Neutrino Quantum Kinetics:
Status and Perspectives in Supernovae and Neutron Star Mergers

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Outline

- Brief introduction and motivation for studying quantum kinetic neutrino transport and collective neutrino oscillations in the context of CCSNe and BNSMs.

  See recent reviews, e.g., Fourcart 2023, Fisher et al. 2024 and Yamada et al. 2025

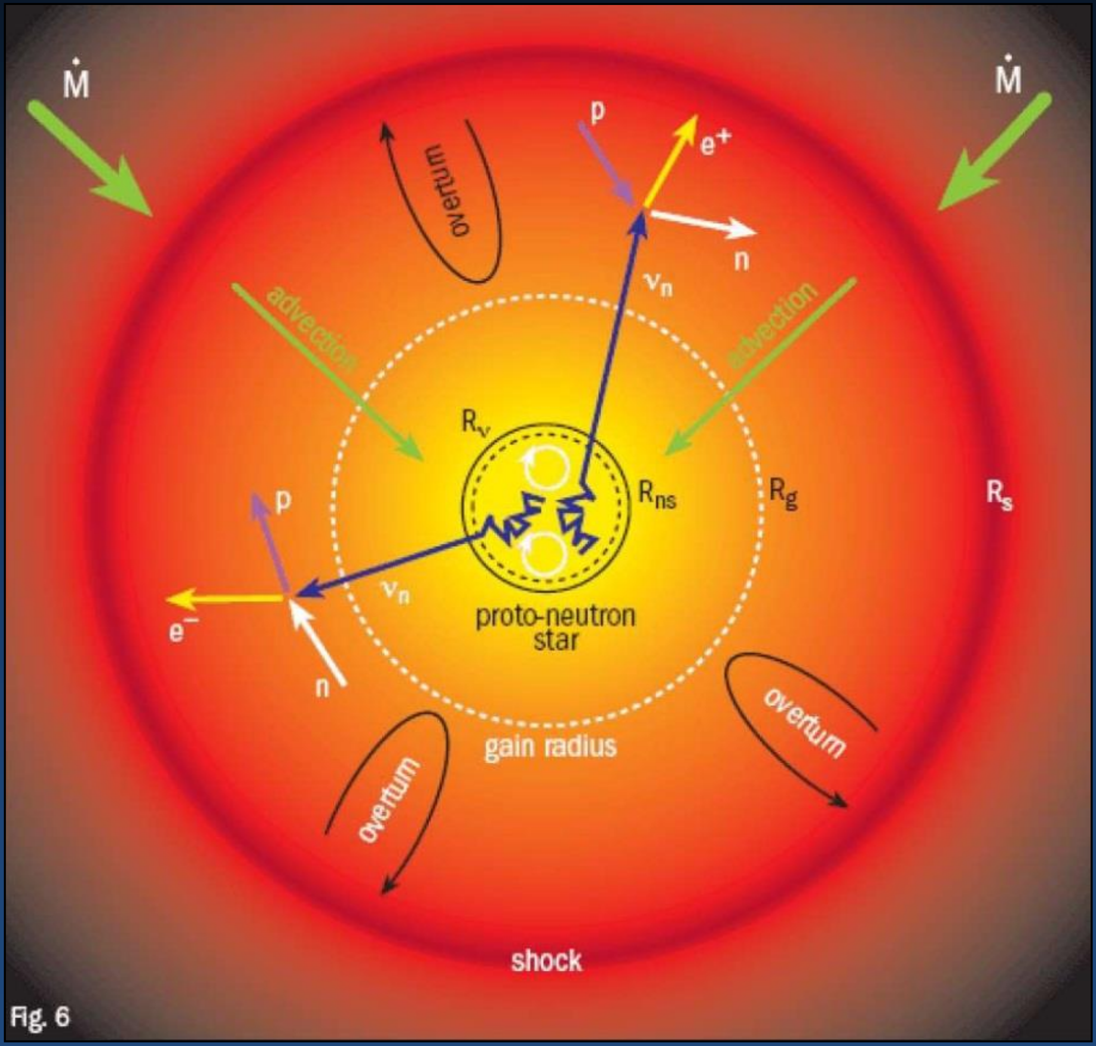
- Local QKE simulations: understanding asymptotic states of flavor conversions.

  See a recent review by Johns et al. 2025

- Global QKE simulations (FFC and CFI)

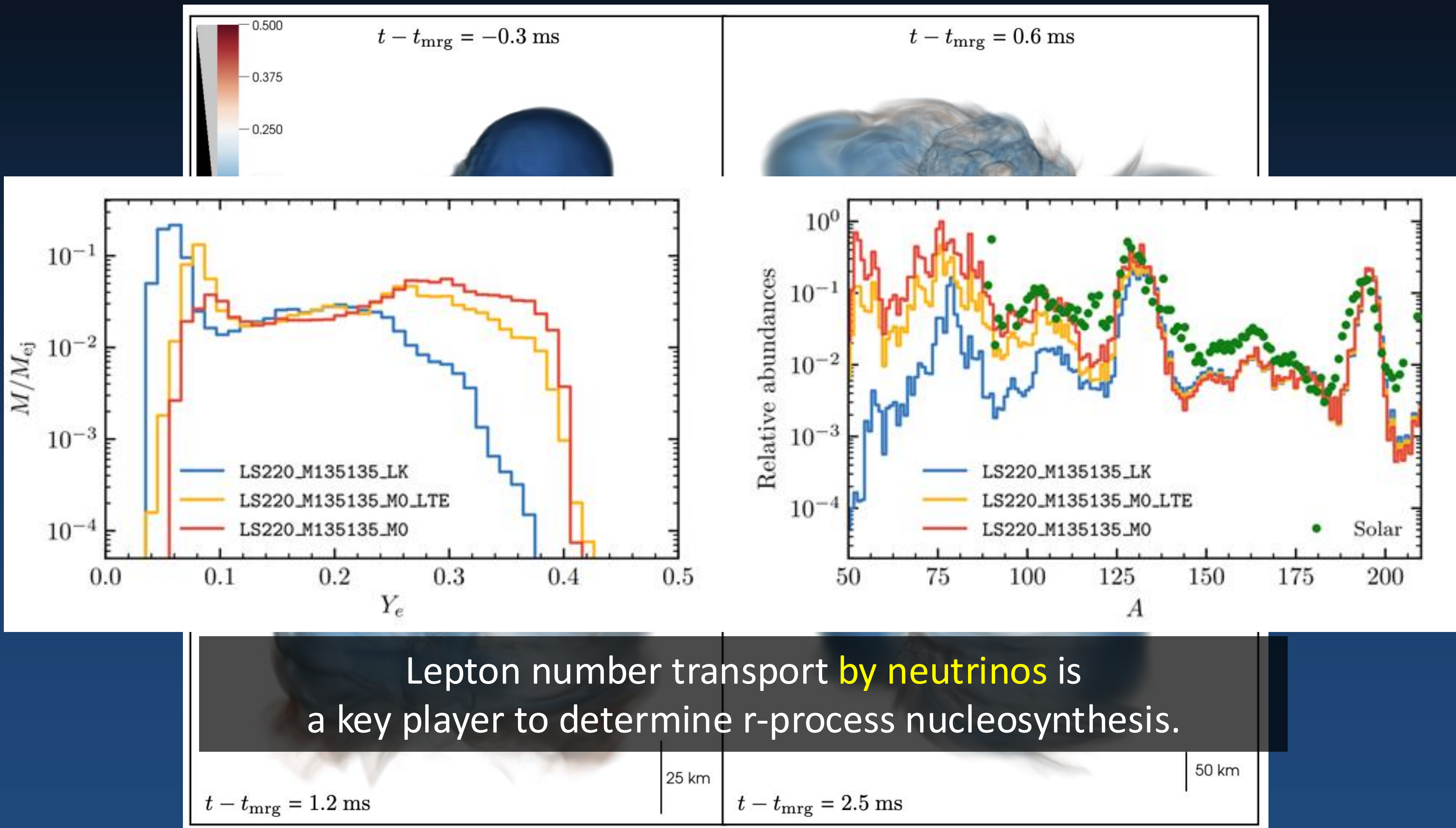
  See, e.g., Shalgar and Tamborra 2023, Xiong et al. 2023, Nagakura and Zaizen 2022

CCSN dynamics hinges on neutrino energy spectrum, angular distributions, and their flavor dependent structures



Cartoon from Thomas Janka

Binary neutron star merger (BNSM)

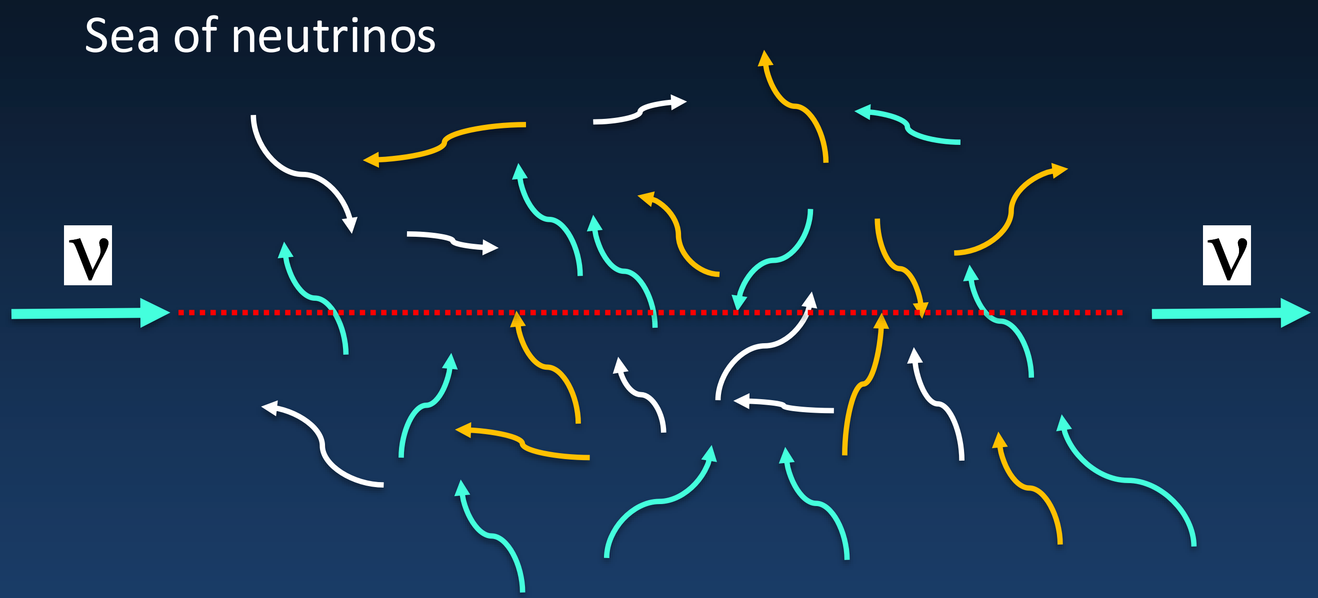


Lepton number transport by neutrinos is
a key player to determine r-process nucleosynthesis.

Radice et al. 2018

Neutrino oscillation induced by self-interactions

Pantalone 1992



- This is a non-equilibrium quantum many-body problem
- Neutrino dispersion relations are modified by refractive effects, analogous to the MSW effect in matter
- Because neutrino flavor (or mass) eigenstates span multiple states (at least three), neutrino self-interactions can enhance flavor coherence

Rich flavor-conversion phenomena driven by neutrino self-interactions

Vacuum: $\omega = \frac{\Delta m^2}{2E_\nu}$,
Matter: $\lambda = \sqrt{2} G_F n_e$,
Self-int: $\mu = \sqrt{2} G_F n_\nu$,

- Slow-mode (Duan et al. 2010)

- Energy-dependent flavor conversion occurs.
- The frequency of the flavor conversion is proportional to $\sqrt{\omega\mu}$

- Fast-mode (FFC) (Sawyer 2005)

- Collective neutrino oscillation in the limit of ω → 0.
- The frequency of the flavor conversion is proportional to μ
- Anisotropy of neutrino angular distributions drives the fast flavor-conversion.

- Collisional flavor instability (CFI) (Johns 2021)

- Asymmetries of matter interactions between neutrinos and anti-neutrinos drive flavor conversion.

$$\mathrm{Im}\,\Omega \simeq \pm \frac{\Gamma - \bar{\Gamma}}{2} \sqrt{\frac{\mu S}{(\mu D)^2 + 4 \mu S}} - \frac{\Gamma + \bar{\Gamma}}{2}$$

Γ: Matter-interaction rate

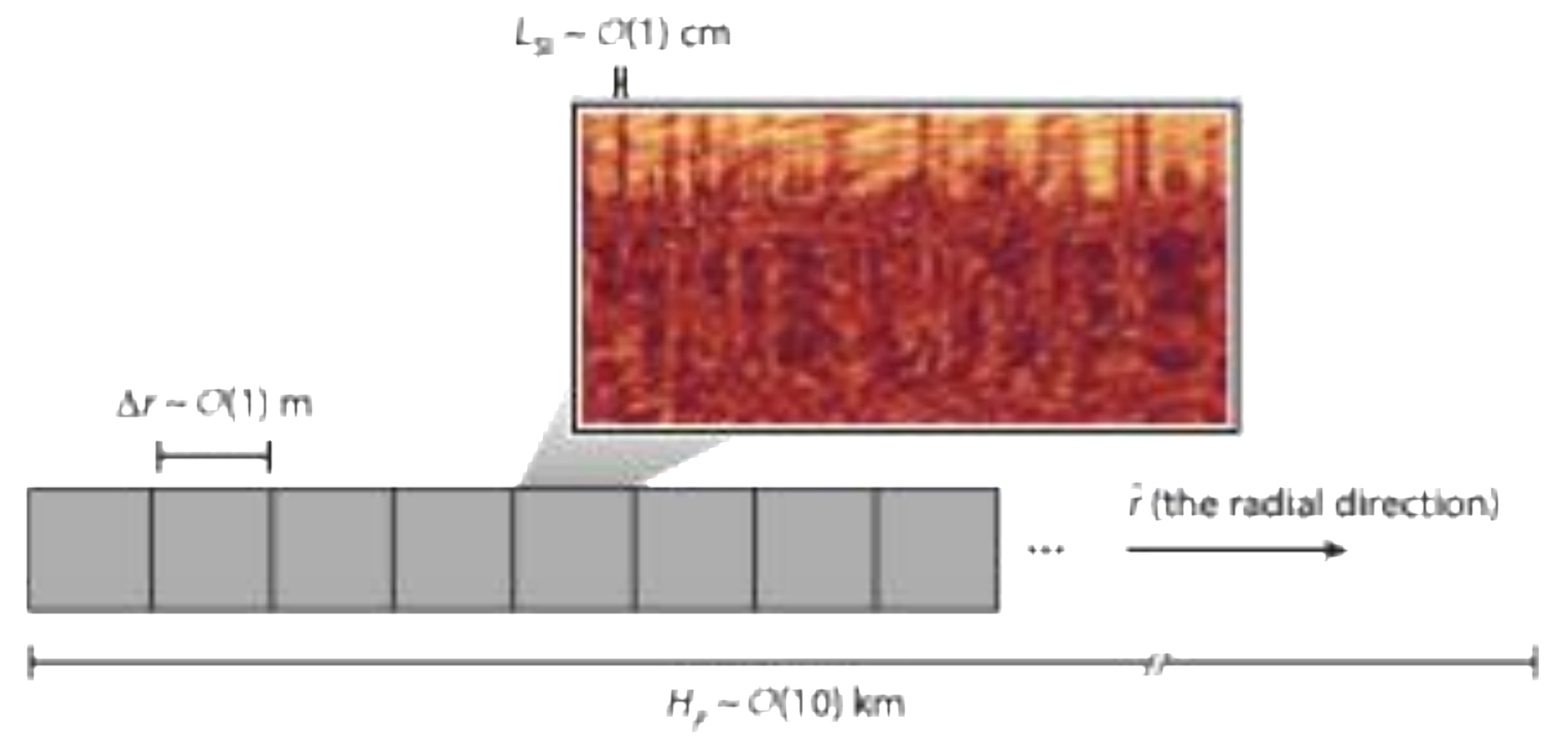
- Matter-neutrino resonance (Malkus et al. 2012)

- The resonance potentially occur in BNSM/Collapsar environment (but not in CCSN).
- Essentially the same mechanism as MSW resonance.

$$|\lambda + \mu| \sim |\omega|$$

Huge scale disparity between neutrino flavor conversions and hydrodynamics

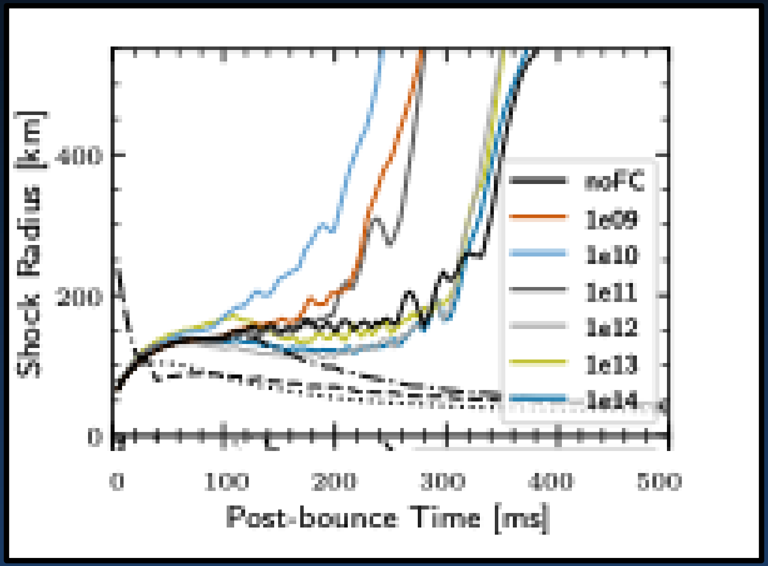
Johns et al. 2025



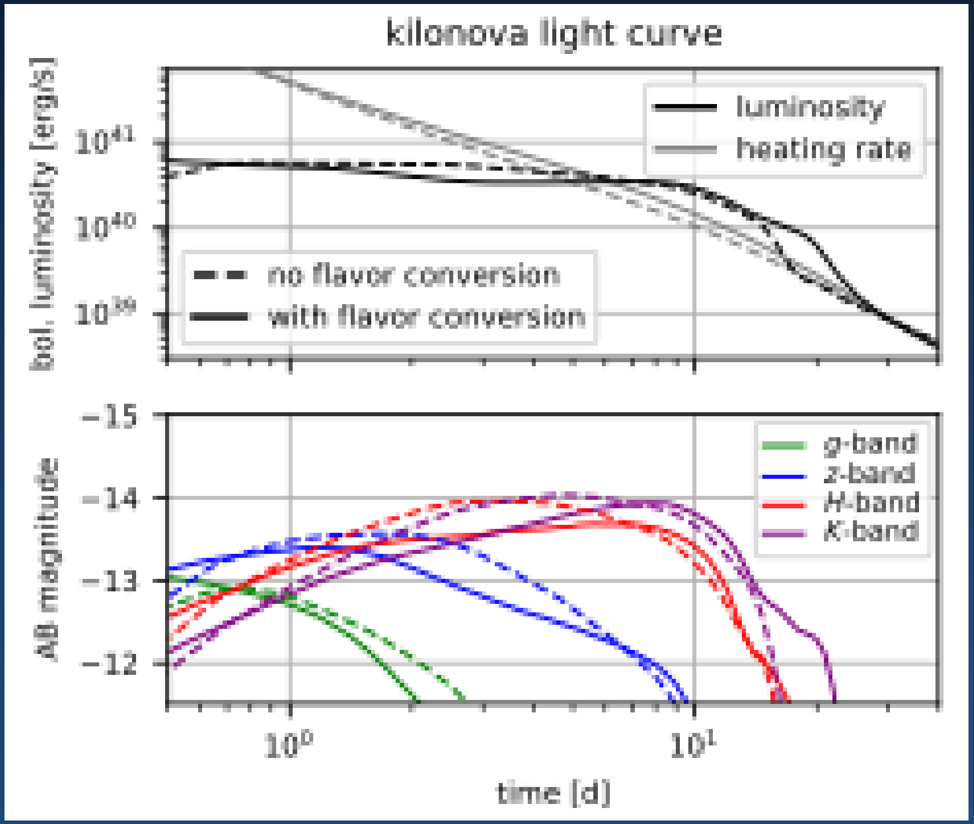
Subgrid model of flavor conversions is indispensable for CCSN and BNSM simulations that incorporate effects of neutrino oscillations.

Phenomenological approach

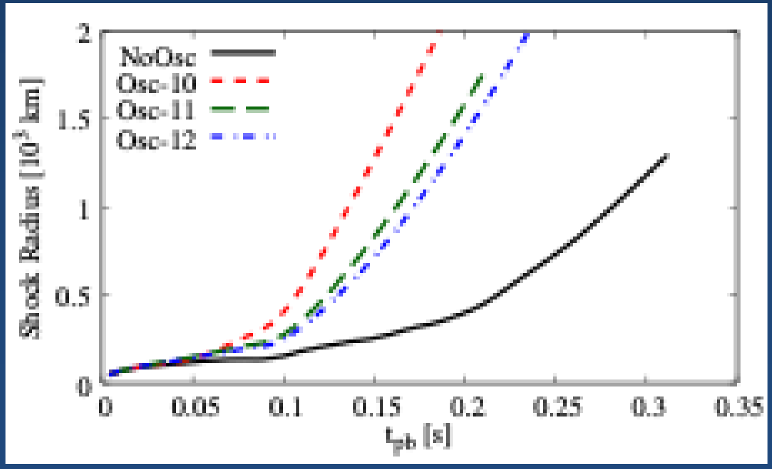
Ehring et al. 2023



Just et al. 2022



Mori et al. 2025

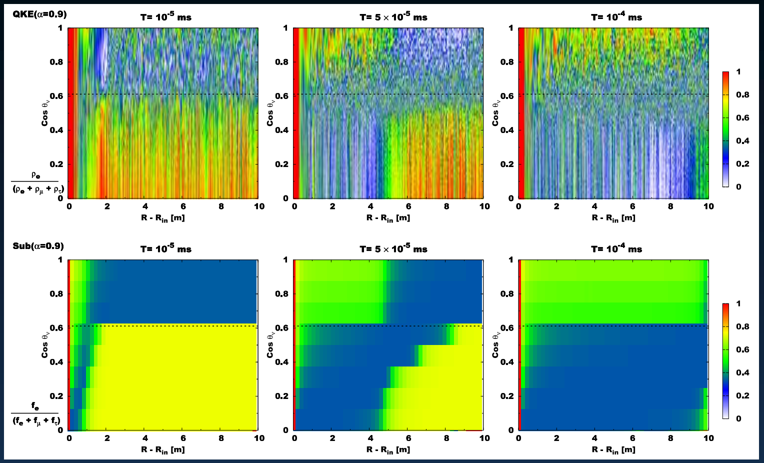


BGK subgrid model

Nagakura et al. 2024 (see also Xiong et al. 2025)

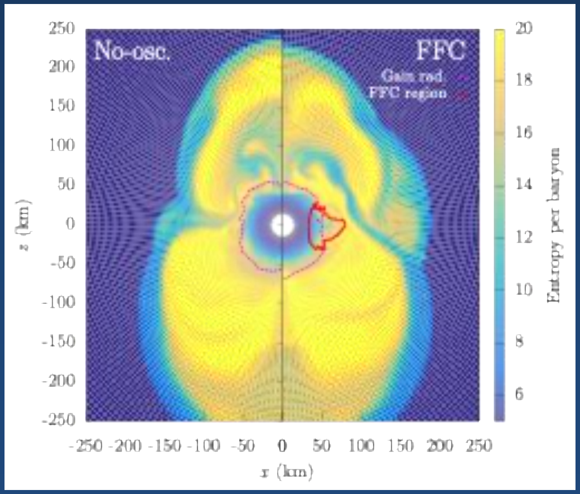
$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + i p^\mu n_\mu [H, f]$$

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + p^\mu n_\mu \frac{1}{\tau_a}(f - f^a)$$

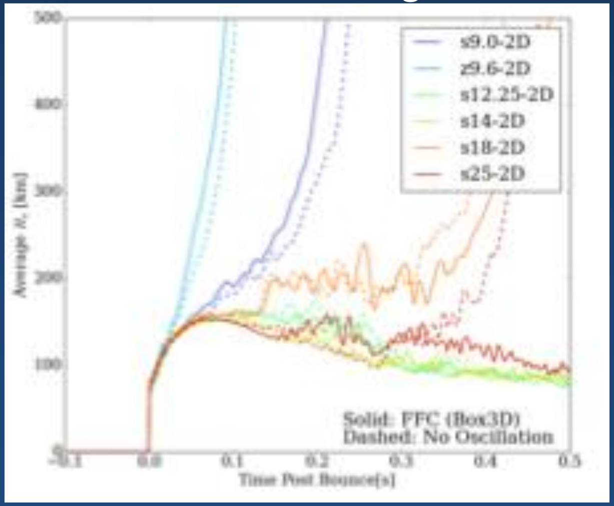


Local QKE simulations are necessary to develop accurate method in determining τ_a and f^a.

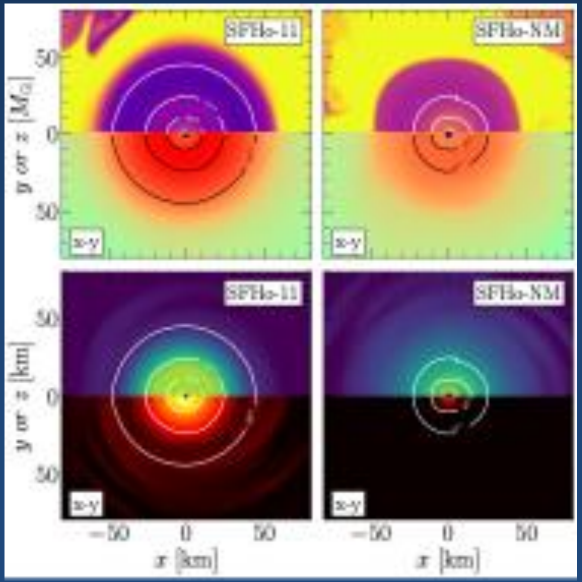
Akaho et al. PRL in press



Wang et al. 2025



Qiu et al. 2025 (PRL and PRD)



Quantum Kinetic Equation (QKE) for neutrino transport

Vlasenko et al. 2014, Volpe 2015,
Blaschke et al. 2016, Richers et al. 2019

$$p^\mu \frac{\partial \overset{(-)}{f}}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^i} = -p^\mu u_\mu \overset{(-)}{S}_{\text{col}} + ip^\mu n_\mu [\overset{(-)}{H}, \overset{(-)}{f}],$$

Advection terms (Same as Boltz eq.)

Collision term

Oscillation term

f is not a "distribution function"

Density matrix

$$\overset{(-)}{f} = \begin{bmatrix} \overset{(-)}{f}_{ee} & \overset{(-)}{f}_{e\mu} & \overset{(-)}{f}_{e\tau} \\ \overset{(-)}{f}_{\mu e} & \overset{(-)}{f}_{\mu\mu} & \overset{(-)}{f}_{\mu\tau} \\ \overset{(-)}{f}_{\tau e} & \overset{(-)}{f}_{\tau\mu} & \overset{(-)}{f}_{\tau\tau} \end{bmatrix}$$

Hamiltonian:

$$\overset{(-)}{H} = \overset{(-)}{H}_{\text{vac}} + \overset{(-)}{H}_{\text{mat}} + \overset{(-)}{H}_{\nu\nu},$$

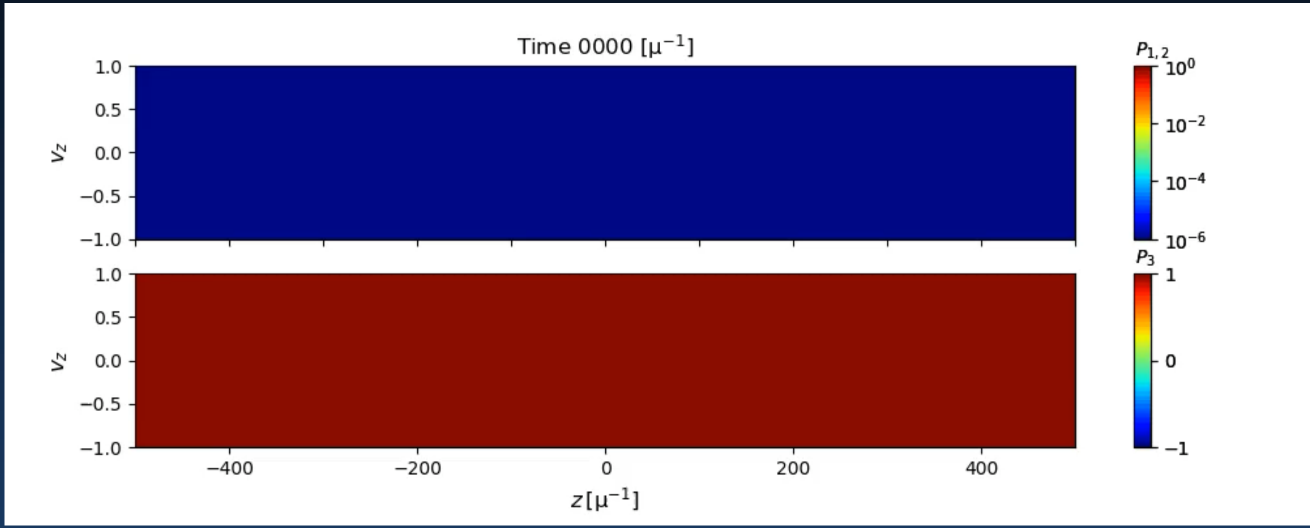
$$H_{\text{vac}} = \frac{1}{2\nu} U \begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix} U^\dagger,$$

$$H_{\text{mat}} = D \begin{bmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau + V_{\mu\tau} \end{bmatrix},$$

$$H_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3 q'}{(2\pi)^3} (1 - \sum_{i=1}^{3} \ell'_{(i)} \ell_{(i)})(f(q') - \bar{f}^*(q')),$$

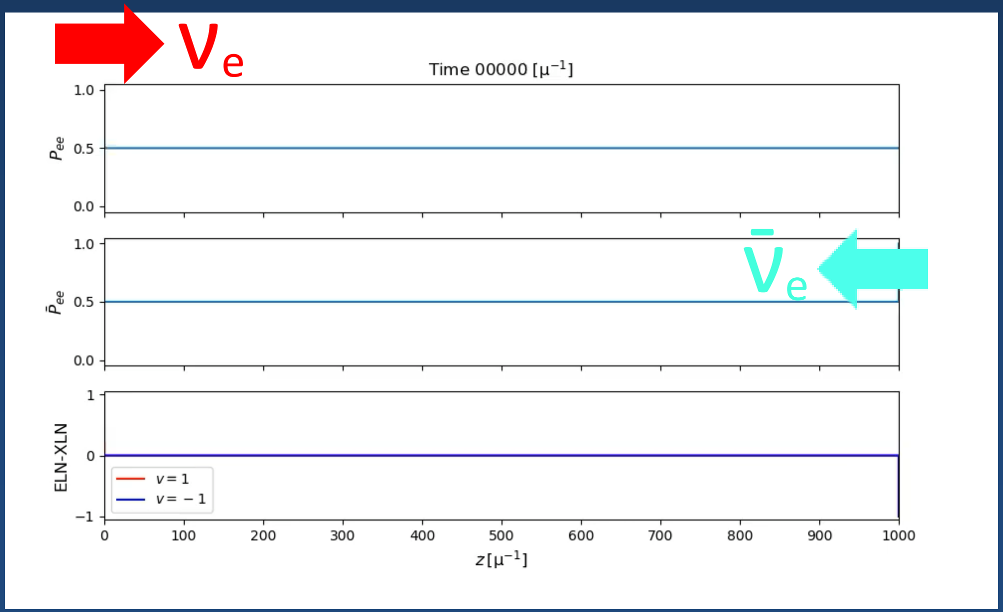
Local QKE simulations

Animations (and simulations) below are created by M. Zaizen



Zaizen and Nagakura 2023

Periodic boundary condition

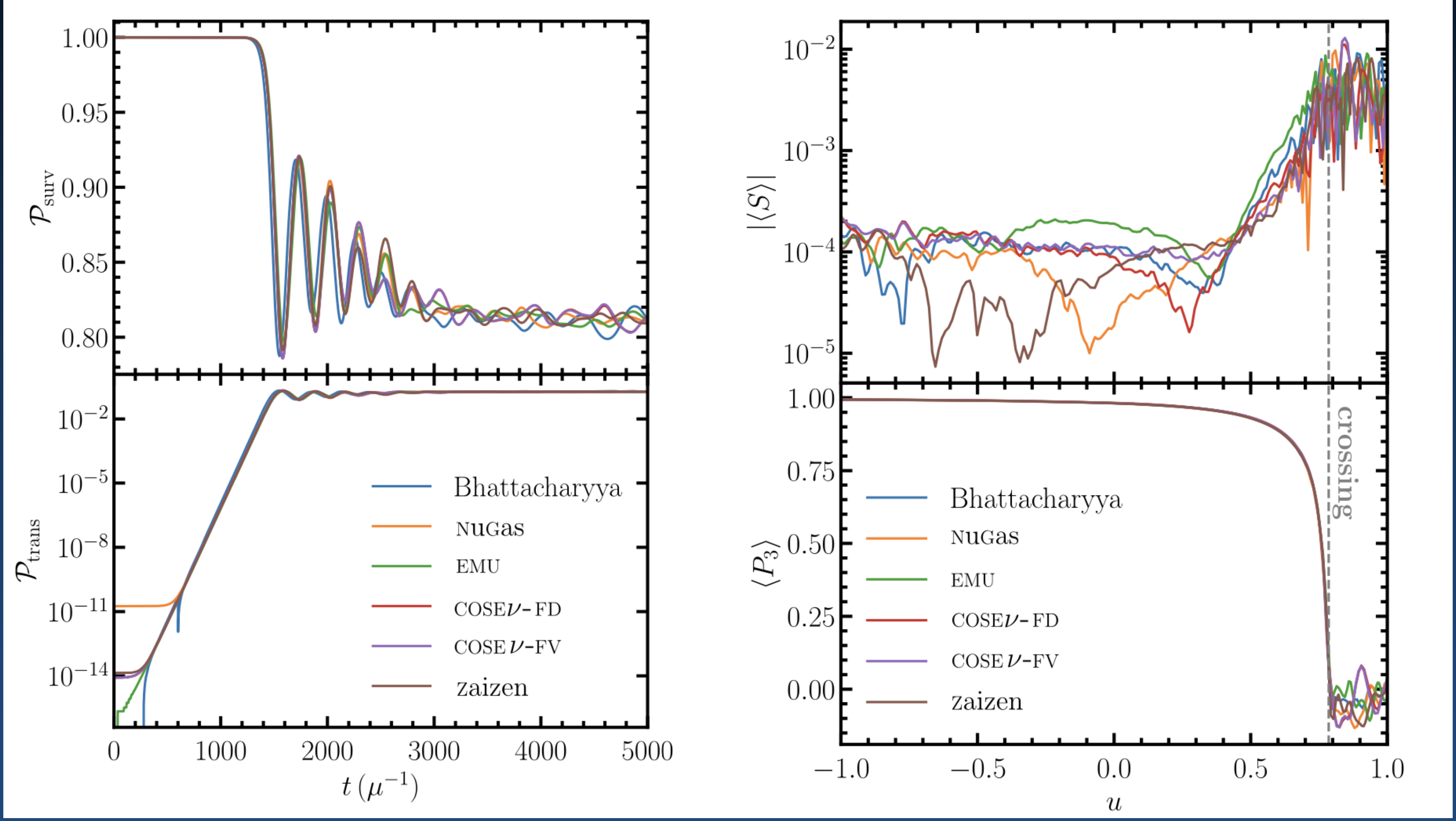


Zaizen and Nagakura 2024

Two-beam colliding model

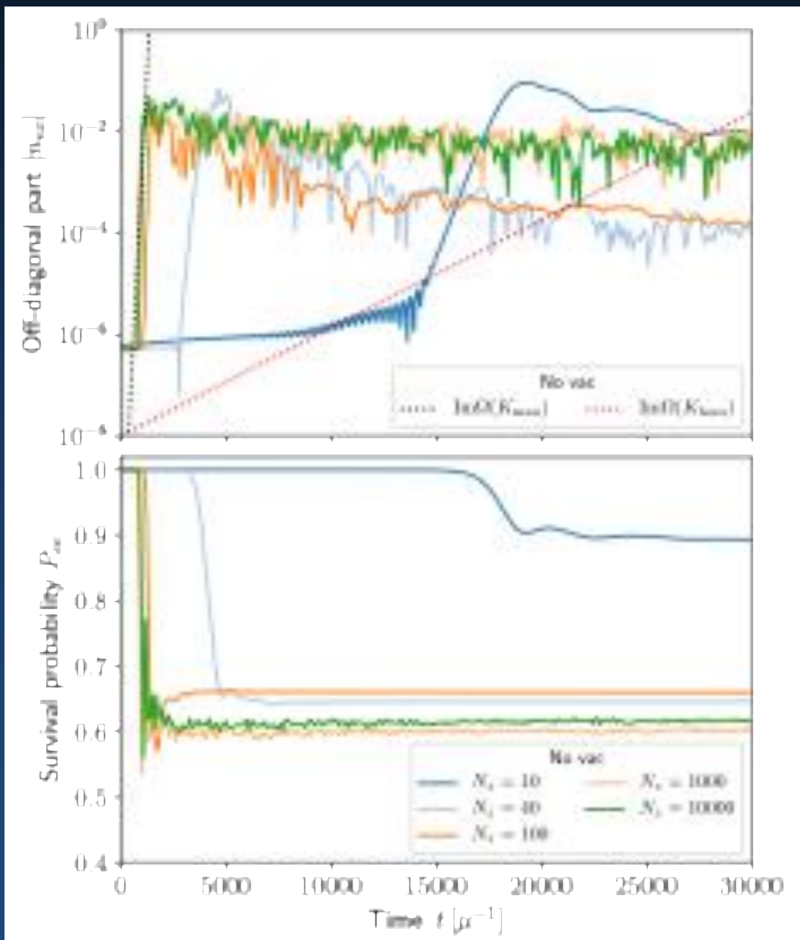
Local QKE simulation codes are well established

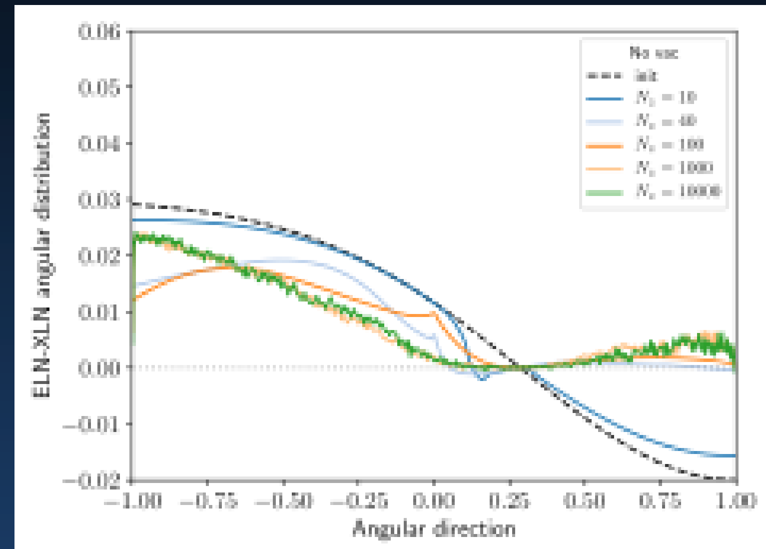
Richers et al. 2022

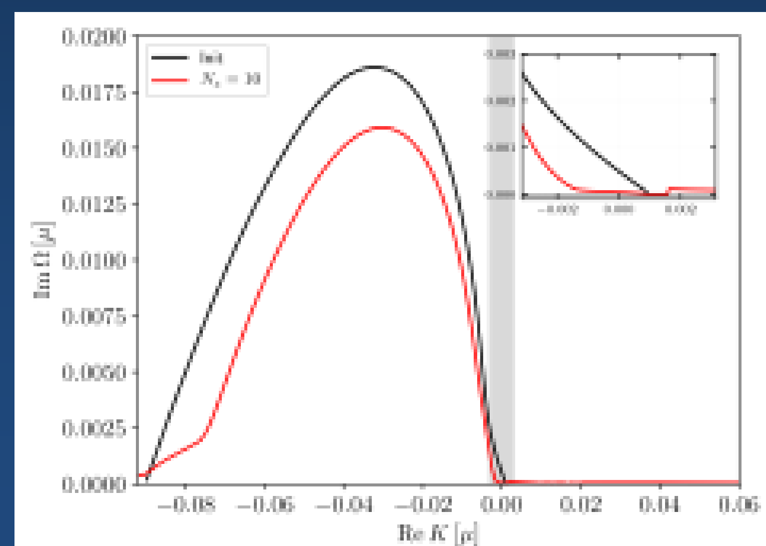


Caveat: resolution dependence

Nagakura et al. 2025







Moment-based QKE simulation

Grohs et al. 2023, 2025

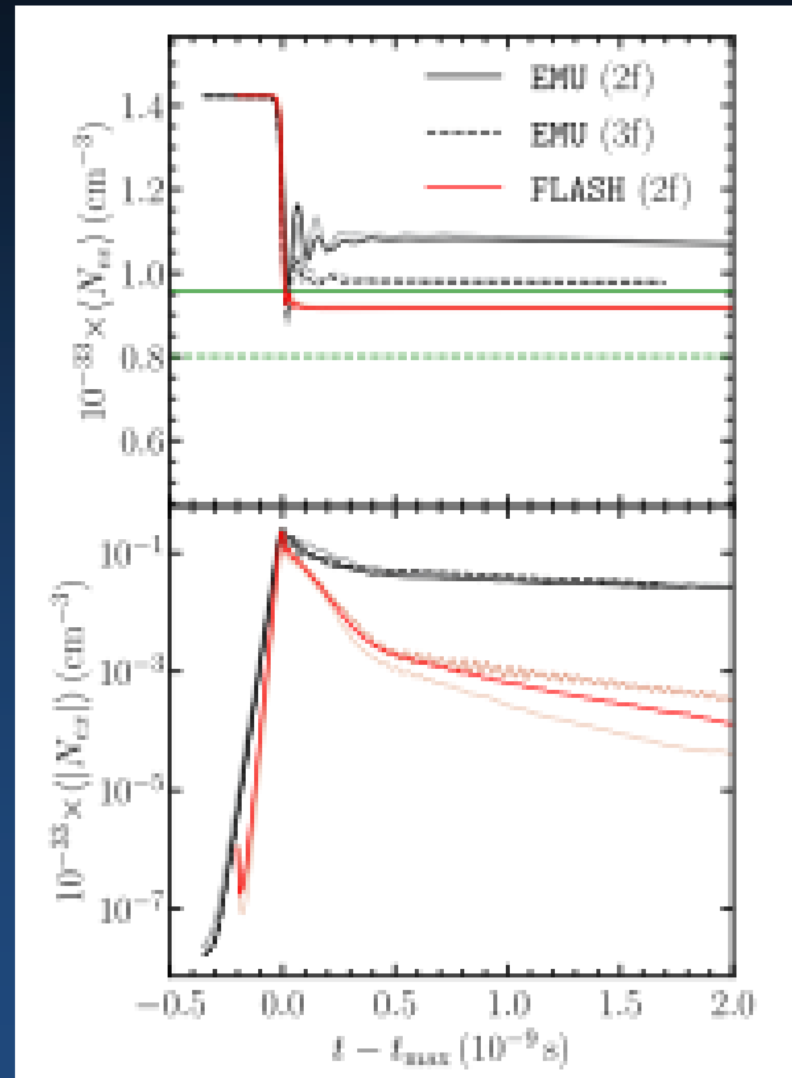
$$E_{ab}(t,\mathbf{x},p) = \frac{p^3}{(2\pi)^3}\int d\Omega_p\, \varrho_{ab}(t,\mathbf{x},\mathbf{p}),$$

$$F^j_{ab}(t,\mathbf{x},p) = \frac{p^3}{(2\pi)^3}\int d\Omega_p\, \frac{p^j}{p}\, \varrho_{ab}(t,\mathbf{x},\mathbf{p}),$$

$$P^{jk}_{ab}(t,\mathbf{x},p) = \frac{p^3}{(2\pi)^3}\int d\Omega_p\, \frac{p^j p^k}{p^2}\, \varrho_{ab}(t,\mathbf{x},\mathbf{p}).$$

$$\frac{\partial E}{\partial t} + \frac{\partial F^j}{\partial x^j} = -i[H_E, E] + i[H^j_F, F_j],$$

$$\frac{\partial F^j}{\partial t} + \frac{\partial P^{jk}}{\partial x^k} = -i[H_E, F^j] + i[H^k_F, P^j_k],$$



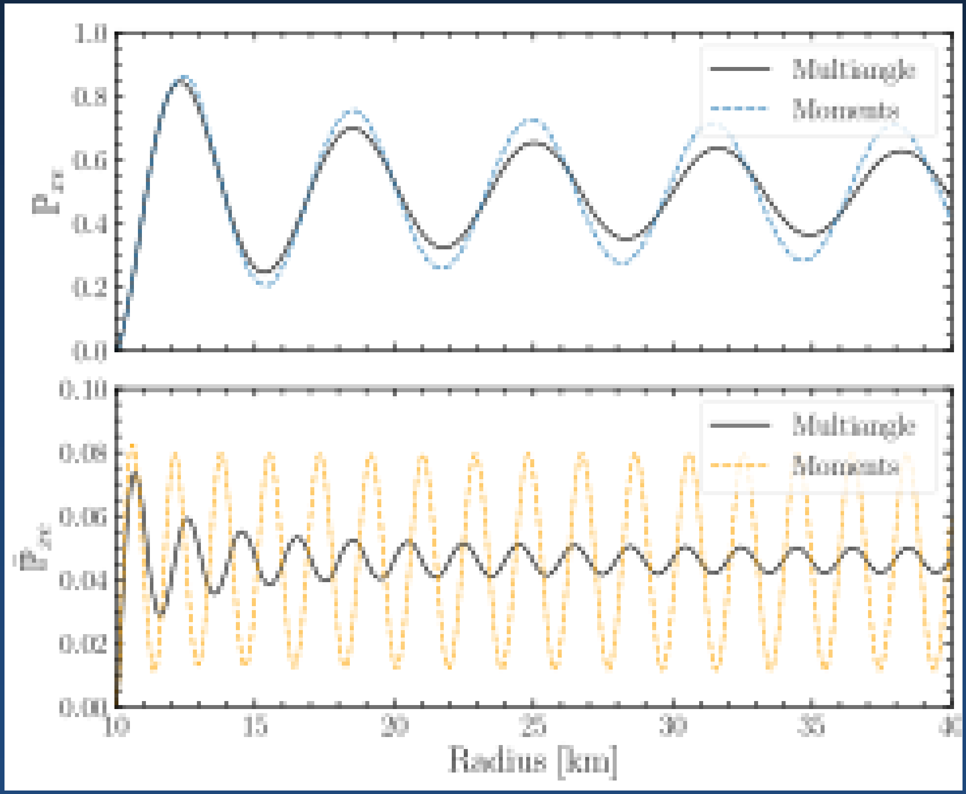
Quantum kinetic closure

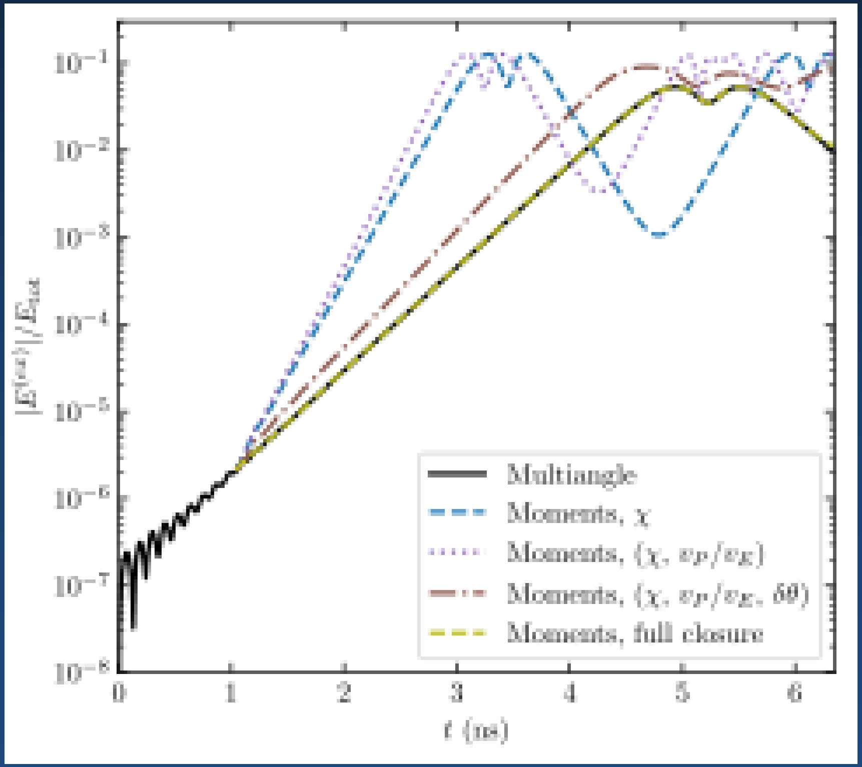
Kneller et al. 2025

A key difference from classical closure problem:

➡ Off-diagonal elements of neutrino density matrix

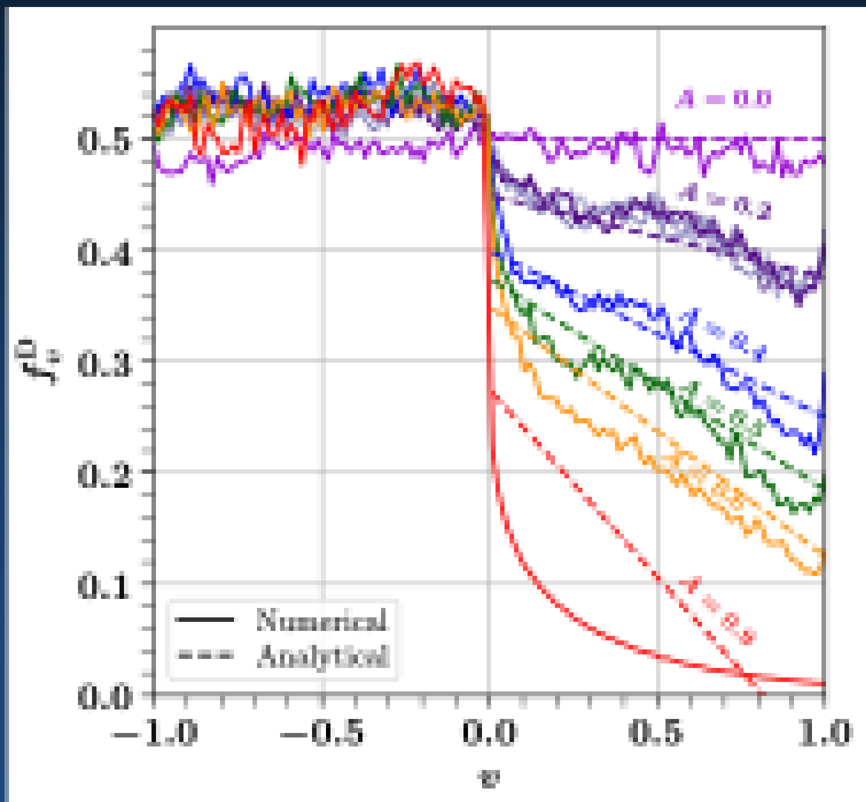
$$P_{rr} = LER$$





Asymptotic states of FFC informed by local QKE simulations

Bhattacharyya and Dasgupta 2021



$$F^{\rm fin}_{\overset{(-)}{\nu}_e,\overset{(-)}{\nu}_\mu}[\vec{p}] = (1 - f^{\rm D}_{\vec{p}})F^{\rm ini}_{\overset{(-)}{\nu}_e,\overset{(-)}{\nu}_\mu}[\vec{p}] + f^{\rm D}_{\vec{p}}F^{\rm ini}_{\overset{(-)}{\nu}_\mu,\overset{(-)}{\nu}_e}[\vec{p}]$$

$$f^{\rm D}_v \approx \begin{cases} \frac{1}{2} - \frac{A}{4} - \frac{3A}{8}v, & \text{if } v > 0\,, \\ \frac{1}{2}, & \text{if } v < 0\,, \end{cases}$$

$A \propto (n_{\nu_e} - n_{\bar{\nu}_e})$: Normalized lepton asymmetry

Asymptotic states of FFC can be characterized by the normalized lepton asymmety

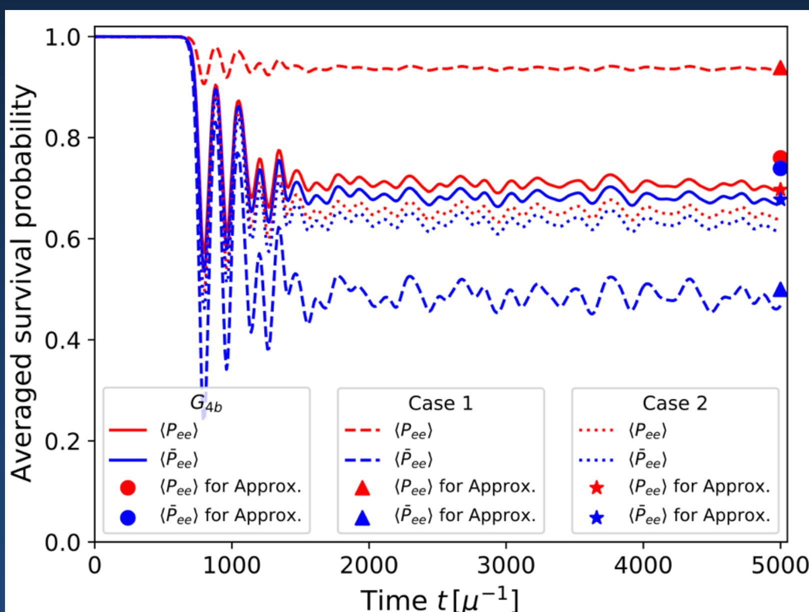
Asymptotic states of FFC informed by local QKE simulations

Zaizen and Nagakura 2022

Analytic scheme (Box-3D)

Conservative law of neutrinos + Stability condition

Time evolution of survival probability



$$f_e^a = \eta f_e + (1-\eta) f_x,$$
$$f_x^a = \frac{1}{2}(1-\eta) f_e + \frac{1}{2}(1+\eta) f_x, \qquad (7)$$

$$A \equiv \left| \int_{G_v<0} d\Gamma G_v \right|,$$
$$B \equiv \int_{G_v>0} d\Gamma G_v, \qquad (28)$$

while G_v is as given in Eq. (18). In cases with $B > A$ (positive ELN-XLN density), we determine η in Eq. (7) as

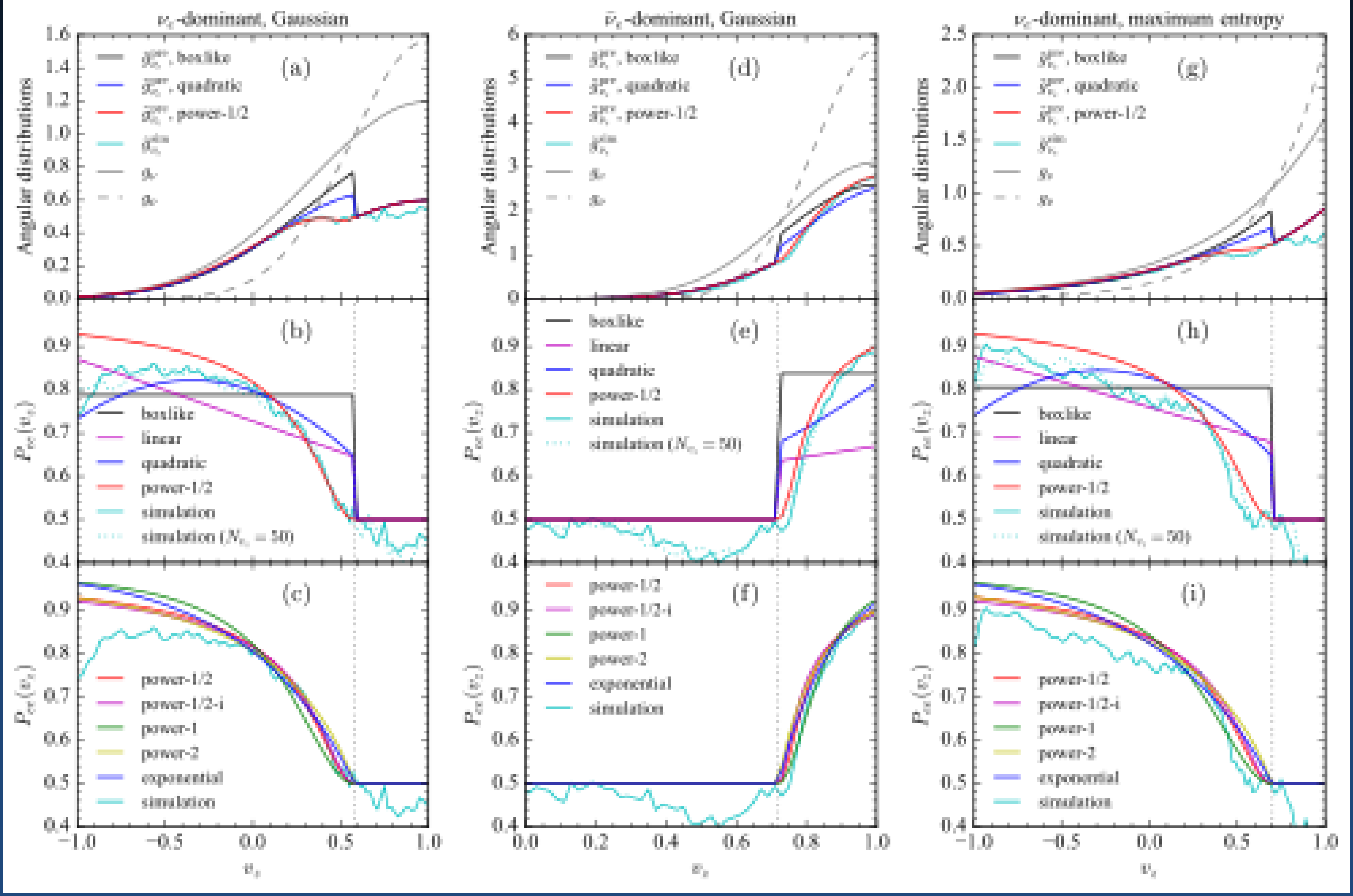
$$\eta = \begin{cases} \frac{1}{3} & (G_v < 0), \\ 1 - \frac{2A}{3B} & (G_v \geq 0). \end{cases} \qquad (29)$$

Meanwhile, η in $B < A$ (negative ELN-XLN density) is determined as

$$\eta = \begin{cases} \frac{1}{3} & (G_v > 0), \\ 1 - \frac{2B}{3A} & (G_v \leq 0). \end{cases} \qquad (30)$$

Another (Better) analytic scheme for FFC asymptotic states

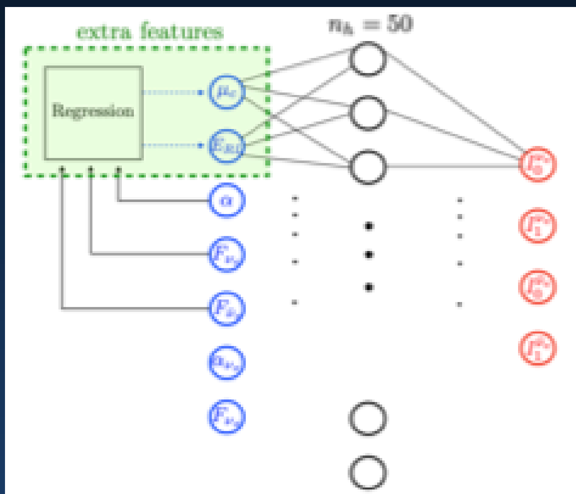
Xiong et al. 2023



Asymptotic states of FFC informed by Machine-learning techniques + local QKE simulations

Abbar et al. 2024

Richers et al. 2024



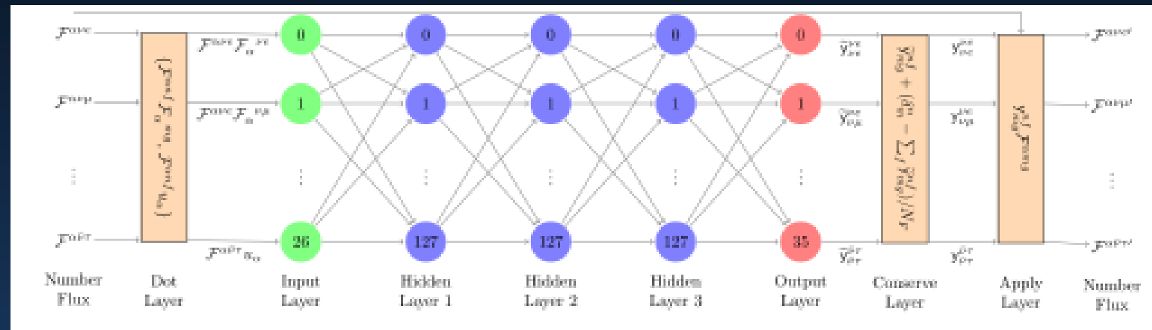
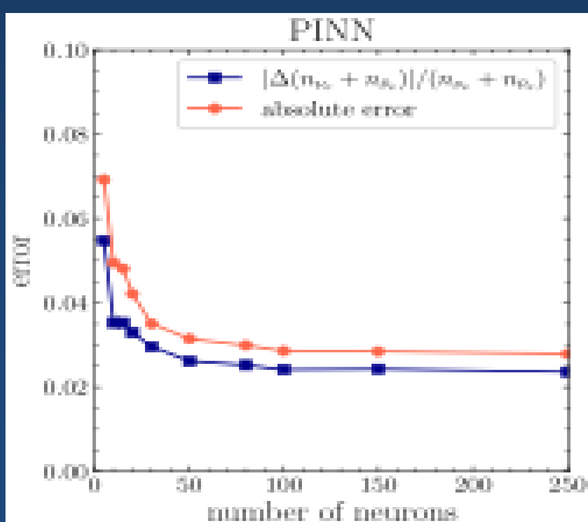
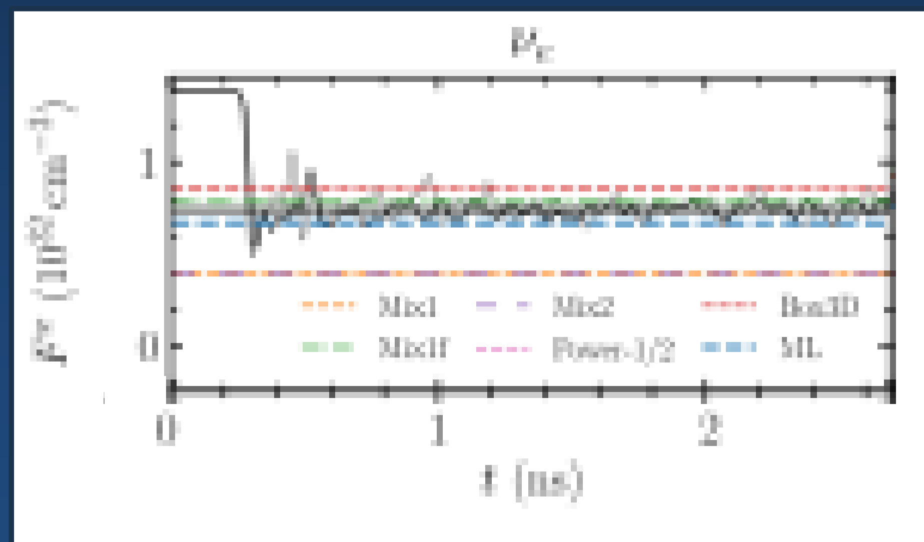


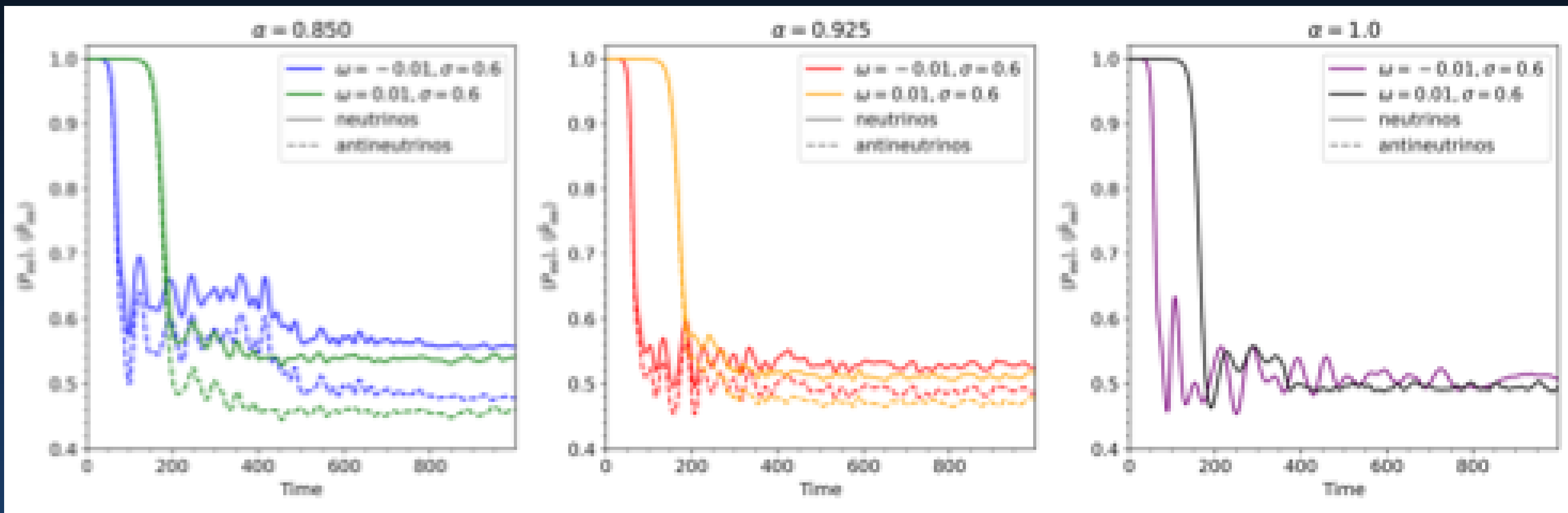
FIG. 1. Neural network architecture. The network is constructed to produce the same result in any reference frame and to exactly preserve the total number of neutrinos.

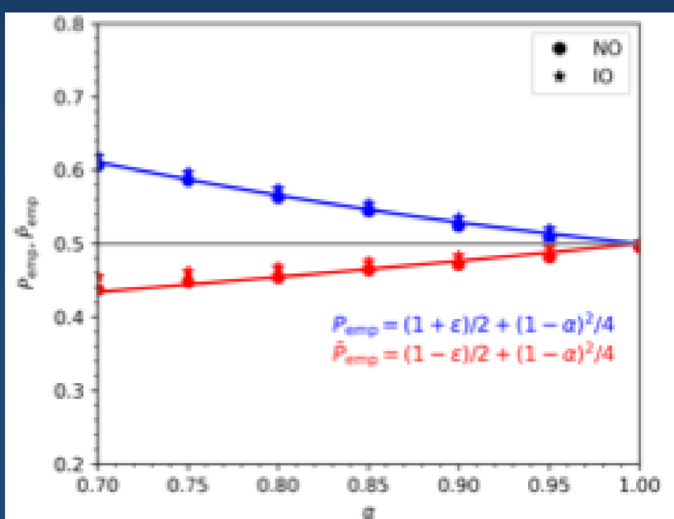




Asymptotic states of SFC informed by local QKE simulations

Padilla-Gay et al. 2025





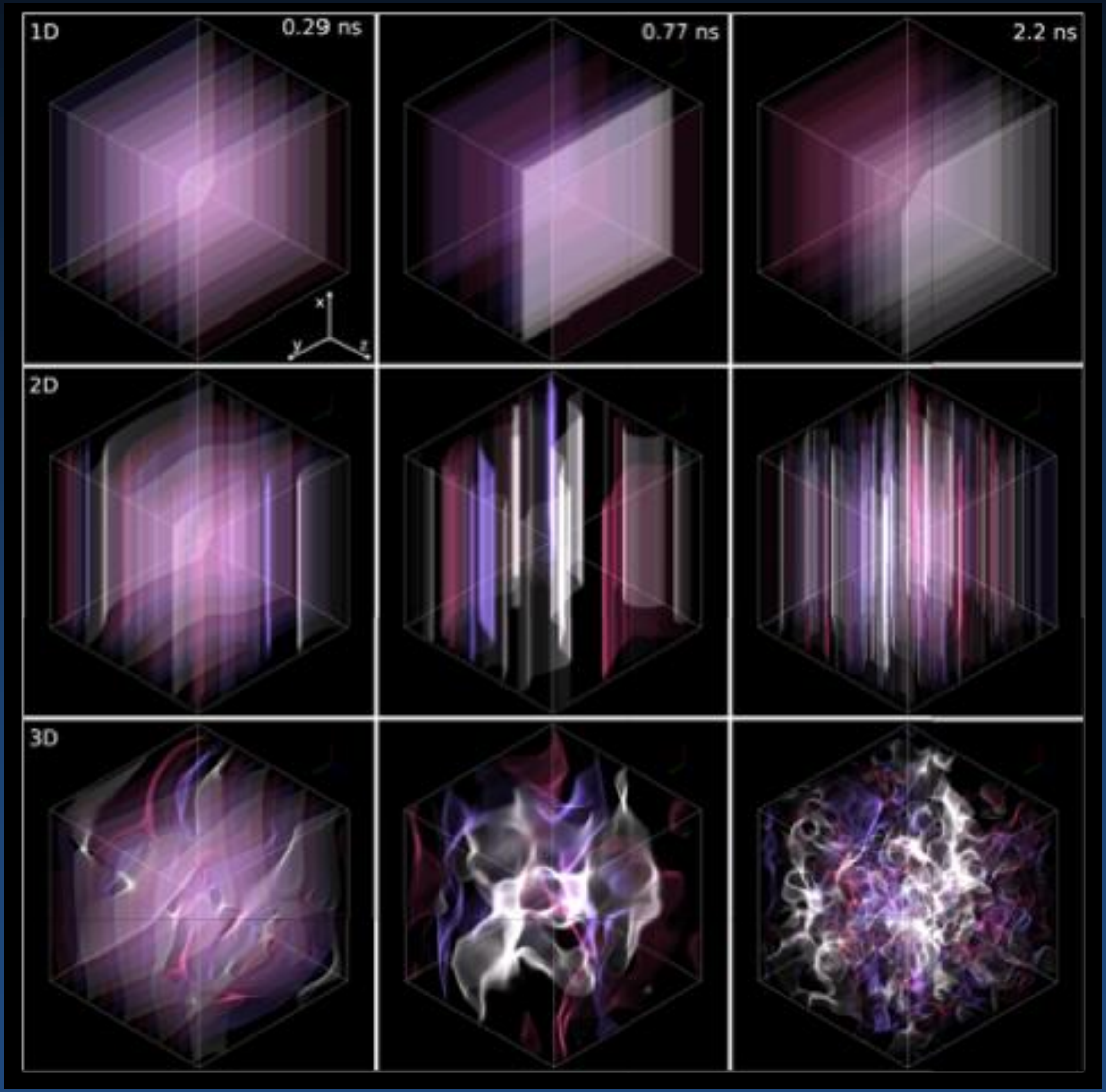
$\alpha = n_{\bar{\nu}_e} / n_{\nu_e}$: asymmetric parameter

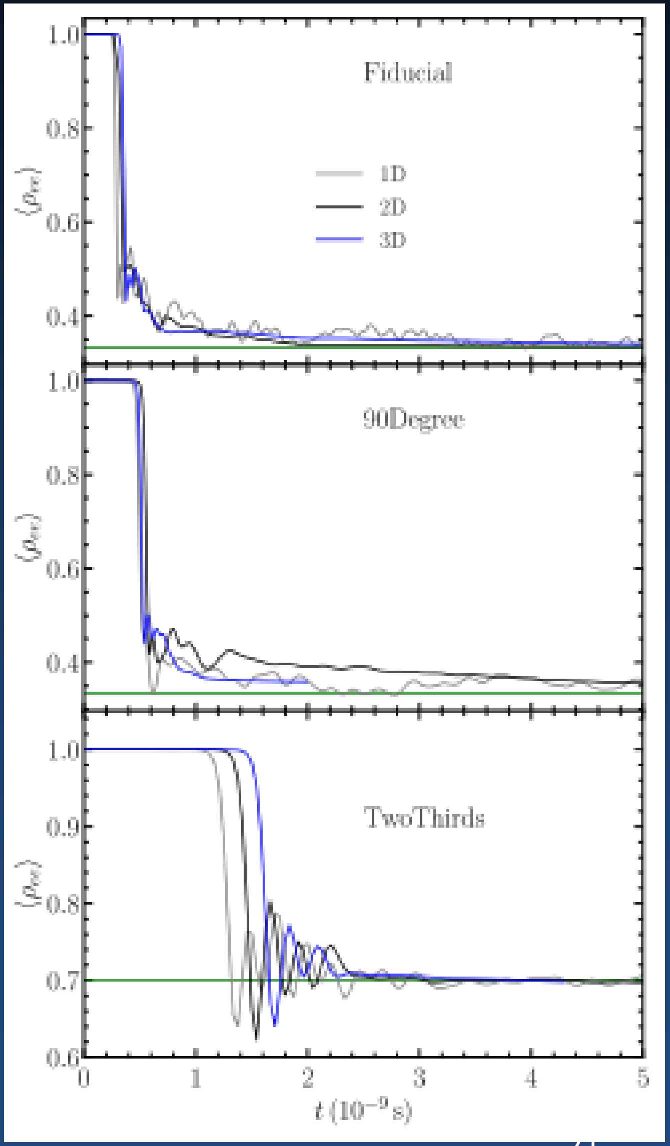
$$P_{\rm emp}(\alpha) = \frac{1+\epsilon}{2} + \frac{(1-\alpha)^2}{4},$$

$$\bar{P}_{\rm emp}(\alpha) = \frac{1-\epsilon}{2} + \frac{(1-\alpha)^2}{4},$$

3D FFC simulations

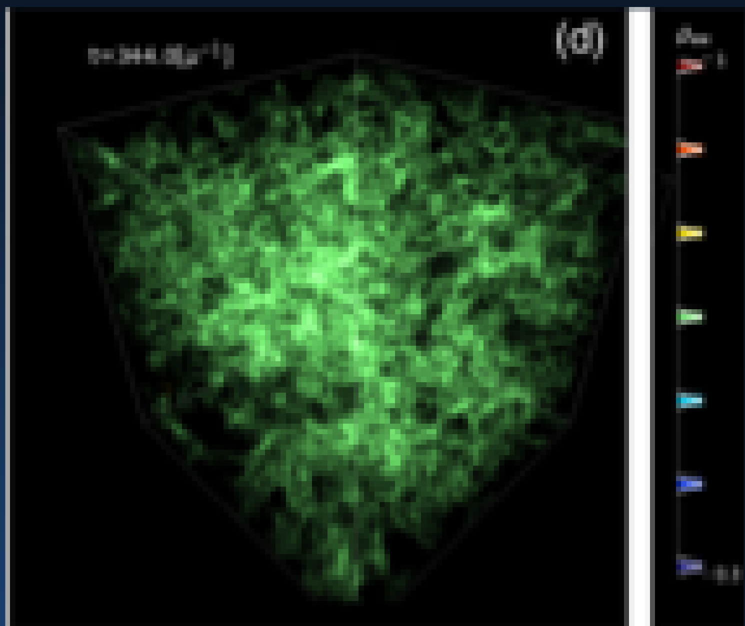
Richers et al. 2021

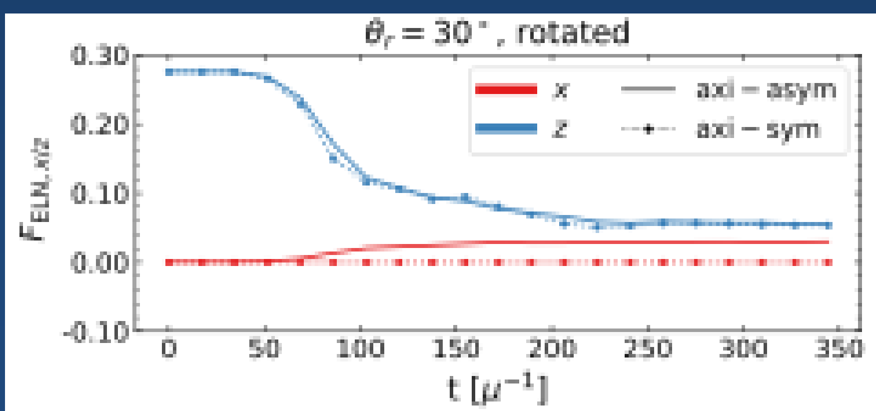


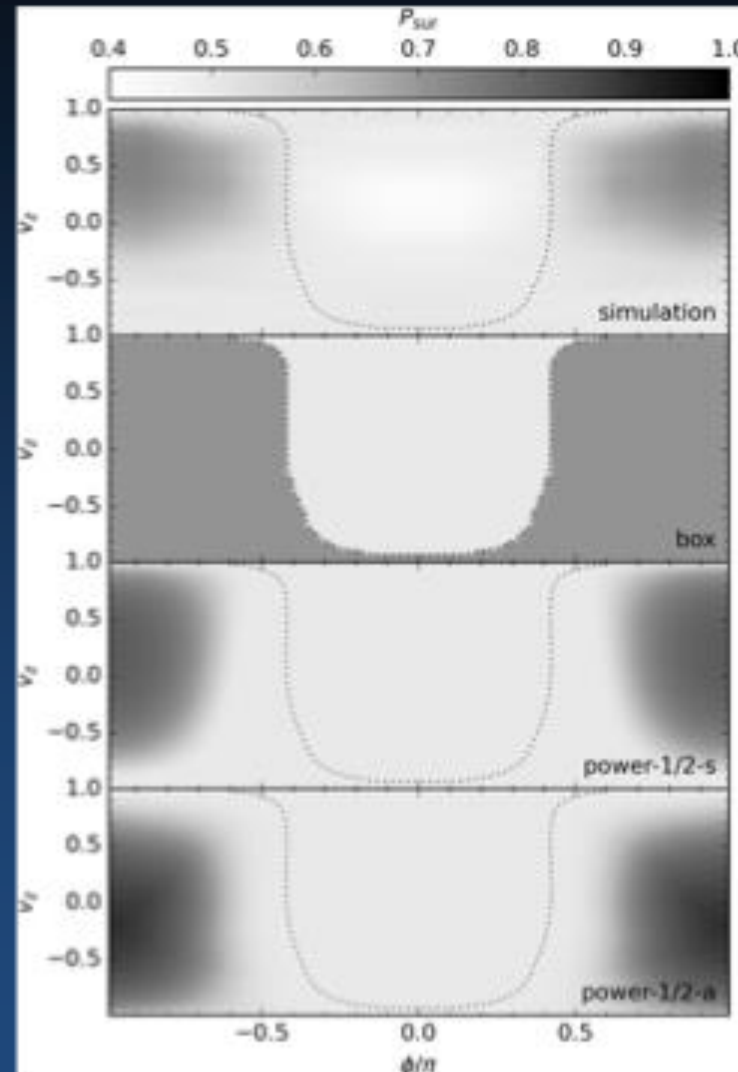


3D FFC simulations

George et al. 2024

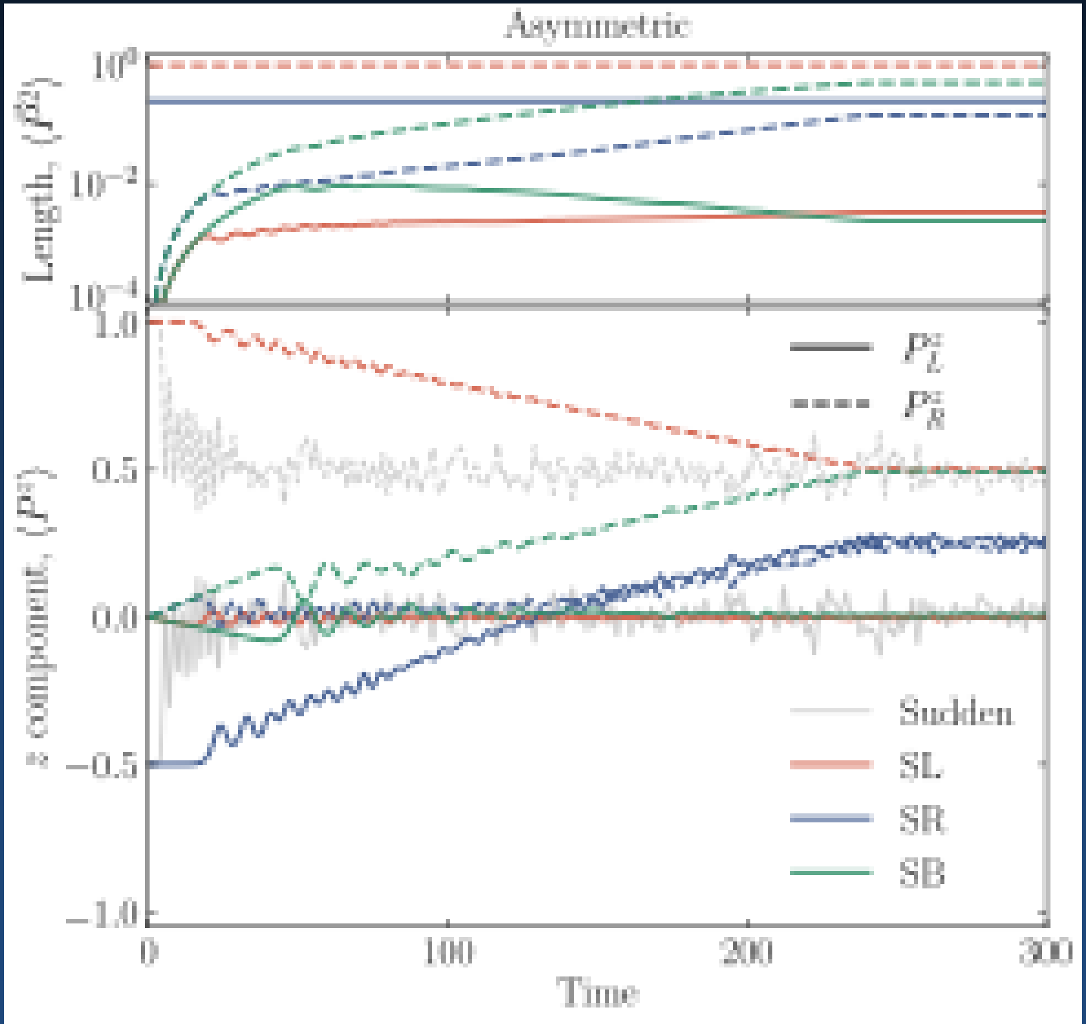






Edge of instability

Fiorillo et al. 2024

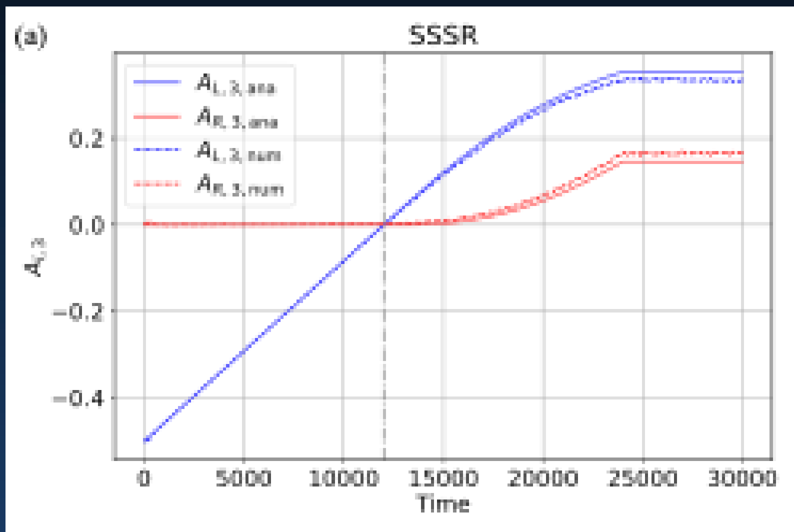


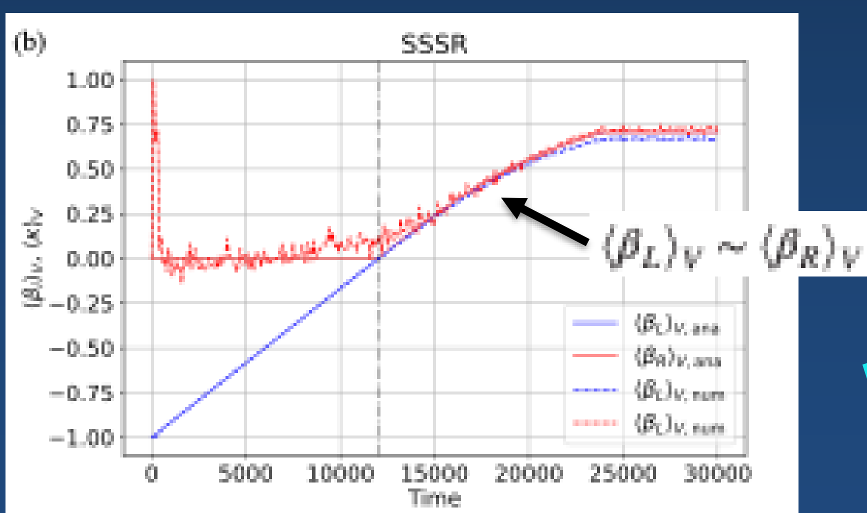
- The time evolution of FFC depends sensitively on the neutrino injection conditions.
- The resulting asymptotic states vary across models.
- Even after the disappearance of ELN–XLN angular crossings, the system continues to exhibit strong flavor conversion.
- This suggests that the stability condition is not a sufficient condition to determine asymptotic states.

See also Urquilla and Johns 2025

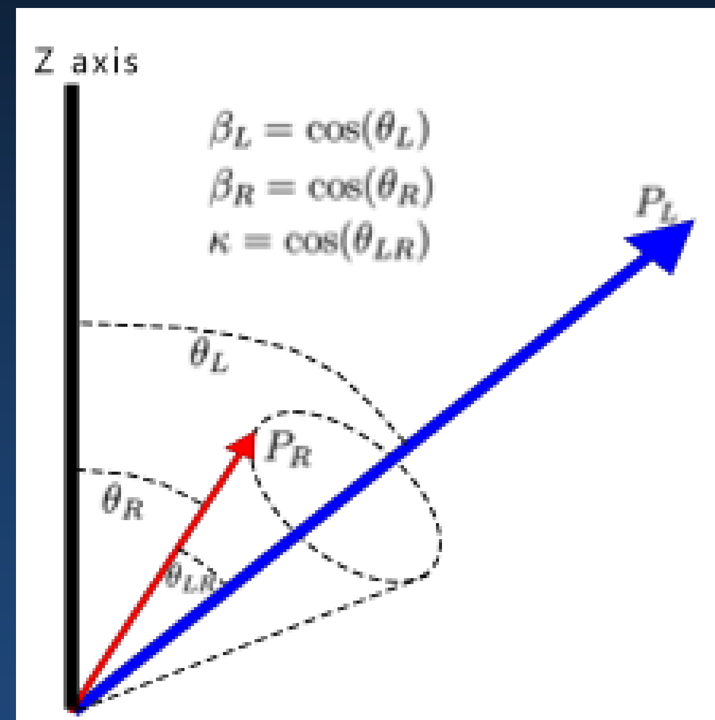
Quasi-steady evolution of FFC

Liu and Nagakura et al. 2025





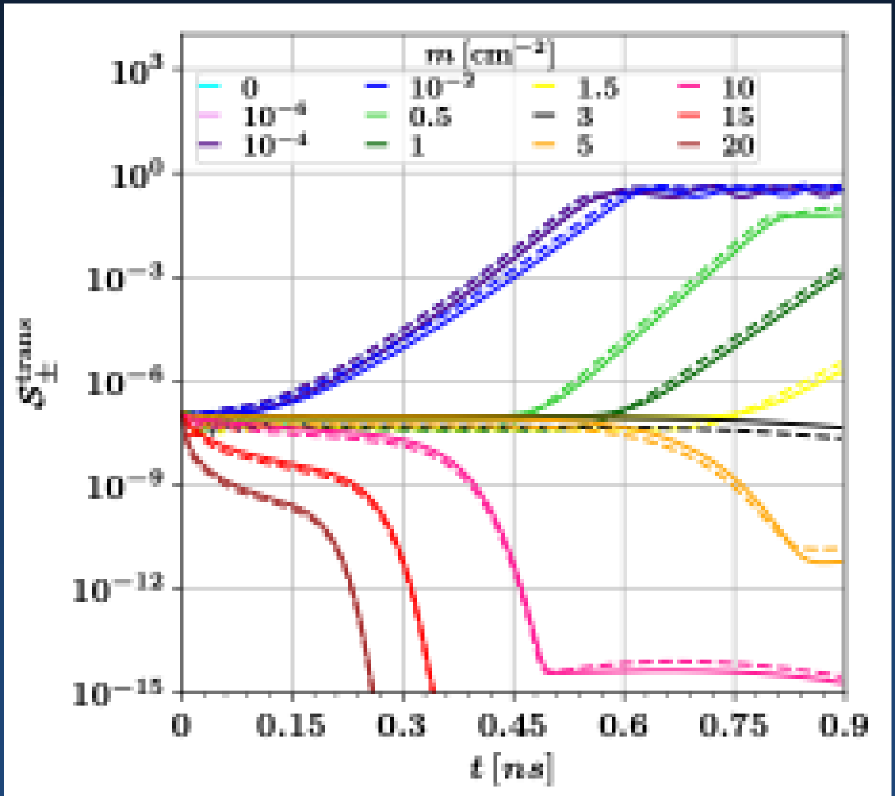
- The quasi-steady evolution of FFC can be modeled analytically.



- See also Liu et al. (arXiv:2509.26418) for a synchronization property of FFC

Effects of matter inhomogeneity in FFC

Bhattacharyya et al. 2025



Due to the phase shift, the dispersion relation of the unstable mode varies spatially, causing the mode to become stable before it has time to grow.

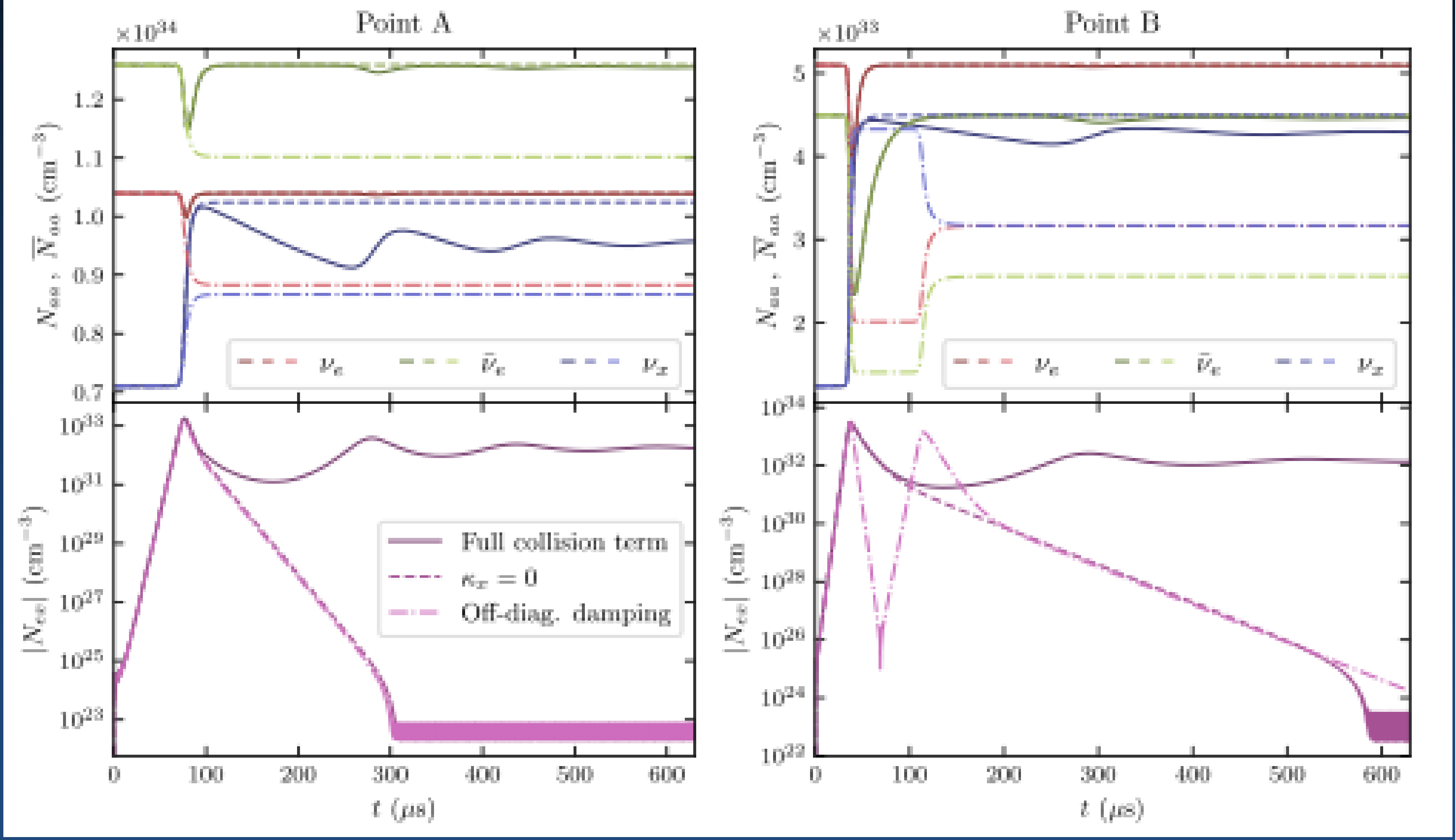
$$Q_{\pm}[k,t] \sim Q_{\pm}[k,0]e^{-i\mu(1-\alpha)t - i\int_0^t \kappa[t]dt}, \tag{6}$$

where

$$\kappa[t] = \sqrt{(k-mt)^2 - 2(1+\alpha)\,\mu\,(k-mt) + \mu^2(1-\alpha)^2}. \tag{7}$$

Asymptotic states of CFI: homogeneous simulations

Froustey 2025

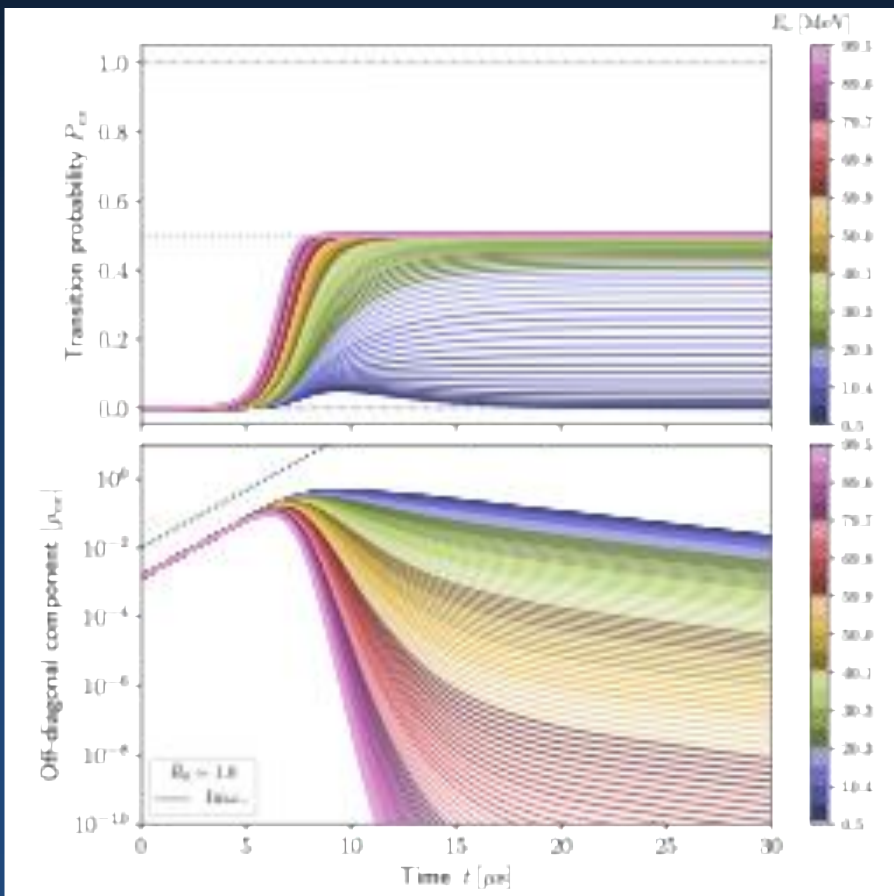


Asymptotic states of CFI: multi-energy dependence

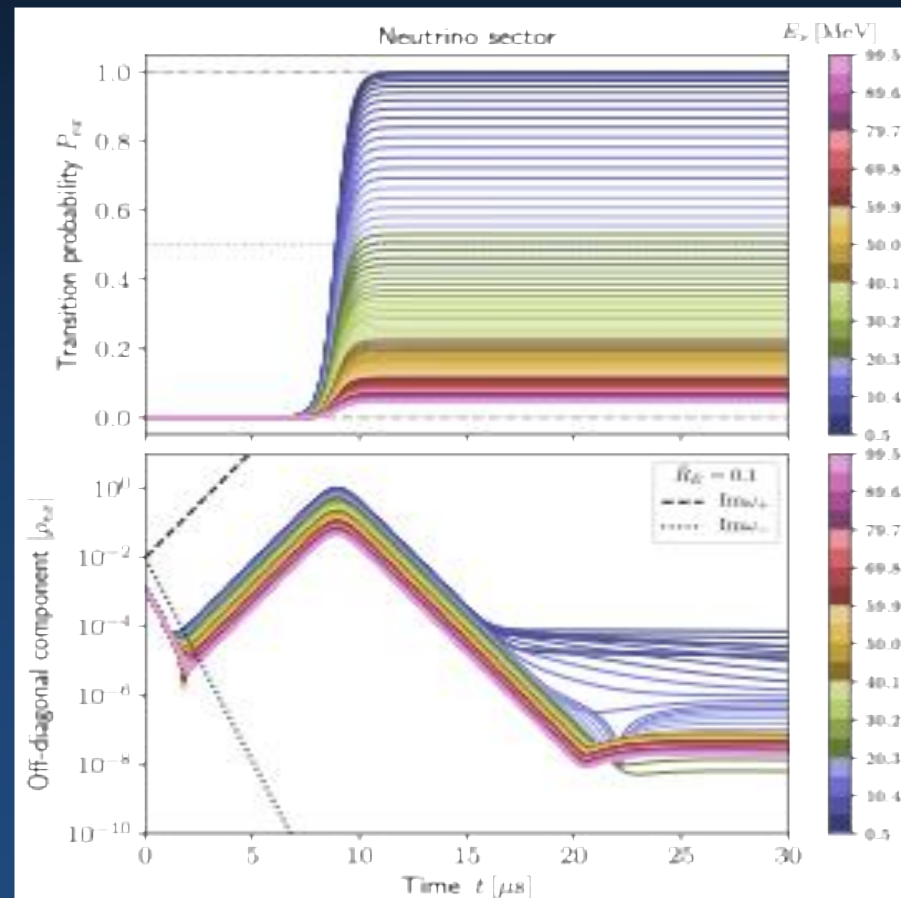
Zaizen 2025

The asymptotic states qualitatively differ between the plus and minus modes.

Driven by minus-mode

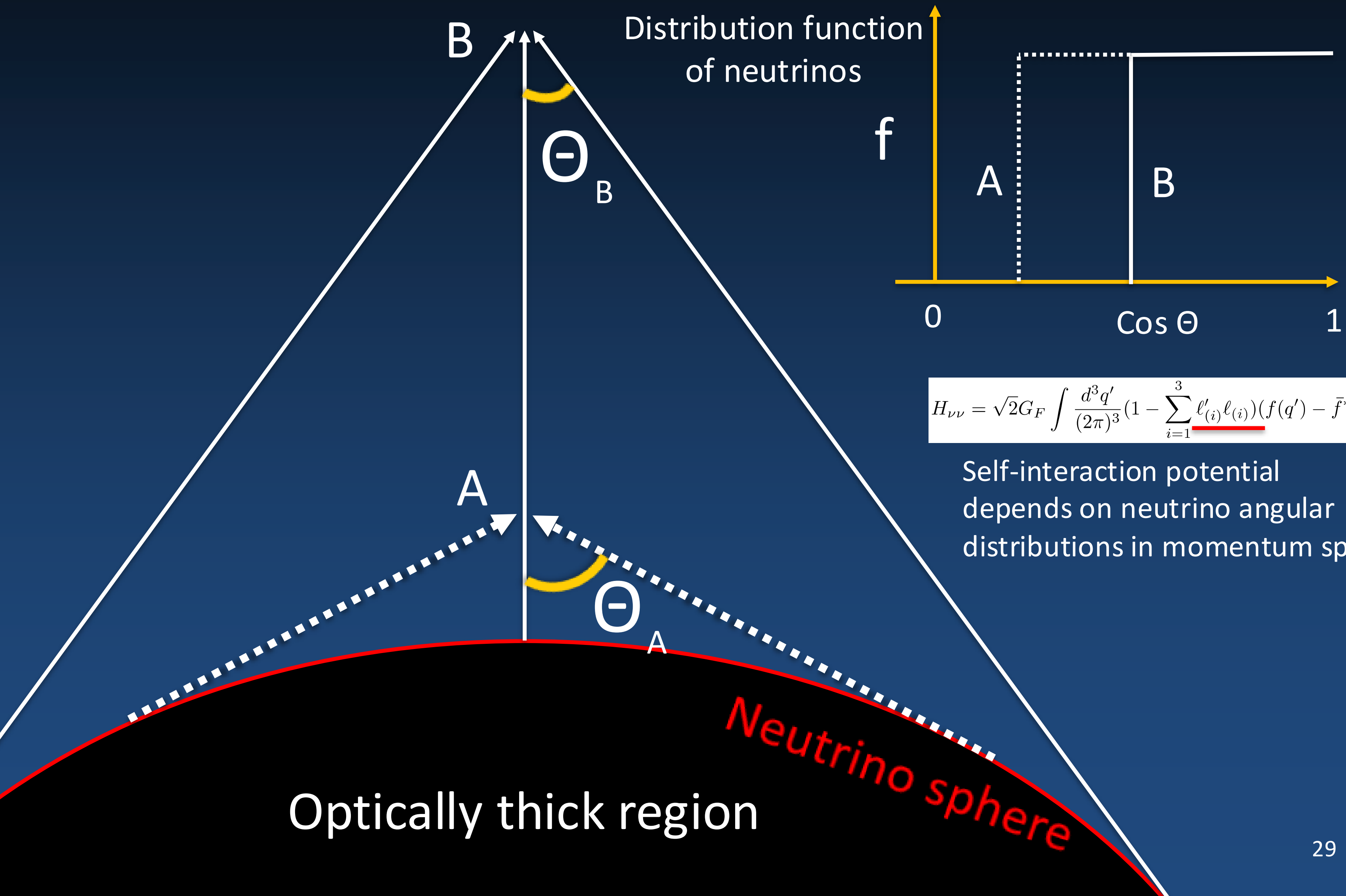


Driven by plus-mode



Global quantum kinetic neutrino transport

- Need of global simulations in the study of flavor conversions in CCSN/BNSM



$$H_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3q'}{(2\pi)^3} \left(1 - \sum_{i=1}^{3} \ell'_{(i)} \ell_{(i)}\right)\left(f(q') - \bar{f}^*(q')\right),$$

Self-interaction potential depends on neutrino angular distributions in momentum space

Time-dependent global simulations of FFC

Nagakura and Zaizen PRL 2022, PRD 2023

- Issue:

$$\ell_{n_\nu} \equiv c\,T_{n_\nu} = 0.235\ \mathrm{cm} \left(\frac{L_\nu}{4\times 10^{52}\mathrm{erg/s}} \right)^{-1} \left(\frac{E_{\rm ave}}{12\mathrm{MeV}} \right) \left(\frac{R}{50\mathrm{km}} \right)^{2} \left(\frac{\kappa}{1/3} \right)$$

Oscillation wavelength is an order of sub-centimeter.

Too short !!!!

How can we make FFC simulations tractable???

- Strategy:

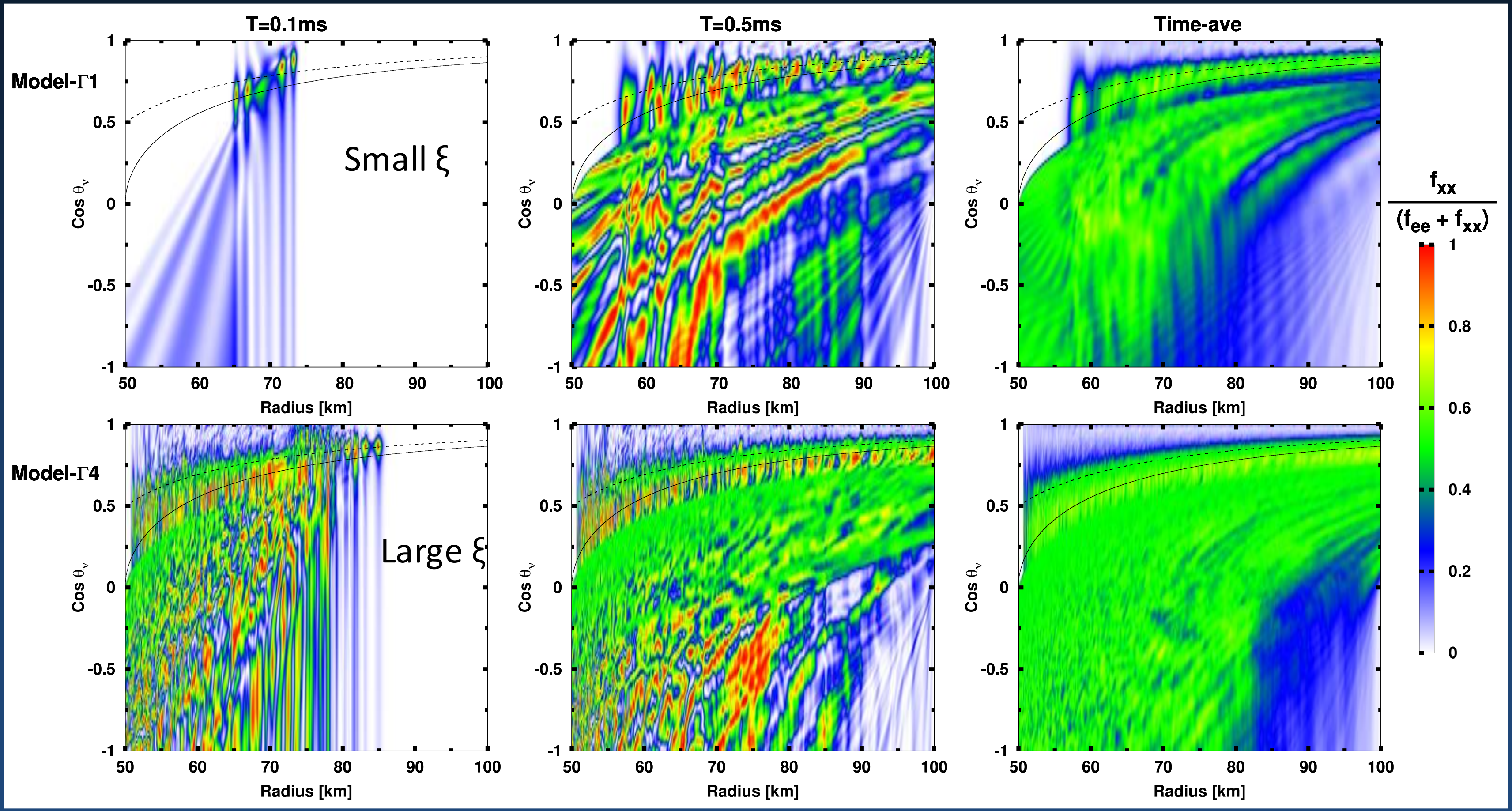
$$\frac{\partial \overset{(-)}{f}}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cos\theta_\nu \overset{(-)}{f}) - \frac{1}{r\sin\theta_\nu}\frac{\partial}{\partial \theta_\nu}(\sin^2\theta_\nu \overset{(-)}{f}) = -i\,\xi\,[\overset{(-)}{H}, \overset{(-)}{f}],$$

Attenuation parameter (0 ≦ ξ ≦ 1)

- Attenuating Hamiltonian makes global QKE simulations tractable.
- Realistic features can be extracted by a convergence study of ξ (→ 1).

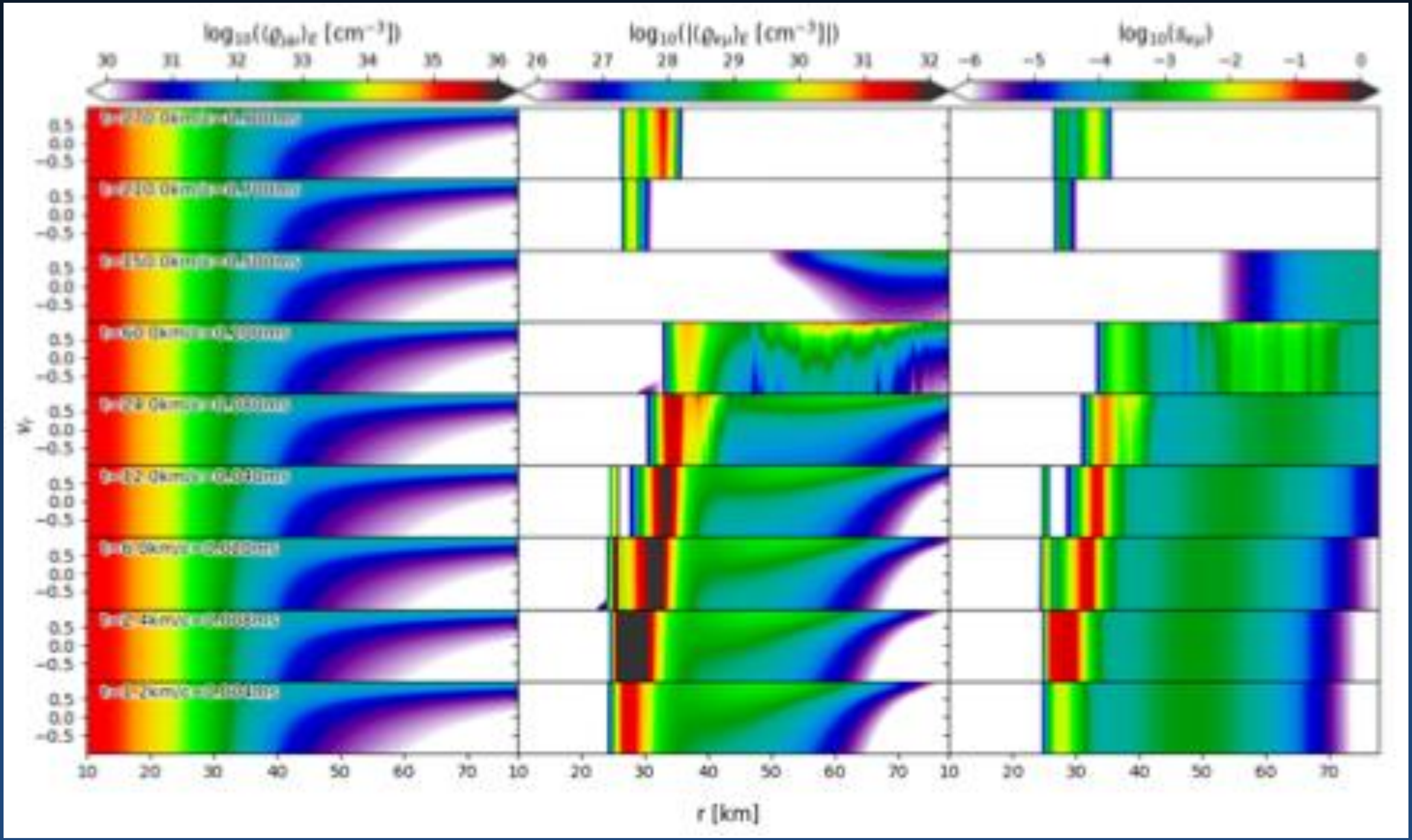
Temporal and quasi-steady features of FFC in global scale (1D in space + 1D angle in momentum space)

Nagakura and Zaizen PRL 2022



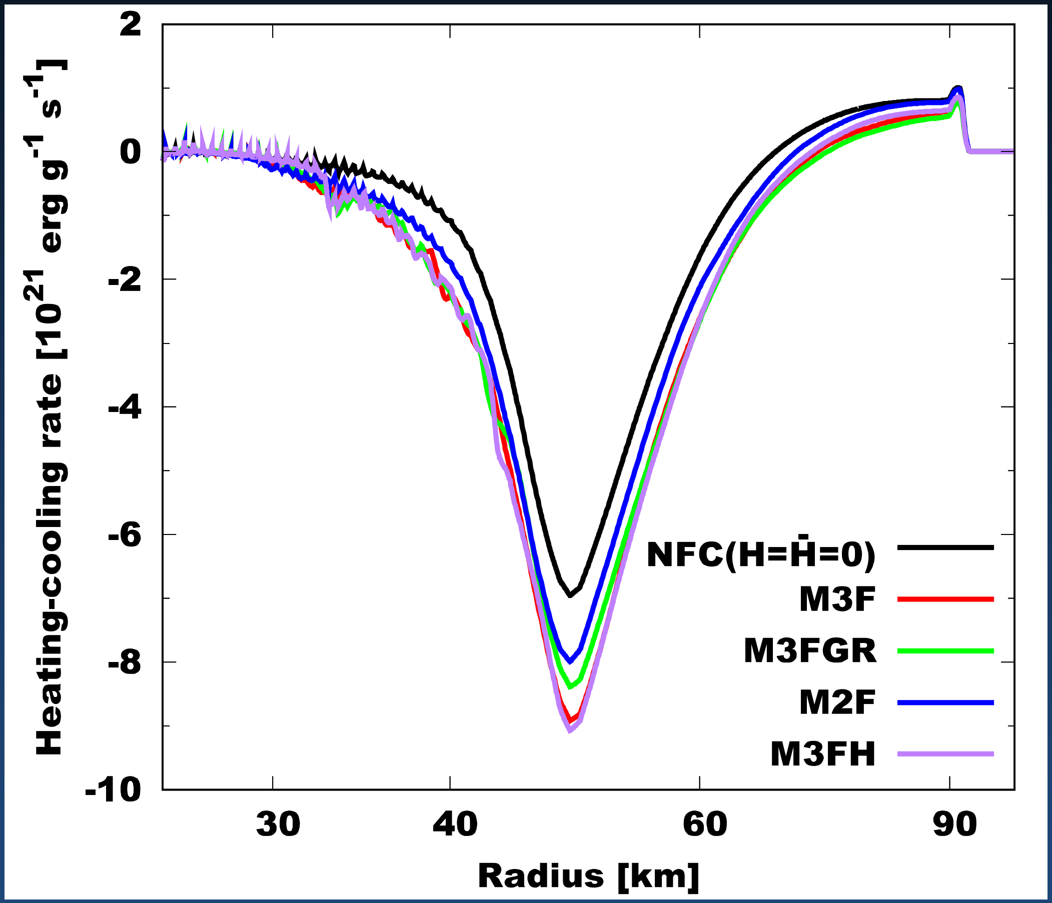
Global simulation for CFI

Xiong et al. 2023



FFC in CCSN

Nagakura 2023, Nagakura and Zaizen 2023



Numerical setup:

Collision terms are switched on.

Fluid-profiles are taken from a CCSN simulation.

General relativistic effects are taken into account.

A wide spatial region is covered.

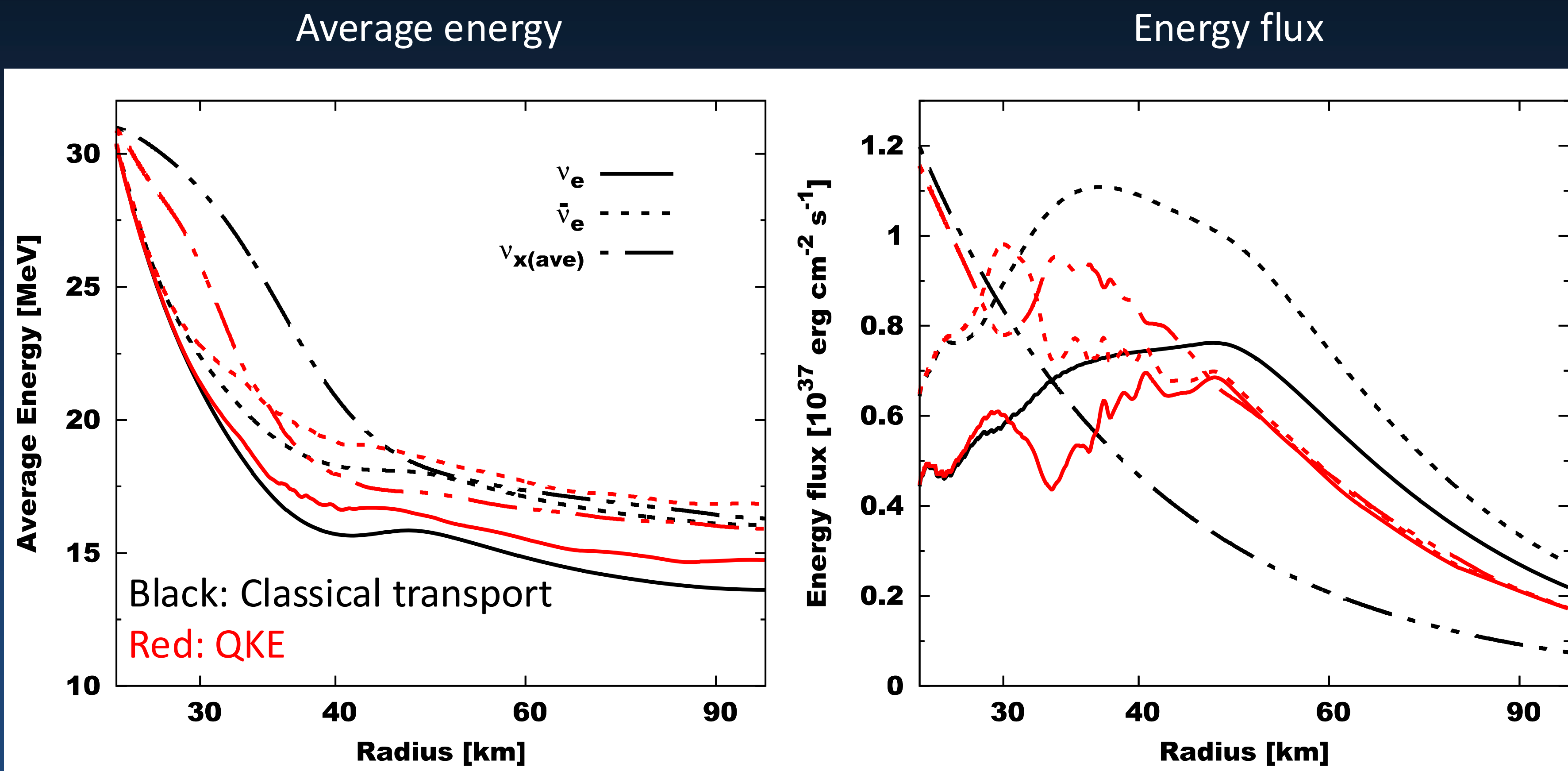
Three-flavor framework

Neutrino-cooling is enhanced by FFCs
Neutrino-heating is suppressed by FFCs

→ Impacts on the explodability of CCSN

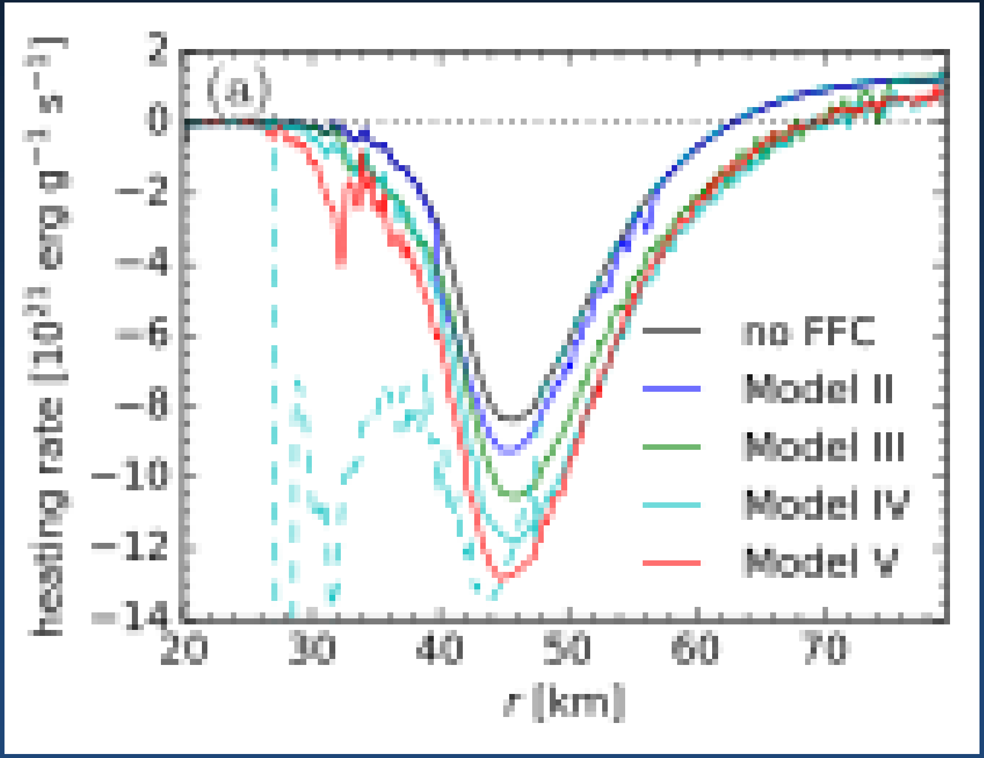
FFC in CCSN

Nagakura 2023, Nagakura and Zaizen 2023

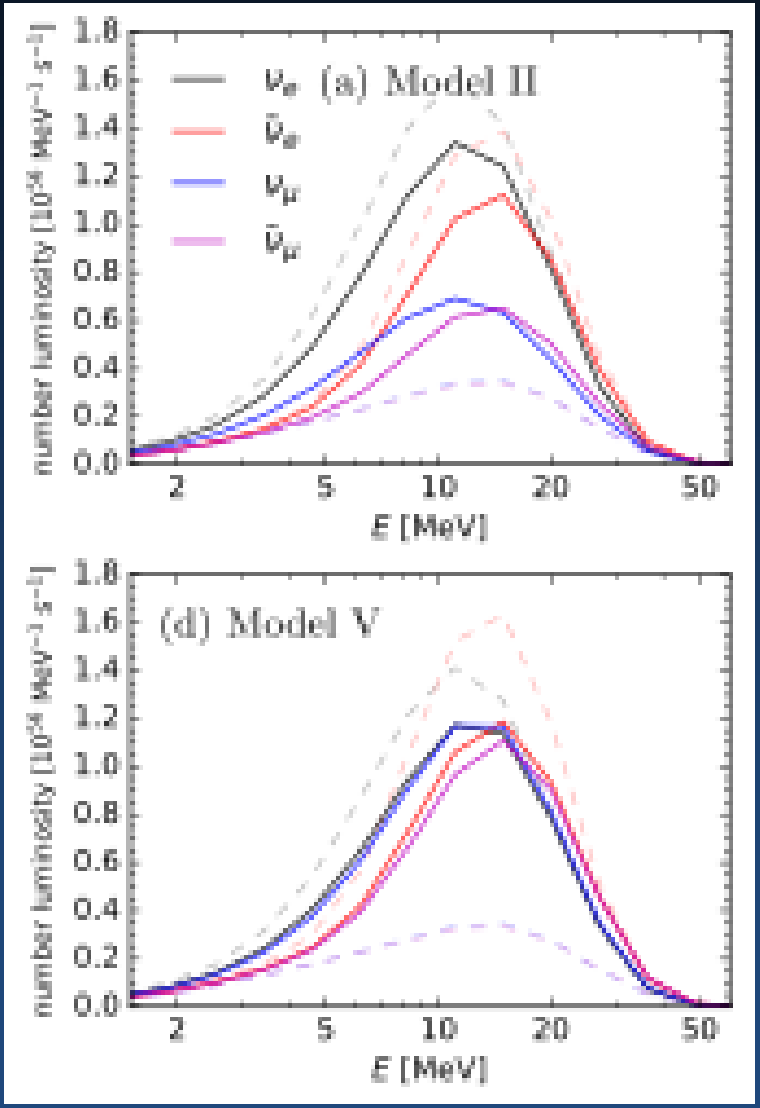


FFC in CCSN

Xiong et al. 2023



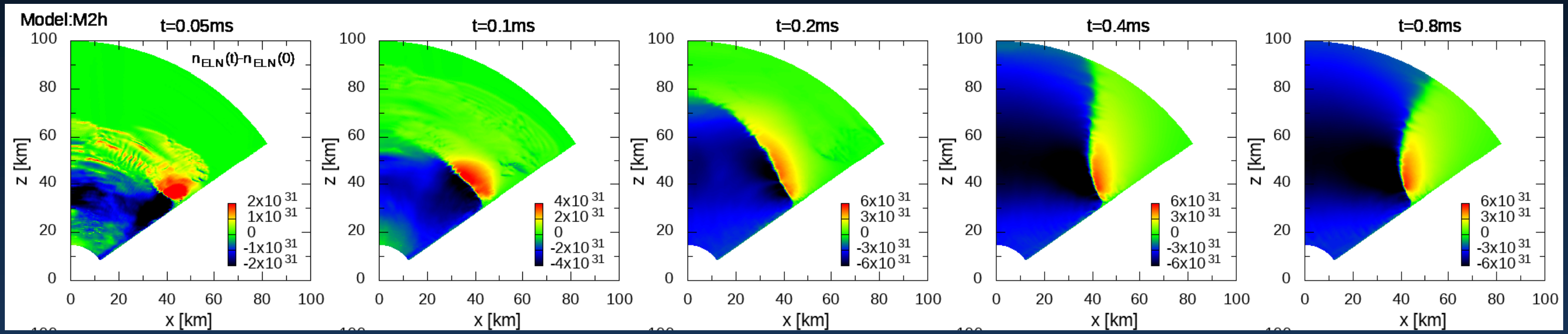
Energy-spectra of neutrinos



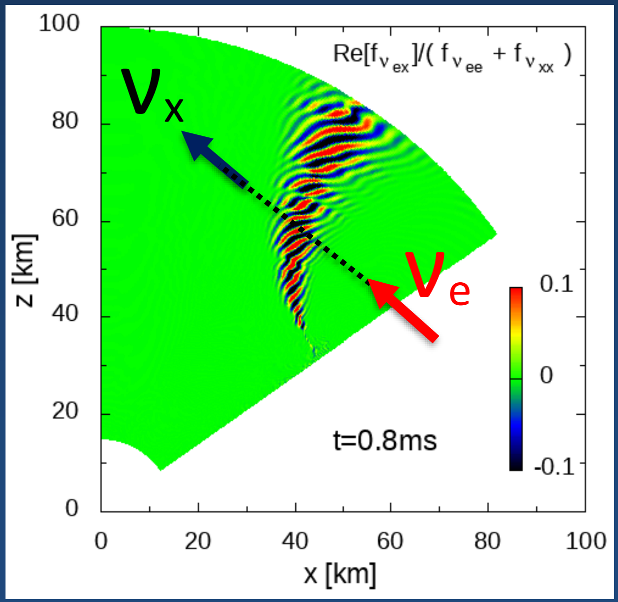
Global Simulations of FFC in a BNSM environment

Nagakura 2023

- Temporal evolution of FFCs in global scale:



- FFC leads to flavor swap rather than flavor equipartition:



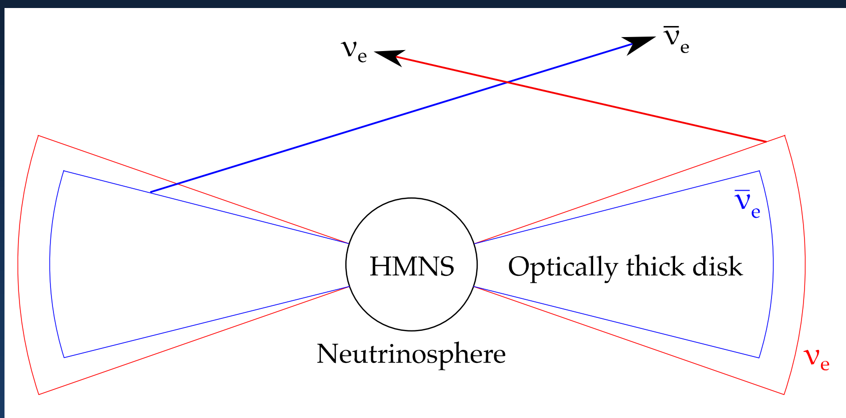
Take-home messages:

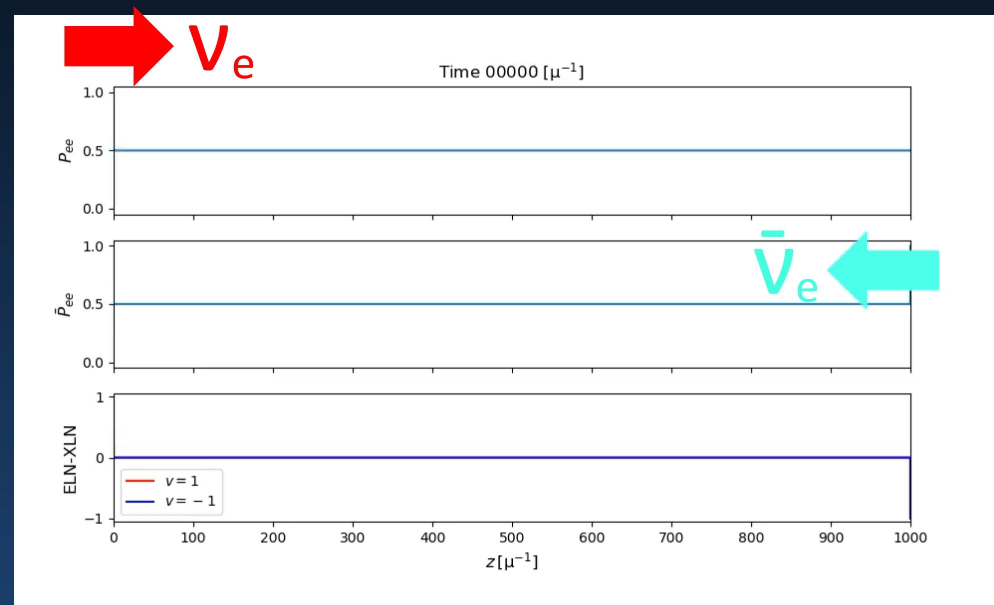
- FFC occurs vividly in a narrow region.
- The converted neutrinos spread in space by advections.
- Neutrinos undergo flavor swap, which is different feature from CCSNe.

Fast flavor swap may be a natural outcome in BNSM

Zaizen and Nagakura 2024

- Colliding-beam model





$$\partial_{t'}^2 P_3 \sim -4\mu^2 \left(1 - (P_3)^2\right)$$

$P_3 = 1$: electron-type → Unstable

$P_3 = 0$: equipartition → Non-steady

$P_3 = -1$: heavy-lepton type → Stable

Head-on colliding neutrinos end up with flavor swaps in asymptotic states.

Summary

- Remarkable progress has been made in numerical modeling of CCSNe and BNSMs that incorporates the effects of neutrino flavor conversions.
- The fidelity of such simulations depends sensitively on the adopted neutrino mixing scheme (or subgrid model).
- The BGK subgrid model for flavor conversions provides a simple, efficient, and physically reasonable approach to incorporating neutrino flavor conversions into conventional CCSN and BNSM simulation codes.
- Local QKE simulations have been widely used to investigate the detailed dynamical behavior and asymptotic states of flavor conversions.
- Global QKE simulations with a fixed matter background have also been developed using attenuation prescriptions.
- Global neutrino transport may lead to outcomes that differ qualitatively from those inferred from local simulations.