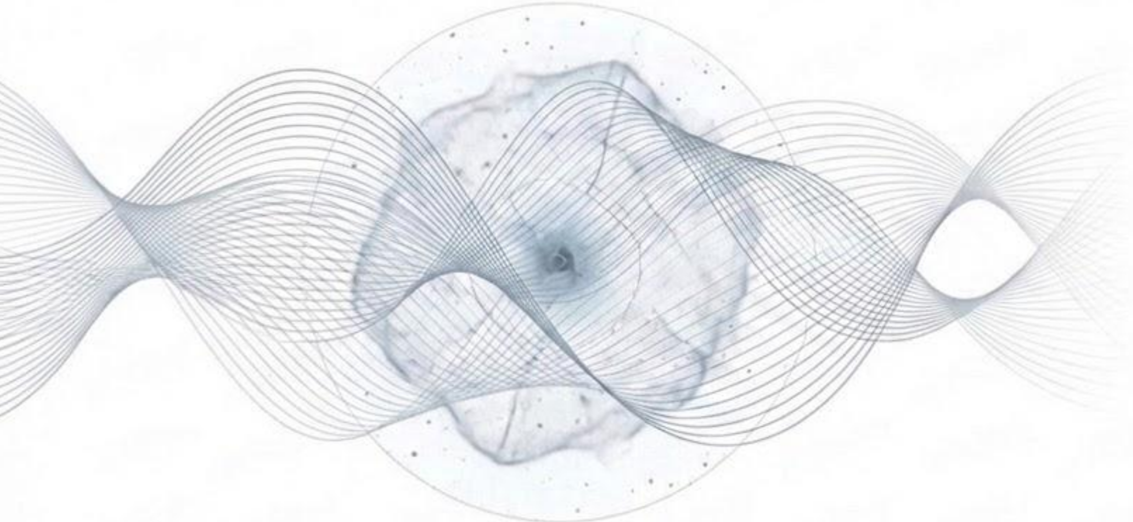
Exploring the Effects of Collective Flavor Oscillation in Core-Collapse Supernova Simulations



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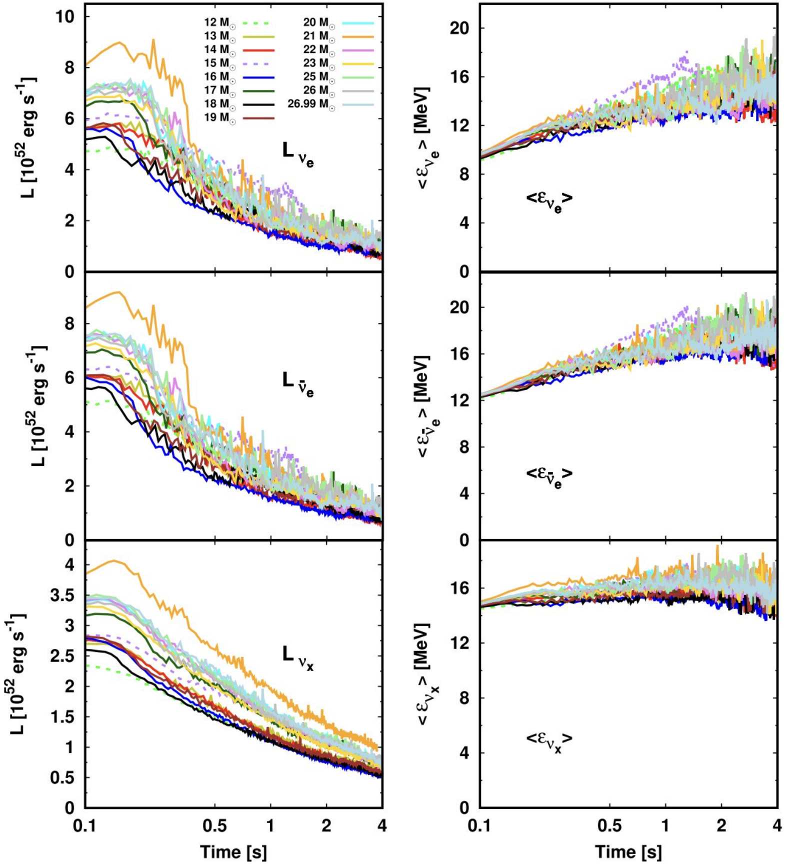
Nagakura+2021

CCSN Neutrino Signals Without Oscillation

Collective flavor conversion could operate faster than CCSN dynamical timescales.

If mixed, i.e., e ⇄ x and anti-e ⇄ anti-x, it will lead to significant changes in neutrino luminosities and spectra. How will CCSN react to such changes?



Challenge: High Computational Cost

Classic CCSN simulations are already quite expensive:

- One-second 3D simulations take ~5,000,000 CPU hours / ~50,000 GPU hours.
- Many observables need many seconds to approach their asymptotic values.

Including collective neutrino conversion (CFC) means:

Short Timescale

- ~1000x shorter timesteps

Neutrino Distribution

- 10~100x more resolved phase space distribution

Quantum Nature

- Density Matrix, 2~3x more variables

Ehring+2023, Akaho+2024, Mori+2025

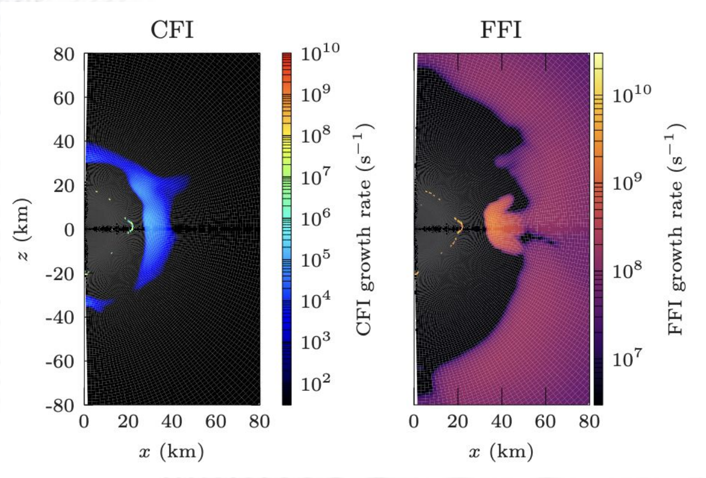
Accuracy

- Instability Detection
- Boltzmann transport
- QKE

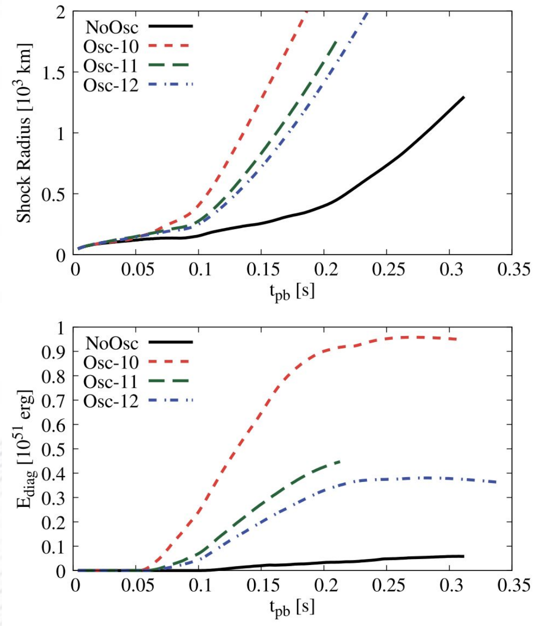
Trade-Off

Affordability

- Phenomenological method with free parameters



Can We Do Something In-Between?



Flavor Equilibrium Estimator + BGK Scheme in Fornax

If instability grows much faster than the CCSN dynamic timescale, a “flavor equilibrium” state is likely to be established. This assumption works for the fast-flavor instability and the collisional instability.

Knowing the growth rates and flavor equilibrium state, the flavor conversion effects can be included in classic CCSN simulations via a BGK scheme:

$$\mathcal{F}'_{\nu_\alpha} - \mathcal{F}_{\nu_\alpha} = -(1 - e^{-\sigma \Delta t})(\mathcal{F}_{\nu_\alpha} - \mathcal{F}^{\mathrm{Equ}}_{\nu_\alpha}) \, .$$

A parameter-free phenomenological recipe with very good affordability, correct instability condition, and reasonable final states. The BGK scheme provides a smooth transition between stable/unstable regions. Suitable for CFC and FFC that have high growth rates.

Example 1:
Collisional Flavor Conversion in CCSNe

Growth Rate Calculation

Isotropic (k=0) CFI $$\mathrm{I} \equiv \sqrt{2}G_F \int_{-\infty}^{\infty} \frac{E^2 dE}{2\pi^2} \frac{f_{\nu_e}(E) - f_{\nu_x}(E)}{\Omega + i\Gamma(E)} = -1 \text{ or } 3$$

If monochromatic: $$f_{\nu_e} - f_{\nu_x} = \frac{2\pi^2}{\sqrt{2}G_F E^2}\left[g\delta(E-\varepsilon) - \bar{g}\delta(E-\bar{\varepsilon})\right],$$

we get $$\frac{g}{\Omega + i\Gamma} - \frac{g}{\Omega + i\bar{\Gamma}} = -1 \text{ or } 3$$ Easy to solve!

Therefore, the monochromatic approximation is widely used in multi-group cases to simply calculation:

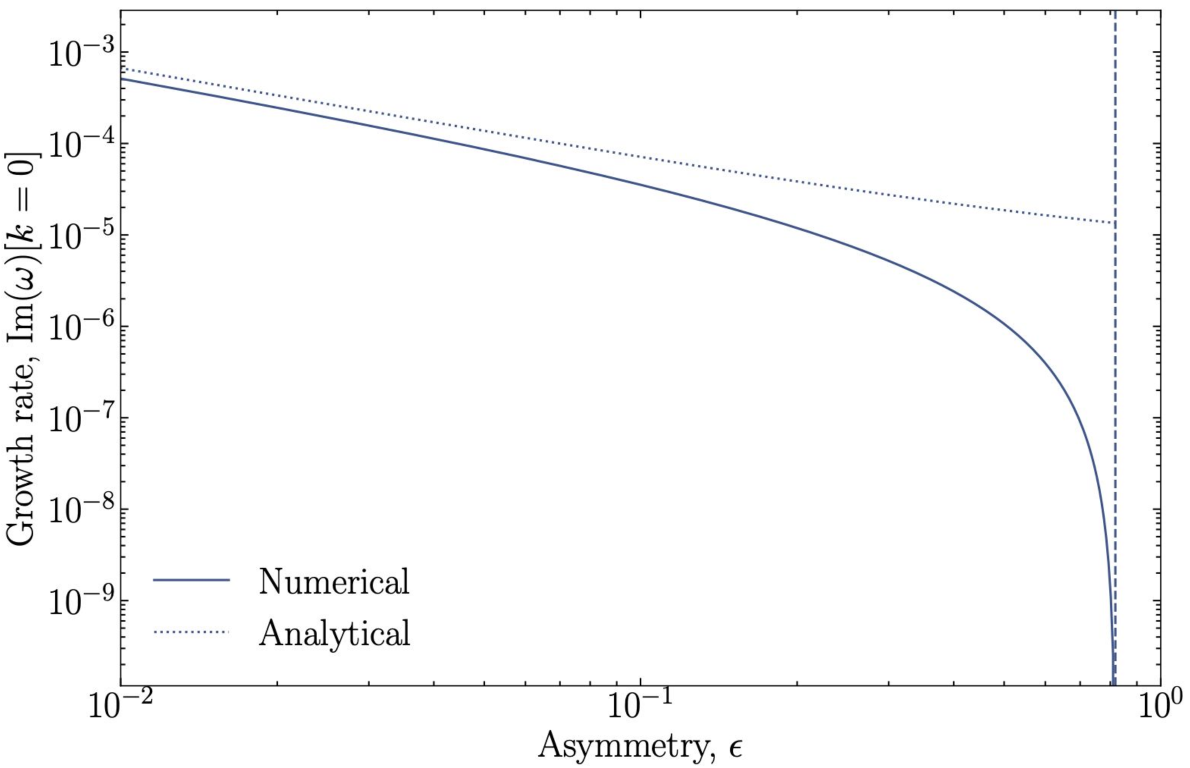
$$g = \sqrt{2}G_F(n_{\nu_e} - n_{\nu_x})$$
$$\bar{g} = \sqrt{2}G_F(n_{\bar{\nu}_e} - n_{\bar{\nu}_x})$$

$$\Gamma = \frac{\int_0^\infty E^2 dE[f_{\nu_e}(E) - f_{\nu_x}(E)]\Gamma(E)}{\int_0^\infty E^2 dE(f_{\nu_e}(E) - f_{\nu_x}(E))}$$
$$\bar{\Gamma} = \frac{\int_0^\infty E^2 dE[f_{\bar{\nu}_e}(E) - f_{\bar{\nu}_x}(E)]\bar{\Gamma}(E)}{\int_0^\infty E^2 dE(f_{\bar{\nu}_e}(E) - f_{\bar{\nu}_x}(E))}$$

And the monochromatic growth rates are

$$\mathrm{Im}(\Omega_+) = \frac{\Gamma\bar{g} - \bar{\Gamma}g}{g - \bar{g}}$$
$$\mathrm{Im}(\Omega_-) = -\frac{\Gamma g - \bar{\Gamma}\bar{g}}{g - \bar{g}}$$

However, the Approximation is Problematic in Some Cases



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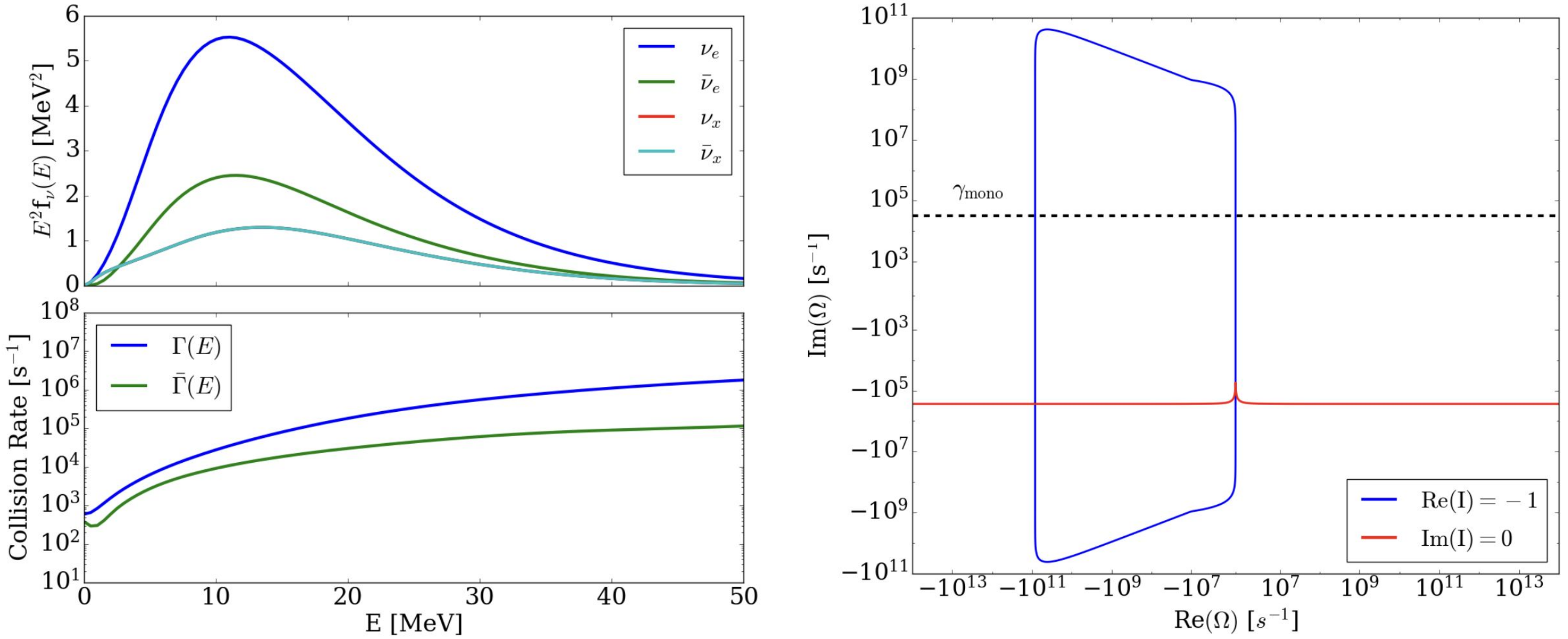
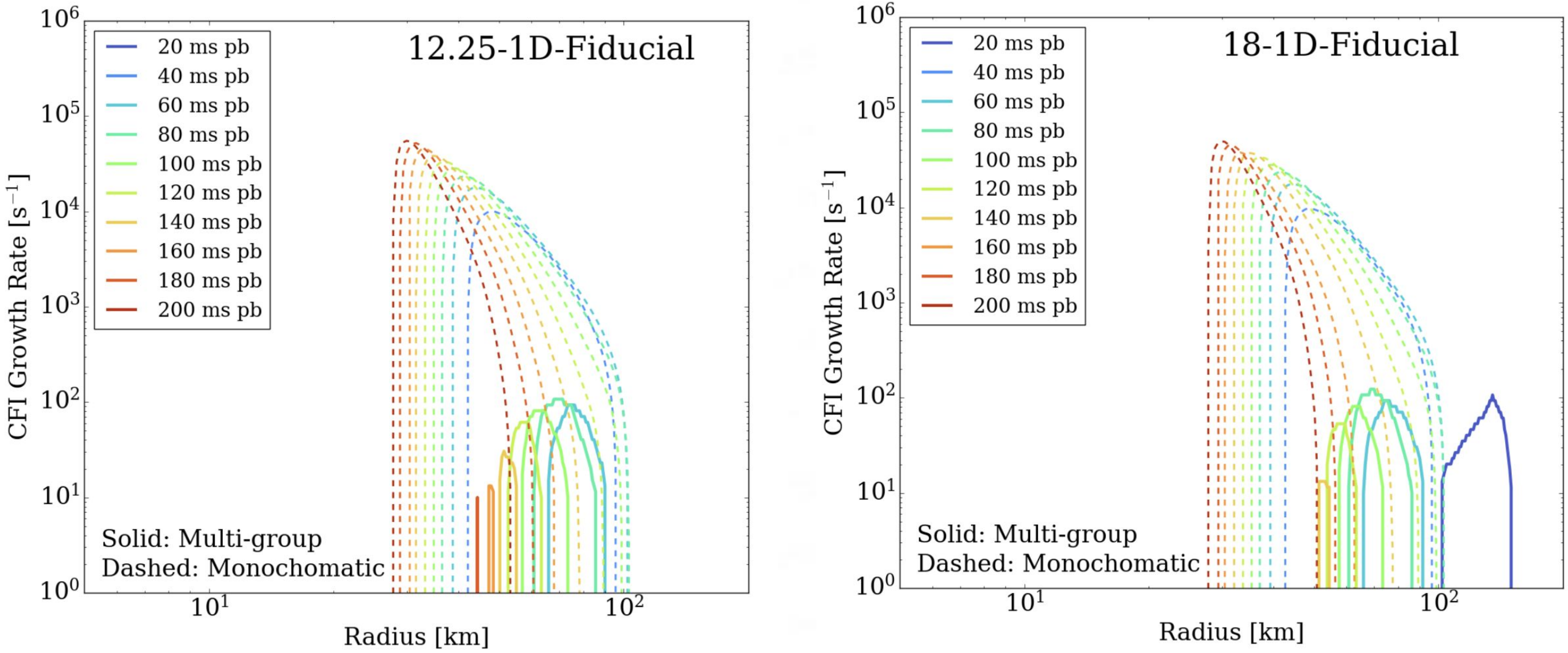


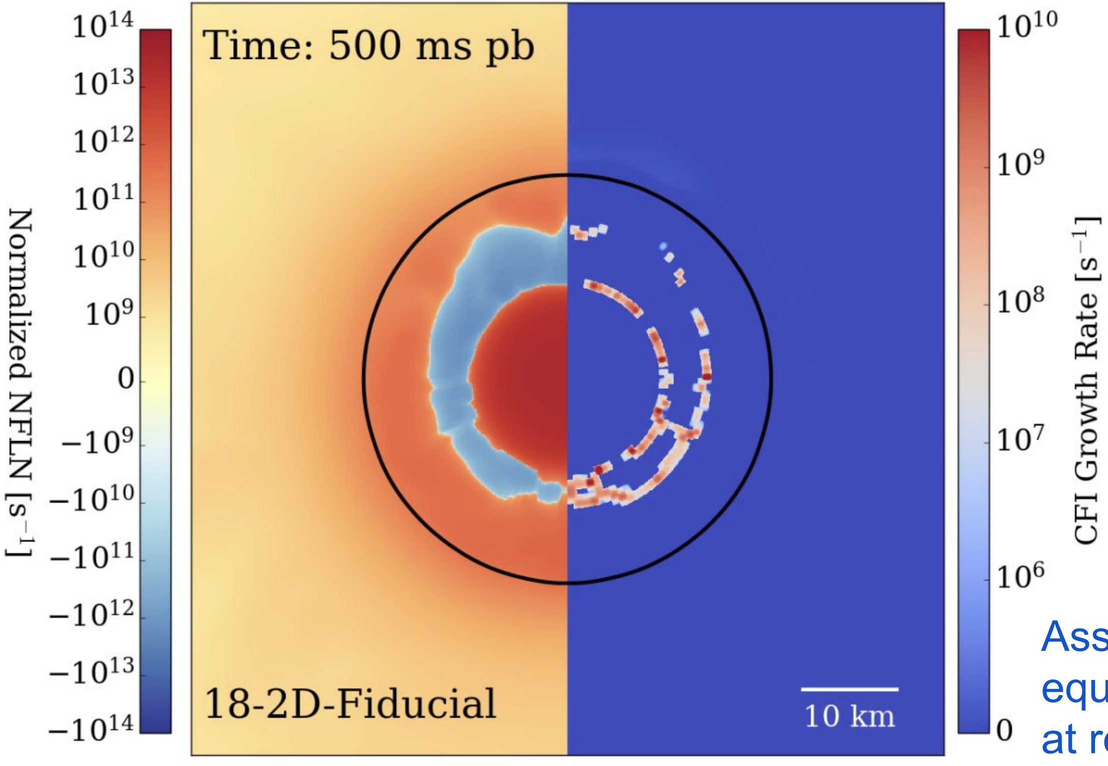
How often does this behavior occur in CCSNe?

Non-Resonance CFI Growth Rates Were Overestimated



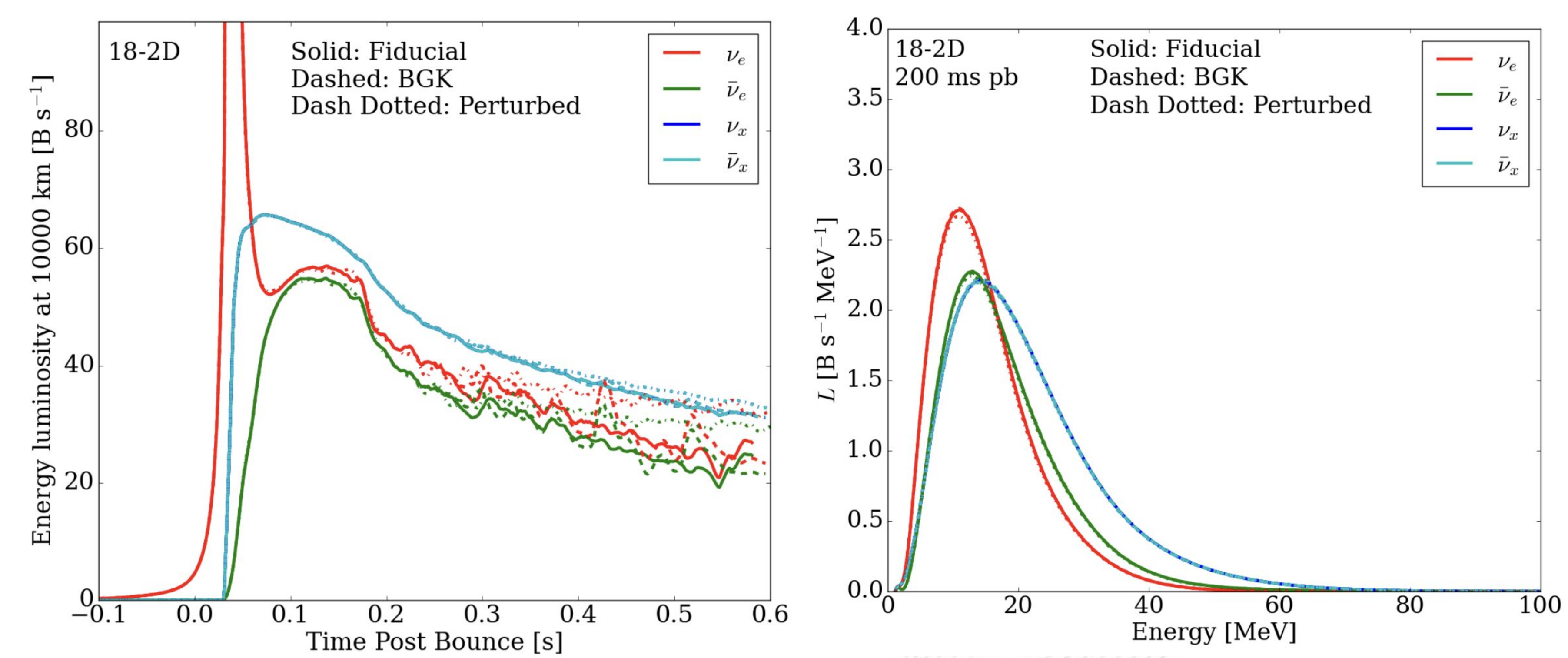
The CFI growth in non-resonance regions is actually much slower than dynamical or light travel time, and can be ignored!

Resonance-Like CFI Is Still There



Assume instant flavor equipartition/flavor swap at resonance, we get …

Luminosity and Spectra with Resonance-Like CFI



Take-Away

The widely-used monochromatic approximation overestimates the CFI growth rates in non-resonance regions by almost three orders of magnitudes.

When the correct multi-group method is used, non-resonance CFI grows much slower than light travel time or CCSN dynamical timescale, and can be ignored.

The resonance-like CFI is still there, but it occurs at too small radii and neutrino-matter interactions erase its effects.

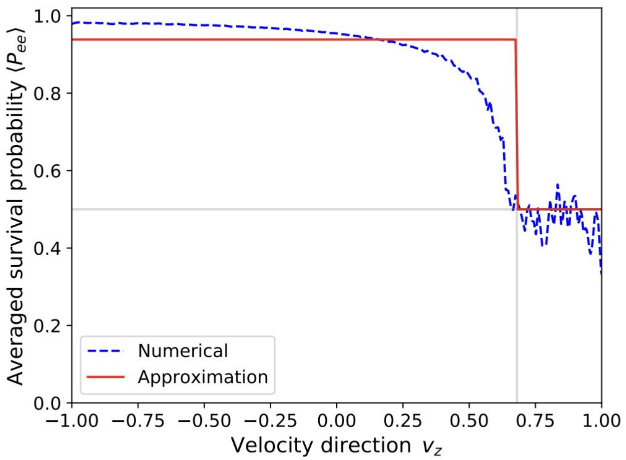
The CFC impact on CCSN outcomes (dynamics and neutrinos) is significantly weaker than the effects of intrinsic randomness of chaotic turbulence in CCSNe.

Example 2:
Fast Flavor Conversion in CCSNe

Zaizen+2023, Richers+2024

The Flavor Equilibrium Estimator: Box3D

Assume piecewise constant conversion probability in angular space and respect the conversions, and capture major behaviors of FFC.



Workflow

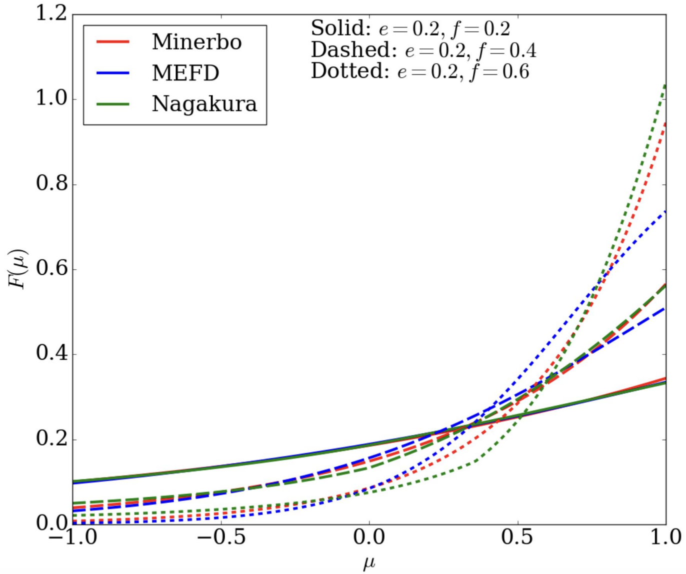
1. Get energy and flux from M1
2. Reconstruct angular distributions
3. Perform FFC calculations
4. Derive energy and flux from updated distributions.

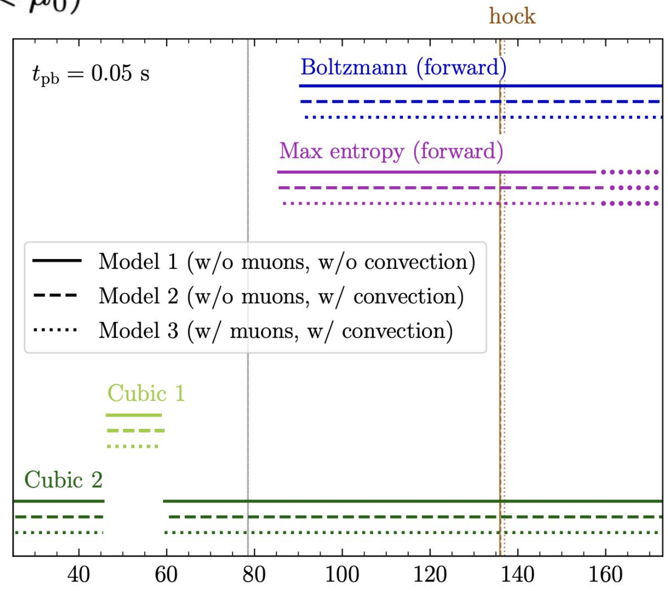
$$P(\hat{n}) = \begin{cases} \frac{1}{3} & I_- < I_+, \hat{n} \in \Gamma_-, \\ 1 - \frac{2I_-}{3I_+} & I_- < I_+, \hat{n} \in \Gamma_+, \\ \frac{1}{3} & I_- > I_+, \hat{n} \in \Gamma_+, \\ 1 - \frac{2I_+}{3I_-} & I_- > I_+, \hat{n} \in \Gamma_-, \end{cases}$$

Wang+2025, Cornelius+2025

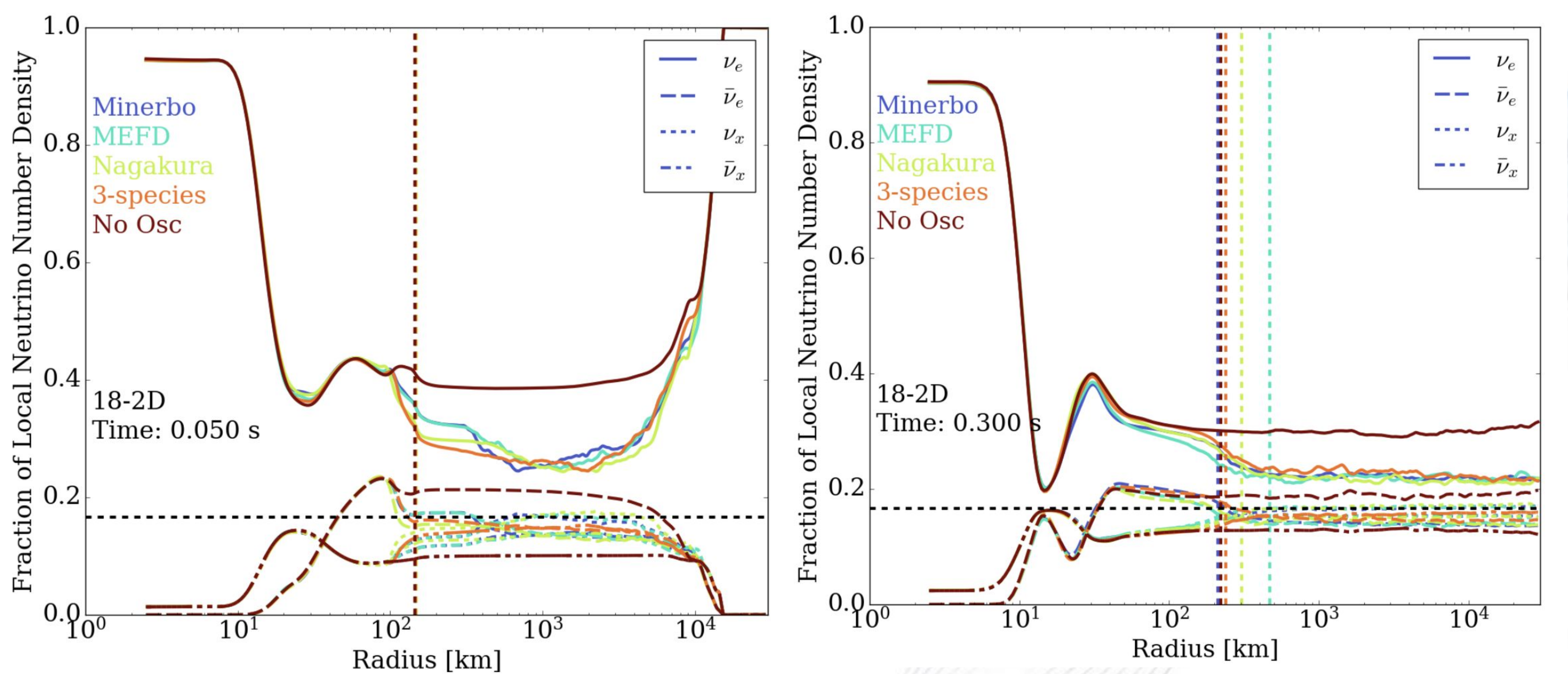
Different Angular Reconstruction Methods

- The Minerbo closure: $\mathcal{F}(\mu) = \exp(-\eta + \alpha\mu)$
- The maximum entropy Fermi-Dirac (MEFD) closure: $\mathcal{F}(\mu) = (1 + \exp(\eta - \alpha\mu))^{-1}$
- The Nagakura closure: $\ln \mathcal{F}(\mu) = \begin{cases} a\mu^2 + b\mu + c & (\mu > \mu_0) \\ d\mu^2 + g\mu + h & (\mu < \mu_0) \end{cases}$

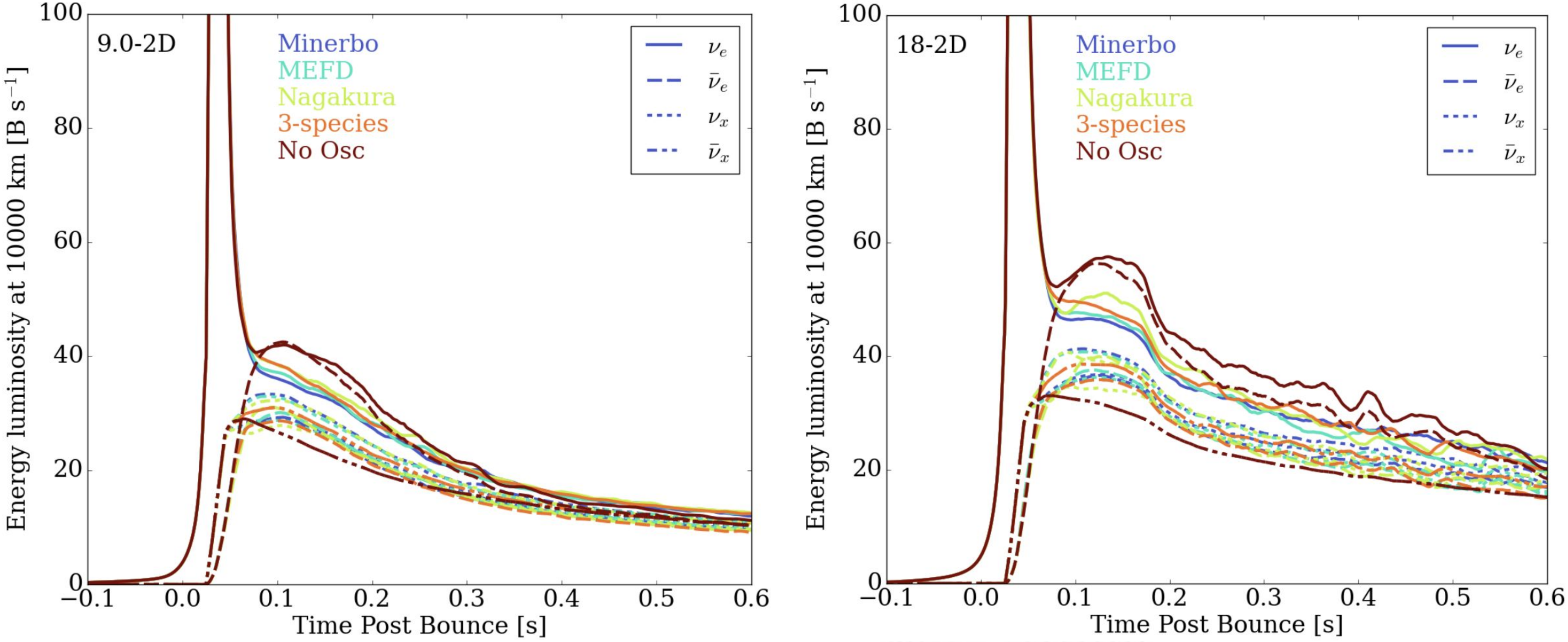




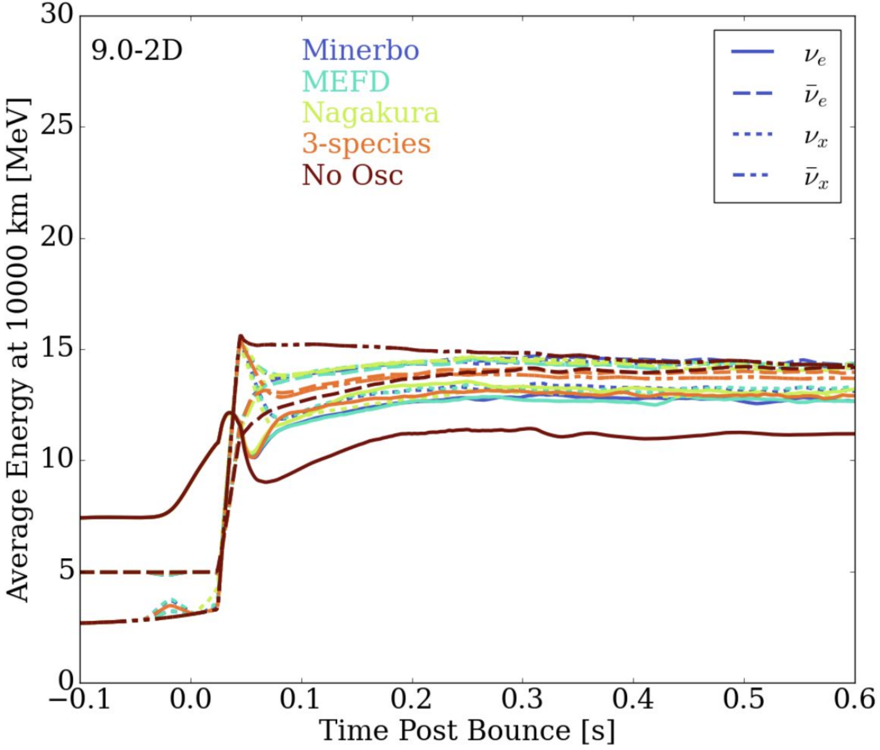
Where is the FFC Operating?

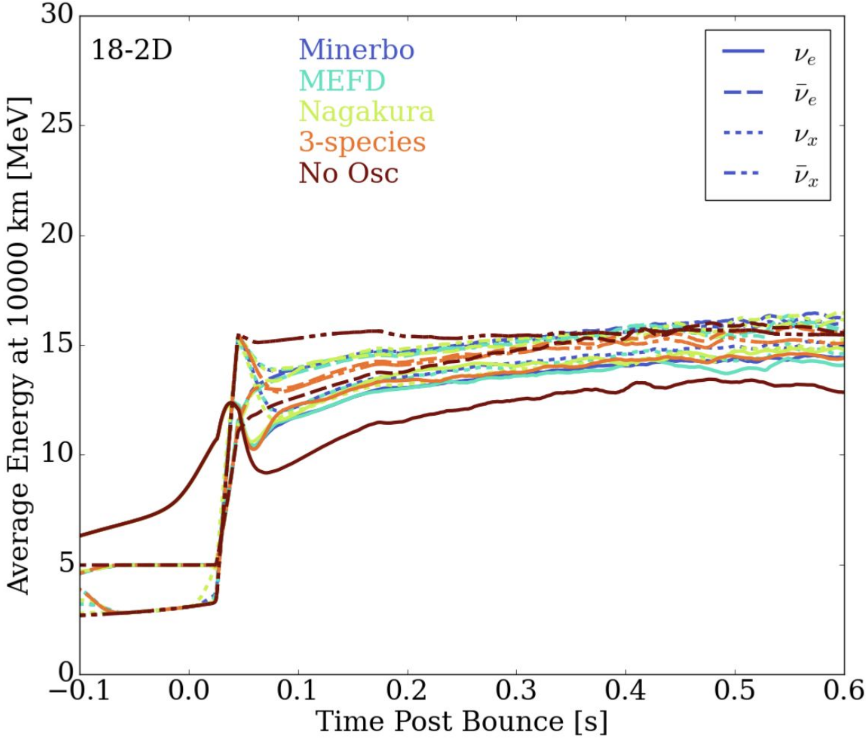


Neutrino Luminosity

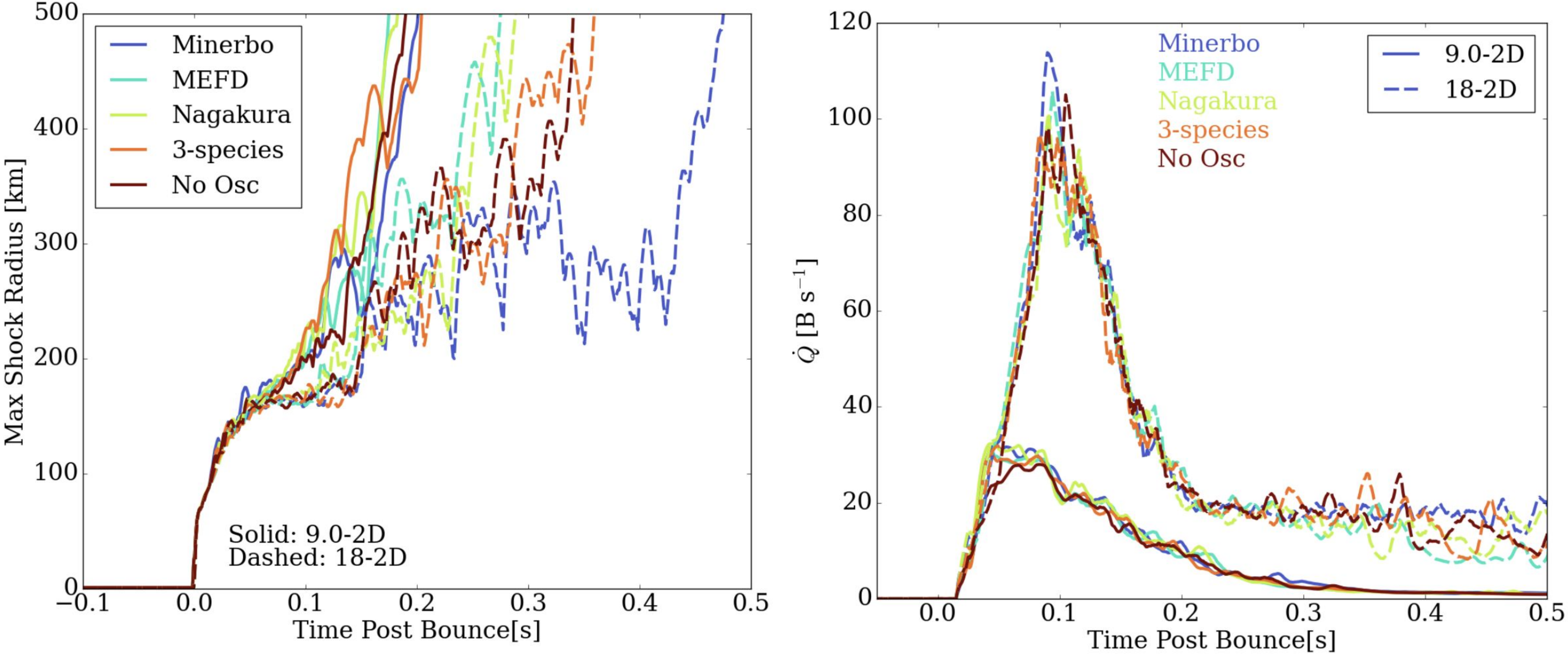


Average Neutrino Energy



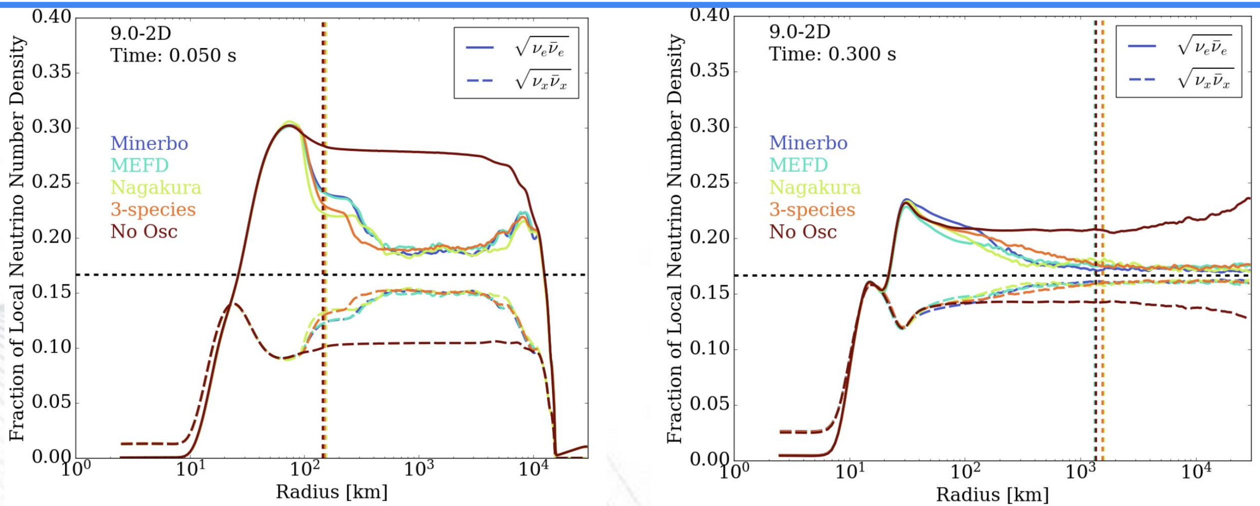


Shock Radius and Heating Rate



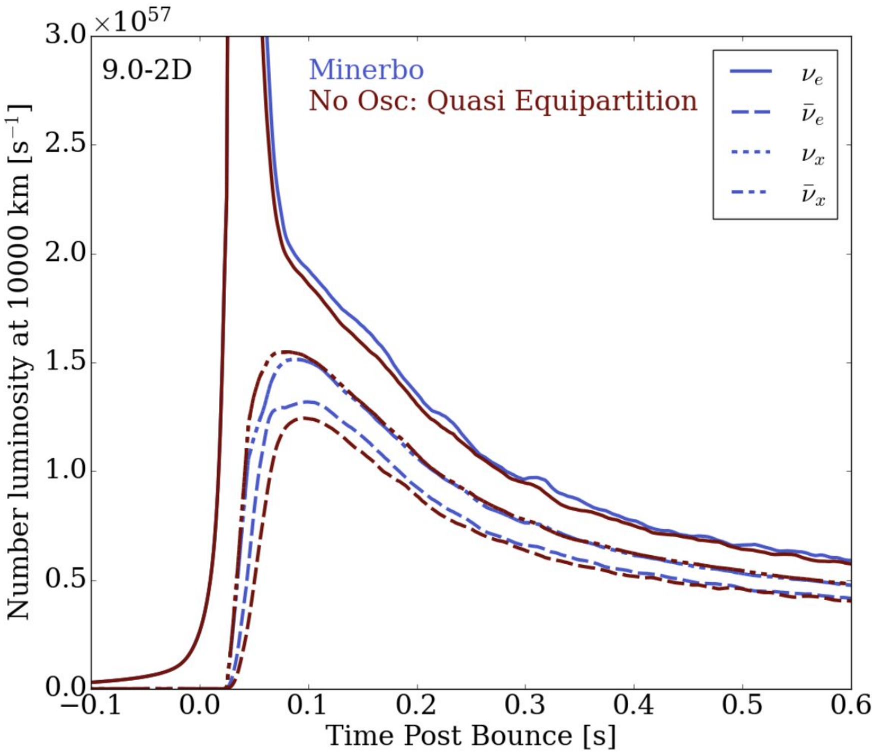
Predict the FFC-Modulated Neutrino Signal

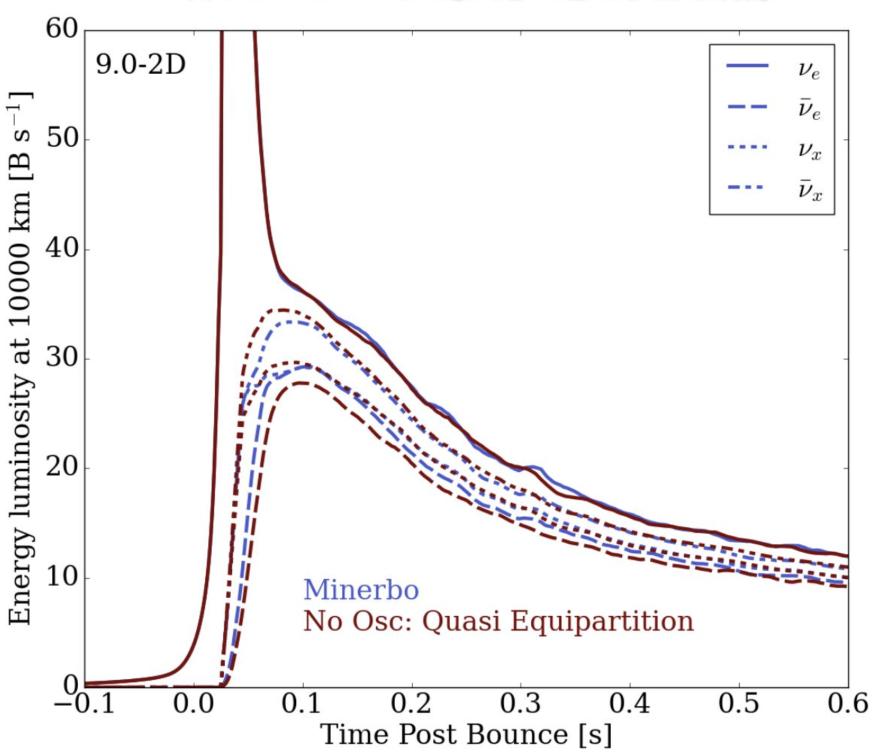
- Neutrino number conservation $N_{\nu_e}^{\mathrm{FFC}} + 2N_{\nu_x}^{\mathrm{FFC}} = N_{\nu_e} + 2N_{\nu_x} = N$
- Anti-neutrino number conservation $N_{\bar{\nu}_e}^{\mathrm{FFC}} + 2N_{\bar{\nu}_x}^{\mathrm{FFC}} = N_{\bar{\nu}_e} + 2N_{\bar{\nu}_x} = \bar{N}$
- Total ELN-XLN conservation $N_{\nu_e}^{\mathrm{FFC}} - N_{\bar{\nu}_e}^{\mathrm{FFC}} - 2N_{\nu_x}^{\mathrm{FFC}} + 2N_{\bar{\nu}_x}^{\mathrm{FFC}} = N_{\nu_e} - N_{\bar{\nu}_e} - 2N_{\nu_x} + 2N_{\bar{\nu}_x} = N_{\mathrm{ELN}}$
- Assumption: $\sqrt{N_{\nu_e}^{\mathrm{FFC}} N_{\bar{\nu}_e}^{\mathrm{FFC}}} = \sqrt{N_{\nu_x}^{\mathrm{FFC}} N_{\bar{\nu}_x}^{\mathrm{FFC}}}$, i.e., flavor equipartition in the sense of geometric mean,



Compared to Simulations

*Actual observed neutrino signal will be further modulated by other effects like MSW.





Take-Away

FFC is commonly seen in CCSN simulations, but their impact on hydrodynamics is weak.

FFC significantly modulates the neutrino signals, but mostly operating at radii of hundreds to thousands of kilometers, decoupled from background matter.

The above conclusion is insensitive to the details in FFC implementation (e.g., angular reconstruction method, 3- vs 4-species, etc).

The quasi-equipartition model provides a good estimation for the FFC-modulated neutrino energy/number luminosities.

Future Work and Conclusion

Joint Analysis of Instabilities

General Dispersion Relation: $\det\left(\varepsilon^{\mu}_{\nu}\right) = 0$

$$\varepsilon^{\mu}_{\nu} = \begin{pmatrix} 1-\mu I_0 & \mu I_1 & 0 & 0 \\ \mu I_1 & -1-\mu I_2 & 0 & 0 \\ 0 & 0 & -1-\frac{\mu}{2}(I_0-I_2) & 0 \\ 0 & 0 & 0 & -1-\frac{\mu}{2}(I_0-I_2) \end{pmatrix},$$

$$I_n = \int \frac{G(v,E)v^n dv dE}{\omega - kv - \tilde{\omega}_E + i\Gamma_E + i\epsilon} - \int \frac{\overline{G}(v,E)v^n dv dE}{\omega - kv + \tilde{\omega}_E + i\overline{\Gamma}_E + i\epsilon},$$

$\mu = \sqrt{2} G_{\mathrm{F}} n_{\nu_e}$ $\quad G = f_e - f_\mu$ $\quad \tilde{\omega}_E = \omega_E \cos 2\theta_{\mathrm{V}}$ $\quad \omega_E = \left|\frac{\delta m^2_\nu}{2E}\right|$

Find a numerically fast way to solve the growth rates and combine with a reasonable final state estimator (equipartition? swap?). All instabilities could be jointly incorporated in CCSN simulations.

Better Final State Estimation

FFC: We use Box3D now, but it's based on QKE sims with periodic bc. It assumes only on crossing in angular space. Need more justification and tests.

CFC: Flavor equipartition? Flavor swap? What about anisotropic CFC?

SFC: Can we use a final state + BGK scheme at all? What should be the final state?

Conclusion:

We include the effects of CFC and FFC in classic CCSN simulations using a flavor equilibrium estimator + BGK scheme. This method could be generalized in future to include instabilities jointly.

We find that FFC leads to lower (anti-)electron neutrino luminosities, higher (anti-)electron neutrino average energies, while the impact on CCSN dynamics is weak. The quasi-equipartition model provide a simple way to estimate the FFC-modulated neutrino signals.

We find that CFI is almost unable to change CCSN outcomes. Resonance-like CFI occurs too deep in and the impact is overwritten by further neutrino-matter interactions, while non-resonance CFI grows too slowly if the correct multi-group growth rate is used.