Role of fast flavor conversion on core-collapse supernovae informed with the classical Boltzmann transport

Ryuichiro Akaho [Waseda University]

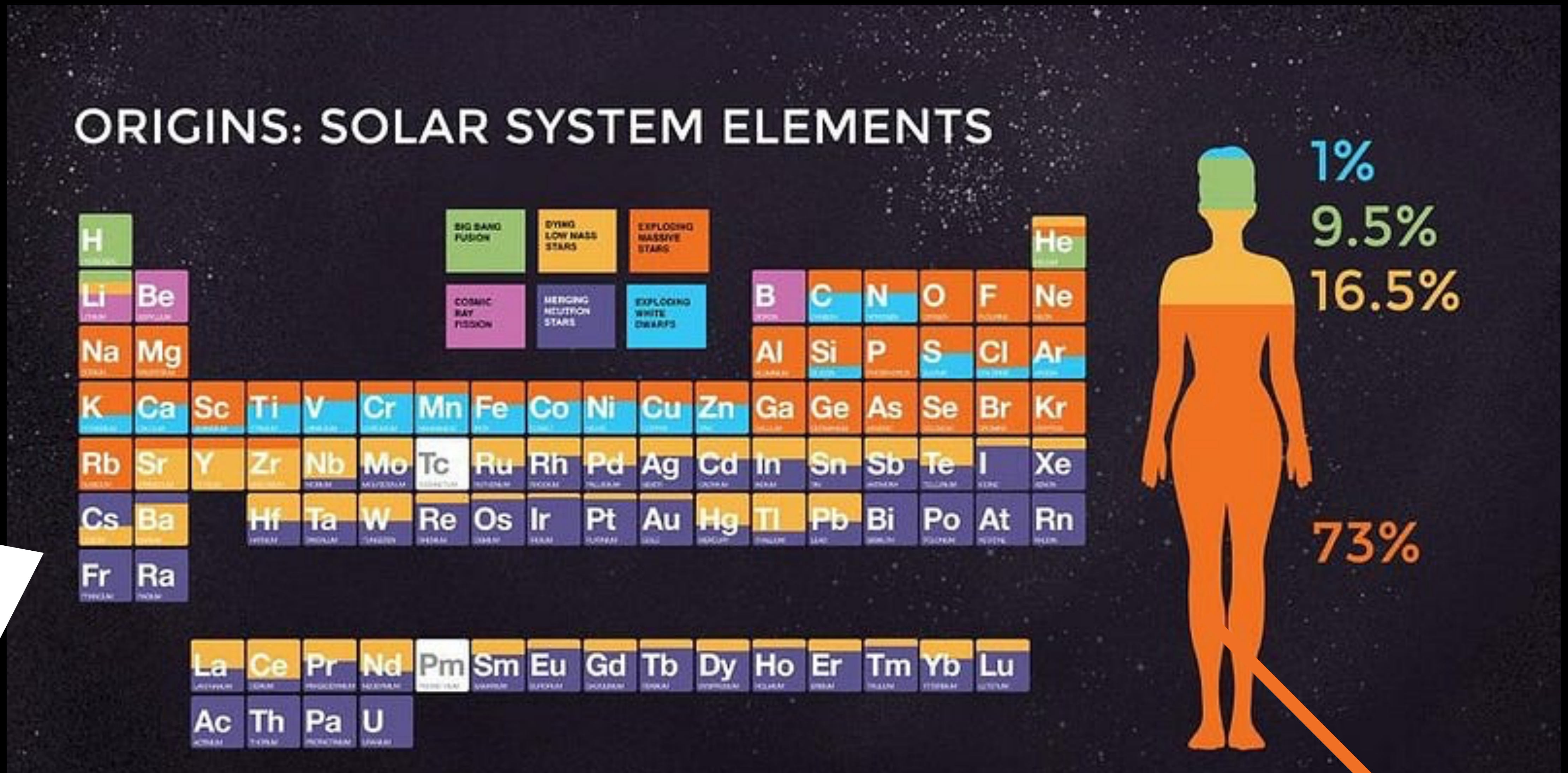
Collaborators: Hiroki Nagakura, Wakana Iwakami, Shun Furusawa, Akira Harada, Hirotada Okawa, Hideo Matsufuru, Kohsuke Sumiyoshi, Shoichi Yamada

Outline

- Introduction
- Part I. Comparison of FFC subgrid methods (in 1D)
  Akaho+ PRD 112, 043015 (2025)
- Part II. 2D CCSN simulations with multi-angle FFC subgrid
  Akaho+ arXiv:2601.08269 accepted to PRL
- Summary

Core-collapse Supernovae (CCSNe)

- Energetic explosion at the end of stellar evolution.
- Plays central role for the evolution of the universe.



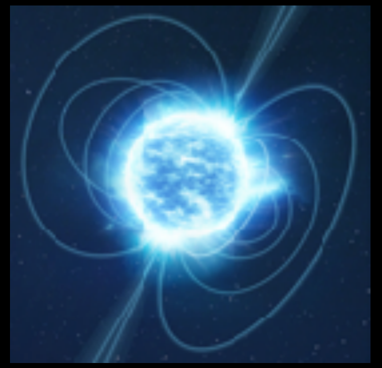
Explosive nucleosynthesis

Supernova yield

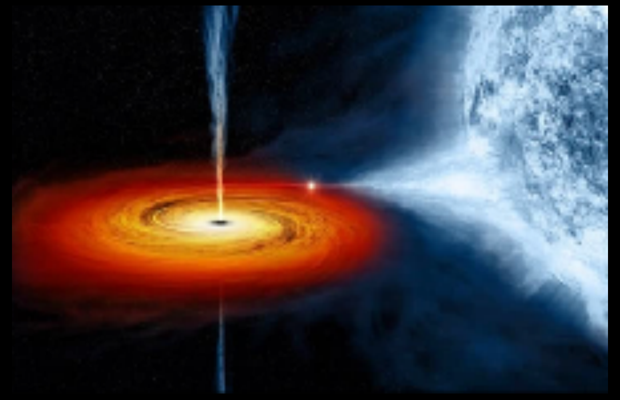
Supernova explosion

Formation of compact stars

Neutron stars



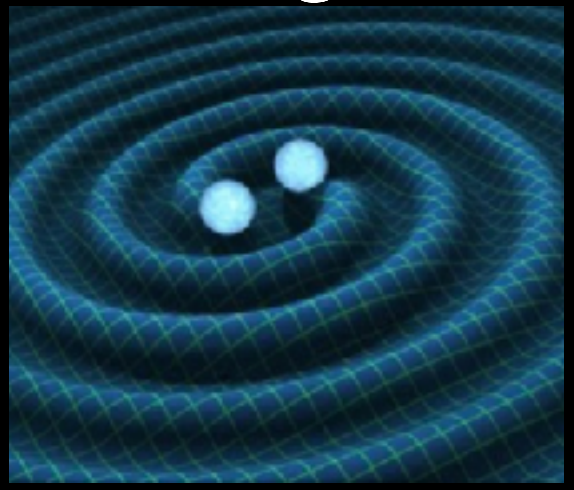
Black holes



High energy astrophysical phenomena

GRB

Compact binary mergers

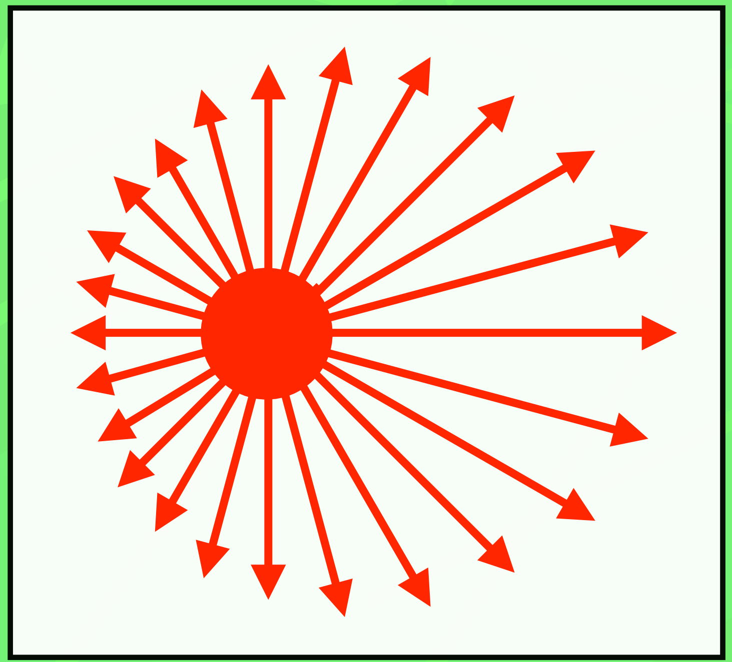


Scenario of CCSN

Neutrinos inside CCSN

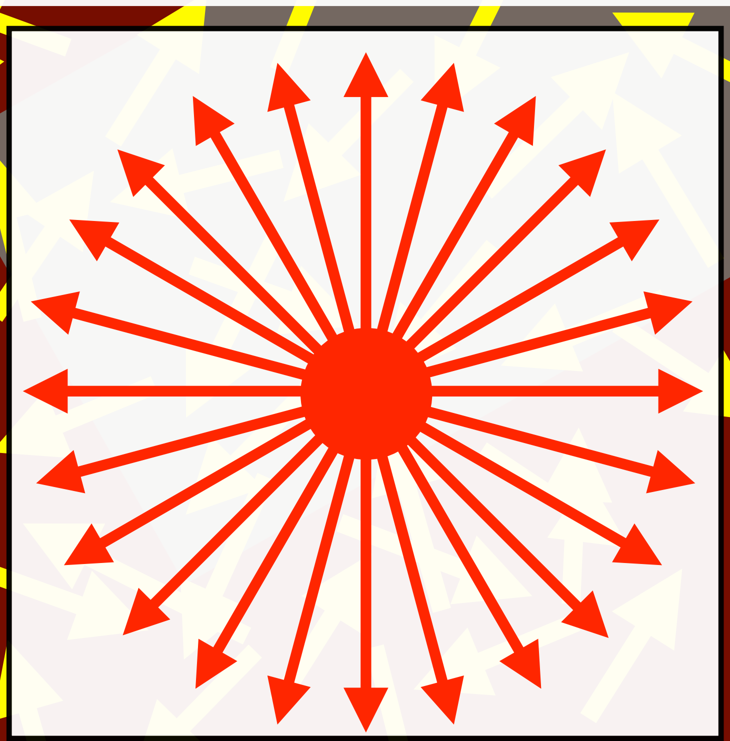
Free streaming

Intermediate: nontrivial



thermal eq. (Fermi-Dirac)

momentum: isotropic



Phase space distribution function $f(x^\mu, p^i)$

Boltzmann equation

$$p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^i_{\alpha\beta} p^\alpha p^\beta \frac{\partial f}{\partial p^i} = \left[\frac{\delta f}{\delta t}\right]_{\mathrm{coll}}$$

Truncated Moment Method

Distribution Function

$$f(r, \theta, \phi, \epsilon, \theta_\nu, \phi_\nu)$$

Boltzmann Equation

$$\frac{\partial f}{\partial t} + p^i \frac{\partial f}{\partial x^i} + \dot{p}^i \frac{\partial f}{\partial p^i} = C$$

Instead of Boltzmann transport, truncated moment method is often used.

Angular moment in momentum space

Moment eqs. (depend on higher moments)

0th $$\frac{\partial E}{\partial t} = L_1(E, M_1^i, M_2^{ij})$$

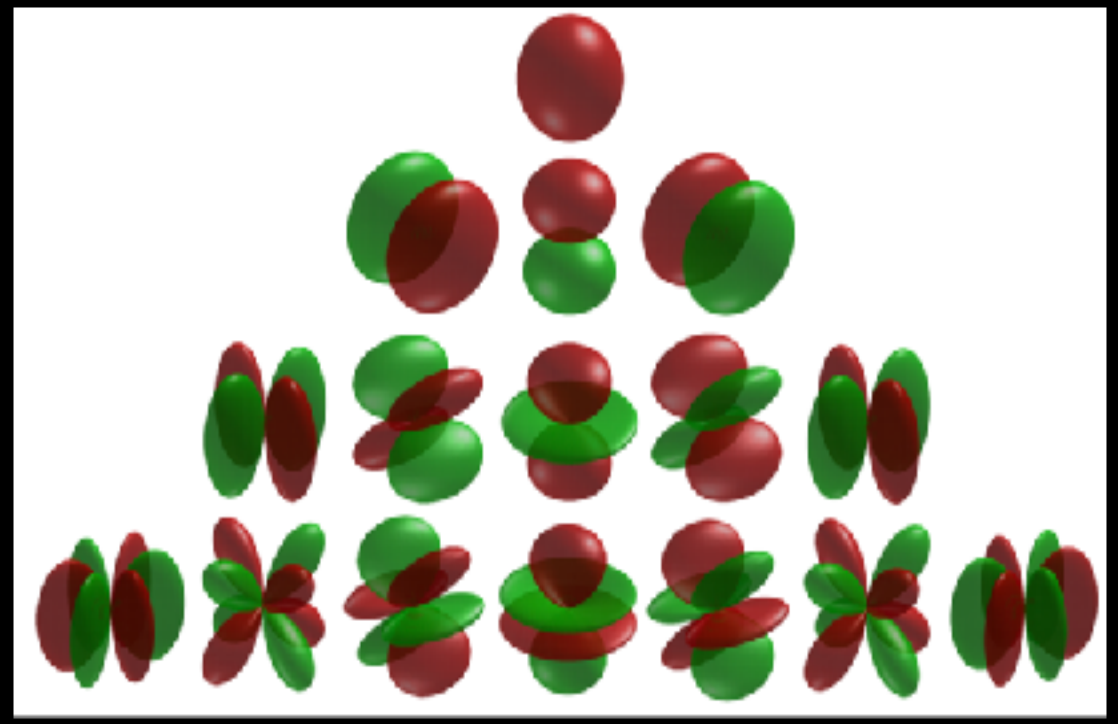
1st $$\frac{\partial M_1^i}{\partial t} = L_2(E, M_1^i, M_2^{ij})$$

Truncation

Part of momentum space information is lost

2nd $$\frac{\partial M_2^{ij}}{\partial t} = L_2(E, M_1^i, M_2^{ij}, M_3^{ijk})$$

⋮

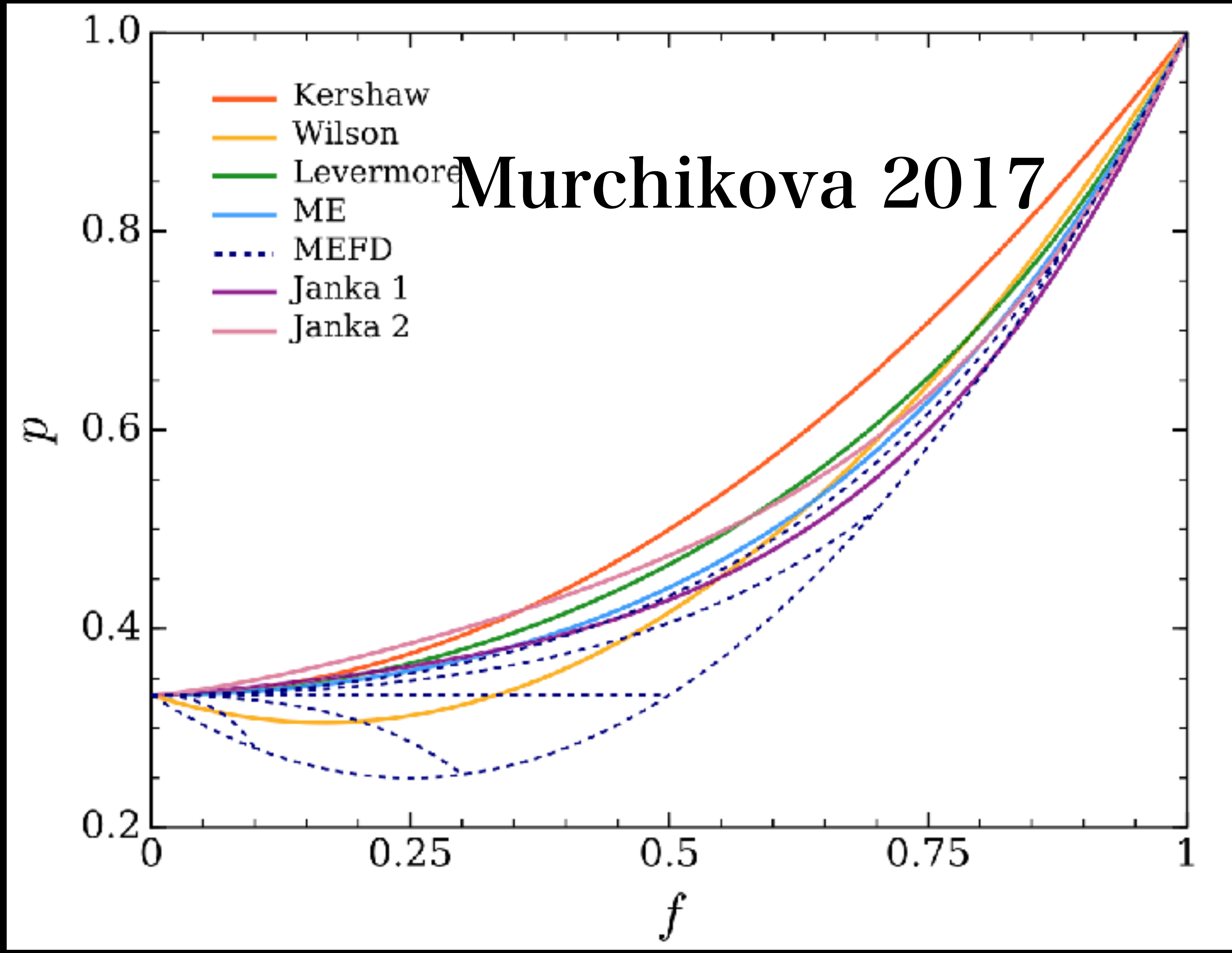


Analytical Closure

Assume closure relation to calculate 2nd moments only from 0th and 1st moments

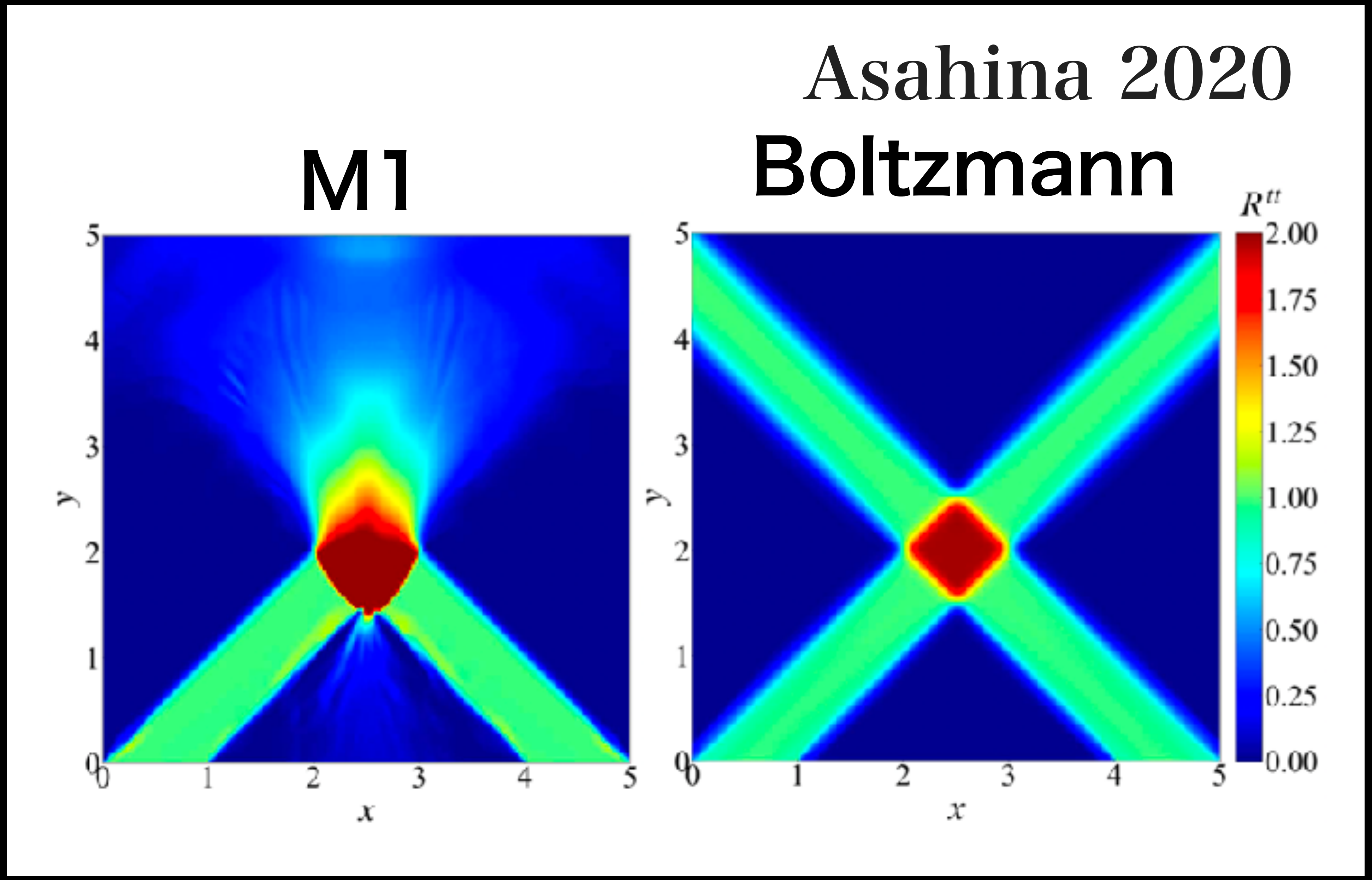
$$P^{ij}_{M1} = \frac{3p-1}{2} P^{ij}_{thin} + \frac{3(1-p)}{2} P^{ij}_{thick}$$

Eddington factor



Flux factor (function of 0th and 1st moment)

Moment method fails to solve ray crossing test



GR Boltzmann Neutrino Radiation Hydrodynamics Code

Boltzmann & hydrodynamics equations are solved together to simulate CCSN

Sumiyoshi (2012), Nagakura+(2014,2017,2019), Akaho+(2021,2023)

Boltzmann equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\Bigg|_{q_i}\left[\left(e^\mu_{(0)}+\sum_{i=1}^{3} l_i e^\mu_{(i)}\right)\sqrt{-g}f\right]-\frac{1}{\epsilon^2}\frac{\partial}{\partial \epsilon}\left(\epsilon^3 f\omega_{(0)}\right)$$
$$+\frac{1}{\sin\theta_\nu}\frac{\partial}{\partial\theta_\nu}\left(\sin\theta_\nu f\omega_{(\theta_\nu)}\right)-\frac{1}{\sin^2\theta_\nu}\frac{\partial}{\partial\phi_\nu}\left(f\omega_{(\phi_\nu)}\right)=S_{\rm rad}$$

Neutrino-matter interactions

Emission/Absorption

$e^- + p \leftrightarrow \nu_e + n$

$e^+ + n \leftrightarrow \bar{\nu}_e + p$

$e^- + A \leftrightarrow \nu_e + A'$

Scattering

$\nu + N \leftrightarrow \nu + N$

$\nu + A \leftrightarrow \nu + A$

$\nu + e^- \leftrightarrow \nu + e^-$

Pair

$e^- + e^+ \leftrightarrow \nu + \bar{\nu}$

$N + N \leftrightarrow N + N + \nu + \bar{\nu}$

Hydrodynamics equation

$$\partial_t \rho_* + \partial_j(\rho_* v^j) = 0$$
$$\partial_t S_i + \partial_j(S_i v^j + \alpha\sqrt{\gamma}P\delta_i^{\ j})$$
$$= -S_0\partial_i\alpha + S_j\partial_i\beta^j - \frac{1}{2}\alpha\sqrt{\gamma}S_{jk}\partial_i\gamma^{jk} - \alpha\sqrt{\gamma}G_i$$
$$\partial_t(S_0-\rho_*) + \partial_k((S_0-\rho_*)v^k + \sqrt{\gamma}P(v^k+\beta^k))$$
$$= \alpha\sqrt{\gamma}S^{ij}K_{ij} - S_iD^i\alpha + \alpha\sqrt{\gamma}n^\mu G_\mu$$

Spacetime metric

$$g_{\mu\nu} = \mathrm{diag}\left[-e^{2\Phi(t,r)}, \left(1-2m(t,r)/r\right)^{-1}, r^2, r^2\sin^2\theta\right]$$
$$\frac{\partial m}{\partial r} = 4\pi r^2(\rho h W^2 - P)$$
$$\frac{\partial \Phi}{\partial r} = \left(1-\frac{2m(t,r)}{r}\right)^{-1}\left(\frac{m(t,r)}{r^2}+4\pi r(\rho h v^2 + P)\right)$$

Neutrino Oscillation

$$i v^\mu \partial_\mu \rho = \left[\frac{m_1^2+m_2^2}{4E}\begin{pmatrix}1&0\\0&1\end{pmatrix} + \frac{m_2^2-m_1^2}{4E}\begin{pmatrix}-\cos2\theta_{12} & \sin2\theta_{12}\\ \sin2\theta_{12} & \cos2\theta_{12}\end{pmatrix} + \sqrt{2}G_F n_e \begin{pmatrix}1&0\\0&0\end{pmatrix} + \sqrt{2}G_F v_\mu \left[\int dP' \rho(x,P') v'^\mu, \rho\right]\right]$$

Vacuum term

Matter term

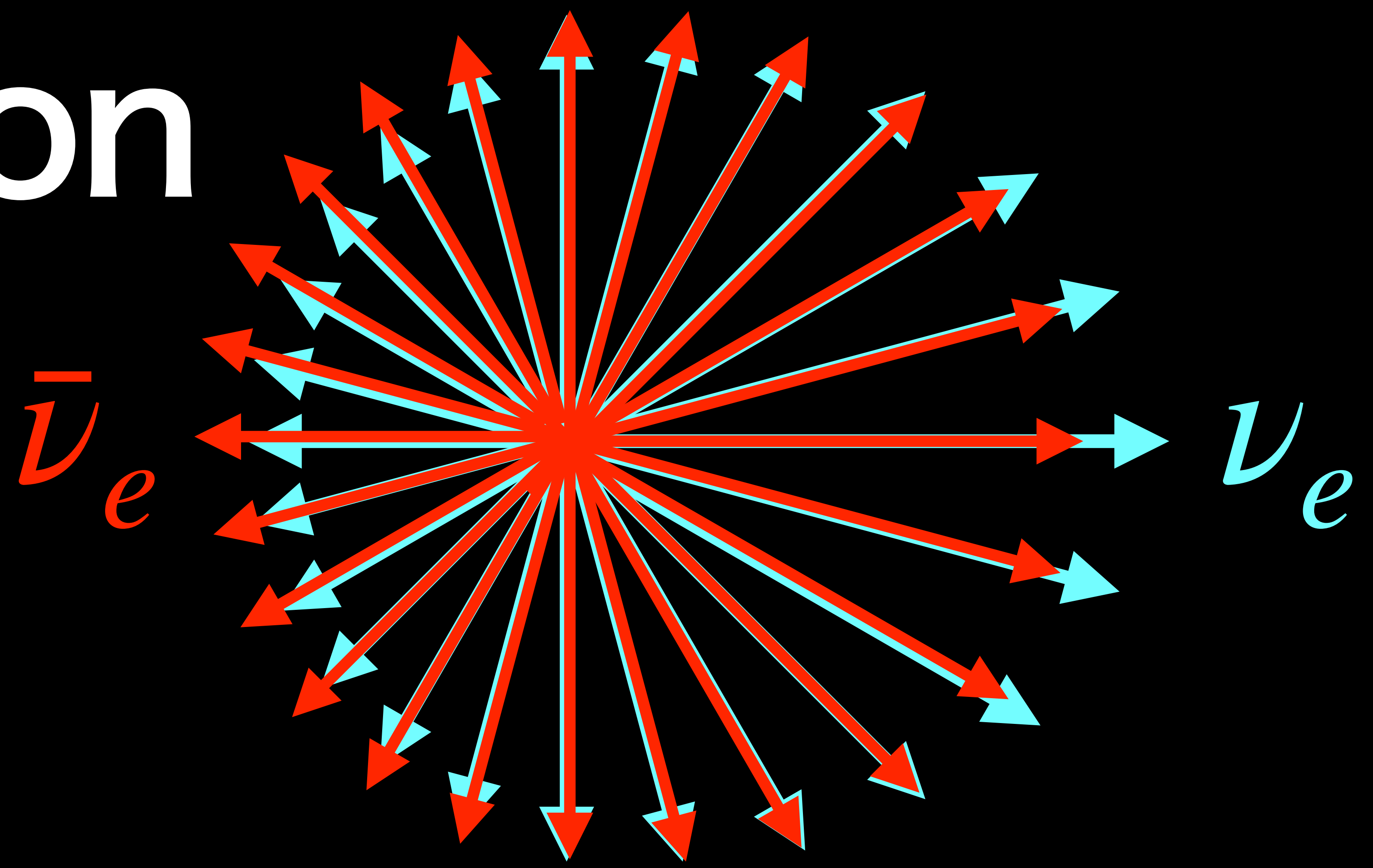
Neutrino self-interaction

Density

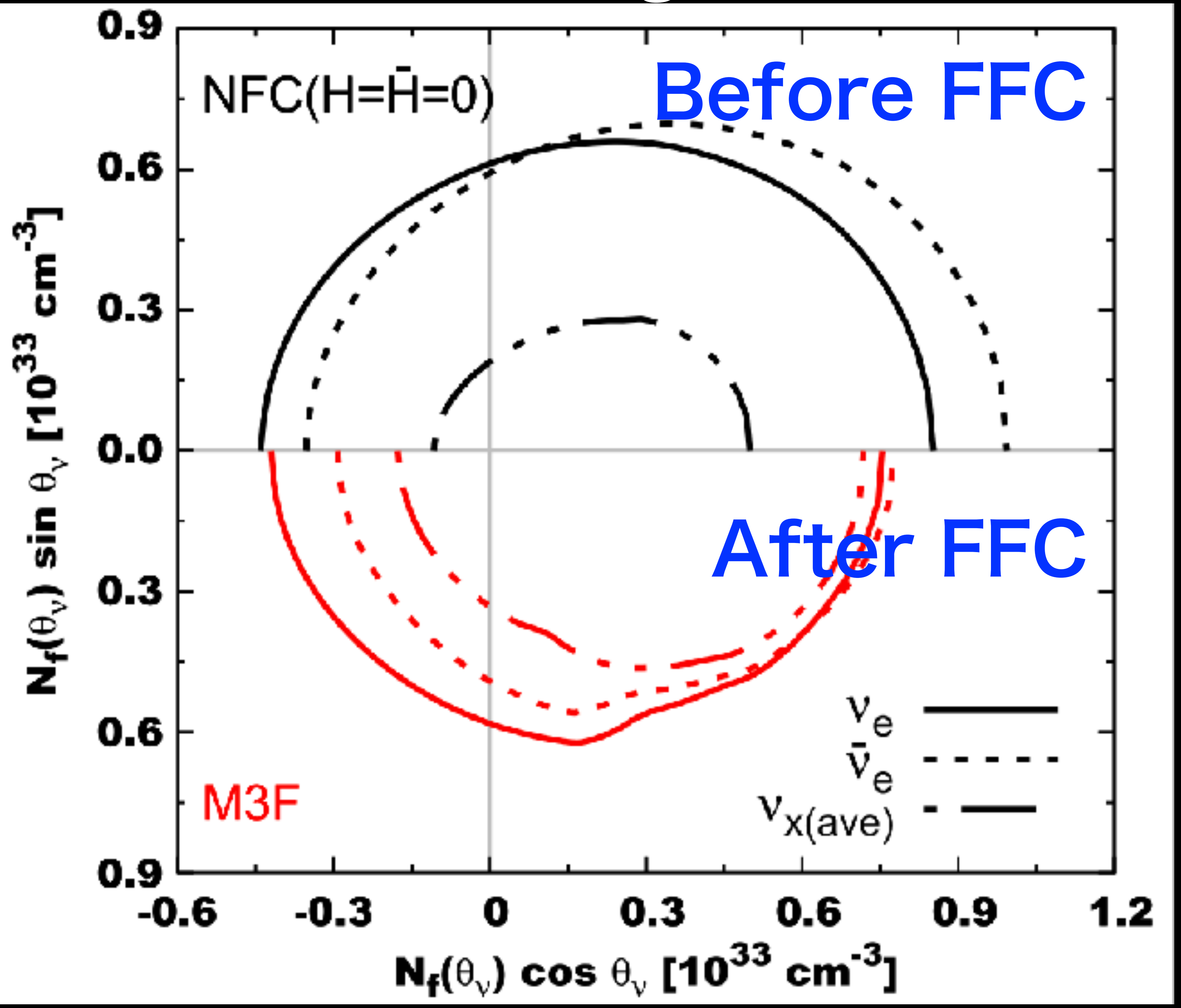
- **Vacuum oscillation** (interstellar region): periodic oscillation with time
- **MSW resonance** (e.g. solar surface): instant conversion
- **Matter suppression** (e.g. inside stars): no neutrino oscillation
- **Collective oscillation** (supernova core): nonlinear, can be very fast (~ns)

Fast Neutrino Flavor Conversion

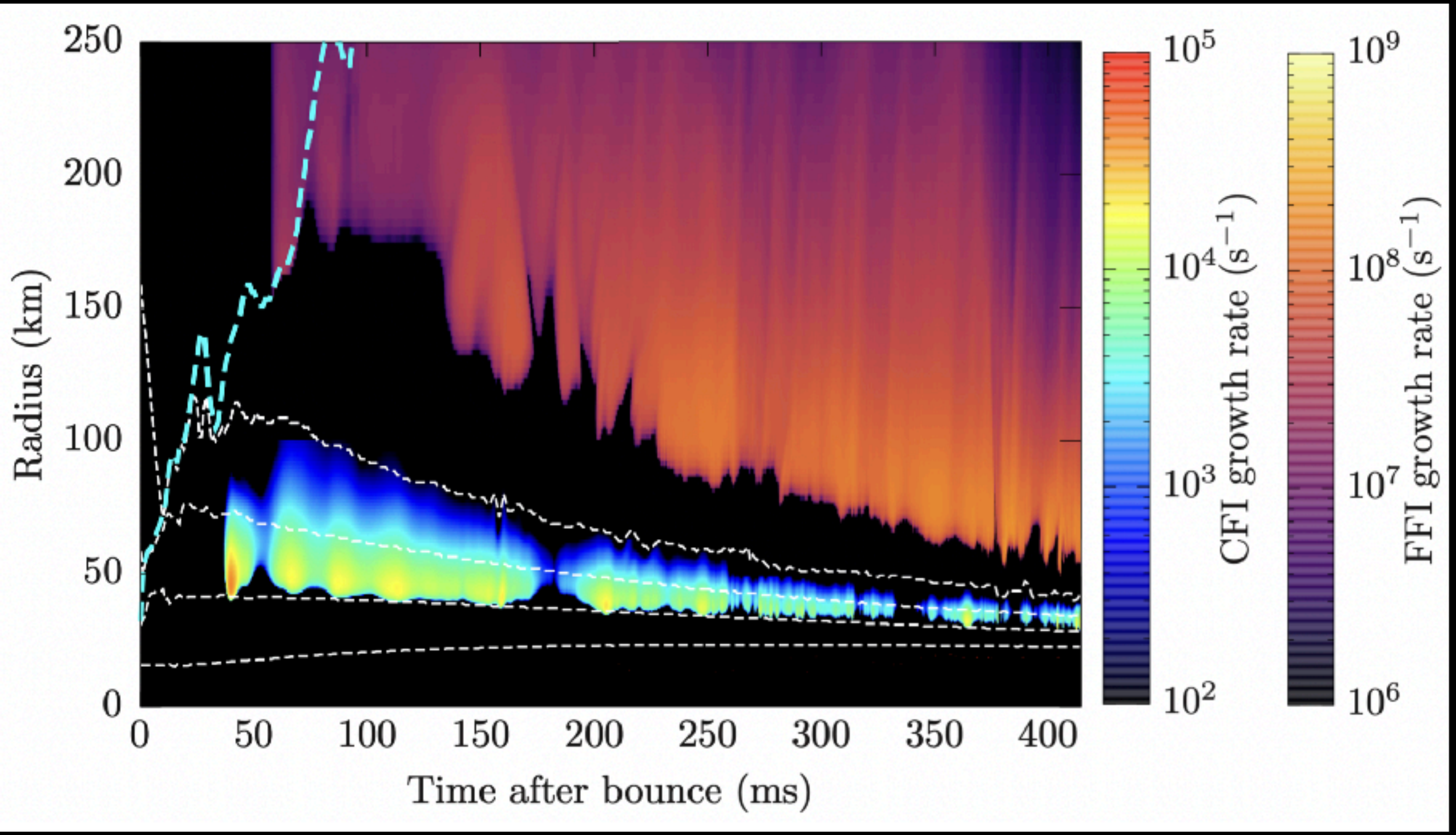
- Fast flavor conversion (FFC) is one of collective oscillation modes
- The conversion timescale can be ~ns, much shorter than the dynamical timescale.
- FFC is induced by angular crossing in momentum space



Nagakura 2023



Akaho+ 2024 PRD 109, 023012



Direct quantum (QKE) simulation is too heavy

➡ Classical transport + FFC subgrid

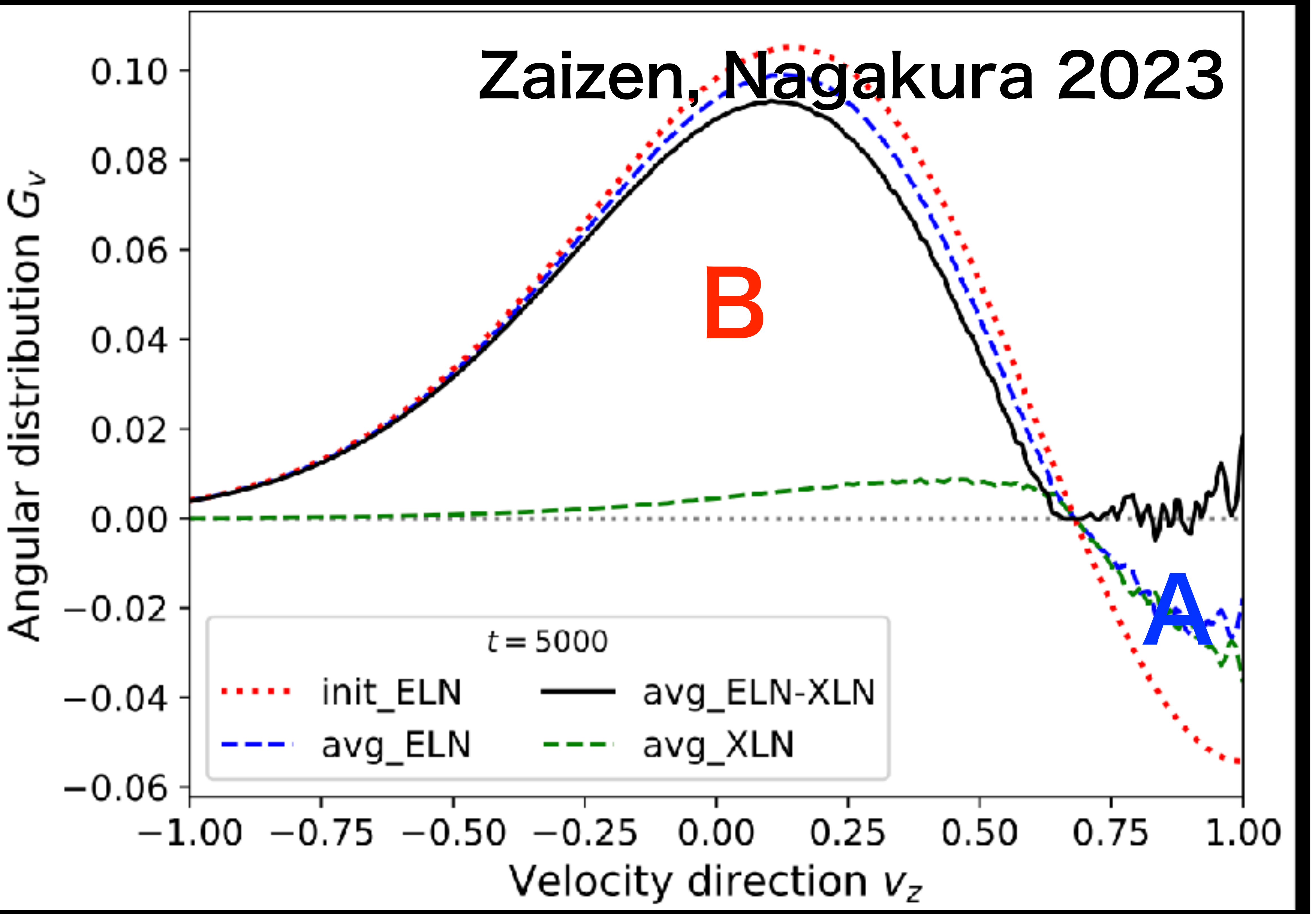
- **Akaho+** PRD 112, 043015 (2025)
- **Akaho+** in prep.

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\bigg|_{q_i}\left[\left(e^{\mu}_{(0)}+\sum_{i=1}^{3} l_i e^{\mu}_{(i)}\right)\sqrt{-g}f\right]-\frac{1}{\epsilon^2}\frac{\partial}{\partial \epsilon}\left(\epsilon^3 f\omega_{(0)}\right)$$

$$+\frac{1}{\sin\theta_\nu}\frac{\partial}{\partial\theta_\nu}\left(\sin\theta_\nu f\omega_{(\theta_\nu)}\right)-\frac{1}{\sin^2\theta_\nu}\frac{\partial}{\partial\phi_\nu}\left(f\omega_{(\phi_\nu)}\right)=S_{\rm rad}-\frac{1}{\tau}(f-f^{\rm asym})$$

Multi-angle Subgrid Model

FFC tries to erase ELNXLN crossing



ELN-XLN

$$\Delta G \equiv \int (f_e - \bar{f}_e - f_x + \bar{f}_x)\epsilon^2 d\epsilon$$

$$A \equiv \left| \frac{1}{8\pi^3} \int_{\Delta G<0} d(\cos\theta_\nu) d\phi_\nu \Delta G \right|$$ negative part

$$B \equiv \frac{1}{8\pi^3} \int_{\Delta G>0} d(\cos\theta_\nu) d\phi_\nu \Delta G,$$ positive part

survival prob.

$$B > A \quad \eta = \begin{cases} 1/3 & (\Delta G < 0) \\ 1 - 2A/(3B) & (\Delta G \geq 0) \end{cases}$$

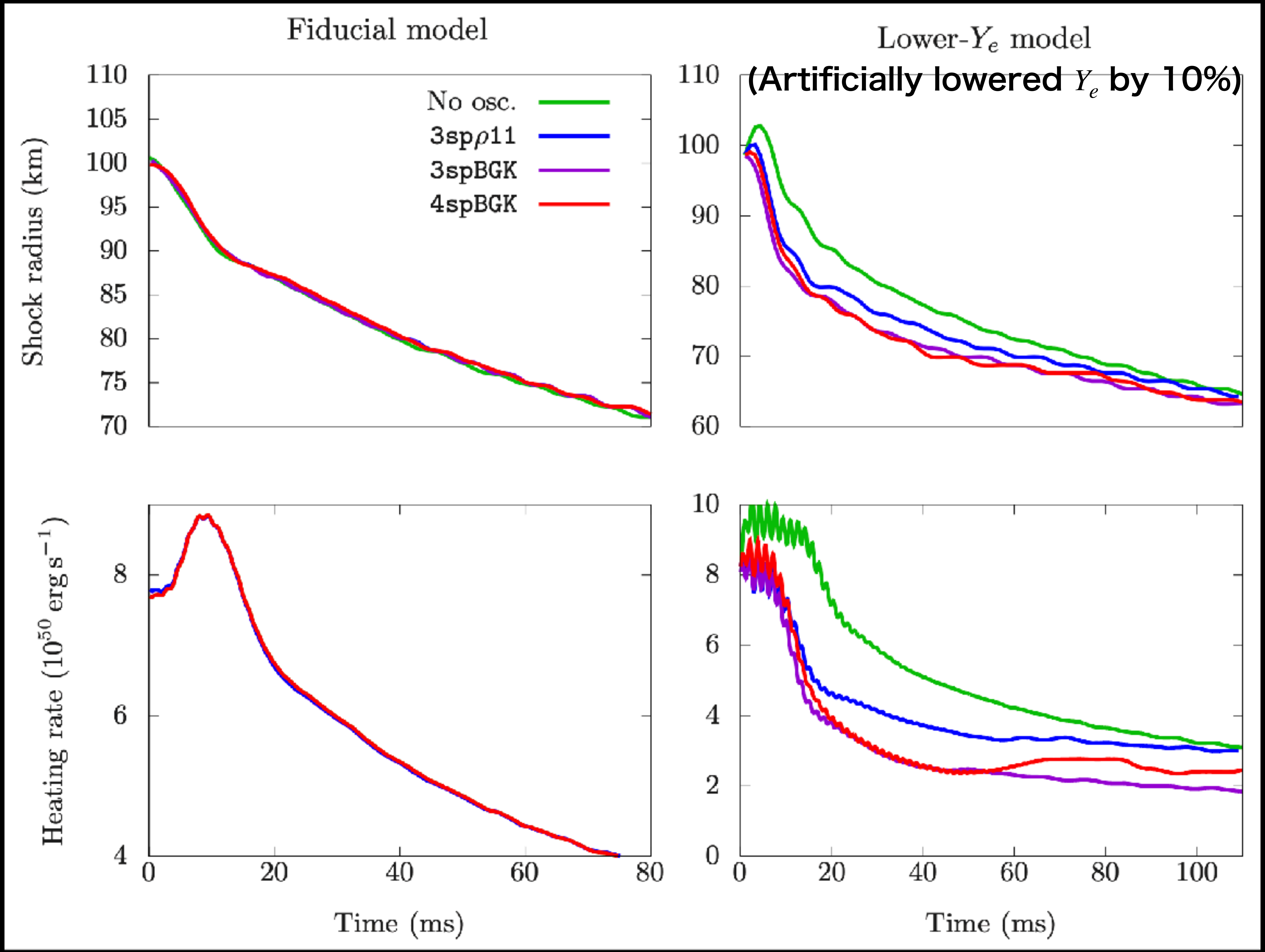
$$B < A \quad \eta = \begin{cases} 1/3 & (\Delta G > 0) \\ 1 - 2B/(3A) & (\Delta G \leq 0) \end{cases}$$

equipartition

ensure total ELN/XLN conservation

Part I. Comparison of the Subgrid Models (1D)

Akaho+ PRD 112, 043015 (2025)

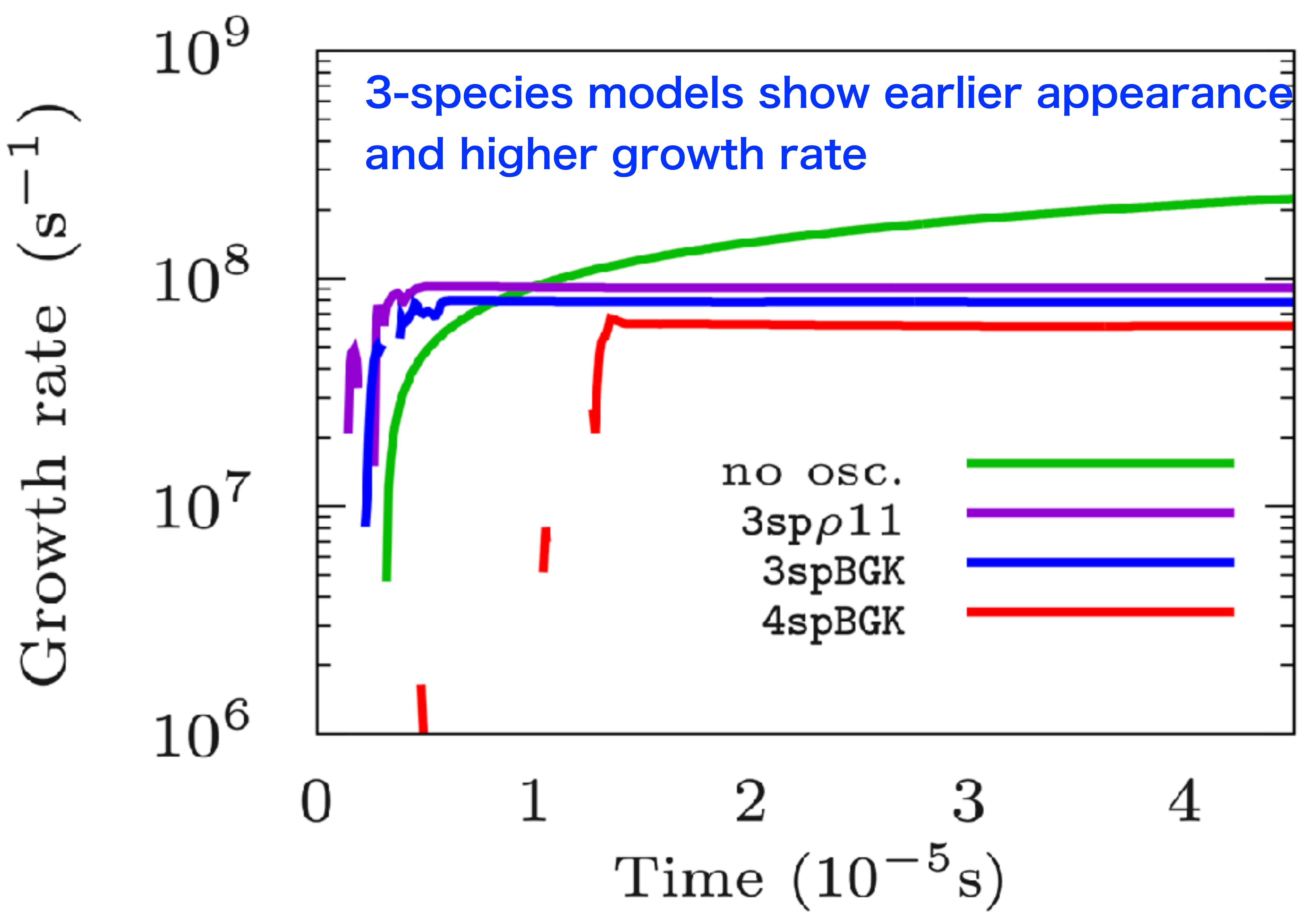


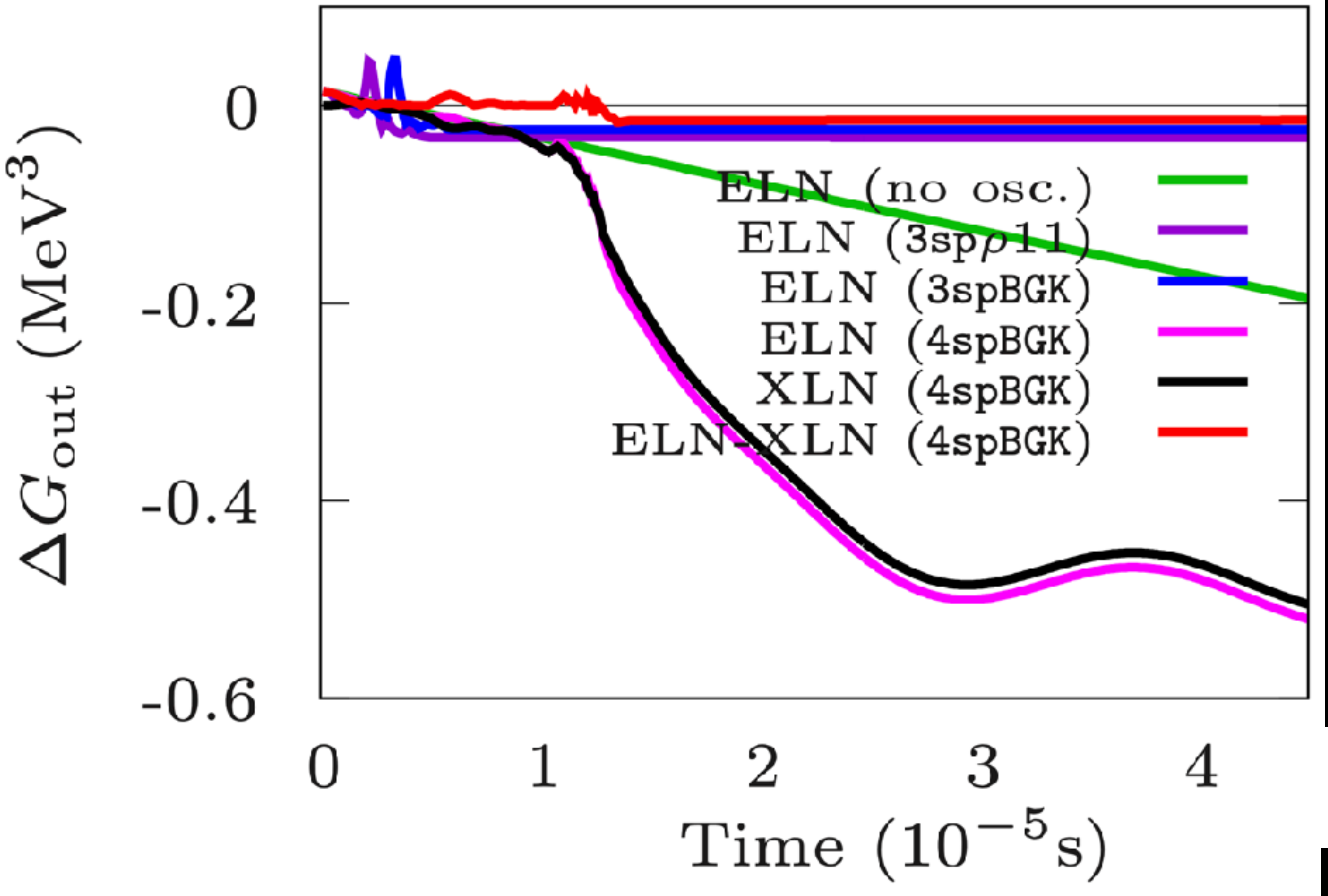
4spBGK: Multi-angle subgrid with 4-species treatment

3spBGK: Multi-angle subgrid with 3-species treatment (artificially force $\nu_x = \bar{\nu}_x$)

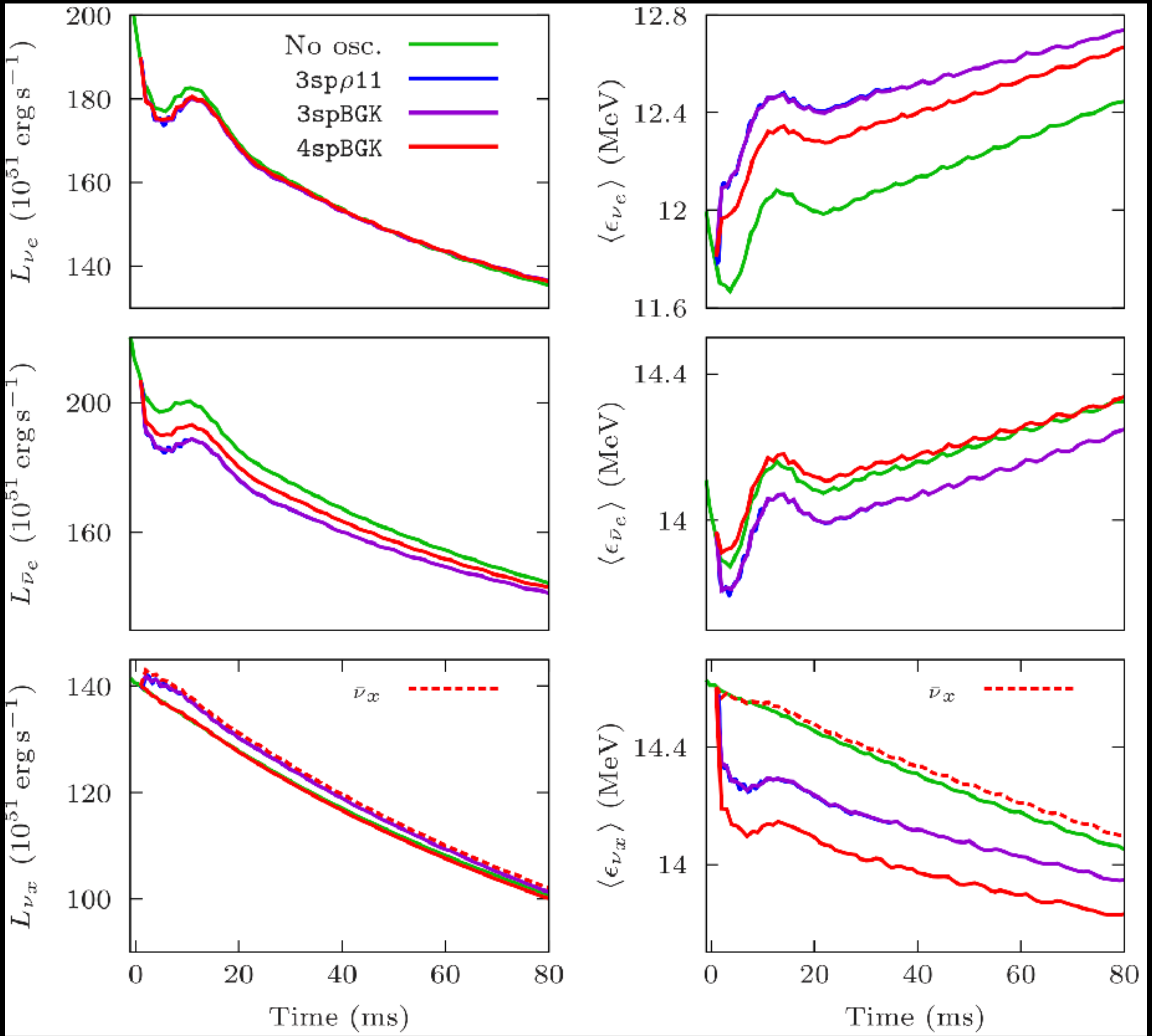
3spρ11: Density threshold method (Ehring+ 2023a, 2023b)

Flavor evolution at the appearance of FFI



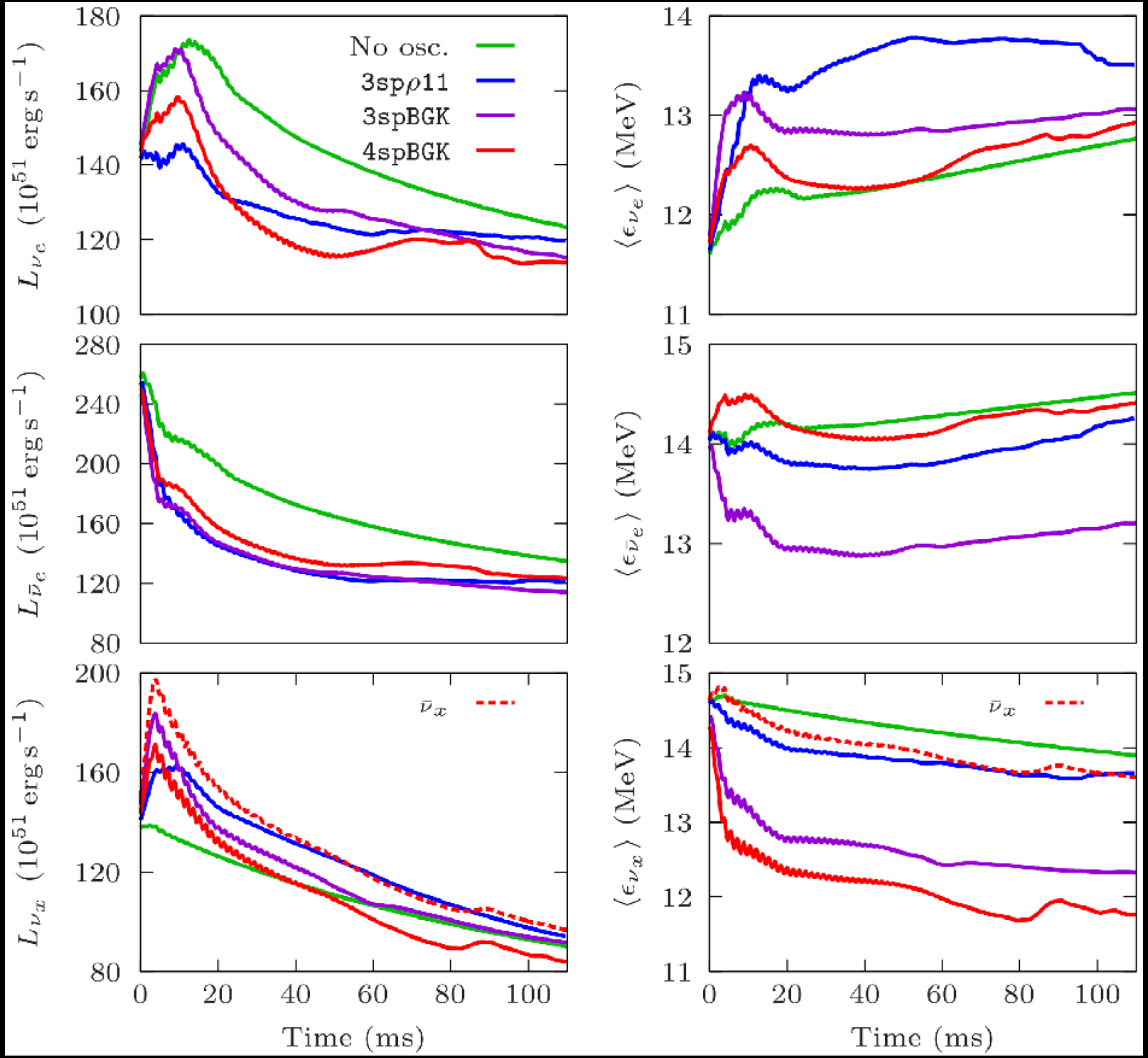


Neutrino Emission Properties (Fiducial)



- 3spBGK and 4spBGK show deviation; the growth rate is overestimated in 3 species case
- 3spBGK and 3spρ11 show reasonable agreement: the former assume flavor equipartition for the outgoing neutrinos, resulting in similar neutrino emission properties with the latter

Neutrino Emission Properties (low-Y_e model)



3spBGK and 3spρ11 also deviate;

the difference of the ingoing neutrinos affect hydro profiles

Difference of energy spectra w.r.t no FFC model

Time Discretization Dependence

Time step width: $10^{-9} \sim 10^{-7}$

Explicit method

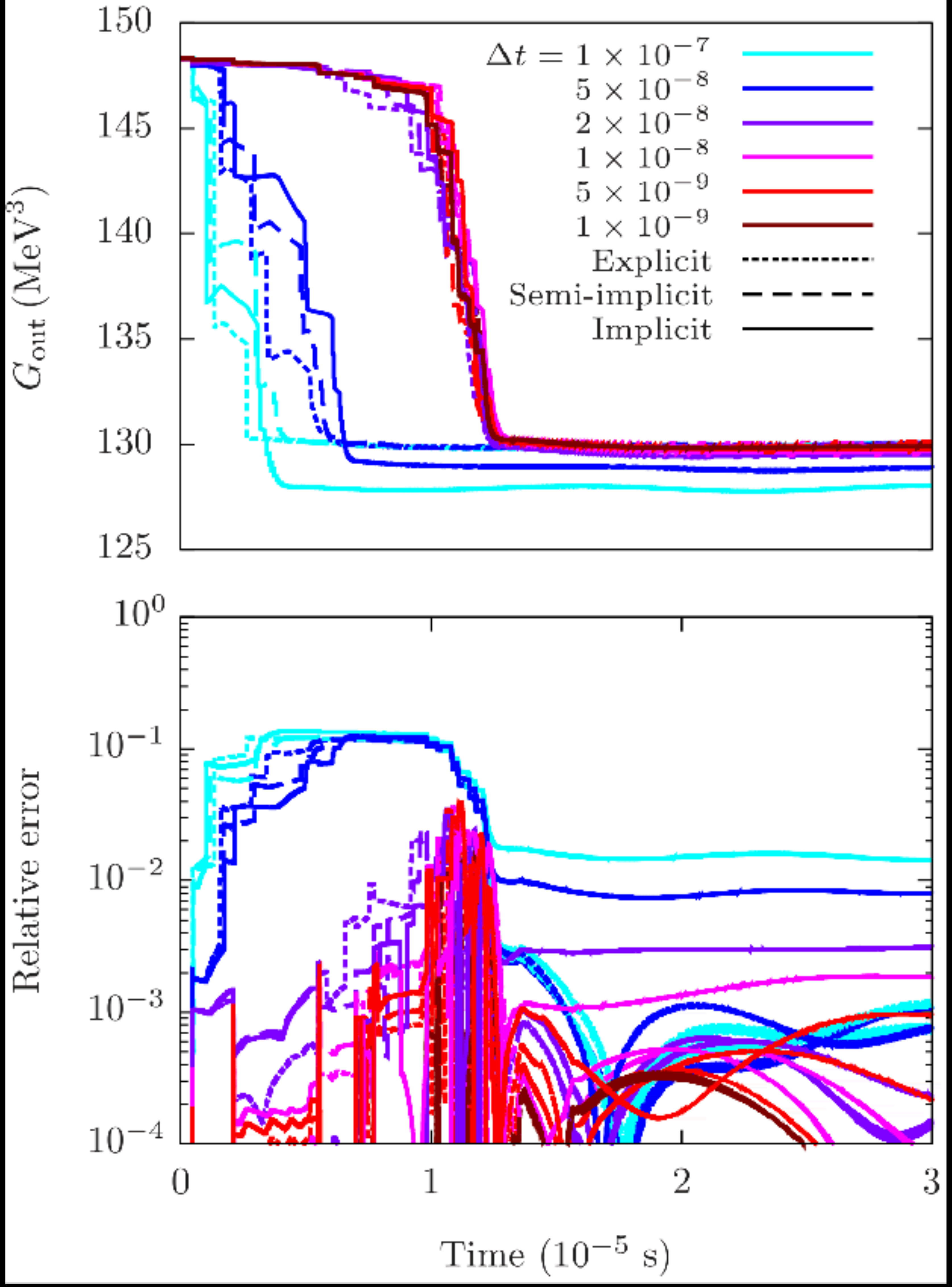
$$f^{n+1} = f^* - \frac{\Delta t}{\tau_{as}}(f^* - f^{as})$$

Semiimplicit method (f^{as} evaluated explicitly)

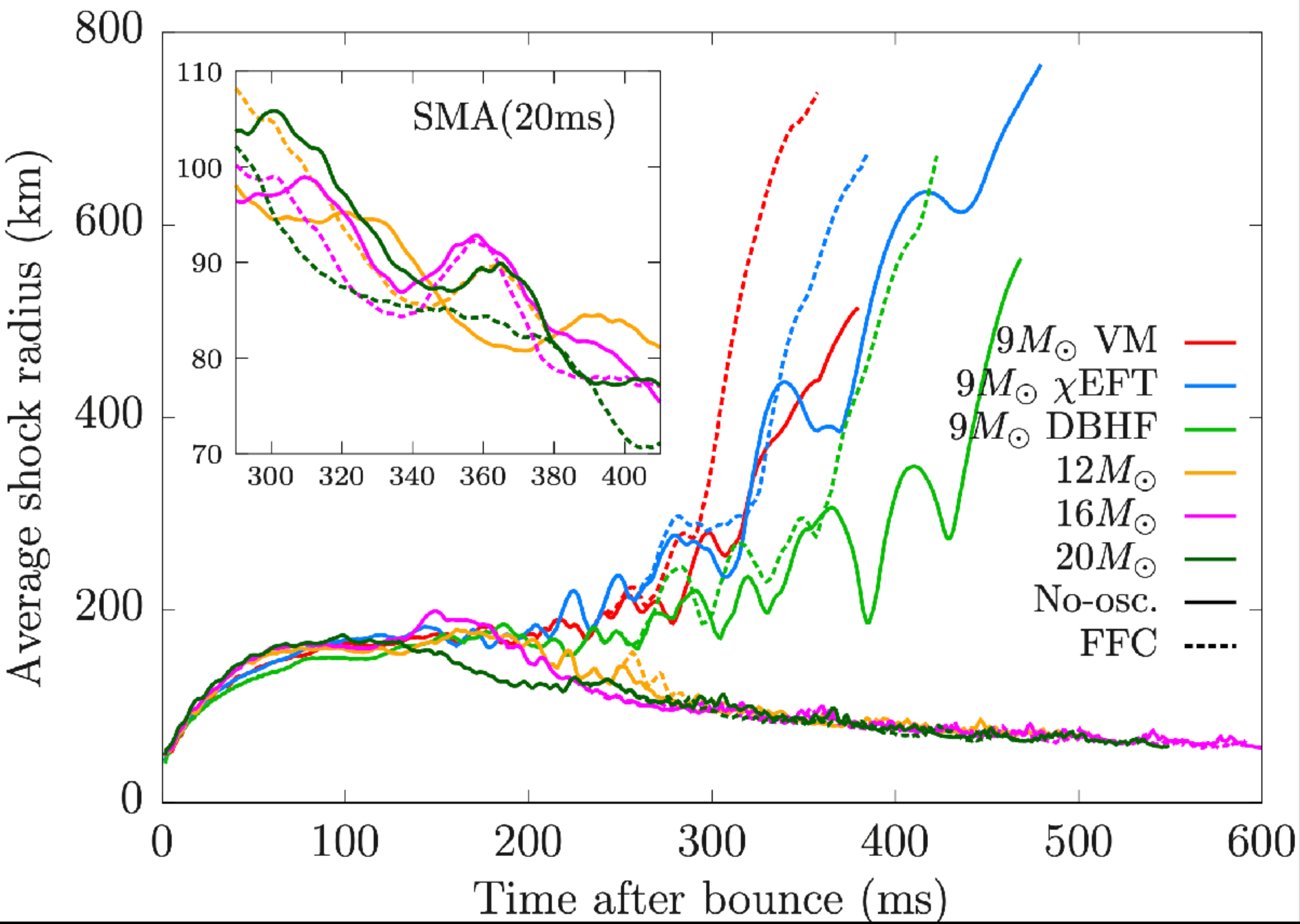
$$f^{n+1} = \left(\frac{1}{\Delta t} + \frac{1}{\tau_{as}}\right)^{-1}\left(\frac{f^*}{\Delta t} + \frac{f^{as}}{\tau_{as}}\right)$$

Implicit method

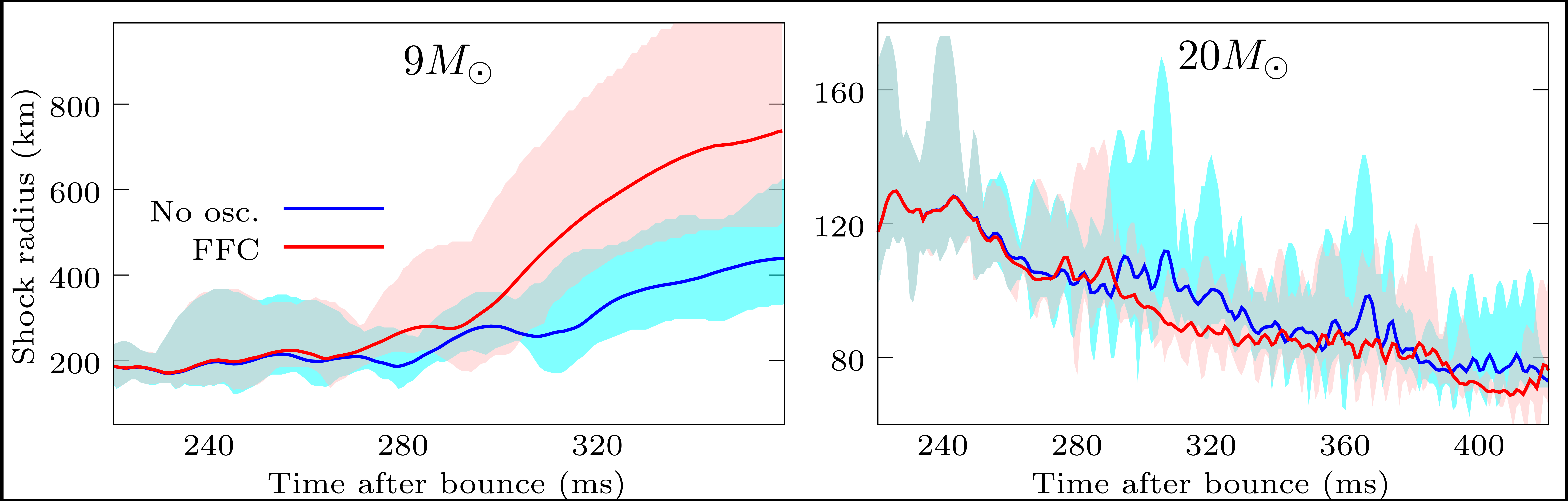
$$\begin{pmatrix} f_e^{n+1} \\ f_x^{n+1} \end{pmatrix} = \begin{pmatrix} \tau^{as} + \Delta t(1-\eta) & -\Delta t(1-\eta) \\ -\Delta t(1-\eta) & \tau^{as} + \Delta t(1-\eta) \end{pmatrix}^{-1} \begin{pmatrix} f_e^* \\ f_x^* \end{pmatrix}$$



Part II. 2D CCSN simulations with FFC



Successful model: FFC has positive effect Failed model: FFC has nagative effect



exploding model: small $\dot{M}$ ➡ weak ν_e emission failed model: large $\dot{M}$ ➡ strong ν_e emission

exploding model: small $\dot{M}$ ➡ weak ν_e emission   failed model: large $\dot{M}$ ➡ strong ν_e emission

exploding model: small $\dot{M}$ → weak ν_e emission   failed model: large $\dot{M}$ → strong ν_e emission

$L_e \sim L_x$

→ Decrease of ν_e by FFC is minor

$\epsilon_e \ll \epsilon_x$

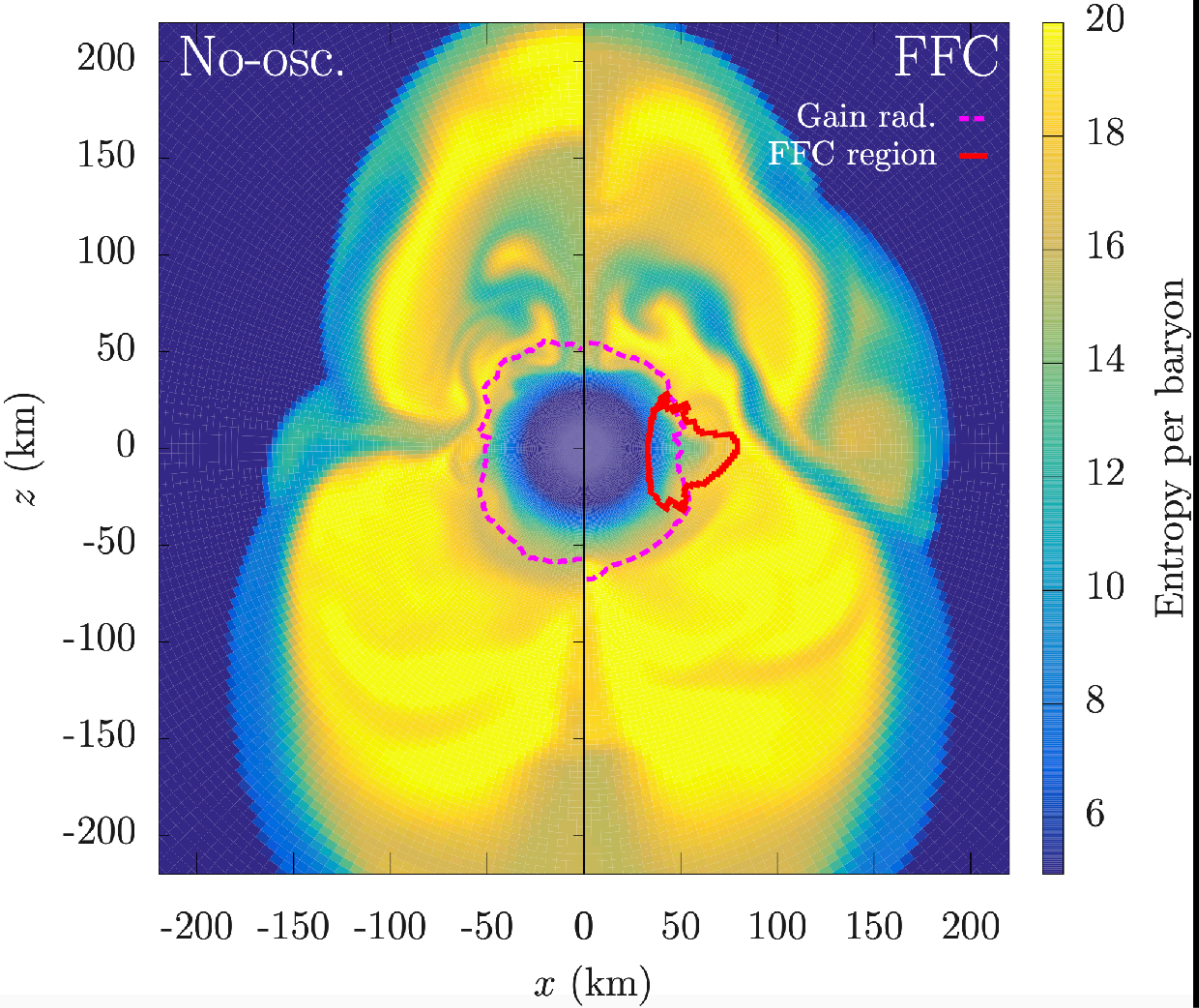
→ ν_e mean energy enhanced by FFC

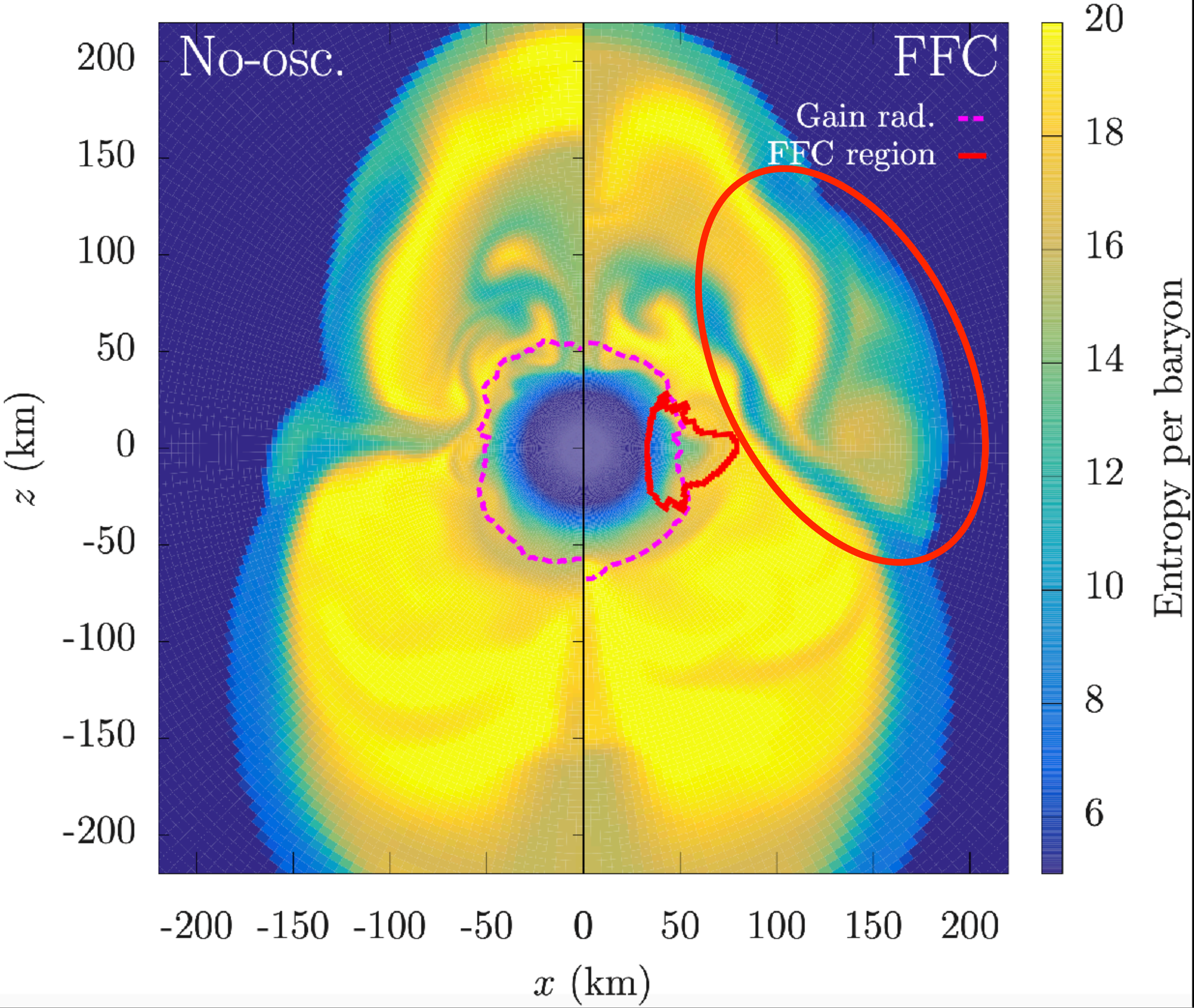
$L_e \gg L_x$

→ ν_e is significantly reduced by FFC

$\epsilon_e \sim \epsilon_x$

→ mean energy change is minor


No-osc.
FFC
Gain rad.
FFC region
200
150
100
50
0
-50
-100
-150
-200
z (km)
-200 -150 -100 -50 0 50 100 150 200
x (km)
20
18
16
14
12
10
8
6
Entropy per baryon


No-osc.
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-200
z (km)
-200 -150 -100 -50 0 50 100 150 200
x (km)
20
18
16
14
12
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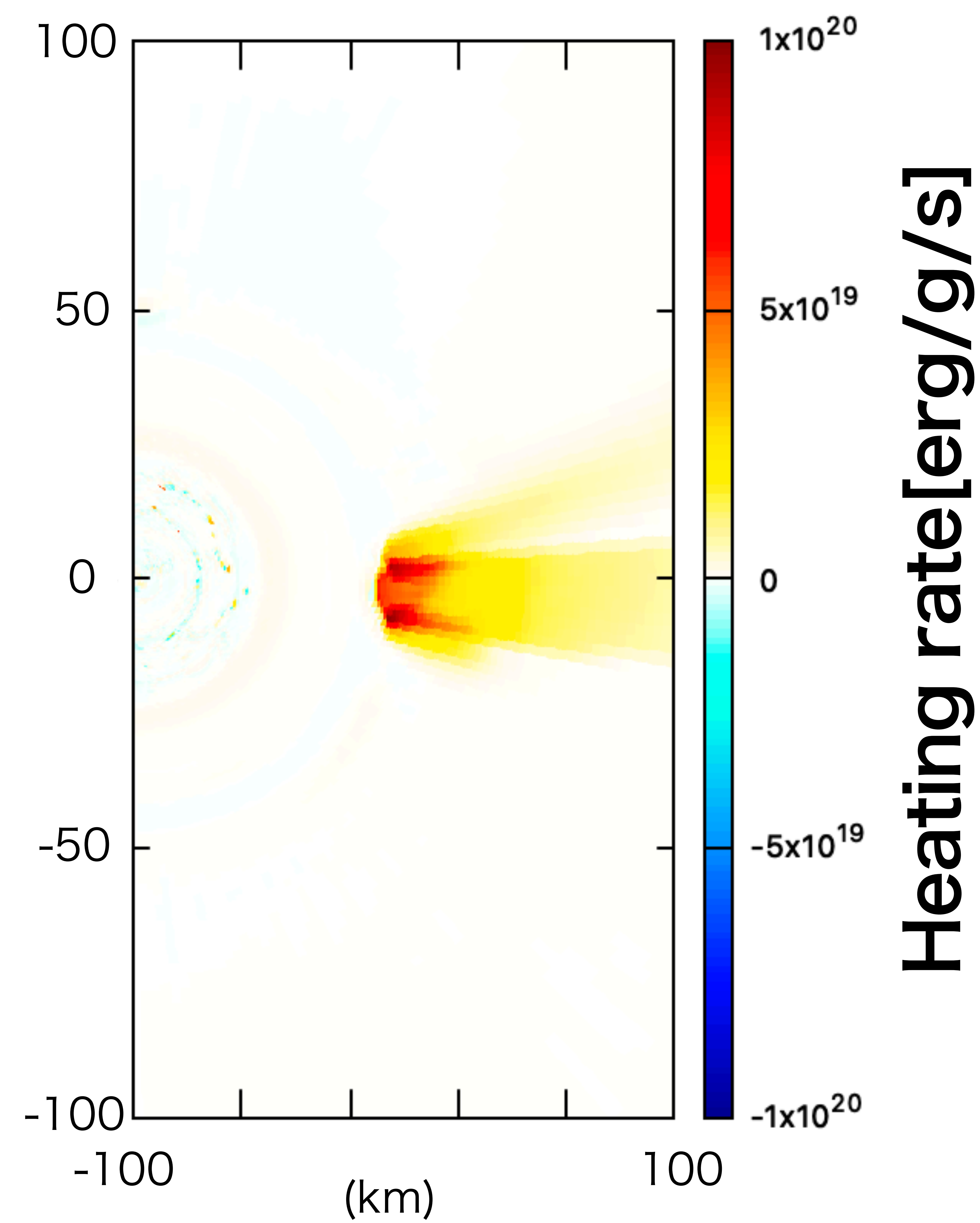
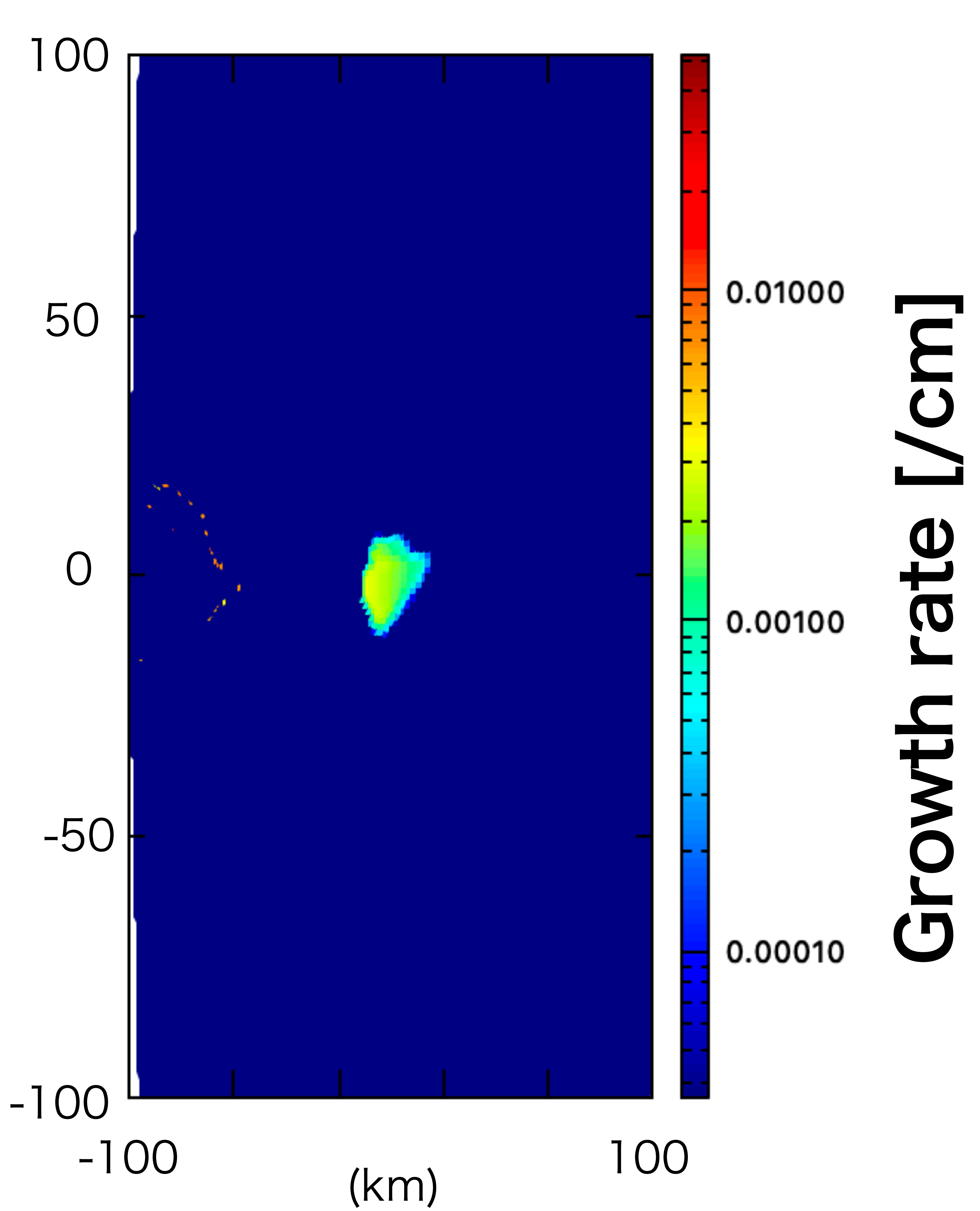
8
6
Entropy per baryon

Difference of heating rate

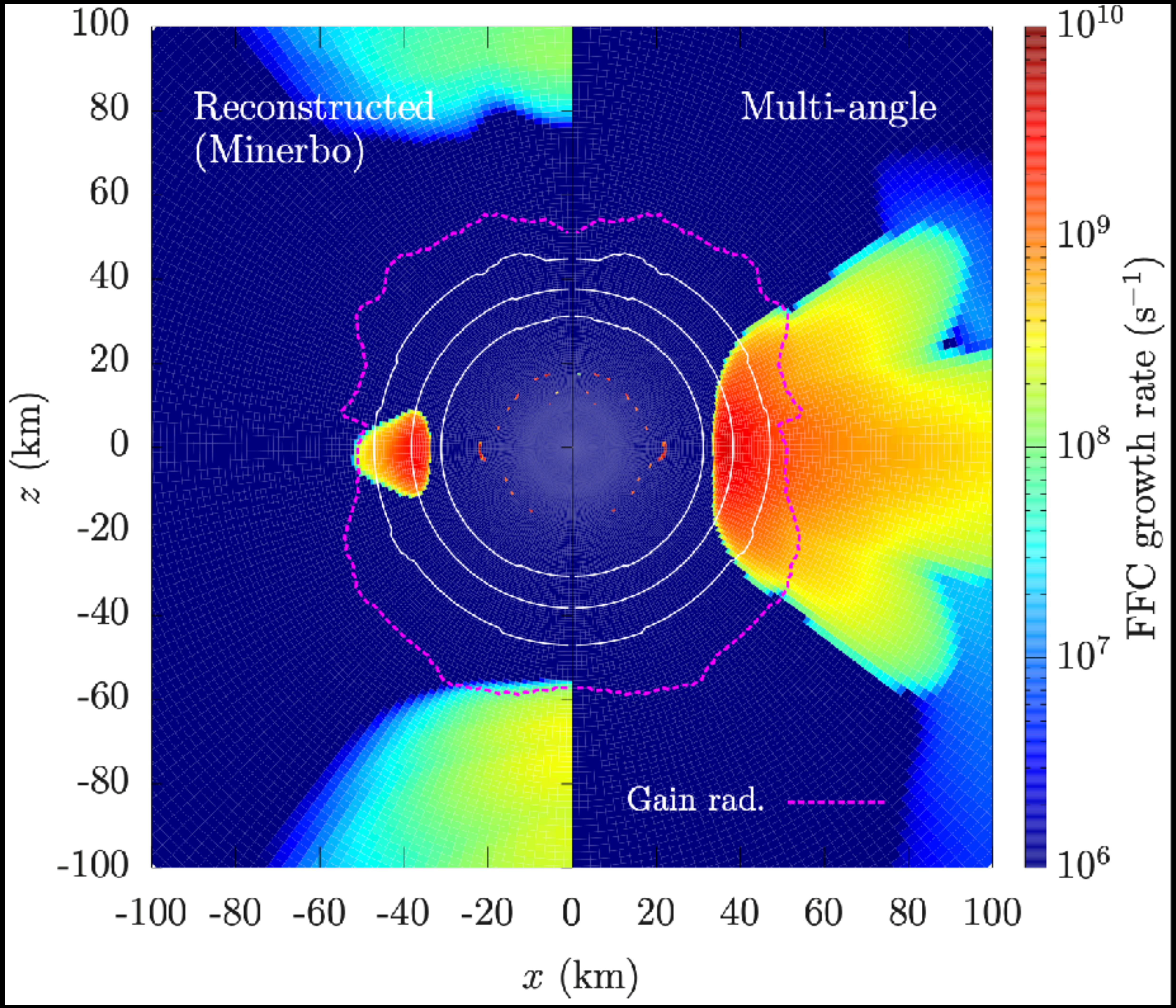


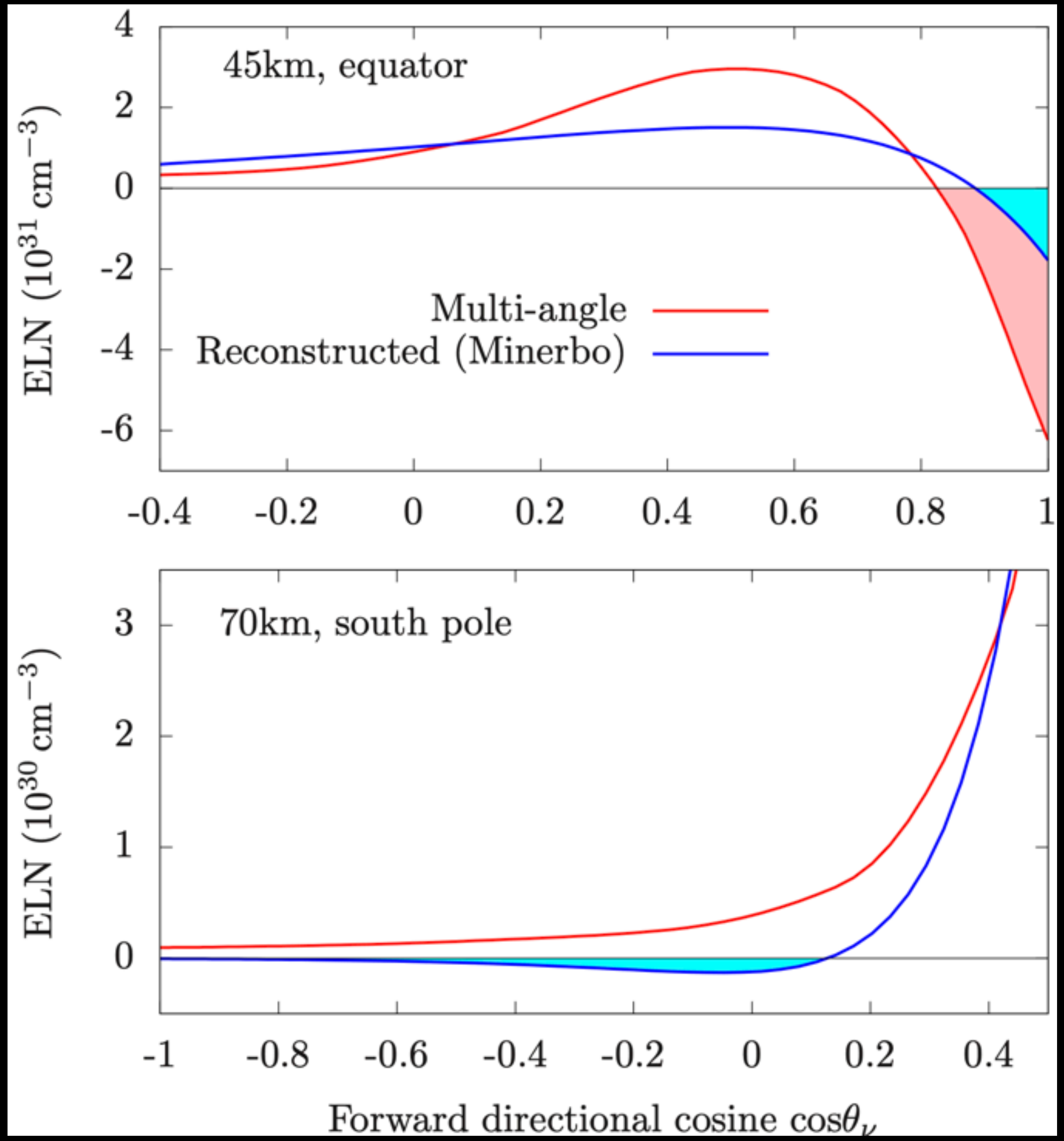
FFC growth rate



Performance of the Angular Reconstruction from Moments

Minerbo reconstruction from the moments greatly underestimate crossings

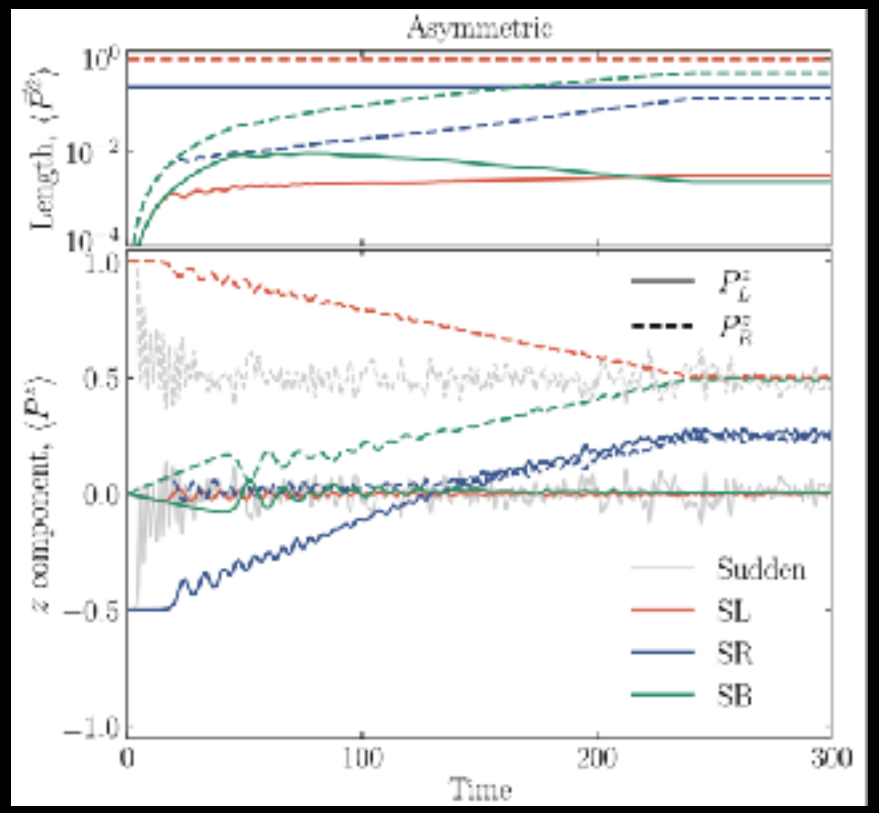




Discussion

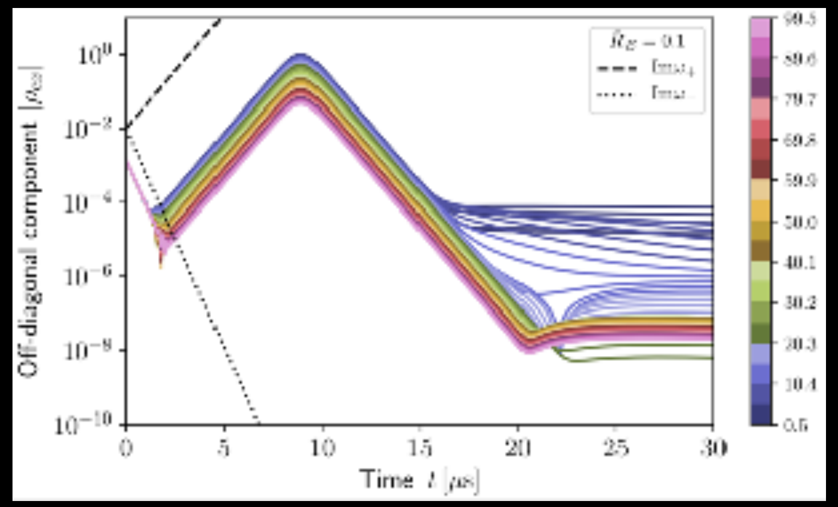
- Does this “bifurcated impact” change the boundary between failed and succesful models?
- Is the asymptotic state always the elimination of crossing? Maybe not

Fiorllio 2024

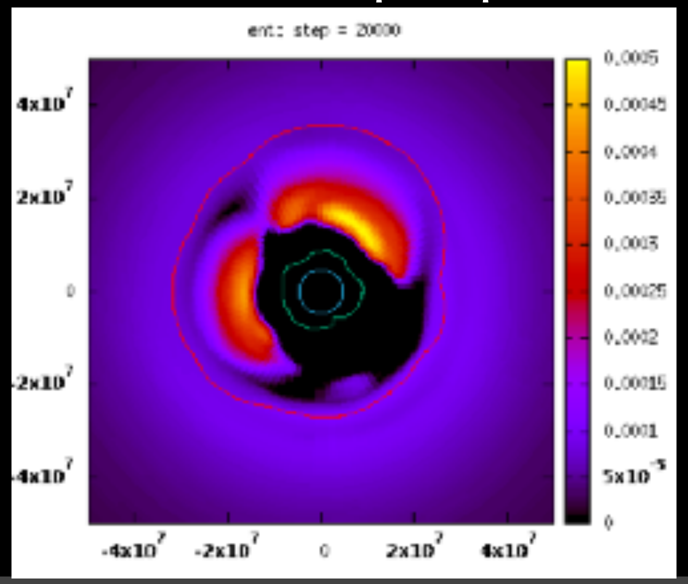


- Effects of collisional flavor instability (intermediate state? asymptotic state?)

Zaizen 2025



Iwakami in prep.



- What about in 3D?