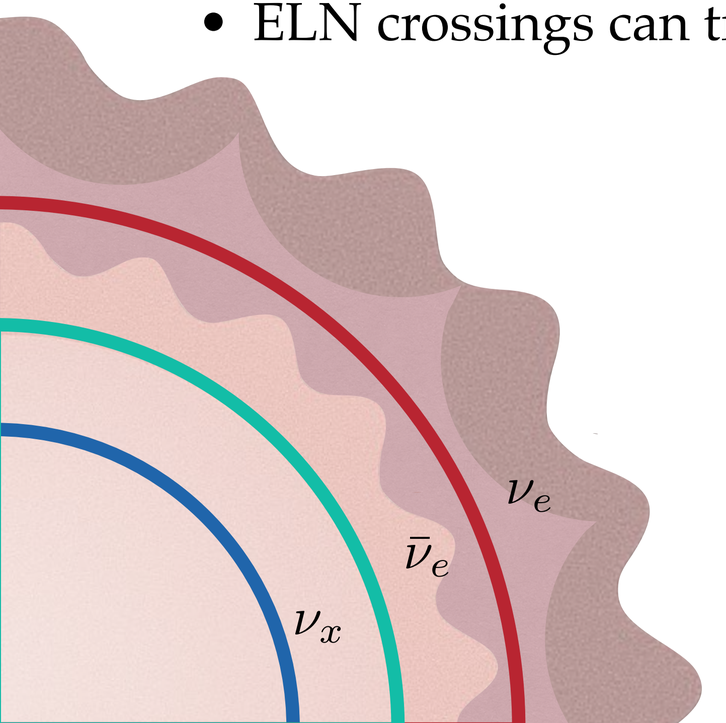
Neutrino decoupling from the supernova core

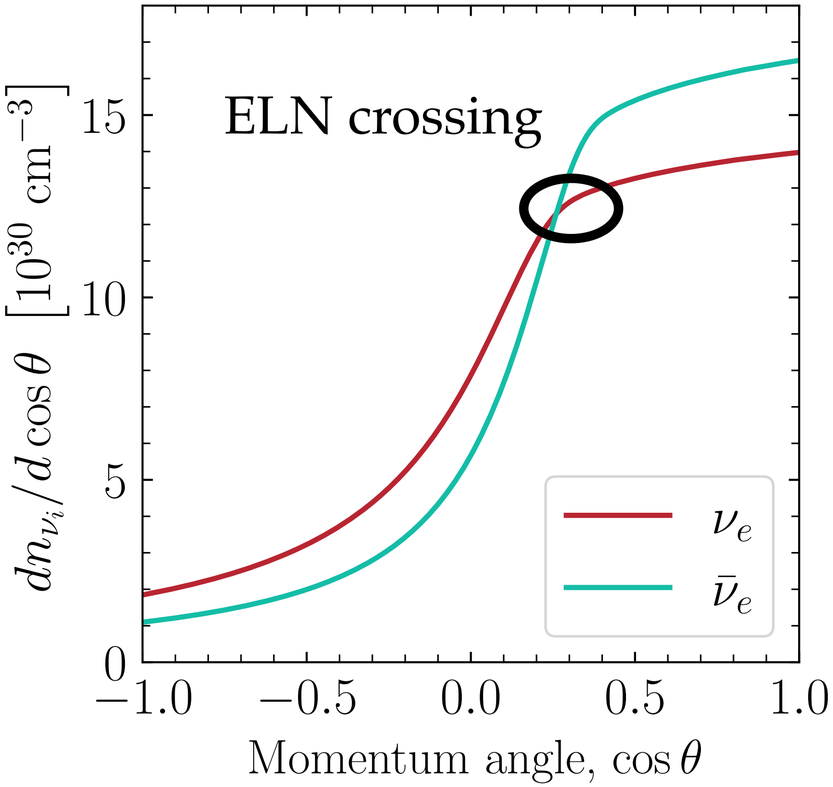
- Neutrinos decouple from matter at radii depending on flavor and energy



Neutrino decoupling from the supernova core

- Neutrinos decouple from matter at radii depending on flavor and energy
- This can lead to **electron-neutrino lepton number (ELN) crossings**
- ELN crossings can trigger fast flavor conversion





Part I

Variations in the existence of ELN
crossings with the supernova core physics

Supernova models

We explore 12 spherically symmetric SN models of 18.6 $M_\odot$ (Prometheus-Vertex)

- 3 equations of state (EoS): LS220, SFHo, DD2
- With (+m) and without muons
- With (+c) and without proto-neutron star (PNS) convection

How is the existence of crossings affected by the microphysics and PNS convection in the SN core?

1.61-LS220 (benchmark)
1.61-LS220+c
1.61-LS220+m
1.62-LS220+c+m
1.62-SFHo
1.62-SFHo+c
1.62-SFHo+m
1.62-SFHo+c+m
1.62-DD2
1.62-DD2+c
1.62-DD2+m
1.62-DD2+c+m

Fiorillo, Heinlein et al., *PRD* **108** 083040 (2023)
Cornelius et al., *PRD* **112** 063006 (2025)

Boltzmann equation for neutrino transport in 1D

Advection

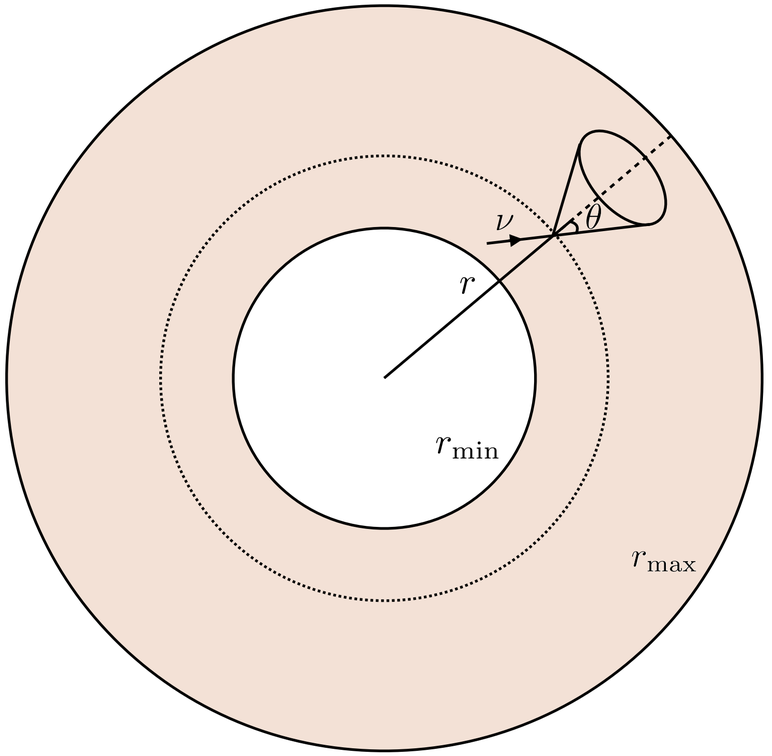
Density matrix

Production, absorption, and scattering of neutrinos

$$\left(\frac{\partial}{\partial t} + \vec{c}\cdot\vec{\nabla}\right)\rho(r, \cos\theta, E, t) = \mathcal{C}[\rho(r, \cos\theta, E, t)]$$

ν_e, ν_x

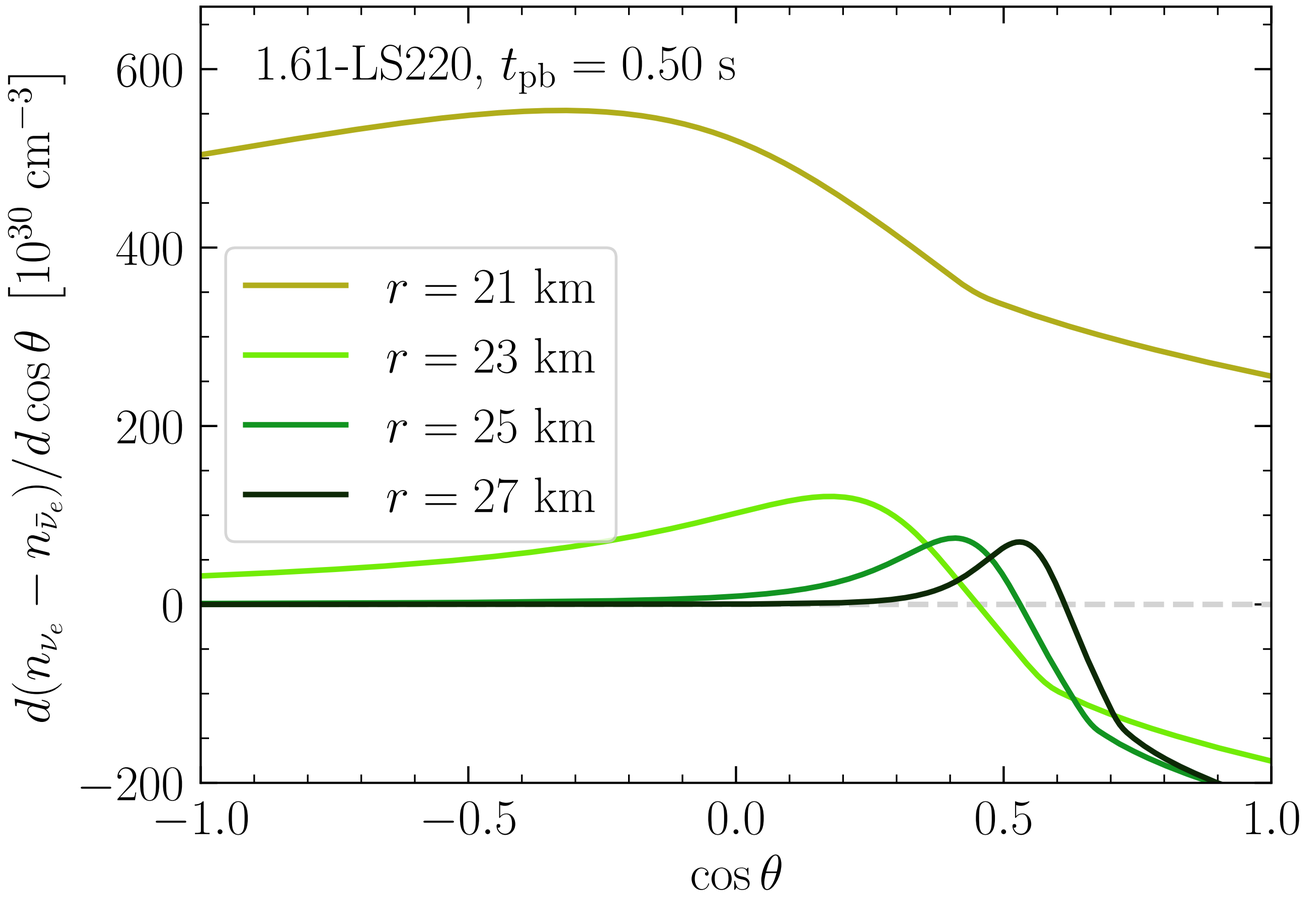
Based on static fluid quantities (e.g. temperature, density, Y_e)



Shalgar & Tamborra, *PRD* **109** 103011 (2024)

Temporal and spatial location of ELN crossings

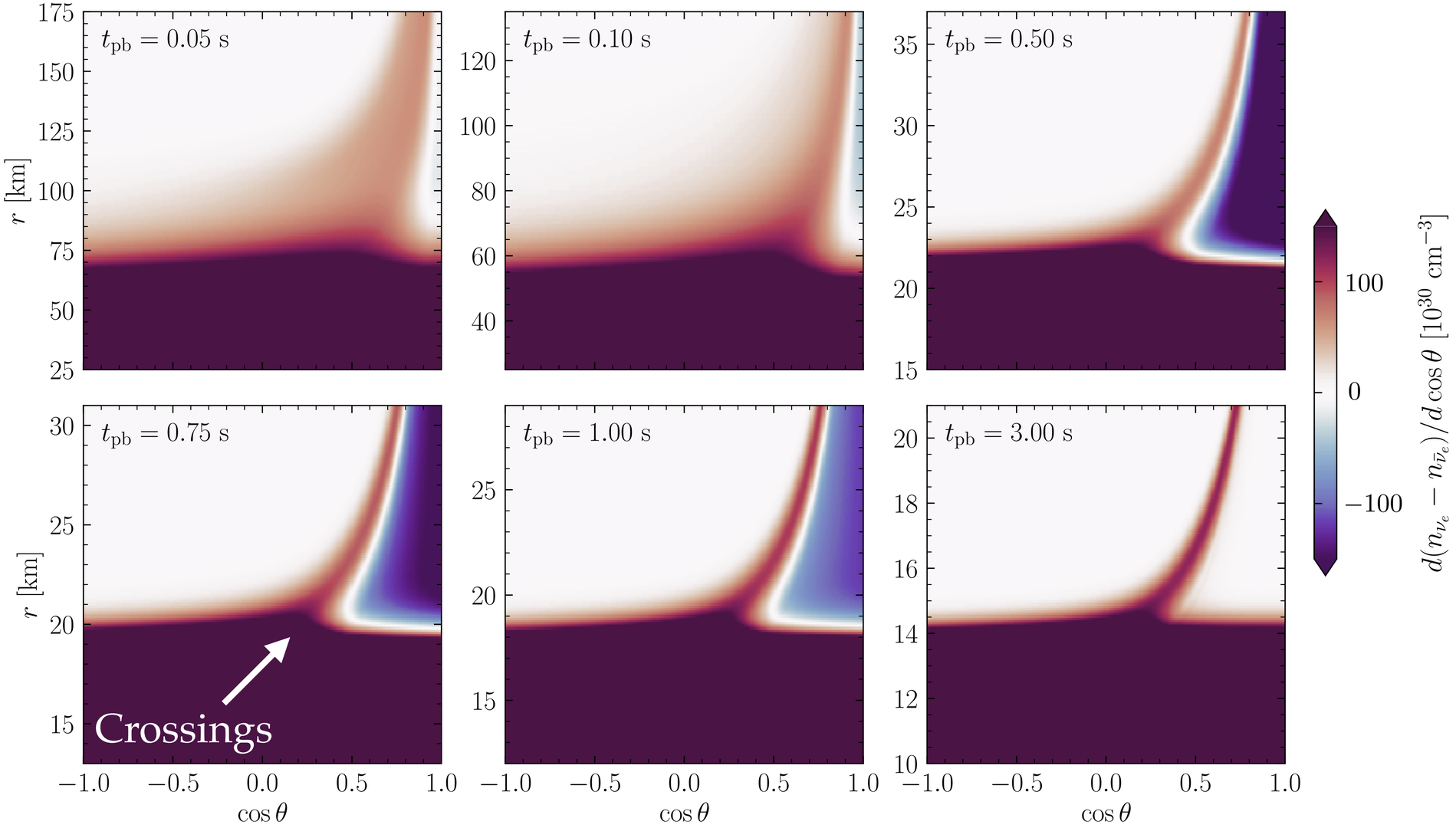
As neutrinos decouple from the medium, their distributions become more forward peaked



Cornelius et al., *PRD* **112** 063006 (2025)

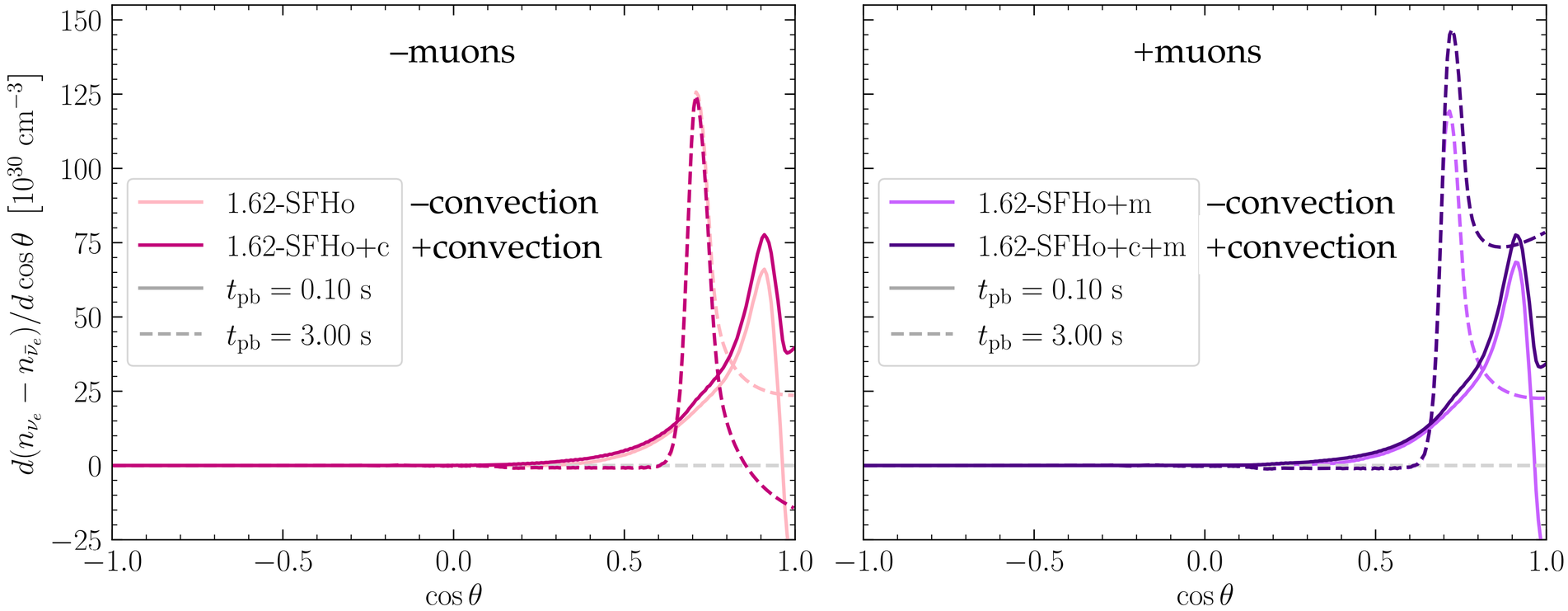
Temporal and spatial location of ELN crossings

Strength of ELN crossings varies with time



Cornelius et al, *PRD* **112** 063006 (2025)

Impact of muons and PNS convection

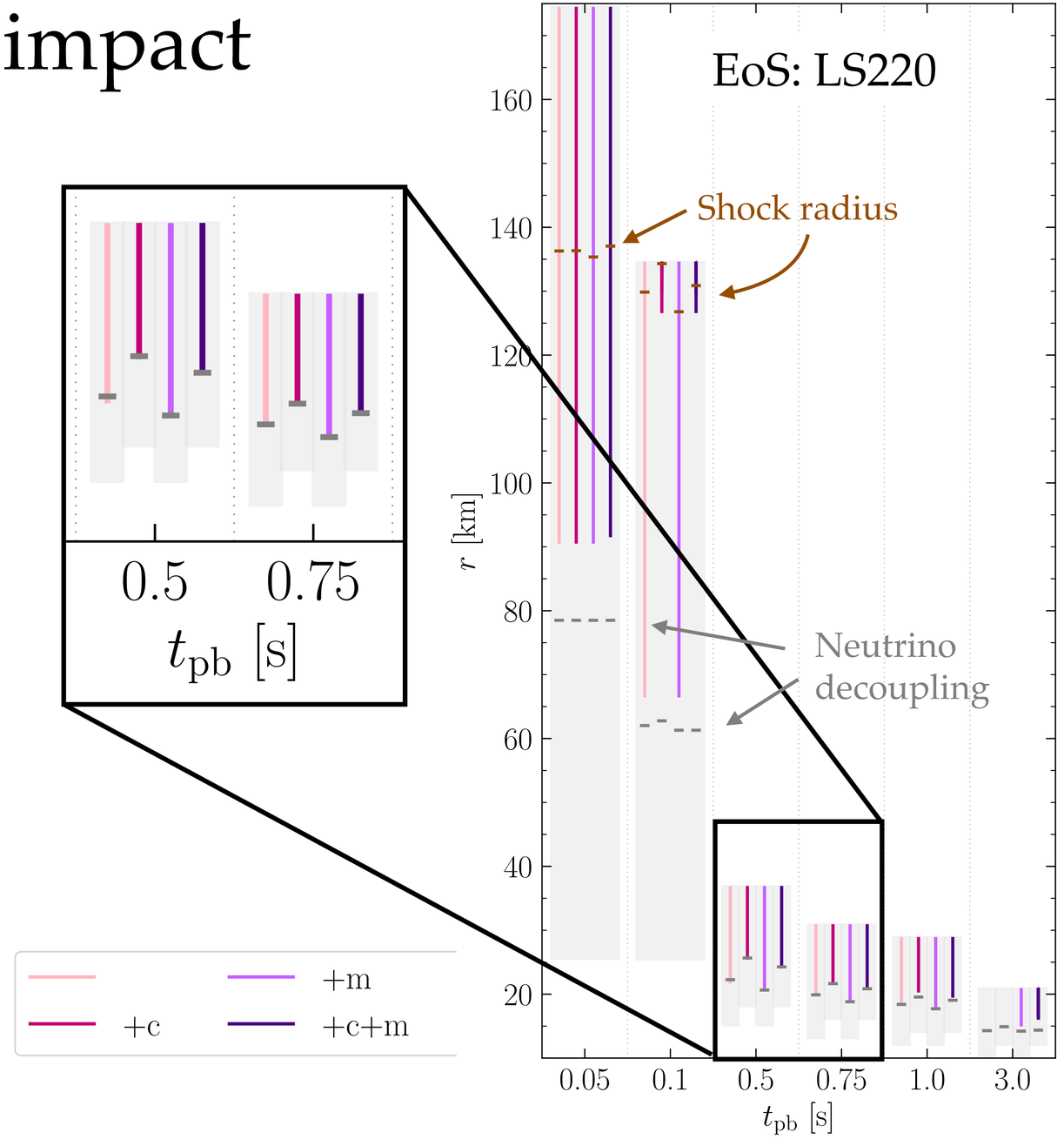


- PNS convection broadens the ELN distribution
- Muons have a minor impact on the shape

Cornelius et al, *PRD* **112** 063006 (2025)

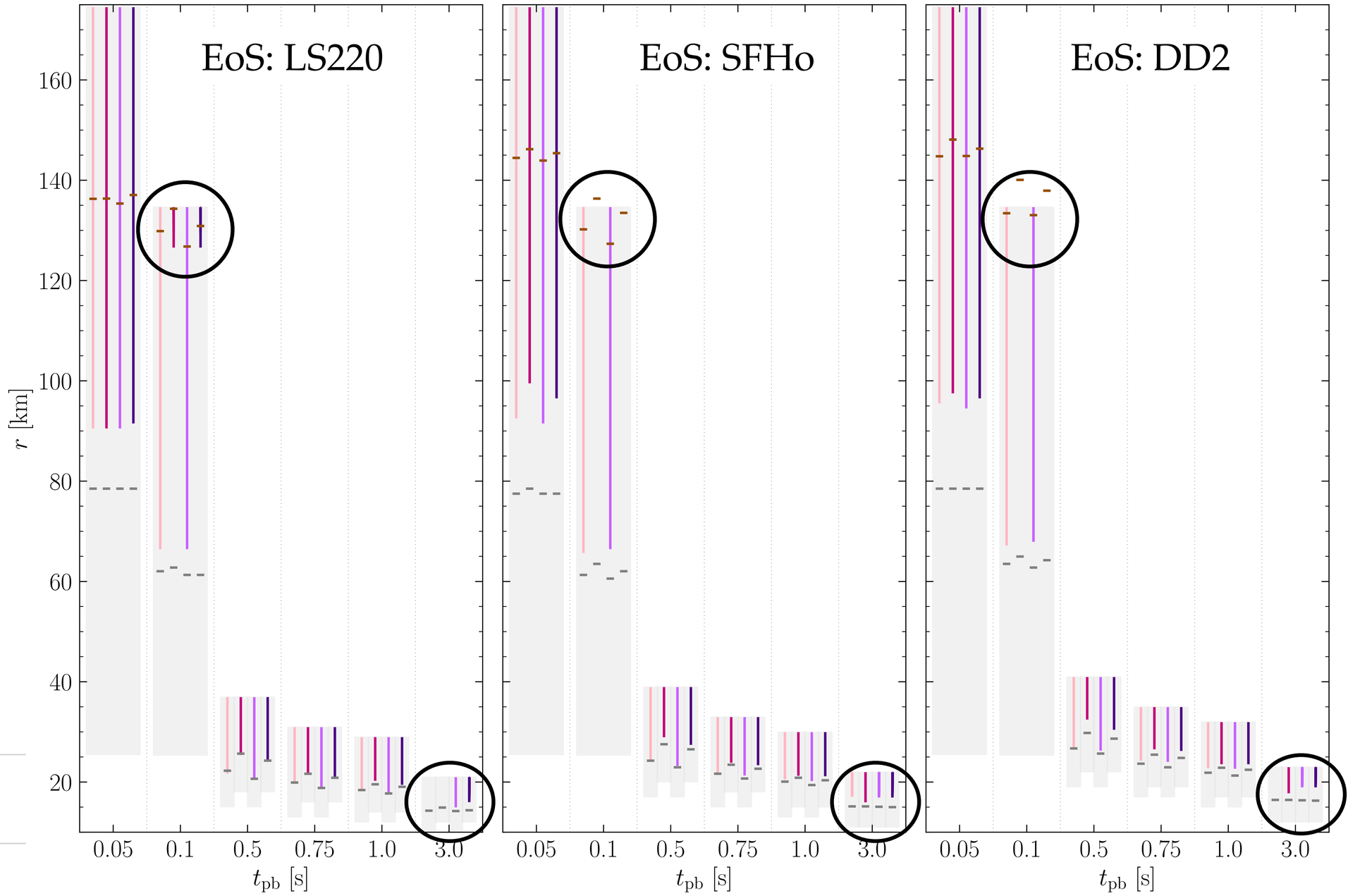
Muon and PNS convection impact

- PNS convection shifts the PNS radius outward → crossings at larger radii
- Muons facilitates contraction of the PNS → crossings at smaller radii



EoS impact

- Subtle differences between the EoS



Cornelius et al, *PRD* **112** 063006 (2025)

Conclusions Part I

How is the existence of crossings affected by the microphysics and PNS convection in the SN core?

- Subtle differences between EoS
- PNS convection shifts PNS radius outward → crossings at larger radii
- Muons facilitates contraction of PNS → crossings at smaller radii

Part II

Diagnosing electron-neutrino lepton number crossings in core-collapse supernovae

Methods for neutrino transport in 1D

Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \vec{c}\cdot\vec{\nabla}\right)\rho(r,\cos\theta,E,t) = \mathcal{C}[\rho(r,\cos\theta,E,t)]$$

Angular dependence

— but expensive

Angular moments

Number density, $n_{\nu_i}(r) = \frac{2\pi}{hc^3}\int dE E^2 \int d\mu f_{\nu_i}(r,\cos\theta,E)$

Flux, $F_{\nu_i}(r) = \frac{2\pi}{hc^3}\int dE E^2 \int d\mu\mu f_{\nu_i}(r,\cos\theta,E)$

No angular dependence

— but cheap

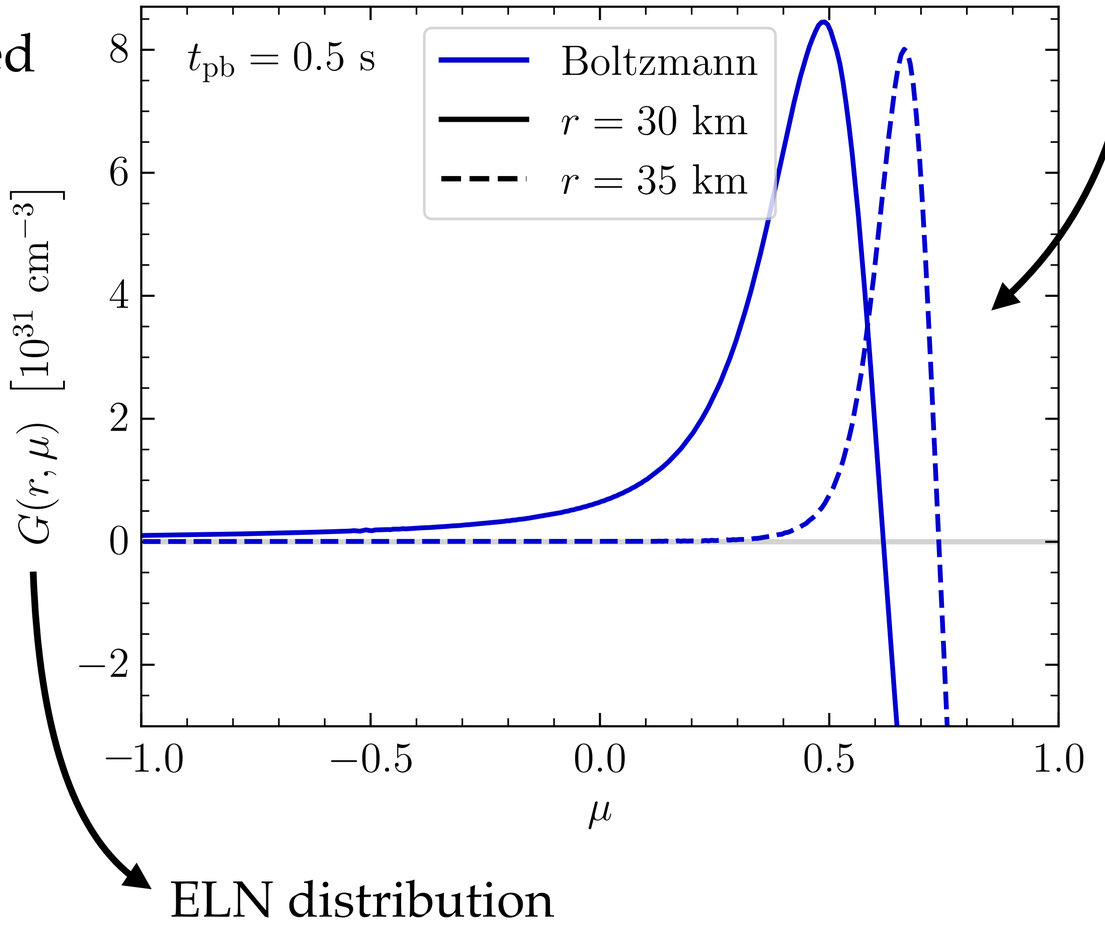
How well can we model ELN crossings without angular information?

Modeling ELN angular distributions

(EoS: LS220)

Boltzmann: angular distributions forward peaked

Forward-peaked distributions



ELN distribution

Cornelius et al., *PRD* **112** 063004 (2025)

Modeling ELN angular distributions

(EoS: LS220)

Forward-peaked distributions

Boltzmann: angular distributions forward peaked

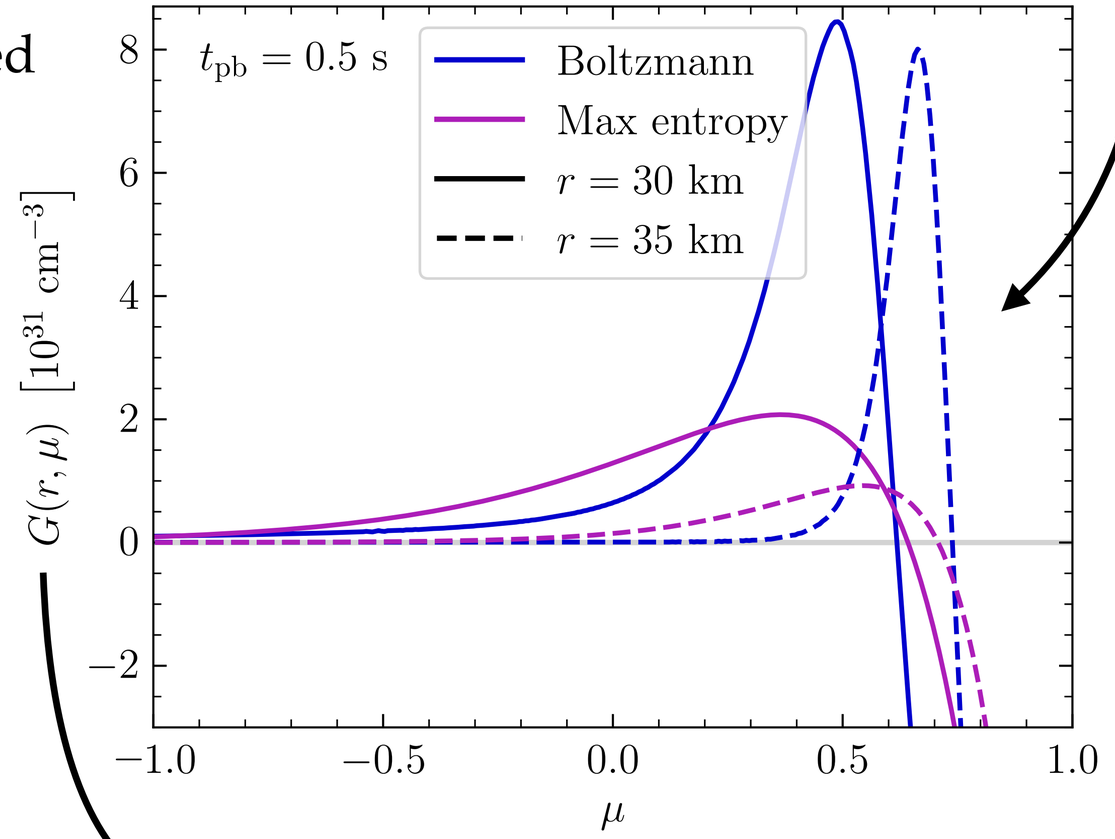
Moment-based methods:

- **Maximum entropy**: somewhat forward peaked

$$f^{\mathrm{ME}}_{\nu_i}(r,\mu) = \frac{n_{\nu_i}(r)}{4\pi}\frac{Z}{\sinh(Z)}e^{Z\mu}$$

$n_{\nu_i}(r)$
$F_{\nu_i}(r)$

Minerbo 1978
Cernohorsky & Bludman 1978
Richers 2022



ELN distribution

Cornelius et al., *PRD* **112** 063004 (2025)

Modeling ELN angular distributions

(EoS: LS220)

Boltzmann: angular distributions forward peaked

Moment-based methods:

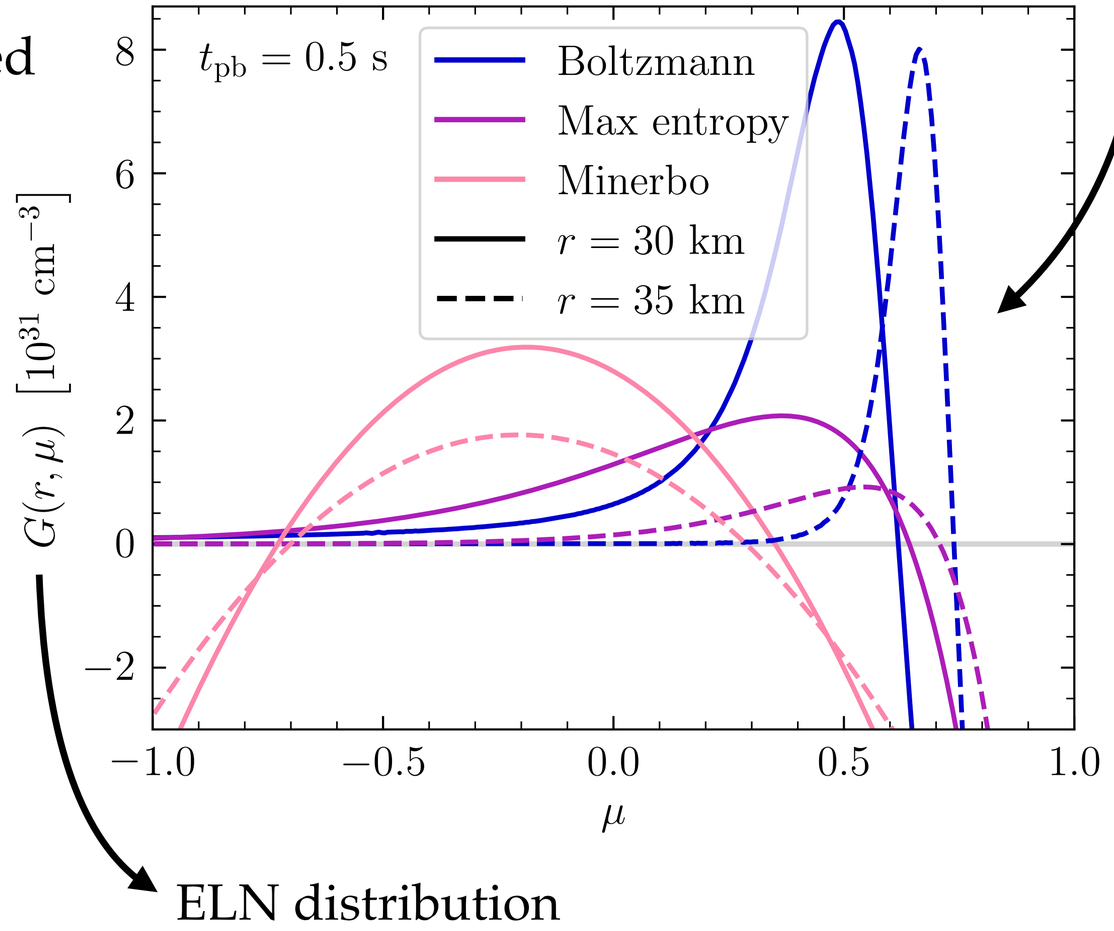
- Maximum entropy: somewhat forward peaked
- **Minerbo closure**: backward peaked

$$f_{\nu_i}^{\mathrm{Minerbo}}(\mu) = a_0 + a_1\mu + a_2\mu^2$$

(a_n coefficients determined by moments)

Minerbo 1978
Just et al. 2015

Forward-peaked distributions



ELN distribution

Cornelius et al., *PRD* **112** 063004 (2025)

Polynomial weighting function

(Polynomial method)

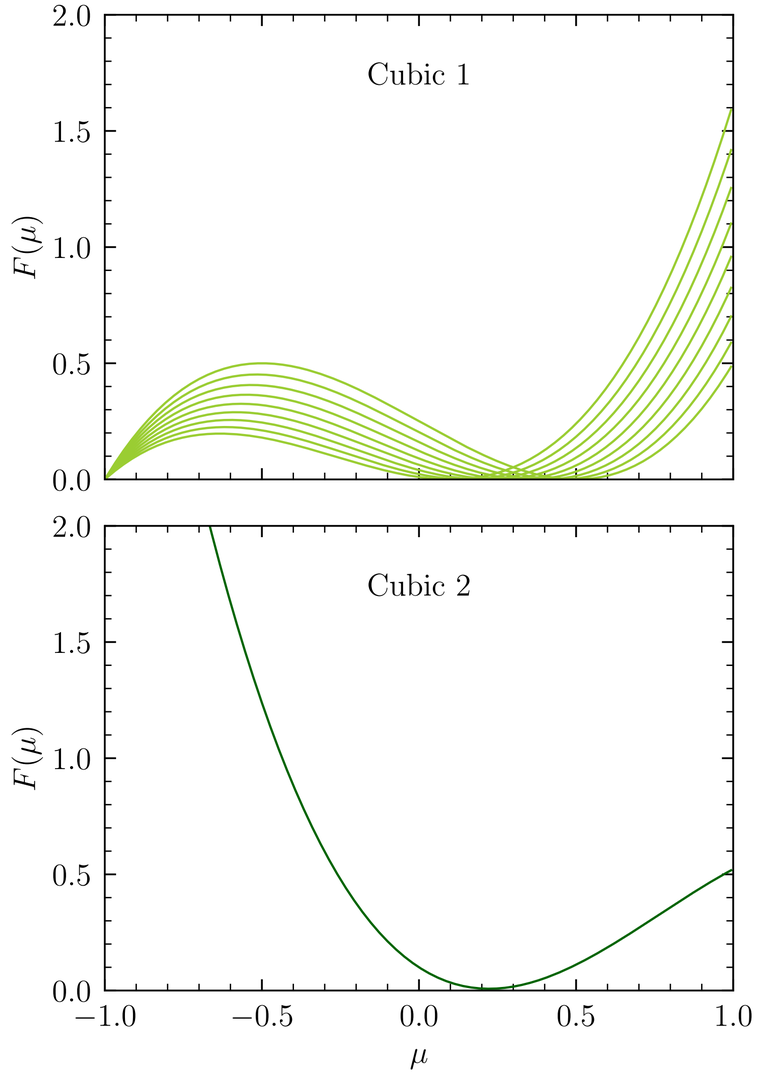
The angular distributions feature an ELN crossing, if there exists a function $\mathcal{F}(\mu) > 0$ for $\mu \in [-1, 1]$ such that

$$I_{\mathcal{F}}(r) I_0(r) < 0$$

With

$$I_{\mathcal{F}}(r) = \int_{-1}^{1} \mathcal{F}(\mu) G(r, \mu) \qquad \mathcal{F}(\mu) = \sum_{n=0}^{N} a_n \mu^n$$

Abbar 2020
Abbar et al. 2020
Capozzi et al. 2020
Just et al. 2022

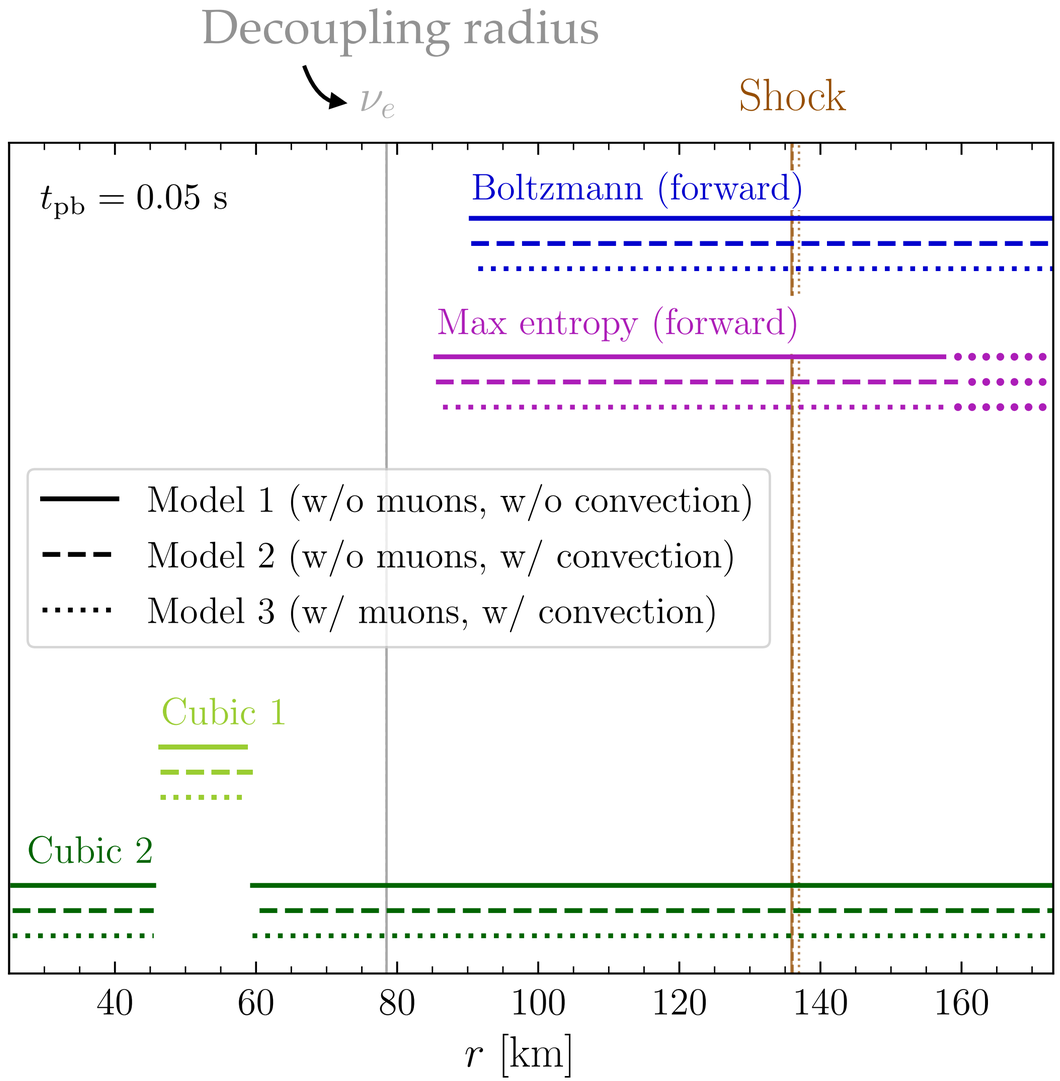


Cornelius et al., *PRD* **112** 063004 (2025)

Performance of each method

$t_{\rm pb} = 0.05$ s (EoS: LS220)

- Boltzmann: crossings occur after neutrino decoupling and also above the shock
- Maximum entropy: reproduces most crossings
- Minerbo: detects no crossings
- Polynomial: crossings everywhere

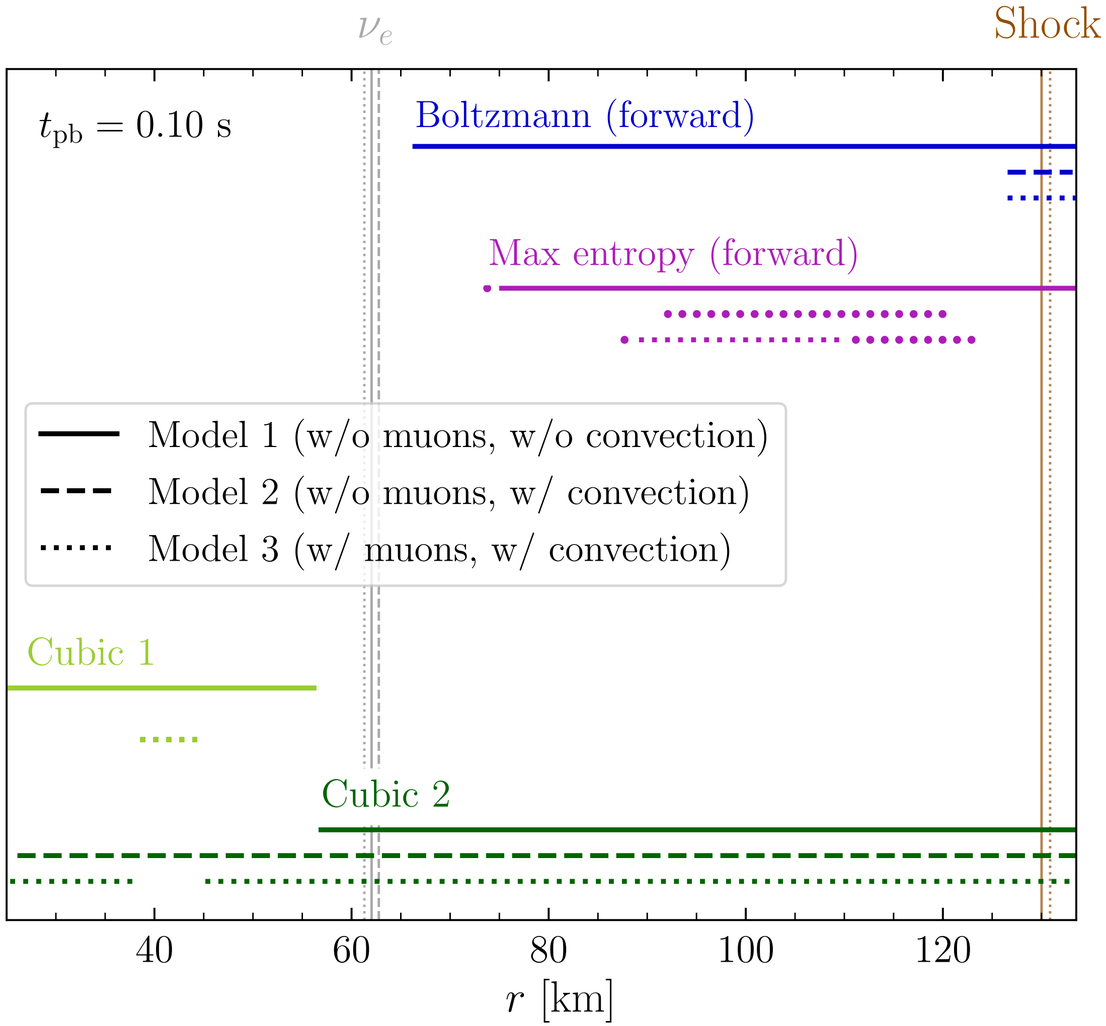


Cornelius et al., *PRD* **112** 063004 (2025)

Performance of each method

$t_{\rm pb} = 0.1$ s (EoS: LS220)

- Boltzmann: crossings occur after neutrino decoupling for Model 1
- Maximum entropy: reproduces crossings for Model 1, misidentifies crossings for Model 2 and 3
- Minerbo: detects no crossings
- Polynomial: crossings everywhere

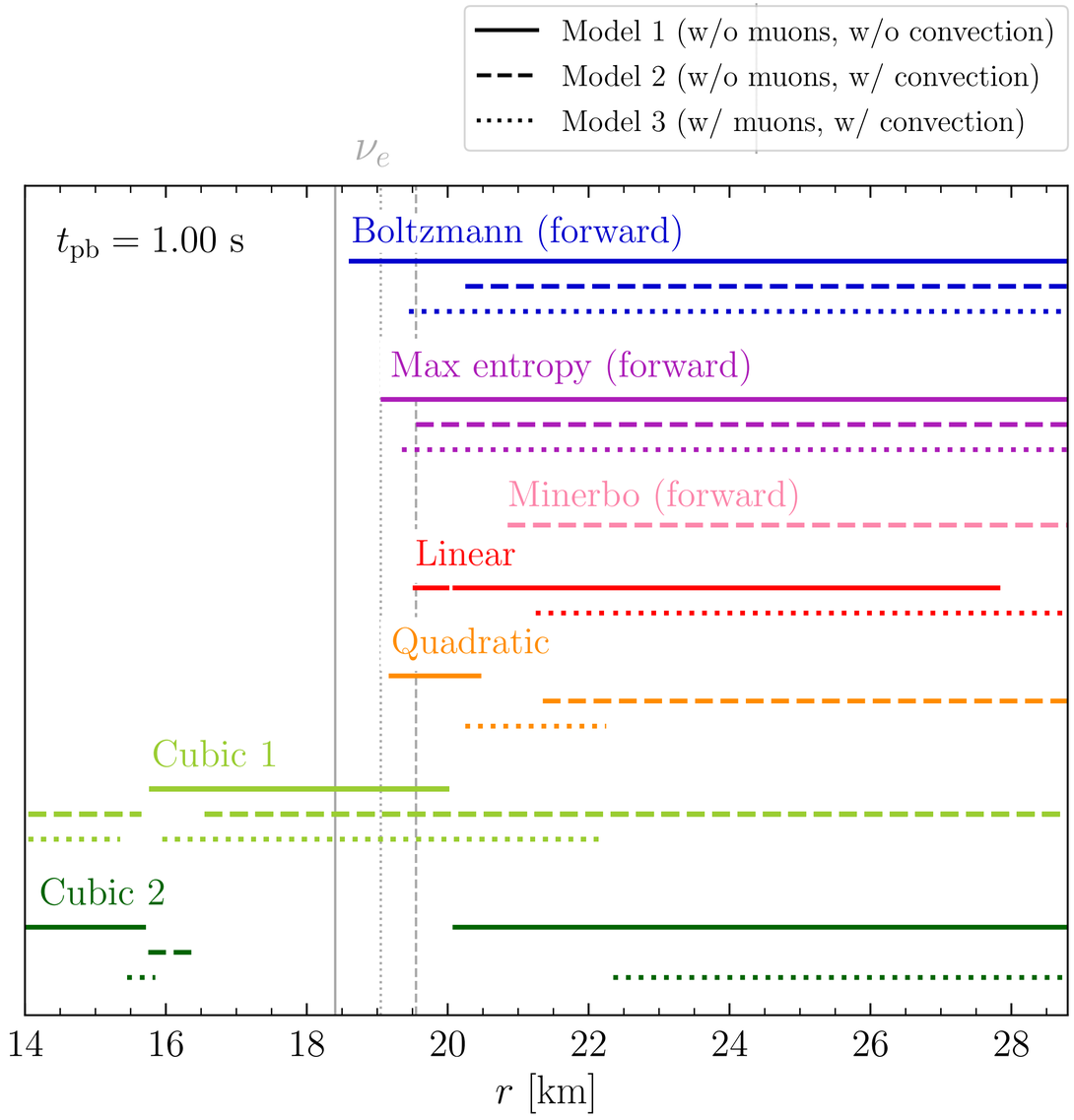


Cornelius et al., *PRD* **112** 063004 (2025)

Performance of each method

$t_{\rm pb} = 1.00$ s (EoS: LS220)

- Boltzmann: crossings after neutrino decoupling
- Maximum entropy: reproduces most crossings
- Minerbo: crossings for only Model 2
- Polynomial: crossings everywhere but now for all polynomials



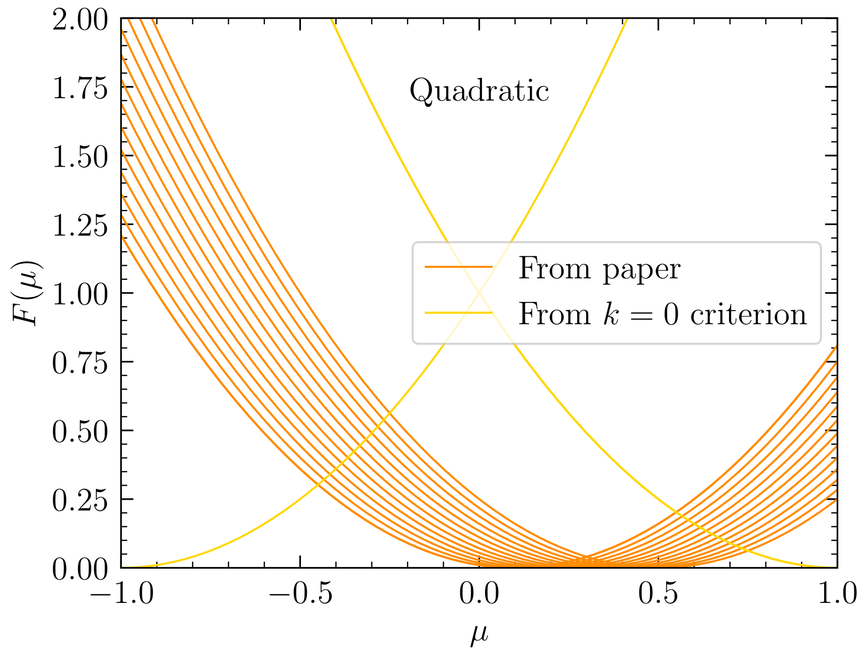
Cornelius et al., *PRD* **112** 063004 (2025)

Other methods

- The Fourier mode $k = 0$ method (Dasgupta et al. 2018, Glas et al. 2019)
- Resonant trajectory (Johns et al. 2019)
- Unstable pendulum (Johns et al. 2019)
- Fitting function from moments/ray-tracing (Nagakura & Johns 2021, Nagakura et al. 2021)

As Johns & Nagakura 2021 point out, the first 3 methods are similar to the polynomial method

e.g.



Comparisons in the literature

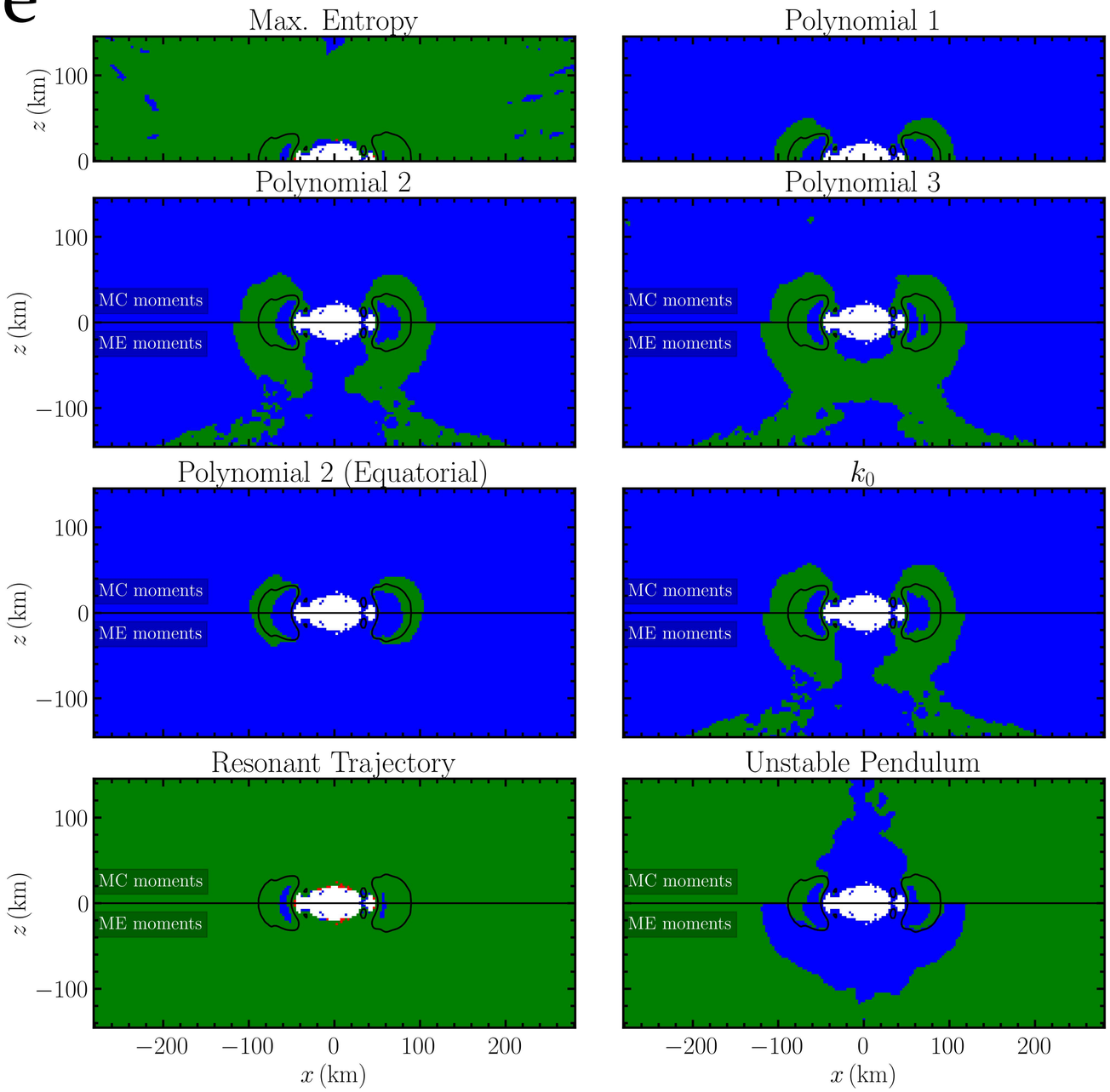
From Richers 2022

(Neutron star merger MC simulation)

Green: MC and method agrees on crossings

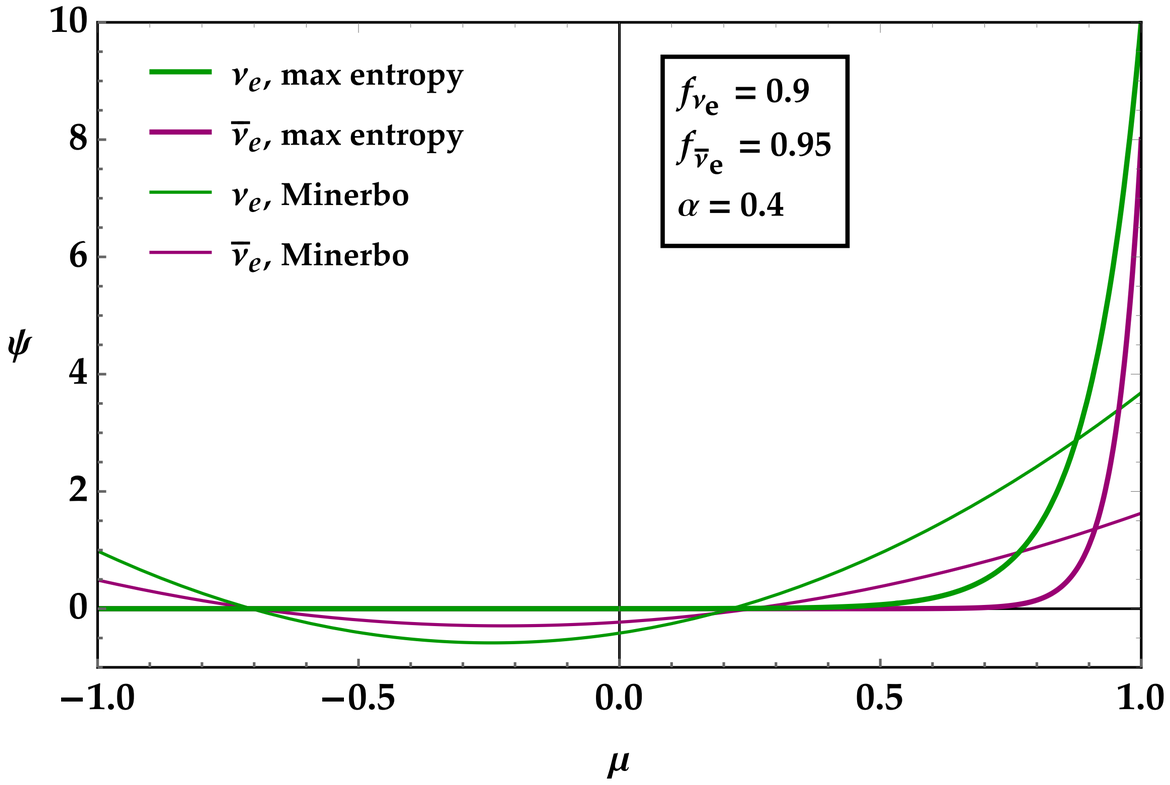
Blue: method does not find the crossings

Red: false crossing

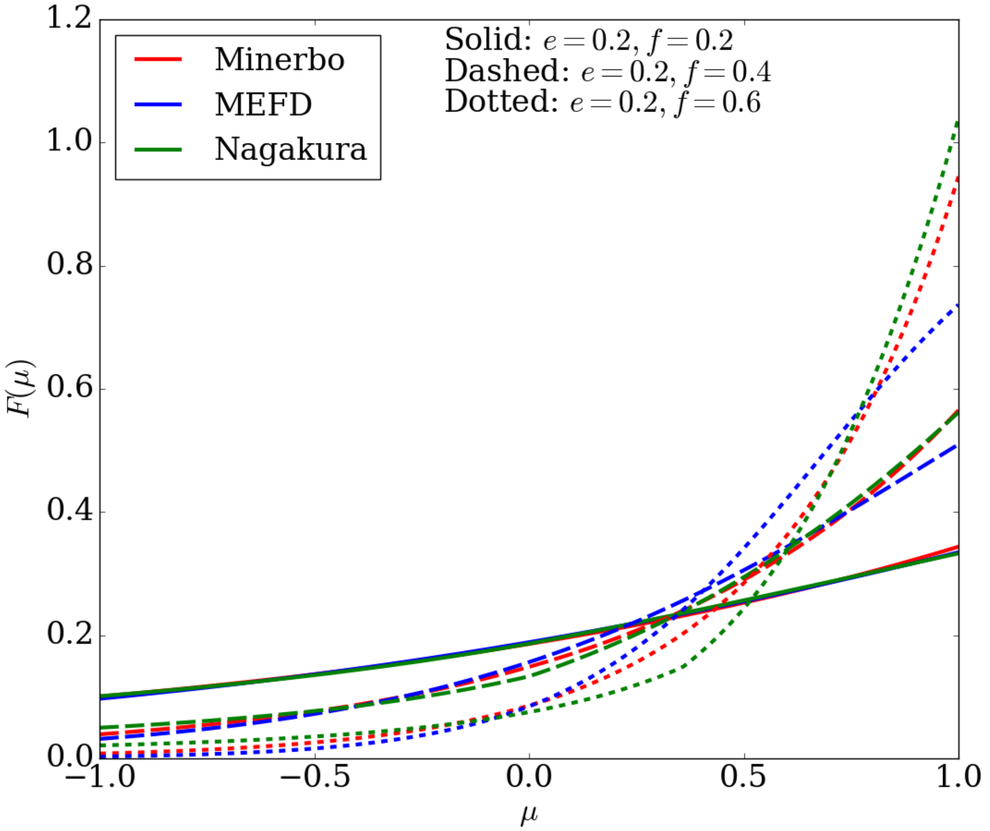


Comparisons in the literature

Johns & Nagakura 2021



Wang & Burrows 2025



Conclusions of part II

How well can we model ELN crossings without angular information?

- Maximum entropy: reproduces the most ELN crossings with some false crossings
- Minerbo: a subset of ELN crossings
- Polynomial: a subset or many false crossings
- Other methods:
  - Similar to the polynomial method
  - Alternative fitting methods might be the way to go

Summary

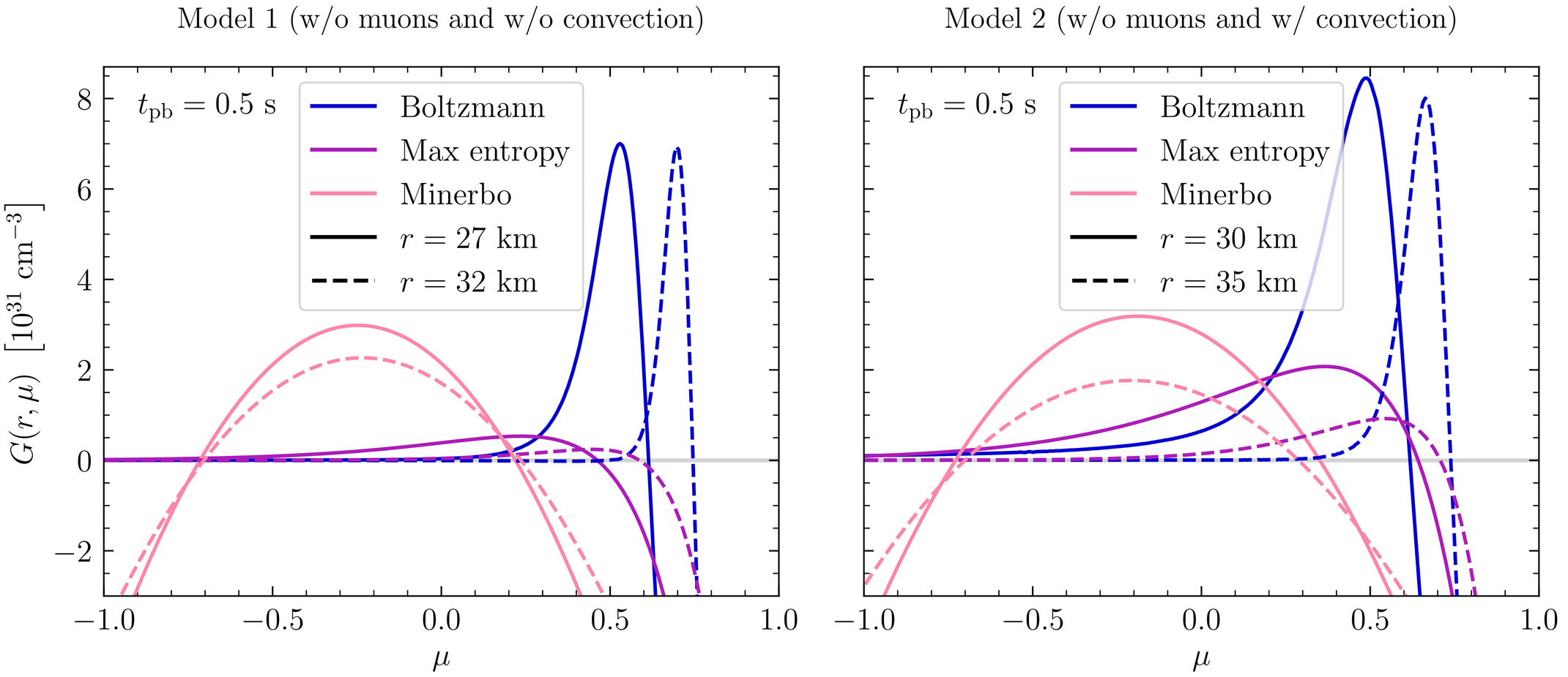
1. The location of ELN crossings varies with the SN core physics:
    - PNS convection moves crossings outward
    - Muons move crossings inward
    - EoS subtle differences

2. Many alternative methods to find crossings:
    - Some untrustworthy, some more reliable
    - We should search for better methods

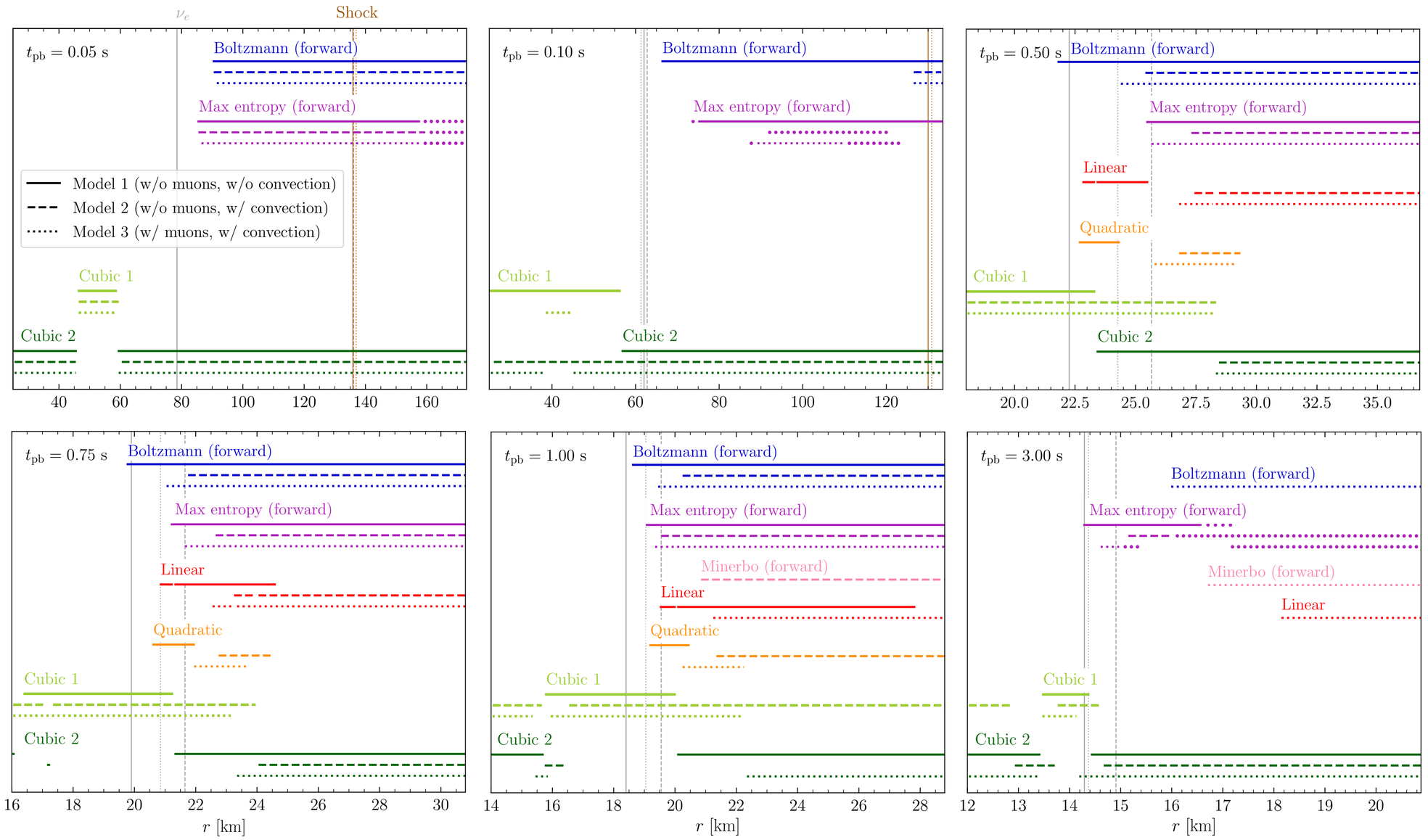
Thank you!

Backup slides

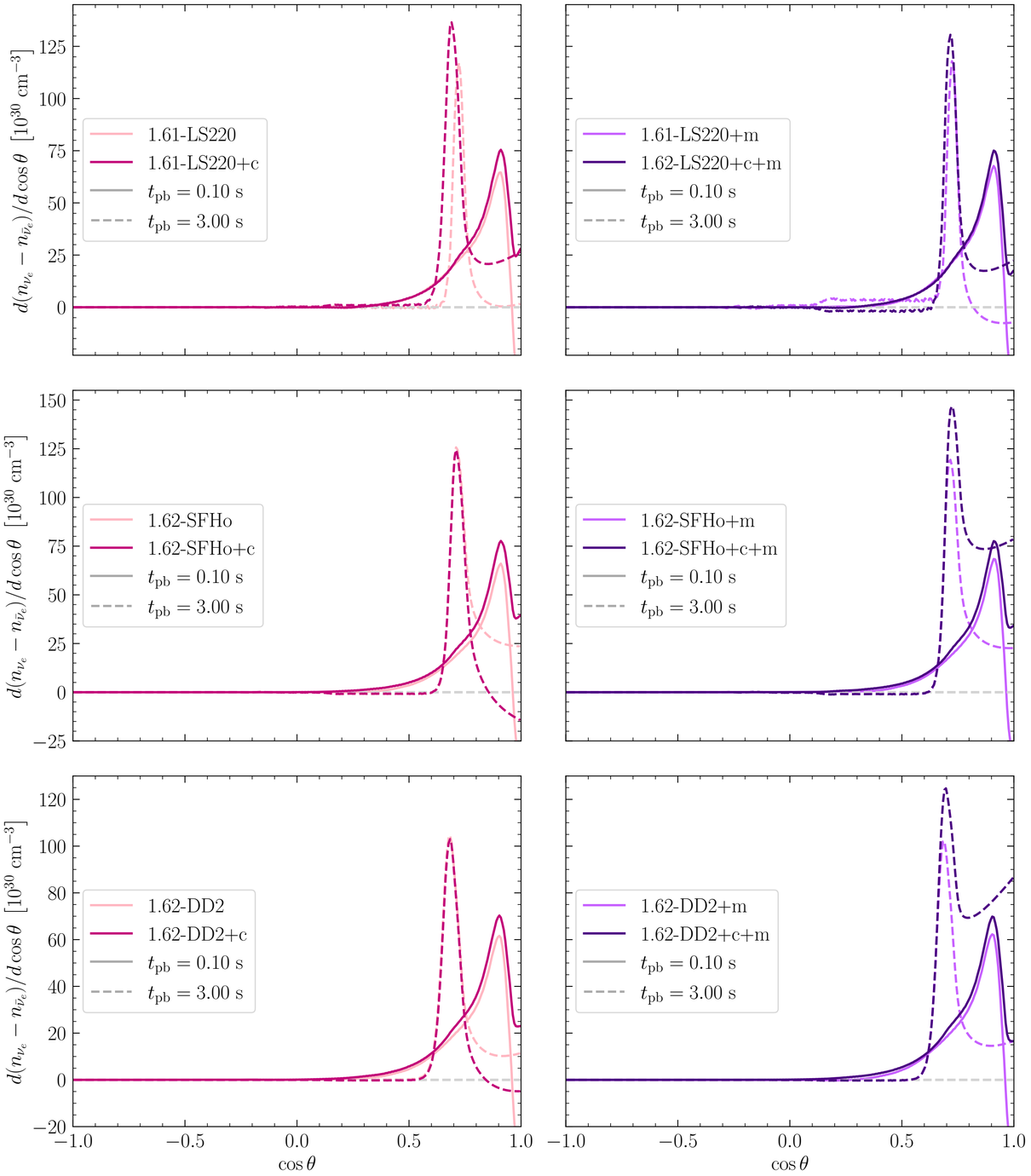
Modeling ELN angular distributions



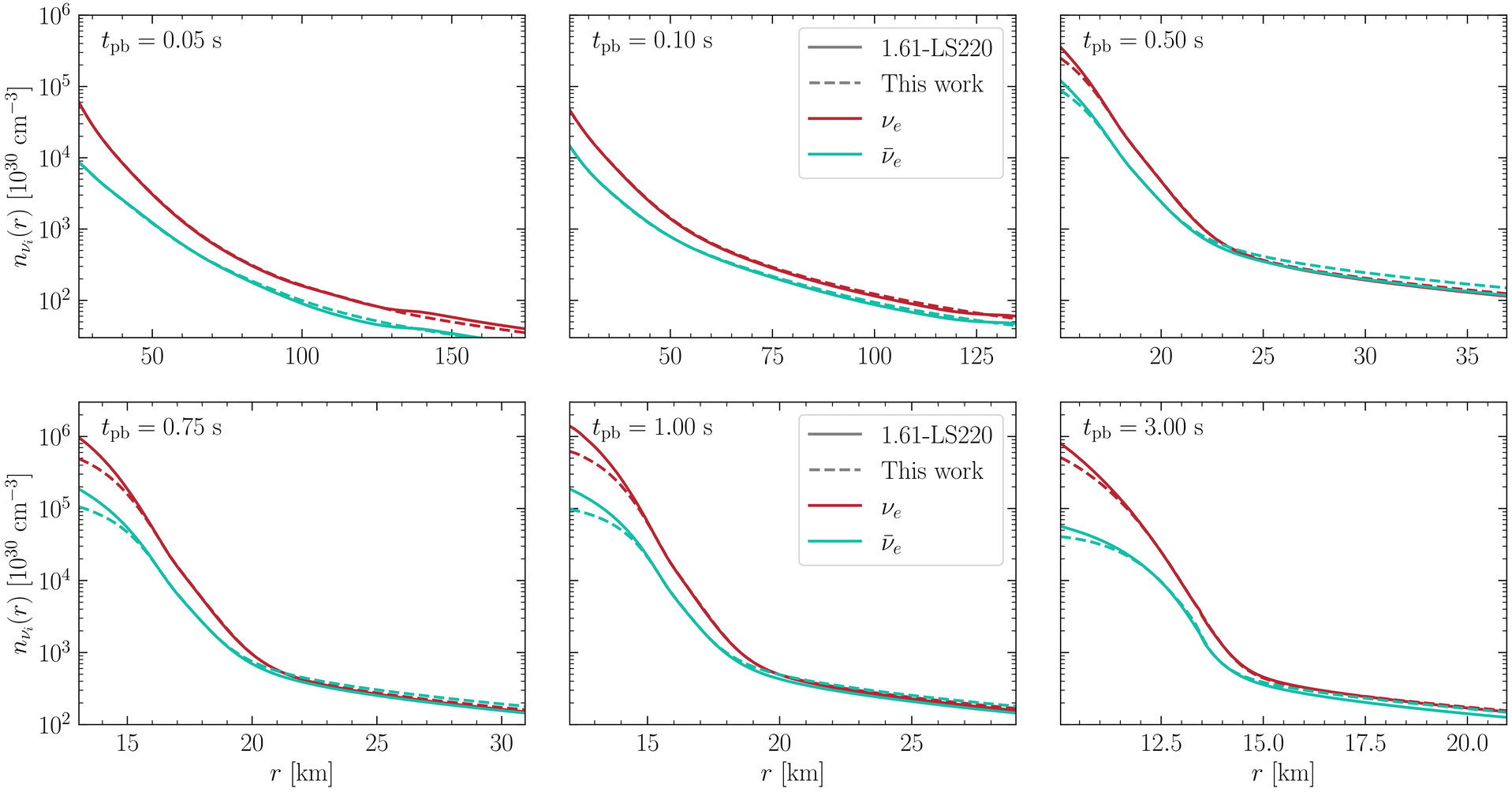
Performance of each method



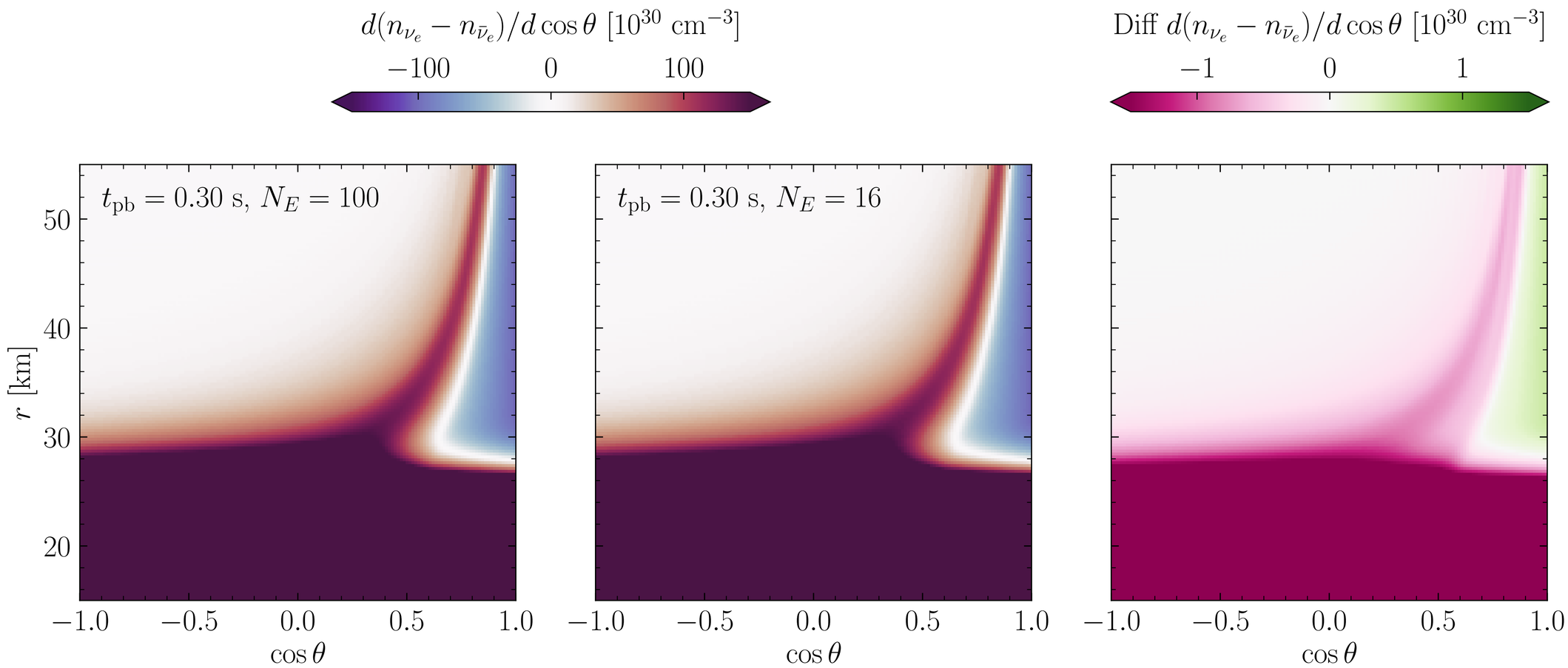
ELN distributions in all 12 SN models



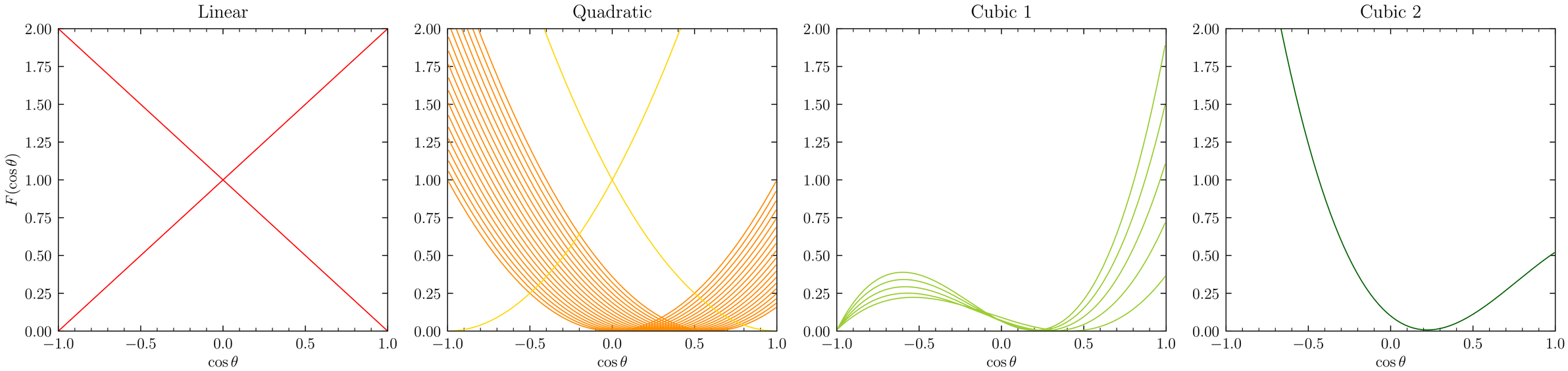
Comparison of number density with the SN simulation



Comparison of energy resolution with the SN simulation



Polynomials



Shock radii in all 12 SN models